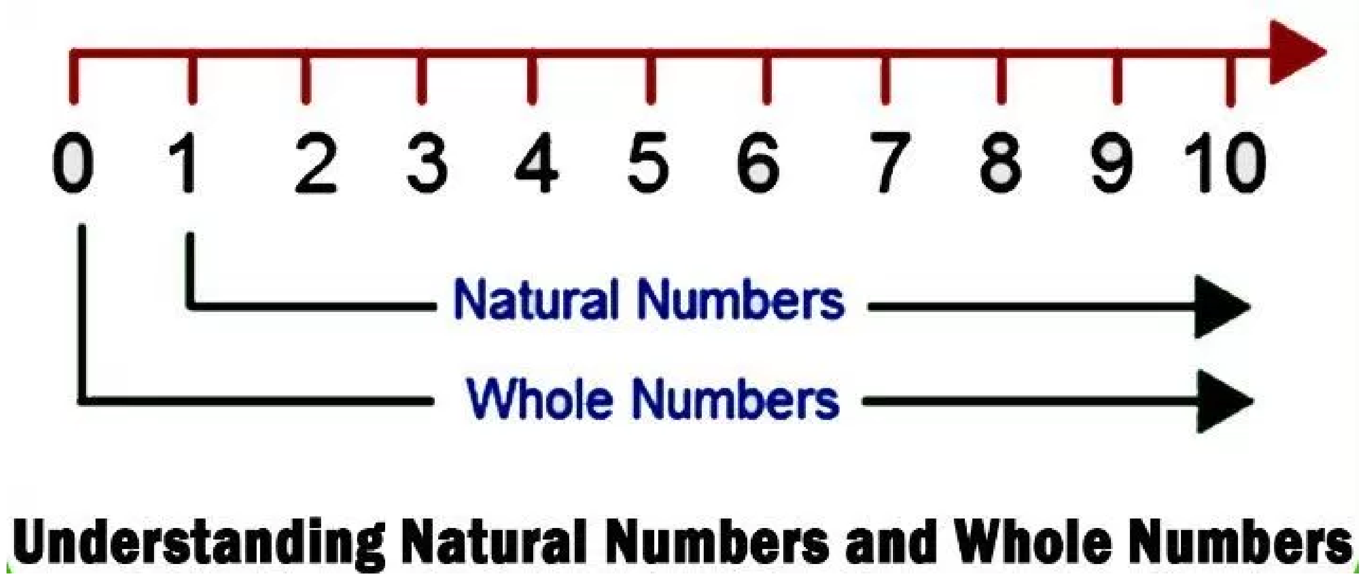
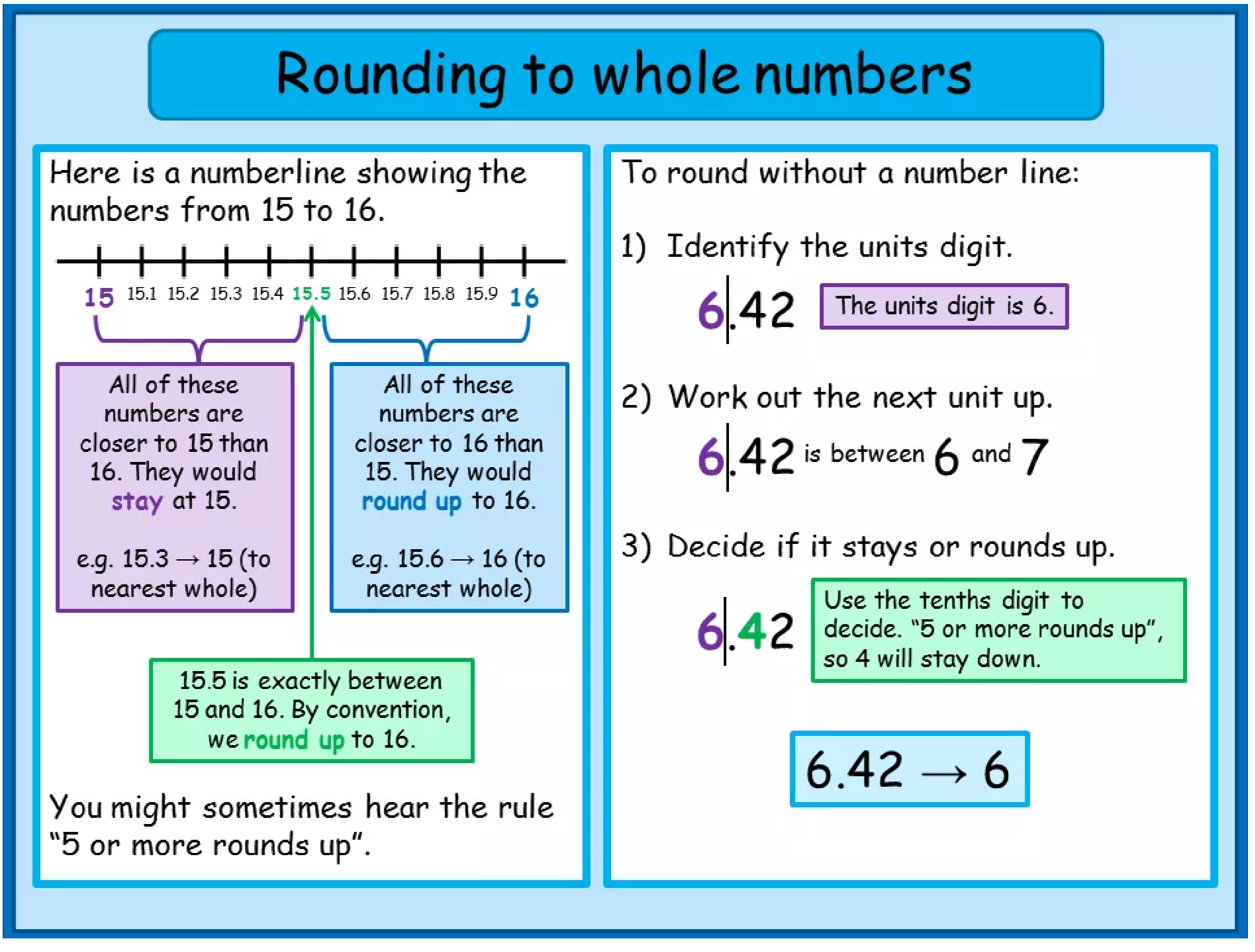
4. Doing Calculations

Whole Numbers And Its Properties

WHOLE NUMBERS

Now if we add zero (0) in the set of natural numbers, we get a new set of numbers called the **whole numbers**. Hence the set of whole numbers consists of zero and the set of natural numbers. It is denoted by W. i.e., W = {0, 1, 2, 3, . . .}. Smallest whole number is zero.





Properties of whole numbers

All the properties of numbers satisfied by natural numbers are also satisfied by whole numbers. Now we shall learn some fundamental properties of numbers satisfied by whole numbers.

Properties of Addition

(a) Closure Property: The sum of two whole numbers is always a whole number. Let a and b be two whole numbers, then a + b = c is also a whole number.
This property is called the closure property of addition

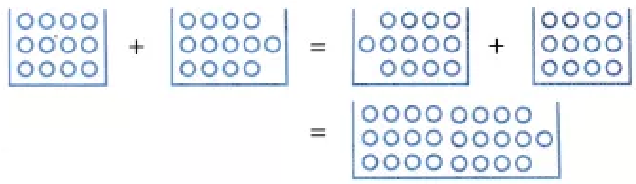
Example: 1 + 5 = 6 is a whole number.

3 + 7 = 10 is a whole number.

(b) Commutative Property: The sum of two whole numbers remains the same if the order of numbers is changed. Let a and b be two whole numbers, then
a + b = b + a
This property is called the commutative property of addition.

Examples: 12 + 13 = 13 + 12

(25) = (25)



0 + 8 = 8 + 0

(8) = (8)

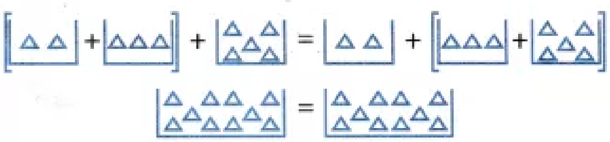
the sum remains same

(c) Associative Property: The sum of three whole numbers remains the same even if the grouping is changed. Let a, b, and c be three whole numbers, then
(a + b) + c = a + (b + c)
This property is called the associative property of addition.

Examples: (2+ 3) + 5 = 2 + (3 +5)

5 + 5 = 2 + 8

10 = 10



12 + (13 + 7) = (12 + 13) + 7

12 + 20 = 25 + 7

32 = 32

(d) Identity Element: If zero is added to any whole number, the sum remains the number itself. As we can see that 0+a=a=a+0 where a is a whole number.

