Pythagoras Theorem

Practice Set 2.1

Q. 1. Identify, with reason, which of the following are Pythagorean triplets.
(i) (3, 5, 4)
(ii) (4, 9, 12)
(iii) (5, 12, 13)
(iv) (24, 70, 74)
(v) (10, 24, 27)
(vi) (11, 60, 61)

Answer : In a triangle with sides (a,b,c), the Pythagoran's theorem states that

$a^2 + b^2 = c^2$. If this condition is satisfied then (a,b,c)are Pythagorean triplets.

1st case: $3^2 + 4^2 = 5^2$. Thus this a triplet.

2nd case: $4^2 + 9^2 \neq 12^2$

3rd case: $5^2 + 12^2 = 13^2$. Thus this is a triplet.

4th case: $24^{2\,+}\, 70^2 = 74^2$. Thus this is a triplet.

5th case: $10^2 + 24^2 \neq 27^2$

6th case: $11^2 + 60^2 = 61^2$. Thus this is a triplet.

Q. 2. In figure 2.17, ∠MNP = 90°, seg NQ ⊥ seg MP, MQ = 9, QP = 4, find NQ.

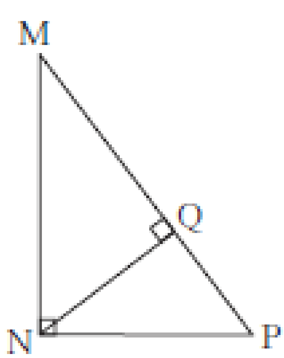


Fig. 2.17

Answer : In ΔMNP, ∠ MNP = 90^0,

$MN^2 + NP^2 = MP^2$

$\Rightarrow MN^2 + NP^2 = (MQ + QP)^2$

$\Rightarrow MN^2 + NP^2 = (13)^2$

$\Rightarrow MN^2 + NP^2 = 169$ … (1)

In ΔMQN, $\angle MQN = 90^0$,

$QN^2 + MQ^2 = MN^2$

$\Rightarrow QN^2 + 9^2 = MN^2$

$\Rightarrow QN^2 + 81 = MN^2$ …(2)

In ΔPQN, $\angle PQN = 90^0$,

$QN^2 + PQ^2 = PN^2$

$\Rightarrow QN^2 + 4^2 = PN^2$

$\Rightarrow QN^2 + 16 = PN^2$ … (3)

Now (2) + (3)

$\Rightarrow QN^2 + 81 + QN^2 + 16 = MN^2 + PN^2$

$\Rightarrow 2QN^2 + 97 = 169$ [from (1)]

$\Rightarrow 2QN^2 = 72$

$\Rightarrow QN^2 = 36$

Thus NQ = 6.

Q. 3. In figure 2.18, $\angle QPR = 90°$, seg PM $\perp$ seg QR and Q – M – R, PM = 10, QM = 8, find QR.

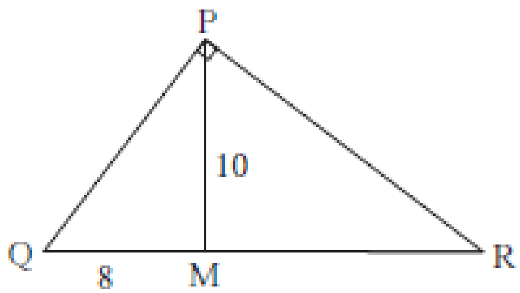


Fig. 2.18

Answer :

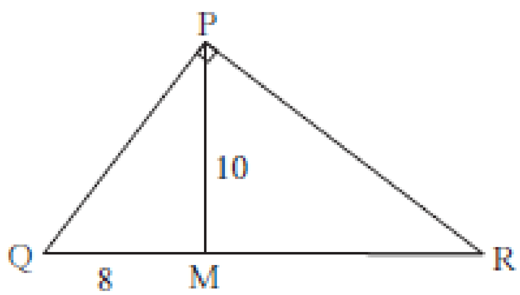


Fig. 2.18

In ∆PMQ, ∠PMQ = 90^0

So $PM^2 + QM^2 = PQ^2$

$\Rightarrow 10^2 + 8^2 = PQ^2$

$\Rightarrow 100 + 64 = PQ^2$

$PQ^2 = 164$ …(1)

In ∆PQR, ∠RPQ = 90^0

So $PQ^2 + PR^2 = QR^2$

$\Rightarrow 164 + PR^2 = QR^2$

$\Rightarrow PR^2 = QR^2 - 164$ …(2)

In ∆PMR, ∠PMR = 90^0

So $PM^2 + MR^2 = PR^2$

$\Rightarrow 10^2 + (QR - QM)^2 = QR^2 - 164$

$\Rightarrow 100 + (QR - QM)^2 = QR^2 - 164$

$\Rightarrow 100 + QR^2 - 2.QR.QM + QM^2 = QR^2 - 164$

$\Rightarrow 100 - 2.QR.8 + 64 = -164$

$\Rightarrow 16QR = 2 \times 164$

$\Rightarrow QR = 20.5$

Thus QR = 20.5

Q. 4. See figure 2.19. Find RP and PS using the information given in ΔPSR.

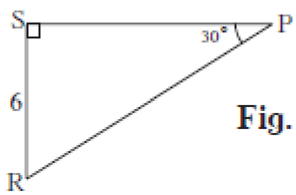


Fig. 2.19

Ans. RP = 12, PS = 6$\sqrt{3}$

Answer : In ΔPSR, $\angle PSR = 90^0$

So $PS^2 + SR^2 = RP^2$

$\Rightarrow 6^2 + (RP\cos(30^0))^2 = RP^2$

$\Rightarrow 6^2 + RP^2 \times \frac{3}{4} = RP^2$

$\Rightarrow 6^2 = \frac{RP^2}{4}$

$\Rightarrow RP^2 = 4 \times 36$

Thus RP = 12.

$PS = RP\cos(30^0)$

$\Rightarrow PS = 12 \times \frac{\sqrt{3}}{2}$

$PS = 6\sqrt{3}$.

Q. 5. For finding AB and BC with the help of information given in figure 2.20, complete following activity.

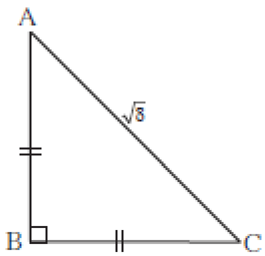


Fig. 2.20

$AB = BC \ldots\ldots\ldots\ldots \square$

$\therefore \quad \angle BAC = \square$

$\therefore AB = BC = \square \times AC$

$= \square \times \sqrt{8}$

$= \square \times 2\sqrt{2}$

$= \square \times$

Answer : In ΔABC, $\angle ABC = 90^0$

So $AB^2 + BC^2 = AC^2$

$\Rightarrow 2AB^2 = 5$

$\Rightarrow AB^2 = \frac{5}{2}$

$\Rightarrow AB = \sqrt{\left(\frac{5}{2}\right)} = X(\text{Say})$

$AB = BC = \sqrt{\left(\frac{5}{2}\right)}$

$\angle BAC = 45^0$ Since AB = BC

Now $\sqrt{\left(\frac{5}{2}\right)} = X\sqrt{5}$

$X = \frac{1}{\sqrt{2}}$

Similarly, X2√2 = $\sqrt{\left(\frac{5}{2}\right)}$

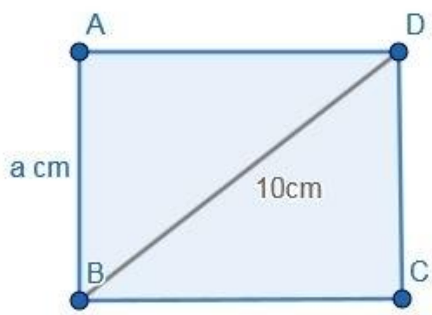
$X = \frac{1}{\sqrt{2}}$

Similarly, X√8 = $\sqrt{\left(\frac{5}{2}\right)}$

$X = \frac{1}{\sqrt{2}}$

Q. 6. Find the side and perimeter of a square whose diagonal is 10 cm.

Answer : In a square of side say a cm, any diagonal divide the square into two right triangles of equal dimensions.



