# MAXIMA AND MINIMA

## SYNOPSIS

**INCREASING FUNCTION:** Let D denote the domain of a real valued function 'f'

- If $\forall\, x_1, x_2 \in D; x_1<x_2 \Rightarrow f(x_1) \leq f(x_2)$ then f is said to be an increasing function in D
- If $\forall\, x_1, x_2 \in D; x_1<x_2 \Rightarrow f(x_1) < f(x_2)$ then f is said to be strictly increasing in D.

**DECREASING FUNCTION:**

- If $\forall\, x_1, x_2 \in D; x_1<x_2 \Rightarrow f(x_1) \geq f(x_2)$ Then f is said to be a decreasing function in D
- If $\forall\, x_1, x_2 \in D; x_1<x_2 \Rightarrow f(x_1) > f(x_2)$ Then f is said to be strictly decreasing in D.

**MONOTONIC FUNCTION**: A function which is either increasing (or) decreasing in its domain is called a monotonic function.

**TEST FOR MONOTONICITY:**

- If $f^1(x) \geq 0\ \forall x \in (a,b)$ then f is increasing in (a, b)
- If $f^1(x) > 0\ \forall x \in (a,b)$ then f is strictly increasing in (a, b)
- If $f^1(x) \leq 0\ \forall x \in (a,b)$ then f is decreasing in (a, b)
- If $f^1(x) < 0\ \forall x \in (a,b)$ then f is strictly decreasing in(a, b)

**NOTE:** If $f^1(a) > 0$ then f is increasing at x = a and if $f^1(a)<0$ then f is decreasing at x = a.

**STATIONARY POINT:** If $f^1(a)=0$ then y=f(x) is said to be stationary at x =a. f(a) is called the stationary value of f at x=a then (a, f(a)) is called a stationary point of 'f'.

**CRITICAL POINT:** If $f^1(a)=0$ (or) $f^1(a)$ does not exist then f(x) is said to have a critical point at x = a.

**FIRST DERIVATIVE TEST:**

- If $f^1(x)$ changes sign from -ve to +ve at 'a' then f has a local minimum (or) relative minimum at 'a'.
- If $f^1(x)$ changes sign from +ve to -ve at 'a'. then f has a local maximum (or) relative maximum at 'a'

**Note I:** maximum, minimum values of f are also called extreme values (or) turning values of f.

**Note II:** A local minimum value may be greater than a local maximum value.

**SECOND DERIVATIVE TEST:**

- If $f^1(a)=0$, $f^{11}(a)<0$ then f has a local maximum at x=a.
- If $f^1(a)=0$; $f^{11}(a)>0$ then f has a local minimum at x=a.

## $n^{th}$ Derivative Test

If f is a continuous function on (a, b), $\alpha \in (a,b)$ and

$$f'(\alpha)=f''(\alpha)=f'''(\alpha)=\ldots\ldots=f^{(n-1)}(\alpha)=0$$

and $f^{(n)}(\alpha) \neq 0$ then

- If 'n' is even and $f^{(n)}(\alpha) > 0$ then $f(\alpha)$ is relative minimum at '$\alpha$'
- If 'n' is odd and $f^{(n)}(\alpha) < 0$ then $f(\alpha)$ is relative maximum at '$\alpha$'
- If 'n' is odd and $f^{(n)}(\alpha) > 0$ then $f$ is increasing at '$\alpha$'
- If 'n' is even and $f^{(n)}(\alpha) < 0$ then $f$ is decreasing at '$\alpha$'

**OTHER POINTS:**

- If the sum of two positive numbers is a constant, then their product will be maximum when the two numbrs are equal.
- If the product of two positive numbers is a constant, then their sum will be least when the two numbers are equal.
- The minimum value of (x-a) (x-b) is $\dfrac{-(a-b)^2}{4}$.
- The least value of each of $a^2\sin^2x+b^2\mathrm{cosec}^2x$, $a^2\sec^2x+b^2\cos^2x$, $a^2\tan^2x+b^2\cot^2x$ is 2ab.
- The least value of $a^2\sec^2x+b^2\mathrm{cosec}^2x$ is $(a+b)^2$ when $x=\tan^{-1}\sqrt{\dfrac{b}{a}}$.
- $\sin^p\theta\ \cos^q\theta$ attains a maximum value at $\theta = \tan^{-1}\sqrt{\dfrac{p}{q}}$ and that max. value is $\left(\dfrac{p^p.q^q}{(p+q)^{p+q}}\right)^{1/2}$
- The minimum value of a secx+bcosecx is $(a^{2/3}+b^{2/3})^{3/2}$ at $x=\tan^{-1}.\left(\dfrac{b}{a}\right)^{1/3}$.
- The minimum value of a cotx+btan x is $2\sqrt{ab}$ at x $= \tan^{-1}\sqrt{\dfrac{a}{b}}$.

- The least value of the portion of tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intercepted between the co-ordinate axes is a+b.
- A normal is drawn at a variable point P of the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then the maximum distance of the normal from the centre of the curve is a-b.
- The minimum distance from the origin to a point on the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ is (a+b).
- The area of greatest isosceles traingle that can be inscribed in a given ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ having its vertex coincident with one extremity of major axis is $\frac{3\sqrt{3}}{4}$ab sq units.
- The area of greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is 2ab sq units.
- The minimum value of $\left(1+\frac{1}{\sin^n \alpha}\right)\left(1+\frac{1}{\cos^n \alpha}\right)$ is $\left(1+2^{n/2}\right)^2$
- When perimeter is given, if sector area is maximum then $\theta=2^c$.
- From the four corners of a square sheet of metal of side 'a', four equal squares are cut off and the remaining edges are folded up to form a rectangular open box. If the volume of the box formed is to be maximum, the side of the square removed is $\frac{a}{6}$.
- From the four corners of rectangular sheet of metal of sides a,b, four equal squares are cut off and the remaining edges are folded up to form an open box. If the volume of the box is to be maximum the side of a square removed is $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$.
- A cone is drawn circumscribing a sphere of radius 'R'. If the volume of the cone is maximum, its height is $\frac{4R}{3}$ and its semivertical angle is $\sin^{-1}\frac{1}{3}$ (If surface area is constant).
- In a right angled triangle, the sum of a side and hypotenuse is given. If the area of the triangle is maximum, then the angle between them is $60^0$.
- The least area of the triangle formed by any line through (p,q) and the co-ordinate axes is 2pq sq units
- The maximum area of rectangle inscribed in a circle of radius r is $2r^2$ sq. units.

## CONCEPTUAL QUESTIONS

1 Let I be an open interval contained in the domain of a real function 'f', then f(x) is called strictly increasing function in I if

1) $x_1 < x_2 \text{ in } I \Rightarrow f(x_1) < f(x_2)$

2) $x_1 < x_2 \text{ in } I \Rightarrow f(x_1) > f(x_2)$

3) $x_1 = x_2 \text{ in } I \Rightarrow f(x_1) = f(x_2)$

4) $x_1 = x_2 \text{ in } I \Rightarrow f(x_1) < f(x_2)$

2 Let I be an open interval contained in the domain of a real function 'f', then f(x) is called strictly decreasing function in I if

1) $x_1 < x_2 \text{ in } I \Rightarrow f(x_1) < f(x_2)$

2) $x_1 < x_2 \text{ in } I \Rightarrow f(x_1) > f(x_2)$

3) $x_1 = x_2 \text{ in } I \Rightarrow f(x_1) = f(x_2)$

4) $x_1 = x_2 \text{ in } I \Rightarrow f(x_1) < f(x_2)$

3 In $\left(0,\frac{\pi}{2}\right)$, f(x) = x sin x + cos x + $\frac{1}{2}\cos^2 x$ is

1. Increasing
2. Decreasing
3. Constant
4. Nothing can be determined

4 $f(x)=\frac{\log x}{x}(x>0)$ is increasing in

1. (0, e) 2. (e, $\infty$)
3. (0, $\infty$) 4. ($-\infty$, $\infty$)

5 $f(x)=\frac{\log x}{x}(x>0)$ is decreasing in

1. (0, e) 2. $(e,\infty)$
3. $(0,\infty)$ 4. ($-\infty$, $\infty$)

6 $f(x)=\frac{1}{2}$ sin x tan x - log sec x is increasing in

1. $\left(\frac{-\pi}{2},0\right)$ 2. $\left(\pi,\frac{3\pi}{2}\right)$ 3. $\left(0,\frac{\pi}{2}\right)$ 4. $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

7 Which of the following function is increasing in $\left(0, \frac{\pi}{2}\right)$

1. sin x 2. cos x 3. cot x 4. cosec x

8 The condition that $f(x) = x^3+ax^2+bx+c$ is an increasing function for all real values of 'x' is

1. $a^2<12b$ 2. $a^2<3b$ 3. $a^2<4b$ 4. $a^2<16b$

9 $f(x) = \frac{a\sin x + b\cos x}{c\sin x + d\cos x}$ is an increasing funtion if

1. ad - bc = 0 2. ad -bc < 0
3. ad - bc > 0 4. ab + cd = 0

10 $f(x) = \frac{a\sin x + b\cos x}{a\cos x - b\sin x}\left(\tan x \neq \frac{a}{b}\right)$ is

1. increasing in dom f
2. Decreasing in dom f
3. Constant
4. Nothing can be determined

11. $f(x) = \cos\frac{\pi}{x}$ is increasing in

1. $\left(\frac{1}{2n+1}, \frac{1}{2n}\right) n \in z^+$
2. $\left(\frac{1}{2n+2}, \frac{1}{2n+1}\right) n \in z^+$
3. R
4. R - {0}

12 If f(x) = kx - sin x is monotonically increasing then

1. k > 1 2. k > -1 3. k < 1 4. k < -1

13 f(x) = sin x - ax is decreasing in R if

1. a > 1 2. a< 1 3. $a > \frac{1}{2}$ 4. $a < \frac{1}{2}$

14. $f(x) = x - e^x$ is decreasing in

1. $(-\infty, 0)$ 2. $(0, \infty)$ 3. $(-\infty, \infty)$ 4. $\varnothing$

15 $f(x) = x - e^x$ is increasing in

1. $(-\infty, 0)$ 2. $(0, \infty)$ 3. $(-\infty, \infty)$ 4. $\varnothing$

16 f(x) = sin h (sin x) is increasing in

1. $\left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$ 2. $\left(2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right)$
3. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ 4. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

17 f(x) = sin h (sin x) is descreasing in

1. $\left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$ 2. $\left(2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right)$
3. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ 4. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

18 $f(x) = \tan^{-1}(\sin x)$ is increasing in

1. $\left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$ 2. $\left(2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right)$
3. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ 4. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

19 $f(x) = \tan^{-1}(\sin x)$ is decreasing in

1. $\left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$ 2. $\left(2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right)$
3. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ 4. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

20 The set of values of 'x' for which f(x) = cos x - x is decreasing in

1. $(-\infty, 0)$ 2. $(0, \infty)$
3. $(-\infty, \infty)$ 4. $\varnothing$

21 $f(x) = \log_a x$ (x > 0) is an increasing function if

1. a > 1 2. 0 < a < 1
3. $a \neq 1$ 4. $a \neq 0$

22 $f(x) = \log_a x$ (x > 0) is decreasing function if

1. a > 1 2. 0 < a < 1
3. $a \neq 1$ 4. $a \neq 0$

23 If a < 0 the function $e^{ax}+e^{-ax}$ is a monotonically decreasing function for values of 'x' given by

1. x > 0 2. x < 0 3. x > 1 4. x < 1

24 $f(x) = \tan^{-1}x$ is

1. Strictly increasing
2. Strictly decreasing
3. Neither increasing nor decreasing
4. Constant

25 $f(x) = \frac{\sin x}{x}$ is

1. increasing in $\left(0, \frac{\pi}{2}\right)$
2. decreasing in $\left(0, \frac{\pi}{2}\right)$
3. Stationary at x = $\frac{\pi}{2}$
4. Stationary at x = 0

26 $f(x) = x^x$ is decreasing for

1. $x > \frac{1}{e}$ 2. x > e
3. 0 < x < e 4. $0 < x < \frac{1}{e}$

27 f(x) = $\frac{1}{x-1}$ is decreasing in

1. (-1, 1) 2. $(-\infty, 1)\cup(1, \infty)$
3. $(-\infty, \infty)$ 4. $(-\infty, 1)$

28 if $f(x) = \sin x - \cos x - ax + b$ decreases for all $x \in R$ then

1. $a < 1$ 2. $a > 1$
3. $a < \sqrt{2}$ 4. $a > \sqrt{2}$

29. The function $x + \log\left(\frac{x}{1+x}\right)$ is increasing when x belongs to the interval

1. $(1, \infty)$ 2. $(2, \infty)$
3. $(3, \infty)$ 4. $(-\infty, -1) \cup (0, \infty)$

30. $f(x) = \tan^{-1} x - x$ is decreasing in

1. $(-\infty, \infty)$ 2. $(0, \infty)$ only
3. $(-1, 1)$ only 4. $(0, 1)$

31. If $x > 0$ then which of the following is true

1. $\tan^{-1} x > \frac{x}{1+x^2}$ 2. $\tan^{-1} x = \frac{x}{1+x^2}$
3. $\tan^{-1} x < \frac{x}{1+x^2}$ 4. $\tan^{-1} x \neq \frac{x}{1+x^2}$

32. The set of all x for which $\sin x \leq x$ is

1. $\left(0, \frac{\pi}{2}\right)$ 2. $\left(\frac{-\pi}{2}, \pi\right)$
3. $\left(-\frac{\pi}{2}, 0\right)$ 4. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

33. Tan x > x when 'x' lies in

1. $\left(0, \frac{\pi}{2}\right)$ 2. $\left(\frac{\pi}{2}, \pi\right)$
3. $\left(-\frac{\pi}{2}, 0\right)$ 4. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

34. Every invertible function is

1) Monotonic function
2) Constant function
3) Identity function
4) not necessarily monotonic function

35. Function $f(x) = a^x$ is increasing on 'R' if

1) $a > 0$ 2) $a < 0$
3) $0 < a < 1$ 4) $a > 1$

36. If f is increasing and g is decreasing functions such that g (f(x)) exists ,then

1)g (f(x)) is an increasing function
2) g(f(x)) is a decreasing function
3) nothing can be said
4) g(f(x)) is a constant function

37. If f,g are increasing functions such that g (f(x)) exists , then

1) g(f(x)) is an increasing function
2) g(f(x)) is a decreasing function
3) g(f(x)) is constant function
4) nothing can be said

38. Let $h(x) = f(x) - [f(x)]^2 + [f(x)]^3$ for every real number 'x' then

1) h is increasing whenever 'f' is increasing
2) h is increasing whenever 'f' is decreasing
3) h is decreasing whenever 'f' is increasing
4) Nothing can be said in general

39. If $f^1(a) = 0$ and $f^{11}(a) < 0$ then the function f(x) at x = a is

1) stationary 2) increasing
3) minimum 4) maximum

40. If $f^1(a) = 0$ and $f^{11}(a) > 0$ then the function f(x) at x = a is

1) stationary 2) increasing
3) minimum 4) maximum

41. Let 'f' be any differentiable function. A point 'p' at which $f^1(p) = 0$ is called a stationary point of 'f'. Which of the following statement is true?

