Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. Let A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to **[1]**

a) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

b) $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$

c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

d) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$

2. If the matrix $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular then x = ?. **[1]**

a) 1

b) 0

c) -1

d) -2

3. If A and B are invertible matrices, then which of the following is not correct? **[1]**

a) $(AB)^{-1} = B^{-1} A^{-1}$

b) $(A + B)^{-1} = B^{-1} + A^{-1}$

c) $\det (A)^{-1} = [\det (A)]^{-1}$

d) adj A = $|A| \cdot A^{-1}$

4. Let f(x) = $[x]^2 + \sqrt{x}$, where [•] and [•] respectively denotes the greatest integer and fractional part functions, then **[1]**

a) f(x) is continuous and differentiable at x = 0

b) f(x) is non differentiable $\forall$ x $\in$ Z

c) f(x) is discontinuous $\forall$ x $\in$ Z - {1}

d) f(x) is continuous at all integral points

5. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$. **[1]**

a) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$, $\lambda \in R$

b) $\vec{r} = \widehat{2i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$

c) $\vec{r} = 4\hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$

d) $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$

6. The order of the differential equation of all circles of given radius a is: **[1]**

a) 4

b) 1

c) 2

d) 3

7. By graphical method solution of LLP maximize Z = x + y subject to x + y $\leq$ 2x; y $\geq$ 0 obtained at **[1]**

a) at infinite number of points

b) only two points

c) only one point

d) at definite number of points

8. The domain of the function $\cos^{-1}(2x - 1)$ is **[1]**

a) $[0, \pi]$

b) $[-1, 1]$

c) $[0, 1]$

d) $(-1, 0)$

9. $\int_0^{\pi/2} \frac{\cos x}{(2+\sin x)(1+\sin x)} dx$ equals **[1]**

a) $\log\left(\frac{3}{4}\right)$

b) $\log\left(\frac{3}{2}\right)$

c) $\log\left(\frac{4}{3}\right)$

d) $\log\left(\frac{2}{3}\right)$

10. If $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}$ = 0, then the value of x is **[1]**

a) $\pm 6\sqrt{5}$

b) $5\sqrt{5}$

c) $\pm 4\sqrt{3}$

d) $\pm 3\sqrt{5}$

11. Objective function of an LPP is **[1]**

a) a function to be optimized

b) a function between the variables

c) a constraint

d) a relation between the variables

12. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is **[1]**

a) $\hat{i} - 2\hat{j} + 2\hat{k}$

b) $3(\hat{i} - 2\hat{j} + 2\hat{k})$

c) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

d) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

13. If A = $\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 3 & -1 & 9 \end{vmatrix}$, then the value of det (Adj (Adj A)) equals **[1]**

a) 14641

b) 121

c) 11

d) 1331

14. If A and B are independent events such that P(A) = $\frac{1}{5}$, P(A $\cup$ B) = $\frac{7}{10}$, then what is P$(\bar{B})$ equal to? **[1]**

a) $\frac{3}{8}$

b) $\frac{7}{9}$

c) $\frac{3}{7}$

d) $\frac{2}{7}$

15. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is **[1]**

a) 2
b) not defined
c) 0
d) 1

16. If $|\vec{a} \times \vec{b}| = 4, |\vec{a} \cdot \vec{b}| = 2$, then $|\vec{a}|^2|\vec{b}|^2 =$ **[1]**

a) 2
b) 20
c) 8
d) 6

17. If $y = \tan^{-1} \frac{\cos x}{1+\sin x}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{1}{2}$
b) 1
c) 0
d) $\frac{-1}{2}$

18. The cartesian equation of a line is given by $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$ **[1]**

The direction cosines of the line is

a) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{-4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
b) $\frac{3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
c) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
d) $\frac{-3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

19. **Assertion (A):** If manufacturer can sell x items at a price of ₹$\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is ₹ $\left(\frac{x}{5} + 500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items. **[1]**

Reason (R): The profit for selling x items is given by $\frac{24}{5}x - \frac{x^2}{100}$ - 300.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** Let A = {1, 5, 8, 9}, B = {4, 6} and f = {(1, 4), (5, 6), (8, 4), (9, 6)}, then f is a bijective function. **[1]**

Reason (R): Let A = {1, 5, 8, 9}, B = {4, 6} and f = {(1, 4), (5, 6), (8, 4), (9, 6)}, then f is a surjective function.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. For the principal value, evaluate $\cot\left[\sin^{-1}\left\{\cos\left(\tan^{-1} 1\right)\right\}\right]$ **[2]**

OR

Which is greater, tan 1 or $\tan^{-1}$ 1?

22. Show that f(x) = sin x - cos x is an increasing function on $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$. **[2]**

23. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/s. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing? **[2]**

OR

Show that the function $f(x) = x^{100} + \sin x - 1$ is increasing on the interval $\left(\frac{\pi}{2}, \pi\right)$

24. Evaluate: $\int \tan^3 x \sec^3 x \, dx$ **[2]**

25. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ. **[2]**

Section C

26. Evaluate the integral: $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$ **[3]**

27. In answering a question on a multiple choice questions test with four choices in each question, out of which only one is correct, a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability the he copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question, given that he correctly answered it. **[3]**

28. Evaluate the definite integral $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$ **[3]**

OR

Evaluate the definite integral: $\int_1^2 e^{2x} \left(\frac{1}{x} - \frac{1}{2x^2}\right) dx$

29. Solve the following differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that y = 1, when x = 0. **[3]**

OR

Find the particular solution of the differential equation $(xe^{x/y} + y)dx = x\,dy$, given that y(1) = 0

30. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being $\perp$ to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$ **[3]**

OR

If $\vec{a} = (\hat{i} - \hat{j}), \vec{b} = (3\hat{j} - \hat{k})$ and $\vec{c} = (7\hat{i} - \hat{k})$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and for which $\vec{c} \cdot \vec{d} = 1$.