Thus $a^2 + a^2 = 10^2$

$\Rightarrow 2a^2 = 100$

$\Rightarrow a^2 = 50$

$a = 5\sqrt{2}$ cm

Perimeter = 4a

$= 4 \times 5\sqrt{2}$

$= 20\sqrt{2}$

Perimeter of square = $20\sqrt{2}$ cm

Q. 7. In figure 2.21, ∠DFE = 90°, FG ⊥ ED, If GD = 8, FG = 12, find (1) EG (2) FD and (3) EF

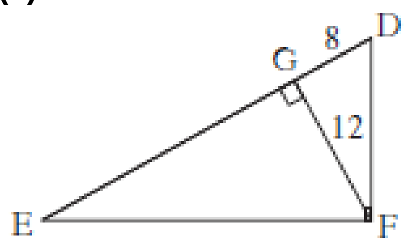


Fig. 2.21

Answer : In ΔDGF, ∠DGF = 90°

$FD^2 = DG^2 + GF^2$

$\Rightarrow FD^2 = 64 + 144$

$\Rightarrow FD^2 = 208$

FD = $4\sqrt{13}$

In ΔDEF, ∠DFE = 90^0

$ED^2 = DF^2 + EF^2$

$\Rightarrow (EG + 8)^2 = 208 + EF^2$ … (1)

In ΔEGF, ∠FGE = 90^0

$EF^2 = EG^2 + GF^2$

$\Rightarrow (EG + 8)^2 - 208 = EG^2 + 144$

$\Rightarrow EG^2 + 2.EG.8 + 64 - 208 = EG^2 + 144$ (As we know $(a+b)^2 = a^2+b^2+2ab$

EG = 18

From (1)

$\Rightarrow (EG + 8)2 = 208 + EF^2$

EF = 6√13.

Q. 8. Find the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.

Answer : The diagonal = $\sqrt{[\text{length}^2 + \text{breadth}^2]}$

$= \sqrt{(35^2 + 12^2)}$

$= \sqrt{(1225 + 144)}$

$= \sqrt{1369}$

$= 37$

Thus the diagonal is 37 cm.

Q. 9. In the figure 2.22, M is the midpoint of QR. ∠PRQ = 90°. Prove that, $PQ^2 = 4PM^2 – 3PR^2$

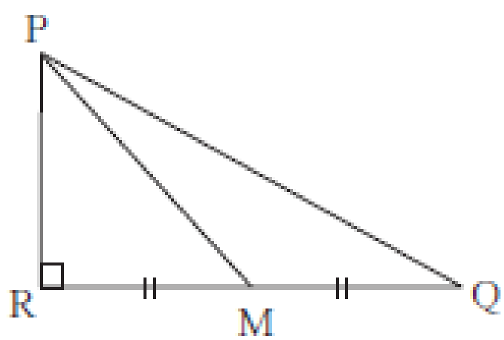


Fig. 2.22

Answer : In ΔPRQ, $\angle PRQ = 90^0$

$PQ^2 = PR^2 + QR^2$ – – – 1

In ΔPRM, $\angle PRM = 90^0$

$PM^2 = PR^2 + MR^2$

$\Rightarrow PM^2 = PR^2 + \left(\frac{QR}{2}\right)^2$ [M is midpoint]

$\Rightarrow 4(PM^2 – PR^2) = QR^2$ – – – 2

1 And 2 implies

$PQ^2 = PR^2 + 4(PM^2 – PR^2)$

$\Rightarrow PQ^2 = 4PM^2 – 3PR^2$

PROVED.

Q. 10. Walls of two buildings on either side of a street are parallel to each other. A ladder 5.8 m long is placed on the street such that its top just reaches the window of a building at the height of 4 m. On turning the ladder over to the other side of the street, its top touches the window of the other building at a height 4.2 m. Find the width of the street.

Answer : Let us consider a distance x m on the street from one building and a distance y m from the other one.

Now according to question,

In the 1^{st} case,

$5.8^2 = 4^2 + x^2$

$\Rightarrow x^2 = 17.64$

$\Rightarrow x = 4.2$

Similarly for the second building,

$5.8^2 = 4.2^2 + y^2$

$\Rightarrow y^2 = 16$

$\Rightarrow y = 4$

Total width = x + y

= 4 + 4.2

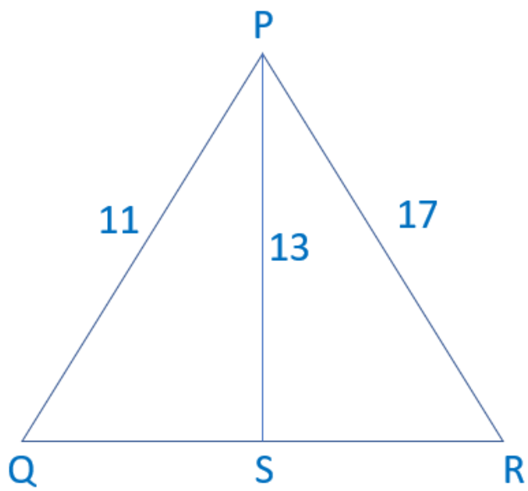
= 8.2

Thus the total width is 8.2m.

Practice Set 2.2

Q. 1. In ΔPQR, point S is the midpoint of side QR. If PQ = 11, PR = 17, PS = 13,find QR.

Answer :



Given PS = 13, PQ = 11, PR = 17

By Apollonius's Theorem,

$PS^2 = \frac{PQ^2 + PR^2}{2} - \frac{QR^2}{4}$

$\Rightarrow 169 = \frac{121 + 289}{2} - \frac{QR^2}{4}$

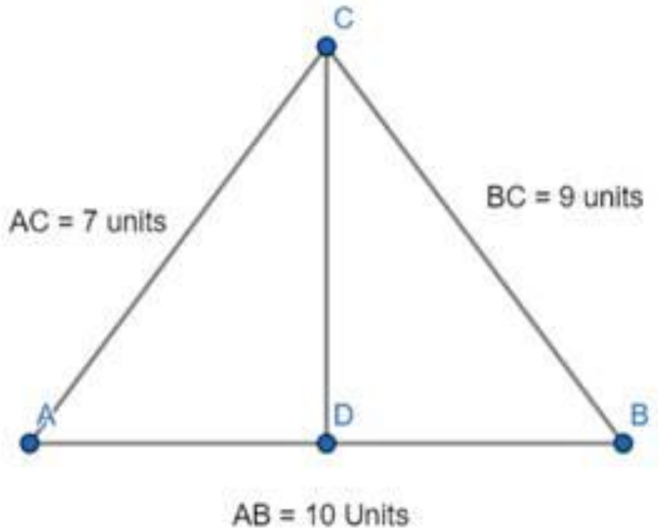
$\Rightarrow \frac{QR^2}{4} = 36$

$\Rightarrow QR^2 = 144$

QR = 12

Q. 2. In ΔABC, AB = 10, AC = 7, BC = 9 then find the length of the median drawn from point C to side AB

Answer : The figure is given below:



According to Pythagoras theorem,

Median2 = $\frac{AC^2+BC^2}{2}-\frac{AB^2}{4}$

⇒ Median2 = $\frac{49+81}{2}-\frac{100}{4}$

⇒ Median2 = 40

Median = 2√10

Thus the median is 2√10

Q. 3. In the figure 2.28 seg PS is the median of ΔPQR and PT ⊥ QR. Prove that,

(1) $PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

(2) $PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2$

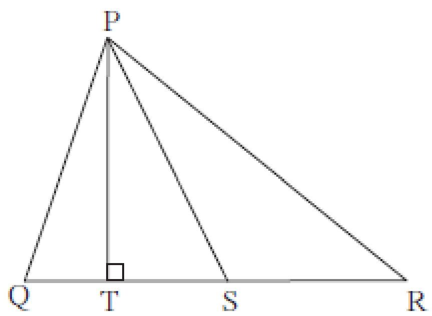


Fig. 2.28

Answer : According to the question,

QS = SR = $\frac{QR}{2}$, $\angle T = 90^0$

Now in ΔPTR, $\angle PTR = 90^0$

$PT^2 + TR^2 = PR^2$

$\Rightarrow PR^2 = PT^2 + (ST + \frac{QR}{2})^2$

$\Rightarrow PR^2 = PT^2 + (ST + \frac{QR}{2})^2$

$\Rightarrow PR^2 = PT^2 + ST^2 + 2.ST.\frac{QR}{2} + \frac{QR^2}{4}$ – – – – 1

Similarly in ΔPTS

$PS^2 = PT^2 + ST^2$ – – – – 2

1 and 2 implies,

$PR^2 = PS^2 - ST^2 + ST^2 + 2.ST.\frac{QR}{2} + \frac{QR^2}{4}$

$\Rightarrow PR^2 = PS^2 + ST.QR + \frac{QR^2}{4}$

PROVED.