Examples: $0 + 3 = 3 = 3 + 0$

$0 + 312 = 312 = 312 + 0$

$0 + 27 = 27 = 27 + 0$

Therefore, the number zero is called the additive identity, as it does not change the value of the number when addition is performed on the number.

Properties of Subtraction

(a) Closure Property: The difference of two whole numbers will not always be a whole number. Let a and b be two whole numbers, then a − b will be a whole number if a > b or a = b. If a < b, then the result will not be a whole number. Hence, closure property does not hold good for subtraction of whole numbers.
Examples
17 − 5 = 12 is a whole number.
5 − 17 = − 12 is not a whole number.

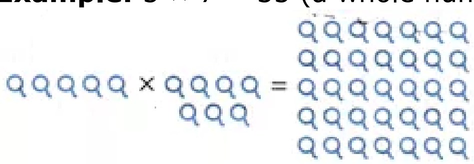
(b) Commutative Property: If a and b are two whole numbers, then a − b ≠ b − a. It shows that subtraction of two whole numbers is not commutative. Hence, commutative property does not hold good for subtraction of whole numbers, i.e.,
a − b ≠ b − a.
Example: 3 − 4 = − 1 and 4 − 3 = 1
∴ 3 − 4 ≠ 4 − 3

(c) Associative Property: If a, b, and c are whole numbers, then (a − b) − c ≠ a − (b − c). It shows that subtraction of whole numbers is not associative. Hence, associative property does not hold good for subtraction of whole numbers.
Example: (40 − 25) − 10 = 15 − 10 = 5
40 − (25 − 10) =40- 15 = 25
∴ (40 − 25) − 10 ≠ 40 − (25 − 10)

(d) Property of Zero: If we subtract zero from any whole number, the result remains the number itself.
Example: 7 − 0 = 7
5 − 0 = 5

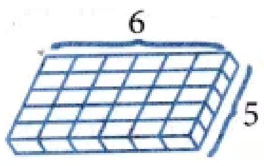
Properties of Multiplication

(a) Closure Property: If a and b are two whole numbers, then a × b = c will always be a whole number. Hence, closure property holds good for multiplication of whole numbers.
Example: 5 × 7 = 35 (a whole number)



6 × 1 = 6 (a whole number)

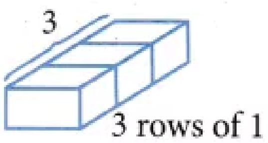
(b) Commutative Property: If a and b are two whole numbers, then the product of two whole numbers remains unchanged if the order of the numbers is interchanged, i.e.,
a × b = b × a.
Example: 6 × 5 = 5 × 6
30 = 30
i. e., 6 rows of 5 or 5 rows of 6 give the same results.

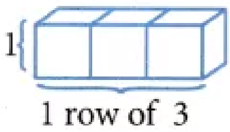


so, 6 × 5 = 30 = 5 × 6

(c) Associative Property: If a, b, and c are whole numbers, then the product of three whole numbers remains unchanged even if they are multiplied in any order. Hence, associative property does hold good for multiplication of whole numbers, i.e.,
(a × b) × c = a × (b × c)
Example:
(4 × 5) × 8 = 4 × (5 × 8)
20 × 8 = 4 × 40
160 = 160

(d) Multiplicative Identity: If any whole number is multiplied by 1, the product remains the number itself. Let a whole number be a, then
a × 1 = a = 1 × a.





3 × 1 = 3 = 1 × 3
Examples
75 × 1 = 75 = 1 × 75
3 × 1 = 3 = 1 × 3
Hence, 1 is called the multiplicative identity.

(e) Multiplicative Property of Zero: Any whole number multiplied by zero gives the product as zero.If a is any whole number, then 0 × a = a × 0 = 0.
Example: 3 × 0 = 0 × 3 = 0

Properties of Division

(a) Closure Property: If a and b are whole numbers, then a ÷ b is not always a whole number. Hence, closure property does not hold good for division of whole numbers.
Example: 7 ÷ 5 = $\frac{7}{5}$ is not a whole number.
7 ÷ 7 = 1 is a whole number.

(b) Commutative Property: If a and b are whole numbers, then a ÷ b ≠ b ÷ a. Hence, commutative property does not hold good for division of whole numbers.
Example: 18 ÷ 3 = 6 is a whole number.
3 ÷ 18 = $\frac{3}{18}$ = $\frac{1}{6}$ is not a whole number.
∴ 3 ÷ 18 ≠ 18 ÷ 3

(c) Associative Property: If a, b, and c are whole numbers then (a ÷ b) ÷ c ≠ a ÷ (b ÷ c). Hence, associative property does not hold good for division of whole numbers.

Example: (15 ÷ 3) ÷ 5 = 5 ÷ 5 = 1
15 ÷ (3 ÷ 5) = 15 ÷ 3/5 = 15 × 5/3
= 25
∴ (15 ÷ 3) ÷ 5 ≠ 15 ÷ (3 ÷ 5)

(d) Property of Zero: If a is a whole number then 0 ÷ a = 0 but a ÷ 0 is undefined.
Example: 6 ÷ 0 is undefined.

Note:

- Product of zero and a whole number gives zero.
  a × 0 = 0

- Zero divided by any whole number gives zero.
  $0 \div a = 0$
  $a \div 0$ = undefined
- Any number divided by 1 is the number itself.
  $a \div 1 = a$

DISTRIBUTIVE PROPERTY

You are distributing something as you separate or break it into parts.

Example: Raj distributes 4 boxes of sweets. Each box comprises 6 chocolates and 10 candies. How many sweets are there in these 4 boxes?
∴ Chocolates in 1 box = 6
Choclolates in 4 boxes = 4 × 6 = 24
Candies in 1 box =10
Candies in 4 boxes = 4 × 10 = 40
Total number of sweets in 4 boxes
= 4 × 6 + 4 × 10 = 4 × (6 + 10)
= 4 × 16 = 64

Hence, we conclude the following:
(a) Multiplication distributes over addition, i.e., a(b + c) = ab + ac, where a, b, c are whole numbers.
Example: 10 × (6 + 5) = 10 × 6 + 10 × 5
10 × 11 = 60 + 50
110 = 110
This property is called the distributive property of multiplication over addition.
(b) Similarly, multiplication distributes over subtraction, i.e., a × (b – c) = ab – ac where a, b, c are whole numbers and b > c.
Example: 10 × (6 – 5) = 10 × 6 – 10 × 5
10 × 1 = 60 – 50
10 = 10
This property is called the distributive property of multiplication over subtraction.

Example 1: Determine the following by suitable arrangement.
2 × 17 × 5
Solution: 2 × 17 × 5 = (2 × 5) × 17
= 10 × 17 = 170

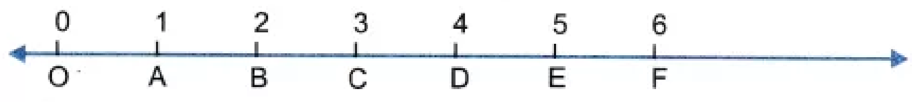
Example 2: Solve the following using distributive property.
97 × 101
Solution: 97 × 101 = 97 × (100 + 1)
= 9700 + 97 = 9797

Example 3: Tina gets 78 marks in Mathematics in the half-yearly Examination and 92 marks in the final Examination. Reenagets 92 marks in the half- yearly Examination and 78 marks in the final Examination in Mathematics. Who has got the higher total marks?
Solution: Tina gets the following marks = 78 + 92 = 170 Total marks
Reena gets the following marks = 92 + 78 = 170 Total marks
So, both of them got equal marks.

Example 4: A fruit seller placed 12 bananas, 10 oranges, and 6 apples in a fruit basket. Tarun buys 3 fruit baskets for a function. What is the total number of fruits in these 3 baskets?
Solution: Number of bananas in 3 baskets = 12 × 3 = 36 bananas
Number of oranges in 3 baskets = 10 × 3 = 30 oranges
Number of apples in 3 baskets = 6×3 = 18 apples
Total number of fruits = 36 + 30+ 18 = 84
Alternative Method
Total number of fruits in 3 baskets
= 3 × [12 +(10 + 6)]
= 3 × [12 + 16]
= 3 × 28 = 84

Representation Of Whole Numbers On A Number Line

We can represent whole numbers-on a straight line. To represent a set of whole numbers on a number line, let's first draw a straight line and mark a point O on it. After that, mark points A, B, C, D, E, F on the line at equal distance, on the right side of point O.



