# Exercise 1.3

## Chapter 1 Functions and Limits Exercise 1.3 1E

(A) $y = f(x) + 3$

(B) $y = f(x) - 3$

(C) $y = f(x - 3)$

(D) $y = f(x + 3)$

(E) $y = -f(x)$

(F) $y = f(-x)$

(G) $y = 3f(x)$

(H) $y = \frac{1}{3} f(x)$

## Chapter 1 Functions and Limits Exercise 1.3 2E

(a) $y = f(x) + 8$

To obtain the graph of $y = f(x) + 8$, shift the graph of $y = f(x)$ distance 8 units upward.

(b) $y = f(x + 8)$

To obtain the graph of $y = f(x + 8)$, shift the graph of $y = f(x)$ distance 8 units left.

(c) $y = 8f(x)$

To obtain the graph of $y = 8f(x)$, stretch the graph of $y = f(x)$ vertically by a factor of 8 units.

(d) $y = f(8x)$

To obtain the graph of $y = f(8x)$, shrink the graph of $y = f(x)$ horizontally by a factor of 8 units.

(e) $y = -f(x) - 1$

To obtain the graph of $y = -f(x) - 1$, first reflect the graph of $y = f(x)$ about $x$-axis and then shift it a distance of 1 unit downward.

(f) $y = 8f\left(\frac{1}{8}x\right)$

To obtain the graph of $y = 8f\left(\frac{1}{8}x\right)$, first stretch the graph of $y = f(x)$ vertically by a factor of 8 and then stretch it horizontally by a factor of 8.

## Chapter 1 Functions and Limits Exercise 1.3 3E

Graph

From the given graph
Given that $y = f(x-4)$
i.e. The graph moves horizontally and right side '4units'

(A) 3

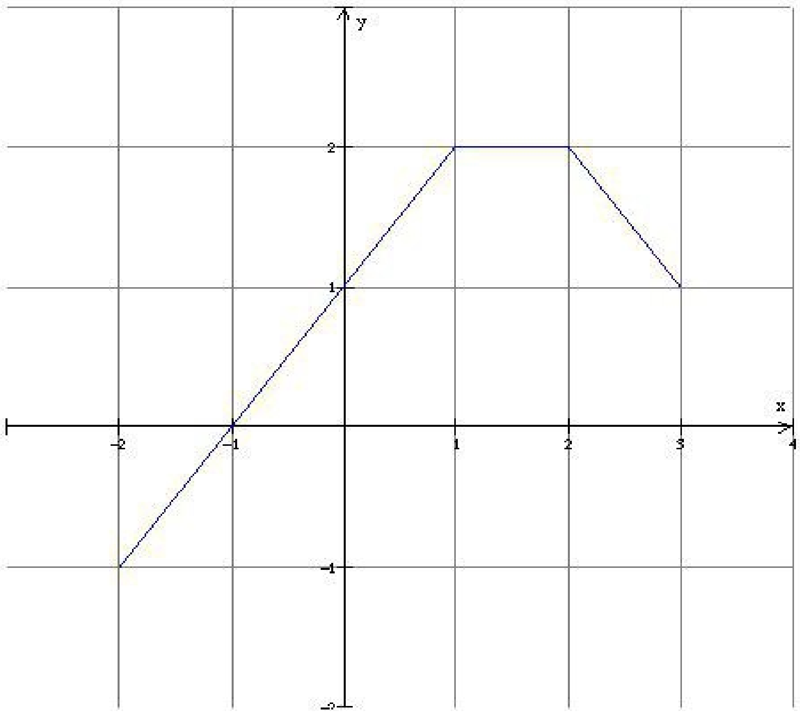
Given that $y = f(x)+3$
i.e. The graph moves vertically and upwards '3units'.
(B) 1

Given that $y = \frac{1}{3}f(x)$
i.e. The graph moves horizontally and left side '6units' and vertically upwards twice.
(C) 4

Given that $y = -f(x+4)$
i.e. The graph moves horizontally and left side '4units'
(D) 5

## Chapter 1 Functions and Limits Exercise 1.3 4E

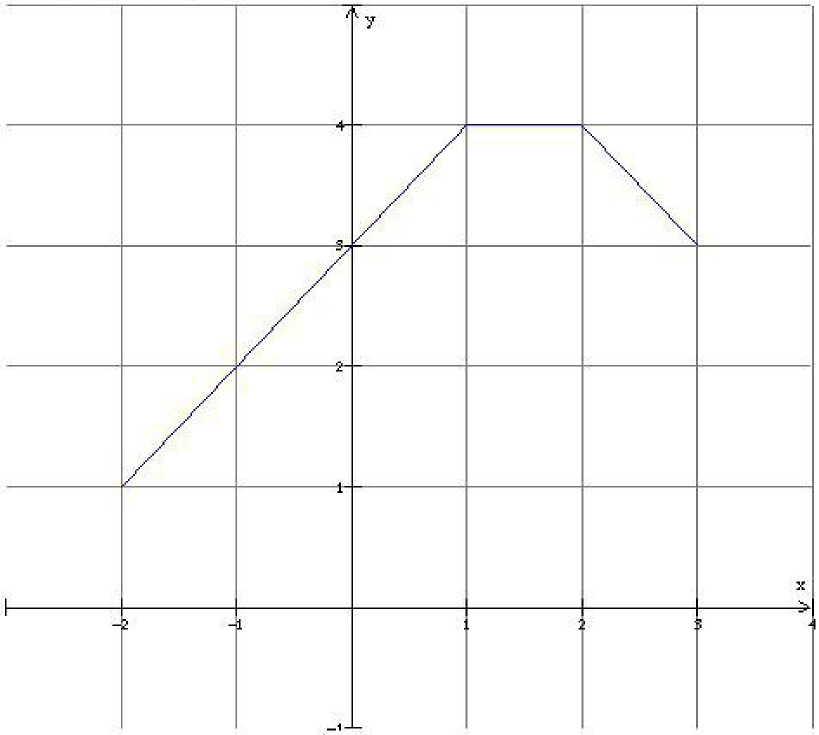
The graph of $f$ is given:




(a) $y = f(x)-2$

To obtain the graph of $y = f(x)-2$, shift the graph of $y = f(x)$ distance 2 units downward.

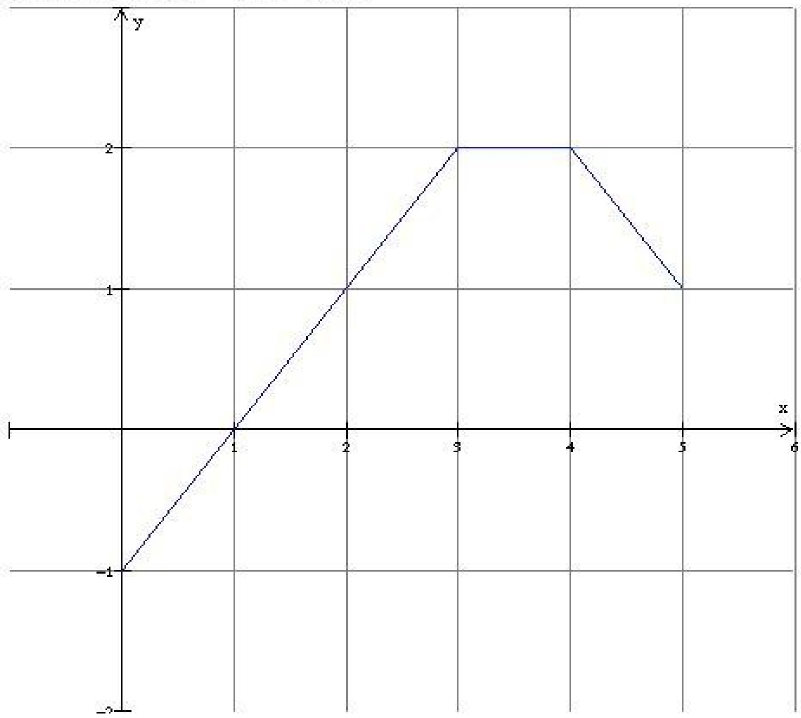
So the graph of $y = f(x) - 2$ is




(b) $y = f(x-2)$

To obtain the graph of $y = f(x-2)$, shift the graph of $y = f(x)$ distance 2 units right.

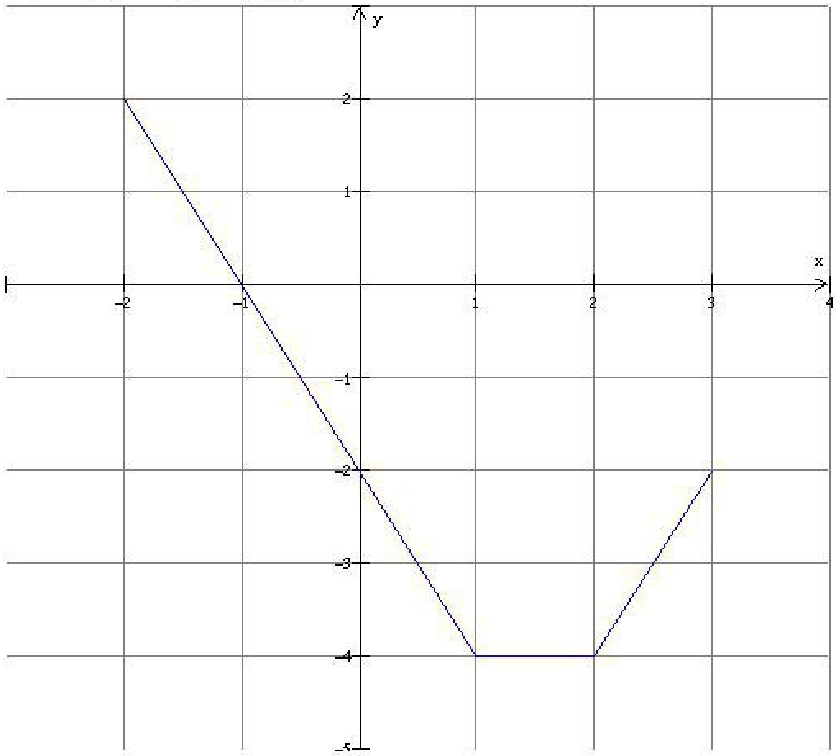
So, the graph of $y = f(x-2)$ is




(c) $y = -2f(x)$

To obtain the graph of $y = -2f(x)$, stretch the graph of $y = f(x)$ vertically by a factor of 2 units and then reflect the graph about $x$-axis.

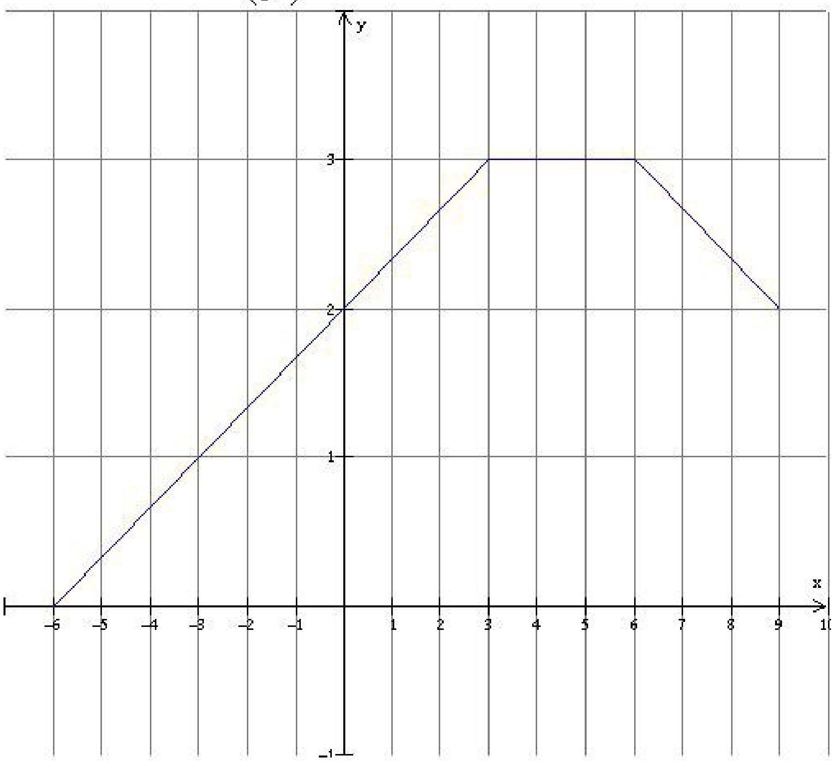
So, the graph of $y = -2f(x)$ is




(d) $y = f\left(\frac{1}{3}x\right) + 1$

To obtain the graph of $y = f\left(\frac{1}{3}x\right) + 1$, stretch the graph of $y = f(x)$ horizontally by a factor of 3 units and then shift it upward by 1 unit.

So, the graph of $y = f\left(\frac{1}{3}x\right) + 1$ is




## Chapter 1 Functions and Limits Exercise 1.3 5E

(A) Given

$y = f(2x)$

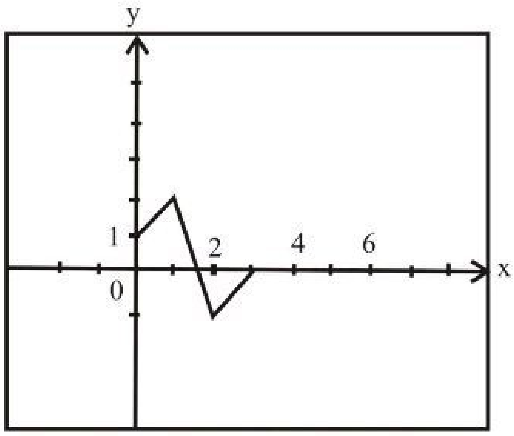



Fig 1

(B) $y = f\left(\frac{1}{2}x\right)$

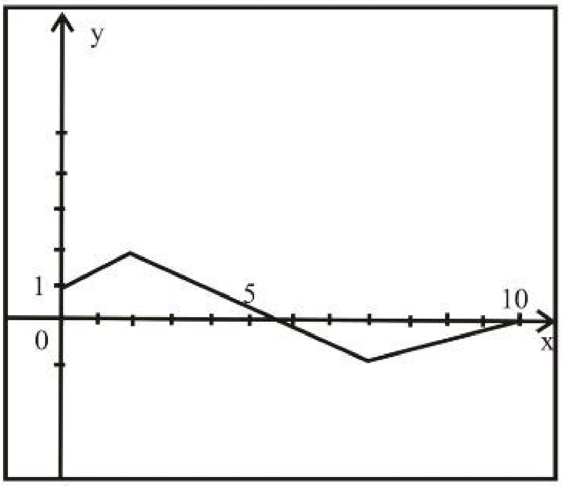
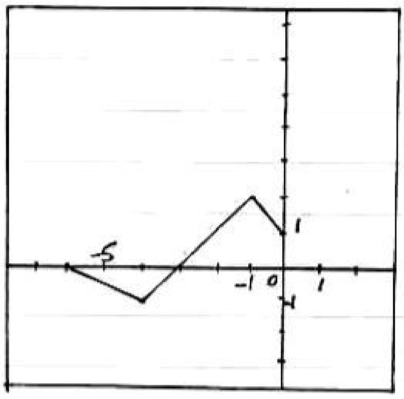



Fig 2

(C) $y = f(-x)$




(D)

$y = -f(-x)$

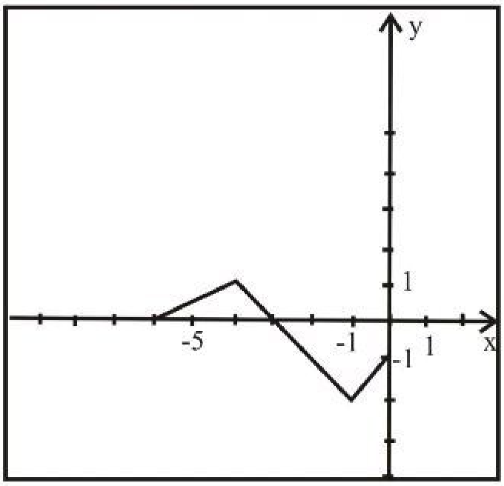
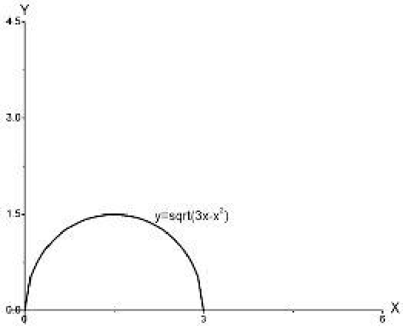
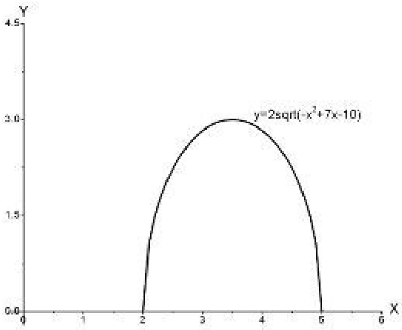



Fig 4

Chapter 1 Functions and Limits Exercise 1.3 6E




Here fig. (1) is given graph and fig. (2) is the graph of transformed function. We can see easily that the graph is shifted to 2 units to the right and it is also stretched upward by the factor of 2 which means that the transformed function will be $y = 2f(x-2)$