1) Each stationary point gives rise to a local maximum
2) Each stationary point need not give a local maximum or a local minimum
3) Each stationary point gives rise to a local minimum
4) Each stationary point gives both local maximum and local minimum

42. The values of 'x' at which f(x)=sin x is stationary are given by

1. $n\pi, \forall n \in Z$ 2. $(2n+1)\frac{\pi}{2}, \forall n \in Z$
3. $\frac{n\pi}{4}, \forall n \in Z$ 4. $\frac{n\pi}{2}, \forall n \in Z$

43. The value of 'x' at which f(x) = cos x is stationary are given by

1. $n\pi, \forall n \in Z$ 2. $(2n+1)\frac{\pi}{2}, \forall n \in Z$
3. $\frac{n\pi}{4}, \forall n \in Z$ 4. $\frac{n\pi}{2}, \forall n \in Z$

44. Stationary point of $y = \frac{\log x}{x} (x > 0)$ is

1. (1, 0) 2. $\left(e, \frac{1}{e}\right)$ 3. $\left(\frac{1}{e}, -e\right)$ 4. $\left(\frac{1}{e}, \frac{1}{e}\right)$

45. The number of stationary points of f(x) = cos x in $[0, 2\pi]$ are

1. 1 2. 2 3. 3 4. 4

46. The number of stationary points of f(x) = sin x in $[0, 2\pi]$ are

1. 1 2. 2 3. 3 4. 4

47. $f(x)=\frac{ax+b}{cx+d}\ (ad-bc\neq 0)$

1. Has a maximum
2. Has a minimum
3. Neither max nor min is true
4. Both max and min are true

48. The maximum of $f(x)=\frac{\log x}{x^2}(x>0)$ occurs at x=

1. e  2. $\sqrt{e}$  3. $\frac{1}{e}$  4. $\frac{1}{\sqrt{e}}$

49. f(x) = sin x (1 + cos x) is maximum at x =

1. $\frac{\pi}{4}$  2. $\frac{\pi}{6}$  3. $\frac{\pi}{3}$  4. $\frac{\pi}{2}$

50. f(x) = sin x + cos 2x (x>0) has minima at x =

1. $(2n+1)\frac{\pi}{2}, \forall n\in Z$  2. $\frac{n\pi}{2}, \forall n\in Z$

3. $n\pi, \forall n\in Z$  4. $\frac{n\pi}{4}, \forall n\in Z$

51. At x = 0, f(x) = cos x + cos hx

1. Has a minimum
2. Has a maximum
3. Does not have an extremum
4. Is not defined.

52. Let $f(x) = a_0 + a_1x^2 + a_2x^4 + .... + a_nx^{2n}$ when $0 < a_0 < a_1 < ...... < a_n$ then f(x) has

1. No extremum  2. Only one maximum
3. Only one minimum  4. Constant

53. At x = 0 f(x) = sin x - x

1. Has a minimum  2. Has a maximum
3. Has no extremum  4. Is not defined

54. If $y=\sum_{i=1}^{n}(x-x_i)^2$, $x_i$ are constants then y has minimum value at x =

1. n  2. $\sum x_i$  3. $\frac{\sum x_i}{n}$  4. $n\sum x_i$

55. If a > b maximum value of a $\sin^2$x + b $\cos^2$x is

1. a  2. b  3. a + b  4. $\sqrt{a^2+b^2}$

56. If a > b, minimum value of a $\sin^2$x + b $\cos^2$x is

1. a  2. b  3. a + b  4. $\sqrt{a^2+b^2}$

57. Minimum value of $\sqrt{e^{x^2}-1}$ is

1. 0  2. e  3. $\frac{1}{e}$  4. $e^{e^2}$

58. Minimum value of $(\sin x)^{\sin x}$ is

1. $(e)^{\frac{-1}{e}}$  2. 1  3. $\frac{\pi}{2}$  4. $\frac{1}{e}$

59. A(0, a), b(0, b) be fixed points. P(x, 0) a variable point. The angle $\angle APB$ is maximum if

1. $x^2$ = ab  2. x = ba
3. $x^2$ = 2ab  4. $2x^2$ = ab

60. When the area of a sector of fixed perimeter is maximum then the arc length ‘l’, radius ‘r’ are connected by the relation

1. $l$ = r  2. $l$ = 2r  3. r = 2$l$  4. r = 4$l$

61. A window is in the shape of a rectangle surmounted by a semi circle. If the perimeter of the window is of fixed length ‘$l$’ then the maximum area of the window is

1. $\frac{l^2}{2\pi+4}$  2. $\frac{l^2}{\pi+8}$  3. $\frac{l^2}{2\pi+8}$  4. $\frac{l^2}{8\pi+4}$

62. The minimum distance from the origin to a point on the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ is

1. 2ab  2. 2(a+b)  3. 2a+4b  4. a + b

63. A wire of length ‘a’ is cut into two parts which are bent in the form of a square and a circle. The least value of the sum of the areas then formed is

1. $\frac{a^2}{\pi+4}$  2. $\frac{a^2}{2(\pi+4)}$  3. $\frac{a^2}{3(\pi+4)}$  4. $\frac{a^2}{4(\pi+4)}$

64. The sum of the hypotenuse and a side of a right angled triangle is constant. If the area of the triangle is maximum then the angle between the hypotenuse and the given side is

1. $\frac{\pi}{2}$  2. $\frac{\pi}{4}$  3. $\frac{\pi}{3}$  4. $\frac{\pi}{6}$

65. Of all isosceles triangles inscribed in a given circle the triangle having maximum area is ..... triangle

1. Equilateral  2. Scalane
3. Right angled isosceles
4. isosceles

66. The height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is

1. $\frac{2R}{3}$  2. $\frac{R}{3}$  3. $\frac{2R}{\sqrt{3}}$  4. $\frac{R}{\sqrt{3}}$

67. The height of the cone of maximum volume inscribed in a sphere of radius R is

1. $\frac{R}{3}$  2. $\frac{2R}{3}$  3. $\frac{4R}{3}$  4. $\frac{4R}{\sqrt{3}}$

68. The height of the cylinder of maximum curved surface area that can be inscribed in a sphere of radius ‘R’ is

1. $\frac{R}{3}$  2. $\sqrt{2}R$  3. $\sqrt{\frac{2}{3}}R$  4. $\frac{3R}{4}$

69. An open cylinder of given surface will have maximum volume if radius =

1. height  2. $\frac{1}{2}$ height
3. 2 height  4. 3 height

70. If PQ is a double ordinate of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and A is one end of the major axis then the maximum area of $\Delta APQ$ is

1. $\frac{\sqrt{3}}{4}ab$ 2. $\frac{3\sqrt{3}}{4}ab$ 3. $\frac{\sqrt{3}}{2}ab$ 4. $\frac{3\sqrt{3}}{2}ab$

71. If the petrol burnt per hour in driving a motor boat varies as the cube of its velocity when going against a current of 'C' KMPH, the most economical speed is

1. $\frac{c}{2}$ 2. $\frac{3c}{2}$ 3. $\frac{\sqrt{3}c}{2}$ 4. c

72. Three sides of a trapezium are equal, each being 'a' cms long. Then its maximum area is

1. $\frac{3\sqrt{3}}{4}a^2$ 2. $\frac{6\sqrt{3}}{4}a^2$ 3. $\frac{\sqrt{3}}{4}a^2$ 4. $\frac{\sqrt{3}}{2}a^2$

73. AB is a diameter of a circle and C is any point on the circumference of the circle then

1. Area of the $\Delta ABC$ is maximum when it is isosceles

2. Area of the $\Delta ABC$ is minimum when it is isosceles

3. The perimeter of $\Delta ABC$ is minimum when it is isosceles

4. The perimeter of $\Delta ABC$ is maximum when it is isosceles

74. The function which has neither maximum nor minimum at x = 0 is

1. $f(x) = x^2$ 2. f(x) = cos x
3. $f(x) = x^3-8$ 4. f(x) = cos hx

75. If f(x) is minimum at x = a then

1. There exists $\delta > 0$ such that
$a-\delta < x < a \Rightarrow f(x) < f(a)$

2. There exists $\delta > 0$ such that
$a < x < a+\delta \Rightarrow f(x) > f(a)$

3. There exists $\delta > 0$ such that
$a-\delta < x < a+\delta \Rightarrow f(x) \geq f(a)$

4. There exists $\delta > 0$ such that
$a-\delta < x < a+\delta \Rightarrow f(x) \leq f(a)$

76. The hypotenuse of a right angled triangle is k cm, if the area is maximum then the sides of the triangle are

1. k, k 2. $k\sqrt{2}, k\sqrt{2}$
3. k/3, 2k/3 4. $k/\sqrt{2}, k/\sqrt{2}$

77. The rectangular sheet of metal sides a, b has four equal square portions removed at the corners and the sides are then turned up so as to form an open rectangular box. When the volume contained in the box is maximum, the depth of the box is

1. $\frac{1}{6}\left[(a+b)+(a^2-ab+b^2)^{1/2}\right]$

2. $\frac{1}{6}\left[(a+b)-(a^2-ab+b^2)^{1/2}\right]$

3. $\frac{1}{6}\left[(a-b)+(a^2-ab+b^2)^{1/2}\right]$

4. $\frac{1}{6}\left[(a-b)-(a^2-ab+b^2)^{1/2}\right]$

78. The sum of two positive numbers is equal to 'a' and if the sum of their cubes is least, the numbers are

1. a/3, a/3 2. a/2, a/2
3. a/4, a/4 4. 3a/3, a/4

79. The volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle $\alpha$ is

1. $\frac{4\pi h^3}{27}\tan^2\alpha$ 2. $4\pi h^2\tan^2\alpha$ 3 .
$\frac{4\pi h^3}{9}\tan^2\alpha$ 4. $\frac{4\pi h^3}{27}\tan^3\alpha$

80. A point P is given on the circumference of a circle of radius r. The chod QR is parallel to the tangent line at P. the maximum area of the triangle PQR is

1. $\frac{3\sqrt{2}}{4}r^2$ 2. $\frac{3\sqrt{3}}{4}r^2$ 3. $\frac{3}{8}r$ 4. $\frac{3\sqrt{2}}{4}r$

81. For 0 < a < x, the minimum value of the function $\log_x a + \log_a x$ is

1. 1 2. 2
3. -2 4. -1/2

82. The minimum value of a sec x + b cosecx is

1. ab 2. $(a^{2/3}-b^{2/3})^{2/3}$
3. $(a^{2/3}+b^{2/3})^{3/2}$ 4. $a^2b^2$

83. The function $a^2\sec^2x + b^2\csc^2x$ attains minimum value when x =

1. $\tan^{-1}(b/a)$ 2. $\tan^{-1}\left(\sqrt{a/b}\right)$

3. $\mathrm{Tan}^{-1}\left(\sqrt{b/a}\right)$ 4. $\mathrm{Tan}^{-1}(a/b)$

84. If a, b $\in Z^+$ then maximum value of $\log[(x-a)^a (x-b)^b]$ occurs at x =

1. $\frac{a+b}{2}$ 2. $\sqrt{ab}$ 3. $\frac{2ab}{a+b}$ 4. a+b

85 The maximum volume of the cone that can be inscribed in a sphere of radius r is

1. $\frac{32}{27}\pi r^3$  2. $\frac{32}{81}\pi r^3$

3. $85\pi r^3$  4. $\frac{32}{9}\pi r^3$

86. A triangle is inscribed in a semi circle of radius 'a' so that one side is the bounding diameter. Then the maximum area of the triangle is

1. $(\pi/4)a^2$  2. $(1/2)a^2$

3. $(1/4)a^2$  4. $a^2$

87. Maximum value of 'r' where $\frac{c^2}{r^2}=\frac{a^2}{\sin^2\theta}+\frac{b^2}{\cos^2\theta}$ where c, a, b are constants is

1) $\frac{c}{a+b}$  2) $\frac{c}{a-b}$

3) $\frac{a^2+b^2}{c^2}$  4) $\frac{c^2}{a^2+b^2}$

88. The area of the quadrilateral in a circle is minimum when the quadrilateral is

1) Rectangle  2) square

3) Parallelogram  4) Rhombus

89. If all the rectangles of a given perimeter 2P, the one having the smallest diagonal is square whose side is

1) $\frac{P}{2}$  2) P  3) 2P  4) $\frac{P}{3}$

90. The maximum value of the area of the triangle with vertices (a, 0) ($a\cos\theta$, $b\sin\theta$) and ($a\cos\theta$, $-b\sin\theta$) is

1) $\frac{3\sqrt{3}ab}{4}$  2) $3\sqrt{ab}$

3) $\frac{\sqrt{3ab}}{4}$  4) $\sqrt{3ab}$

## KEY

| | | | | |
|---|---|---|---|---|
| 1. 1 | 2. 2 | 3. 1 | 4. 1 | 5. 2 |
| 6. 3 | 7. 1 | 8. 2 | 9. 3 | 10. 1 |
| 11. 1 | 12. 1 | 13. 1 | 14. 2 | 15. 1 |
| 16. 1 | 17. 2 | 18. 1 | 19. 2 | 20. 3 |
| 21. 1 | 22. 2 | 23. 2 | 24. 1 | 25. 2 |
| 26. 4 | 27. 2 | 28. 4 | 29. 4 | 30. 1 |
| 31. 1 | 32. 1 | 33. 1 | 34. 1 | 35. 4 |
| 36. 2 | 37. 1 | 38. 1 | 39. 4 | 40. 3 |
| 41. 2 | 42. 2 | 43. 1 | 44. 2 | 45. 3 |
| 46. 2 | 47. 3 | 48. 2 | 49. 3 | 50. 1 |
| 51. 1 | 52. 3 | 53. 3 | 54. 3 | 55. 1 |
| 56. 2 | 57. 1 | 58. 1 | 59. 1 | 60. 2 |
| 61. 3 | 62. 4 | 63. 4 | 64. 3 | 65. 1 |
| 66. 3 | 67. 3 | 68. 2 | 69. 1 | 70. 2 |
| 71. 2 | 72. 1 | 73. 1 | 74. 3 | 75. 3 |
| 76. 4 | 77. 2 | 78. 2 | 79. 1 | 80. 2 |
| 81. 2 | 82. 3 | 83. 3 | 84. 3 | 85. 2 |
| 86. 4 | 87. 1 | 88. 2 | 89. 1 | 90. 1 |