31. Find $\frac{dy}{dx}$ of the function $(\cos x)^y = (\cos y)^x$. **[3]**

Section D

32. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$. **[5]**

33. Let A = {1, 2, 3, ...9} and R be the relation in A×A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A×A. Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)]. **[5]**

OR

Let A = R – {3} and B = R – {1}. Consider the function f: A ⇒B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

34. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 5A + 7I = 0$ and hence find A^4. **[5]**

35. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. **[5]**

OR

A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

i. Calculate the probability that a randomly chosen seed will germinate. (1)

ii. Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates. (1)

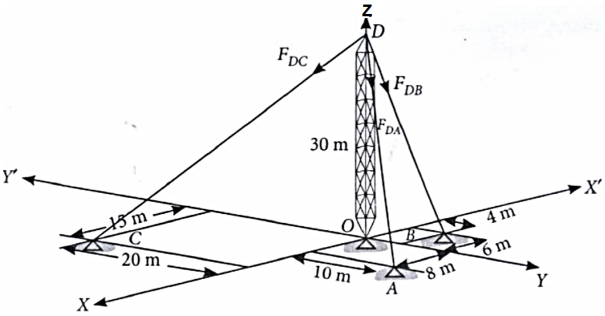
iii. A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card. (2)

OR

If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then find P(A|B). (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Consider the following diagram, where the forces in the cable are given.



i. What is the equation of the line along cable AD? (1)

ii. What is length of cable DC? (1)

iii. Find vector DB (2)

OR

What is sum of vectors along the cable? (2)

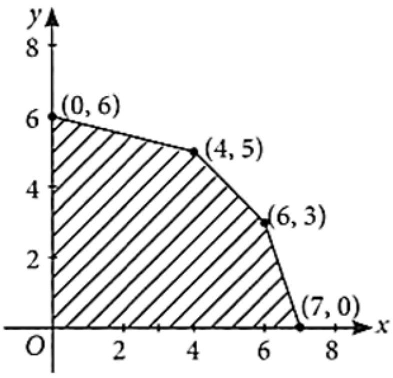
38. **Read the following text carefully and answer the questions that follow:** **[4]**

Linear programming is a method for finding the optimal values (maximum or minimum) of quantities subject to the constraints when a relationship is expressed as linear equations or inequations.

i. At which points is the optimal value of the objective function attained? (1)

ii. What does the graph of the inequality 3x + 4y < 12 look like? (1)

iii. Where does the maximum of the objective function Z = 2x + 5y occur in relation to the feasible region shown in the figure for the given LPP? (2)



OR

What are the conditions on the positive values of p and q that ensure the maximum of the objective function $Z = px + qy$ occurs at both the corner points (15, 15) and (0, 20) of the feasible region determined by the given system of linear constraints? (2)

Solution

Section A

1.

(c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

Explanation: A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

A^n = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ ....{n times, (where n $\in$ N)}

A^n = $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

2. **(a)** 1

Explanation: When a given matrix is singular then the given matrix determinant is 0.

|A| = 0

Given, $A = \begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$

|A| = 0

4(3 - 2x) - 2(x + 1) = 0

12 - 8x - 2x - 2 = 0

10 - 10x = 0

10(1 - x) = 0

x = 1

3.

(b) $(A + B)^{-1} = B^{-1} + A^{-1}$

Explanation: Since, A and B are invertible matrices.

So, we can say that

$(AB)^{-1} = B^{-1} A^{-1}$ ...(i)

We know that, $A^{-1} = \frac{1}{|A|}$ (adj A)

$\Rightarrow$ adj A = $|A| \cdot A^{-1}$ ...(ii)

Also, $\det (A)^{-1} = [\det (A)]^{-1}$

$\Rightarrow \det (A)^{-1} = \frac{1}{[\det(A)]}$

$\Rightarrow \det (A) \cdot \det (A)^{-1} = 1$ ...(iii)

Which is true,

So, only option d is incorrect.

4.

(c) f(x) is discontinuous $\forall$ x $\in$ Z - {1}

Explanation: f(x) is discontinuous $\forall$ x $\in$ Z - {1}

5. **(a)** $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$ $+\lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$, $\lambda \in R$

Explanation: The equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$, let vector $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$ and vector $\overrightarrow{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$, the equation of line is :

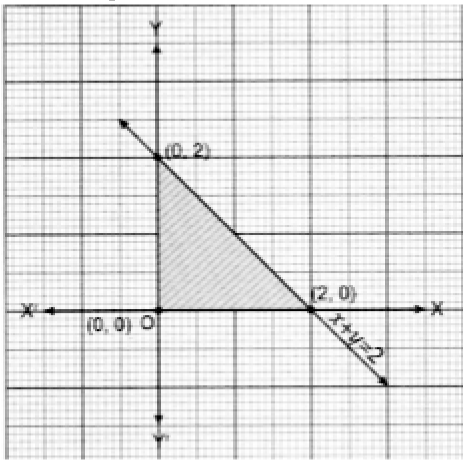
$\overrightarrow{a}+\lambda\overrightarrow{b}=(\hat{i}+\hat{j}+\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$

6.

(c) 2

Explanation: Let the equation of given family be $(x - h)^2 + (y - k)^2 = a^2$. It has two arbitrary constants h and k. Therefore, the order of the given differential equation will be 2.

7. **(a)** at infinite number of points

Explanation: 

Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2)

Z(0, 0) = 0

Z(2, 0) = 2⟵ maximise

Z(0, 2) = 2 ⟵ maximise

Z_{max} = 2 obtained at (2, 0) and (0, 2) so is obtained at any point on line segment joining (2, 0) and (0, 2).

8.