So $f(x) = \sqrt{3x - x^2}$

Then $f(x-2) = \sqrt{3(x-2) - (x-2)^2}$

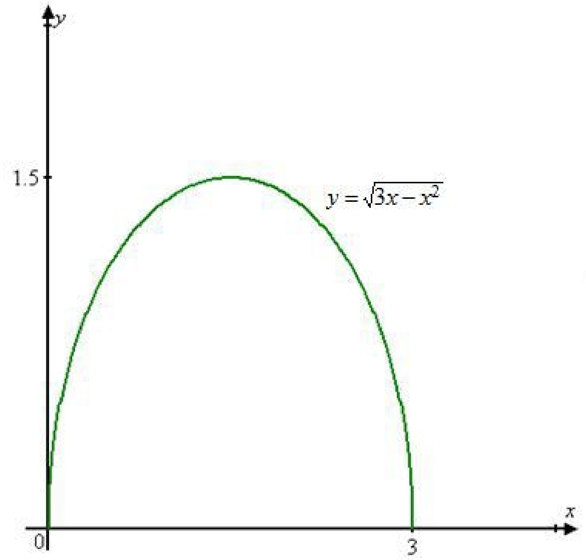
$$= \sqrt{3x - 6 - (x^2 - 4x + 4)}$$
$$= \sqrt{3x - 6 - x^2 + 4x - 4}$$
$$= \sqrt{7x - x^2 - 10}$$

Then the problem figure is the graph of the function

$$\boxed{y = 2.\sqrt{7x - x^2 - 10}}$$

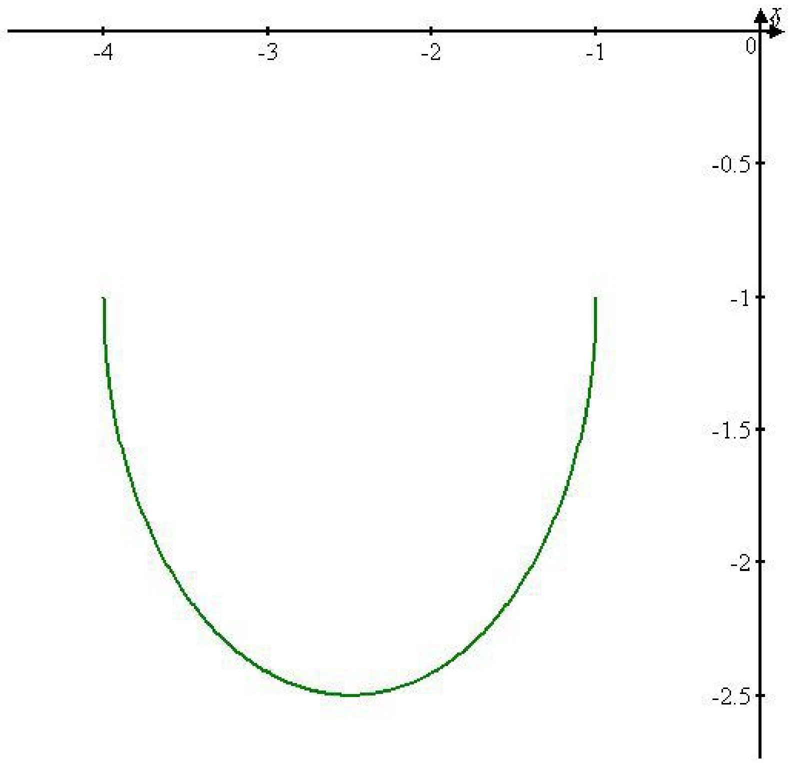
## Chapter 1 Functions and Limits Exercise 1.3 7E

Consider the graph of $y=\sqrt{3x-x^2}$




Vertical and Horizontal Shifts and Reflecting: Suppose $c>0$. To obtain the graph of $y=f(x)+c$, shift the graph of $y=f(x)$ a distance *c* units upward. $y=f(x)-c$, shift the graph of $y=f(x)$ a distance *c* units downward. $y=f(x-c)$, shift the graph of $y=f(x)$ a distance *c* units to the right. $y=f(x+c)$, shift the graph of $y=f(x)$ a distance *c* units to the left. $y=-f(x)$, reflect the graph of $y=f(x)$ about the *x*-axis.

Use transformations to find a function whose graph is shown:




From the graph observe that $y=\sqrt{3x-x^2}$ has been shifted 4 units to the left, reflected about the *x*-axis, and shifted downward 1 unit.

Therefore, a function describing the graph is

$$\begin{aligned} y &= -\sqrt{3(x+4)-(x+4)^2}-1 \\ &= -\sqrt{3x+12-x^2-8x-16}-1 \\ &= \boxed{-\sqrt{-x^2-5x-4}-1} \end{aligned}$$

Chapter 1 Functions and Limits Exercise 1.3 8E

(A)

The graph of $y = 2\sin x$, we get by stretching the graph of $y = \sin x$ by a factor of 2. (Upward)

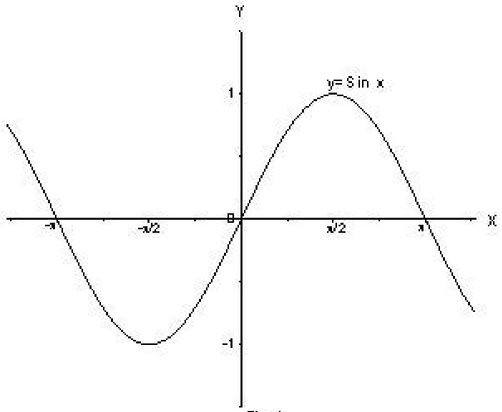



Fig.1

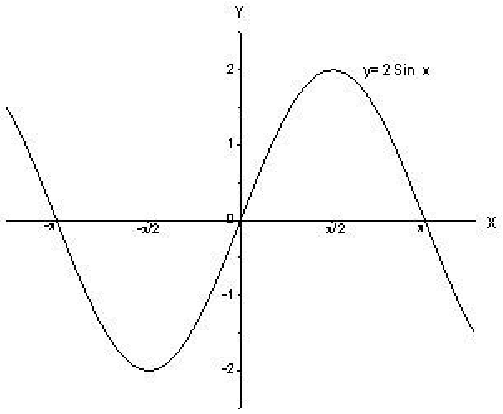



Fig.2

(B)

Here the graph of $y = 1 + \sqrt{x}$, we can get by shifting the graph $y = \sqrt{x}$, a distance 1 unit upward

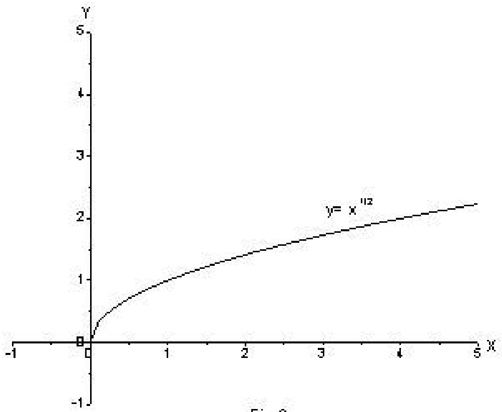



Fig.3

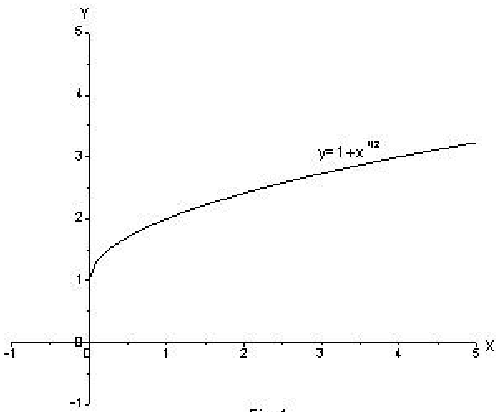



Fig.4

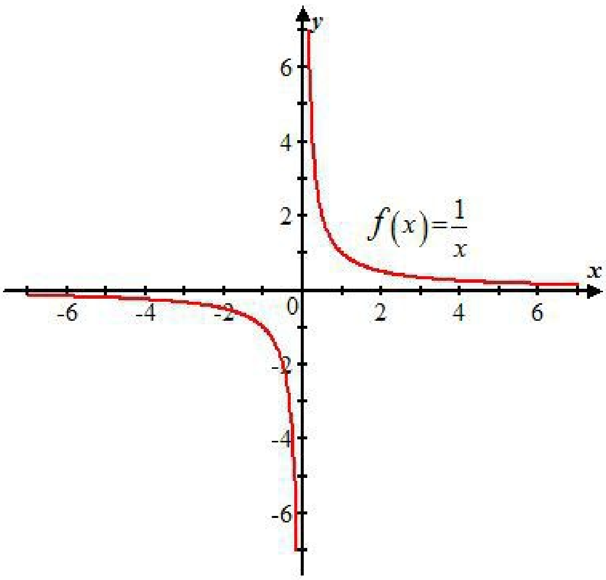
Consider the function $y = \frac{1}{x+2}$

To draw the graph of original function we plot the graph of standard function.

The standard function is $f(x) = \frac{1}{x}$.

Its graph has the equation $y = \frac{1}{x}$, or $xy = 1$, and is a hyperbola with the coordinate axes as its asymptotes.

The graph of the standard function $f(x) = \frac{1}{x}$ is shown below:

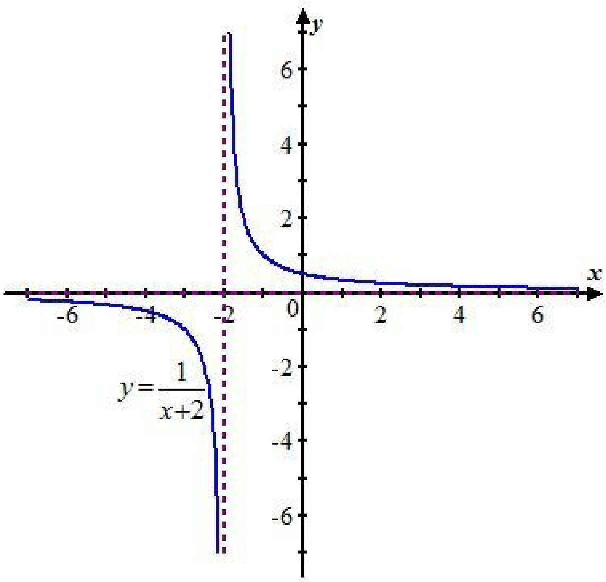



Observe that, the function

$$\begin{aligned} y &= \frac{1}{x+2} \\ &= f(x+2) \end{aligned}$$

This means that, the graph of the function $y = \frac{1}{x+2}$ is obtained by starting with the reciprocal function $f(x) = \frac{1}{x}$ and shifting 2 units to the left. And also $x = -2$ and $x$-axis are its asymptotes.

Therefore, the required graph of the given function is shown below:



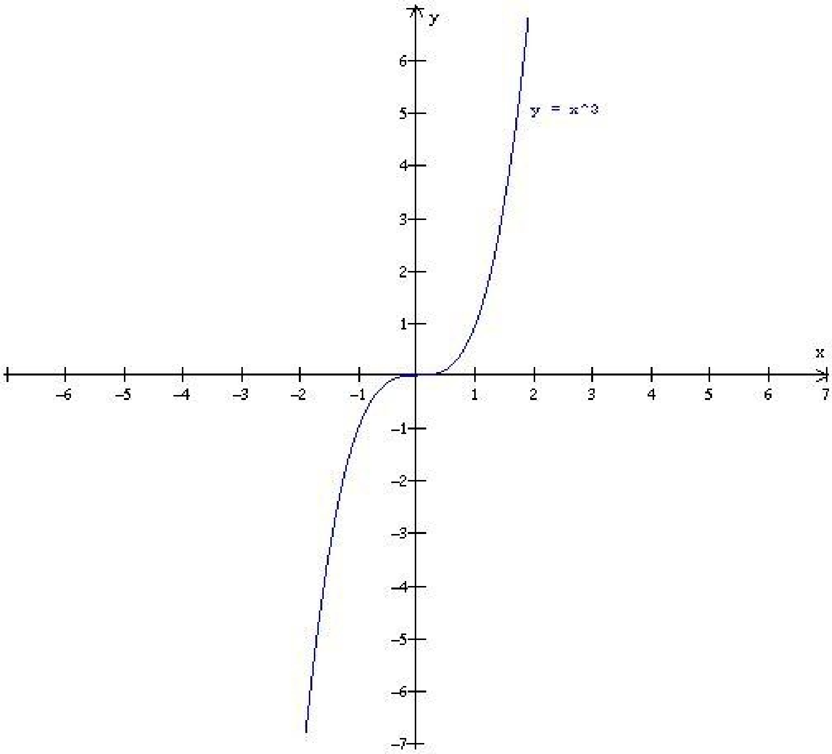

Chapter 1 Functions and Limits Exercise 1.3 10E

Given function

$y = (x-1)^3$

To draw the graph of given function we plot the graph of standard function ( power function)

$f(x) = x^3$




Now, to draw the graph of given function

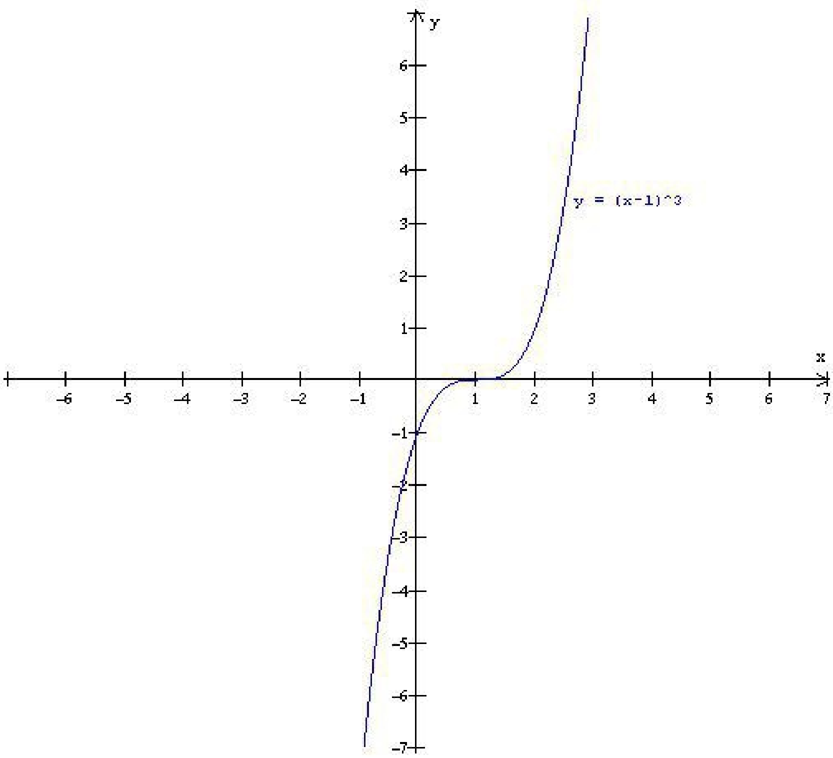
$y = f(x-1) = (x-1)^3$

we shift the graph of

$f(x) = x^3$

a distance 1 unit to the right.

So, the required graph of the given function is

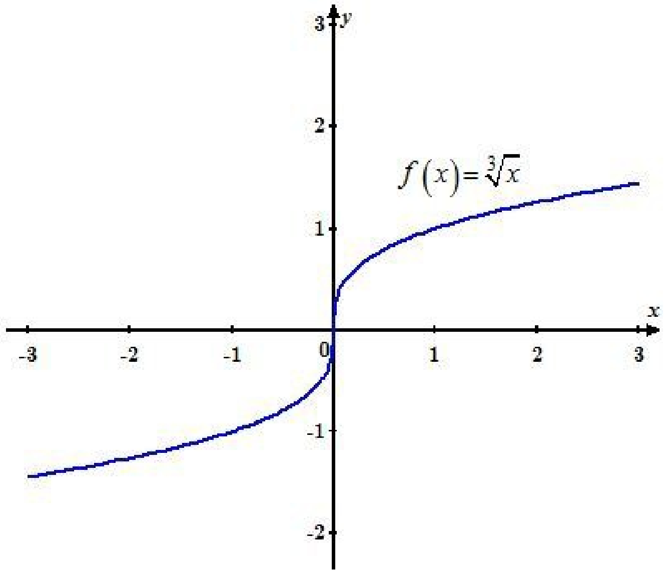



Chapter 1 Functions and Limits Exercise 1.3 11E

Consider the function

$y = -\sqrt[3]{x}$

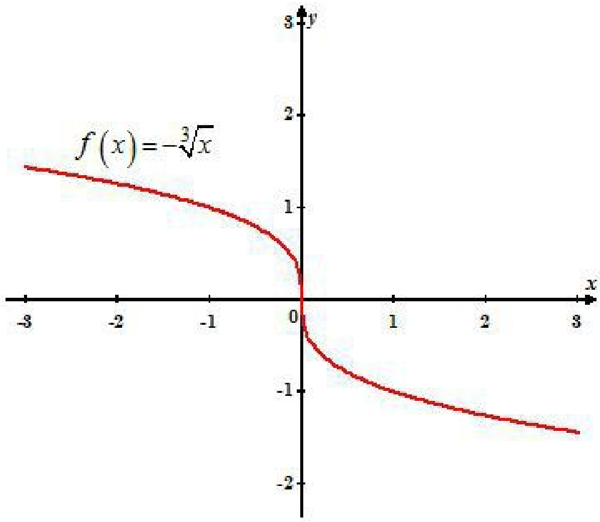
First draw the graph of $f(x) = \sqrt[3]{x}$ and how the present function can be graphed with the help of this graph.




Observe that the change in the sign of the function $f(x) = \sqrt[3]{x}$ will give the present function $f(x) = -\sqrt[3]{x}$.

Note that, the change in the sign of *f* will give a symmetric graph about *x*-axis and that in *x* will give a symmetric graph about *y*-axis.