## NUMERICAL QUESTIONS
## LEVEL-1

1. $f(x) = 2x^3-15x^2+36x+5$ is decreasing in

1. (1, 2)  2. (2, 3)

3. $(-\infty, 2)$  4. $(3, \infty)$

2. $f(x)=\frac{x}{5}+\frac{5}{x}(x\neq 0)$ is increasing in

1. (-5, 0)  2. (0, 5)

3. $(-\infty,-5)\cup(5,\infty)$  4. (-5, 5)

3. f(x) = x + 2 cos x is increasing in

1. $\left(0,\frac{\pi}{2}\right)$  2. $\left(\frac{-\pi}{2},\frac{\pi}{6}\right)$

3. $\left(\frac{\pi}{2},\pi\right)$  4. $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

4. $f(x)=2|x-3|+|x-4|$ is decreasing in

1. (3, 4)  2. $(4,\infty)$  3. $(-\infty, 3)$  4. $(-\infty,\infty)$

5. The interval of $x$ for which the function $y=-x(x-2)^2$ increases is

1) $\left(\frac{2}{3},2\right)$  2) $\left[\frac{2}{3},2\right)$

3) $\left[\frac{2}{3},2\right]$  4) $\left(\frac{2}{3},2\right]$

6. In the interval $(7,\infty)$, $f(x)=|x-5|+2|x-7|$ is

1. Increasing  2. Decreasing

3. Constant  4. Cannot be estimated

7. $f(x) = x^2 (x-2)^2$ is increasing in

1. $(-\infty,0)$  2. $(2,\infty)$  3. R  4. (1, 2)

8. If 15 < X < 30 then $f(x)=|x-15|+|x-30|$ is
1. Increasing  2. Decreasing
3. Constant  4. cannot be estimated

9. The intervals of monotonicity of $f(x) = x^2 e^{-x}$ is
1. (2, 4)  2. $(-\infty,0)\cup(2,\infty)$
3. (0, 4)  4. $(-\infty,0)\cup(4,\infty)$

10. The set of values of 'x' for which $f(x)=2x^3-9x^2+12x+10$ is increasing is
1. $(-\infty,\infty)$  2. (1, 2)
3. $(-\infty,1)\cup(2,\infty)$  4. [1, 2]

11. $f(x)= x^3+6x^2+24x-17$ is increasing for
1. $(-\infty,\infty)$  2. $R-\{0\}$  3. (1, 2)  4. [1, 2]

12. If x > 0 then the interval of monotonically increasing of $f(x)=2x^2-\log x$ is
1. $\left(-\infty,\frac{-1}{2}\right)$  2. $\left(\frac{-1}{2},0\right)$  3. $\left(0,\frac{1}{2}\right)$  4. $\left[\frac{1}{2},\infty\right)$

13. The set of values of 'a' for which $f(x)= x^3 - ax^2 + 48x + 1$ is increasing for all real values of 'x' is
1. (-12, 12)  2. $(-\infty,-12)$
3. $(12,\infty)$  4. $(-\infty, \infty)$

14. $f(x)=x+\frac{1}{x}$ is decreasing when $x \in$
1. (-1, 1)  2. (-1, 0) U (0, 1)
3. $(-\infty,-1)$  4. $(1,\infty)$

15. The set of values 'x' for which $f(x) = x^3-6x^2+27x+10$ is increasisng is
1. (1, 2)  2. $(-\infty, 1)$ U $(2, \infty)$
3. $(-\infty, \infty)$  4. $(-\infty, 1)$

16. f(x) = x ln x - x is an increasing function in the interval
1. $(0, \infty)$  2. (0, 1) U (1,2)
3. $(1, \infty)$  4. $(-\infty, \infty)$

17. $f(x) = x^2 - 8x + 3$ is increasing in
1. $(0, \infty)$  2. $(-\infty, 4)$  3. $(4, \infty)$  4. $(-\infty, \infty)$

18. $f(x) = |x|$ is increasing in
1. $(-\infty, \infty)$  2. $(-\infty, 0)$  3. $(0, \infty)$  4. $(-\infty, -1)$

19. $y=\sqrt{x-x^2}$ increases in the interval
1) $\left(0,\frac{1}{2}\right)$  2) $\left(-\frac{1}{2},0\right)$  3) $\left[0,\frac{1}{2}\right]$  4) $\left[0,\frac{1}{2}\right)$

20. $f(x) = x.e^{-x}$ is decreasing in
1. $(-\infty, 1)$  2. $(1, \infty)$  3. $(-\infty, \infty)$  4. $(0, \infty)$

21. $f(x)=\sqrt{x^2-4}$ is decreasing in
1. (-2, 2)  2. $(2, \infty)$  3. $(-\infty, -2)$  4. $(-\infty, \infty)$

22. $f(x)=\sqrt{100-x^2}$ is increasing in
1. (0, 10)  2. (-10, 0)  3. $(-\infty,-10)$  4. $(10,\infty)$

23. $f(x) = 3x^2-6x+2$ is decreasing in
1. (2, 3)  2. (1, 2)  3. (3, 4)  4. $(-\infty,1)$

24. $f(x) = x^9 + 3x^7 + 6$ is increasing in
1. $(-\infty, \infty)$  2. $(-\infty, 0)$  3. $(0, \infty)$  4. $\varnothing$

25. $f(x)=x^3 - 27x + 5$ is monotonically increasing for
1. x < -3  2. $|x| > 3$  3. $|x| < 3$  4. x > 3

26. f(x) = ln (ln x) (x > 1) is increasing in
1. (0, 1)  2. $(1, \infty)$  3. (0, 2)  4. $(-\infty, 1)$

27. $f(x) = x^3+4x+45$ is increasing in
1. $(0, \infty)$  2. $(-\infty, 0)$  3. $(-\infty, \infty)$  4. $\varnothing$

28. $f(x) = x^5+2x^3+x$ is increasing in
1. $(0, \infty)$  2. $(-\infty, 0)$  3. $(-\infty, \infty)$  4. $\varnothing$

29. $f(x) = \log (1-x^2)$ is increasing in
1. (-1, 0)  2. (0, 1)  3. (-1, 1)  4. $\varnothing$

30. $f(x) = \log (1-x^2)$ is decreasing in
1. (-1, 0)  2. (0, 1)  3. (-1, 1)  4. $\varnothing$

31. $f(x)=12+x^{\frac{2}{3}}$ is increasing in
1. $(0, \infty)$  2. $(-\infty, 0)$  3. 0  4. $(-\infty, \infty)$

32. $f(x)=12+x^{\frac{2}{3}}$ is decreasing in
1. $(0, \infty)$  2. $(-\infty, 0)$  3. 0  4. $(-\infty, \infty)$

33. $f(x)=25+|x-9|$ is increasing in
1. $(-\infty, 9)$  2. $(9, \infty)$  3. $(-\infty, \infty)$  4. $\varnothing$

34. $f(x)=25+|x-9|$ is decreasisng in
1. $(-\infty, 9)$  2. $(9, \infty)$  3. $(-\infty, \infty)$  4. $\varnothing$

35. $f(x) = x - e^x$ is increasing in
1. $(-\infty, 0)$  2. $(0, \infty)$  3. $(-\infty, \infty)$  4. $\varnothing$

36. $f(x) = 1 - x^3$ is decreasing in
1. R  2. $(0, \infty)$  3. $(-\infty, 0)$  4. $\varnothing$

37. $f(x)=\frac{x}{\log x}-\frac{\log 5}{5}$ is increasing in
1. $(e, \infty)$  2. (0, 1) U (1, e)
3. (0, 1)  4. (1, e)

38. $f(x)=\frac{x}{\log x}-\frac{\log 5}{5}$ is decreasing in
1. $(e, \infty)$  2. (0, 1) U (1, e)
3. (0, 1)  4. (1, e)

39. $f(x) = x^3 - 27x + 5$isstrictly increasing when

1. $x < -3$ 2. $|x| > 3$

3. $x \le -3$ 4. $|x| < 3$

40. The function $2x^2 - \log x$ is a decreasing function if 'x' belongs to the interval

1. $\left(-\frac{1}{2}, \frac{1}{2}\right)$ 2. $\left(-\frac{1}{2}, 0\right)$ 3. $\left(0, \frac{1}{2}\right)$ 4. (-2, 2)

41. The set of all 'x' for which log (1+x) $\le$ x is

1. $(-\infty, 1)$ 2. $(-1, \infty)$ 3. [-1, 1] 4. $(-\infty, \infty)$

42. $y = \frac{x^2}{2} - \log x$ decreases in the interval

1) (0,1) 2) [0,1) 3) [0,1] 4) (0,1]

43. A stationary point of $f(x) = \sqrt{16 - x^2}$ is

1. (4, 0) 2. (-4, 0) 3. (0, 4) 4. (-4, 4)

44. A stationary value of $f(x) = x(\ln x)^2$ is

1. $2e^{-2}$ 2. $4e^{-2}$ 3. $2e^2$ 4. $4e^2$

45. The stationary point of $f(x) = x^2 - 10x + 43$ is

1. (5, 18) 2. (18, 5) 3. (5, 5) 4. (5, 15)

46. The critical point of $f(x) = |2x + 7|$ at x =

1. 0 2. 7 3. $\frac{-7}{2}$ 4. -7

47. $f(x) = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$ is stationary at

1. $x = \frac{1}{\sqrt{2}}$ 2. $x = \frac{\pi}{4}$ 3. x = 1 4. x = 0

48. The stationary point of $f(x) = e^x + e^{-x}$ is

1. (1, 2) 2. (2, 3) 3. (1, 3) 4. (0, 2)

49. The minimum value of $f(x) = x + \frac{4}{x+2}$ is

1. -1 2. -2 3. 1 4. 2

50. $f(x) = x^3 + 27x + 5$

1. Has a maximum 2. Has a minimum

3. Neither max nor min is true

4. Both max and min are true

51. f(x) = $|x|$ is minimum at x =

1. 1 2. 0 3. -1 4. 2

52. The maximum of $f(x) = \frac{x^4}{4} - \frac{3}{2}x^2$ occurs at x =

1. 3 2. 0 3. -1 4. 2

53. The minimum value of 16 cot x + 9 tan x is

1. 12 2. 6 3. 24 4. 25

54. in $(0, 2\pi)$, f(x)= x + sin 2x is

1. Minimum at $x = \frac{2\pi}{3}$ 2.Maximum at $x = \frac{2\pi}{3}$

3. Maximum at $x = \frac{\pi}{4}$ 4.Minimum atr $x = \frac{\pi}{6}$

55. The minimum value of $f(x) = 4\sec^2 x + 9\text{cosec}^2 x$ is

1. 5 2. 25 3. 13 4. $13^2$

56. $f(x) = \frac{x}{1 + x\tan x}$ is maximum when x =

1. sin x 2. cos x 3. tan x 4. cot x

57. The maximum value of

$5\cos\theta + 3\left(\cos\left(\theta + \frac{\pi}{3}\right)\right) + 3$ is

1. 5 2. 10 3. 11 4. -1

58. The minimum value of $f(x) = x^2 + \frac{250}{x}$ is

1. 15 2. 25 3. 45 4. 75

59. The minimum of $f(x) = (x-2)^{\frac{2}{3}}$ occurs at x =

1. 2 2. 1 3. 0 4. -2

60. The maximum value of f(x) =100-$|45 - x|$ is

1. 100 2. 145 3. 55 4. 45

61. Minimum value of $2^{x^2+8x+6}$ occurs at x =

1. 2 2. -2 3. 4 4. -4

62. $f(x) = |3x - 4|$ has least value at x =

1. $\frac{3}{4}$ 2. $\frac{4}{3}$ 3. 3. 4. 4

63. The least, greatest values of $f(x) = xe^{-x}$ on $[0, \infty)$ are

1. $0, \frac{1}{e}$ 2. $\frac{1}{e}, \frac{2}{e^2}$ 3. $\frac{2}{e^2}, \frac{3}{e^3}$ 4. $\frac{1}{e}, \frac{2}{e}$

64. The value of 'a' for which f(x) = a sin $x + \frac{1}{3}$ sin 3x has an extremum at $x = \frac{\pi}{3}$ is

1. 1 2. -1 3. 0 4. 2

65. The absolute maximum of $y = x^3 - 3x + 2$ in $0 \le x \le 2$ is

1. 4 2. 6 3. 2 4. 0

66. Greatest value of $\left(\frac{1}{x}\right)^x$ is

1. e 2. $(e)^{\frac{1}{e}}$ 3. $\left(\frac{1}{e}\right)^e$ 4. $\frac{1}{e}$

67. The profit P of a function of the selling price p is given by $P = 1 + 36000p - 6000p^2$. At what price is the profit maximum

1. Rs.3 2. 1.15
3. Rs.0.50 4. Rs. 1.50

68. The sum of two +ve numbers is 20. If the sum of their square is minimum then one of the number is

1. 6 2. 4 3. 10 4. 8

69. If the sum of the +ve numbers is 18 then the maximum value of their product is

1. 81 2. 85 3. 72 4. 80

70. If the product of two + ve numbers is 256 then the least value of their sum is

1. 32 2. 16 3. 48 4. 40

71. Two sides of a triangle are given. If the area of the triangle is maximum then the angle between the given sides is

1. $45^{\circ}$ 2. $30^{\circ}$ 3. $60^{\circ}$ 4. $90^{\circ}$

72. The sides of a rectangle are (6 - x) cm and (x - 3) cm. If its area is maximum then x =

1. 4 2. 4.5 3. 4.8 4. 4. 6

73. A triangle of maximum area is inscribed in a circle. If a side of the trinagle is $20\sqrt{3}$ then the radius of the circle is

1. 20 2. 30 3. 40 4. 60

74. x and y are two +ve numbers suchs that xy = 1. Then the minimum value of x + y is

1. 4 2. $\frac{1}{4}$ 3. $\frac{1}{2}$ 4. 2

75. The minimum value of $|x-3|+|x-7|$ is

1. 3 2. 7 3. 4 4. 5

76. The point on the curve $x^2 = 2y$ which is closest to the point (0, 5) is

1. $(2\sqrt{2}, 4)$ 2. (4, 8) 3. $(\sqrt{2}, 1)$ 4. (2, 2)

77. At x = 0, $f(x) = \cos x - 1 + \frac{x^2}{2} - \frac{x^3}{6}$

1. Has a minimum 2. Has a maximum
3. Does not have an extremum
4. Is not defined

78. At x = 0, $f(x) = \sin x - x + \frac{x^3}{6} - \frac{x^4}{24}$

1. Has a minimum 2. Has a maximum
3. Does not have an extremum
4. Is not defined

79. The least and the greatest value of $f(x) = x^2 \log x$ in [1, e] are

1. log 2, log 4 2. 0, $e^2$
3. $e^2$, $e^4$ 4. $\frac{1}{e}$, e

80. The greatest value of f(x) = x - 2 log x in [1, e] is attained at x =

1. 1 2. $\sqrt{e}$ 3. 2 4. $\frac{e}{2}$

81. The maximum possible area that can be enclosed by a wire of length 20 cms by bending it into the form of a sector in sq. cms is