(c) [0, 1]

Explanation: We have $f(x) = \cos^{-1}(2x - 1)$

Since, $-1 \leq 2x - 1 \leq 1$

$\Rightarrow 0 \leq 2x \leq 2$

$\Rightarrow 0 \leq x \leq 1$

$\therefore x \in [0,1]$

9.

(c) $\log\left(\frac{4}{3}\right)$

Explanation: $\log\left(\frac{4}{3}\right)$

Let $I=\int_0^{\frac{\pi}{2}} \frac{\cos x}{(2+\sin x)(1+\sin x)}dx$

Let sin x = t, then cos x dx = dt

When x = 0, t = 0 x = $\frac{\pi}{2}$, t= 1

Therefore the integral becomes

$I=\int_0^1 \frac{dt}{(2+t)(1+t)}$

$=\int_0^1\left[\frac{-1}{2+t}+\frac{1}{1+t}\right]dt$

$=[-\log(2+t)+\log(1+t)]_0^1$

$=[\log(1+t)-\log(2+t)]_0^1$

$=$ log 2 - log 3 - log 1 + log 2

$=\log\frac{4}{3}$

10.

(c) $\pm4\sqrt{3}$

Explanation: Given, $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}=0$

$\Rightarrow$ x $\times$ 1 + (-5) $\times$ 0 + (-1) $\times$ 2 x $\times$ 0 + (-5) $\times$ 2 + (-1) $\times$ 0

$x \times 2 + (-5) \times 1 + (-1) \times 3] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow$ [(x - 2) $\times$ x + (-10) $\times$ 4 + (2x - 8) $\times$ 1] = 0

$\Rightarrow$ x^2 - 2x - 40 + 2x - 8 = 0

$\Rightarrow$ x^2 = 48

$\Rightarrow x = \pm\sqrt{48} = \pm 4\sqrt{3}$

11. **(a)** a function to be optimized

Explanation: a function to be optimized

The objective function of a linear programming problem is either to be maximized or minimized i.e. objective function is to be optimized.

12.

(b) $3(\hat{i} - 2\hat{j} + 2\hat{k})$

Explanation: Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Unit vector in the direction of a vector $\vec{a}$

$= \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1^2+2^2+2^2}} = \frac{i - 2\hat{j} + 2\hat{k}}{3}$

$\therefore$ Vector in the direction of $\vec{a}$ with magnitude 9

$= 9 \cdot \frac{i-2j+2k}{3} = 3(i - 2j + 2k)$.

13. **(a)** 14641

Explanation: We know that, for a square matrix of order n, if $|A| \neq 0$

Adj(Adj A) = $|A|^{n - 2}$ A ($\because$ n = 3)

$\therefore$ Adj(Adj A) = $|A|^{3 - 2}$ A ($\because$ n = 3)

= |A| A

$\therefore$ |Adj(Adj A) = ||A| A|| = $|A|^3$ det A $|A|^4$

= 11^4 = 14641

14. **(a)** $\frac{3}{8}$

Explanation: Given that,

P(A) = $\frac{1}{5}$, P(A $\cup$ B) = $\frac{7}{10}$

Also, A and B are independent events,

$\therefore$ P(A $\cap$ B) = P(A) $\cdot$ P(B)

$\Rightarrow$ P(A) + P(B) - P(A $\cup$ B) = P(A) $\cdot$ P(B)

$\Rightarrow$ $\frac{1}{5}$ + P(B) - $\frac{7}{10} = \frac{1}{5} \times$ P(B)

$\Rightarrow$ P(B) - $\frac{P(B)}{5} = \frac{7}{10} - \frac{1}{5} = \frac{5}{10}$

$\Rightarrow$ $\frac{4P(B)}{5} = \frac{1}{2} \Rightarrow$ P(B) = $\frac{5}{8}$

$\therefore$ P$(\bar{B})$ = 1 - P(B) = 1 - $\frac{5}{8} = \frac{3}{8}$

15.

(b) not defined

Explanation: not defined

16.

(b) 20

Explanation: We know that

$(\vec{a} \cdot \vec{b})^2 + (\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

$|\vec{a}|^2 \cdot |\vec{b}|^2$ = $2^2 + 4^4$

$|\vec{a}|^2 \cdot |\vec{b}|^2$ = 20

17.

(d) $\frac{-1}{2}$

Explanation: Given that $y=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

Using $\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}$, $\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta+\sin^2\theta=1$

Therefore,

$y=\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\right)=\tan^{-1}\left(\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$

$\Rightarrow \mathrm{y}=\tan^{-1}\frac{\cos\frac{\mathrm{x}}{2}-\sin\frac{\mathrm{x}}{2}}{\cos\frac{\mathrm{x}}{2}+\sin\frac{\mathrm{x}}{2}}$

Dividing by cos $\frac{x}{2}$ in numerator and denominator, we get

$y=\tan^{-1}\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

Using $\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}$, we obtain

$y=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$=\frac{\pi}{4}-\frac{x}{2}$

Differentiating with respect to x, we

$\frac{dy}{dx}=-\frac{1}{2}$

18.

(c) $\frac{\sqrt{3}}{\sqrt{55}},\frac{4}{\sqrt{55}},\frac{6}{\sqrt{55}}$

Explanation: Rewrite the given line as

$r\frac{2\left(x-\frac{1}{2}\right)}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$

or $\frac{x-\frac{1}{2}}{\sqrt{3}}=\frac{y+2}{4}=\frac{z-3}{6}$

$\therefore$ DR's of line are $\sqrt{3}$, 4 and 6

Therefore, direction cosines are:

$\frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2+4^2+6^2}},\frac{4}{\sqrt{(\sqrt{3})^2+4^2+6^2}},\frac{6}{\sqrt{(\sqrt{3})^2+4^2+6^2}}$ or $\frac{\sqrt{3}}{\sqrt{55}},\frac{4}{\sqrt{55}},\frac{6}{\sqrt{55}}$

19.