The graph of $f(x) = -\sqrt[3]{x}$ is




The transformation used to change the standard function $f(x) = \sqrt[3]{x}$ to $f(x) = -\sqrt[3]{x}$ is the reflection about the *x*-axis.

## Chapter 1 Functions and Limits Exercise 1.3 12E

Given function

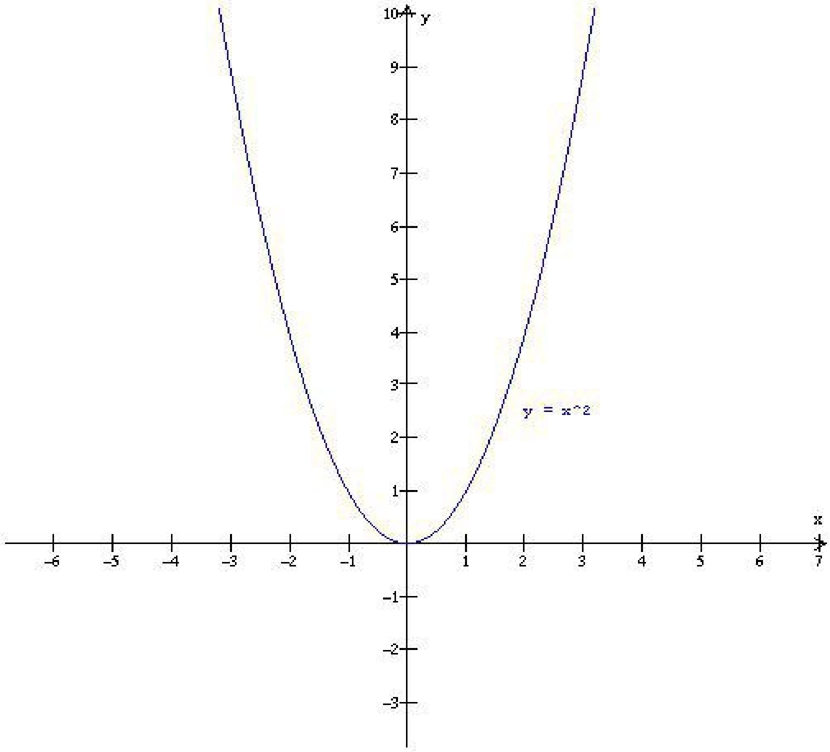
$y = x^2 + 6x + 4$

$\Rightarrow y = x^2 + 6x + 9 - 5$

$\Rightarrow y = (x+3)^2 - 5$

To draw the graph of given function we plot the graph of standard function ( power function)

$f(x) = x^2$




Now, to draw the graph of given function

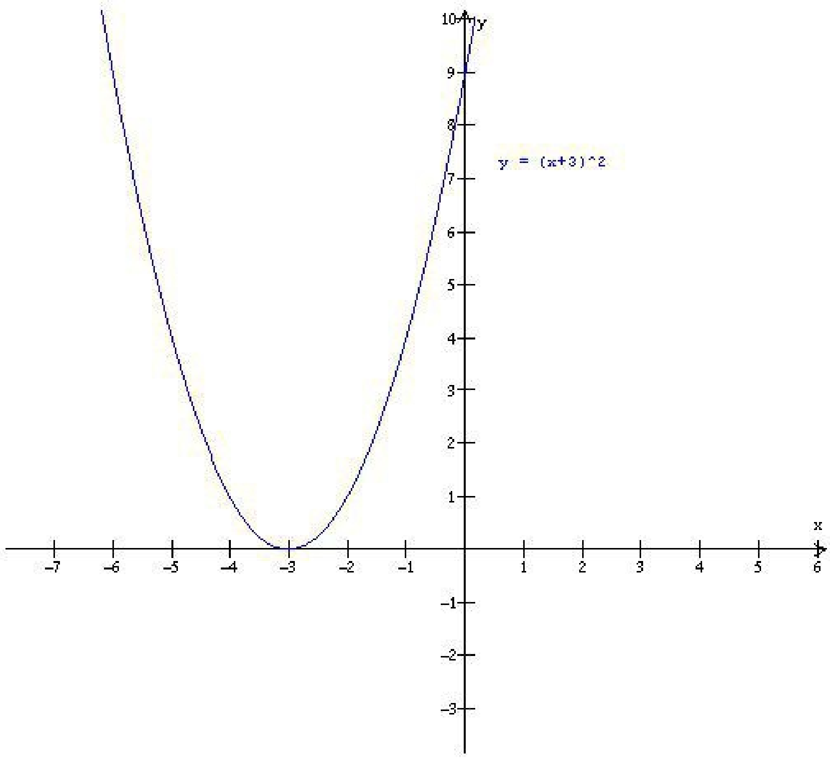
$g(x) = f(x+3) = (x+3)^2$

we shift the graph of

$f(x) = x^2$

a distance of 3 units to the left.

So, the graph of $g(x) = f(x+3) = (x+3)^2$ is




Now, to draw the graph of given function

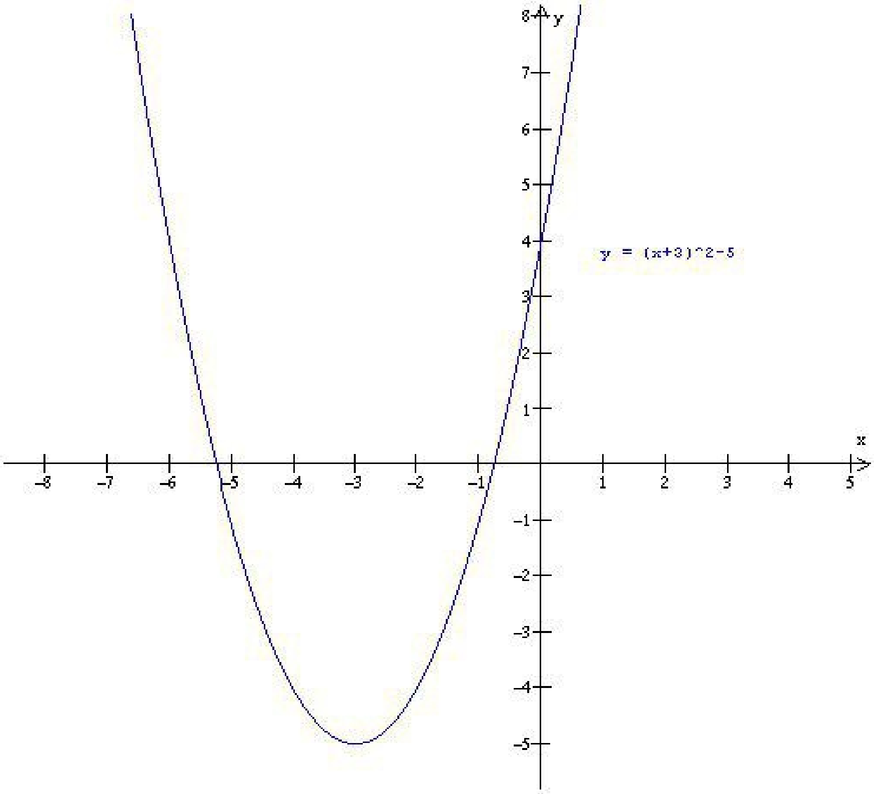
$y = f(x+3) - 5 = (x+3)^2 - 5$

we shift the graph of

$g(x) = f(x+3) = (x+3)^2$

a distance of 5 units downward.

So, the graph of $y = f(x+3) - 5 = (x+3)^2 - 5$ is



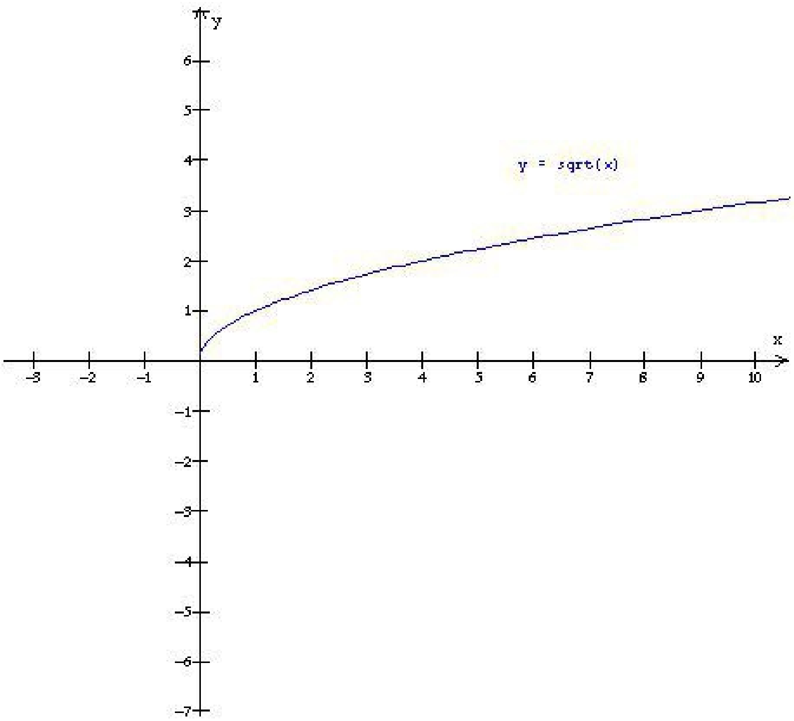

## Chapter 1 Functions and Limits Exercise 1.3 13E

Given function

$y=\sqrt{x-2}-1$

To draw the graph of given function we plot the graph of standard function ( root function)

$f(x)=\sqrt{x}$




Now, to draw the graph of function

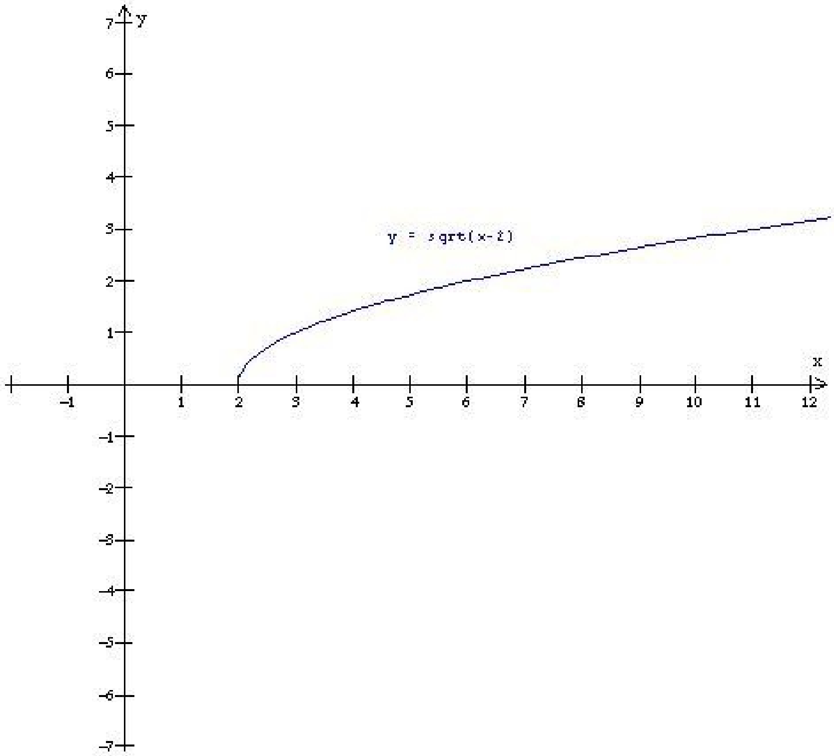
$g(x)=f(x-2)=\sqrt{x-2}$

we shift the graph of

$f(x)=\sqrt{x}$

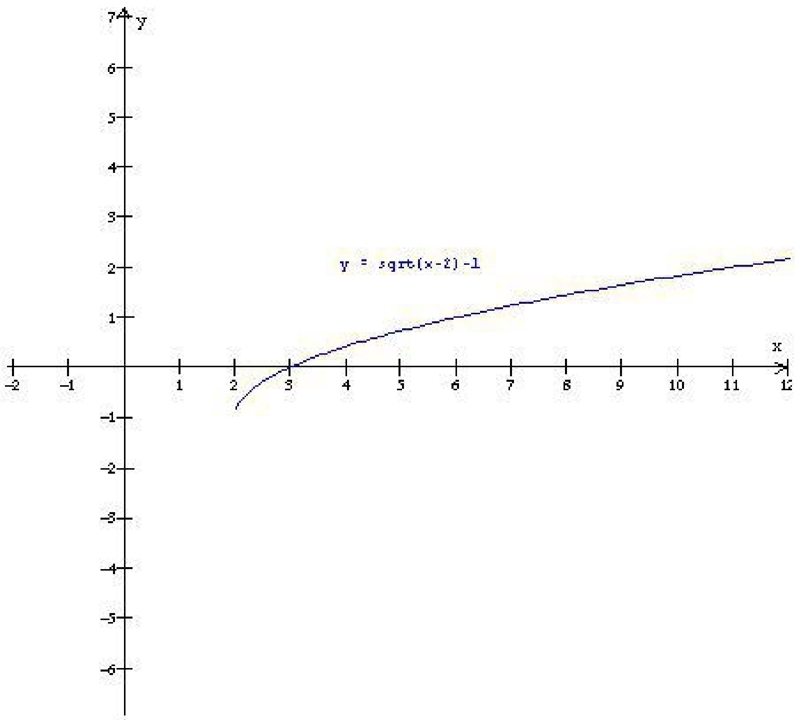
A distance of 2 units right.

So, the graph of $f(x-2)=\sqrt{x-2}$ is




Now, to draw the graph of function
$y=f(x-2)-1=\sqrt{x-2}-1$
we shift the graph of
$g(x)=f(x-2)=\sqrt{x-2}$
A distance of 1 units downward.

So, the required graph of the given function is




## Chapter 1 Functions and Limits Exercise 1.3 14E

$y=4\sin 3x$
Here we use the graph of $y=\sin x$

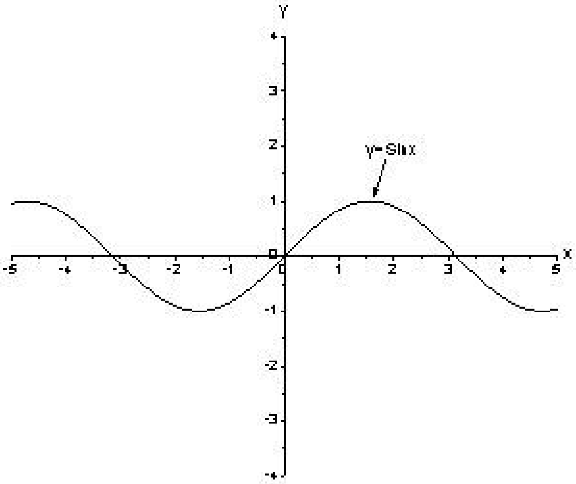



Fig.1

Compress the graph of $y = \sin x$ by the factor of 3 horizontally for getting the graph of $y = \sin 3x$

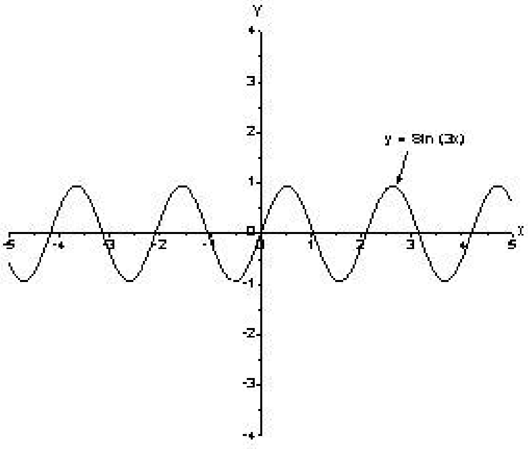



Fig.2

Stretch the graph of $y = \sin 3x$, by factor 4 vertically for getting the graph of $y = 4\sin 3x$. Fig 3 is the final graph of $y = 4\sin 3x$.

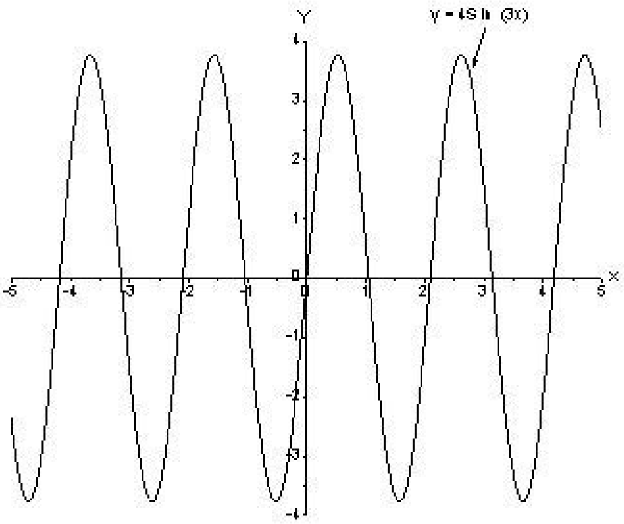
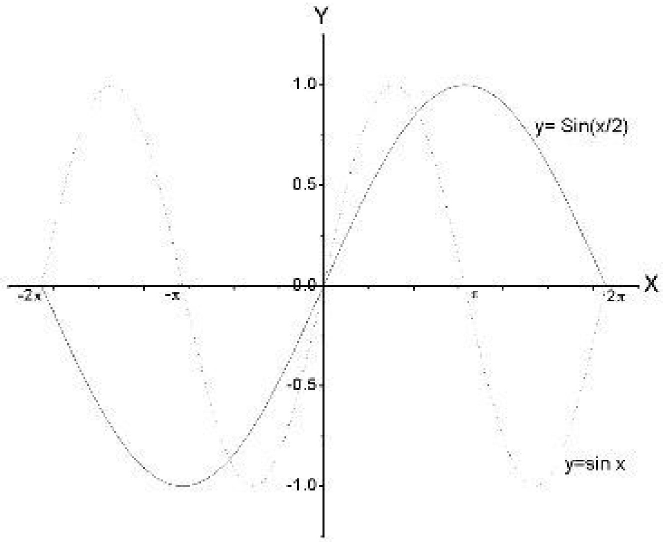



Fig.3

## Chapter 1 Functions and Limits Exercise 1.3 15E

We can find the graph of $y = \sin\left(x/2\right)$ by stretching the graph of y = Sin x horizontally by a factor of 2.