Now in ΔPTQ, $\angle PTQ = 90^0$

$PT^2 + TQ^2 = PQ^2$

$\Rightarrow PR^2 = PT^2 + (ST + \frac{QR}{2})^2$

$\Rightarrow PR^2 = PT^2 + (\frac{QR}{2} - ST)^2$

$\Rightarrow PR^2 = PT^2 + ST^2 - 2.\ ST.\ \frac{QR}{2} + \frac{QR^2}{4}$ …1

Similarly in ∆PTS

$PS^2 = PT^2 + ST^2$ …2

1 and 2 implies,

$PR^2 = PS^2 - ST^2 + ST^2 - 2.ST.\ \frac{QR}{2} + \frac{QR^2}{4}$

$\Rightarrow PR^2 = PS^2 - ST.QR + \frac{QR^2}{4}$

PROVED.

Q. 4. In ΔABC, point M is the mid pointof side BC.

If, $AB^2 + AC^2 = 290\ cm^2$, AM = 8 cm, find BC.

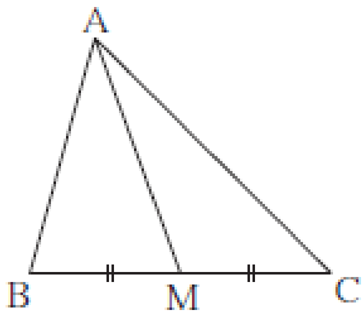


Fig. 2.29

Answer : Given $AB^2 + AC^2 = 290\ cm^2$,AM = 8 cm, BM = MC

According to formula,

$AM^2 = \frac{AB^2 + AC^2}{2} - \frac{BC^2}{4}$

$\Rightarrow 64 = \frac{290}{2} - \frac{BC^2}{4}$

$\Rightarrow 64 - \frac{290}{2} = -\frac{BC^2}{4}$

$\Rightarrow BC^2 = 324$

BC = 18.

Thus BC = 18 cm.

Q. 5. In figure 2.30, point T is in the interior of rectangle PQRS, Prove that, $TS^2 + TQ^2 = TP^2 + TR^2$(As shown in the figure, draw seg AB || side SR and A – T – B)

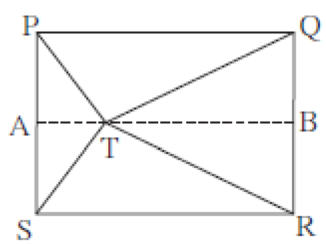


Fig. 2.30

Answer : From figure,

In ΔPAT, $\angle PAT = 90^0$

$TP^2 = AT^2 + PA^2$ …1

In ΔAST, $\angle SAT = 90^0$

$TS^2 = AT^2 + SA^2$ …2

In ΔQBT, $\angle QBT = 90^0$

$TQ^2 = BT^2 + QB^2$ …3

In ΔBTR, $\angle RBT = 90^0$

$TR^2 = BT^2 + BR^2$ …4

$TS^2 + TQ^2 = AT^2 + SA^2 + BT^2 + QB^2$ [Adding 2 and 3]

$\Rightarrow TS^2 + TQ^2 = AT^2 + PA^2 + BT^2 + BR^2$ [SA = BR, QB = AP]

$\Rightarrow TS^2 + TQ^2 = TP^2 + TR^2$ [From 1 and 4]

PROVED.

Problem Set 2

Q. 1. A. Some questions and their alternative answers are given. Select the correct alternative.

Out of the following which is the Pythagorean triplet?
A. (1, 5, 10)

B. (3, 4, 5)
C. (2, 2, 2)
D. (5, 5, 2)

Answer : A Pythagorean triplet consists of three positive integers (l, b, h) such that

$l^2 + b^2 = h^2$

And (3, 4, 5) is a Pythagorean triplet as,

$5^2 = 3^2 + 4^2$

Q. 1. B. Some questions and their alternative answers are given. Select the correct alternative.

In a right-angled triangle, if sum of the squares of the sides making right angle is 169 then what is the length of the hypotenuse?
A. 15

B. 13
C. 5
D. 12

Answer : Given,

Sum of the squares of the sides making right angle = 169

$\Rightarrow (\text{base})^2 + (\text{perpendicular})^2 = 169$

But we know, By Pythagoras's theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$\Rightarrow (\text{Hypotenuse})^2 = 169$

$\Rightarrow$ Hypotenuse = 13 units.

Q. 1. C. Some questions and their alternative answers are given. Select the correct alternative.

Out of the dates given below which date constitutes a Pythagorean triplet?
A. 15/08/17

B. 16/08/16
C. 3/5/17
D. 4/9/15

Answer : A Pythagorean triplet consists of three positive integers (l, b, h) such that

$l^2 + b^2 = h^2$

And 15/08/17 is a Pythagorean triplet as,

$15^2 + 8^2 = 17^2$

i.e. 225 + 64 = 289

Q. 1. D. Some questions and their alternative answers are given. Select the correct alternative.

If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle.
A. Obtuse angled triangle

B. Acute angled triangle
C. Right angled triangle
D. Equilateral triangle

Answer : As, the sides of right-angled triangles satisfies the Pythagoras theorem, i.e.

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

Q. 1. E. Some questions and their alternative answers are given. Select the correct alternative.

Find perimeter of a square if its diagonal is 10 2 cm.
A. 10 cm
B. $40\sqrt{2}$ cm
C. 20 cm
D. 40 cm

Answer : We know that,

Diagonal of a square = $\sqrt{2}\,a$

Where 'a' is the side of the triangle.

⇒ $\sqrt{2}$ a = 10$\sqrt{2}$

⇒ a = 10 cm

Also, we know

Perimeter of square = 4a

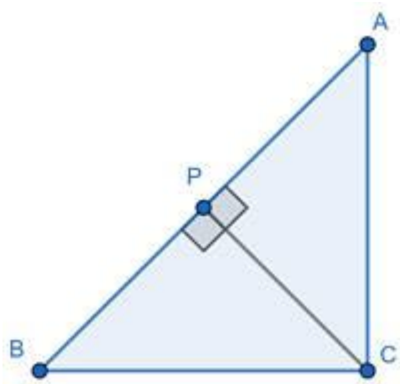
Where 'a' is the side of the triangle

∴ Perimeter of given square = 4(10) = 40 cm

Q. 1. F. Some questions and their alternative answers are given. Select the correct alternative.

Altitude on the hypotenuse of a right angled triangle divides it in two parts of lengths 4 cm and 9 cm. Find the length of the altitude.
A. 9 cm
B. 4 cm
C. 6 cm
D. $2\sqrt{6}$ **cm**

Answer :



Let ABC be a right-angled triangle, at B, and BP be the altitude on hypotenuse that divides it in two parts such that,

AP = 4 cm

PC = 9 cm

As, ABC, ABP and CBP are right-angled triangles, therefore they all satisfy Pythagoras theorem i.e.

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

∴ In ΔABC

$AB^2 + BC^2 = AC^2$

$\Rightarrow AB^2 + BC^2 = (AP + CP)^2$

$\Rightarrow AB^2 + BC^2 = (4 + 9)^2 = 13^2$

$\Rightarrow AB^2 + BC^2 = 169$ [1]

∴ In ΔABP

$AP^2 + BP^2 = AB^2$

$AP^2 + 4^2 = AB^2$ [2]

∴ In ΔCBP

$CP^2 + BP^2 = BC^2$

$\Rightarrow 9^2 + BP^2 = BC^2$ [3]

Adding [2] and [3], we get

$AP^2 + 4^2 + 9^2 + BP^2 = AB^2 + BC^2$

$\Rightarrow 2AP^2 + 16 + 81 = 169$ [From 1]

$\Rightarrow 2AP^2 = 72$

$\Rightarrow AP^2 = 36$

$\Rightarrow AP = 6$ cm

Hence, length of Altitude is 6 cm.