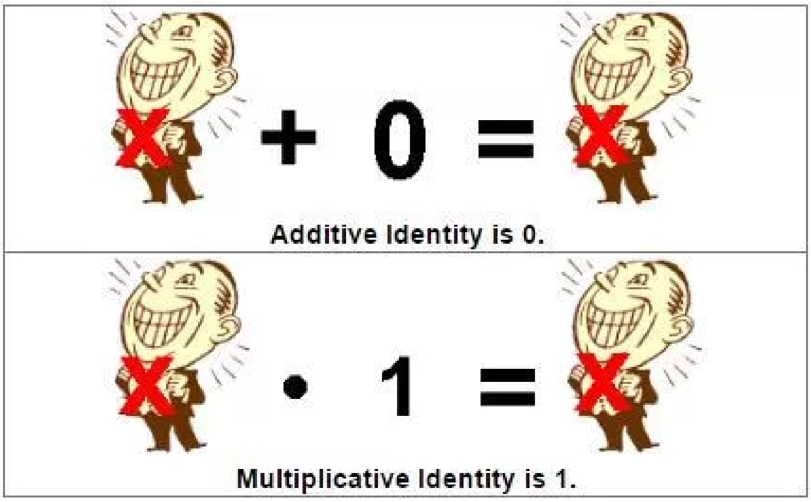
Now, OA = AB = BC = CD and so on
Let OA = 1 unit
OB = OA + AB = 1 + 1 = 2 units
OC = OB + BC = 2 + 1 = 3 units
OD = OC + CD = 3 + 1 = 4 units and so on.
Let the point O correspond to the whole number 0, then points A, B, C, D, E, ….. correspond to the whole numbers 1, 2, 3, 4, 5,…. In this way every whole number can be represented on the number line.

Hints for Remembering the Properties of Real Numbers

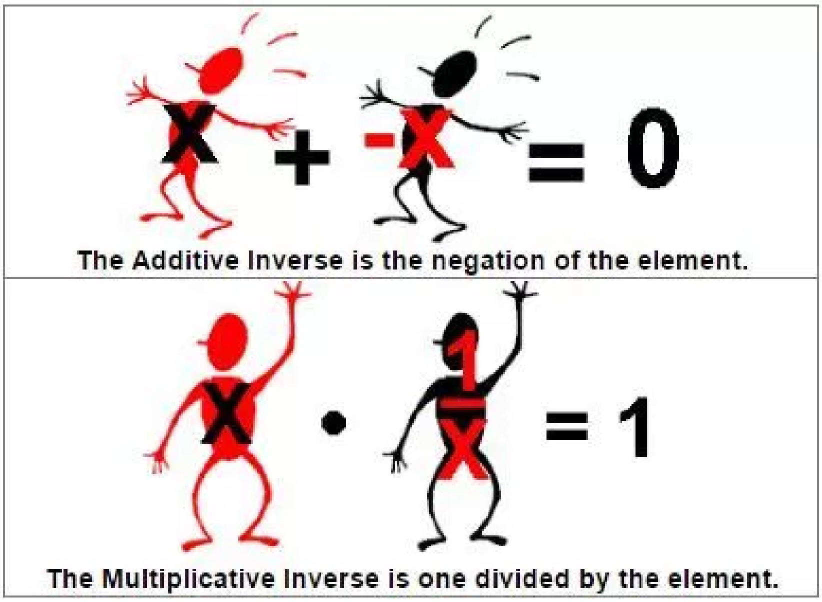
Commutative Property – interchange or switch the elements
Example shows commutative property for addition:
X + Y = Y + X
Think of the elements as "commuting" from one location to another. "They get in their cars and drive to their new locations." This explanation will help you to remember that the elements are "moving" (physically changing places).

Associative Property – regroup the elements
Example shows associative property for addition:
(X + Y) + Z = X + (Y + Z)
The associative property can be thought of as illustrating "friendships" (associations). The parentheses show the grouping of two friends. In the example below, the red girl (y) decides to change from the blue boyfriend (x) to the green boyfriend (z). "I don't want to associate with you any longer!" Notice that the elements do not physically move, they simply change the person with whom they are "holding hands" (illustrated by the parentheses.)

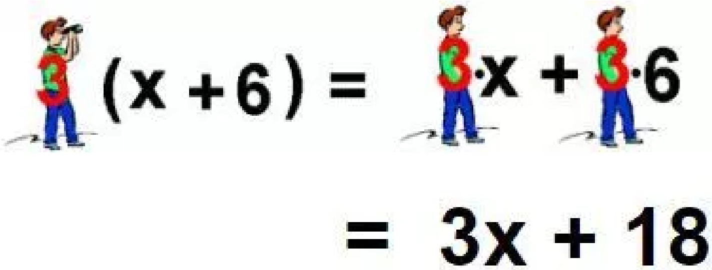
Identity Property – What returns the input unchanged?
X + 0 = X   Additive Identity
X • 1 = X   Multiplicative Identity
Try to remember the "I" in the word identity. Variables can often times have an "attitude". "I am the most important thing in the world and I do not want to change!" The identity element allows the variable to maintain this attitude.



Inverse Property – What brings you back to the identity element using that operation?
X + -X = 0   Additive Inverse
X • 1/X = 1   Multiplicative Inverse
Think of the inverse as "inventing" an identity element. What would you need to add (multiply) to this element to turn it into an identity element?



Distributive Property – multiply across the parentheses. Each element inside the parentheses is multiplied by the element outside the parentheses.
a(b + c) = a•b + a•c
Let's consider the problem 3(x + 6). The number in front of the parentheses is "looking" to distribute (multiply) its value with all of the terms inside the parentheses.



What Are The Four Basic Operations In Mathematics

OPERATIONS ON NUMBERS

Addition

Addition of two or more numbers gives us another number. The numbers which are added are called addends and the new number thus obtained is called the sum. For example,
34670 + 12345 = 47015
Here, 34670 and 12345 are called addends and 47015 is called the sum of 34670 and 12345. Large numbers are added in the same way as the small numbers.

Addition by grouping

When we add three numbers, we can add any two numbers first and then add the third number to the sum. In other words, we can group any two of the three numbers in order to find the sum of the three numbers. If we have to add more than three numbers, we can similarly group any two of the given numbers in several ways and add them. For example, suppose we want to add 234523, 123098, 555623, and 876543. We can find the sum in any of the following ways:

- [(234523 + 123098) + 555623] + 876543
  = (357621 + 555623) + 876543
  = 913244 + 876543 = 1789787
- [(234523 + 123098) + 876543] + 555623
  = (357621 + 876543) + 555623
  = 1234164 + 555623 = 1789787

So, we can add three or more numbers by grouping them in any way we feel convenient.

Subtraction

Subtraction of one number from another number gives us a third number. The new number thus obtained is called the difference of the two numbers. For example,
70000 – 67429 =2571
Here, 2571 is the difference of 70000 and 67429. Large numbers are subtracted in the same way as the small numbers.

Multiplication

Multiplication of two or more numbers gives us another number. The new number thus obtained is called the product of those numbers. For example, 11 × 13 = 143.
Here, 143 is the product of 11 and 13. 11 and 13 are called the/actors of 143. Note that 1 is always a factor of any number.
Let us take another example, say, 855 × 73 = 62415. Here, 855 and 73 are factors of 62415, and 62415 is the product of 855 and 73.
Large numbers are multiplied in the same way as the small numbers.

Multiplication using zeros

When we multiply a number by 10,100,1000, etc. we simply place those many zeros on the right of that number. For example, if we want to multiply 15 by 10, then the answer will be 150. Similarly, if we want to multiply 15 by 100, then the answer will be 1500, and so on. Zeros have been added on the right side of 15.

Division

Division of a number by another number gives two new numbers—the **quotient** and the **remainder**. The number that is divided is called the **dividend** and the number that divides is called the **divisor**. For example,

12) 160 (13
12
40
36
4

Here, **12** is the **divisor**; **160** is the **dividend**; **13** is the **quotient**, and **4** is the **remainder**.

Divide 69205 by 432 and find the quotient and remainder.

432) 69205 (160
432
2600
2592
85

Here, the quotient is 160 and remainder is 85.

Mixed operations involving +, –, ×, and ÷

Till now we have solved problems involving only one type of operation, that is, one of the following: addition, subtraction, division, and multiplication. But what do we do if a problem involves two or more operations together? Consider the following problem:
Example: Simplify: 16 – 6 + 2 – 3

1st Case	2nd Case
16 – 6 + 2 – 3	16 – 6 + 2 – 3
= 10 + 2 – 3	= 16 – 8 – 3
= 12 – 3	= 8 – 3
= 9	= 5

In the 1st case, the answer is 9 and in the 2nd case, it is 5. We get different answers depending on the order in which the operations are carried out. But one of the two answers we got is wrong. In order to avoid this kind of ambiguity, an international convention has been accepted.

