CBSE Test Paper 03

Chapter 12 Area Related to Circle

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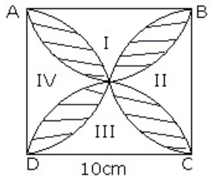
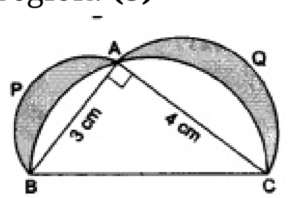
1. If the radius of the circle is $\frac{7}{\sqrt{\pi}}$ cm, then its area is **(1)**
   a. 98sq. cm
   b. 45sq. cm
   c. 22 sq. cm
   d. 49 sq. cm
2. If the circumference of a circle and the perimeter of a square are equal, then **(1)**
   a. Area of the circle = $\frac{1}{2}$ Area of the square
   b. area of the circle > area of the square
   c. area of the circle < area of the square
   d. area of the circle = area of the square
3. If a chord subtends an angle of 60^0 at the centre, then the area of the corresponding segment is **(1)**
   a. $\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right) r^2\ sq.units$
   b. $\left(\frac{\pi}{2}-\frac{\sqrt{3}}{2}\right) r^2\ sq.units$
   c. $\left(\frac{\pi}{6}+\frac{\sqrt{3}}{2}\right) r^2\ sq.units$
   d. $\left(\frac{\pi}{2}+\frac{\sqrt{3}}{2}\right) r^2\ sq.units$
4. The length of the minute hand of a clock is 14cm. The area swept by the minute hand in 1 hour is **(1)**
   a. 616 sq. cm
   b. 516 sq. cm
   c. 628 sq. cm
   d. 542 sq. cm
5. A horse is tied to a peg at one corner of a square-shaped gross field of side 25 m by means of a 14m long rope. The area of that part of the field in which the horse can graze is **(1)**
   a. 156 sq. cm
   b. 142 sq. cm
   c. 102 sq. cm

d. 128 sq. cm

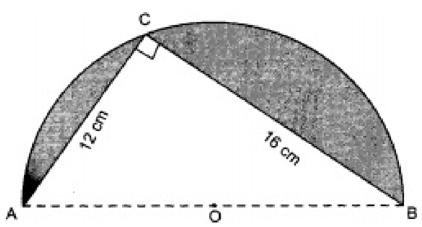
6. What is the perimeter of a sector of angle 45° of a circle with radius 7 cm? **(1)**
7. Radius of a circle is 1m. If diameter is increased by 100% then find the percentage increase in its area. **(1)**
8. If the perimeter of a semicircular protactor is 66 cm, find the radius of the protactor. **(1)**
9. If the circumference is numerically equal to 3 times the area of a circle, then find the radius of the circle. **(1)**
10. The area of two concentric circles forming a ring are 154 cm^2 and 616 cm^2. Find the breadth of the ring. **(1)**
11. The area of a sector of a circle of radius 2 cm is π cm^2. Find the angle contained by the sector. **(2)**
12. Find the area of the shaded region in the given figure where ABCD is a square of side 10cm and semi-circles are drawn with each side of the square as diameter. $[\pi = 3.14]$ **(2)**



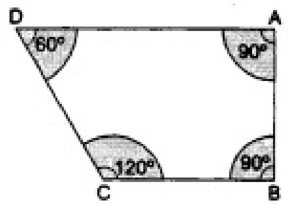
13. Find the area of a quadrant of a circle , whose circumference is 22 cm. **(2)**
14. In the given figure, $\triangle ABC$ is right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. It is given that AB = 3 cm and AC = 4 cm. Find the area of the shaded region. **(3)**



15. In figure, O is the centre of a circular arc and AOB is a straight line. Find the perimeter and the area of the shaded region correct to one decimal place. (Take π = 3.142) **(3)**

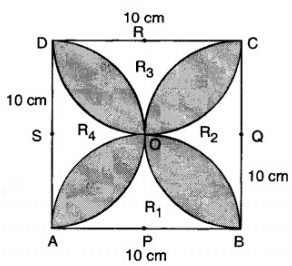


16. ABCD is a field in the shape of a trapezium, AD II BC, $\angle ABC$ = 90° and $\angle ADC$ = 60°. Four sectors are formed with centres A, B, C and D, as shown in the figure. The radius of each sector is 14 m. Find the following: **(3)**