(c) A is true but R is false.

Explanation: Let S(x) be the selling price of x items and let C(x) be the cost price of x items.

Then, we have

S(x) = (5 - $\frac{x}{100}$)x = 5x - $\frac{x^2}{100}$

and C(x) = $\frac{x}{5}$ + 500

Thus, the profit function P(x) is given by

P(x) = S(x) - C(x) = 5x - $\frac{x^2}{100}-\frac{x}{5}$ - 500

i.e. P(x) = $\frac{24}{5}x-\frac{x^2}{100}$ - 500

On differentiating both sides w.r.t. x, we get

P'(x) = $\frac{24}{5}-\frac{x}{50}$

Now, P'(x) = 0 gives x = 240.

Also, P'(x) = $\frac{-1}{50}$.

So, P'(240) = $\frac{-1}{50}$ < 0

Thus, x = 240 is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

20.

(d) A is false but R is true.

Explanation: We have, A = {1, 5, 8, 9}, B = {4, 6} and f = {(1, 4), (5, 6), (8, 4), (9, 6)}

So, all elements of B has a domain element on A or we can say elements 1 and 8 & 5 and 9 have some range 4 & 6, respectively.

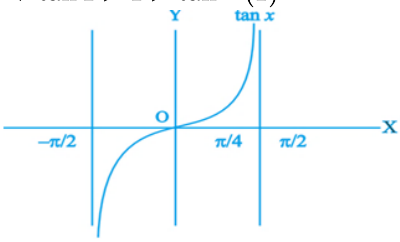
Therefore, f : A → B is a surjective function not one to one function.
Also, for a bijective function, f must be both one to one onto.

Section B

21. We know that $\tan^{-1} 1 = \frac{\pi}{4}$.
$\therefore \quad \cot\left[\sin^{-1}\left\{\cos\left(\tan^{-1} 1\right)\right\}\right]$
$= \cot\left\{\sin^{-1}\left(\cos\frac{\pi}{4}\right)\right\} = \cot\left(\sin^{-1}\frac{1}{\sqrt{2}}\right) = \cot\frac{\pi}{4} = 1$

OR

From Fig. we note that tan x is an increasing function in the interval $\left(\frac{-\pi}{2},\ \frac{\pi}{2}\right)$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan\frac{\pi}{4}$. This gives tan 1 > 1
$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$
$\Rightarrow \tan 1 > 1 > \tan^{-1}(1)$



22. Given: f(x) = sin x – cos x
f'(x) = cos x + sin x
$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$
$= \sqrt{2}\left(\frac{\sin\pi}{4}\cos x + \frac{\cos\pi}{4}\sin x\right)$
$= \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$
Now,
$x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$
$\Rightarrow -\frac{\pi}{4} < x < \frac{\pi}{4}$
$\Rightarrow 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$
$\Rightarrow \sin 0^\circ < \sin\left(\frac{\pi}{4} + x\right) < \sin\frac{\pi}{2}$
$\Rightarrow 0 < \sin\left(\frac{\pi}{4} + x\right) < 1$
$\Rightarrow \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) > 0$
= f'(x)>0
Hence, f(x) is an increasing function on $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$

23. Let A be the area of the circle of radius r.
Then, A = πr^2
Therefore, the rate of change of area A with respect to time 't' is
$\frac{d\mathbf{A}}{dt} = \frac{d}{dt}\left(\pi r^2\right) = \frac{d}{dr}\left(\pi r^2\right)\cdot\frac{dr}{dt} = 2\pi r\frac{dr}{dt}$ ...(By Chain Rule)
Given that $\frac{dr}{dt}$ = 4cm/s
Therefore, when r = 10, $\frac{dA}{dt}$ = $2\pi \times 10 \times 4$ = 80π
Thus, the enclosed area is increasing at a rate of $80\pi\ cm^2/s$, when r = 10 cm.

OR

Given interval: $x \in (\pi/2, \pi)$
$\Rightarrow \quad \pi/2 < x < \pi$
$x^{99} > 1$
$100x^{99} > 100$
Again, $x \in (\pi/2, \pi) \Rightarrow -1 < \cos x < 0 \Rightarrow 0 > \cos x > -1$
$100x^{99} > 100 \text{ and } \cos x > -1$
$100x^{99} + \cos x > 100 - 1 = 99$
$100x^{99} + \cos x > 0$
$f'(x) > 0$
Thus f(x) is increasing on $(\pi/2, \pi)$

24. Let I = $\int \tan^3 x \sec^3 x\, dx$, then we have

I = $\int \tan^2 x \sec^2 x (\sec x \tan x)\, dx = \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x)\, dx$

Substituting sec x = t and sec x tan x dx = dt, we obtain

I = $\int (t^2 - 1) t^2\, dt = \int (t^4 - t^2)\, dt = \frac{t^5}{5} - \frac{t^3}{3} + C = \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

25. Let $\Delta = \begin{bmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{bmatrix}$

Expanding along first row,

$\Delta = x\begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\theta\begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$

$\Rightarrow \Delta = x\left(-x^2 - 1\right) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$\Rightarrow \Delta = -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$\Rightarrow \Delta = -x^3 - x + x\left(\sin^2\theta + \cos^2\theta\right)$ = $-x^3 - x + x = -x^3$ which is independent of θ

Section C

26. We have,

$I = \int_0^\pi \frac{x}{a^2\cos^2 x + b^2\sin^2 x}dx$ ...(i)

Using property of definite integral we have,

$= \int_0^\pi \frac{(\pi - x)}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}dx$

$= \int_0^\pi \frac{\pi - x}{a^2\cos^2 x + b^2\sin^2 x}dx$ ...(ii)