1. 10 2. 25 3. 30 4. 15

82. The sum of two numbers is 6. The minimum value of the sum of their reciprocals is

1. $\frac{3}{4}$ 2. $\frac{6}{5}$ 3. $\frac{2}{3}$ 4. $\frac{2}{5}$

83. The minimum value of $f(x) = 9x + \frac{25}{x}$ is

1. 34 2. -34 3. 30 4. -30

84. The least value of $9\sec^2 x + 16\operatorname{cosec}^2 x$ is

1. 25 2. 49 3. 81 4. 64

85. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the point x =

1. 0 2. $\frac{1}{4}$ 3. $\frac{1}{2}$ 4. $\frac{1}{3}$

86. If A > 0, B > 0, and A + B = $\frac{\pi}{3}$ then the maximum value of tan A tan B is

1. $\frac{1}{\sqrt{3}}$ 2. $\frac{1}{3}$ 3. 3 4. $\sqrt{3}$

87. The maximum value of $f(x) = \frac{1}{2e^x + e^{-x}}$ is

1. $\frac{1}{2\sqrt{2}}$ 2. $2\sqrt{2}$ 3. $\frac{1}{2}$ 4. $\frac{e}{2e^2+1}$

88. The perimeter of a sector is given. The area is maximum when the angle of the sector is

1. 1radian 2. 2radians
3. 3radians 4. 4 radians

89. The maximum area of a recetangle inscribed in a circle of radius 5 cm is

1. 25 sq. cm 2. 50 sq. cm
3. 100 sq.cm 4. $\frac{25}{2}$ sq.cm

90. The maximum height of the curve y = 6 cos x - 8 sin x above the X - axis

1. 6 2. 8 3. 14 4. 10

91. N characters of information are held on a magnetic tape in batches of x characters each, the batch processing time being $(4+9x^2)$ seconds. The optimal value of x for fast processing is

1. $\frac{4}{9}$ 2. $\frac{9}{4}$ 3. $\frac{2}{3}$ 4. $\frac{3}{2}$

92. The greatest value of the function $f(x) = \sin^2 x - 20\cos x + 1$ is

1. 20 2. 11 3. 21 4. 0

93. Minimum value of $\frac{(6+x)(11+x)}{2+x}$ is

1. 5 2. 15 3. 45 4. 25

94. The minimum value of $4e^x + 9e^{-x}$ is

1. 5 2. 25 3. 12 4. 13

95. The least value of $f(x) = (\sin^{-1}x)^3 + (\cos^{-1}x)^3$

1. $\frac{\pi}{2}$ 2. $\frac{\pi^3}{8}$ 3. $\frac{\pi^3}{32}$ 4. $\frac{7\pi^3}{8}$

96. The least value of $f(x) = \frac{x^3}{3} - abx$ occurs at $x =$

1) G.M of a and b 2) A.M. of a and b
3) H.M. of a and b 4) sum of a and b

97. The points on the hyperbola $x^2 - y^2 = 2$ closest the point (0, 1) are

1) $\left(\pm\frac{3}{2}, \frac{1}{2}\right)$ 2) $\left(\frac{1}{2}, \pm\frac{3}{2}\right)$

3) $\left(\frac{1}{2}, \frac{1}{2}\right)$ 4) $\left(\pm\frac{3}{4}, \pm\frac{3}{2}\right)$

98. Which fraction exceeds its $P^{th}$ power by the greatest number possible is?

1) $P^P$ 2) $\left(\frac{1}{P}\right)^{P-1}$ 3) $P^{\frac{1}{1-P}}$ 4) $P^{\frac{1}{P}}$

99. The least value of $5^{\sin x+1} + 5^{-\sin x+1}$ is

1) 5 2) 10 3) 15 4) 11

100. If $x, y$ are two real numbers such that $x^2 + y^2 = 1$, then the maximum value of $x + y$ is

1) $\sqrt{2}$ 2) $\sqrt{5}$ 3) 2 4) 6

101. If a, b, c are the lengths of sides of a trangle, then the minimum value of $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$

1) 1 2) 2 3) 5 4) 3

102. If a, b, c are the sides of a triangle such that $a^2 + b^2 + c^2 = 1$, then the maximum value of ab + bc + ca is

1) 1 2) $\frac{1}{2}$ 3) $\frac{2}{3}$ 4) $\frac{5}{6}$

103. The greatest value of the function f(x) = $\sin\left\{x[x] + e^{[x]} + \frac{\pi}{2} - 1\right\}$ for all $x \in [0, \infty)$ is

1) -1 2) 0 3) 1 4) 2

104. A particle is moving in a straight line such that its distance at any time 't' is given by S = $\frac{t^4}{4} - 2t^3 + 4t^2 + 7$ then its acceleration is minimum at t =

1) 1 2) 2 3) 1 / 2 4) 3 / 2

105. If $-4 \le x \le 4$ then the critical points of $f(x) = x^2 - 6|x| + 4$ are

1) 3, -2 2) 6, -6 3) 0, 1 4) 3, -3

106. The distance 'S' described in time 't' by a particle moving in a straight line is given by S = $t^5 - 40t^3 + 30t^2 + 80t - 250$ then the maximum acceleration is

1) 260 units/sec$^2$ 2) -260 units/sec$^2$
3) 130 units/sec$^2$ 4) 380 units/sec$^2$

107. For a particle moving on a straight line it is observed that the distance 'S' at time 't' is given by S = 6t - $\frac{t^3}{2}$, the maximum velocity during the motion is

1) 3 2) 6 3) 9 4) 12

108. The number of critical points of f(x) = $\frac{|x-1|}{x^2}$ is

1) 1 2) 2 3) 3 4) 0

109. The absolute minimum of y = c cosh x/c is

1) 1/c 2) c/2 3) c 4) 2c

## KEY

| | | | | |
|---|---|---|---|---|
| 1. 2 | 2. 3 | 3. 2 | 4. 3 | 5. 1 |
| 6. 1 | 7. 2 | 8. 3 | 9. 2 | 10. 3 |
| 11. 1 | 12. 4 | 13. 1 | 14. 2 | 15. 3 |
| 16. 3 | 17. 3 | 18. 3 | 19. 1 | 20. 2 |
| 21.3 | 22. 2 | 23. 4 | 24. 1 | 25. 2 |
| 26. 2 | 27. 3 | 28. 3 | 29. 1 | 30. 2 |
| 31. 1 | 32. 2 | 33. 2 | 34. 1 | 35. 1 |
| 36. 1 | 37. 1 | 38. 2 | 39. 2 | 40. 3 |

| | | | | |
|---|---|---|---|---|
| 41. 2 | 42. 1 | 43. 3 | 44. 2 | 45. 1 |
| 46. 3 | 47. 1 | 48. 4 | 49. 4 | 50. 3 |
| 51. 2 | 52. 2 | 53. 3 | 54. 1 | 55.2 |
| 56. 2 | 57. 2 | 58. 4 | 59. 1 | 60. 1 |
| 61. 4 | 62. 2 | 63.1 | 64. 4 | 65. 1 |
| 66. 2 | 67. 1 | 68. 3 | 69. 1 | 70. 1 |
| 71. 4 | 72. 2 | 73. 1 | 74. 4 | 75. 3 |
| 76. 1 | 77. 3 | 78. 2 | 79. 2 | 80. 1 |
| 81. 2 | 82. 3 | 83. 3 | 84. 2 | 85. 2 |
| 86. 2 | 87. 1 | 88. 2 | 89. 2 | 90. 4 |
| 91. 3 | 92. 3 | 93.4 | 94.3 | 95.3 |
| 96.1 | 97.1 | 98.3 | 99.2 | 100.1 |
| 101.4 | 102.1 | 103.3 | 104.2 | 105.4 |
| 106.2 | 107.2 | 108.3 | 109.3 | |

## HINTS

1. On verification $f^1(x) < 0, \ \forall x \in (2, 3)$
2. On verification $f^1(x) > 0$ v $\forall x \in (-\infty, -5) \cup (5, \infty)$
3. On verification $f^1(x) > 0, \ \forall x \in \left(\frac{-\pi}{2}, \frac{\pi}{6}\right)$
4. $\forall x_1, x_2 \in (-\infty, 3) x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$
6. $\forall x_1, x_2 \in (7, \infty); x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$
7. On verification $f^1(x) > 0, \ \forall x \in (2, \infty)$
8. $\forall x_1, x_2 \in (15, 30); f(x_1) = f(x_2)$
9. On verification $f^1(x) > 0$ (or) $f^1(x) < 0$,
   $\forall x \in (-\infty, 0) \cup (2, \infty)$
10. On verification $f^1(x) > 0, \ \forall x \in (-\infty, 1) \cup (2, \infty)$
11. On verification $f^1(x) > 0, \ \forall x \in (-\infty, \infty)$
12. $x > 0$, on verification $f^1(x) \geq 0, \ \forall x \in \left[\frac{1}{2}, \infty\right)$
13. $f^1(x) > 0$ in that $\Delta < 0$ then $a \in (-12, 12)$
14. On verification $f^1(x) < 0, \ \forall x \in (-1, 0)$ U$(0, 1)$
15. On verification $f^1(x) > 0, \ \forall x \in (-\infty, \infty)$
16. $f^1(x) > 0$ then $x \in (1, \infty)$
17. On verification $f^1(x) > 0, \ \forall x \in (4, \infty)$
18. $\forall x_1, x_2 \in (0, \infty), x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$
20. On verification $f^1(x) < 0, \ \forall x \in (1, \infty)$
21. On verification $f^1(x) < 0, \ \forall x \in (-\infty, -2)$
22. On verification $f^1(x) > 0, \ \forall x \in (-10, 0)$
23. On verification $f^1(x) < 0, \ \forall x \in (-\infty, 1)$
24. On verification $f^1(x) > 0, \ \forall x \in (-\infty, \infty)$
25. On verification $f^{I}(x) > 0$ for $|x| > 3$
26. $f^{I}(x) > 0$ at $x > 1$
27. On verification $f^{I}(x) > 0, \ \forall x \in (-\infty, \infty)$
28. On verification $f^{I}(x) > 0, \ \forall x \in (-\infty, \infty)$
29. On verification $f^{I}(x) > 0, \ \forall x \in (-1, 0)$
30. On verification $f^{I}(x) < 0, \ \forall x \in (0, 1)$
31. On verification $f^{I}(x) > 0 \ \forall x \in (0, \infty)$
32. On verification $f^{I}(x) < 0 \ \forall x \in (-\infty, 0)$
33. On verification
34. On verification
35. $f^{I}(x) > 0$
    $1 - e^x > 0$ ; $e^x < 1$ at $(-\infty, 0)$ it is increasing
36. On verification $f^{I}(x) < 0$ in R.
37. On verification $f^1(x) > 0, \ \forall x \in (e, \infty)$
38. On verification $f^1(x) < 0, \ \forall x \in (0, 1) \cup (1, e)$
40. $f^1(x) < 0$
41. Verification.
43. Clearly $f^1(x) = 0$ at $x = 0$, and
    $f(0) = \sqrt{16 - 0} = \sqrt{16} = 4$
    stationary point = (0, f(0) = (0, 4)
44. Clearly $f^1(x) = 0$ at $x = e^{-2}$ Therefore stationary value $= f(e^{-2})$.
45. Clearly $f^1(x) = 0$ at $x = 5$, Therefore Stationary point = (5, f(5))
46. at $x = -7/2$
47. $\sin^{-1}x = \cos^{-1}x$ Then $x = \frac{1}{\sqrt{2}}$
48. $f^1(x) = 0 \Rightarrow x = 0$; Stationary point (0, f(0))
49. Clearly at $x = 0$, $f^{I}(x) = 0$ and $f^{II}(x) > 0$
    $\therefore$ Minimum value = f(0)
50. For any value of x, $f^{I}(x) \neq 0$
51. On verification
52. On verification at $x = 0$, $f^{I}(x) = 0$ and $f^{II}(x) < 0$
53. Minimum value of $a \cot x + b \tan x$ is $2\sqrt{ab}$
54. On verification at $x = 2\pi / 3$, $f^{I}(x) = 0$
    $f^{II}(x) > 0$
    $\therefore$ Minimum at $x = 2\pi / 3$
55. Minimum value of $a^2\sec^2 x + b^2 \operatorname{cosec}^2 x$ is $(a+b)^2$
56. Clearly at $x = \cos x$ $f^{I}(x) = 0$ and $f^{II}(x) < 0$
    $\therefore$ f(x) has maximum at $x = \cos x$

57. 5 cos $\theta$ + $3\left(\cos\left(\theta+\frac{\pi}{3}\right)\right)+3$

$=\frac{13}{2}\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta+3$

Maximum value of a cos$\theta$ + bsin$\theta$ + c is

$c+\sqrt{a^2+b^2}$

58. Clearly at x = 5, $f^{I}(x) = 0$ and $f^{II} > 0$
Minimum value f (5).

59. $f(x)$ *is* minimum *at* $x=\frac{-b}{2a}=2$

60. at x = 45 f takes maximum value

61. Clearly at x = -4, $f^{I}(x) = 0$ and $f^{II}(x) > 0$

62. f (x) = 0 at x = 4/3

63. at x = 0 min
at x = 1 max

64. f(x) has an extremum at x = $\pi$/3
$\Rightarrow f^{I}(x) = 0$ at x = $\pi$/3

65. $\therefore$ absolute maximum = f(2) = 4

66. $f^{I}(x) = 0$ at x = 1/e

67. Derivative = 0 and find P value

68. Half of the sum of numbers (It is minimum)

69. x = y = 18/2 = 9 then product = 81.

70. x = y = $\sqrt{256}=16$ Then sum = 2(16) = 32.