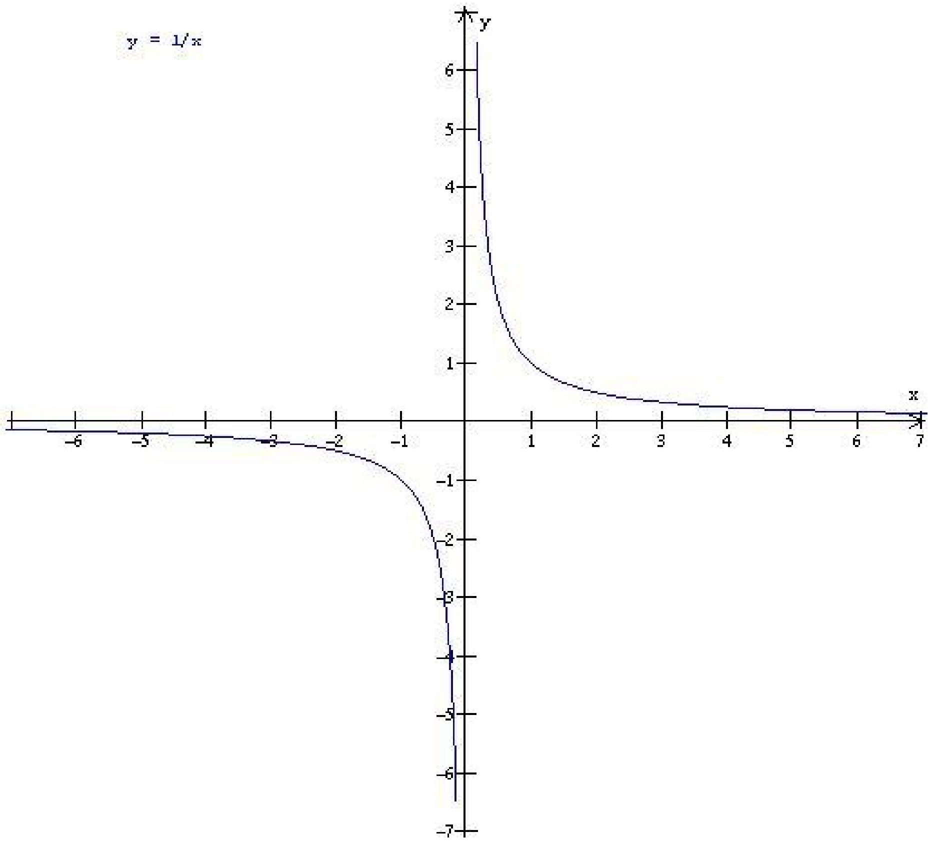

Chapter 1 Functions and Limits Exercise 1.3 16E

Given function

$$y=\frac{2}{x}-2$$

To draw the graph of given function we plot the graph of standard function (reciprocal function)

$$f(x)=\frac{1}{x}$$




Now, to draw the graph of function

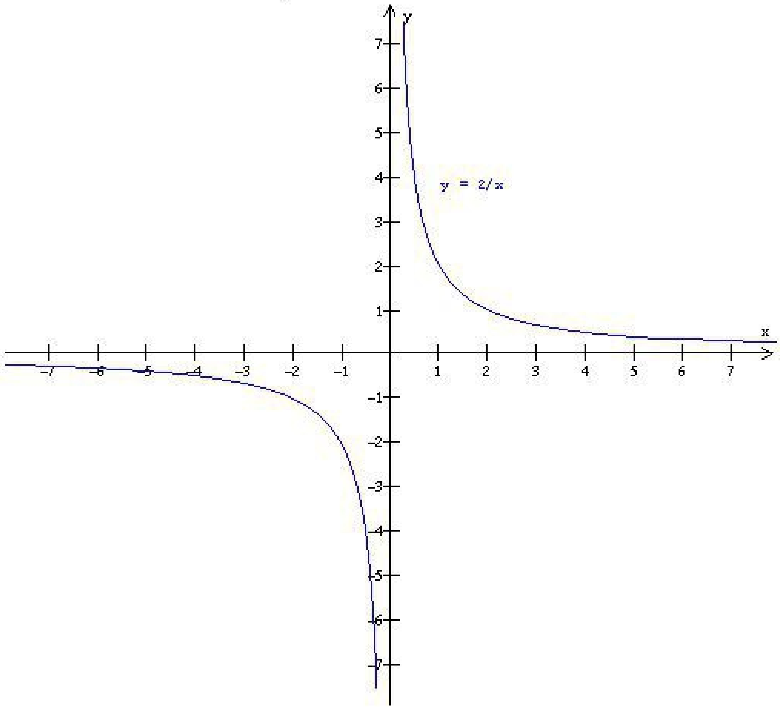
$$g(x)=2f(x)=\frac{2}{x}$$

we stretch the graph of

$$f(x)=\frac{1}{x}$$

vertically by a factor of 2.

So, the graph of $g(x) = 2f(x) = \frac{2}{x}$ is




Now, to draw the graph of function

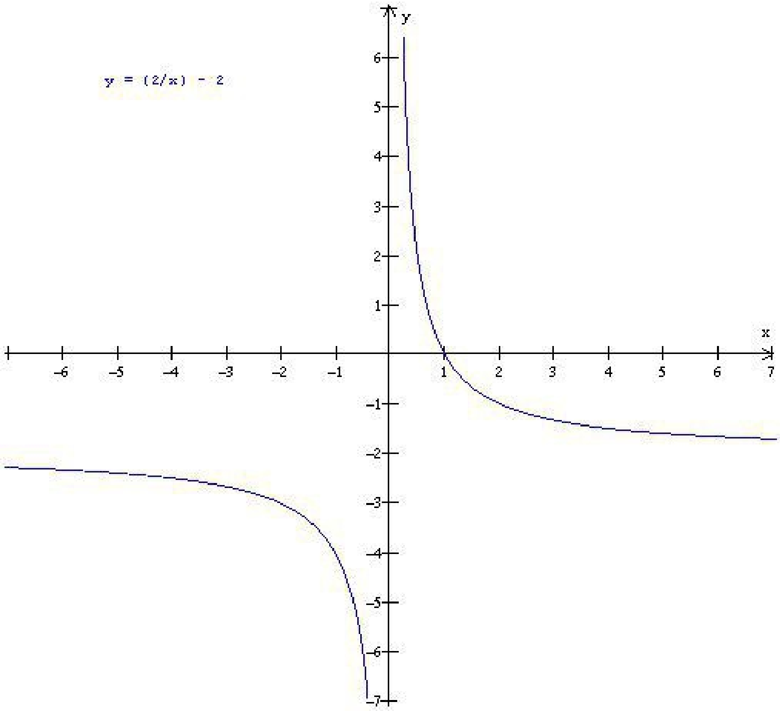
$y = 2f(x) - 2 = \frac{2}{x} - 2$

we shift the graph of

$g(x) = f(x-2) = \sqrt{x-2}$

a distance of 2 units downward.

So, the required graph of the given function is



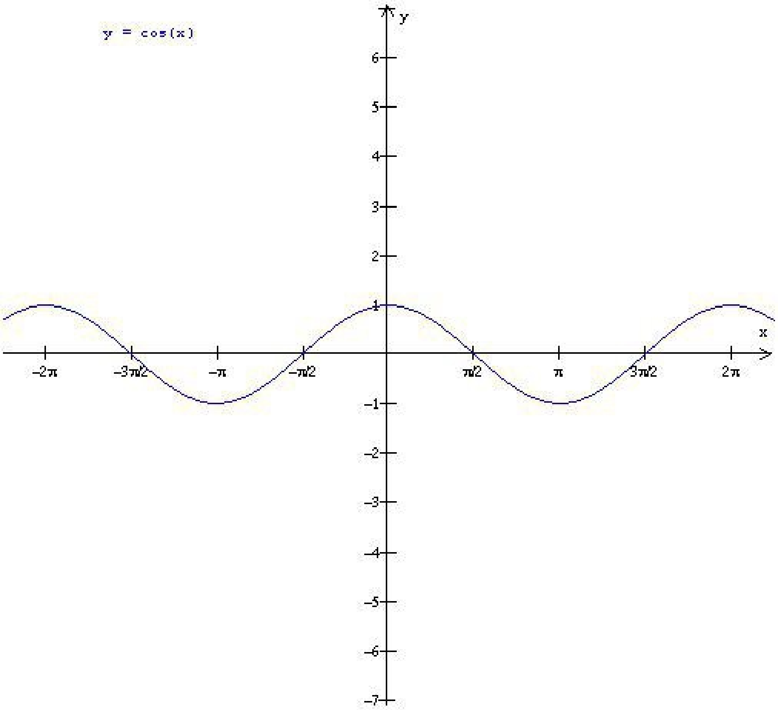

## Chapter 1 Functions and Limits Exercise 1.3 17E

Given function

$y = \frac{1}{2}(1 - \cos x)$

To draw the graph of given function we plot the graph of standard function (Trigonometric function)

$f(x) = \cos x$




Now, to draw the graph of function

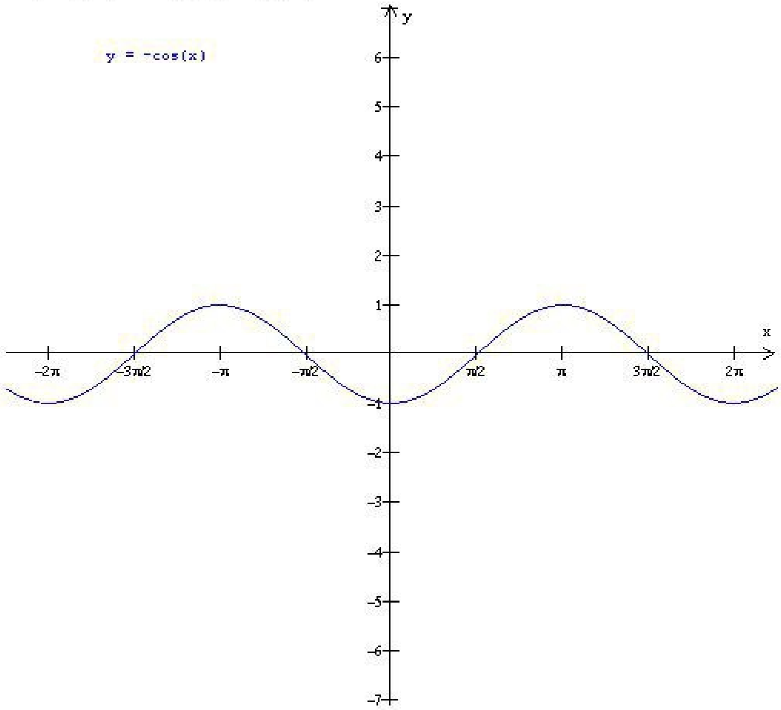
$$g(x) = -f(x)$$
$$= -\cos x$$

we reflect the graph of

$f(x) = \cos x$

about x-axis.

So, the graph of $g(x) = -f(x) = -\cos x$ is




Now, to draw the graph of function

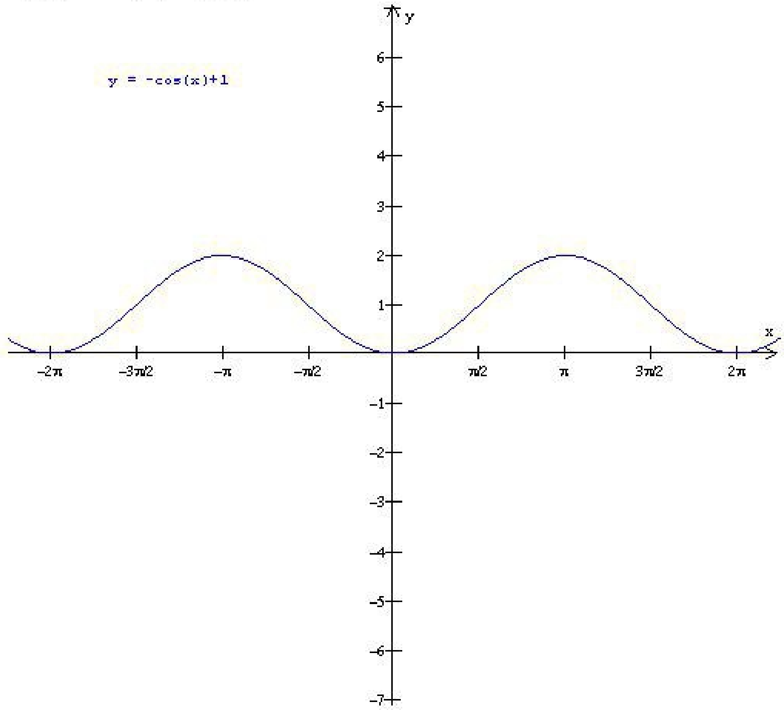
$h(x) = -f(x) + 1 = 1 - \cos x$

we shift the graph of

$g(x) = -f(x) = -\cos x$

a distance of 1 unit upward.

So, the graph of $h(x) = -f(x) + 1 = 1 - \cos x$ is




Now, to draw the graph of function

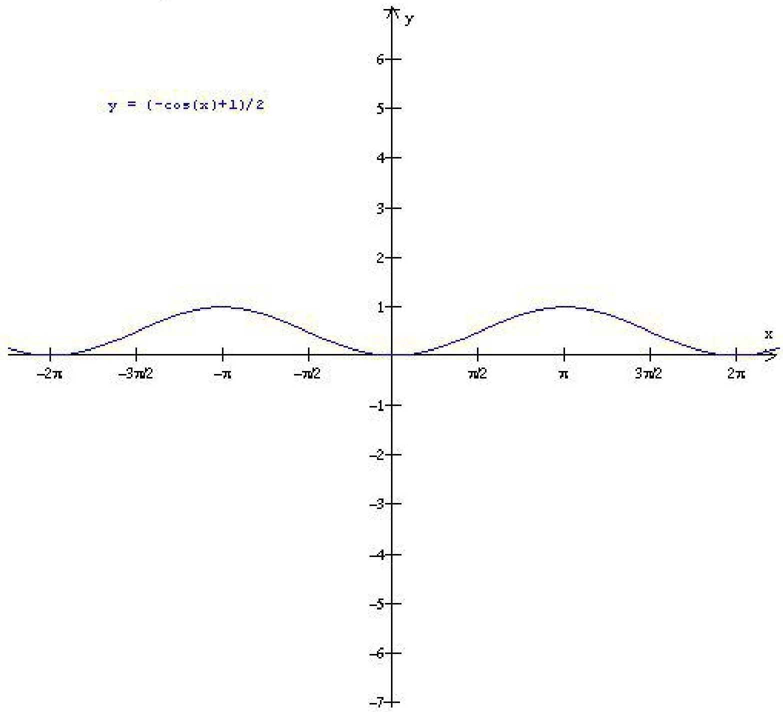
$y = \frac{h(x)}{2} = \frac{1}{2}(1 - \cos x)$

We shrink the graph of

$h(x) = -f(x) + 1 = 1 - \cos x$

Vertically by a factor of 2.