Adding (i) and (ii)

$2I = \int_0^\pi \frac{x + \pi - x}{a^2\cos^2 x + b^2\sin^2 x}dx$

$= \pi\int_0^\pi \frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx$

$= \pi\int_0^\pi \frac{\sec^2 x}{a^2 + b^2\tan^2 x}dx$ ... (Dividing numerator and denominator by $\cos^2 x$)

$= 2\pi\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2\tan^2 x}dx$ ... $\left[\text{Using } \int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a - x)dx\right]$

Put tan x = t

$\Rightarrow \sec^2 x dx = dt$

When $x \to 0; t \to 0$

and $x \to \frac{\pi}{2}; t \to \infty$

$\therefore 2I = 2\pi\int_0^{\frac{x}{2}} \frac{dt}{a^2 + b^2t^2}$

$\Rightarrow I = \frac{\pi}{b^2}\int_0^{\frac{\pi}{2}} \frac{dt}{\frac{a^2}{b^2} + t^2}$

$= \frac{\pi}{b^2} \times \frac{b}{a}\left[\tan^{-1}\left(\frac{bt}{a}\right)\right]_0^\infty$

$= \frac{\pi}{ab}\left[\frac{\pi}{2} - 0\right]$

$= \frac{\pi}{ab} \times \frac{\pi}{2}$

$= \frac{\pi^2}{2ab}$

Hence, $I = \frac{\pi^2}{2ab}$

27. Let E_1 = Student guesses the answer

E_2 = Student copies the answer

E_3 = Student knows the answer

A = Student answers the question correctly.

$\mathrm{P}(\mathrm{E}_1) = \frac{1}{4}, \mathrm{P}(\mathrm{E}_2) = \frac{1}{4}, \mathrm{P}(\mathrm{E}_3) = 1 - \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{1}{2}$

$\mathrm{P}(\mathrm{A} \mid \mathrm{E}_1) = \frac{1}{4}, \mathrm{P}(\mathrm{A} \mid \mathrm{E}_2) = \frac{3}{4}, \mathrm{P}(\mathrm{A} \mid \mathrm{E}_3) = 1$

The required probability

$= P(E_3 \mid A) = \frac{P(E_3) \times P(A|E_3)}{\sum_{i=1}^{3} P(E_i) \times P(A|E_i)}$

$= \frac{\frac{1}{2}\times 1}{\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times 1}$
$= \frac{1}{\frac{1}{8}+\frac{3}{8}+1} = \frac{8}{12} = \frac{2}{3}$

28. $I = \int_0^{\pi/4} \frac{\sin x.\cos x}{\cos^4 x+\sin^4 x} dx$

Dividing Nr. and Dr. by $\cos^4 x$

$= \int_0^{\pi/4} \frac{\frac{\sin x.\cos x}{cos^4 x}}{\frac{\cos^4 x}{\cos^4 x}+\frac{\sin^4 x}{\cos^4 x}} dx$
$= \int_0^{\pi/4} \frac{\tan x.\sec^2 x}{1+\tan^4 x} dx$
$= \int_0^{\pi/4} \frac{\tan x.\sec^2 x}{1+(\tan^2 x)^2} dx$

Put $\tan^2 x = t$

$2\tan x.\sec^2 x dx = dt$

When x = 0, t= 0 and when $x = \frac{\pi}{4}, t = 1$

$\therefore I = \frac{1}{2}\int_0^1 \frac{dt}{1+t^2}$
$= \frac{1}{2}\left[\tan^{-1} t\right]_0^1$
$= \frac{1}{2}.\frac{\pi}{4} = \frac{\pi}{8}$

OR

We have,

I = $\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right) dx$

I = $\int_1^2 \frac{1}{x}\cdot e^{2x}$ - $\int_1^2 \frac{1}{2x^2}\cdot e^{2x} dx$

⇒ I = I_1 - I_2

Now, I_1 = $\int_1^2 \frac{1}{x} e^{2x}$ (By parts we have)

⇒ I_1 = $[\frac{1}{x}]_1^2 \cdot \int_1^2 e^{2x} dx$ - $\int_1^2 -\frac{1}{x^2}\frac{e^{2x}}{2} dx$

⇒ I_1 = $\left[\frac{1}{x}\cdot\frac{e^{2x}}{2}\right]_1^2$ + $\int_1^2 \frac{1}{2x^2} e^{2x} dx$

⇒ I_1 = $\left[\frac{1}{2x} e^{2x}\right]_1^2$ + I_2

As, I = I_1 - I_2

⇒ I = $\left[\frac{1}{2x} e^{2x}\right]_1^2$ - I_2 + I_2

⇒ I = $\left[\frac{1}{2x} e^{2x}\right]_1^2$ = $\frac{1}{2}\left[\frac{1}{2}e^4 - e^2\right]$

⇒ I = $\frac{1}{4} e^2 (e^2 - 1)$

29. According to the question ,

Given differential equation is ,

$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2 y^2$

$\Rightarrow \quad \frac{dy}{dx} = 1\left(1+x^2\right) + y^2\left(1+x^2\right)$

$\Rightarrow \quad \frac{dy}{dx} = \left(1+x^2\right)\left(1+y^2\right)$

$\Rightarrow \quad \frac{dy}{1+y^2} = \left(1+x^2\right) dx$

On integrating both sides, we get

$\int \frac{dy}{1+y^2} = \int\left(1+x^2\right) dx$

$\Rightarrow \quad \tan^{-1} y = x + \frac{x^3}{3} + C$ ...(i)

Given that y = 1, when x = 0.

