Sample Question Paper - 14
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. **[2]**

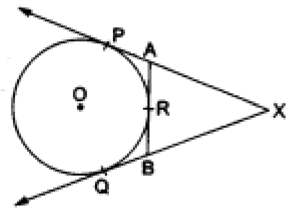
OR

In an A.P, if S_5+ S_7 = 167 and S_{10} = 235, then find the A.P., where Sndenotes the sum of first n terms.

2. Solve the quadratic equation by factorization: **[2]**

$\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

3. In the given figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that XA +AR = XB + BR. **[2]**



4. If a metallic cube of edge 1 cm is drawn into a wire of diameter 4 mm, then find the length of the wire. **[2]**

5. Candidates of four schools appear in a mathematics test. The data were as follow: **[2]**

Schools	**No. of Candidates**	**Average Score**
I	60	75
II	48	80
III	Not available	55
IV	40	50

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

6. Find whether the equation has real roots. If real roots exist, find them: $8x^2 + 2x - 3 = 0$ **[2]**

OR

If 2 is a root of the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q.

Section B

7. Find median for the following data: **[3]**

Marks	**Number of students**
More than 150	Nil
More than 140	12
More than 130	27
More than 120	60
More than 110	105
More than 100	124
More than 90	141
More than 80	150

8. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle. **[3]**

9. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows: **[3]**

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

10. A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point B which is at 'b' meters above A is β. **[3]**
Prove that the height of the tower is b tan α cot β.

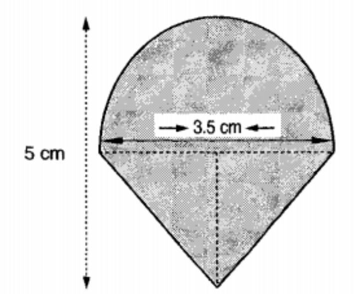
OR

The angle of elevation of an aeroplane from a point on the ground is 45^o. After flying for 15 s, the angle of elevation changes to 30^o. If the aeroplane if flying at a constant height of 2500 m, then find the average speed of the aeroplane.

Section C

11. Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. find the **[4]**

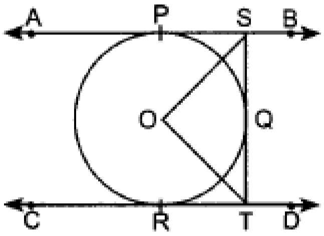
area he has to colour. (Take π = 22/7).



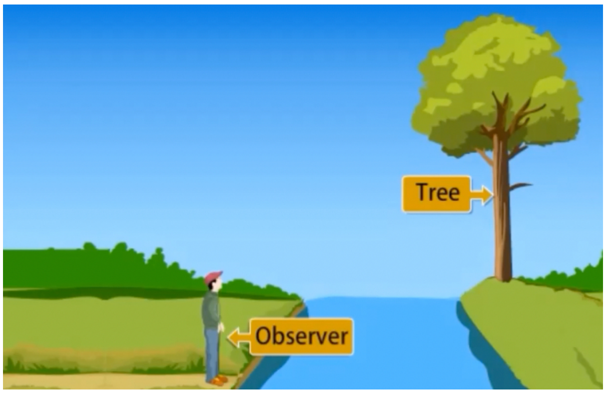
12. O is the centre of a circle. PA and PB are tangents to touch the circle from a point P. Prove that (i) quadrilateral PAOB is a cyclic quadrilateral (ii) PO is the bisector of $\angle$APB (iii) $\angle$OAB = $\angle$OPA. **[4]**

OR

In figure AB and CD are two parallel tangents to a circle with centre O. ST is tangent segment between the two parallel tangents touching the circle at Q. Show that $\angle$SOT = 90^o



13. Akshat is a class 10 student. He went to his grandparent's home in a village. His grandfather took him to the bank of a nearby river. Akshat was very happy to see the pollution-free village environment. He was standing on the bank of the river observed that the angle of elevation of the top of a tree standing on the opposite bank was 60°. When he moved 30 metres away from the bank, he found the angle of elevation to be 30°. **[4]**



i. Find the height of the tree.
ii. Find the width of the river. [Take $\sqrt{3}$ = 1.732.]

14. Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves Rs. 27.5 On each succeeding day he increases his saving by Rs. 2.5. **[4]**

Find

i. the amount saved by Sehaj on 10th day,
ii. the amount saved by Sehaj on 25th day, and the total amount saved by Sehaj in 30 days.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section A

1. We have,
   $a_4 = 0$
   $a + 3d = 0$
   $3d = -a$
   or $-3d = a$..........(i)
   Now,
   $a_{25} = a + 24d$
   $= -3d + 24d$ [Putting value of a from eq(i)]
   $= 21d$..........(ii)
   $a_{11} = a + 10d$
   $= -3d + 10d$
   $= 7d$.........(iii)
   From eq(ii) and (iii), we get
   $a_{25} = 21$ d
   $a_{25} = 3(7d)$
   $a_{25} = 3a_{11}$
   Hence Proved

   OR

   $S_n = \frac{n}{2}[2a + (n - 1)d$
   $S_5 + S_7 = 167$
   or, $\frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$
   or, $\frac{1}{2}$ (10a + 20d + 14a + 42d) = 167
   or, 24a + 62d = 334
   or 12a + 31d = 167.......(i)
   $S_{10} = 235$
   or, 5(2a + 9d) = 235
   or 2a + 9d = 47 ...(ii)
   multiply (ii) by 6 and subtract from (i)
   12a+ 31d - (12a + 54d) = 167 - 282
   12a + 31d - 12a - 54d = -115
   -23d = -115
   d = 5
   Put d= 5 in (ii)
   2a + 9d = 47
   2a + 9(5) = 47
   2a + 45 = 47
   2a = 2
   a = 1
   So, a = 1, d = 5
   Here A.P. = 1,6,11,...
2. According to the question,
   $\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$
   $\Rightarrow \frac{m}{n}x^2 + 2x + \frac{n}{m} - 1 = 0$
   $\Rightarrow x^2 + \frac{2nx}{m} + \frac{n^2}{m^2} - \frac{n}{m} = 0$ [multiplying both sides by 'n' and dividing both sides by 'm']
   $\Rightarrow x^2 + \frac{2nx}{m} + \frac{n^2 - mn}{m^2} = 0$

To factorize $x^2+\frac{2nx}{m}+\frac{n^2-mn}{m^2}$, we have to find two numbers $'a'\ and\ 'b'$ such that.

$a+b=\frac{2n}{m}$ and $ab=\frac{n^2-mn}{m^2}$

Clearly, $\frac{n+\sqrt{mn}}{m}+\frac{n-\sqrt{mn}}{m}=\frac{2n}{m}$ and $\frac{(n+\sqrt{mn})}{m}\times\frac{(n-\sqrt{mn})}{m}=\frac{n^2-mn}{m^2}$ $(\because a=\frac{n+\sqrt{mn}}{m}$ and $b=\frac{n-\sqrt{mn}}{m})$

$\Rightarrow x^2+\frac{2nx}{m}+\frac{n^2-mn}{m^2}=0$

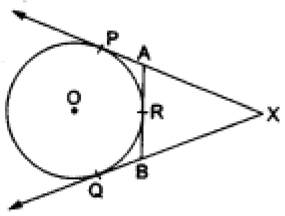
$\Rightarrow x^2+\frac{(n+\sqrt{mn})}{m}x+\frac{(n-\sqrt{mn})}{m}x+\frac{n^2-mn}{m^2}=0$

$\Rightarrow x\left[x+\frac{n+\sqrt{mn}}{m}\right]+\frac{n-\sqrt{mn}}{m}\left[x+\frac{n+\sqrt{mn}}{m}\right]=0$

$\Rightarrow\left(x+\frac{n-\sqrt{mn}}{m}\right)\left(x+\frac{n+\sqrt{mn}}{m}\right)=0$

$\Rightarrow x+\frac{n-\sqrt{mn}}{m}=0$ or $x+\frac{n+\sqrt{mn}}{m}=0$

$\Rightarrow x=\frac{-n-\sqrt{mn}}{m}$ or $x=\frac{-n+\sqrt{mn}}{m}$

3. 