Q. 1. G. Some questions and their alternative answers are given. Select the correct alternative.

Height and base of a right angled triangle are 24 cm and 18 cm find the length of its hypotenuse

A. 24 cm
B. 30 cm
C. 15 cm
D. 18 cm

Answer : By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

Given,

Base = 18 cm

Perpendicular = Height = 24 cm

$\Rightarrow \text{Hypotenuse}^2 = 24^2 + 18^2$

$\Rightarrow \text{Hypotenuse}^2 = 576 + 324$

$\Rightarrow \text{Hypotenuse}^2 = 900$

⇒ Hypotenuse = 30 cm

Q. 1. H. Some questions and their alternative answers are given. Select the correct alternative.

In D ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm, BC = 6 cm. Find measure of $\angle A$.
A. 30°

B. 60°
C. 90°
D. 45°

Answer : As,

$(6\sqrt{3})^2 + 6^2 = 12^2$

$\Rightarrow AB^2 + BC^2 = AC^2$

i.e. sides of the triangle ABC satisfy the Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

∴ ABC is a right-angled triangle with hypotenuse as AC

Now,

$$BC = \frac{1}{2}AC$$

By converse of 30°-60°-90° triangle theorem i.e.

In a right-angled triangle, if one side is half of the hypotenuse then the angle

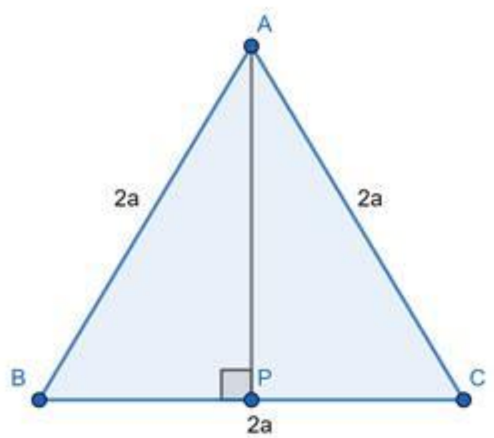
Opposite to that side is 30°.

∠A = 30°

Q. 2. A. Solve the following examples.

Find the height of an equilateral triangle having side 2a.

Answer :



Let ABC be an equilateral triangle,

Let AP be a perpendicular on side BC from A.

To find : Height of triangle = AP

As, ABC is an equilateral triangle we have

AB = BC = CA = 2a

Also, we know that Perpendicular from a vertex to corresponding side in an equilateral triangle bisects the side

$\Rightarrow BP = CP = \frac{1}{2}BC = 'a'$

Now, In ΔABP, By Pythagoras theorem

$(Hypotenuse)^2 = (base)^2 + (Perpendicular)^2$

$\Rightarrow AB^2 = BP^2 + AP^2$

$\Rightarrow (2a)^2 = a^2 + AP^2$

$\Rightarrow AP^2 = 4a^2 - a^2$

$\Rightarrow AP^2 = 3a^2$

$\Rightarrow AP = a\sqrt{3}$

Q. 2. B. Solve the following examples.

Do sides 7 cm , 24 cm, 25 cm form a right angled triangle ? Give reason.

Answer : Yes,

Because

$7^2 + 24^2 = 25^2$ [i.e. 49 + 576 = 625]

As, sides satisfy the Pythagoras theorem, i.e.

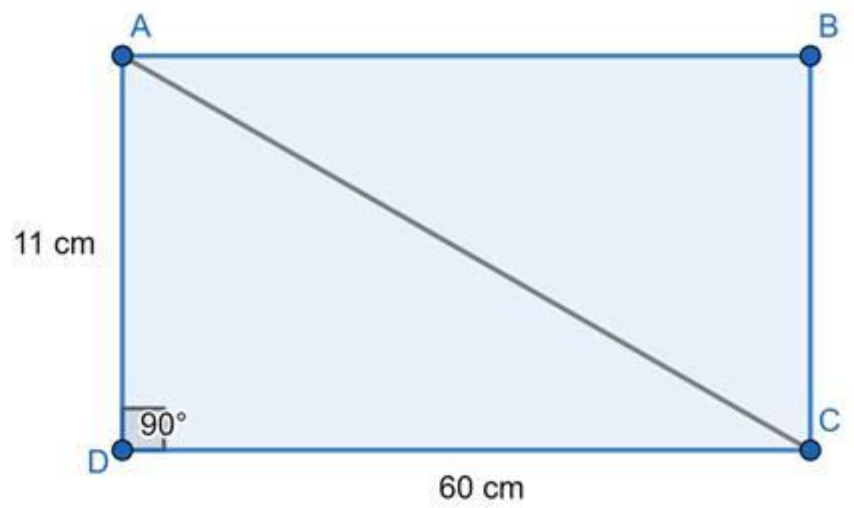
$(Hypotenuse)^2 = (base)^2 + (Perpendicular)^2$

They do form a right-angled triangle.

Q. 2. C. Solve the following examples.

Find the length a diagonal of a rectangle having sides 11 cm and 60cm.

Answer :



Let ABCD be a rectangle, with

AB = CD = 60 cm

BC = DA = 11 cm

And AC be a diagonal.

As, ∠A = 90°

ADC is a right-angled triangle, By Pythagoras Theorem i.e.

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$AC^2 = (CD)^2 + (DA)^2$

$\Rightarrow AC^2 = 60^2 + 11^2$

$\Rightarrow AC^2 = 3600 + 121$

$\Rightarrow AC^2 = 3721$

$\Rightarrow AC = 61$ cm

Q. 2. D. Solve the following examples.

Find the length of the hypotenuse of a right angled triangle if remaining sides are 9 cm and 12 cm.

Answer : In a right-angled triangle

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

Given,

Other sides are 9 cm and 12 cm

⇒ $\text{Hypotenuse}^2 = 9^2 + 12^2$

⇒ $\text{Hypotenuse}^2 = 81 + 144$

⇒ $\text{Hypotenuse}^2 = 225$

⇒ Hypotenuse = 15 cm

Q. 2. E. Solve the following examples.

A side of an isosceles right angled triangle is x. Find its hypotenuse.

Answer : In a right-angled triangle

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

As, the triangle is isosceles

Base = Perpendicular = x

[Hypotenuse can't be equal to any of the sides, because hypotenuse is the greatest side in a right-angled triangle and it must be greater than other two sides]

⇒ $(\text{Hypotenuse})^2 = x^2 + x^2$

⇒ $(\text{Hypotenuse})^2 = 2x^2$

⇒ Hypotenuse = $x\sqrt{2}$

Q. 2. F. Solve the following examples.

In Δ PQR; PQ = $\sqrt{8}$, QR = $\sqrt{5}$, PR = $\sqrt{3}$ Is Δ PQR a right angled triangle? If yes, which angle is of 90°?

Answer : As,

$(\sqrt{5})^2 + (\sqrt{3})^2 = (\sqrt{8})^2$

$\Rightarrow QR^2 + PR^2 = PQ^2$

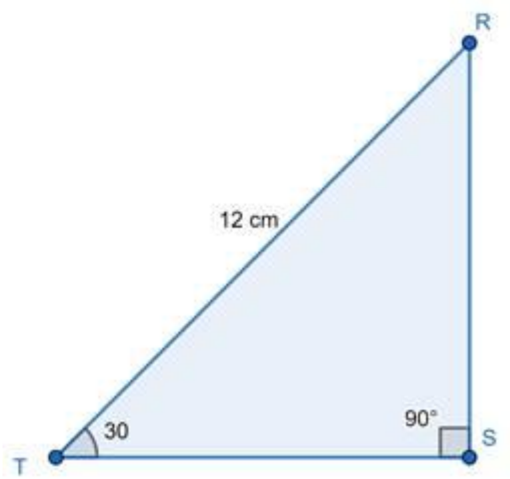
i.e. sides of the triangle ABC satisfy the Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

∴ PQR is a right-angled triangle at R [As hypotenuse is PQ].

Q. 3. In $\Delta RAT, \angle S = 90^{\circ}, \angle T = 30^{\circ}, RT = 12 \text{ cm}$ then find RS and ST.

Answer :



As, ∠S = 90°, and ∠T = 30° and RT = 12 cm is given.

Clearly, RTS is a 30°-60°-90° triangle.

We know, Property of 30°-60°-90° triangle i.e.

If acute angles of a right angled-triangle are 30° and 60°, then the side opposite

30° angle is half of the hypotenuse and the side opposite to 60° angle is $\frac{\sqrt{3}}{2}$ times of hypotenuse.