On putting x = 0 and y = 1 in Eq. (i), we get

$\tan^{-1} 1 = C$

$\Rightarrow \quad \tan^{-1}(\tan\pi/4) = C \quad \left[\because \tan\frac{\pi}{4} = 1\right]$

$\Rightarrow \quad C = \frac{\pi}{4}$

On putting the value of C in Eq. (i), we get

$\tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$

$\therefore \quad y = \tan\left(x + \frac{x^3}{3} + \frac{\pi}{4}\right)$

which is the required solution of differential equation.

OR

The given differential equation can be rewritten as,

$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$

$\Rightarrow x\frac{dy}{dx} = y - xe^{\frac{y}{x}}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right) - e^{\frac{y}{x}}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{f}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put y = vx

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{v} + \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}$

$v + x\frac{dv}{dx} = \left(\frac{vx}{x}\right) - e^{\frac{vx}{x}}$

$\Rightarrow \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} = -e^{v}$

$\Rightarrow \frac{\mathrm{dv}}{\mathrm{e^v}} = \frac{-\mathrm{dx}}{\mathrm{x}}$

Integrating both the sides we get:

$\Rightarrow \int \frac{\mathrm{d}v}{\mathrm{e^v}} = -\int \frac{\mathrm{dx}}{\mathrm{x}} + c$

$\Rightarrow -e^{-v} = -\ln|x| + c$

Resubstituting the value of y = vx we get

$\Rightarrow -\mathrm{e}^{-\left(\frac{y}{x}\right)} = -\ln|x| + c$

Now, y(1) = 0

$\Rightarrow -e^{-(0)} = -\ln|1| + c$

$\Rightarrow$ c = -1

$\Rightarrow \log|x| + e^{-y/x} = 1$

30. $\vec{a}.\left(\vec{b}+\vec{c}\right) = 0, \vec{b}.\left(\vec{c}+\vec{a}\right) = 0, \vec{c}.\left(\vec{a}+\vec{b}\right) = 0$ (Given)

$\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)$

$= \vec{a}.\vec{a} + \vec{a}.\left(\vec{b}+\vec{c}\right) + \vec{b}.\vec{b} + \vec{b}.\left(\vec{a}+\vec{c}\right) + \vec{c}.\vec{c} + \vec{c}.\left(\vec{a}+\vec{b}\right)$

$= |\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2$

= 9 + 16 + 25

= 50

$\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{50}$

$= 5\sqrt{2}$

OR

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{d} \perp \vec{a}, \vec{d}\cdot\vec{a} = 0 \quad \Rightarrow (a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{x})(\hat{i} - \hat{j}) = 0$

$\Rightarrow a_1 - a_2 = 0$ ...(i)

$\vec{d} \perp \vec{b}, \vec{a}\cdot\vec{b} = 0 \quad \Rightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)(3j - \hat{k}) = 0$

$\Rightarrow 3a_2 - a_2 = 0$ ...(ii)

$\vec{d}\vec{c} = 1 \quad \Rightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)(7\hat{i} - \hat{k}) = 1$

$\Rightarrow 7a_1 - a_3 = 1$ ...(iii)

Solving equation (i) and (ii) we get $3a_1 - a_3 = 0$ ...(iv)

Again solving equation (iii) & (iv) we get $a_1 = \frac{1}{4}$

From equation (i), $a_1 - a_2 = 0$ or $a_1 = a_2 = \frac{1}{4}$

From equation (ii), $3a_2 - a_2 = 0 \Rightarrow 3 \cdot \frac{1}{4} = a_3 \Rightarrow a_3 = \frac{3}{4}$

Hence, $\vec{d} = \frac{1}{4}\hat{i} + \frac{1}{4}\hat{j} + \frac{3}{4}\hat{k}$

31. We have, $(\cos x)^y = (\cos y)^x$

On taking log both sides, we get

$\log(\cos x)^y = \log(\cos y)^x$

$\Rightarrow$ y log(cos x) = x log(cos y)

On differentiating both sides w.r.t x, we get

$y \cdot \frac{d}{dx}\log(\cos x) + \log\cos x \cdot \frac{d}{dx}(y)$

$= x\frac{d}{dx}\log\left(\cos y\right) + \log(\cos y)\frac{d}{dx}(x)$ [by using product rule of derivative]

$\Rightarrow \quad y \cdot \frac{1}{\cos x}\frac{d}{dx}(\cos x) + \log(\cos x)\frac{dy}{dx} = x \cdot \frac{1}{\cos y}\frac{d}{dx}$ (cos y) + log cos y.1

$\Rightarrow y \cdot \frac{1}{\cos x}(-\sin x) + \log(\cos x) \cdot \frac{dy}{dx} = x \cdot \frac{1}{\cos y}$ (-sin y)$\frac{dy}{dx}$ + log cos y.1

$\Rightarrow$ - y tanx + log(cos x)$\frac{dy}{dx}$ =-x tan y$\frac{dy}{dx}$+ log(cos y)

$\Rightarrow$[x tan y + log (cos x)] $\frac{dy}{dx}$= log(cos y) + y tan x

$\therefore \quad \frac{dy}{dx} = \frac{\log(\cos y) + y\tan x}{x\tan y + \log(\cos x)}$

Section D

32. According to Given question , Region is $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$.

The above region has a circle with equation $x^2 + y^2 = 4$ .....(i)

centre of the given circle is (0, 0)

Radius of given circle = 2

The above region consists of line whose equation is

x + y = 2 ....(ii)

Point of intersection of line and circle is

$\Rightarrow x^2 + (2 - x)^2 = 4$ [from Eq. (ii)]

$\Rightarrow x^2 + 4 + x^2 - 4x = 4$

$\Rightarrow 2x^2 - 4x = 0$

$\Rightarrow$ 2x (x - 2) = 0

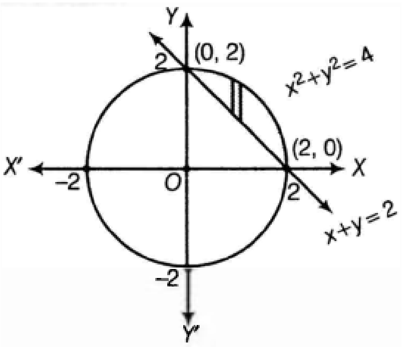
$\Rightarrow$ x = 0 or 2

When x = 0, then y = 2 - 0 = 2

When x = 2, then y = 2 - 2 = 0

So, points of intersection are (0, 2) and (2, 0).