1. If any mathematical expression has symbols of addition and subtraction both, we first add and then subtract. For example, consider the following case: 16 – 3 + 4 – 5
   = 20 – 3 – 5 (Addition: 16 + 4 = 20)
   = 17 – 5 (Subtraction: 20 – 3 = 17)
   = 12 (Subtraction: 17 – 5 = 12)
2. If in three operations [(+, – , and ×) or (+, – , and ÷)], that is, besides addition and subtraction, a problem involves multiplication or division, we first multiply or divide, and then go for addition and subtraction respectively. For example, consider the following cases.
   (a) Simplify: 7 + 3 × 4 – 3
   In the above example, the three operations involved are +, – , and ×. To solve this problem, we first multiply the numbers, then we go for addition, and at the end we subtract.
   7 + 3 × 4 – 3
   = 7 + 12 – 3 (Multiplication: 3 × 4 = 12)
   = 19 – 3 (Addition: 7 + 12 = 19)
   = 16 (Subtraction: 19 – 3 = 16)
   (b) Simplify: 16 – 6 ÷ 2 + 8
   In the above example, the three operations involved are +, – , and ÷. To solve this problem, we first divide, then add, and at the end subtract the number. 16 – 6 ÷ 2 + 8
   = 16 – 3 + 8 (Division: 6 ÷ 2 = 3)

= 24 − 3 (Addition: 16 + 8 = 24)
= 21 (Subtraction: 24 − 3 = 21)

3. When a problem involves all the operations namely, +, − , × , and ÷, then there is an agreed formula denoted by 'DMAS', which mathematicians follow. In 'DMAS', D stands for division, M for multiplication, A for addition, and S for subtraction. DMAS represents the order of operations. For example, consider the following cases.
   (a) Simplify: 5 + 4 × 3 − 9 ÷ 3
   In the above example, all the four operations are there, so we have to use DMAS rule, as shown below: 5 + 4 × 3 − 9 ÷ 3
   = 5 + 4 × 3 − 3 (Division: 9 ÷ 3 = 3)
   = 5 + 12 − 3 (Multiplication: 4 × 3 = 12)
   = 17 − 3 (Addition: 5 + 12 = 17)
   = 14 (Subtraction: 17 − 3 = 14)
   (b) Simplify: 7 × 3 − 4 + 60 ÷ 10
   In this example too, all the four operations are there, hence to simplify this we have to use DMAS rule. 7 × 3 − 4 + 60 ÷ 10
   = 7 × 3 − 4 + 6 (Division: 60 ÷ 10 = 6)
   = 21 − 4 + 6 (Multiplication: 7 × 3 = 21)
   = 27 − 4 (Addition: 21 + 6 = 27)
   = 23 (Subtraction: 27 − 4 = 3)

Using the operation 'Of'

Sometimes we need to find the value of '$\frac{1}{2}$ of 16' or '3 of 5'.
This means we need to find the value of $\frac{1}{2}$ × 16 or 3 × 5.
So 'of' means multiplication.
Hence, $\frac{1}{2}$ of 16 = $\frac{1}{2}$ × 16
= 8
and 3 of 5 =3 × 5
= 15.
When the operation 'of' appears in any mathematical expression, then it must be performed before any other operation. To solve such kind of expression we use ODMAS rule, in which O stands for of, D for division, M for multiplication, A for addition, and S for subtraction.
Consider the following examples.

Example 1: Simplify 36 ÷ 2 of 3 + 6 × 2.

Solution: To solve this, we first solve the operation 'of'.
36 ÷ 2 of 3 + 6 × 2
= 36 ÷ 6 + 6 × 2 (Of. 2 of 3 = 2 × 3 = 6)
= 6 + 6 × 2 (Division: 36 ÷ 6 = 6)
= 6 + 12 (Multiplication: 6 × 2 = 12)
= 18 (Addition: 6 + 12 = 18)

Example 2: Simplify 42 ÷ 6 × 2 + $\frac{1}{7}$ of 35 × 2.

Solution: 42 ÷ 6 × 2 + $\frac{1}{7}$ of 35 × 2
= 42 ÷ 6 × 2 + 5 × 2 (Of: $\frac{1}{7}$ of 35 = $\frac{1}{7}$ × 35 = 5)
= 7 × 2 + 5 × 2 (Division: 42 ÷ 6 = 7)
= 14 + 10 (Multiplication: 7 × 2 = 14 and 5 × 2 = 10)
= 24 (Addition: 14 + 10 = 24)

Use of brackets and the BODMAS rule

Let us consider an example to illustrate the use of brackets.
Rima bought 35 chocolates and ate 5 of them. She distributed the remaining chocolates equally among 6 of her friends. How many chocolates did she give to each of them?

In this problem we have to subtract 5 chocolates that Rima ate, from 35 chocolates she had, before dividing them among 6 of her friends. So we have to first perform the operation of subtraction and then do division. In such cases, we use brackets around the part that has to be done first, that is

(35 – 5) ÷ 6 (First solve bracket, i.e., 35 – 5 = 30)
= 30 ÷ 6   (Division: 30 ÷ 6 = 5)
= 5
Consider another example.

Example 3: Solve 2 of 3 × (5 + 2).
Solution: 2 of 3 × (5 + 2)
= 2 of 3 × 7 (First bracket: 5 + 2 = 7)
= 6 × 7   (Of: 2 of 3 = 2 × 3 = 6)
= 42   (Multiplication: 6 × 7 = 42)
Hence, when problems involve brackets, of, ×, ÷, +, and – then

First work out	–	brackets	**(B)**
then perform	–	'of'	**(O)**
then	–	÷ (division)	**(D)**
after that	–	× (multiplication)	**(M)**
next	–	+ (addition)	**(A)**
at the end	–	– (subtraction)	**(S)**

To make it easy to remember this order, we remember the word **BODMAS**, where **B** stands for brackets, **O** for of', **D** for division, **M** for multiplication, **A** for addition, and **S** for subtraction. This is called the '**BODMAS**' rule.
Sometimes numerical expressions may involve different types of brackets. These brackets are

- Vinculum or bar —
- Parentheses or small brackets ( )
- Braces or curly brackets { }
- Square brackets or big brackets [ ]

We simplify expressions by starting with the innermost bracket. Usually the vinculum is the innermost bracket, next is the parentheses, then the braces, and finally the square brackets. Let us now consider some examples.

Example 4: Simplify $25 - [20 - \{10 - (7 - \overline{5 - 3})\}]$.

Solution:

$$25 - [20 - \{10 - (7 - \overline{5 - 3})\}]$$
$$= 25 - [20 - \{10 - (7 - 2)\}]$$
(*Simplifying vinculum:* $5 - 3 = 2$)
$$= 25 - [20 - \{10 - 5\}]$$
(*Simplifying parentheses:* $7 - 2 = 5$)
$$= 25 - [20 - 5]$$
(*Simplifying curly brackets:* $10 - 5 = 5$)
$$= 25 - 15$$
(*Simplifying square brackets:* $20 - 5 = 15$)
$$= 10$$ (*Subtraction:* $25 - 15 = 10$)

Example 5: Simplify [72 -12 ÷ 3 of 2] + (18 − 6) ÷ 4.

Solution:

$$[72 - 12 \div 3 \text{ of } 2] + (18 - 6) \div 4$$
$$= [72 - 12 \div 3 \text{ of } 2] + 12 \div 4$$
(*Solving parentheses:* $18 - 6 = 12$)
$$= [72 - 12 \div 6] + 12 \div 4$$
(*Solving 'of' inside square bracket:* $3 \text{ of } 2 = 6$)
$$= [72 - 2] + 12 \div 4$$
(*Division inside square bracket:* $12 \div 6 = 2$)
$$= 70 + 12 \div 4$$
(*Solving square brackets:* $72 - 2 = 70$)
$$= 70 + 3$$ (*Division:* $12 \div 4 = 3$)
$$= 73$$ (*Addition:* $70 + 3 = 73$)

RULES FOR SIMPLIFICATION

1. Order of operation: The use of brackets take us to a new order of operation. The operation inside the brackets comes before the ODMAS. There are different types of brackets already mentioned here.

2. If there is no sign between a number and the bracket, then it is implied that the operation to be performed is multiplication.