So, the graph of $y = \frac{1}{2}(1 - \cos x)$ is



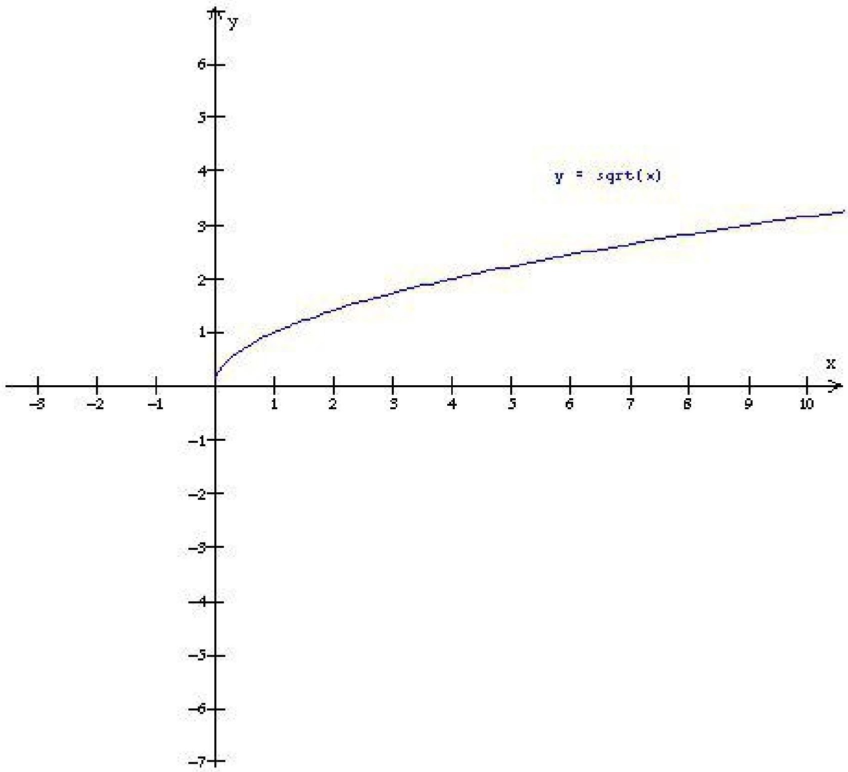

## Chapter 1 Functions and Limits Exercise 1.3 18E

Given function

$y = 1 - 2\sqrt{x+3}$

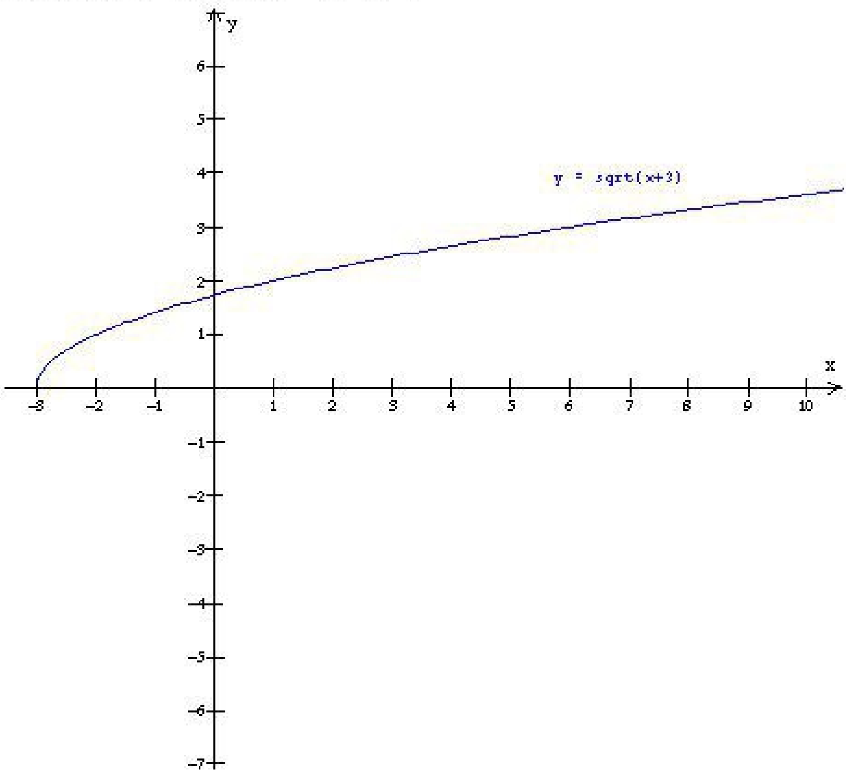
To draw the graph of given function we plot the graph of standard function ( root function)

$f(x) = \sqrt{x}$



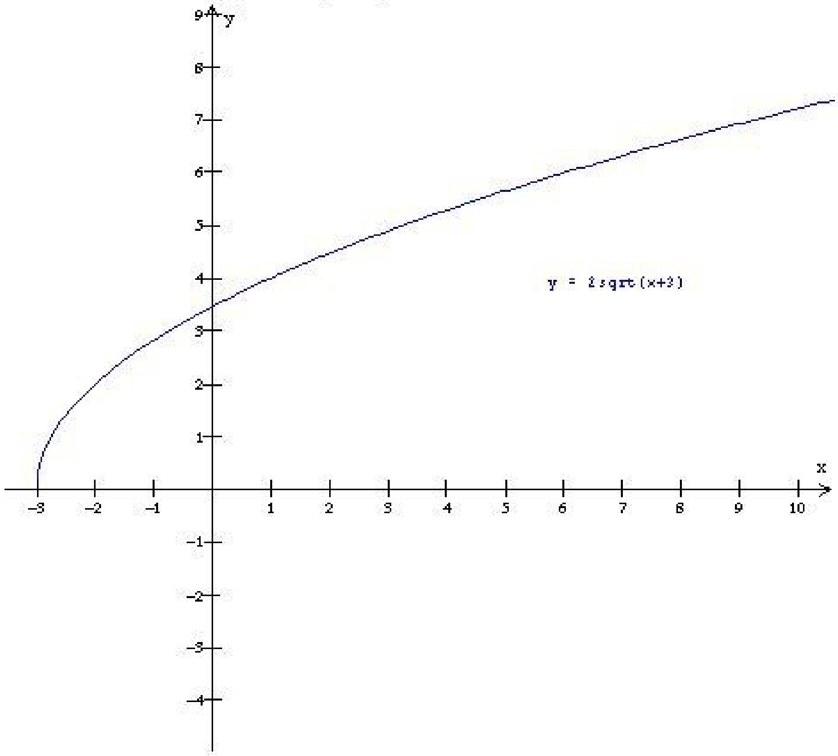

Now, to draw the graph of function
$g(x) = f(x+3) = \sqrt{x+3}$
we shift the graph of
$f(x) = \sqrt{x}$
A distance of 3 units to the left.

So, the graph of $g(x) = f(x+3) = \sqrt{x+3}$ is




Now, to draw the graph of function
$h(x) = 2f(x+3) = 2\sqrt{x+3}$
we stretch the graph of
$g(x) = f(x+3) = \sqrt{x+3}$
vertically by a factor of 2.

So, the graph of $h(x) = 2f(x+3) = 2\sqrt{x+3}$ is




Now, to draw the graph of function

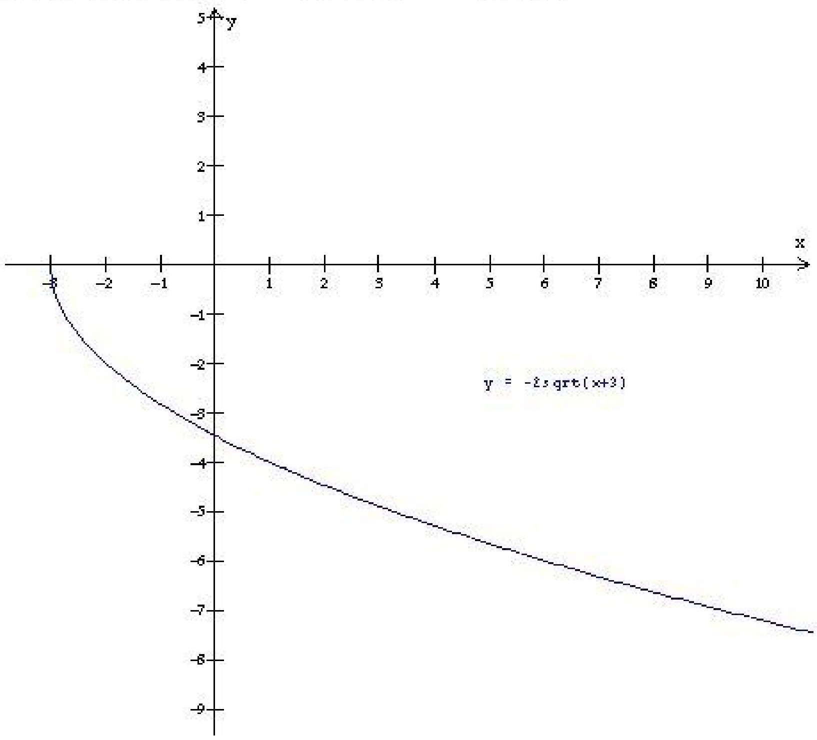
$u(x) = -2f(x+3) = -2\sqrt{x+3}$

we reflect the graph of

$h(x) = 2f(x+3) = 2\sqrt{x+3}$

about x-axis.

So, the graph of $u(x) = -2f(x+3) = -2\sqrt{x+3}$ is




Now, to draw the graph of function

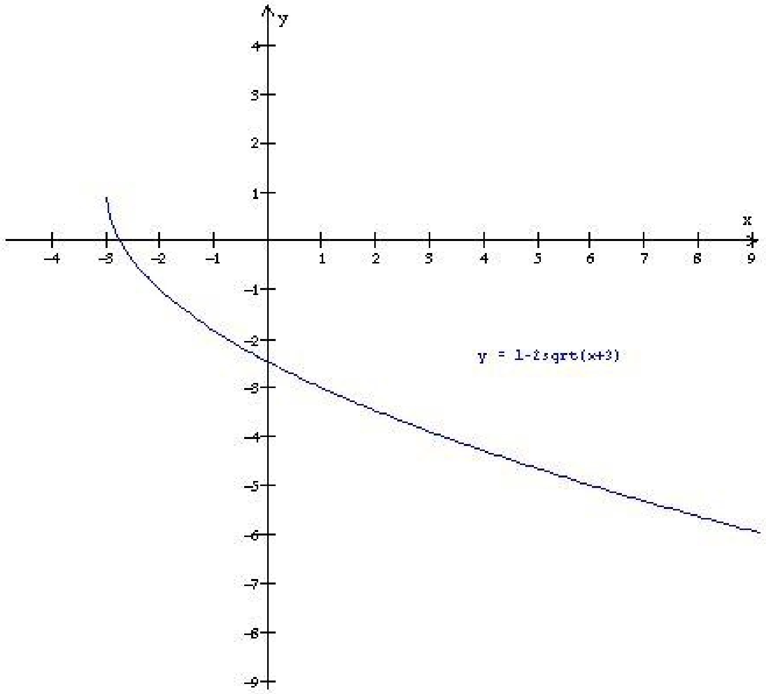
$y = 1 - 2f(x+3) = 1 - 2\sqrt{x+3}$

we shift the graph of

$u(x) = -2f(x+3) = -2\sqrt{x+3}$

a distance of 1 unit upward.

So, the graph of $y = 1 - 2f(x+3) = 1 - 2\sqrt{x+3}$ is




## Chapter 1 Functions and Limits Exercise 1.3 19E

Consider the following function:

$$y = 1 - 2x - x^2$$

The objective is to sketch the graph of the given function using transformation.

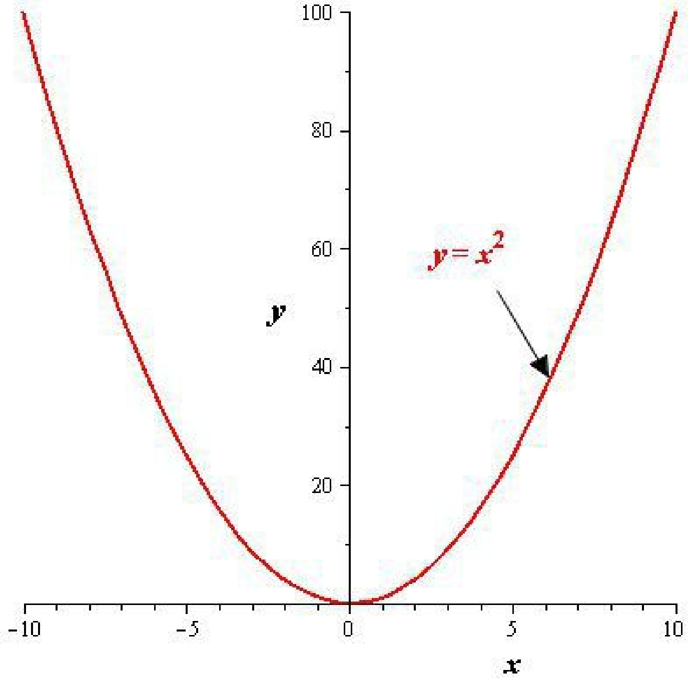
Complete the square to rewrite the above function as,

$$\begin{aligned} y &= 1 - 2x - x^2 \\ &= -(x^2 + 2x + 1) + 2 \\ &= -(x+1)^2 + 2 \end{aligned}$$

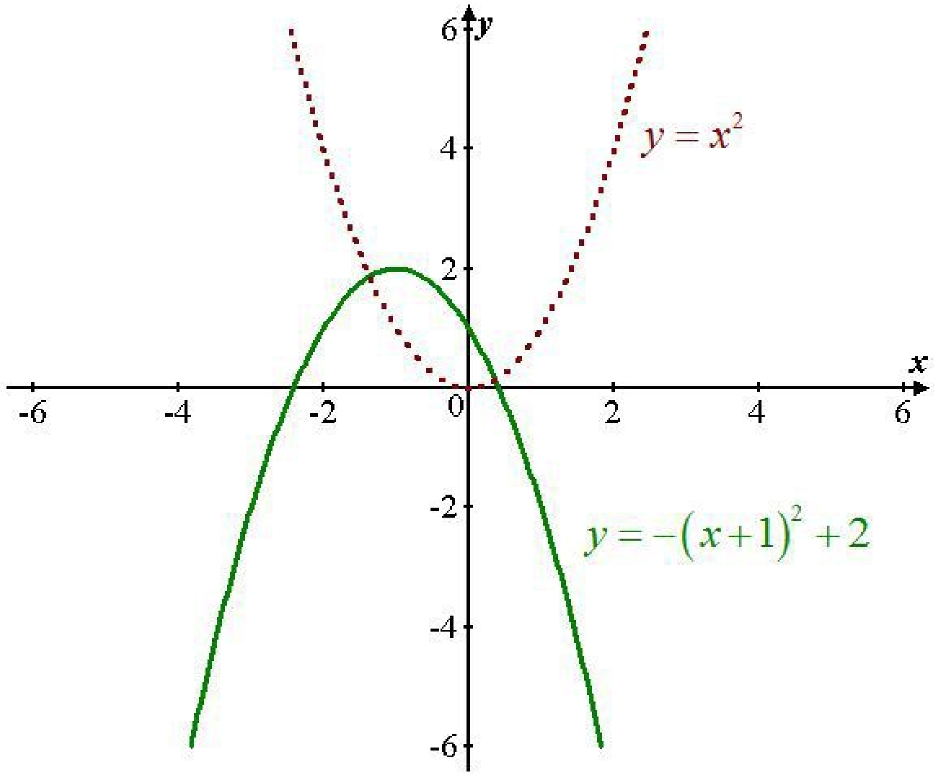
To plot the graph of the function above, consider the following function,

$$y = x^2$$

The graph of the function $y = x^2$ is a parabola.



The graph of the function $y=-(x+1)^2+2$ is as shown below.



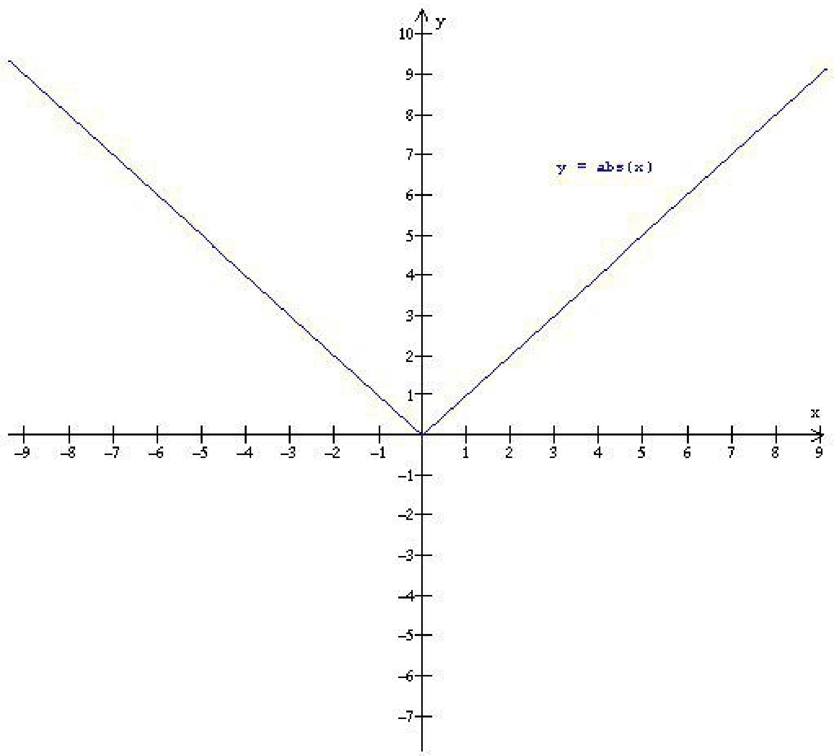

Chapter 1 Functions and Limits Exercise 1.3 20E

Given function

$y=|x|-2$

To draw the graph of given function we plot the graph of standard function ( modulus function)

$f(x)=|x|$




Now, to draw the graph of function

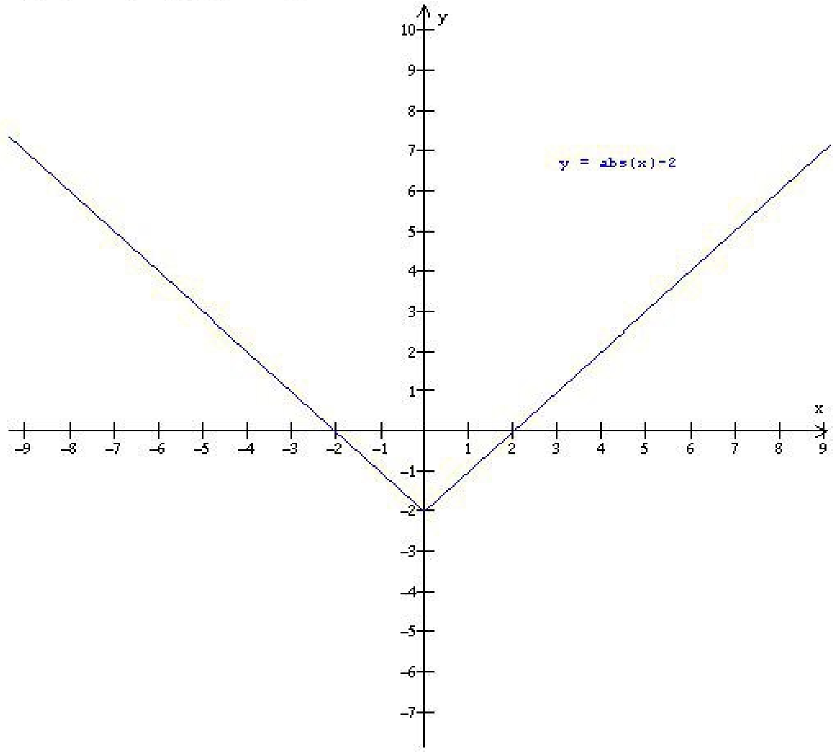
$y=f(x)-2=|x|-2$

we shift the graph of

$f(x)=|x|$

a distance of 2 units downward.