$\Rightarrow RS = \frac{1}{2} \times RT = \frac{1}{2}(12) = 6 \text{ cm}$

And

$$\mathrm{ST} = \frac{\sqrt{3}}{2} \times \mathrm{RT} = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}\ \mathrm{cm}$$

Q. 4. Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq.cm.

Answer : Given,

Length of rectangle, l = 16 cm

Breadth of rectangle = b

Area of rectangle = length × breadth

⇒ 192 = 16b

⇒ b = 12 cm

Also, we know that

Length of diagonal = $\sqrt{(l^2 + b^2)}$

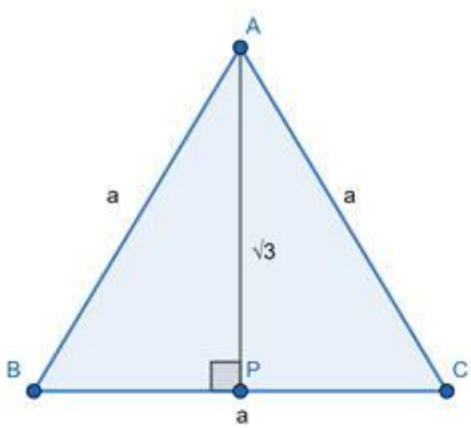
Where, l = length and b = breadth

⇒ Length of diagonal = $\sqrt{(16^2 + 12^2)}$

= √(256 + 144) = 20 cm

Q. 5. Find the length of the side and perimeter of an equilateral triangle whose height is $\sqrt{3}$ cm.

Answer :



Let ABC be an equilateral triangle,

Let AP be a perpendicular on side BC from A.

To find : Height of triangle = AP

As, ABC is an equilateral triangle we have

AB = BC = CA = 'a'

Also, we know that Perpendicular from a vertex to corresponding side in an equilateral triangle bisects the side

$$\Rightarrow BP = CP = \frac{1}{2}BC = \frac{1}{2}a$$

Now, In ΔABP, By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$\Rightarrow AB^2 = BP^2 + AP^2$

$$\Rightarrow a^2 = \left(\frac{1}{2}a\right)^2 + AP^2$$

$$\Rightarrow AP^2 = a^2 - \frac{1}{4}a^2 = \frac{3}{4}a^2$$

$$\Rightarrow AP = \frac{\sqrt{3}}{2}a$$

Given,

Height = $\sqrt{3}$

$$\Rightarrow \frac{\sqrt{3}}{2}a = \sqrt{3}$$

$\Rightarrow a = 2$ cm

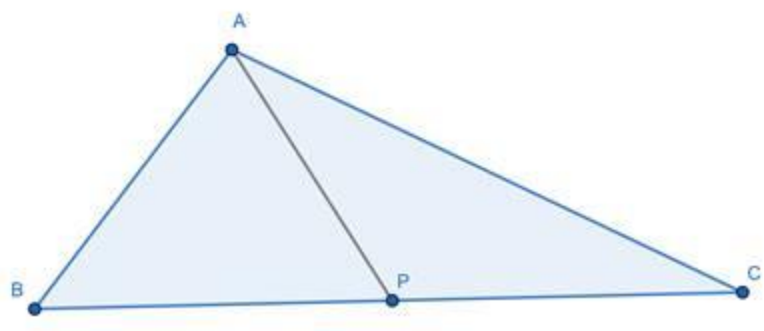
Also, Perimeter of equilateral triangle = 3a

Where 'a' depicts side of equilateral triangle.

∴ Perimeter = 3(2) = 6 cm

Q. 6. In ΔABC seg AP is a median. If BC = 18, $AB^2 + AC^2 = 260$ Find AP.

Answer :



We know, By Apollonius theorem

In ΔABC**,**

If P is the midpoint of side BC, then $AB^2 + AC^2 = 2AP^2 + 2BP^2$

Given that, AP is median i.e. P is the mid-point of BC

$$BP = CP = \frac{1}{2}BC = 9$$

And BC = 18 cm

And $AB^2 + AC^2 = 260$

⇒ $260 = 2AP^2 + 2(9)^2$

⇒ $260 = 2AP^2 + 162$

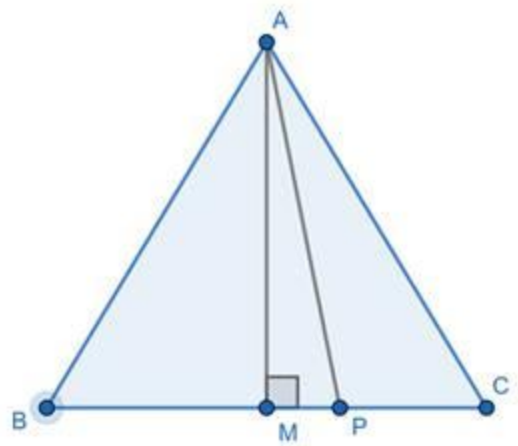
⇒ $98 = 2AP^2$

⇒ $AP^2 = 49$

⇒ AP = 7 units

Q. 7. Δ ABC is an equilateral triangle. Point P is on base BC such that $PC = -\frac{1}{3}BC$. if AB = 6 cm find AP.

Answer :



ABC be an equilateral triangle,

Point P is on base BC, such that

$$PC = \frac{1}{3}BC$$

Let us construct AM perpendicular on side BC from A.

As, ABC is an equilateral triangle we have

AB = BC = CA = 6 cm

Also, we know that Perpendicular from a vertex to corresponding side in an equilateral triangle bisects the side

$$\Rightarrow BM = CM = \frac{1}{2}BC = 3 \text{ cm}$$

Now, In ΔACM, By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$\Rightarrow CA^2 = CM^2 + AM^2$

$\Rightarrow (6)^2 = (3)^2 + AM^2$

$\Rightarrow 36 = 9 + AM^2$

$\Rightarrow AM^2 = 27$ [1]

As,

$$PC = \frac{1}{3}BC$$

$$CM = \frac{1}{2}BC$$

We have,

CM - PC = PM

$$\Rightarrow PM = \frac{1}{2}BC - \frac{1}{3}BC$$

$$\Rightarrow PM = \frac{1}{6}BC = \frac{1}{6}(6)$$

$\Rightarrow$ PM = 1 cm

Now, In right angled triangle AMP, By Pythagoras theorem

$(AP)^2 = (AM)^2 + (PM)^2$

$\Rightarrow (AP)^2 = 27 + 1^2$

$\Rightarrow AP^2 = 28$

$\Rightarrow AP = 2\sqrt{7}$ cm

Q. 8. From the information given in the figure 2.31, prove that PM = PN = $\sqrt{3} \times a$

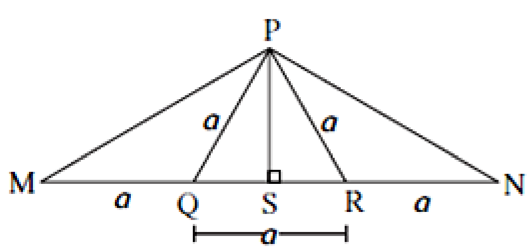


Fig. 2.31

Answer : In ΔPQS and ΔPSR, By Pythagoras theorem i.e. $(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$PQ^2 = QS^2 + PS^2$ [1]

$PR^2 = SR^2 + PS^2$ [2]

Subtracting [2] from [1],

$PQ^2 - PR^2 = QS^2 - SR^2$

$\Rightarrow a^2 - a^2 = QS^2 - SR^2$

$\Rightarrow QS^2 = SR^2$

$\Rightarrow QS = SR$

$$\Rightarrow QS = SR = \frac{1}{2}QR = \frac{a}{2}$$

Also,

$MS = MQ + QS$

$$\Rightarrow MS = a + \frac{a}{2} = \frac{3a}{2}$$

And

$SN = SR + RN$

$$\Rightarrow SN = \frac{a}{2} + a = \frac{3a}{2}$$

In ΔPSM and ΔPSN, By Pythagoras theorem

$PM^2 = PS^2 + MS^2$

$$\Rightarrow PN^2 = PS^2 + \left(\frac{3a}{2}\right)^2 \text{ [4]}$$

$PN^2 = PS^2 + SN^2$

$$\Rightarrow PN^2 = PS^2 + \left(\frac{3a}{2}\right)^2 \text{ [4]}$$

From [3] and [4]

$PM^2 = PN^2$

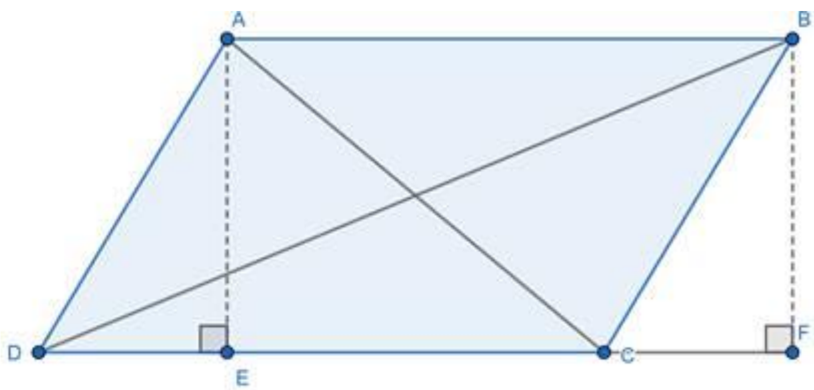
⇒ PM = PN

Hence Proved.