Examples

(*a*) $100 - 3[20 + \{50 - 40\}]$

$= 100 - 3[20 + 10]$

(*Solving curly brackets*: $50 - 40 = 10$)

$= 100 - 3[30]$

(*Solving square brackets*: $20 + 10 = 30$)

$= 100 - 90$

(*Solving 3[30]*: $3 \times 30 = 90$)

$= 10$ (*Subtraction:* $100 - 90 = 10$)

(*b*) $3\{(-6) + 20\} + (-7 + 9)(2) - 12$

$= 3\{-6 + 20\} + (2)(2) - 12$

[*Solving parentheses:* $(-7 + 9) = 2$]

$= 3\{14\} + 4 - 12$

[*Solving parentheses:* $(-6 + 20) = 14$]

$= 42 + 4 - 12$

[*Solving curly brackets:*

$3\{14\} = 3 \times 14 = 42$]

$= 46 - 12$ (*Addition:* $42 + 4 = 46$)

$= 34$ (*Subtraction:* $46 - 12 = 34$)

3. When there is '+' sign before a bracket, you can simply remove the bracket.

Examples:

(*a*) $7 + (8 - 3 \text{ of } 2)$

$= 7 + (8 - 6)$

(*Solving 'of'*: $3 \text{ of } 2 = 3 \times 2 = 6$)

$= 7 + 2$

(*Solving small brackets:* $8 - 6 = 2$)

$= 9$ (*Addition:* $7 + 2 = 9$)

(*b*) $5 + [6\{5 - 4 + 1\}]$

$= 5 + [6\{6 - 4\}]$

(*Inside curly brackets addition:* $5 + 1 = 6$)

$= 5 + [6\{2\}]$

(*Inside curly brackets subtraction:* $6 - 4 = 2$)

$= 5 + [6 \times 2]$

(*Multiplication:* $6 \times 2 = 12$)

$= 5 + 12$

$= 17$ (*Addition:* $5 + 12 = 17$)

4. When there is a '–' sign before a bracket, then all signs within the bracket change while removing the bracket.

Examples:

(*a*) $20 - (10 - 3 + 2)$ or $20 - (10 - 3 + 2)$

$= 20 - 10 + 3 - 2$ $\quad$ $20 - (7 + 2)$

$= 23 - 12$ $\quad$ $20 - 9$

$= 11$ $\quad$ $= 11$

(*b*) $50 - [20 + \{30 - (20 - 5)\}]$

$= 50 - [20 + \{30 - 15\}]$

(*Solving*: $20 - 5 = 15$)

$= 50 - [20 + 15]$

(*Solving*: $30 - 15 = 15$)

$= 50 - 35$ (*Solving*: $20 + 15 = 35$)

$= 15$ (*Solving*: $50 - 35 = 15$)

Order of Operations and Evaluating Expressions

Order of Operations

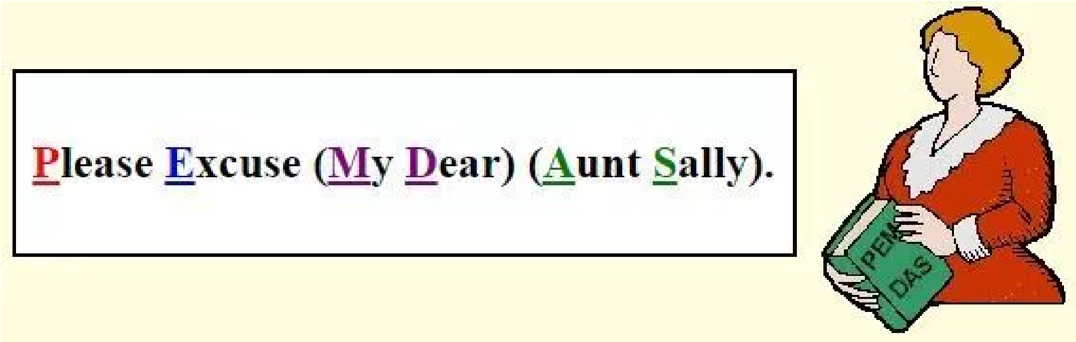
When a numerical expression involves two or more operations, there is a specific order in which these operations must be performed.

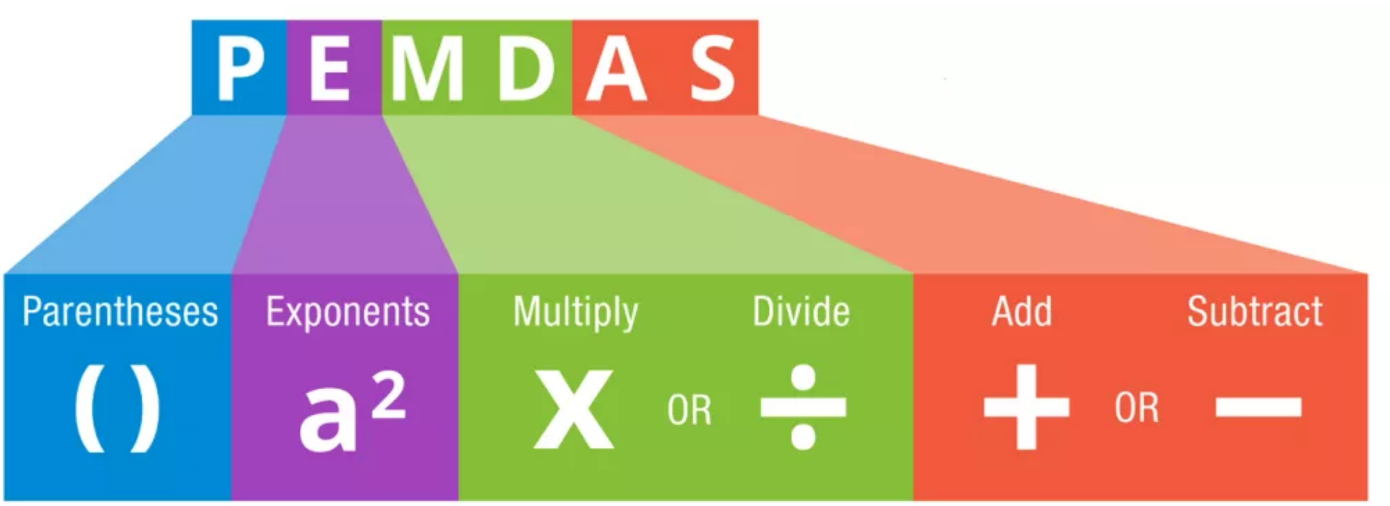
When evaluating an expression, proceed in this order:

1. parentheses are done first.
2. exponents are done next.
3. multiplication and division are done as they are encountered from left to right.
4. addition and subtraction are done as they are encountered from left to right.

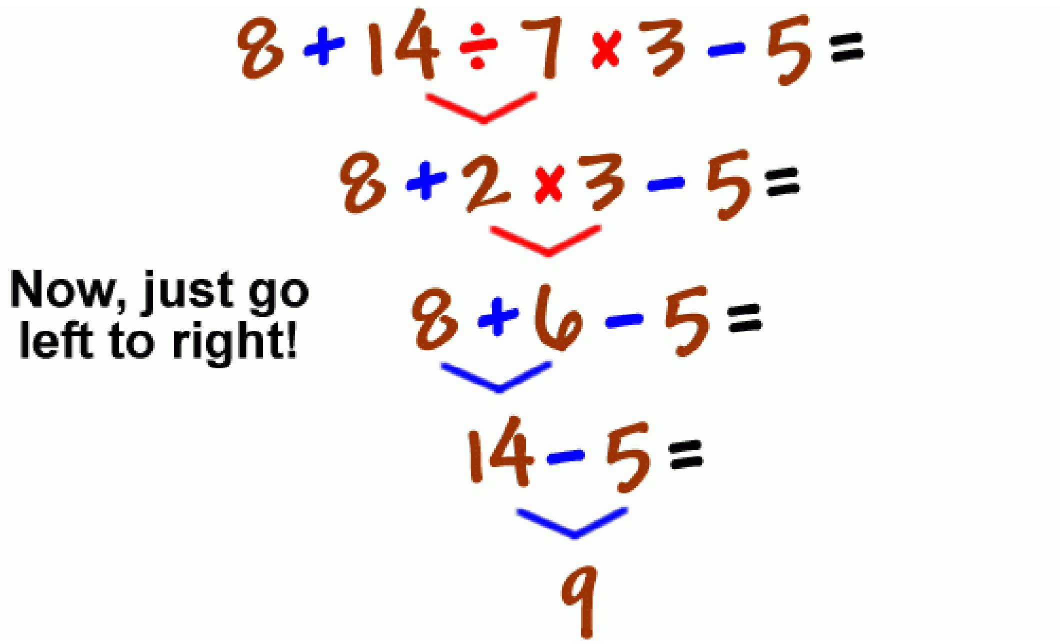
The proper application of “order of operations” is needed when working with such mathematical topics as evaluating formulas, solving equations, evaluating algebraic expressions, and simplifying monomials and polynomials.

There is a phrase that may help you to remember this order: **PEMDAS**
Parenthesis, Exponents, (Multiplication/Division), (Add/Subtract)





The reason (multiplication & division – MD) and (add & subtract – AS) are “grouped” in sets of parentheses is that when those operations are next to each other you do the math from **left to right.** You do not always do multiplication or addition first. It may be the case where division will be done BEFORE multiplication or subtraction will be done BEFORE addition.



Be very careful when listening to Aunt Sally!!!

If you forget to take MD and AS in order as you come to them from left to right, Aunt Sally's advice is toast!

Example:

$8 - 6 + 2$

$2 + 2$ **Subtraction is done first !**

4

Example: When there are two or more parenthesis, or grouping symbols, perform the inner most grouping symbol first.

$2 + 3[\ 5 + (4 - 1)^2]$

$2 + 3[\ 5 + (3)^2]$ inner most parentheses are done first

$2 + 3[\ 5 + 9]$ then work your way out

$2 + 3[\ 14]$

$2 + 42$

44

Example: Simplify: $2 + 6\ (3+1)^2$

It may be helpful to build a PEMDAS table. Check off the operation after it has been performed. For operations that are not part of the problem, place a hyphen.