So, the graph of $y = f(x) - 2 = |x| - 2$ is



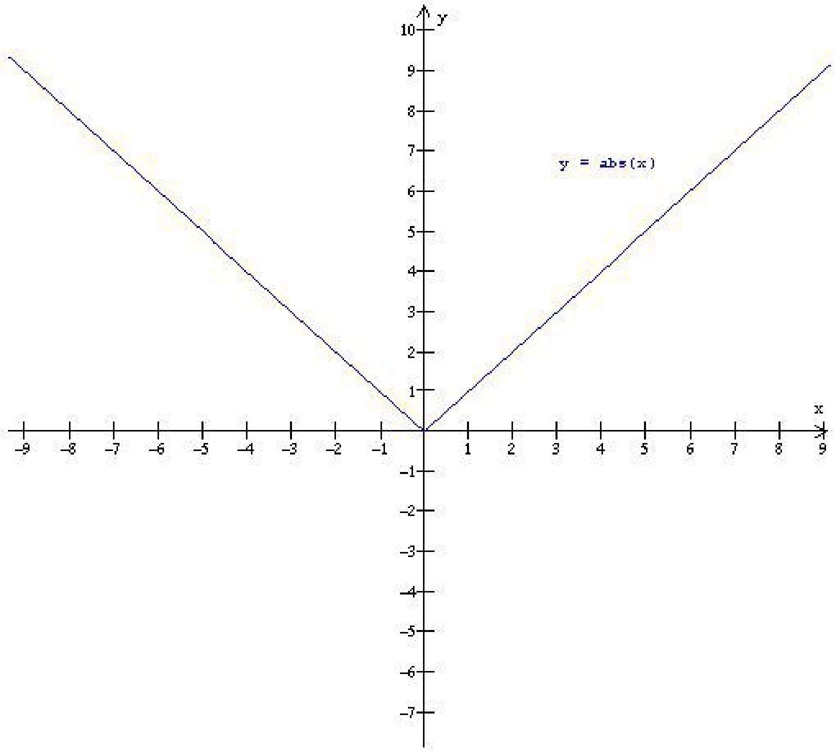

## Chapter 1 Functions and Limits Exercise 1.3 21E

Given function

$y = |x - 2|$

To draw the graph of given function we plot the graph of standard function ( modulus function)

$f(x) = |x|$




Now, to draw the graph of function

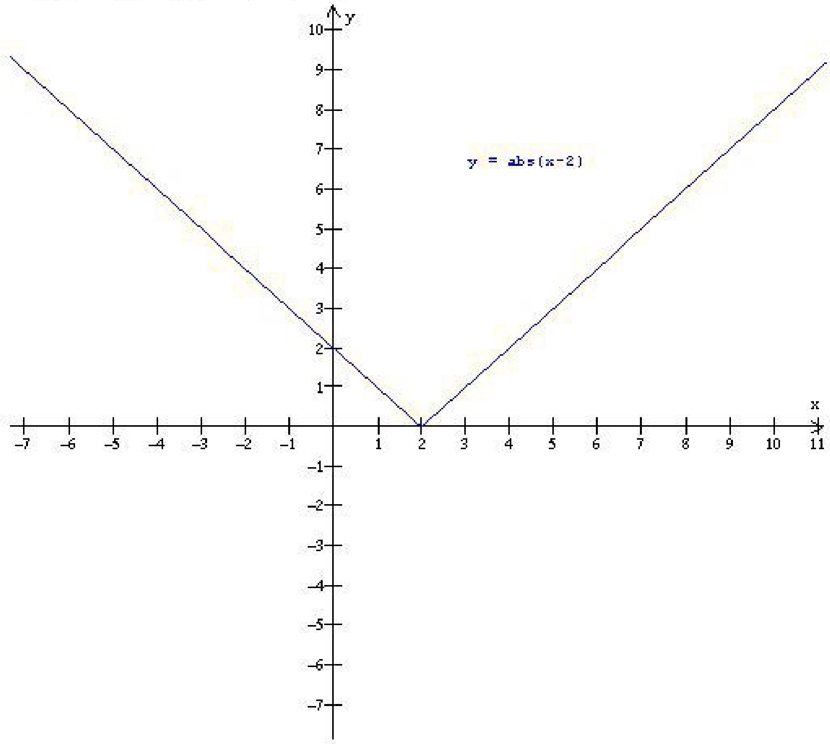
$y = f(x - 2) = |x - 2|$

we shift the graph of

$f(x) = |x|$

a distance of 2 units to right.

So, the graph of $y = f(x-2) = |x-2|$ is




## Chapter 1 Functions and Limits Exercise 1.3 22E

$y = \frac{1}{4}\tan\left(x - \frac{\pi}{4}\right)$

Here we will use the graph of $y = \tan x$

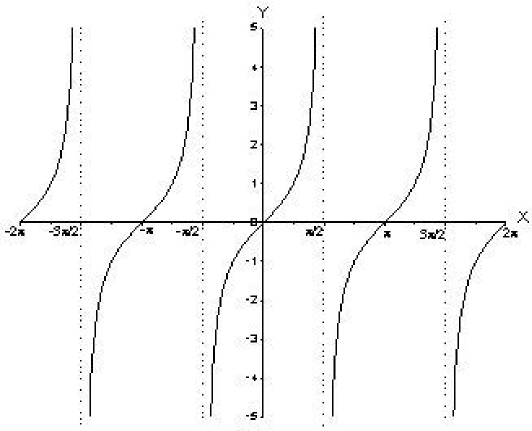


Fig.1

Shift the graph of $y = \tan x$, to the right a distance $\frac{\pi}{4}$, for getting the graph of $y = \tan\left(x - \frac{\pi}{4}\right)$

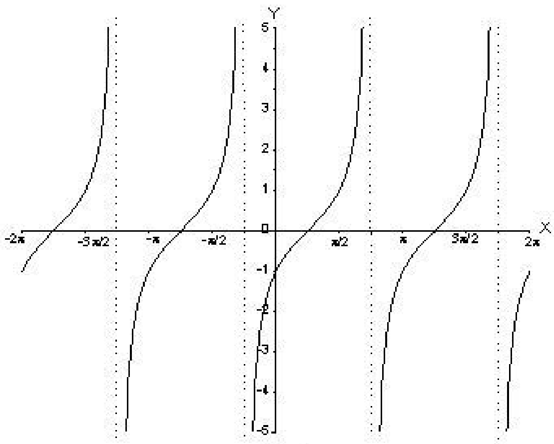


Fig.2

Then compress the graph of $y = \tan\left(x - \frac{\pi}{4}\right)$ by the factor of 4 vertically for getting the graph of $y = \frac{1}{4}\tan\left(x - \frac{\pi}{4}\right)$

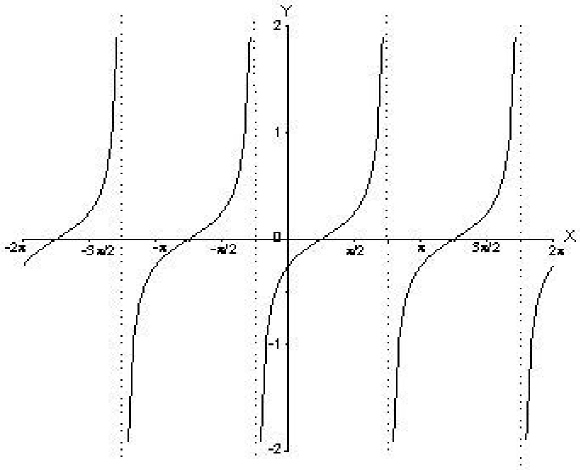


Fig.3

Fig. 3 is the final graph of $y = \frac{1}{4}\tan\left(x - \frac{\pi}{4}\right)$

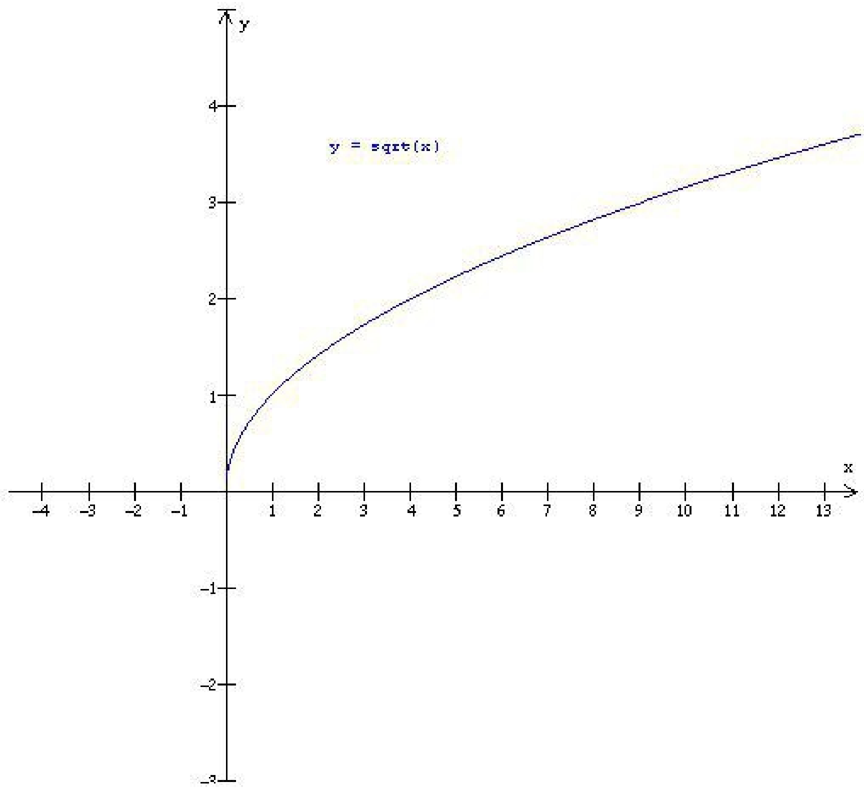
## Chapter 1 Functions and Limits Exercise 1.3 23E

Given function

$y = \left|\sqrt{x} - 1\right|$

To draw the graph of given function we plot the graph of standard function ( root function)

$f(x) = \sqrt{x}$




Now, to draw the graph of function

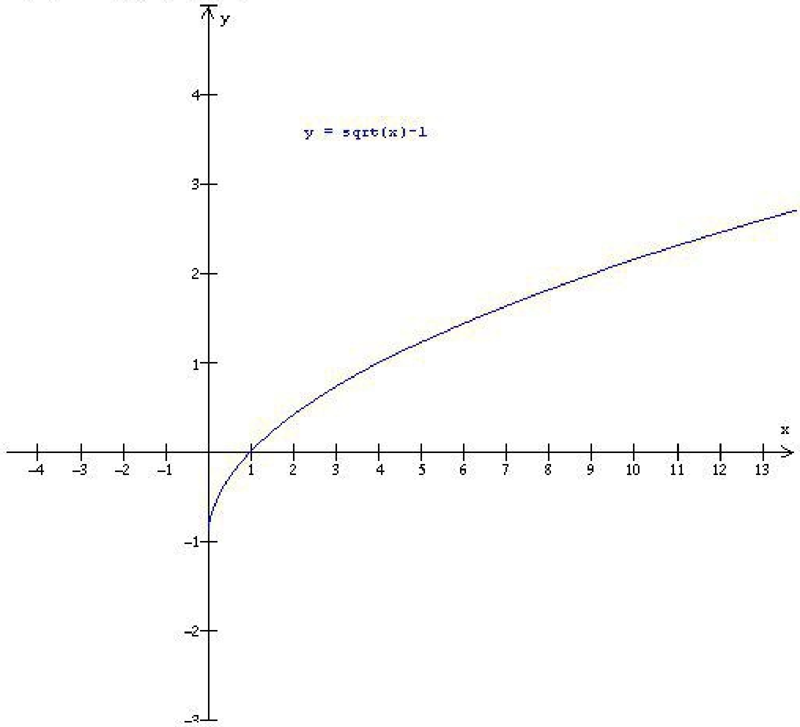
$g(x) = f(x) - 1$

we shift the graph of

$f(x) = \sqrt{x}$

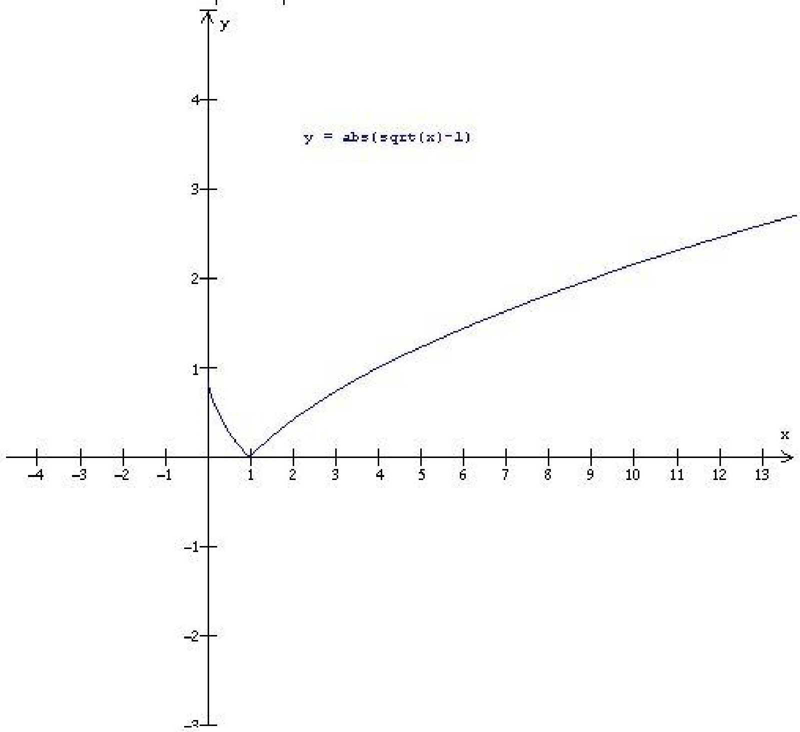
a distance of 1 units downward.

So, the graph of $g(x) = f(x) - 1$ is



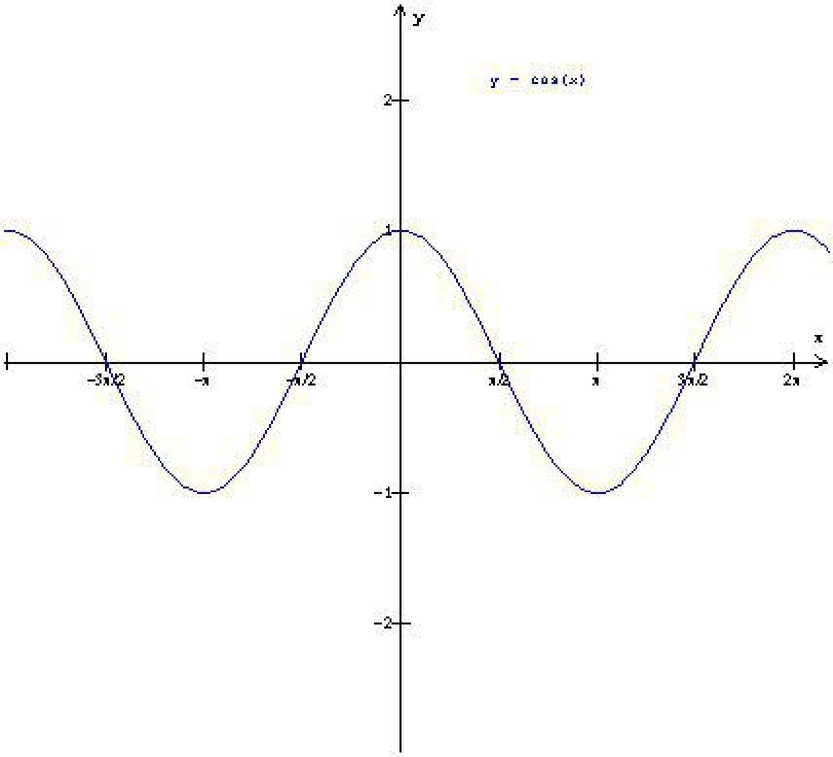

Now, to draw the graph of function
$y = |g(x)| = |\sqrt{x} - 1|$
we reflect that portion of the graph of
$g(x) = f(x) - 1$
about x-axis, where it is below x-axis.