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

XP = XQ, ...(i) [tangents from X]

AP = AR, ... (ii) [tangents from A]

BR = BQ. ... (in) [tangents from B]

Now, XP = XQ $\Rightarrow$ XA + AP = XB + BQ

XA + AR = XB + BR [using (ii) and (iii)]

4. a = 1 cm, d = 4 mm = 0.4 cm, r = 0.2 cm

Volume of cube = volume of wire or cylinder

$a^3 = \pi r^2 h$

$a^3 = \frac{22}{7}\times 0.2\times 0.2\times h$

$h=\frac{7\times 1^3}{22\times 0.2\times 0.2}$

h = 7.95

or h = 8 cm (approx)

So length of wire is 8 cm.

5. Let the number of candidates from school III = P

Schools	**No. of candidates N_i**	**Average scores (x_i)**
I	60	75
II	48	80
III	P	55
IV	40	50

Given

Average score for all schools = 66

$\frac{N_1\overline{x_1}+N_2\overline{x_2}+N_3\overline{x_3}+N_4\overline{x_4}}{N_1+N_2+N_3+N_4}$ = 66

$\frac{4500+3840+55p+2000}{60+48+p+40}$ = 66

$\Rightarrow$ 4500 + 3840 + 55p + 2000 = 66 (60 + 48 + p + 40)

$\Rightarrow$ 10340 + 55p = 66p + 9768

$\Rightarrow$ 10340 – 9768 = (66 – 55)p

$\Rightarrow$ P = $\frac{572}{11}$

$\Rightarrow$ P = 52

6. For real roots of quadratic equation, $b^2 - 4ac > 0$

We have, $8x^2 + 2x - 3 = 0$

$b^2 - 4ac > 0$

$\Rightarrow (2)^2 - 4(8)(-3) > 0$ (using: a = 8, b = 2, c = -3)

$\Rightarrow 4 + 96 > 0 \Rightarrow 100 > 0$

As D > 0, so roots are real.

Now, Discriminant $\sqrt{D} = 10$

So, roots are $x = \frac{-b\pm\sqrt{D}}{2a} = \frac{-2\pm10}{2\times8} = \frac{-2\pm10}{16}$

$\Rightarrow x_1 = \frac{-2+10}{16}$ and $x_2 = \frac{-2-10}{16}$

$\Rightarrow x_1 = \frac{8}{16}$ and $x_2 = \frac{-12}{16}$

$\Rightarrow x_1 = \frac{1}{2}$ and $x_2 = \frac{-3}{4}$

So, the roots of the given equation are $\frac{1}{2}$ and $\frac{-3}{4}$.

OR

we are given that 2 is a root of the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q.

If 2 is the root of $x^2 + kx + 12 = 0$,then

$(2)^2 + 2k + 12 = 0$

or, 2k + 16 = 0

k = - 8

Put k = -8, in $x^2 + kx + q = 0$,we get

$x^2 - 8x + q = 0$

For equal roots

$(-8)^2 - 4(1)q = 0$

64 -4q = 0

4q = 64

q = 16

Section B

7.

C.I.	**f**	**c.f.**
80 - 90	9	9
90 - 100	17	26
100 - 110	19	45
110 - 120	45	90
120 - 130	33	123
130 - 140	15	138
140 - 150	12	150

n = 150 $\Rightarrow \frac{n}{2}$ = 50

Median Class = 110 - 120

l = 110, f = 45, c.f. = 45, h = 10

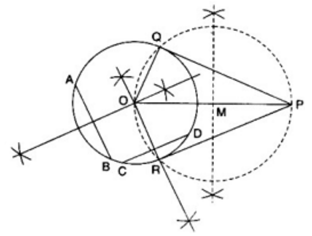
we know that, Median = l + $\frac{\frac{n}{2}-cf}{f} \times$ h

= 110 + $\frac{75-45}{45} \times$ 10

= 116.67

8. Steps of construction:
   i. Draw a circle with the help of a bangle.
   ii. Take two non-parallel chords AB and CD of this circle.
   iii. Draw the perpendicular bisectors of AB and CD. Let these intersect at O.
   Then O is the centre of this circle drawn.

iv. Take a point P outside the circle.
v. Join PO and bisect it. Let M be the mid-point of PO.