P	E	M	D	A	S

1. Simplify any parenthesis first, starting with the inner most group, and check off the "P" box.
2. Simplify any powers (exponents) and check off the "E" box.
3. Perform the multiplication and division in order from left to right and check off the "M" & "D" boxes.
4. Do the addition and subtraction last. Remember, if the operations are written next to each other work from left to right and check off the last two boxes.

P	E	M	D	A	S

Draw a PEMDAS table.

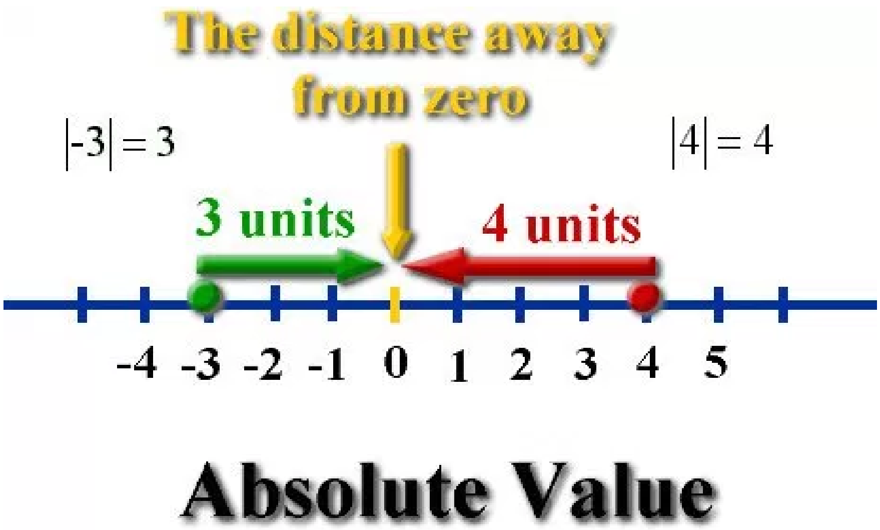
P	E	M	D	A	S	$2 + 6\ (3+1)^2$
x						
P	E	M	D	A	S	$2 + 6\ (4)^2$
x	x					
P	E	M	D	A	S	$2 + 6\ (16)$
x	x	x				
P	E	M	D	A	S	
x	x	x	-			$2 + 96$
P	E	M	D	A	S	
x	x	x	-	x	-	98

It is very important to understand that it DOES make a difference if the order is not performed correctly!!!

70 - 2(5+3)	70 - 2(5+3)
70 - 2(8)	70 - 2(8)
68(8)	70 - 16
544 **incorrect** (subtraction was done before multiplication)	54 **correct**

Absolute Value

The **absolute value** of a number can be considered as the **distance** between 0 and that number on the real number line.



Remember that distance is always a positive quantity (or zero).
The distance in the diagram above from +4 to 0 is 4 units and the distance from -3 to 0 is 3 units. These units are never negative values.

Using absolute value, we write this as:

$|4| = 4$ and $|-3| = 3$

The rule for computing absolute value is:

$$|a| = a \text{ if } a \geq 0$$

$$|a| = -a \text{ if } a < 0$$

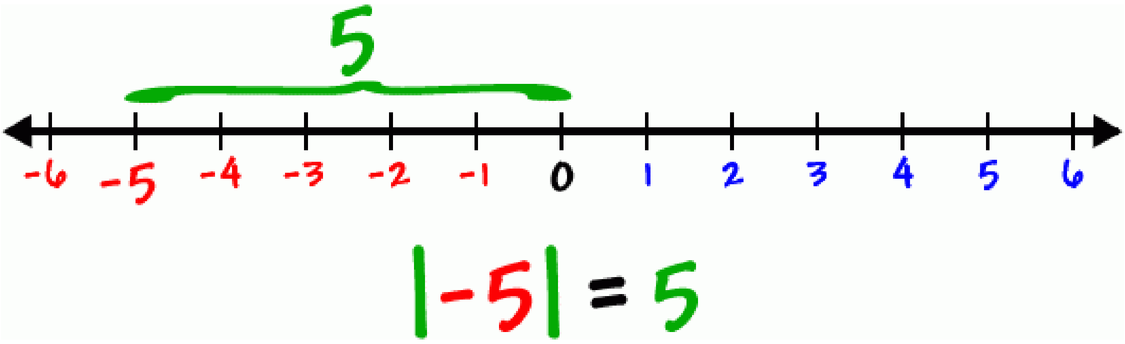
Examples:

$|25| = 25$

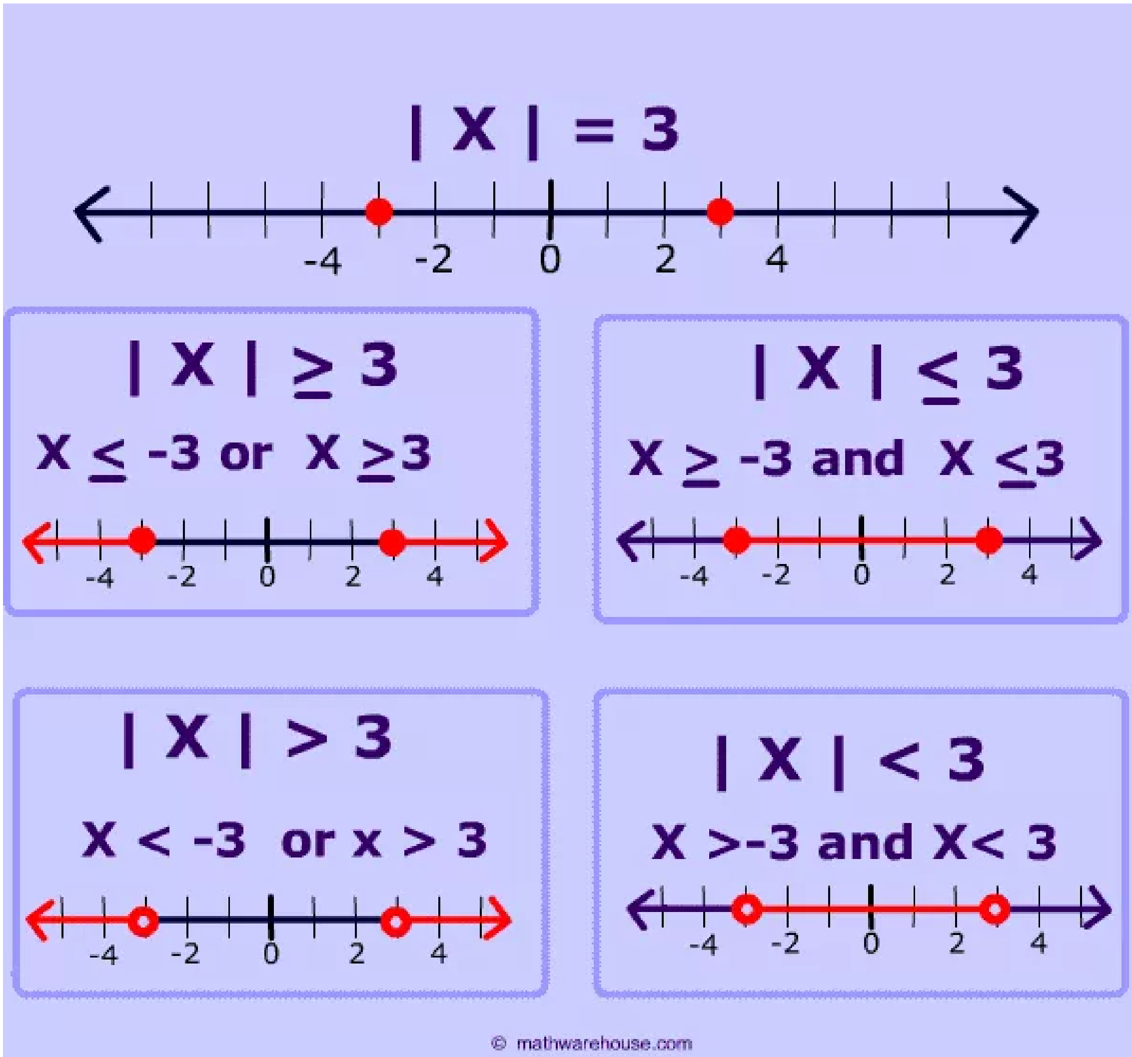
$|-25| = -(-25) = 25$

Absolute Value of an Integer:

The absolute value of an integer is the numerical value (magnitude) of an integer regardless of its sign (direction). It is denoted by the symbol **? ?.** The absolute value of an integer is either zero or positive. Also, the corresponding positive and negative integers have the same absolute value.
Examples:
The absolute value of -2 is ?-2? = 2.
The absolute value of 5 is ?5? = 5.
The absolute value of 0 is ?0? = 0