So, the graph of $y = |\sqrt{x} - 1|$ is




## Chapter 1 Functions and Limits Exercise 1.3 24E

Given function
$y = |\cos \pi x|$
To draw the graph of given function we plot the graph of standard function (trigonometric function)
$f(x) = \cos x$




Now, to draw the graph of function

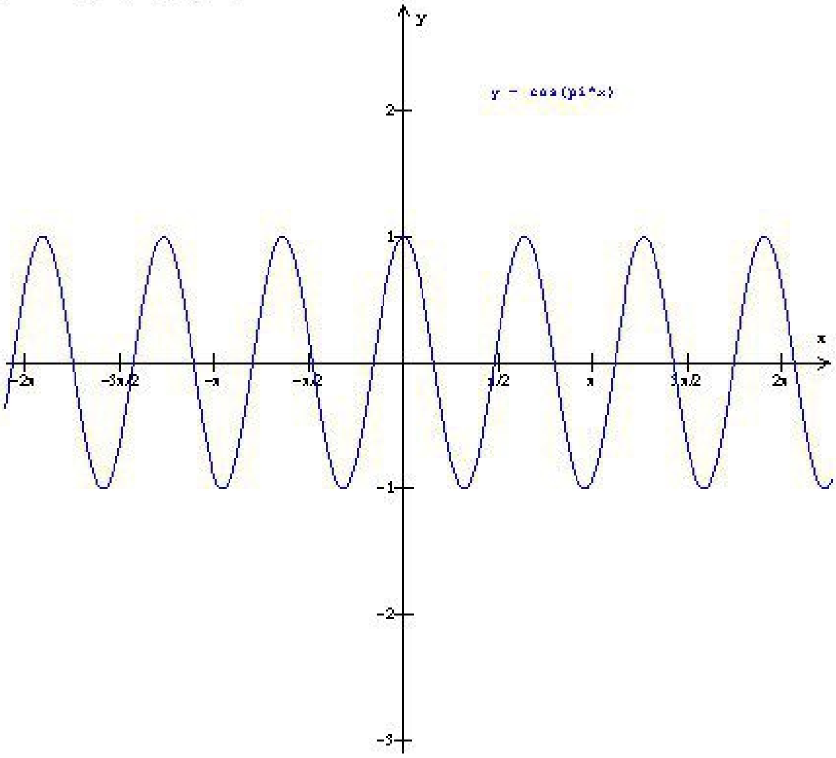
$g(x) = f(\pi x) = \cos \pi x$

We shrink the graph of

$f(x) = \cos x$

Horizontally by a factor of $\pi$.

So, the graph of $g(x) = f(\pi x) = \cos \pi x$ is




Now, to draw the graph of function

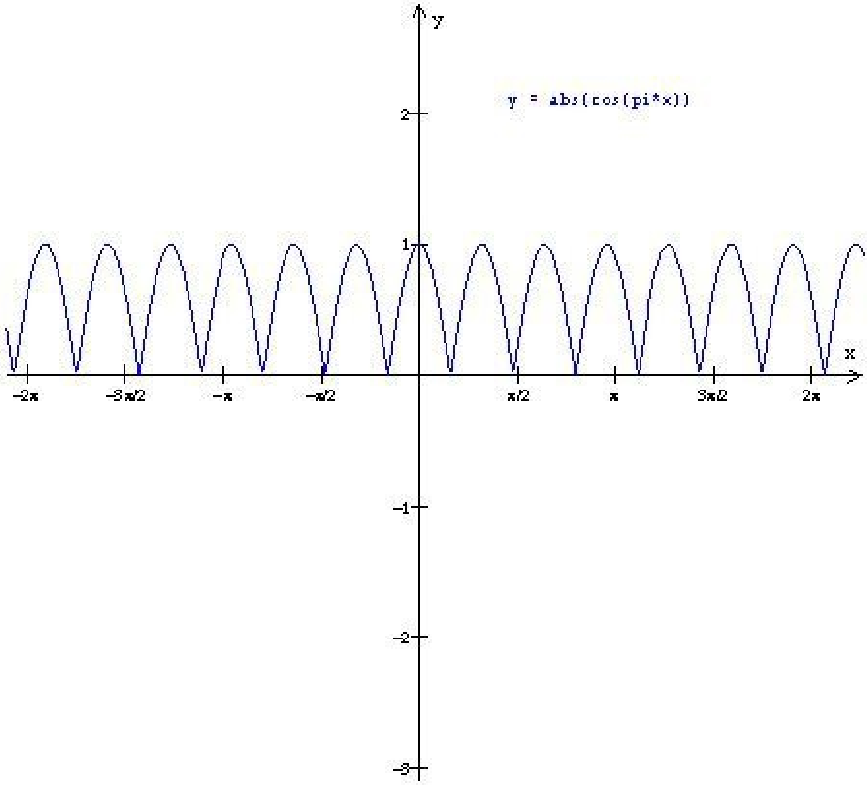
$y = |g(x)| = |\cos \pi x|$

we reflect that portion of the graph of

$g(x) = f(\pi x) = \cos \pi x$

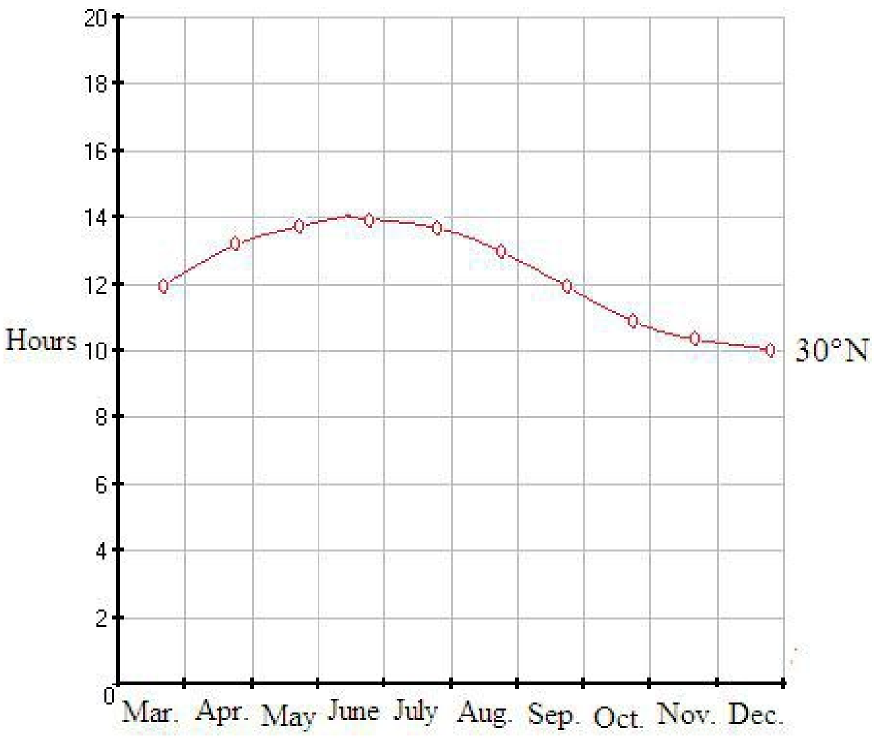
About x-axis, where it is below x-axis.

So, the graph of $y = |\cos \pi x|$ is




## Chapter 1 Functions and Limits Exercise 1.3 25E

Consider the following is the figure:




The above figure shows the graph of the number of hours of daylight as functions of the time of the year at the latitude $30°\text{N}$.

The city of New Orleans is located at $30°\text{N}$ latitude.

Use the figure to find a function that models the number of hours of daylight at New Orleans as a function of the time of the year.

Use the fact that on March 31, the sun rises at 5:51 AM and sets at 6:18 PM in New Orleans.

From the above figure, the curve resembles a shifted and stretched sine function.

At the latitude of New Orleans, daylight lasts about 14 hours and on June 21, and 10 hours on December 21.

So the amplitude of the curve is,

$$\frac{1}{2}(14-10)=\frac{4}{2}$$
$$=2$$

In a year there are 365 days.

The period of the time model is 365.

But the period of $y=\sin t$ is $2\pi$.

So the horizontal stretching factor is $c=\frac{2\pi}{365}$.

The curve begins its cycle on March 21, the 80th day of the year.

So shift the curve 80 units right.

In addition shift the curve 12 units upward.

Therefore, the model the length of daylight in New Orleans on the $t$th day of the year by the function,

$$\boxed{L(t)=12+2\sin\left[\frac{2\pi}{365}(t-80)\right]}.$$

The curve begins its cycle on March 21, the 80th day of the year.

So shift the curve 80 units right.

In addition shift the curve 12 units upward.

Therefore, the model the length of daylight in New Orleans on the $t$th day of the year by the function,

$$\boxed{L(t)=12+2\sin\left[\frac{2\pi}{365}(t-80)\right]}.$$

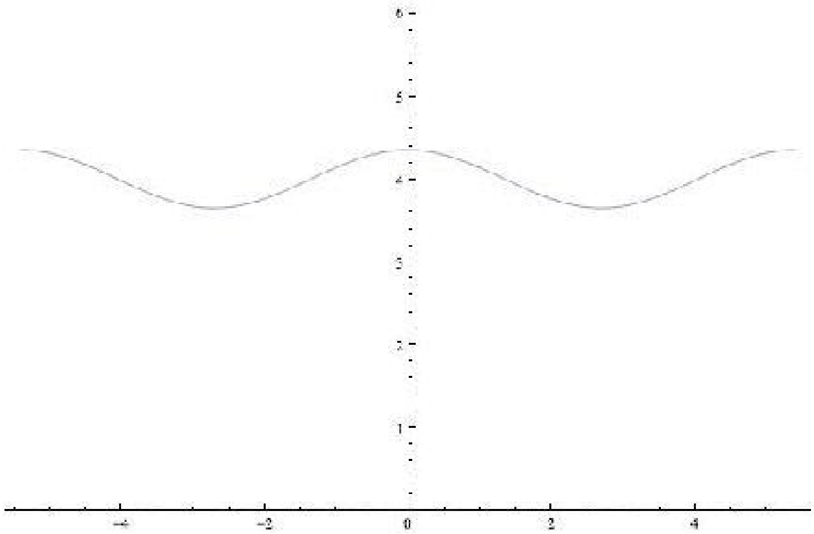
## Chapter 1 Functions and Limits Exercise 1.3 26E

Because the brightness of this star varies periodically, we know that we should use a trigonometric function to model it. Start with $y=\cos(x)$.

We know that the average brightness of the star is 4.0, so the function should be varying around the horizontal line $y=4$. Therefore we need to translate our function up by 4 units, which we can do by adding 4 outside of the cosine term: $y=\cos(x)+4$.

We also know that the magnitude variation in brightness is 0.35. To give our wave this amplitude, we need to shrink it vertically by a factor of .35, which we can do by multiplying the cosine term by 0.35: $y=.35\cos(x)+4$.

Finally, we know that the period of our wave is 5.4 days. Because the period of a standard cosine wave is $2\pi$, to shrink the wave period to 5.4 we need to multiply our $x$-term by $\frac{2\pi}{5.4}$, giving us our final equation: $y = 0.35\cos\left(\frac{2\pi x}{5.4}\right) + 4$.



## Chapter 1 Functions and Limits Exercise 1.3 27E

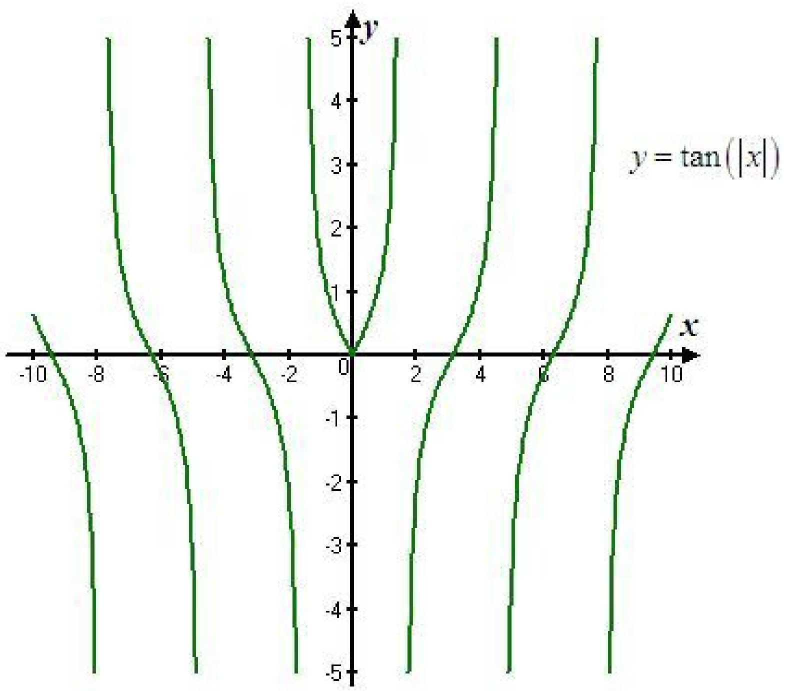
(a)

How the graph of is $y = f(|x|)$ related to the graph of $f$.

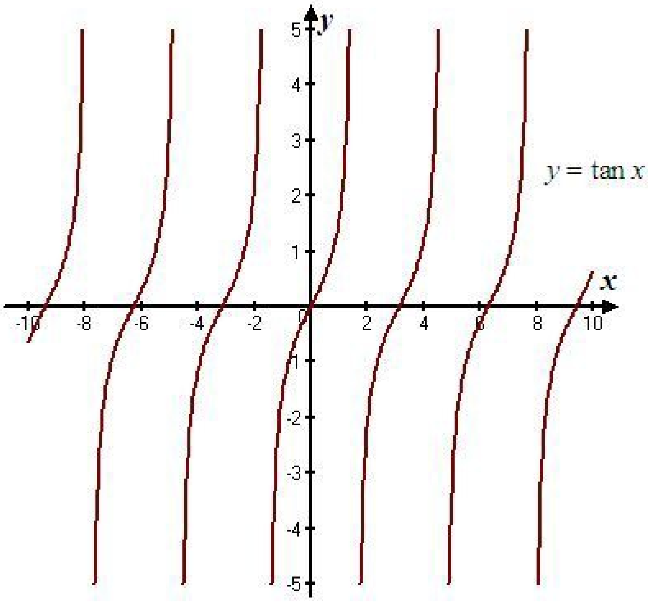
Let $y = \tan(|x|)$, $y = \tan x$.

Now graph $y = \tan(|x|)$, $y = \tan x$ to find the difference.

The following is the graph of $y = \tan(|x|)$



The following is the graph of $y = \tan x$.

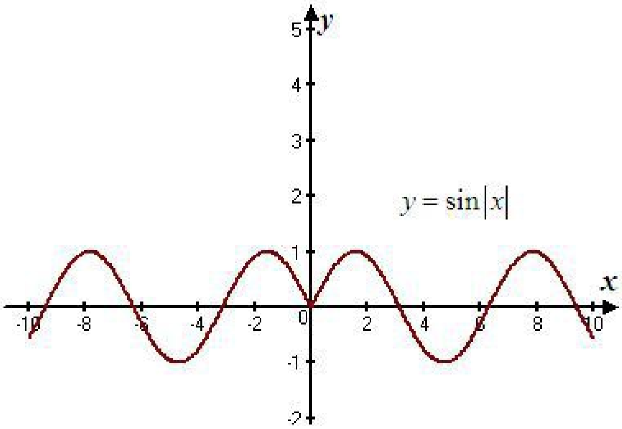



From the above two graphs, it is observed that the portion of the graph of $y = f(x)$ to the right of the *y*-axis is reflected about the *y*-axis in $y = f(|x|)$.

(b)

Sketch the graph of $y = \sin|x|$.

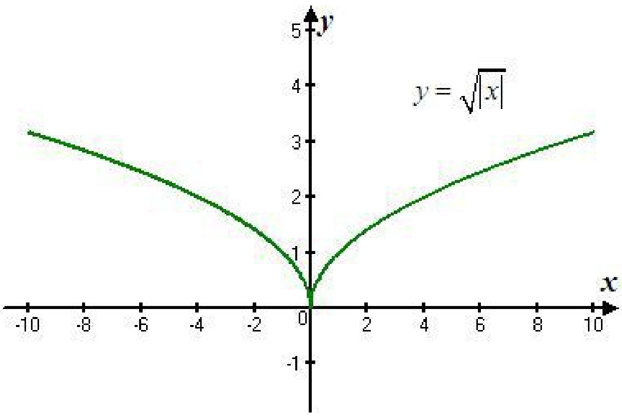
The following is the graph of $y = \sin|x|$.



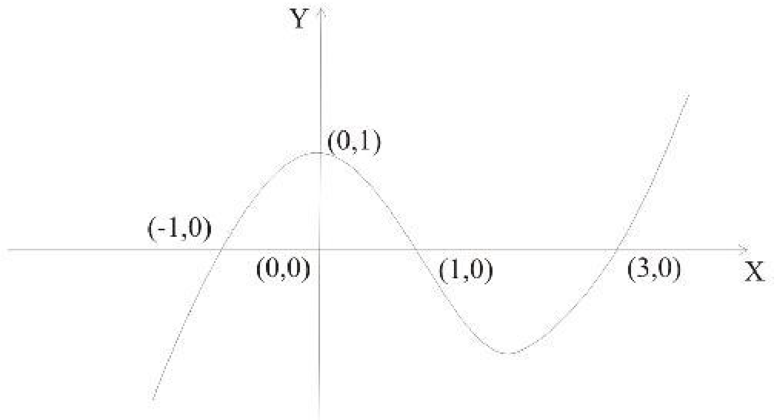

(c)

Sketch the graph of $y = \sqrt{|x|}$.

The following is the graph of $y = \sqrt{|x|}$.