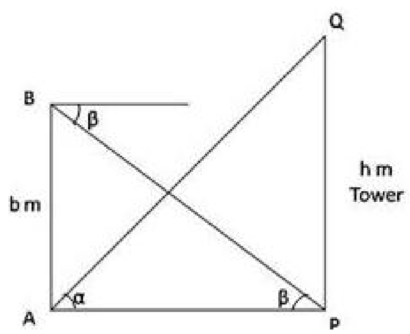
vi. Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at the points Q and R.
vii. Join PQ and PR.
Then, PQ and PR are the required two tangents.
Then $\angle PQO$ is an angle in the semicircle and therefore,
$\angle PQO = 90^\circ$
$\Rightarrow PQ \perp OQ$
Since OQ is a radius of the given circle, PQ has to be a tangent to the circle.
Similarly, PR is also a tangent to the circle.

9. First, we will convert the graph given into tabular form as shown below:

Class interval	**Frequency (f_i)**	**Mid value (x_i)**	**f_ix_i**	**Cumulative Frequency**
1 – 4	6	2.5	15	6
4 – 7	30	5.5	165	36
7 – 10	40	8.5	340	76
10 – 13	16	11.5	184	92
13 – 16	4	14.5	58	96
16 – 19	4	17.5	70	100
	N = $\sum f_i$ = 100		Σf_ix_i = 832	

i. N = 100
Mean = $\frac{\Sigma f_ix_i}{N} = \frac{832}{100}$ = 8.32
ii. $\frac{N}{2} = \frac{100}{2}$ = 50
The cumulative frequency just greater than $\frac{N}{2}$ is 76, then the median class is 7 - 10 such that
l = 7, h = 10 - 7 = 3, f = 40, F = 36
Median = l + $\frac{\frac{N}{2}-F}{f} \times$ h
= 7 + $\frac{50-36}{40} \times$ 3
$= 7 + \frac{42}{40}$ = 7 + 1.05 = 8.05
iii. Mode = 3 Median - 2 Mean
= 3 $\times$ 8.05 - 2 $\times$ 8.32 = 7.51

10.

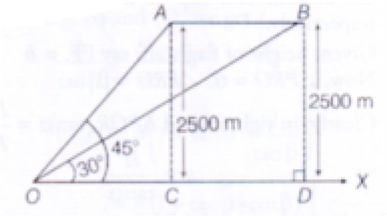


Let height of tower = QP = h m
In ΔBAP
$\tan\beta = \frac{BA}{AP}$

$\Rightarrow \tan\beta = \frac{b}{AP}$
$\Rightarrow AP = \frac{b}{\tan\beta}$
$\Rightarrow AP = b \times \cot\beta$ .... (i)
In ΔQPA
$\tan\alpha = \frac{QP}{AP}$
$\Rightarrow QP = AP \times \tan\alpha$
$\Rightarrow QP = b\cot\beta \times \tan\alpha$From (i)

OR

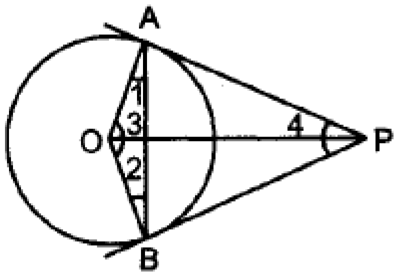
Let OX be the horizontal ground; A and B be the two positions of the plane and O be the point of observation.