Absolute Value Inequality Graph



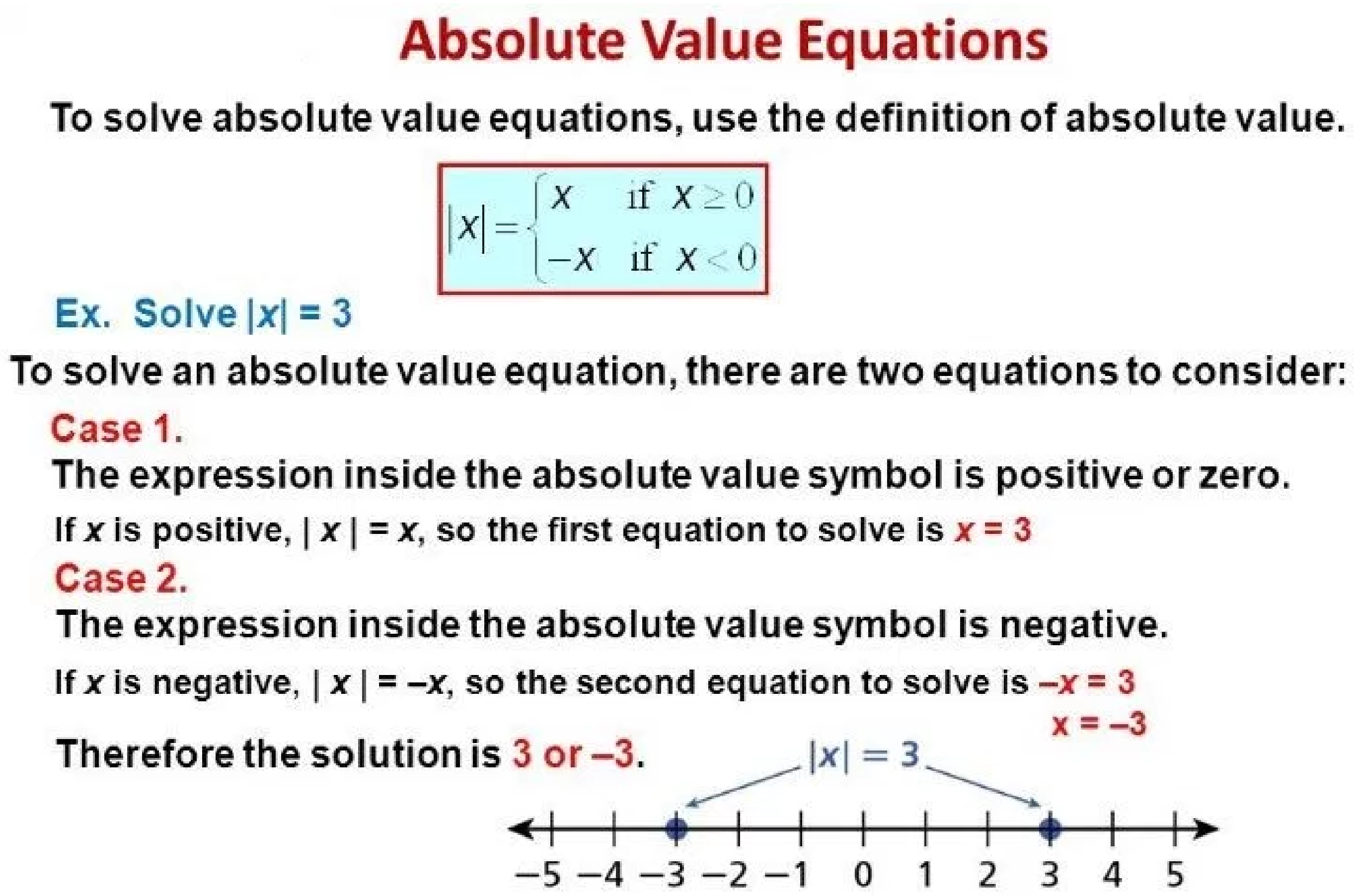
Absolute Value Equations

To solve an absolute value equation, isolate the absolute value on one side of the equal sign, and establish two cases:

Case 1:

|a| = b set a = b
Set the expression inside the absolute value symbol equal to the other given expression.

Case 2:
|a| = b set a = – b
Set the expression inside the absolute value symbol equal to the negation of the other given expression.



Always Check your answers.

The two cases create “derived” equations. These derived equations may not always be true equivalents to the original equation. Consequently, the roots of the derived equations MUST BE CHECKED in the original equation so that you do not list extraneous roots as answers.

Case 1: $x-10=6$ $x=16$	Case 2: $x-10=-6$ $x=4$	Answer: $x=16$, $x=4$
Check: $\lvert 16-10\rvert=6$ $\lvert 6\rvert=6$ $6=6$	*Check:* $\lvert 4-10\rvert=6$ $\lvert -6\rvert=6$ $6=6$	The solutions are 16 or 4. On a number line, these value are each 6 units away from 10.

Example 2: (No solution)

As soon as you isolate the absolute value expression, you observe:

There is no need to work out the two cases in this problem. Absolute value is NEVER equal to a negative value. This equation is never true. The answer is the empty set .

Example 3: $|3x+2| = 4x+5$ (Two cases with one solution)

Case 1:	Case 2:	
$\lvert 3x+2\rvert = 4x+5$ $3x+2 = 4x+5$ $2 = x+5$ $-3 = x$	$\lvert 3x+2\rvert = 4x+5$ $3x+2 = -(4x+5)$ $3x+2 = -4x-5$ $7x = -7$ $x = -1$	**Answer:** $x = -1$
Check: $\lvert 3(-3)+2\rvert = 4(-3)+5$ $\lvert -9+2\rvert = -12+5$ $\lvert -7\rvert = -7$ $7 \neq -7$ *Not an answer!*	*Check:* $\lvert 3(-1)+2\rvert = 4(-1)+5$ $\lvert -3+2\rvert = -4+5$ $\lvert -1\rvert = 1$ $1 = 1$	The check shows that $x = -3$ is NOT a solution to this absolute value equation. There is only ONE answer, $x = -1$. **Always check!!!!**

Example 4: A machine fills Quaker Oatmeal containers with 32 ounces of oatmeal. After the containers are filled, another machine weighs them. If the container's weight differs from the desired 32 ounce weight by more than 0.5 ounces, the container is rejected. Write an equation that can be used to find the heaviest and lightest acceptable weights for the Quaker Oatmeal container. Solve the equation.

Solution: Let x = the weight of the container $\lvert x-32\rvert = 0.5$ **Case 1:** $x-32 = 0.5$ $x = 32.5$ *Check:* $\lvert 32.5-32\rvert = 0.5$ $\lvert 0.5\rvert = 0.5$ $0.5 = 0.5$ **Case 2:** $x-32 = -0.5$ $x = 31.5$ *Check:* $\lvert 31.5-32\rvert = 0.5$ $\lvert -0.5\rvert = 0.5$ $0.5 = 0.5$ Answer: $x = 31.5$ ounces (lightest) $x = 32.5$ ounces (heaviest)	When setting up a word problem involving absolute value, remember that absolute value can represent "distance" from a given point. The difference between the answer (x) and the desired point (32) is placed under the absolute value symbol. This absolute value is then set equal to the desired "distance" (0.5).

Absolute Value Inequalities

Solving an absolute value inequality problem is similar to solving an absolute value equation.

Start by isolating the absolute value on one side of the inequality symbol, then follow the rules below:

If the symbol is > (or >=) : (or)
If a > 0, then the solutions to |x| > a
are x > a or x < − a.

If a < 0, all real numbers will satisfy . |x| > a

Think about it: absolute value is always positive (or zero), so, of course, it is greater than any negative number.

If the symbol is < (or <=) : (and)
If a > 0, then the solutions to |x| < a
are x < a and x > − a.
Also written: − a < x < a.

If a < 0, there is no solution to .|x| < a

Think about it: absolute value is always positive (or zero), so, of course, it cannot be less than a negative number.

Remember:

When working with any absolute value inequality,
you must create two cases.
If <, the connecting word is “and”.
If >, the connecting word is “or”.

To set up the two cases:

x < a
Case 1: Write the problem without the absolute value sign, and solve the inequality.

x > -a
Case 2: Write the problem without the absolute value sign, reverse the inequality, negate the value NOT under the absolute value, and solve the inequality.