Q. 9. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Answer : Let ABCD be a parallelogram, with AB = CD ; AB || CD and BC = AD ; BC || AD.

Construct AE ⊥ CD and extend CD to F such that, BF ⊥ CF.



In ΔAED and ΔBCF

AE = BF [Distance between two parallel lines i.e. AB and CD]

AD = BC [opposite sides of a parallelogram are equal]

∠AED = ∠BFC [Both 90°]

ΔAED ≅ ΔBCF [By Right Angle - Hypotenuse - Side Criteria]

⇒ DE = CF [Corresponding sides of congruent triangles are equal] [1]

In ΔBFD, By Pythagoras theorem i.e.

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$BD^2 = DF^2 + BF^2$

$\Rightarrow BD^2 = (CD + CF)^2 + BF^2$ [2]

In ΔAEC, By Pythagoras theorem

$AC^2 = AE^2 + CE^2$

$\Rightarrow AC^2 = AE^2 + (CD - AE)^2$

$\Rightarrow AC^2 = BF^2 + (CD - CF)^2$ [As, AE = BF and CF = AE] [2]

In ΔBCF, By Pythagoras theorem,

$BC^2 = BF^2 + CF^2$

$BF^2 = BC^2 - CF^2$ [3]

Adding [2] and [3]

$BD^2 + AC^2 = 2BF^2 + (CD + CF)^2 + (CD - CF)^2$

$\Rightarrow BD^2 + AC^2 = 2BC^2 - 2CF^2 + CD^2 + CF^2 + 2CD.CF + CD^2 + CF^2 - 2CD.CF$

$\Rightarrow BD^2 + AC^2 = 2BC^2 + 2CD^2$

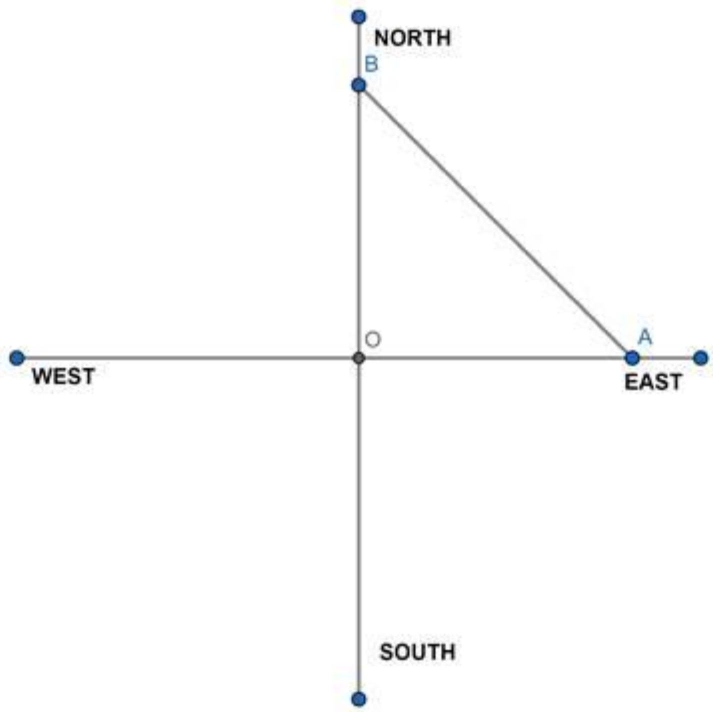
$\Rightarrow BD^2 + AC^2 = BC^2 + BC^2 + CD^2 + CD^2$

$\Rightarrow BD^2 + AC^2 = AB^2 + BC^2 + CD^2 + AD^2$ [since BC = AD and AB = CD]

Hence, the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Q. 10. Pranali and Prasad started walking to the East and to the North respectively, from the same point and at the same speed. After 2 hours distance between them was 15 $\sqrt{2}$ **km. Find their speed per hour.**

Answer :



Let their speed be 'x' km/h

We know, distance = speed × time

In two hours,

Distance travelled by both = '2x' km

Let their starting point be 'O', and Pranali and Prasad reach the point A in the East and point B in the north direction respectively.

Clearly, AOB is a right-angled triangle, So By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$(AB)^2 = (OA)^2 + (OB)^2$

As, AB = distance between them = $15\sqrt{2}$ km

And OA = OB = distance travelled by each = 2x

$\Rightarrow (15\sqrt{2})^2 = (2x)^2 + (2x)^2$

$\Rightarrow 450 = 8x^2$

$\Rightarrow x^2 = 56.25$

$\Rightarrow x = 7.5$ km/h

Q. 11. In $\Delta ABC, \angle BAC = 90^{\circ}$, seg BL and seg CM are medians of Δ ABC. Then prove that : $4(BL^2 + CM^2) = 5\,BC^2$

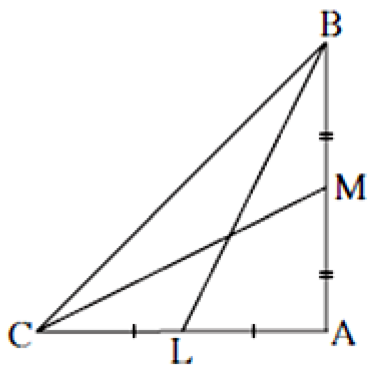


Fig. 2.32

Answer :

We know, By Apollonius theorem

In ΔABC, if L is the midpoint of side AC, then $AB^2 + BC^2 = 2BL^2 + 2AL^2$

Given that, BL is median i.e. L is the mid-point of CA

$$CL = AL = \frac{1}{2}AC$$

$\Rightarrow AB^2 + BC^2 = 2BL^2 + 2AL^2$

$$\Rightarrow AB^2 + BC^2 = 2BL^2 + 2\left(\frac{AC}{2}\right)^2$$

$$\Rightarrow AB^2 + BC^2 = 2BL^2 + \frac{AC^2}{2} \quad [1]$$

Also, if M is the midpoint of side AB, then $AC^2 + BC^2 = 2CM^2 + 2BM^2$

Given that, CM is median i.e. M is the mid-point of BA

$AM = BM = \frac{1}{2}AB$

$\Rightarrow AC^2 + BC^2 = 2CM^2 + 2BM^2$

$\Rightarrow AC^2 + BC^2 = 2CM^2 + 2\left(\frac{AB}{2}\right)^2$

$\Rightarrow AC^2 + BC^2 = 2CM^2 + \frac{AB^2}{2}$ [2]

Also, In ΔABC, By Pythagoras theorem i.e.