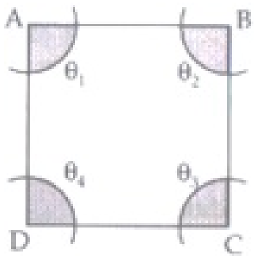
i. total area of the four sectors,
ii. area of the remaining portion, given that AD = 55 m, BC = 45 m and AB = 30 m

17. In Fig. ABCD is a square of side 10 cm. Semi-circles are drawn with each side of square as diameter. Find the area of



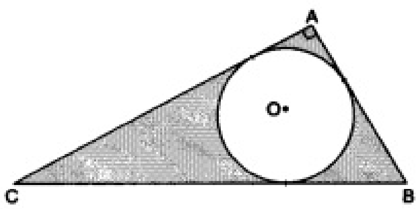
i. the unshaded region (R_1, R_2, R_3, R_4)
ii. the shaded region **(3)**

18. In the given figure, arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centre. Find the area of shaded region. **(4)**



19. A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions. **(4)**

20. In the given figure, $\triangle ABC$ is right-angled at A. Find the area of the shaded region if AB = 6 cm, BC = 10 cm and O is the centre of the incircle of ΔABC. [Take π= 3.14.] **(4)**



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Solution

1. d. 49 sq. cm

   Explanation: Area of the circle = πr^2

   $\Rightarrow$ Area of the circle = $\pi\left(\frac{7}{\sqrt{\pi}}\right)^2$ = $\pi \times \frac{49}{\pi}$ = 49 sq. cm

2. b. area of the circle > area of the square

   Explanation: Let the radius of the circle be r and side of the square be a.

   Then, according to the question,

   $2\pi r = 4a$

   $\Rightarrow a = \frac{2\pi r}{4} = \frac{\pi r}{2}$ ....(i)

   Now, the ratio of their areas,

   πr^2 and a^2

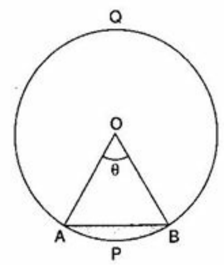
   $\Rightarrow \pi r^2 \text{and} \left(\frac{\pi r}{2}\right)^2$ [From eq. (i)]

   $\Rightarrow \pi r^2$ and $\frac{\pi^2 r^2}{4}$

   Therefore, Area of the circle > Area of the square

3. a. $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) r^2 \ sq. units$

   Explanation:



   Area of segment = $\frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

   $\Rightarrow$ Area of segment = $\frac{60^\circ}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin 60^\circ$

   $\Rightarrow$ Area of segment = $\frac{\pi r^2}{6} - \frac{r^2}{2} \times \frac{\sqrt{3}}{2}$

   $\Rightarrow$ Area of segment = $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) r^2$ sq. units

4. a. 616 sq. cm

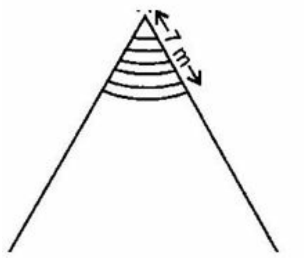
   Explanation: $\because$ Angle describe by minute hand in 1 hour = 360°

$\therefore \theta = 360^\circ$

$\therefore$ Area of the sector = $\frac{\theta}{360^\circ} \times \pi r^2$ = $\frac{360^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$ = 616 sq. cm

5. c. 102 sq. cm

Explanation:



Area of the shaded region = $\frac{\theta}{360^\circ} \times \pi r^2$

$\Rightarrow$ Area of the shaded region = $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$\Rightarrow$ Area of the shaded region = 102 sq. cm

6. Given radius r of circle = 7 cm.

l = length of the arc = $\frac{\theta \pi r}{180^\circ}$

$= \frac{45^\circ}{180^\circ} \times \frac{22}{7} \times 7 = \frac{11}{2}\text{cm}$

Hence, perimeter of sector = (r + r+ l)