On drawing the graph, we get the shaded region as shown below:



Required area$=\int_0^2 \left[y_{(\text{ circle })} - y_{(\text{line})}\right] dx$

$= \int_0^2 \left[\sqrt{4 - x^2} - (2 - x)\right] dx$ [From Eq(i) and (ii)]

$= \int_0^2 \sqrt{4 - x^2}dx - \int_0^2 (2 - x)dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2 \left[\because \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$

$= \left[0 + 2\sin^{-1}\left(\frac{2}{2}\right) - 0 - 2\sin^{-1}0\right] - \left(4 - \frac{4}{2} - 0\right)$

$= \left(2\sin^{-1} - 0\right) - \left(4 - \frac{4}{2}\right)$

$= 2 \cdot \frac{\pi}{2} - 2$

$= (\pi - 2)$ sq units

33. Given that A = {1, 2, 3, ...9} (a, b) R (c, d) a + d = b + c for (a, b) $\in$ A$\times$A and (c, d) $\in$ A$\times$A.

Let (a, b) R (a, b)

$\Rightarrow$ a + b = b + a, $\forall$a, b $\in$ A

Which is true for any a, b ∈ A

Hence, R is reflexive.

Let (a, b) R (c, d)

a+d = b+c

c + b = d + a ⇒ (c, d) R (a, b)

So, R is symmetric.

Let (a, b) R (c, d) and (c, d) R (e, f)

a + d = b + c and c + f = d + e

a + d = b + c and d + e = c + f(a + d) - (d + e) = (b + c) - (c + f)

(a - e) = b - f

a + f = b + e

(a, b) R (e, f)

So, R is transitive.

Hence R is an equivalence relation.

Let (a,b) R (2,5),then

a+5=b+2

a=b-3

If b<3 ,then a does not belong to A.

Therefore,possible values of b are >3.

For b=4,5,6,7,8,9

a=1,2,3,4,5,6

Therefore, equivalence class of (2,5) is

{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9).

OR

A = R – {3} and B = R – {1} and $f(x)=\left(\frac{x-2}{x-3}\right)$

Let $x_1, x_2 \in$ A, then $f(x_1)=\frac{x_1-2}{x_1-3}$ and $f(x_2)=\frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-3}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3)=(x_2-2)(x_1-3)$

$\Rightarrow x_1x_2-3x_1-2x_2+6=x_1x_2-2x_1-3x_2+6$

$\Rightarrow -3x_1-2x_2=-2x_1-3x_2$

$=x_1=x_2$

$\therefore f$ is one-one function.

Now $y=\frac{x-2}{x-3}$

$\Rightarrow y(x-3)=x-2$

$\Rightarrow xy-3y=x-2$

$\Rightarrow x(y-1)=3y-2$

$\Rightarrow x=\frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right)=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}=\frac{3y-2-2y+2}{2y-2-3y+3}=y$

$\Rightarrow f(x)=y$

Therefore, f is an onto function.

34. Given $A^2 – 5A + 7I = 0$

L.H.S $=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ = R.H.S

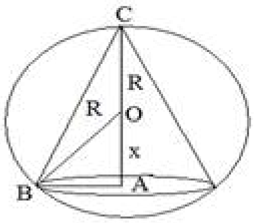
$A^2 = 5A – 7I$

$A^3 = A^2. A$

$= 5A^2 – 7AI$

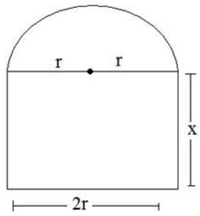
$= 5A^2 – 7A$ (Since AI = A)

= 5(5A – 7I) – 7A
= 25A – 35I – 7A
A^3= 18A – 35I
A^4 = A^3.A
= (18A – 35I).A
=18A^2 – 35IA
= 18(5A – 7I) – 35A
= 90A – 126I – 35A
= 55A – 126I
$= 55\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$= \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} + \begin{bmatrix} -126 & 0 \\ 0 & -126 \end{bmatrix}$
$A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$.

35. 

$v = \frac{1}{3}\pi r^2 h\ \left[r^2 = \sqrt{R^2 - x^2}\right]$
$V = \frac{1}{2}\pi.\left(R^2 - x^2\right).(R + x)$
$\frac{dy}{dx} = \frac{1}{3}\pi\left[\left(R^2 - x^2\right)(1) + (R + x)(-2x)\right]$
$= \frac{1}{3}\pi\left[(R + x)(R - x) - 2x(R + x)\right]$
$= \frac{1}{3}\pi(R + x)[R - x - 2x]$
$= \frac{1}{3}\pi(R + x)(R - 3x)$....(1)
Put $\frac{dv}{dr} = 0$
R = - x (neglecting)
R = 3x
$\frac{R}{3} = x$
On again differentiating equation (1)
$\frac{d^2v}{dx^2} = \frac{1}{3}\pi\left[(R + x)(-3) + (R - 3x)(1)\right]$
=$\left.\frac{d^2v}{dx^2}\right]_{x=\frac{R}{3}} = \frac{1}{3}\pi\left[\left(R + \frac{R}{3}\right)(-3) + \left(R - 3.\frac{R}{3}\right)\right]$
$\frac{1}{3}\pi\left[\frac{4R}{3} \times -3 + 0\right]$
$= \frac{-1}{3}\pi 4R$
$\frac{d^2v}{dx^2} < 0$ Hence maximum
Now $v = \frac{1}{3}\pi\left[\left(R^2 - x^2\right)(R + x)\right]\ \left[x = \frac{R}{3}\right]$
$v = \frac{1}{3}\pi\left[\left(R^2 - \left(\frac{R}{3}\right)^2\right)\left(R + \left(\frac{R}{3}\right)\right)\right]$
$= \frac{1}{3}\pi\left[\frac{8R^2}{9} \times \frac{4R}{3}\right]$
$v = \frac{8}{27}\left(\frac{4}{3}\right)\pi R^3$
$v = \frac{8}{27}$ Volume of sphere
Volume of cone $= \frac{8}{27}$ of volume of sphere.