$(Hypotenuse)^2 = (base)^2 + (Perpendicular)^2$

$\Rightarrow BC^2 = AC^2 + AB^2$ [3]

Adding [1] and [2]

$\Rightarrow AB^2 + BC^2 + AC^2 + BC^2 = 2BL^2 + \frac{AC^2}{2} + 2CM^2 + \frac{AB^2}{2}$

$\Rightarrow \frac{AB^2}{2} + \frac{AC^2}{2} + 2BC^2 = 2BL^2 + 2CM^2$

$\Rightarrow AB^2 + AC^2 + 4BC^2 = 4(BL^2 + CM^2)$

$\Rightarrow BC^2 + 4BC^2 = 4(BL^2 + CM^2)$ [From 3]

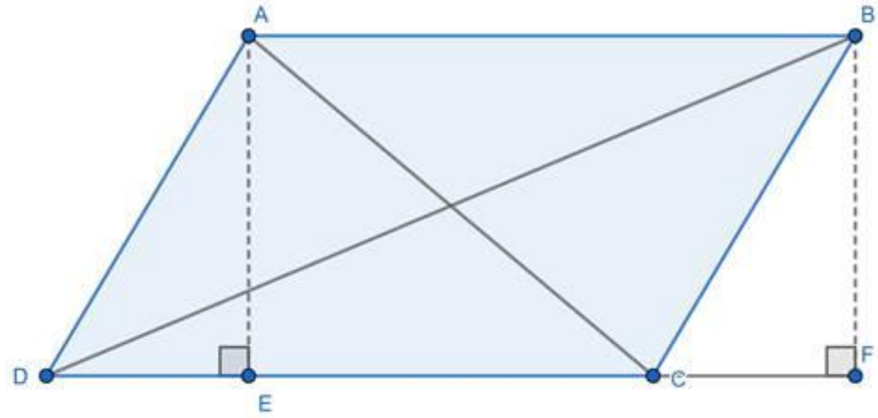
$\Rightarrow 5BC^2 = 4(BL^2 + CM^2)$

Hence Proved.

Q. 12. Sum of the squares of adjacent sides of a parallelogram is 130 sq.cm and length of one of its diagonals is14 cm. Find the length of the other diagonal.

Answer : Let ABCD be a parallelogram, with AB = CD ; AB || CD and BC = AD ; BC || AD.

Construct AE ⊥ CD and extend CD to F such that, BF ⊥ CF.



Given: sum of squares of adjacent side = 130

⇒ $CD^2 + BC^2 = 130$ and

Length of one diagonal = 14 cm [let it be AC]

To Find: length of the other diagonal, BD

In ΔAED and ΔBCF

AE = BF [Distance between two parallel lines i.e. AB and CD]

AD = BC [opposite sides of a parallelogram are equal]

∠AED = ∠BFC [Both 90°]

ΔAED ≅ ΔBCF [By Right Angle - Hypotenuse - Side Criteria]

⇒ DE = CF [Corresponding sides of congruent triangles are equal] [1]

In ΔBFD, By Pythagoras theorem i.e.

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$BD^2 = DF^2 + BF^2$

⇒ $BD^2 = (CD + CF)^2 + BF^2$ [2]

In ΔAEC, By Pythagoras theorem

$AC^2 = AE^2 + CE^2$

⇒ $AC^2 = AE^2 + (CD - AE)^2$

$\Rightarrow AC^2 = BF^2 + (CD - CF)^2$ [As, AE = BF and CF = AE] [2]

In ΔBCF, By Pythagoras theorem,

$BC^2 = BF^2 + CF^2$

$BF^2 = BC^2 - CF^2$ [3]

Adding [2] and [3]

$BD^2 + AC^2 = 2BF^2 + (CD + CF)^2 + (CD - CF)^2$

$\Rightarrow BD^2 + AC^2 = 2BC^2 - 2CF^2 + CD^2 + CF^2 + 2CD.CF + CD^2 + CF^2 - 2CD.CF$

$\Rightarrow BD^2 + AC^2 = 2BC^2 + 2CD^2$

$\Rightarrow BD^2 + 14^2 = 2(130)$

$\Rightarrow BD^2 + 196 = 260$ [Using given data]

$\Rightarrow BD^2 = 64$

$\Rightarrow BD = 8$ cm

Hence, length of other diagonal is 8 cm.

Q. 13. In △ ABC, seg AD ⊥ seg BC DB = 3CD. Prove that : $2AB^2 = 2AC^2 + BC^2$

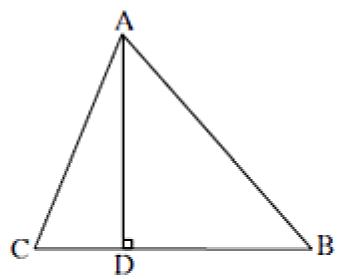


Fig. 2.33

Answer : Given,

DB = 3CD

Also,

BC = CD + DB = CD + 3CD

$\Rightarrow BC = 4CD$ [1]

As, $AD \perp BC$, By Pythagoras theorem i.e.

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

In Δ ACD

$AC^2 = AD^2 + CD^2$ [2]

In ΔABD

$AB^2 = AD^2 + DB^2$

$\Rightarrow AB^2 = AD^2 + (3CD)^2$

$\Rightarrow AB^2 = AD^2 + 9CD^2$ [3]

Subtracting [2] from [3]

$\Rightarrow AB^2 - AC^2 = 9CD^2 - CD^2$

$\Rightarrow AB^2 = AC^2 + 8CD^2$

$\Rightarrow 2AB^2 = 2AC^2 + 16CD^2$

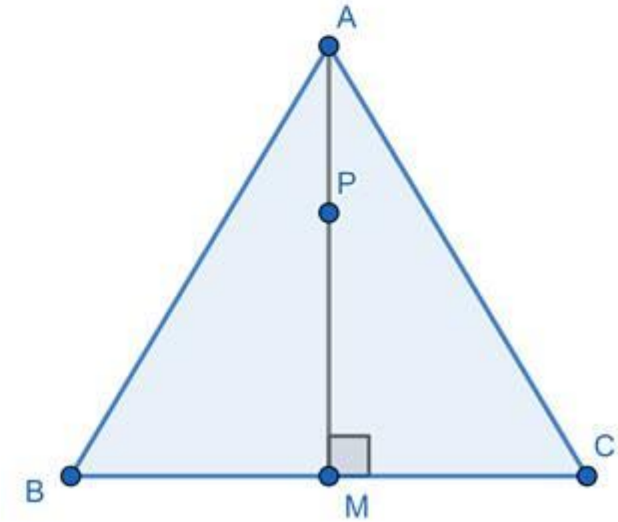
$\Rightarrow 2AB^2 = 2AC^2 + (4CD)^2$

$\Rightarrow 2AB^2 = 2AC^2 + BC^2$ [From 1]

Hence Proved.

Q. 14. In an isosceles triangle, length of the congruent sides is 13 cm and its base is 10 cm. Find the distance between the vertex opposite the base and the centroid.

Answer :



Let ABC be an isosceles triangle, In which AB = AC = 13 cm

And BC = 10 cm

Let AM be median on BC such that

$$BM = CM = \frac{1}{2}BC = 5\ cm$$

Let P be centroid on median BC

To Find : AP [Distance between vertex opposite the base and centroid]

We know, By Apollonius theorem

In ΔABC**,** if M is the midpoint of side BC, then $AB^2 + AC^2 = 2AM^2 + 2BM^2$

Putting values, we get

$(13)^2 + (13)^2 = 2AM^2 + 2(5)^2$

$\Rightarrow 169 + 169 = 2AM^2 + 50$

$\Rightarrow 2AM^2 = 288$

$\Rightarrow AM^2 = 144$

$\Rightarrow AM = 12$ cm

Let P be the centroid

As, Centroid divides median in a ratio 2 : 1

⇒ AP : PM = 2 : 1

⇒ AP = 2PM

Now, AM = AP + PM

$$\Rightarrow AM = AP + \frac{AP}{2} = \frac{3}{2}AP$$

$$\Rightarrow AP = \frac{2}{3}AM = \frac{2}{3}(12) = 8\ cm$$

Q. 15. In a trapezium ABCD, seg AB || seg DC seg BD $\perp$ seg AD, seg AC $\perp$ seg BC, If AD = 15, BC = 15 and AB = 25. Find $A(\square ABCD)$

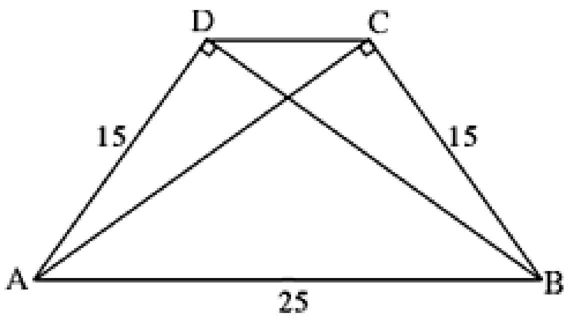
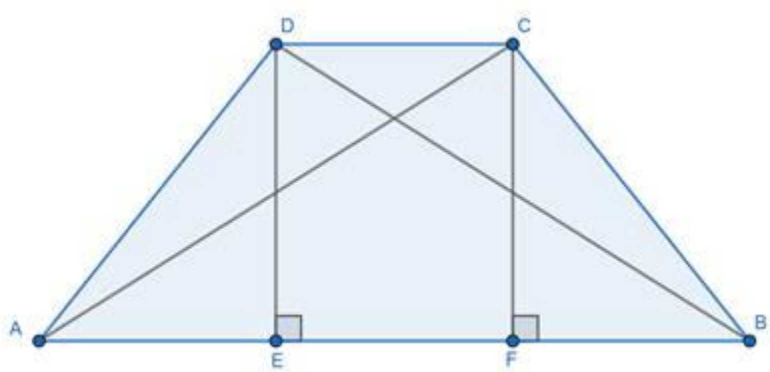


Fig. 2.34

Answer :



Construct DE ⊥ AB and CF ⊥ AB

In ΔADB, as $BD \perp AD$, By Pythagoras theorem i.e.