71. $\Delta=\frac{1}{2}$ ab Sin C. Then derivative is Zero;

cos C = 0; C = $90^0$

72. 6-x = x-3 $\Rightarrow x=\frac{9}{2}$

73. Equilateral Triangle A = B = C = $60^0$ .
Then 2R sin60 = $20\sqrt{3}\Rightarrow$ R= 20

74. x = y = 1 Then x+y=2

75. The min value of $|x-a|+|x-b|$ is $|a-b|$

76. Calculate distance and Verify

77. At x = 0, $f^1(x) = 0$, $f^{11}(x) = 0$ and $f^{111}(x) = -1 \neq 0$

78. $f'(x)=f''(x)=f'''(x)=0, f^{iv}(x)=-1<0$

79. Put x = 1 and x = e

80. Verification

81. l+2r = 20 $\Rightarrow$ l = 10 and r = 5;
Area = (1/2)lr = 25

82. x = y = 6/2 = 3, 1/x+1/y = 2/3

83. For ax+b/x, The min value is $2\sqrt{ab}$

84. For $a^2 \sec^2x+b^2 \operatorname{Cosec}^2x$;
The min value is $(a+b)^2$

85. $f^1(x) = 0 \Rightarrow x = 1/4$

86. A = B = $\frac{\pi}{6}$. Then TanA .TanB = 1/3

87. $f^1(x) = 0 \Rightarrow e^x=\frac{1}{\sqrt{2}}$. Then max value = $\frac{1}{2\sqrt{2}}$

88. $r\theta+2r=k\Rightarrow r=\frac{k}{\theta+2}; A=\frac{1}{2}r^2\theta=\frac{1}{2}\frac{k^2\theta}{(\theta+2)^2}$

$\frac{dA}{d\theta}=0\Rightarrow\theta=2$ Radians

89. $2r^2 = 2(5)^2 = 50$ Sq.cm

90. $\sqrt{a^2+b^2}=\sqrt{36+64}=10$

91. $\sqrt{\frac{a}{b}}=\sqrt{\frac{4}{9}}=\frac{2}{3}$

92. $f^1(x) = 0$ ; x= $\pi$ substitute

93. $f^1(x) = 0$ Then put x = 4

94. $f^1(x) = 0 \Rightarrow e^x = 3/2$

95. $\sin^{-1}x = \cos^{-1}x \Rightarrow x=\frac{1}{\sqrt{2}}, f(x)=\frac{\pi^3}{64}+\frac{\pi^3}{64}=\frac{\pi^3}{32}$

## LEVEL-2

1. $f(x) = x - \log\left(\frac{x+1}{x}\right)$ (x > 0) is increasisng in

1. (0, $\infty$ )  2. (1, 2)
3. (1, 2) U (2, $\infty$ )  4. (-$\infty$ , $\infty$ )

2. $f(x)=\log(1+x)-\frac{2x}{x+2}(x>0)$ is increasing in interval

1. (1, 3)  2. (0, $\infty$ )  3. (0, 1)  4. (1, $\infty$ )

3. The least integral value of ‘a’ such that the function $x^2+ax+1$ is strictly increasing on $[1,2]$ is

1) -1  2) -3  3) -4  4) -2

4. f(x) = 2x-$\tan^{-1}$x-$\log\left(x+\sqrt{1+x^2}\right)(x>0)$ is increasing in

1. (1, 2)  2. (0, 1) U (2, $\infty$ )
3. (0, $\infty$ )  4. (-$\infty$ , $\infty$ )

5. The function $f(x)=\cos x-2\lambda x$ is monotonically decreasing when

1) $\lambda>\frac{1}{2}$  2) $\lambda<\frac{1}{2}$
3) $\lambda<2$  4) $\lambda>2$

6. The function f(x) = 2 log(x-2) - $x^2$+4x+1 increases in the interval

1. (1, 2)  2. (2, 3)  3. (2, 4)  4. (0, $\infty$ )

7. $f(x)=\dfrac{x}{x^2+5x+4}$ is maximum at x =

1. 2 2. -2 3. 6 4. -6

8. The minimum of $f(x)=\dfrac{1+x+x^2}{1-x+x^2}$ occurs at x=

1. -1 2. 1 3. 2 4. -2

9. The minimum ordinate of the curve $y=\dfrac{x^2+1}{4x-3}$ is

1. 1 2. -1 3. 2 4. 4

10. At x = .... $f(x) = 12x^5 - 45x^4 + 40x^3 + 7$ is maximum

1. 1 2. 0 3. 2 4. -2

11. The maximum value of $f(x) = (x-2)^2(x-3)$ is

1. 2 2. 4 3. -4 4. 0

12. If x+y = 28 then the maximum value of $x^3y^4$ is

1. $4^3 . 24^4$ 2. $12^3 .16^4$ 3. 4321 4. 1234

13. The minimum value of $f(x)=\dfrac{x}{1+x^2}$ is

1. $\dfrac{1}{2}$ 2. $\dfrac{2}{5}$ 3. $\dfrac{-1}{2}$ 4. $\dfrac{-2}{5}$

14. If 2x+y = 5 then the maximum value of $x^2+3xy+y^2$ is

1. $\dfrac{125}{4}$ 2. $\dfrac{4}{125}$ 3. $\dfrac{625}{4}$ 4. $\dfrac{4}{625}$

15. If $f(x) = a \log x + bx^2 + x$ has extreme values at x = -1, x = 2 then a = ....., b = ......

1. $2,\dfrac{-1}{2}$ 2. $\dfrac{-1}{2},2$ 3. $\dfrac{1}{2},2$ 4. $2,\dfrac{1}{2}$

16. $f(x) = \sin x. \cos^3x$ has a maximum at x =

1. $\dfrac{\pi}{4}$ 2. $\dfrac{\pi}{3}$ 3. $\dfrac{\pi}{6}$ 4. $\dfrac{\pi}{2}$

17. The maximum value of $12 \sin\theta - 9 \sin^2\theta$ in $\left[0,\dfrac{\pi}{2}\right]$ is

1. 2 2. 4 3. 6 4. 0

18. At x = 0, $f(x) = (3 - x)e^{2x} - 4xe^x - x$

1. Has a minimum 2. Has a maximum
3. Has no extremum 4. Is not defined

19. The minimum values of $f(x)=2^{\left(x^2-3\right)^3+27}$

1. $2^{27}$ 2. 1 3. 4 4. 16

20. The least and the greatest values of $f(x) = 2 \sin x + \sin 2x$ on $\left[0,\dfrac{3\pi}{2}\right]$ are

1. $\dfrac{-3\sqrt{3}}{2.},2$ 2. $-2,\dfrac{3\sqrt{3}}{2.}$

3. $\dfrac{-3\sqrt{3}}{2.},\dfrac{3\sqrt{3}}{2}$ 4. -2, 2

21. The maximum value of $f(x) = 1+ 2 \sin x + 2 \cos^2x$ in $\left[0,\dfrac{\pi}{2}\right]$ is

1. $\dfrac{7}{2}$ 2. $\dfrac{5}{2}$ 3. $\dfrac{3}{2}$ 4. $\dfrac{1}{2}$

22. The function $f(x) = 4x^5 - 25x^4 + 40x^3 - 10$ has

1. One maximum and one minimum
2. One maximum and two minimum
3. Two maximum, one minimum
4. Two maximum, two minimum

23. The sum of two +ve numbers is 100. If the product of the square of one number and the cube of the other is maximum then the numbers are

1. 60, 40 2. 20, 80 3. 80, 20 4. 40, 60

24. A straight line segment through the point (3, 4) in the first quadrant meets the coordinate axes in A and B. The minimum area of $\Delta AOB$ is

1. 42 2. 64 3. 48 4. 24

25. The slope of the curve represented by $y = 3x^3+5x^2+4x+9$ is minimum at x =

1. $\dfrac{-5}{9}$ 2. $\dfrac{5}{9}$ 3. $\dfrac{-5}{3}$ 4. $\dfrac{-5}{8}$

26. The least intercept made by the cordinate axes on a tangent to the ellipse $\dfrac{x^2}{64}+\dfrac{y^2}{49}=1$ is

1. 40 2. 30 3. 15 4. 100

27. A rod AB of length 10 cms slides between two perpendicular lines OX, OY. The maximum area of the $\Delta OAB$

1. 50 2. 20 3. 25 4. 60

28. The point on the curve $xy^2 = 1$ which is nearest to the origin is

1. (1, 1) 2. (1, -1) 3. $\left(4,\dfrac{1}{2}\right)$ 4. $\left(\dfrac{1}{2^{\frac{1}{3}}},2^{\frac{1}{6}}\right)$

29. Through the point A(2, 3) a straight line is drawn making +ve intercepts on the coordinates axes. If the area of the triangle so formed is least then the ratio of the x, y intercepts is

1. 2 : 3 2. 3 : 2 3. 4 : 9 4. 9 : 4

30. The maximum distance of the normal to the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ from its centre is

1. $\dfrac{1}{2}$ 2. 2 3. 1 4. 4

31. The area of the rectangle of maximum area inscribed in the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$ is

1. 48 2. 41 3. 40 4. 50

32. The diagonal of the rectangle of maximum area having perimeter 100 cm is

1. $10\sqrt{2}$ 2. 10 3. $25\sqrt{2}$ 4. 15

33. A box is made from a piece of metal sheet 24 cms square by cutting equal small squares from each corner and turning up the edges. If the volume of the box is maximum then the dimensions of the box are

1. 16,16,4 2. 9, 9, 6 3. 8, 8, 8 4. 9, 9, 8

34. From a given metal sheet a cylindrical vessel without lid having maximum capacity is made then the ratio of the height and the base radius is

1. 1 : 2 2. 2 : 1 3. 1 : 3 4. 1 : 1

35. An open tank is to be constructed with a square base and vertical sides to hold a given capacity of water. The expense of lining the tank with lead is least when the ratio of the depth and the width is

1. 1 : 1 2. 1 : 2 3. 2 : 1 4. 1 : 3

36. If 40 sq.meters of sheet is used in the construction of an open tank with a sq.base having maximum capacity then the area of the base in sq.meters is

1. $\frac{30}{7}$ 2. $\frac{20}{3}$ 3. $\frac{40}{3}$ 4. $\frac{40}{7}$

37. The vertical angle of a cone of maximum volume and given slant height is

1. $\tan^{-1}\sqrt{2}$ 2. $2\tan^{-1}\sqrt{2}$

3. $\tan^{-1}\sqrt{3}$ 4. $2\tan^{-1}\sqrt{3}$

38. A closed cylinder of given volume will have least surface area when the ratio of its height and base radius is

1. 2 : 1 2. 1 : 2 3. 2 : 3 4. 3 : 2

39. Maximum area of the rectangle inscribed in a circle of radius 10 cms is

1. 100 2. 200 3. 400 4. 1600

40. The dimensions of a rectangle of maximum area inscribed in a semi circle of radius 8 cms is

1. $8\sqrt{2},4\sqrt{2}$ 2. $8\sqrt{2},8\sqrt{2}$

3. $4\sqrt{2},4\sqrt{2}$ 4. $4\sqrt{2},\sqrt{2}$

41. The in-radius of the triangle of maximum area inscribed in a circle of radius 46 cm is

1. 46 2. 23 3. 92 4. 45

42. A box without lid having maximum volume is made out of square metal sheet of edge 60 cms by cutting equal square pieces from the four corners and turning up the projecting pieces to make the sides of the box. The height of the box is

1. 60 2. 10 3. 15 4. 12

43. A quadratc function in 'x' has the value 10 when x = 1 and has minimum value '1' when x = -2 the function is

1. $2x^2 + 3x + 5$ 2. $3x^2 + 2x + 5$

3. $x^2 + 3x + 6$ 4. $x^2 + 4x + 5$

44. A running track 440 ft. is to be laid out enclosing foot ball field the shape of which a rectangle with a semi circle at each end. If the area o f the rectangular position is to be maximum then the dimensions of the rectangle are

1. 100, 70 2. 110, 70

3. 100, 80 4. 110, 60

45. if xy(y-x) = 16 then y has a minimum value when x=

1. 1 2. 3 3. 2 4. 4

46. ABCD is a rectangle in which AB = 10 cms, BC = 8 cms. A point P is taken on AB such that PA = x. Then the minimum value of $PC^2+PD^2$ is obtained when x =

1. 10 2. 5 3. 8 4. 4

47. AB is a line segment of length 24 cm. P is a point on AB such that $AP^2 + PB^2$ is minimum then AP=

1. 12 2. 6 3. 18 4. 24

48. The difference between two +ve numbers is 10. If the difference between the square of the greater and twice the square of the smaller is maximum then the numbers are

1. 20,10 2. 10, 20 3. 30, 20 4. 20, 30

49. The total cost of producing x pocket radio sets per day is Rs. $\left(\frac{1}{4}x^2 + 35x + 25\right)$ and the price per set at which they may be sold is Rs. $\left(50 - \frac{x}{2}\right)$ to obtain maximum profit the daily out put should be .... radio sets

1. 10 2. 5 3. 15 4. 20

50. The equation of the line through (3, 4) which cuts from the first quadrant a triangle of minimum area is

1. 4x + 3y - 24 = 0 2. 3x + 4y - 12 = 0

3. 2x + 3y - 12 = 0 4. 3x + 2y - 24 = 0

51. The value of a for which the difference of the roots of the equation $ax^2 + (a - 1)x + 2 = 0$ is minimum is given by

1. $\frac{1}{5}$ 2. 5 3. $\frac{-1}{5}$ 4. -5

52. $y = (x(x-3))^2$ increases for all values of 'x' lying in the interval

1. $0 < x < \frac{3}{2}$ 2. $0 < x < \infty$

3. $-\infty < x < 0$ 4. $1 < x < 3$

53. The image of the interval [-1, 3] under the mapping $f(x) = 4x^3 - 12x$ is

1. [-2, 0] 2. [-8, 72] 3. [-8, 0] 4. [-8, -2]

54. The maximum of $f(x) = 2x^3 - 9x^2 + 12x + 4$ occurs at x =

1. 1 2. 2 3. -1 4. -2

55. $f(x) = 4 + 5x^2 + 6x^4$ has
1. Only one minimum
2. Neither maximum nor minimum
3. Only one maximum
4. Can not be determined

56. $f(x) = (x-1)(x-2)(x-3)$ is minimum at x =
1. $3+\frac{1}{\sqrt{2}}$  2. $3-\frac{1}{\sqrt{2}}$
3. $2+\frac{1}{\sqrt{3}}$  4. $2-\frac{1}{\sqrt{3}}$

57. $f(x) = (x-2)^2(11-x)$ is maximum at x =
1. 5  2. 6  3. 7  4. 8

58. Maximum value of $(x+5)^4(13-x)^5$ is
1. $7^4.11^5$  2. $6^4.14^5$  3. $8^4.10^5$  4. $7^5.10^5$

59. Minimum value of $f(x) = (x-1)^2 + (x-2)^2 + .. + (x-10)^2$ occurs at x=
1. 7  2. 6  3. 4  4. 5.5

60. The point on the parabola $x^2=4y$ which is nearest to the point (8, 2) is
1. (4, 4)  2. (2, 1)  3. (8, 16)  4. (8, 8)

61. P(3, 4), Q(-7, 6). The point A on x-axis for which PA + AQ is least is
1. (-2, 0)  2. (-1, 0)  3. (3, 0)  4. (2, 0)

62. The least length of the thread required to construct a rectangle of area 256 $cm^2$ is
1. 32  2. 64  3. 40  4. 58

63. Maximum value of $(\sin x)^{\sin x}$ is
1. $\frac{7}{3}$  2. 7  3. $\frac{\pi}{2}$  4. 1

64. The maximum slope of the curve $y = -x^3+3x^2+9x-27$ is
1. 1  2. 12  3. 6  4. 9

65. $f(x)=\sqrt{3}\sin x + \cos x$ has a greatest when x=
1. $\frac{\pi}{6}$  2. $\frac{\pi}{3}$
3. $\frac{\pi}{4}$  4. $\frac{\pi}{2}$

