MIND MAP : LEARNING MADE SIMPLE

CHAPTER - 13

Probability

Random Variable

Real valued function whose domain is the sample space of a random experiment.

The probability distribution of a random variable x is the system of numbers x: x_1 x_2 . . . . , x_n
$P(x)$: p_1 p_2 . . . . p_n where , $pi>0$,
$\sum_{i=1}^{n} p_i=1$, $i=1.2,....,$ n.

Mean of a random variable

Let x be a R.V. whose possible values $x_1, x_2, \ldots, x_n$ occur with probabilities $p_1, p_2, \ldots, p_n$ resp. Then, mean of $x, \mu = \sum_{i=1}^{n} x_i p_i$ It is also called the expectation of x, denoted by $E(x)$

Variance and standard deviation

Let x be a R.V. whose possible values $x_1, x_2, \ldots, x_n$ occurs with probabilities $p(x_1), p(x_2), \ldots, p(x_n)$ respectively. Let, $\mu = E(x)$ be the mean of x. The variance of x, var (x) or $\sigma_x^2 = \sum_{i=1}^{n}(x_i-\mu)^2 p(x_i)$ or $E(x-\mu)^2$

The non-negative number

$6x=\sqrt{\text{var}(x)}=\sqrt{\sum_{i=1}^{n}(x_i-\mu)^2 p(x_i)}$ is called the standard deviation of the R.V. 'X'. Also, $\text{var}(x)=E(x^2)-[E(x)]^2$ For eg: $E(x) = 3$ and $E(x^2) = 10$, then var x= 10–9 = 1 and SD=$\sqrt{1}$ =1.

Bernoulli's Trials

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(i)There should be a finite no. of trials.

(ii)The trials should be independent.

(iii)Each trial has exactly two outcomes: success or failure.

(iv)The probability of success remains the same in each trial.

For Binomial distribution,B (n,p), $P(X=x)=({}^nC_x q^{n-x} p^x, x=0,1,....,n)$ $(q=1-p)$

Conditional Probability

The probability of the event E is called the conditional probability of E given that F has already occurred, and is denoted by $P(E/F)$. Also,

$P(E/F)=\frac{P(E\cap F)}{P(F)}, P(F)\neq 0.$

Properties

(i) $0 \le P(E/F) \le 1$, $P(E'/F)=1-P(E/F)$

(ii) $P((E\cup F)/G)=P(E/G)+P(F/G)-P((E\cap F)/G)$

(iii) $P(E\cap F)=P(E)P(F/E), P(E)\neq 0$

(iv) $P(E\cap F)=P(F)P(E/F), P(F)\neq 0$

For eg : if $P(A)=\frac{7}{13}$, $P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$, then

$P(A/B)=\frac{p(A\cap B)}{P(B)}=\frac{\frac{4}{13}}{\frac{9}{13}}=\frac{4}{9}.$

If E and F are independent, then $P(E\cap F)=P(E)P(F)$, $P(E|F)=P(E), P(F)\neq 0$ and $P(F|E)=P(F), P(E)\neq 0$.

Theorem of total probability

Let, $\{E_1, E_2 \ldots .E_n\}$ be a partition of a sample space 'S' and suppose that each of $E_1, E_2, \ldots .E_n$ has non-zero probability. Let 'A' be any event associated with S, then $P(A) = P(E_1)P(A|E_1) + P(E_2) P(A|E_2) + \ldots .+ P(E_n) P(A|E_n)$.

Bayes' Theorem

If $E_1, E_2, \ldots, E_n$ are events which constitute a partition of sample space S, *i.e.*, $E_1, E_2, \ldots E_n$ are pair wise dis joint and $E_1\cup E_2\cup ....\cup E_n=S$ and A be any event with non-zero probability, then $P(E_i|A)=\frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n}P(E_j)P(A|E_j)}$