$= \left(7 + 7 + \frac{11}{2}\right)$ cm

$= \left(14 + \frac{11}{2}\right)$ cm $= \left(\frac{28+11}{2}\right)$ cm

$= \frac{39}{2}\text{cm}$= 19.5 cm

7. Area of circle = π m^2

New diameter = 2 m + $\frac{100}{100} \times 2\text{m}$= 4 m

New radius = 2 m

New area = 4π m^2

Increase in area = 4π - π = 3π m^2

% increase in area = $\frac{3\pi}{\pi} \times 100$= 300%

8. Let radius of the protractor be r cm.

Perimeter = $\left[\frac{1}{2} \times 2\pi r + 2r\right]$

66 cm = $\left[\pi r + 2r\right]$

$\Rightarrow \quad 66 = (\pi + 2)r$

$\Rightarrow \quad 66 = \left(\frac{22}{7} + 2\right) r = \left(\frac{22+14}{7}\right) r$

$= \left(\frac{36}{7}\right)$r

$\Rightarrow \quad \frac{7 \times 66}{36} = r \Rightarrow r = \frac{77}{6}$cm

9. Let $radius = r\ units$

   According to the question,

   $2\pi r = 3\pi r^2$

   $\Rightarrow r = \frac{2}{3}$ units

10. Area of bigger circle = πR^2 = 616

    $\Rightarrow$ R = 14 cm

    and area of smaller circle = πr^2 = 154

    $\Rightarrow$ r = 7 cm

    Breadth of the ring = 14 - 7 = 7 cm

11. Radius of circle = 2cm

    Area of sector $= \pi cm^2$

    $\therefore$ Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

    $\Rightarrow \pi = \frac{\theta}{360^\circ} \times \pi \times 2 \times 2$

    $\Rightarrow \theta = \frac{\pi \times 360^\circ}{\pi \times 2 \times 2} = 90^\circ$

12. Area of region I + II = area of ABCD - area of 2 semicircles of each radius 5cm

    $= 10 \times 10 - 2 \times \frac{1}{2} \times \pi \times 5^2$

    $= 100 - 25\pi = 100 - 25 \times 3.14$

    $= 21.5\text{cm}^2$

    Similarly, the area of III + area of IV = 21.5cm^2

    Area of the region I, II, III, and IV $= 2 \times 21.5 = 43\text{cm}^2$

    Thus, the area of shaded region = Area ABCD - Area of (I, II, III, IV)

    $= 100 - 43 = 57\text{cm}^2$

13. Given , Circumference = 22 cm

    $\Rightarrow 2\pi r = 22$

    $\Rightarrow r = \frac{7}{2} = 3.5cm$

    Area of Circle = πr^2 = $\frac{22}{7} \times (3.5)^2 = 38.5cm^2$

Area of quadrant of circle = $\frac{Area\ of\ circle}{4}$
$= \frac{38.5}{4} = 9.625cm^2$
∴Area of the quadrant of cicle = 9.625 cm^2

14. In triangle ABC, by pythagoras theorm,we have,
$BC = \sqrt{AB^2 + AC^2}$
$BC = \sqrt{9 + 16}$
$BC = \sqrt{25}$
= 5 cm
Ar(shaded part) = Ar($\triangle ABC$) + Ar(semicircle APB) + Ar.(semicircle AQC) - Ar.(semicircle BAC)
$= \left(\frac{1}{2} \times 3 \times 4\right) + \left(\frac{1}{2}\pi \times 1.5 \times 1.5\right) + \left(\frac{1}{2}\pi \times 2 \times 2\right) - \left(\frac{1}{2}\pi \times 2.5 \times 2.5\right)$
$= 6 + \frac{1}{2}\pi\left(4 + \frac{9}{4} - \frac{25}{4}\right)$
= 6 + 0
= 6 cm^2