66. The minimum value of $\frac{7}{4\sin x+3\cos x+2}$ is
1. 1  2. $\frac{7}{9}$
3. $\frac{7}{5}$  4. $\frac{7}{3}$

67. The volume of the largest cone that can be inscribed in a sphere of radius 9 cm is
1. $\frac{32}{3}\pi$c.c  2. $72\pi$c.c  3. $288\pi$c.c  4. $\frac{288}{3}\pi$c.c

68. A cubic function of x has maximum value 10 and minimum $\frac{-5}{2}$ when x = -3, x= 2 then the function is
1. $\frac{1}{5}x^3+\frac{3}{10}x^2-\frac{18}{5}x+\frac{19}{10}$
2. $x^3 + 3x^2 - 18x + 19$
3. $2x^3 + 3x^2 - 36x + 10$
4. $x^3 + x^2 + x + 1$

69. The maximum value of $y = 2x^3 - 3x^2 - 36x+10$ is
1. 51  2. 52  3. 53  4. 54

70. The rectangle of perimeter 36 cm is revolved about one side to form a cylinder of maximum volume. The length and breath of the rectangle are in the ratio.
1. 1 : 1  2. 1 : 3  3. 2 : 3  4. 2 : 1

71. A box is made with square base and open top. The area of the material used is 192 sq. cms. If the volume of the box is maximum, the dimensions of the box are
1. 4,4,8  2. 2, 2,4  3. 8, 8, 4  4. 2, 2, 2

72. The height of the maximum cone that can be obtained by revolving a right angled triangle of hypotenuse 1 units about a side is
1. 1/3  2. $1/\sqrt{2}$  3. $1/\sqrt{3}$  4. 1/2

73. Point of extremum of $f(x) = \int_0^x (t-2)^2(t-1)\,dt$ is
1. maximum at x = 1  2. minimum at x = 1
3. maximum at x = 2  4. minimum at x = 2

74. The maximum value of $\sin^2 x \cos^3 x$ is
1. $\frac{6\sqrt{3}}{25\sqrt{5}}$  2. $\frac{9\sqrt{3}}{25\sqrt{5}}$  3. $\frac{9\sqrt{2}}{6\sqrt{5}}$  4. $\frac{\sqrt{2}}{\sqrt{5}}$

75. The minimum value of $(px+qy)$ when $xy=n^2$ is equal to
1) $2n\sqrt{pq}$  2) $2pq\sqrt{n}$
3) $2\sqrt{npq}$  4) $2pqn$

76. The area of the triangle formed by the tangent at any point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the axes is minimum at the point
1) $\left(\frac{a}{\sqrt{2}},\frac{b}{\sqrt{2}}\right)$  2) $\left(\frac{a}{2},\frac{b}{2}\right)$
3) (a, b)  4) $\left(\frac{\sqrt{a}}{2},\frac{\sqrt{b}}{2}\right)$

77. The coordinates of a point P(x, y) lying in the first quadrant of the ellipse $\frac{x^2}{8}+\frac{y^2}{18}=1$ so that the area of the triangle formed by the tangent at 'P' and the axes is the smallest are

1) (3,2) 2) (-2,3) 3) (2,-3) 4) (2,3)

78. The sum of the positive intercepts a, b made by a line on the axes is constant. If the area of the triangle formed by the line and the axes is least, then a : b =

1) 2 : 1 2) 1 : 2 3) 1 : 1 4) 1 : 3

79. The positive number $x$ that exceeds its square by largest amount is

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) 1

80. The sides of a rectangle of greatest area which can be inscribed in an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$

1) 5, 3 2) 2, $\sqrt{2}$

3) $5\sqrt{2}, 3\sqrt{2}$ 4) $3, \sqrt{2}$

81. Let 'P' be a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci $S(ae,0)$ and $S^{/}(-ae,0)$. If A is the area of the triangle PSS$^1$, then the maximum value of A (where e is eccentricity and $b^2=a^2(1-e^2)$) is

1) $\frac{ab}{2}$ 2) 2 abe 3) abe 4) 4abe

82. The difference between the greatest and the least values of the function f(x) = $\int_0^x \left(t^2+t+1\right)dt$ on [2, 3] is

1) $\frac{6}{49}$ 2) $\frac{49}{6}$ 3) $\frac{6}{59}$ 4) $\frac{59}{6}$

83. If $a^2x^4+b^2y^4=c^6$ then the maximum value of xy is

1) $\frac{c^3}{2ab}$ 2) $\frac{c^3}{\sqrt{2ab}}$

3) $\frac{c^3}{ab}$ 4) $\frac{c^3}{\sqrt{ab}}$

84. The maximum value of $4\sin^2x+3\cos^2x+\sin\frac{x}{2}+\cos\frac{x}{2}$ is

1) $4+\sqrt{2}$ 2) $2+\sqrt{2}$

3) $6+\sqrt{2}$ 4) $7+\sqrt{2}$

85. The minimum distance from origin to any point on $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ is

1) a 2) a/2

3) 2a 4) $a^{2/3}$

86. The greatest distance of a normal to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from its centre is

1) a - b 2) a + b 3) $\frac{a-b}{2}$ 4) $\frac{a+b}{2}$

87 An open rectangular tank with a square base and 32c.c. of capacity has least surface area in sq. cms. is

1) 48 2) 16

3) 32 4) 12

## KEY

| | | | | |
|---|---|---|---|---|
| 01. 1 | 02. 2 | 03. 1 | 04. 3 | 05.2 |
| 06. 2 | 07. 1 | 08. 1 | 09. 1 | 10.1 |
| 11. 4 | 12. 2 | 13. 3 | 14. 1 | 15.1 |
| 16. 3 | 17. 2 | 18. 3 | 19. 2 | 20.2 |
| 21. 1 | 22. 1 | 23. 4 | 24.4 | 25.1 |
| 26. 3 | 27. 3 | 28. 4 | 29. 1 | 30.3 |
| 31. 3 | 32. 3 | 33. 1 | 34. 4 | 35.2 |
| 36. 3 | 37. 2 | 38. 1 | 39. 2 | 40.1 |
| 41. 2 | 42. 2 | 43. 4 | 44. 2 | 45.3 |
| 46. 2 | 47. 1 | 48. 1 | 49. 1 | 50.1 |
| 51. 1 | 52. 1 | 53. 2 | 54. 1 | 55.1 |
| 56. 3 | 57. 4 | 58. 3 | 59. 4 | 60.1 |
| 61. 2 | 62. 2 | 63. 4 | 64. 2 | 65.2 |
| 66. 1 | 67. 3 | 68. 1 | 69. 4 | 70.4 |
| 71.3 | 72. 3 | 73.2 | 74.1 | 75.1 |
| 76.1 | 77.4 | 78.3 | 79.1 | 80.3 |
| 81.3 | 82.4 | 83.2 | 84.1 | 85.1 |
| 86.1 | 87.1 | | | |

# HINTS

1. On verification $f^1(x) > 0$
2. On verification $f^1(x) > 0$
4. On verification $f^1(x) > 0$
6. On verification $f^1(x) > 0$
7. $f^1(x) = 0$
8. $f^1(x) = 0$ and substitute and verify
9. $f^1(x) = 0$ and substitute
10. $f^1(x) = 0$ and $f^{11}(x) < 0$
11. $f^1(x) = 0$ and substitute and verify
12. x = 3 (28/7) = 12, y = 16.
13. $f^1(x) = 0$ and substitute
14. y = 5-2x and substitute and derivative is zero and substitute
15. $f^1(-1) = 0$ and $f^1(2) = 0$
16. $x = \tan^{-1}\sqrt{\frac{p}{q}}$ for $\sin^p x.\cos^q x$
17. a = -9; b= 12; c=0 Then max value $\frac{4ac-b^2}{4a}$
18. at x = 0; $f^1(x)= f^{11}(x)$ ....... = 0
19. $f^1(x) = 0 \Rightarrow x = 0, \sqrt{3}$ at x = 0 min value is $2^0 = 1$
20. min value = $f\left(\frac{3\pi}{2}\right); \max value = f\left(\frac{\pi}{3}\right)$
21. max value $f\left(\frac{\pi}{6}\right)$
22. $f^1(x) = 0$ and observe
23. $x = 2\left(\frac{100}{2+3}\right) = 40$; y = 60
24. area = $2x_1.y_1$
25. $d^2y/dx^2 = 0$
26. (a+b)
27. $a^2 + b^2 = 100 \Rightarrow a^2 = b^2 = 50 \Rightarrow$

    $a = b = 5\sqrt{2} \Rightarrow$ area = 25 Sq.units
28. $D^2 = x^2 + y^2$

    $D^2 = x^2 + 1/x$

    $\frac{d}{dx}(D^2) = 2x - \frac{1}{x^2} = 0$

    $x = \frac{1}{2^{\frac{1}{3}}}, y = 2^{\frac{1}{6}}$
29. $x_1 : y_1$
30. a - b
31. 2ab
32. r = 100/4 = 25 (Square),

    Length of hypotenuse = $r\sqrt{2}$
33. a = 24, K = 24/6 = 4; measurements a-2k, a-2k, k
34. h = r, Then ratio is 1 : 1
35. Standard.
37. Standard result
38. Volume = K(Constant)

    Surface area = $2\pi rh + 2\pi r^2$ minimum

    Then h = 2r
39. $2r^2$
40. $r\sqrt{2}, \frac{r}{\sqrt{2}}$
41. Equilateral trinagle r = R/2
42. $h = \frac{a}{6} = \frac{60}{6} = 10$
43. f(1) = 10, $f^1(-2) = 0$ f(-2) = 1 verify
45. y min $\Rightarrow$ y = 2x then x = 2
47. AP = BP = $\frac{24}{2}$ = 12
48. x - y = 10 and $D = x^2 - 2y^2$ is max. derivative is zero Then x = 20, y = 10
50. a = $2x_1$, b=$2y_1$ and find intercept form
51. $\alpha+\beta = \frac{1-a}{a}; \alpha\beta = \frac{2}{a}, (\alpha-\beta)^2$ is min then

    a= 1/5
52. $f^1(x) > 0$
53. [f(-1), f(3)]
54. $f^1(x) = 0$ and $f^{11}(x)<0$
55. $f^{11}(x) > 0$ at x =0
56. $f^1(x) = 0$ and $f^{11}(x) > 0$
57. $f^1(x) = 0$ and $f^{11}(x) < 0$
58. Sum of numbers = 18; x = $4.\frac{18}{9}$ = 8, y = 10.
59. $x = \frac{\Sigma n}{n} = \frac{\Sigma 10}{10} = \frac{55}{10} = 5.5$
60. Find distance and verify
61. y= 0, verify the distance
62. $x = \sqrt{256} = 16$ (Square).

    Min length = 4(16) = 64
63. $f^1(x) = 0 \Rightarrow x = \frac{\pi}{2}$ and sin x = 1/e.

    max obtained at x = $\frac{\pi}{2}$
64. $\frac{d^2y}{dx^2} = 0$ then find x and substitute in $\frac{dy}{dx}$
65. $f^1(x) = 0 \Rightarrow x = \frac{\pi}{3}$

66. min = $\frac{7}{\max} = \frac{7}{2+5} = 1$

67. R = 9; $\frac{32}{81}\pi R^3$

68. $f^1(-3) = 0$, $f^1(2) = 0$

$f(-3) = 10$ $f(2) = -\frac{5}{2}$ verify

69. $f^1(x) = 0$ and $f^{11}(x) < 0$, and substitute

71. Verify the formula $x^2+4xy$

72. Standard

73. $f^1(x) = 0$; $f^{11}(x) > 0$ at x = 1

74. For $\sin^p x \cos^q x$. The max value is $\left(\frac{p^p.q^q}{(p+q)^{p+q}}\right)^{1/2}$

## LEVEL-3

1. Given the total surface area of a cone its volume is maximum when the semi vertical angle is

1. $\sin^{-1}\frac{1}{3}$ 2. $\sin^{-1}\frac{1}{\sqrt{3}}$

3. $\tan^{-1}\frac{1}{3}$ 4. $\tan^{-1}\frac{1}{\sqrt{3}}$

2. A conical tent of given capacity will require the least amount of canvas when the height is .... times the base radius

1. $\sqrt{3}$ 2. 3 3. 2 4. $\sqrt{2}$

3. The sum of the surface areas of a sphere and a cube is given. When sum of their volumes is least, the ratio of the radius of the sphere and the edge of the cube is

1. 1 : 2 2. 2 : 1 3. 2 : 3 4. 3 : 2

4. Height of the cylinder of maximum volume that can be inscribed in a sphere of radius 12 cm is

1. $8\sqrt{3}$ cm 2. 8 cm 3. $12\sqrt{3}$ cm 4. 24 cm

5. The base radius of a cone of maximum volume, inscribed ina sphere of radius 6 cms is

1. $8\sqrt{2}$ cms 2. $7\sqrt{2}$ cms

3. $4\sqrt{2}$ cms 4. $6\sqrt{2}$ cms

6. A closed pan with a square base and vertical sides can hold 64 cubic metres of water. For the total surface area to be minimum, The dimensions of the pan in metres are

1) 4,4,4 2) 4,6,2 3) 4,4,2 4) 2,2,2

7. The radius of a right circular cylinder of maximum volume which can be inscribed in a sphere of radius R, is

1) R 2) $\frac{R}{2}$ 3) $\sqrt{\frac{2}{3}}R$ 4) $\sqrt{\frac{3}{2}}R$

8. $P(x, y)$ is a point on a straight line which makes intercepts a and b on the x,y axes respectively, then the minimum value of $x^2 + y^2 =$

1) $\frac{a^2+b^2}{a^2b^2}$ 2) $\frac{a^2-b^2}{a^2+b^2}$

3) $\frac{a^2-b^2}{2(a^2+b^2)}$ 4) $\frac{a^2b^2}{a^2+b^2}$

9. $20\pi$ sq.ft. of metal sheet is used to construct an open top cylinder. Relation between height and radius 'r' when it has maximum volume is

1) h = 2r 2) h = r

3) h = $\sqrt{2}r$ 4) r = $\sqrt{2}h$

10. The least value of $(x + 100)^2 + (x + 99)^2 + .....+ (x + 1)^2 + x^2 + (x-1)^2 + (x - 2)^2 + ..... +(x-100)^2$ is

1) 67 67 2) 67 670

3) 676700 4) 767600

## KEY

| | | | | |
|---|---|---|---|---|
| 1.1 | 2.4 | 3.1 | 4.1 | 5.3 |
| 6.1 | 7.2 | 8.4 | 9.2 | 10.3 |