OR



Let P be the perimeter of window

$P = 2x + 2r + \frac{1}{2} \times 2\pi r$

$10 = 2x + 2r + \pi r$ [P = 10]

$x = \frac{10-2r-\pi r}{2}$

Let A be area of window

$A = 2rx + \frac{1}{2}\pi r^2$

$= 2r\left[\frac{10-2r-\pi r}{2}\right] + \frac{1}{2}\pi r^2$

$= 10r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$

$= 10r - 2r^2 - \frac{\pi r^2}{2}$

$\frac{dA}{dr} = 10 - 4r - \pi r$

$\frac{d^2A}{dr^2} = -(\pi + 4)$

$\frac{dA}{dr} = 0$

$r = \frac{10}{\pi+4}$

$\frac{d^2A}{dr^2} < 0$ maximum

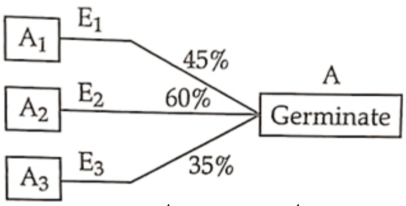
$x = \frac{10-2r-\pi r}{2}$

$x = \frac{10}{\pi+4}$

Length of rectangle = 2r $= \frac{20}{\pi+4}$

width $= \frac{10}{\pi+4}$

Section E

36. i. 

Here, P(E_1) = $\frac{4}{10}$, P(E_2) = $\frac{4}{10}$, P(E_3) = $\frac{2}{10}$

$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$

$\therefore$ P(A) = P(E_1) $\cdot$ P$\left(\frac{A}{E_1}\right)$ + P(E_2) $\cdot$ $P\left(\frac{A}{E_2}\right)$ + P(E_3) $\cdot$ $P\left(\frac{A}{E_3}\right)$

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$

$= \frac{490}{1000}$ = 4.9

ii. Required probability = $P\left(\frac{E_2}{A}\right)$

$= \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(A)}$

$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$

$= \frac{240}{490} = \frac{24}{49}$

iii. Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore$ P(E_1) = $\frac{3}{6}$

$= \frac{1}{2}$

and P(E_2) = $\frac{13}{52} = \frac{1}{4}$

Then, $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$= \frac{1}{2}, \frac{1}{4} = \frac{1}{8}$

OR

P(A) + P(B) - P(A and B) = P(A)

$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A)$

$\Rightarrow P(B) - P(A \cap B) = 0$

$\Rightarrow P(A \cap B) = P(B)$

$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{P(B)}{P(B)}$

= 1

37. i. Clearly, the coordinates of A are (8, 10, 0) and D are (0, 0, 30)

$\therefore$ Equation of AD is given by

$\frac{x-0}{8-0} = \frac{y-0}{10-0} = \frac{z-30}{-30}$

$\Rightarrow \frac{x}{4} = \frac{y}{5} = \frac{30-z}{15}$

ii. The coordinates of point C are (15, - 20, 0) and D are (0, 0, 30)

$\therefore$ Length of the cable DC

$= \sqrt{(0-15)^2 + (0+20)^2 + (30-0)^2}$

$= \sqrt{225 + 400 + 900} = \sqrt{1525} = 5\sqrt{61}$ m

iii. Since, the coordinates of point B are (-6, 4, 0) and D are (0, 0, 30), therefore vector DB is

$(-6-0)\hat{i} + (4-0)\hat{j} + (0-30)\hat{k}$, i.e., $-6\hat{i} + 4\hat{j} - 30\hat{k}$

OR

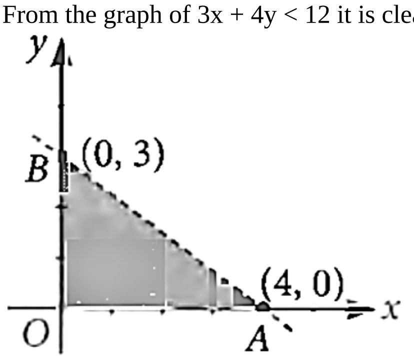
Required sum

$= (8\hat{i} + 10\hat{j} - 30\hat{k}) + (-6\hat{i} + 4\hat{j} - 30\hat{k}) + (15\hat{i} - 20\hat{j} - 30\hat{k})$

$= 17\hat{i} - 6\hat{j} - 90\hat{k}$

38. i. When we solve an L.P.P. graphically, the optimal (or optimum) value of the objective function is attained at corner points of the feasible region.

ii. From the graph of 3x + 4y < 12 it is clear that it contains the origin but not the points on the line 3x + 4y = 12.



iii. Maximum of objective function occurs at corner points.

Corner Points	**Value of Z = 2x + 5y**
(0, 0)	0
(7, 0)	14
(6, 3)	27
(4, 5)	33 ← Maximum
(0, 6)	30

OR

Value of Z = px + qy at (15, 15) = 15p + 15q and that at (0, 20) = 20q. According to given condition, we have

$15p + 15q = 20q \Rightarrow 15p = 5q \Rightarrow q = 3p$