Example 1: (solving with "greater than")

Solve: $|x - 20| > 5$

Case 1:		Case 2:
$x - 20 > 5$	or	$x - 20 < -5$
$x > 25$		$x < 15$

$x < 15$ or $x > 25$

Example 2: (solving with "less than or equal to"

Solve: $|x - 3| \le 4$

Case 1:		Case 2:
$x - 3 \le 4$	and	$x - 3 \ge -4$
$x \le 7$		$x \ge -1$

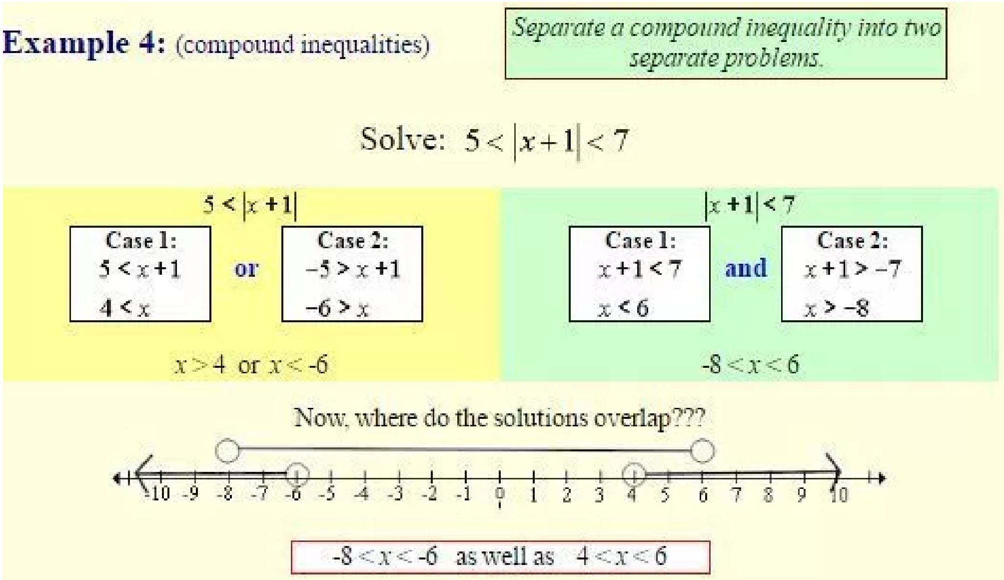
$x \ge -1$ and $x \le 7$
also written as:
$-1 \le x \le 7$

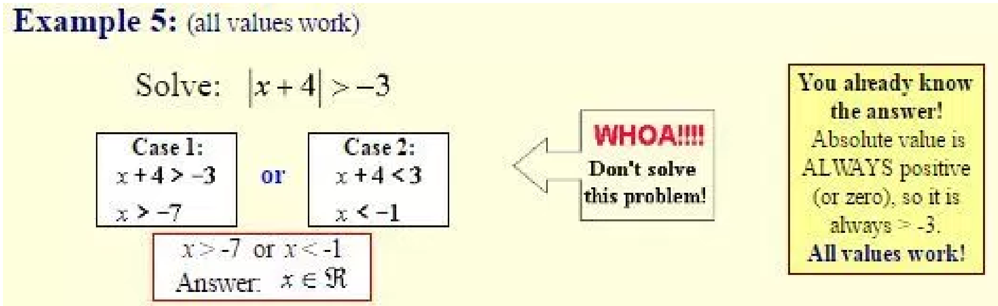
Example 3: (isolating the absolute value first)

Solve: $|3 + x| - 4 < 0$

Case 1:		Case 2:
$\lvert 3 + x \rvert < 4$		$\lvert 3 + x \rvert < 4$
$3 + x < 4$	and	$3 + x > -4$
$x < 1$		$x > -7$

$x < 1$ and $x > -7$
also written as:
$-7 < x < 1$





Absolute Value of Complex Numbers

Geometrically, the absolute value of a complex number is the number's distance from the origin in the complex plane.

The absolute value of a complex number

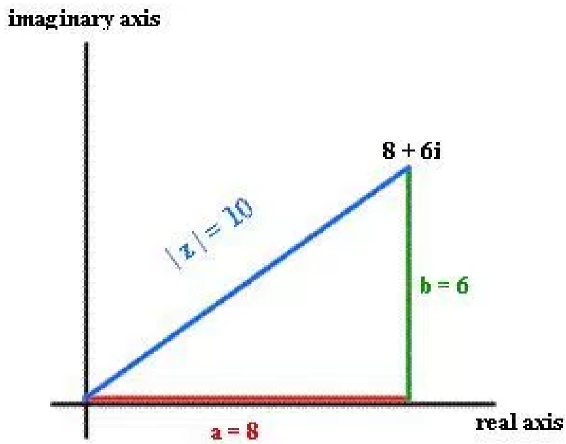
$z = a + bi$ is written as $|z|$.

It is a nonnegative real number defined as:

$$|z| = \sqrt{a^2 + b^2}$$

In the diagram at the left, the complex number 8 + 6i is plotted in the complex plane on an Argand diagram (where the vertical axis is the imaginary axis). For this problem, the distance from the point 8 + 6i to the origin is 10 units. Distance is a positive measure.

Notice the Pythagorean Theorem at work in this problem.



A complex number can be represented by a point, or by a vector from the origin to the point. When thinking of a complex number as a vector, the absolute value of the complex number is simply the length of the vector, called the magnitude.

The formula for finding the absolute value of a complex number,

$$|a+bi| = |z| = \sqrt{a^2+b^2}$$

can be derived from the Pythagorean theorem,

$$c^2 = a^2 + b^2$$ (see example 2 below).

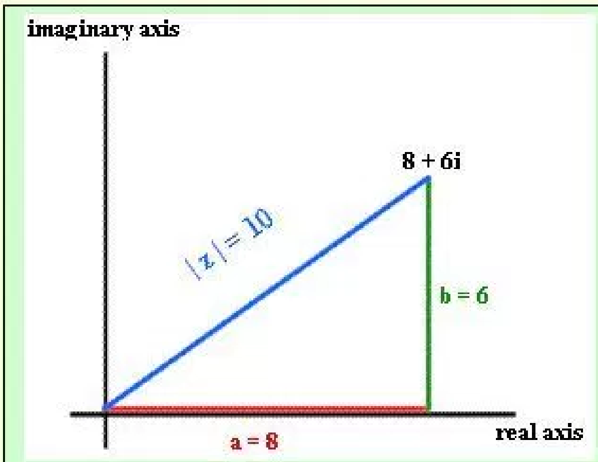
In the Pythagorean Theorem, c is the hypotenuse and when represented in the coordinate plane, is always positive. This same idea holds true for the distance from the origin in the complex plane. Using the absolute value in the formula will always yield a positive result.

To find the absolute value of a complex number a + bi:
1. Be sure your number is expressed in a + bi form
2. Pick out the coefficients for a and b
3. Substitute into the formula

$$|z| = \sqrt{a^2+b^2}$$

Example 1:

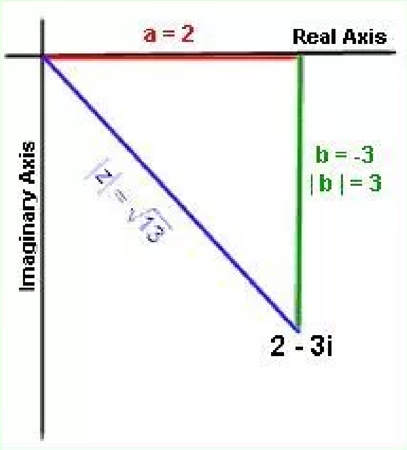
Plot z = 8 + 6i on the complex plane, connect the graph of z to the origin (see graph below), then find | z | by appropriate use of the definition of the absolute value of a complex number.



$$z = 8+6i$$
$$|z| = \sqrt{a^2+b^2}$$
$$|z| = \sqrt{8^2+6^2}$$
$$|z| = \sqrt{64+36}$$
$$|z| = \sqrt{100}$$
$$|z| = 10$$

Example 2:

Find the | z | by appropriate use of the Pythagorean Theorem when z = 2 – 3i.



You can find the distance | z | by using the Pythagorean theorem. Consider the graph of **2 - 3*i*** shown at the left. The horizontal side of the triangle has length | a |, the vertical side has length | b |, and the hypotenuse has length | z |. By applying the Pythagorean Theorem, you have, $|z|^2 = a^2 + b^2$.

Notice: you can drop the absolute value symbols for a and b since $|a|^2 = a^2$ and $|b|^2 = b^2$. You must keep the absolute value symbol for z to insure that the final answer will be positive.

Solving this equation for | z |, you have just derived the formula for the absolute value of a complex number:

$$|z| = \sqrt{a^2 + b^2}$$

$$|z| = \sqrt{2^2 + (-3)^2}$$

$$|z| = \sqrt{4 + 9}$$

$$|z| = \sqrt{13}$$

Example 3:

If z = – 8 – 15i, find | z |.

$$z = -8 - 15i$$

$$|z| = \sqrt{a^2 + b^2}$$

$$|z| = \sqrt{(-8)^2 + (-15)^2}$$

$$|z| = \sqrt{64 + 225}$$

$$|z| = \sqrt{289}$$

$$|z| = 17$$