## LEVEL - IV

**MATCHING TYPE QUESTIONS**

1. Let $f : I \to R$ be a function, where I is an intervel subsect of domain of f, then match the following lists.

| **List - I** | **List - II** |
|---|---|
| A) If $x_1, x_2 \in I$ and $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$ then f is | 1) Strictly increasing |
| B) If $x_1, x_2 \in I$ and $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ then f is | 2) Strictly decreasing |
| C) If $x_1, x_2 \in I$ and then f is | 3) decreasing |
| D) If $x_1, x_2 \in I$ and $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$ then f is | 4) increasing |

*The correct matching for list - I from list - II*

| | A | B | C | D |
|---|---|---|---|---|
| 1 | 4 | 3 | 2 | 1 |
| 2 | 4 | 1 | 2 | 3 |
| 3 | 2 | 3 | 1 | 4 |
| 4 | 2 | 3 | 4 | 1 |

2. Observe the following lists

| List - I | List - II |
|---|---|
| A) If f(x) is increasing at x = a then | 1) $f^1(a) < 0$ |
| B) If f(x) is decreasing at x = a then | 2) $f^1(a) = 0$ |
| C) If f(x) is monotonic at x = a then | 3) $f^1(a) > 0$ or $f^1(a) < 0$ |
| D) If f(x) is stationary at x = a then | 4) $f^1(a) > 0$ |
| | 5) $f^1(a) > 0$ and $f^1(a) < 0$ |

*The correct matching for list - I from list - II*

| | A | B | C | D |
|---|---|---|---|---|
| 1. | 4 | 1 | 2 | 3 |
| 2. | 4 | 1 | 3 | 2 |
| 3. | 1 | 4 | 3 | 2 |
| 4. | 4 | 1 | 2 | 3 |

3. Observe the following lists

| List - I | List - II |
|---|---|
| Let f(x) be any function | |
| A) $f^1(a) = 0$ and $f^{11}(a) < 0$ then | 1) f(x) is increasing at x = a |
| B) $f^1(a) = 0$ and $f^{11}(a) > 0$ then | 2) f(x) has maximum value at x = a |
| C) $f^1(a) \neq 0$ then | 3) f(x) has neither maximum nor minimum |
| D) $f^1(a) > 0$ | 4) f(x) has minimum value at x = a |
| | 5) f(x) is decreasing at x = a |

*The correct matching for list - I from list - II*

| | A | B | C | D |
|---|---|---|---|---|
| 1. | 4 | 2 | 3 | 5 |
| 2. | 2 | 4 | 3 | 1 |
| 3. | 2 | 4 | 3 | 5 |
| 4. | 2 | 4 | 5 | 1 |

4. Observe the following lists

| List - I | List - II |
|---|---|
| A) minimum value of $a^2\sin^2 x + b^2 \operatorname{cosec}^2 x$ | 1) $2\sqrt{ab}$ |
| B) minimum value of acotx + b tanx | 2) $(a+b)^2$ |
| C) minimum value of $a^2\sec^2 x + b^2 \operatorname{cosec}^2 x$ | 3) $-\sqrt{a^2+b^2}$ |
| D) minimum value of $a\cos x + b\sin x$ | 4) ab |
| | 5) 2ab |

*The correct matching for list - I from list - II*

| | A | B | C | D |
|---|---|---|---|---|
| 1) | 5 | 1 | 2 | 3 |
| 2) | 5 | 4 | 1 | 2 |
| 3) | 4 | 1 | 2 | 3 |
| 4) | 4 | 5 | 1 | 3 |

5. Observe the following lists

| List - I | List - II |
|---|---|
| A) minimum value of $\lvert x-5 \rvert$ is | 1) -1 |
| B) minimum value of $\lvert x-3 \rvert + \lvert x-7 \rvert$ is | 2) 0 |
| C) maximum value of $\lvert \sin 5x \rvert$ | 3) $\sqrt{2}$ |
| D) maximum value of $\lvert \sin x + \cos x \rvert$ | 4) 1 |
| | 5) 4 |

| | A | B | C | D |
|---|---|---|---|---|
| 1) | 2 | 5 | 3 | 2 |
| 2) | 2 | 5 | 4 | 3 |
| 3) | 2 | 3 | 1 | 4 |
| 4) | 2 | 3 | 5 | 1 |

6. Observe the following lists

| List-I | List-II |
|---|---|
| (A) Maximum value of xy subject to $x+y=7$ is | 1) 72 |
| (B) If $l^2+m^2=1$, then the maximum value of $l+m$ is | 2) 1 |
| (C) If $x+y=12$, then the minimum Value of $x^2+y^2$ is | 3) $\sqrt{2}$ |
| (D) Minimum value $x^2-8x+17$ is | 4) $\frac{49}{4}$ |
| | 5) 0 |

| | A | B | C | D |
|---|---|---|---|---|
| 1) | 4 | 3 | 1 | 2 |
| 2) | 4 | 3 | 2 | 1 |
| 3) | 2 | 3 | 5 | 4 |
| 4) | 2 | 3 | 1 | 4 |

7. Observe the following list

| List -I | List - II |
|---|---|
| (A) $f(x)=x^2-2x+5$ is increasing in | 1) $\phi$ |
| (B) $f(x)=e^{-x}$ is increasing in | 2) $(-\infty,1)\cup(2,\infty)$ |
| (C) $f(x)=\log_e x$ is increasing in | 3) R |
| (D) $f(x)=\frac{x^3}{3}-\frac{3x^2}{2}+2x+5$ is increasing in | 4) $(0,\infty)$ |
| | 5) $(1,\infty)$ |

***The correct matching for list - I from list - II***

| | A | B | C | D |
|---|---|---|---|---|
| 1) | 5 | 1 | 4 | 2 |
| 2) | 5 | 1 | 2 | 3 |
| 3) | 4 | 1 | 5 | 2 |
| 4) | 4 | 1 | 3 | 5 |

8. Observe the following lists

| List -I | List -II |
|---|---|
| (A) $f(x)=x+\frac{1}{x}$ is decreasing in | 1) R |
| (B) $f(x)=1-x^3$ is decreasing in | 2) $\left(0,\frac{1}{e}\right)$ |
| (C) $f(x)=xe^{-x}$ is decreasing in | 3) $(-1,0)\cup(0,1)$ |
| (D) $f(x)=x^x$ is decreasing in | 4) $(1,\infty)$ |
| | 5) $(0,\infty)$ |

***The correct matching for list - I from list - II***

| | A | B | C | D |
|---|---|---|---|---|
| 1) | 3 | 4 | 2 | 5 |
| 2) | 2 | 3 | 5 | 4 |
| 3) | 3 | 1 | 4 | 2 |
| 4) | 2 | 3 | 5 | 4 |

9. Observe the following Lists

| List - I | List - II |
|---|---|
| (A) The number of stationary points of f(x) = cos2x in $(0,2\pi)$ | 1) 1 |
| (B) The number of stationary points of f(x) = sinx in $(0,2\pi)$ | 2) 3 |
| (C) The number of stationary points of $f(x)=e^{-x}$ | 3) 2 |
| (D) The number of stationary points of f(x) = $e^x+e^{-x}$ | 4) 5 |
| | 5) 0 |

***The correct matching for list - I from list - II***

| | A | B | C | D |
|---|---|---|---|---|
| 1) | 2 | 3 | 1 | 5 |
| 2) | 4 | 3 | 2 | 1 |
| 3) | 4 | 1 | 5 | 2 |
| 4) | 2 | 3 | 5 | 1 |

## KEY

| | | | | |
|---|---|---|---|---|
| 1.2 | 2.2 | 3.2 | 4.1 | 5.2 |
| 6.1 | 7.1 | 8.3 | 9.4 | |

### ASSERTION AND REASON TYPE

1. Assertion (A) : The maximum area of the triangle inscribed in a circle of radius '2' units is $3\sqrt{3}$ square units

   Reason (R): The maximum triangle which is inscribed in a circle of given radius is equilateral

   1) Both A and R are true and R is the correct explanation of A
   2) Both A and R are true and R is not the correct explanation of A
   3) A is true and R is false
   4) A is false and R is true

2. Assertion (A): The maximum area of the rectangle inscribed in a circle of radius '5' units is 50 square units

   Reason (R): The maxium area of the rectangle inscribed in a circle is a square

   1) Both A and R are true and R is the correct explanation of A
   2) Both A and R are true and R is not the correct explanation of A
   3) A is true and R is false
   4) A is false and R is true

3. Assertion (A): If f(x) = |x|, then f has minimum value at x = 0

   Reason (R): A function f(x) has minimum value at x = a if $f^1(a) = 0$ and $f^{11}(a)>0$

   1) Both A and R are true and R is the correct explanation of A
   2) Both A and R are true and R is not the correct explanation of A
   3) A is true and R is false
   4) A is false and R is true

4. Assertion (A): The function $f(x) = 2x^3-3x^2-12x+8$ has minimum value -12 at x = 2

   Reason (R): For the fucntion $f(x) = 2x^3-3x^2-12x+8$, $f^1(2)=0$ and $f^{11}(2)>0$

   1) Both A and R are true and R is the correct explanation of A
   2) Both A and R are true and R is not the correct explanation of A
   3) A is true and R is false
   4) A is false and R is true

5. Assertion(A): The minimum value of $16\cot x + 9\tan x$ is 3

   Reason (R): For two positve real numbers a and b, A.M $\geq$ G.M

   1) Both A and R are true and R is the correct explanation of A
   2) Both A and R are true and R is not the correct explanation of A
   3) A is true and R is false
   4) A is false and R is true

6. Assertion (A): If x+y=12 then the minimum value of $x^2+y^2$ is 72

   Reason (R): If x+y=k, then the maximum value of xy is $k^2$

   1) Both A and R are true and R is the correct explanation of A
   2) Both A and R are true and R is not the correct explanation of A
   3) A is true and R is false
   4) A is false and R is true

7. Assertion (A): The minimum value of $f(x) = a^2\sec^2x + b^2\operatorname{cosec}^2x$ is $(a+b)^2$

   Reason (R): For positive real numbers a and b, A.M. $\geq$ G.M.

   1) Both A and R are true and R is the correct explanation of A
   2) Both A and R are true and R is not the correct explanation of A
   3) A is true and R is false
   4) A is false and R is true

8. Assertion(A): The minimum radius vector of the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ is of length a + b

   Reason (R) : The minimum value of $a^2\sec^2\theta + b^2\cos ec^2\theta$ is $(a+b)^2$

   1) Both A and R are true and R is the correct explanation of A
   2) Both A and R are true and R is not the correct explanation of A
   3) A is true and R is false
   4) A is false and R is true

9. Assertion (A): The maximum value of quadratic function $-x^2+3x+1$ is $\frac{11}{4}$

Reason (R): The maximum value of $ax^2+bx+c$ is $\frac{4ac-b^2}{4a}$ if a < 0

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true and R is not the correct explanation of A
3) A is true and R is false
4) A is false and R is true

10. Assertion (A): sin $1^0$ < sin 1

Reason (R): f(x) = sinx is increasing function in $(0, \frac{\pi}{2})$

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true and R is not the correct explanation of A
3) A is true and R is false
4) A is false and R is true

11. Assertion (A): cos $1^0$ > cos 1

Reason (R): f(x) = cos x is decreasing function in $(0, \frac{\pi}{2})$

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true and R is not the correct explanation of A
3) A is true and R is false
4) A is false and R is true

12. Assertion (A): The values of x at which f(x) = sinx is stationary are $(2n+1)\frac{\pi}{2}$ where $n \in z$

Reason (R): If f(x) = sinx then $f^1(x) = 0$ at x = $(2n+1)\frac{\pi}{2}$

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true and R is not the correct explanation of A
3) A is true and R is false
4) A is false and R is true

## KEY

| | | | | |
|---|---|---|---|---|
| **1.1** | **2.1** | **3.2** | **4.1** | **5.4** |
| **6.3** | **7.1** | **8.1** | **9.4** | **10.1** |
| **11.1** | **12.1** | | | |

**Linked Type Questions**

1. I: f(x) = $\log_a x$ (x>0) is an increasing function if a>1

II: f(x) = $\log_a x$ (x>0) is a decreasing function if 0<a<1

Which of the above statements are true

1) only I 2) only II
3) both I and II 4) neither I nor II

2. I: If $f^1(a) < 0$ then the function f is decreasing at x = a

II: If f is decreasing at x = a then $f^1(a)<0$

Which of the above statements are true

1) onlyI 2) only II
3) both I and II 4) neither I nor II

3. I : If $f^1(a) > 0$ then f is increasing at x = a

II : I f f is increasing at x= a then $f^1(a)$ need not to be positive

which of the above statements are true

1) only I 2)only II
3) both I and II 4)neither I nor II

4. I : A rod AB of length 10cms. slides between the perpendicular lines OX, OY .The maximum area of the $\Delta$ OAB is 50 sq. cms.