Here, AC = BD = 2500 m,
$\angle AOC = 45^o\ and\ \angle BOD = 30^o$
In right angled $\triangle OCA,\ \cot 45^o = \frac{B}{P} = \frac{OC}{AC}$
$\Rightarrow 1 = \frac{OC}{AC}$
$\Rightarrow OC = AC = 2500\ m$
In right angled $\triangle ODB,\ \cot 30^o = \frac{B}{P} = \frac{OD}{BD}$
$\Rightarrow \sqrt{3} = \frac{OD}{2500}$
$\Rightarrow OD = 2500\sqrt{3}\ m$
Now, CD = OD - OC $= 2500\sqrt{3} - 2500$
$\Rightarrow$ CD = 2500(1.732 - 1)
$\Rightarrow$ CD = 2500(0.732)
$\Rightarrow$ CD = 1830 m
Thus, distance covered by plane in 15 s is 1830 m.
$\therefore$ Speed of the plane= $\frac{Distance}{Time}$ = $\frac{1830}{15} \times \frac{60\times60}{1000} = 439.2\ km/h$

Section C

11. Surface area to colour = surface area of hemisphere + curved surface area of cone
Diameter of hemisphere = 3.5 cm
So radius of hemispherical portion of the lattu = r = $\frac{3.5}{2}$cm = 1.75
r = Radius of the concial portion = $\frac{3.5}{2}$ = 1.75
Height of the conical portion = height of top - radius of hemisphere = 5 - 1.75 = 3.25 cm
Let I be the slant height of the conical part. Then,
$l^2 = h^2 + r^2$
$l^2 = (3.25)^2 + (1.75)^2$
$\Rightarrow l^2 = 10.5625 + 3.0625$
$\Rightarrow l^2 = 13.625$
$\Rightarrow l = \sqrt{13.625}$
$\Rightarrow l = 3.69$
Let S be the total surface area of the top. Then,
$S = 2\pi r^2 + \pi rl$
$\Rightarrow \quad S = \pi r(2r + l)$
$\Rightarrow S = \frac{22}{7} \times 1.75(2 \times 1.75 + 3.7)$
$= 5.5(3.5 + 3.7)$
$= 5.5(7.2)$
$= 39.6\ cm^2$

12. 

Given,O is the centre of a circle. PA and PB are tangents to touch the circle from a point P.

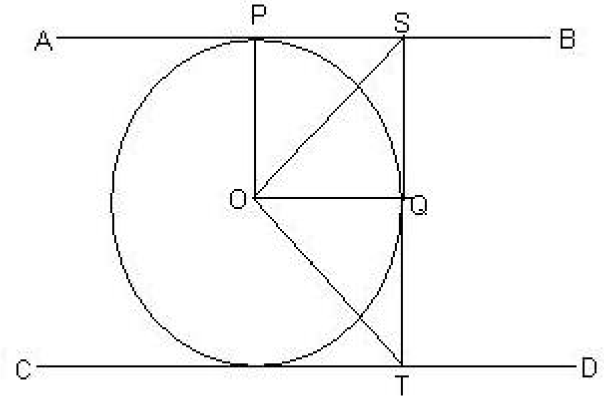
i. PA and PB are tangents to the circle at the points A and B respectively.
$\Rightarrow \angle \mathrm{OAP} = \angle \mathrm{OBP} = 90^\circ$ ..(i)
$\Rightarrow \angle \mathrm{OAP} + \angle \mathrm{OBP} = 90^\circ + 90^\circ = 180^\circ$...(ii)
In quadrilateral OAPB,
$\angle \mathrm{AOB} + \angle \mathrm{OAP} + \angle \mathrm{OBP} + \angle \mathrm{APB} = 360^\circ$[Angle sum property of a quadrilateral]
$\Rightarrow \angle AOB + \angle APB + 180^\circ = 360^\circ$ [From (ii)]
$\Rightarrow \angle APB + \angle AOB$
= 360°-180° = 180° ...(iii)
From (ii) and (iii), we have
$\angle OAP + \angle OBP = 180^\circ$
and $\angle APB + \angle AOB = 180^\circ$

ii. In $\triangle$OAP and $\triangle$OBP, r
AP = BP [Tangents from the same external point are equal]
OP = OP [Common]
OA = OB [Radii of the same circle]
$\Rightarrow \Delta \mathrm{OAP} \cong \triangle \mathrm{OBP}$ [Using SSS cong.]
$\Rightarrow \angle APO = \angle BPO$ [CPCT]
$\Rightarrow$ PO bisects $\angle \mathrm{APB}$