15. AC = 12 cm, BC = 16 cm
In $\triangle$ACB, by ythagoras theorem
$AB^2 = AC^2 + BC^2$
$\Rightarrow AB^2 = 12^2 + 16^2$
$\Rightarrow \quad AB^2 = 144 + 256 = 400$
$\Rightarrow \quad AB = \sqrt{400} = 20\text{cm}$
∴ radius of semi-circle $= \frac{20}{2} = 10\text{cm}$
∴ Area of shaded region = Area of semi-circle - Area of $\triangle$ACB
$= \frac{1}{2}\pi(10)^2 - \frac{1}{2}AC \times BC$
$= \frac{1}{2} \times 3.142 \times 100 - \frac{1}{2} \times 12 \times 16$
= 157.1 - 96
= 61.1 cm^2
Perimeter of shaded region
= AC + BC + circumference of semi-circle
$= 12 + 16 + \pi(10)$
$= 28 + 3.142 \times 10$
= 28 + 31.42

= 59.42 cm

16. i. Total area of 4 sectors $= \left\{\frac{22}{7} \times (14)^2 \times \left(\frac{90}{360} + \frac{90}{360} + \frac{120}{360} + \frac{60}{360}\right)\right\} \mathrm{m}^2$

$= \left\{22 \times 2 \times 14 \times \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{3} + \frac{1}{6}\right)\right\} \mathrm{m}^2$

$= \left\{616 \times \frac{3+3+4+2}{12}\right\} m^2$

$= \left\{616 \times \frac{12}{12}\right\} \mathrm{m}^2$

= 616 m^2

ii. Area of trapezium ABCD $= \frac{1}{2} \times (AD + BC) \times AB$

$= \frac{1}{2} \times (55 + 45) \times 30$

$= 100 \times 15$

= 1500 m^2

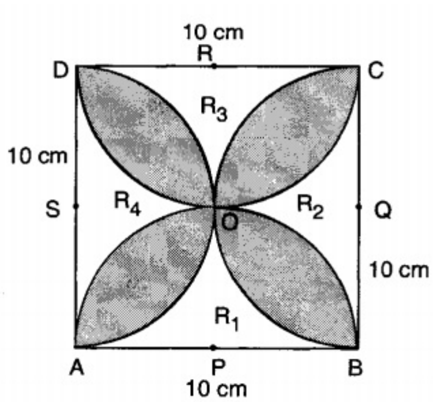
Therefore, required area

= Ar (trapezium ABCD) - Total area of 4 sectors

= (1500-616) m^2

= 884 m^2

17.



Clearly,

Area of R_1 + Area of R_3

= Area of square ABCD - Area of two semi-circles having centres at Q and S

$= \left(10 \times 10 - 2 \times \frac{1}{2} \times 3.14 \times 5^2\right) \mathrm{cm}^2$ [∵ Radius = AP = 5 cm]

= (100 - 3.14 × 25) cm^2 = (100 - 78.5) cm^2 = 21.5 cm^2

Similarly, we have

Area of R_2 + Area of R_4 = 21.5 cm^2

i. Area of the unshaded region = Area R_1 + Area R_2 + Area R_3 + Area R_4

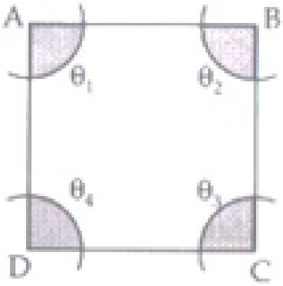
= (Area R_1 + Area R_3) + (Area R_2 + Area R_4)

= 2 (21.5) cm^2 = 43 cm^2

ii. Area of the shaded region

= Area of square ABCD - (Area of R_1 + Area of R_2 + Area of R_3 + Area of R_4)

= (100 - 2 × 21.5) cm^2 = 57 cm^2

18. 

Specification of quadrilateral are not given, so quadrilateral may be of any shape.

As the radius of all 4 arcs are same equal to r = 21 cm

but not different angles ∠A, ∠B, ∠C and ∠D.

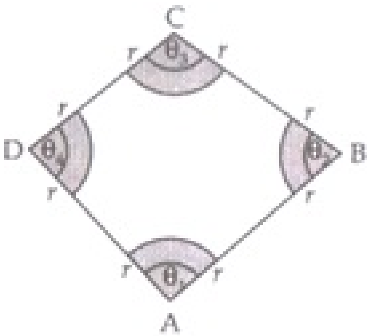
Therefore, there are four sectors of ∠A, ∠B, ∠C and ∠D with r = 21 cm.