II: Through a point A(2,3) a straight line drawn making positive intercepts on the co-ordinate axes, then the maximum area of triangle with the axes is 12

which of the above statements are true

1) only I 2)only II
3) both I and II 4)neither I nor II

5. I: Minimum of value of $|x-3|+|x-7|$ is 4

II: Minimum value of $|x-3|+|x-7|$ is 10

which of the above statements are true

1) only I 2)only II
3) both I and II 4)neither I nor II

6. I: $f(x)=\sin x$ is neither increasing nor decreasing in the interval $(0,\pi)$

II: $f(x)=\tan x$ is increasing in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

which of the above statements are true

1) only I
2)only II
3) both I and II
4) neither I nor II

7. I: The greatest value of the function $f(x)=\sin^2 x-20\cos x+1$ is 25

II: The minimum value of $\frac{7}{4\sin x+3\cos x+2}$ is 1

which of the above statements are true

1) only I 2)only II
3) both I and II
4)neither I nor II

8. I: The function $f(x)=2x^3-3x^2-12x+8$ attains minimum value at x=2

II: The function of $f(x)=x^4-6x^2+8x+11$ attains minimum value at x=2

which of the above statements are true

1) only I 2)only II
3) both I and II 4)neither I nor II

9. I: If $f^{I}(a)=f^{II}(a)=f^{III}(a)=0$ and $f^{IV}(a)\neq 0$ then $f(x)$ attains maximum if $f^{IV}(a)$ is negative

II: Every maximum or minimum value must be stationary value

which of the above statements are true

1) only I 2)only II
3) both I and II 4)neither I nor II

10. I: The function $f(x)=9x^2-15x-x^3+10$ is increasing in (1,5)

II: The function $f(x)=9x^2-15x-x^3+10$ is decresing in (1,5)

which of the above statements are true

1) only I 2)only II
3) both I and II 4)neither I nor II

11. I: The sides of a rectangle are (6-x) cm and (x-3) cm. If its area is maximum , then x =4.5

II: The maximum area of the rectangle inscribed in a circle of radius 5cm is 50sq.cm.

which of the above statements are true

1) only I 2)only II
3) both I and II 4)neither I nor II

12. I: The real number x when added to its inverse gives the minimum value of the sum at x = 1

II: If product of the two positive numbers is 400, then the minimum value of their sum is 20

which of the above statements are true

1) only I is true 2) only II is true
3) both I and II are true 4)niether I nor II are true

## KEY

1) 3 2) 1 3) 3 4) 2 5) 1
6) 3 7) 2 8)1 9) 3 10) 1
11) 3 12) 1

## SEQUENCE TYPE QUESTIONS

1) If A, B, C are respectively the minimum values of the functions $\frac{x}{1+x^2},x^2+\frac{16}{x},\frac{x}{\log x}$, then the ascending order of A,B,C is

1) A, B, C 2)A, C, B
3) B, A, C 4)B, C, A

2) The functions $y=xe^x$ has minimum at x = A, $y=x^3-3x$ has minimum at x = B and $y=2x^3-3x^2-12x+5$ has minimum at x = C, then the ascending order of A, B, C is

1) A, B, C 2)B, C, A
3)A, C, B 4) B, A, C

3) Arrange A, B, C, D in ascending order

A) Maximum value of sin5x

B) Minimum value of $x+\frac{1}{x}(x>0)$

C) Minimum value of $2^{(x^2-3)^3+27}$

D) Minimum value of $5\sin^2 x+3\cos^2 x$

1) A, B, D,C 2) A, C, B, D
3) A, D, B, C 4) B, D, A, C

## KEY

**1)2 2)1 3)1**

## LEVEL - V

I Let 'f' be a continuous function at 'a'. If 'f' changes sign from +ve to -ve at 'a' then f(a) is relative maximum. If 'f' changes sign from -ve to +ve at 'a' then f(a) is relative minimum. If the sign of 'f' does not change at 'a' then f(a) is neither relative maximum nor minimum. If $f^1(a)=0$ then f(a) is called stationary value of 'f'. If 'f' is defined on [a,b] and $f(x_1), f(x_2)....f(x_n)$ are local maxima or local minimaof 'f' in [a,b], then maximum of $\{f(a), f(x_1), f(x_2)....f(x_n), f(b)\}$ is absolute maximum of 'f' in [a,b], minimum of $\{f(a), f(x_1), f(x_2)....f(x_n), f(b)\}$ is called absolute minimum of 'f' in [a,b]

1. The absolute minimum of $f(x)=4x^3-8x^2$ on [-1,1] is

   1.1 2.-11 3.$\frac{-128}{27}$ 4.10

2. The stationary values of $f(x)=\frac{x^2}{x+2}$ are

   1.0,8 2.0,-8 3.0,4 4.0,-4

3. The function $f(x)=3x^4-4x^3+1$ has

   1. relative minimum only
   2. relative maximum only
   3. Both 1 and 2
   4. neither relative minimum nor relative maximum

II. If $f^1(a)\geq 0$ then f(x) is an increasing function at 'a' and $f^1(a)\leq 0$ then f(xd) is decreasing function at 'a' the converse is not true

1. $f(x)=x^3(x-2)^2$ is decreasing at x=

   1.1 2.$\frac{3}{2}$ 3.$\frac{2}{3}$ 4.4

2. $f(x)=\log(\log x)$ is increasing at x=

   1.2 2.1 3.3 4.$x=\frac{1}{2}$

3. $f(x)=|x|$ is decreasing for all $x\in$

   1.$R^+$ 2.$R^-$
   3.R 4.$R^-\cup\{0\}$

4. $f(x)=x^x$ is increasing for all

   1.$x>\frac{1}{e}$ 2.$0<x<\frac{1}{e}$
   3.$x\in R$ 4.$x<\frac{1}{e}$

### KEY

| | | | | |
|---|---|---|---|---|
| I | 1.1 | 2.2 | 3.1 | |
| II | 1.2 | 2.3 | 3.2 | 4.1 |

## PREVIOUS AIEEE QUESTIONS

1) Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is **(2005)**

   1.ab 2.2ab 3. a/b 4) $\sqrt{ab}$

2) If $U=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}$ $+\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$, then the difference between the maximum and minimum values of $U^2$ is given by **(2004)**

   1) $2(a^2+b^2)$ 2) $2\sqrt{a^2+b^2}$
   3) $(a+b)^2$ 4) $(a-b)^2$

3) If the function $f(x)=2x^3-9ax^2+12a^2x+1$ where a>0 attains its maximum and minimum at p and q respectively such that, $p^2=q$ then a = **(2003)**

   1) 1 2)2 3) 1/2 4)3

4) The greatest value of f(x) = $(x+1)^{1/3}-(x-1)^{1/3}$ on [0, 1] is **(2002)**

   1) 1 2) 2 3) 3 4) 1/3

### KEY

1) 2 2) 4 3) 2 4) 2

## QUESTIONS FROM PREVIOUS EAMCET EXAMINATIONS

**EAMCET - 2005**

1. The extreme values of $4\left(\cos x^2\right)\cos\left(\frac{\pi}{3}+x^2\right)\cos\left(\frac{\pi}{3}-x^2\right)$ on R are

   1. -1, 1 2. -2, 2 3. -3, 3 4. -4, 4

**EAMCET - 2003**

2. Minimum value of $\int_0^x t\left(e^{-t^2}\right)dt$

1.1 2.2 3.3 4.0

**EAMCET - 2002**

3. If log (1+x) - $\frac{2x}{2+x}$ is increasing then

1. $0<x<\infty$ 2. $-\infty<x<0$
3. $-\infty<x<\infty$ 4. $1<x<2$

4. The function $f(x) = x.e^{-x}$ ( $x \in R$) attains a maximum value at x =

1. 2 2. $\frac{1}{e}$ 3. 1 4. 3

**EAMCET - 2001**

5. The minimum value of $(x-\alpha)(x-\beta)$ is

1. 0 2. $\alpha\beta$
3. $\frac{1}{4}(\alpha-\beta)^2$ 4. $\frac{-1}{4}(\alpha-\beta)^2$

6. The maximum value of xy subject to x + y = 7 is

1. 12 2. 10 3. $\frac{49}{4}$ 4. $\frac{55}{4}$

**EAMCET - 2000**

7. If $l^2 + m^2 = 1$, then the max values of l + m is

1. 1 2. $\sqrt{2}$ 3. $\frac{1}{\sqrt{2}}$ 4. 2

**EAMCET - 1999**

8. Maximum value of $1+8\sin^2(x^2)\cos^2(x^2)$ is

1. 3 2. -1 3. -8 4. 9

9. If x + y = 12, then minimum value of $x^2 + y^2$ is

1. 72 2. 144 3. 48 4. 36

10. The minimum distance from the point (4, 2) to the parabola $y^2 = 8x$ is

1. $\sqrt{2}$ 2. $2\sqrt{2}$ 3. $3\sqrt{2}$ 4. $4\sqrt{2}$

**EAMCET-1998**

11. Maximum and Minimum value of $\sin^2(120+\theta)+\sin^2(120-\theta)$ are

1. $\frac{3}{2},\frac{1}{2}$ 2. $\frac{1}{2},0$ 3. $\frac{3}{2},0$ 4. $\frac{3}{2},\frac{1}{3}$

12. If x and y ae +ve and x + y = 1 then the minimum value of x log x + y log y is

1. log 2 2. -log 2 3. 2 log 2 4. 0

13. The minimum value of $x^2 - 8x + 17$ is

1. 17 2. -1 3. 1 4. 2

14. The maximum value of $f(x) = 2x^3 - 21x^2 + 36x + 20$ in $0 \le x \le 2$ is

1. 37 2. 44 3. 32 4. 30

15. The point on the curve $y^2 = 4x$ which is nearest to the point (2, 1) is

1. (4, 4) 2. (1, 2) 3. (9, 6) 4. (4, 6)

**EAMCET - 1997**

16. The minimum value of $x^2 + 2bx + c$ is

1. $cb^2$ 2. $c^2b$ 3. $c + b^2$ 4. $c - b^2$

**EAMCET - 1995**

17. The minimum value of $a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta$ is

1. $a^2 - b^2$ 2. $2(a^2+b^2)$ 3. $(a - b)^2$ 4. $(a + b)^2$

**EAMCET - 1996**

18. The function x(x-1)(x-2) attains its maximum value when x is

1. 1 2. $1+\frac{1}{\sqrt{3}}$ 3. $1-\frac{1}{\sqrt{3}}$ 4. $1\pm\sqrt{3}$

19. The point on the curve $y = x^2$ which is nearest to (3, 0) is

1. (1, -1) 2. (-1, 1) 3. (-1, -1) 4. (1, 1)

**EAMCET - 1994**

20. maximum area of the rectangle that can be inscribed in a circle of radius 'r' is

1. $\frac{\pi r^2}{2}$ 2. $r^2$ 3. $\frac{\pi r^2}{4}$ 4. $2r^2$

21 The maximum value of $x^3 - 3x$ in the interval [0, 2] is

1. -2 2. 0 3. 2 4. 1

22. The function $\frac{\log x}{x}$ is increasing in the interval

1. (1, 2e) 2. (0, e) 3. (e, 2e) 4. $\left(\frac{1}{e},2e\right)$

**EAMCET - 1993**

23. If $\alpha,\beta$ are roots of the quadratic equation $x^2-x(a-2)-(a+1)=0$ when 'a' is a virable then the least value of $\alpha^2 + \beta^2$

1. 5 2. 3 3. 4 4. 1

**EAMCET - 1992**

24. The greatest value of $\sin^3x + \cos^3x$ in $\left[0,\frac{\pi}{2}\right]$ is

1. 1 2. 2 3. 3 4. 4

25. The minimum value of $64\sec\theta + 27\operatorname{cosec}\theta$ when $\theta$ lies in $\left(0,\frac{\pi}{2}\right)$ is ............

1. 125 2. 625 3. 25 4. 1025

26. P(-2, 3), Q(3, 7). The point A on the x-axis for which PA + AQ is least is .............

1. $\left(\frac{-1}{2},0\right)$ 2. $\left(\frac{1}{2},0\right)$ 3. $\left(0,\frac{-1}{2}\right)$ 4. $\left(0,\frac{1}{2}\right)$

27. If l, m, n are the direction cosines of a half line OP then the maximum value of l.m.n is .........

1. $\frac{1}{\sqrt{3}}$ 2. $\frac{1}{3\sqrt{3}}$ 3. $\frac{1}{3}$ 4. 1/4

**EAMCET - 1991**

28. The maximum value of $\frac{\log x}{x}$ is .........

1. e  2. 1/e  3. $e^2$  4. 2e

29. The smallest value of $x^2$ - 3x + 3 in the interval $\left(-3, \frac{3}{2}\right)$ is

1. $\frac{3}{4}$  2. 5  3. -15  4. -20

**EAMCET - 1990**

30. The minimum of $2x^3 - 3x^2 - 12x + 8$ occurs at x=

1. -1  2. 2  3. $\sqrt{6}$  4. $-\sqrt{6}$

31. The maximum value of (x - 1) (x -2) (x - 3) is

1. $\frac{2}{3\sqrt{3}}$  2. 2/3  3. $\frac{1}{\sqrt{3}}$  4. $\frac{1}{3\sqrt{3}}$

**EAMCET - 1989**

32. The values for which $x^3 - 6x^2 - 36x + 7$ increase with x are...........

1. x<-2 (or) x > 6  2. x > -2 (or) x < 6
3. x = -2 (or) x = 6  4. x = 0

**EAMCET - 1988**

33. The maximum value of $x^{-x}$ is .........

1. $e^e$  2. $e^{1/e}$  3. $e^{-e}$  4. 1/e

34. Which of the following function is increasing in $\left(0, \frac{\pi}{2}\right)$

1. cos x - sin x  2. cos x + sin x3 .
$\frac{\sin x}{x}$  4. $\frac{x}{\sin x}$

**EAMCET - 1987**

35. If x > 0 then the minimum value of $x^x$ is .........

1. $e^{-1/e}$  2. $e^{1/e}$
3. $e^e$  4. e

36. The cordinates of the point on the parabola $y = x^2 + 4x + 3$ which is closest to the straight line y = 3x + 2 is .........

1. $\left(\frac{-1}{2}, \frac{5}{4}\right)$ 2. $\left(\frac{1}{2}, \frac{5}{4}\right)$  3. $\left(\frac{1}{2}, \frac{-5}{4}\right)$ 4. $\left(\frac{-1}{2}, \frac{-5}{4}\right)$

37. The strength of a beam varies as the product of its breadth 'b' and square of its depth 'd' A beam cut out of a circular log of radius 'r' would be stiffest when

1. $b = d = \frac{r}{\sqrt{2}}$  2. $b^2 = \frac{r}{\sqrt{2}} = d$ 3 .
$d = \sqrt{2}b = 2\sqrt{\frac{2}{3}}r$  4. $d = \sqrt{3}b = 2\sqrt{\frac{2}{3}}r$

**EAMCET - 1986**

38. The sum of two positive numbers is 20. If the product of the square of one and the cube of the other is maximum then they are........

1. 8, 12  2. 12, 9  3. 4, 16  4. 7, 13

**EAMCET - 1984**

39. When x lies between -6 and 8. The maximum value of $(x + 6)^4 (8 - x)^3$ is .......

1. $6^4 . 8^3$  2. $8^4 . 7^3$  3. $8^4 . 6^3$  4. $7^4 . 5^3$

**EAMCET - 1983**

40. The point on the curve $y = x^2 + 7x + 2$ closest to the line y = 3x - 3 is.......

1. (-2, -8)  2. (-8, -2)  3. (2, 8)  4. (8, 2)

41. The maximum possible area that can be enclosed by a wire of length 20 cm by bending it into the form of a sector in sq. cms. is

1. 20  2. 25  3. 30  4. 15

42. A box without lid having maximum possible volume is to be made from a rectangular piece of tin 32 cm x 20 cms by cutting of equal square pieces from the four corners and turning up the projecting pieces to make the sides of the box. The height of the box is equal to

1. 4  2. 2  3. 1  4. 8

## KEY

| | | | | |
|---|---|---|---|---|
| 01. 1 | 02. 4 | 03. 1 | 04. 3 | 05. 4 |
| 06. 3 | 07. 2 | 08. 1 | 09. 1 | 10. 2 |
| 11. 1 | 12. 2 | 13. 3 | 14. 1 | 15. 2 |
| 16. 4 | 17. 4 | 18. 3 | 19. 4 | 20. 4 |
| 21. 3 | 22. 2 | 23. 1 | 24. 1 | 25. 1 |
| 26. 1 | 27. 2 | 28. 2 | 29. 1 | 30. 2 |
| 31. 1 | 32. 1 | 33. 2 | 34. 4 | 35. 1 |
| 36. 1 | 37. 3 | 38. 1 | 39. 3 | 40. 1 |
| 41. 2 | 42. 1 | | | |