$(Hypotenuse)^2 = (base)^2 + (Perpendicular)^2$

$(AB)^2 = (AD)^2 + (BD)^2$

$\Rightarrow 25^2 = 15^2 + BD^2$

$\Rightarrow BD^2 = 625 - 225 = 400$

$\Rightarrow$ BD = 20 cm

Similarly,

AC = 20 cm

Now, In ΔAED and ΔABD

$\angle AED = \angle ADB$ [Both 90°]

$\angle DAE = \angle DAE$ [Common]

$\Delta AED \sim \Delta ABD$ [By Angle-Angle Criteria]

$\Rightarrow \frac{DE}{BD} = \frac{AD}{AB} = \frac{AE}{AD}$ [Property of similar triangles]

As AD = 15 cm, BD = 20 cm and AB = 25 cm

$$\Rightarrow \frac{DE}{20} = \frac{15}{25}$$

$\Rightarrow$ DE = 12 cm

Also,

$$\frac{DE}{BD} = \frac{AE}{AD}$$

$$\Rightarrow \frac{12}{20} = \frac{AE}{15}$$

$\Rightarrow$ AE = 9 cm

Similarly, BF = 9 cm

Now,

DC = EF [By construction]

DC = AB - DE - AE

DC = 25 - 9 - 9 = 7 cm

Also, we know

Area of trapezium $= \frac{1}{2} \times (\text{Sum of Parallel Sides}) \times \text{Height}$

$$= \frac{1}{2} \times (DC + AB) \times DE$$

$$= \frac{1}{2} \times (7 + 25) \times 12$$

$$= 192 \text{ cm}^2$$

Q. 16. In the figure 2.35, △ P20QR is an equilateral triangle. Point S is on seg QR such that $QS = \frac{1}{3}QR.$

Prove that : 9 PS^2 = 7 PQ^2

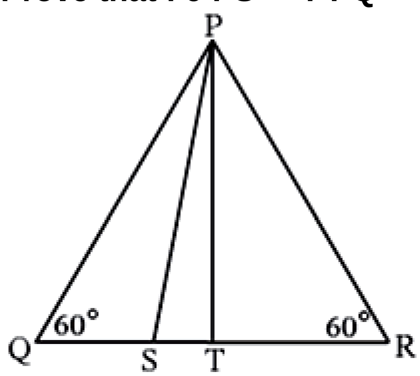


Fig. 2.35

Answer : As, PQR is an equilateral triangle,

Point S is on base QR, such that

$$QS = \frac{1}{3}QR$$

PT is perpendicular on side QR from P.

As, PQR is an equilateral triangle we have

PQ = QR = PR [1]

Also, we know that Perpendicular from a vertex to corresponding side in an equilateral triangle bisects the side

$$\Rightarrow QT = TR = \frac{1}{2}QR = \frac{1}{2}PQ$$

Now, In ΔPTQ, By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$\Rightarrow PQ^2 = PT^2 + QT^2$

$$\Rightarrow PQ^2 = PT^2 + \left(\frac{1}{2}PQ\right)^2$$

$$\Rightarrow PQ^2 = PT^2 + \frac{1}{4}PQ^2$$

$$\Rightarrow PT^2 = \frac{3}{4}PQ^2 \quad [2]$$

As,

$$QS = \frac{1}{3}QR$$

$$QT = \frac{1}{2}QR$$

We have,

QT – QS = ST

$$\Rightarrow ST = \frac{1}{2}QR - \frac{1}{3}QR$$

$$\Rightarrow ST = \frac{1}{6}QR = \frac{1}{6}PQ$$

Now, In right angled triangle PST, By Pythagoras theorem

$(PS)^2 = (ST)^2 + (PT)^2$

$$\Rightarrow PS^2 = \left(\frac{1}{6}PQ\right)^2 + \frac{3}{4}PQ^2 \text{ [From 2]}$$

$$\Rightarrow PS^2 = \frac{PQ^2}{36} + \frac{3}{4}PQ^2$$

$$\Rightarrow PS^2 = \frac{PQ^2 + 27PQ^2}{36}$$

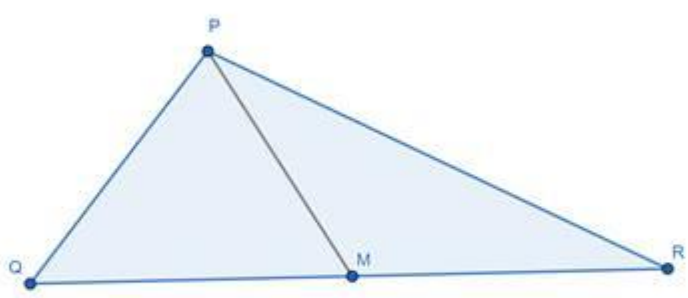
$\Rightarrow 36\ PS^2 = 28\ PQ^2$

$\Rightarrow 9\ PS^2 = 7\ PQ^2$

Hence Proved.

Q. 17. Seg PM is a median of △ PQR. If PQ = 40, PR = 42 and PM = 29, find QR.

Answer :



We know, By Apollonius theorem

In ΔPQR**,** if M is the midpoint of side QR, then $PQ^2 + PR^2 = 2PM^2 + 2QM^2$

Given that, PM is median i.e. M is the mid-point of QR

$$QM = MR = \frac{1}{2}QR$$

And PQ = 40, PR = 42, PM = 29

Putting values,

⇒ $(40)^2 + (42)^2 = 2(29)^2 + 2(QM)^2$

⇒ $1600 + 1764 = 1682 + 2QM^2$

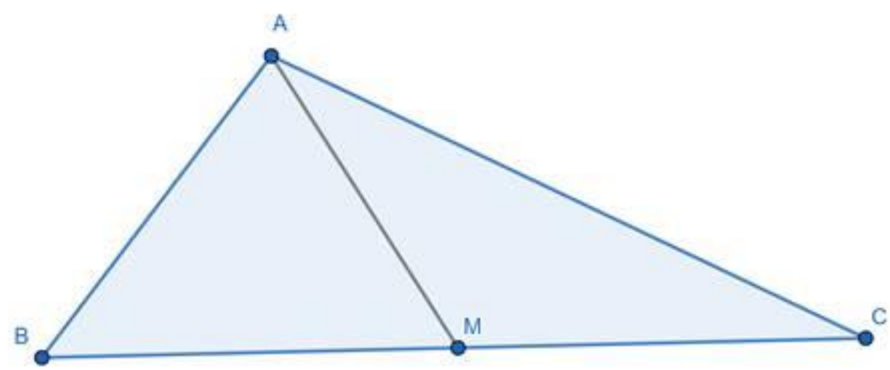
⇒ $QM^2 = 1682$

⇒ $QM = 29$

⇒ $QR = 2(29) = 58$

Q. 18. Seg AM is a median of △ ABC. If AB = 22, AC = 34, BC = 24, find AM

Answer :



We know, By Apollonius theorem

In ΔABC**,** if M is the midpoint of side BC, then $AB^2 + AC^2 = 2AM^2 + 2BM^2$

Given that,

AB = 22, AC = 34, BC = 24

AP is median i.e. P is the mid-point of BC

$$\Rightarrow BP = CP = \frac{1}{2}BC = 12$$

Putting values in equation

⇒ $22^2 + 34^2 = 2AM^2 + 2(12)^2$

$\Rightarrow 484 + 1156 = 2AM^2 + 288$

$\Rightarrow 1352 = 2AM^2$

$\Rightarrow AM^2 = 676$

$\Rightarrow AM = 26$