iii. $\angle 3 + \angle 4 = 180^\circ$
$\angle 1 + \angle 2 + \angle 3 = 180^\circ$[Angle sum property] ...(iv)
In $\triangle$OAB,
OA =OB [Radii]
$\angle 1 = \angle 2$[Angles opposite to equal sides of a A are equal] ...(v)
From (iii) and (iv),
$\angle 3 + \angle 4 = \angle 1 + \angle 2 + \angle 3$
$\Rightarrow \angle 4 = \angle 1 + \angle 2$
$= \angle 1 + \angle 1$
$= \angle 2 + \angle 2$
$\Rightarrow \angle 4 = 2\angle 1$
$= 2\angle 2$
$\Rightarrow \frac{1}{2}\angle 4 = \angle 1$
$\Rightarrow \angle \mathrm{OPA} = \angle \mathrm{OAB}$
Hence proved

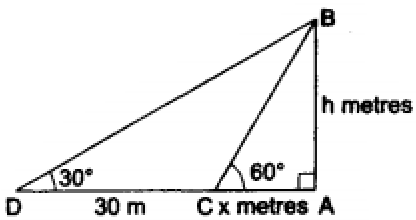
OR

Given, AB and CD are two parallel tangents to a circle with centre O.



From the figure we get,
AB⊥ST then $\angle$ASQ = 90° and
CD⊥TS then $\angle$CTQ = 90°
$\angle$ASO = $\angle$QSO = $\frac{90^\circ}{2}$ = 45°
Similarly, $\angle$OTQ = 45°
Consider ΔSOT,
$\angle$OTS = 45° and $\angle$OST = 45°
$\angle$SOT + $\angle$OTS + $\angle$OST = 180° (angle sum property)
$\angle$SOT = 180° - ($\angle$OTS + $\angle$OST) = 180° - (45° +45°)
= 180° - 90° = 90°
$\therefore \angle$SOT = 90^0

13. Let AB be the tree of height h m and AC be the river.
Let C be the position of a man standing on the opposite bank of the river . After moving 30 m away from point C. Let new position of man be D i.e CD = 30m
Then, $\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$, $\angle DAB = 90^\circ$ and CD = 30m.
Let AB = h metres and AC = x metres.



From right ΔCAB, we have
$\frac{AC}{AB} = \cot 60^\circ = \frac{1}{\sqrt{3}}$
$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{h}{\sqrt{3}}$...(i)
From right ΔDAB, we have
$\frac{AD}{AB} = \cot 30^\circ = \sqrt{3}$
$\Rightarrow \frac{x+30}{h} = \sqrt{3} \Rightarrow x = \sqrt{3}h - 30$....(ii)
Equating the values of x from (i) and (ii), we get
$\frac{h}{\sqrt{3}} = \sqrt{3}h - 30 \Rightarrow h = 3h - 30\sqrt{3}$
$\Rightarrow 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3} = 15 \times 1.732 = 25.98$
Putting $h = 15\sqrt{3}$ in (i), we get $x = \frac{15\sqrt{3}}{\sqrt{3}} = 15.$
Hence, the height of the tree is 25.98m and the width of the river is 15 metres.

14. i. Money saved on 1st day = Rs. 27.5
$\because$ Sehaj increases his saving by a fixed amount of Rs. 2.5
$\therefore$ His saving form an AP with a = 27.5 and d = 2.5
$\therefore$ Money saved on 10th day,

a_{10} = a + 9d = 27.5 + 9(2.5)

= 27.5 + 22.5 = Rs. 50

ii. a_{25} = a = 24d

= 27.5 + 24(2.5)

= 27.5 + 60 = Rs. 87.5

iii. Total amount saved by Sehaj in 30 days.

= $\frac{30}{2}[2 \times 27.5 + (30 - 1) \times 2.5]$

= 15(55 + 29(2.5)

= Rs. 1912.5