Therefore, Area of shaded region

$$= \frac{\pi r_1^2(\theta_1)}{360^\circ} + \frac{\pi r_2^2(\theta_2)}{360^\circ} + \frac{\pi r_3^2(\theta_3)}{360^\circ} + \frac{\pi r_4^2(\theta_4)}{360^\circ}$$

because $r_1 = r_2 = r_3 = r_4 = r$ and

$\angle\theta_1 + \angle\theta_2 + \angle\theta_3 + \angle\theta_4 = 360^0$ [Interior ∠s of a quad.]



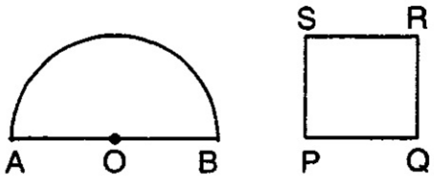
Therefore, Area of shaded region $= \frac{\pi r^2(\theta_1)}{360^\circ} + \frac{\pi r^2(\theta_2)}{360^\circ} + \frac{\pi r^2(\theta_3)}{360^\circ} + \frac{\pi r^2(\theta_4)}{360^\circ}$

$$= \frac{\pi r^2}{360^\circ}(\theta_1 + \theta_2 + \theta_3 + \theta_4)$$

$$= \frac{\pi r^2}{360^\circ}(360^\circ)$$

$= \pi r^2 = \frac{22}{7} \times 21 \times 21 = 22 \times 63 = 1386\ cm^2$

Therefore, the area of shaded region = 1386 cm^2.

19. 

Let radius of semicircular region be r units.

Perimeter = 2r + πr

Let side of square be x units

Perimeter = 4x units.

A.T.Q, 4x = 2r + πr $\Rightarrow x = \frac{2r+\pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \left(\frac{2r+\pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$

$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$

$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$

$\Rightarrow r^2(\pi - 2)^2 = 64$

$\Rightarrow r = \sqrt{\frac{64}{(\pi-2)^2}}$

$\Rightarrow r = \frac{8}{\pi-2} = \frac{8}{\frac{22}{7}-2}$ = 7 cm

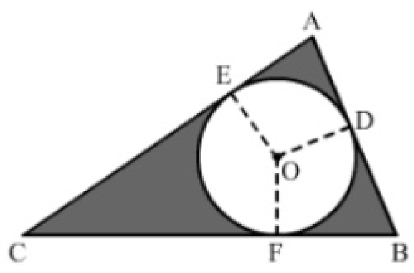
Perimeter of semicircle = $2 \times 7 + \frac{22}{7} \times 7$ = 36 cm

Perimeter of square = 36 cm

Side of square = $\frac{36}{4}$ = 9 cm

Area of square = 9$\times$ 9 = 81 cm^2

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22}{2\times7} \times 7 \times 7$ = 77 cm^2

20. 

Using Pythagoras theorem for triangle ABC, we have:

$CA^2 + AB^2 = BC^2$

$CA^2 = BC^2 - AB^2$

$$CA = \sqrt{BC^2 - AB^2}$$

$$CA = \sqrt{100 - 36}$$

$$CA = \sqrt{64}$$

= 8 cm

Now, we must find the radius of the incircle. Draw OE, OD and OF Perpendicular to AC, AB and BC respectively.

Here,

EO = OD (Both are radii)

Because the circle is an incircle. AE and AD are tangents to the circle

$$\angle AEO = \angle ADO = 90^\circ$$

Also,

$$\angle A = 90^\circ$$

Therefore, AEOD is a square.

Thus, we can say that AE = EO = OD = AD = r

CE = CF

= 8 - r

BD = DB

= 6 - r

CF + BF = 10

$$\Rightarrow (8 - r) + (6 - r) = 10$$

$\Rightarrow$ 14 - 2r

= 10

$\Rightarrow$ r = 2 cm

Area of the shaded portion

= Area of the triangle - Area of the circle

$$= \left(\frac{1}{2} \times 6 \times 8\right) - (\pi \times 2 \times 2)$$

= 24 - 12.56

= 11.44 cm^2