Chapter 1 Functions and Limits Exercise 1.3 28E




In the given graph, the curve is intersecting the x-axis at x = -1 and x = 3.
If also intersects the y-axis at y = 1
So that $y = f(x) = 0$ where $x = -1,1,3$ and $y = 1 = f(0)$

Since $f(x) = 0$ at $x = -1,1\,\&\,3,$ the graph of $y = \dfrac{1}{f(x)}$ is not defined at $x = -1,1,3$

For drawing the graph of the function $\dfrac{1}{f(x)}$ using the given graph of $f(x)$, we will consider the portion between the intervals $(-\infty,-1), (-1,1), (1,3)$ and $(3,\infty)$.
Let us take the interval -1, 1. The shape of the curve resembles inverted parabola with vertex at (0, 1). Hence it equation can be taken as $y-1 = -x^2$ or $y = 1-x^2$

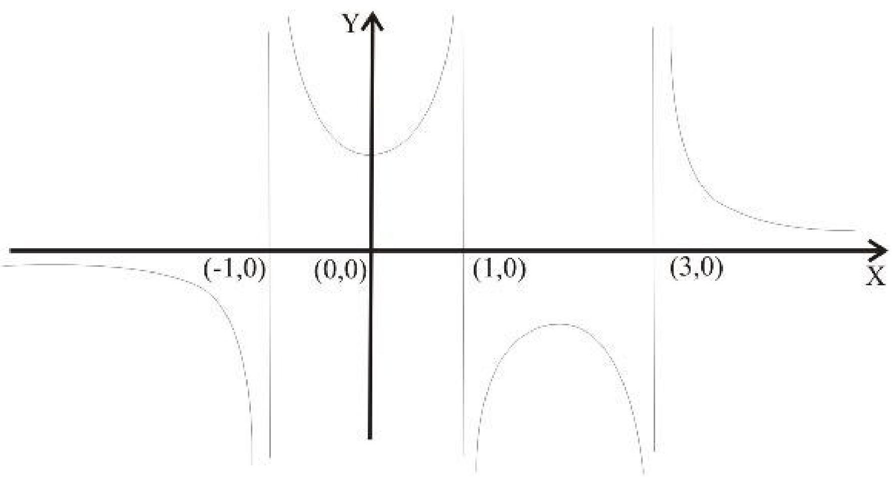
$$\text{Then } \frac{1}{y} = \frac{1}{f(x)} = \frac{1}{1-x^2}$$

From this relation we find that for values of $x$ between -1 and +1, the value of $\dfrac{1}{y}$ increases and as $x$ is close to -1 or +1, $\dfrac{1}{y}$ is very very large.The lines $x$ = -1 and $x$ = +1 are the asymptotes to the curve between -1 and +1.

Similarly between the interval (1, 3) the curve again resembles parabola with vertex at (2, -1). Hence its equation can be taken as $y+1 = (x-2)^2$ or $y = (x-2)^2 - 1$ and $\dfrac{1}{y} = \dfrac{1}{(x-2)^2 - 1}$.
Here again for values of $x$ near 1 or near 3 , values of $1/y$ are very very large.
Hence $x = 1$ and $x = 3$ are the asymptotes to the curve. The point (2,-1) lies on the graph of $1/y$.

Next for $x < -1$ or $x > 3$, y increases rapidly So that the function $\dfrac{1}{y}$ will decreases rapidly, and when $y$ is very large, $\dfrac{1}{y}$ will be very very small

The graph of $\dfrac{1}{y}$ will be as follows:



## Chapter 1 Functions and Limits Exercise 1.3 29E

Here $f(x) = x^3 + 2x^2$ and $g(x) = 3x^2 - 1$

$\text{domain}(f) = \mathbb{R}$ and $\text{domain}(g) = \mathbb{R}$

Now $(f+g)(x) = f(x) + g(x) = x^3 + 2x^2 + 3x^2 - 1$

$$= x^3 + 5x^2 - 1$$

$$\text{dom}(f+g) = \text{dom}(f) \cap \text{dom}(g) = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

$$(f-g)(x) = f(x) - g(x) = x^3 + 2x^2 - 3x^2 + 1 = x^3 - x^2 + 1$$

$$\text{dom}(f-g) = \text{dom}(f) \cap \text{dom}(g) = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

$$(f.g)(x) = f(x).g(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2$$

$$\text{dom}(fg) = \text{dom}(f) \cap \text{dom}(g) = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^3 + 2x^2}{3x^2 - 1}$$

$$\text{dom}\left(\frac{f}{g}\right) = \text{dom}(f) \cap \text{dom}(g) - \{x : g(x) = 0\} = \mathbb{R} - \{x : g(x) = 0\}$$

Now $3x^2 - 1 = 0$

When $x^2 = \frac{1}{3} \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

Hence, $\text{dom}\left(\frac{f}{g}\right) = \{x : x \neq \pm 1/\sqrt{3}\}$

## Chapter 1 Functions and Limits Exercise 1.3 30E

Consider the following functions:

$$f(x) = \sqrt{3-x},\ g(x) = \sqrt{x^2 - 1}$$

(a).

Find $f+g$:

The sum function $(f+g)(x)$ is defined by $(f+g)(x) = f(x) + g(x)$

Using the above formula, $(f+g)(x)$ can be obtained as,

$$(f+g)(x) = f(x) + g(x) = \boxed{\sqrt{3-x} + \sqrt{x^2 - 1}}$$

Domain of $f+g$ is given by the intersection of the respective domains of the individual functions.

Find the domains of $f$ and $g$:

Since the square root of a negative number is not defined (as a real number), the domain of $f$ consist of all values of $x$ such that

$$3-x\geq 0$$
$$x-3\leq 0$$
$$x\leq 3$$

Therefore the domain of $f(x)$ is $\{x \mid x\leq 3\}$ or $(-\infty,3]$.

Since the square root of a negative number is not defined (as a real number), the domain of $g$ consist of all values of $x$ such that

$$x^2-1\geq 0$$
$$x^2\geq 1$$
$$|x|\geq 1$$
$$x\leq -1 \text{ or } x\geq 1$$

Therefore the domain of $g(x)$ is $\{x \mid x\leq -1 \text{ or } x\geq 1\}$ or $(-\infty,-1]\cup[1,\infty)$

The domain of the function $f(x)+g(x)=\sqrt{3-x}+\sqrt{x^2-1}$ is the intersection of the intervals $(-\infty,3]$ and $(-\infty,-1]\cup[1,\infty)$.

These intervals overlap on the interval $(-\infty,-1]\cup[1,3]$.

Hence the domain of $f(x)+g(x)=\sqrt{3-x}+\sqrt{x^2-1}$ is $\boxed{(-\infty,-1]\cup[1,3]}$.

(b).

Find $(f-g)(x)$:

The difference function $(f-g)(x)$ is defined by $(f-g)(x)=f(x)-g(x)$

Using the above formula, $(f-g)(x)$ can be obtained as,

$$(f-g)(x)=f(x)-g(x)$$
$$=\boxed{\sqrt{3-x}-\sqrt{x^2-1}}$$

The domain of $(f-g)(x)$ is also given by the intersection of the respective domains of the individual functions.

Hence, the domain of $f-g$ is identical to $f+g$,

Therefore, the domain of the function $(f-g)(x)$ is $\boxed{(-\infty,-1]\cup[1,3]}$.

(c).

Find $(fg)(x)$:

The product function $(fg)(x)$ is defined by $(fg)(x)=f(x)g(x)$

Using the above formula, $(fg)(x)$ can be obtained as,

$$(fg)(x)=f(x)g(x)$$
$$=\boxed{\sqrt{(3-x)(x^2-1)}}$$

The domain of $(fg)(x)$ is also given by the intersection of the respective domains of the individual functions.

Hence, the domain of $(fg)(x)$ is identical to $f+g$,

Therefore, the domain of the function $(fg)(x)$ is $\boxed{(-\infty,-1]\cup[1,3]}$.

(d). Find $\left(\frac{f}{g}\right)(x)$:

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$$

$$=\boxed{\frac{\sqrt{3-x}}{\sqrt{x^2-1}}}$$

The domain of $\left(\frac{f}{g}\right)(x)$ is given by the intersection of the respective domains of the individual functions, and also exclude those values of *x* that make the denominator zero.

The values of *x* that make the denominator 0, satisfy the equation is,

$$x^2-1=0$$
$$x=\pm 1$$

Thus, the domain of $\left(\frac{f}{g}\right)(x)$ is $\boxed{(-\infty,1)\cup(1,3)}$

## Chapter 1 Functions and Limits Exercise 1.3 31E

(a)

Consider the functions,

$f(x)=x^2-1$ and $g(x)=2x+1$.

The object is to find $f\circ g$.

The composite function $f\circ g$ is calculated as,

$$f(g(x))=(g(x))^2-1$$

$$=(2x+1)^2-1$$

$$=4x^2+4x+1-1$$

$$=\boxed{4x^2+4x}.$$

So, the domain of $f\circ g$ is all real numbers.

Therefore, the domain of the function expressed in the interval notation is,

$\boxed{(-\infty,\infty)}$.

(b)

Consider the functions,

$f(x)=x^2-1$ and $g(x)=2x+1$.

The object is to find $g\circ f$.

The composite function $g\circ f$ is calculated as,

$$g(f(x))=2(f(x))+1$$

$$=2(x^2-1)+1$$

$$=2x^2-2+1$$

$$=\boxed{2x^2-1}.$$

So, the domain of $g\circ f$ is all real numbers.

Therefore, the domain of the function expressed in interval notation is,

(c)

Consider the functions,

$f(x) = x^2 - 1$ and $g(x) = 2x + 1$.

The object is to find $f \circ f$.

The composite function $f \circ f$ is calculated as,

$$f(f(x)) = (f(x))^2 - 1$$
$$= (x^2 - 1)^2 - 1$$
$$= x^4 - 2x^2 + 1 - 1$$
$$= \boxed{x^4 - 2x^2}.$$

So, the domain of $f \circ f$ is all real numbers.

Therefore, the domain of the function expressed in interval notation is,

$\boxed{(-\infty, \infty)}$.

(d)

Consider the functions,

$f(x) = x^2 - 1$ and $g(x) = 2x + 1$.

The object is to find $g \circ g$.

The composite function $g \circ g$ is calculated as,

$$g(g(x)) = 2(g(x)) + 1$$
$$= 2(2x + 1) + 1$$
$$= 4x + 2 + 1$$
$$= \boxed{4x + 3}.$$

So, the domain of $g \circ g$ is all real numbers.

Therefore, the composite function $g \circ g$ is calculated as,

$\boxed{(-\infty, \infty)}$.

## Chapter 1 Functions and Limits Exercise 1.3 32E

Consider the functions:

$f(x) = x - 2, g(x) = x^2 + 3x + 4$.

(a) Find the function $(f o g)(x)$.

$$(f o g)(x) = f(g(x))$$
$$= f(x^2 + 3x + 4) \qquad \text{Since } g(x) = x^2 + 3x + 4$$
$$= x^2 + 3x + 4 - 2 \qquad \text{Since } f(x) = x - 2$$
$$= x^2 + 3x + 2 \qquad \text{Simplify}$$

Therefore, $(f o g)(x) = \boxed{x^2 + 3x + 2}$.

The domain of $f o g$ is, $\boxed{(-\infty, \infty)}$.

(b) Find the function $(gof)(x)$.

$$(gof)(x) = g(f(x))$$
$$= g(x-2) \quad \text{Since } f(x) = x-2$$
$$= (x-2)^2 + 3(x-2) + 4 \quad \text{Since } g(x) = x^2 + 3x + 4$$
$$= x^2 - 4x + 4 + 3x - 6 + 4 \quad \text{Simplify}$$
$$= x^2 - x + 2$$

Therefore, $(gof)(x) = \boxed{x^2 - x + 2}$.

The domain of $gof$ is, $\boxed{(-\infty, \infty)}$.

(c) Find the function $(fof)(x)$.

$$(fof)(x) = f(f(x))$$
$$= f(x-2) \quad \text{Since } f(x) = x-2$$
$$= (x-2) - 2 \quad \text{Simplify}$$
$$= x - 2 - 2$$
$$= x - 4$$

Therefore, $(fof)(x) = \boxed{x-4}$.

The domain of $fof$ is, $\boxed{(-\infty, \infty)}$.

(d) Find the function $(gog)(x)$.

$$(gog)(x) = g(g(x))$$
$$= g(x^2 + 3x + 4) \quad \text{Since } g(x) = x^2 + 3x + 4$$
$$= (x^2 + 3x + 4)^2 + 3(x^2 + 3x + 4) + 4 \quad \text{Simplify}$$
$$= x^4 + 9x^2 + 16 + 6x^3 + 24x + 8x^2 + 3x^2 + 9x + 12 + 4$$
$$= x^4 + 6x^3 + 20x^2 + 33x + 32$$

Therefore, $(gog)(x) = \boxed{x^4 + 6x^3 + 20x^2 + 33x + 32}$.

The domain of $gog$ is, $\boxed{(-\infty, \infty)}$.

## Chapter 1 Functions and Limits Exercise 1.3 33E

Given that $f(x) = 1 - 3x$ and $g(x) = \cos x$

a)

We have to find $f \circ g$ and its domain.

$$f \circ g(x) = f(g(x))$$
$$= f(\cos x)$$
$$= 1 - 3\cos x$$

Since cosine function is defined for all values of $x$, the domain of this function is $(-\infty, \infty)$.

b)

We have to find $g \circ f$ and its domain.

Given $f(x) = 1 - 3x$ and $g(x) = \cos(x)$

$$g \circ f(x) = g(f(x))$$
$$= g(1 - 3x)$$
$$= \cos(1 - 3x)$$

Cosine is defined for all values, thus the domain of this function is $(-\infty, \infty)$.

c)

We have to find $f \circ f$ and its domain.

Given $f(x) = 1 - 3x$ and $g(x) = \cos(x)$

$$
\begin{aligned}
f \circ f(x) &= f(f(x)) \\
&= f(1-3x) \\
&= 1-3(1-3x) \\
&= 1-3+9x \\
&= 9x-2
\end{aligned}
$$

$9x-2$ is defined for all values of $x$, thus the domain of this function is $(-\infty,\infty)$

d)

We have to find $g \circ g$ and its domain.

Given $f(x)=1-3x$ and $g(x)=\cos x$

$$
\begin{aligned}
g \circ g(x) &= g(g(x)) \\
&= g(\cos x) \\
&= \cos(\cos x)
\end{aligned}
$$

Cosine is defined for all values, thus the domain of this function is $(-\infty,\infty)$. The fact that there is a cosine inside a cosine is irrelevant because no matter the value of $x$ both cosines will be defined.

## Chapter 1 Functions and Limits Exercise 1.3 34E

Consider the functions:

$f(x)=\sqrt{x}, g(x)=\sqrt[3]{1-x}$.

(a) Find the function $(fog)(x)$.

$$
\begin{aligned}
(fog)(x) &= f(g(x)) \\
&= f\left(\sqrt[3]{1-x}\right) && \text{Since } g(x)=\sqrt[3]{1-x} \\
&= \sqrt{\sqrt[3]{1-x}} && \text{Since } f(x)=\sqrt{x} \\
&= (1-x)^{\frac{1}{6}}
\end{aligned}
$$

Therefore, $(fog)(x)=\boxed{(1-x)^{\frac{1}{6}}}$.

The domain of $fog$ is, $\boxed{\{x \mid x \le 1\}}$.

(b) Find the function $(gof)(x)$.

$$
\begin{aligned}
(gof)(x) &= g(f(x)) \\
&= g\left(\sqrt{x}\right) && \text{Since } f(x)=\sqrt{x} \\
&= \sqrt[3]{1-\sqrt{x}} && \text{Since } g(x)=\sqrt[3]{1-x}
\end{aligned}
$$

Therefore, $(gof)(x)=\boxed{\sqrt[3]{1-\sqrt{x}}}$.

The domain of $gof$ is, $\boxed{\{x \mid 0 \le x \le 1\}}$.

(c) Find the function $(fof)(x)$.

$$
\begin{aligned}
(fof)(x) &= f(f(x)) \\
&= f\left(\sqrt{x}\right) && \text{Since } f(x)=\sqrt{x} \\
&= \sqrt{\sqrt{x}} \\
&= \sqrt[4]{x}
\end{aligned}
$$

Therefore, $(fof)(x)=\boxed{\sqrt[4]{x}}$.

The domain of $fof$ is, $\boxed{\{x \mid x \ge 0\}}$.

(d) Find the function $(gog)(x)$.

$$\begin{aligned}(gog)(x) &= g(g(x)) \\ &= g\left(\sqrt[3]{1-x}\right) \qquad \text{Since } g(x)=\sqrt[3]{1-x} \\ &= \sqrt[3]{1-\sqrt[3]{1-x}}\end{aligned}$$

Therefore, $(gog)(x)=\boxed{\sqrt[3]{1-\sqrt[3]{1-x}}}$.

The domain of $gog$ is, $\boxed{\{x \mid 0 \le x \le 1\}}$.

## Chapter 1 Functions and Limits Exercise 1.3 35E

Consider the functions:

$$f(x)=x+\frac{1}{x}, g(x)=\frac{x+1}{x+2}.$$

(a) Find the function $(fog)(x)$.

$$\begin{aligned}(fog)(x) &= f(g(x)) \\ &= f\left(\frac{x+1}{x+2}\right) \qquad \text{Since } g(x)=\frac{x+1}{x+2} \\ &= \frac{x+1}{x+2}+\frac{1}{\frac{x+1}{x+2}} \qquad \text{Since } f(x)=x+\frac{1}{x} \\ &= \frac{x+1}{x+2}+\frac{x+2}{x+1}\end{aligned}$$

The above is simplified to,

$$\begin{aligned}&= \frac{(x+1)^2+(x+2)^2}{(x+2)(x+1)} \\ &= \frac{x^2+2x+1+x^2+4x+4}{(x+2)(x+1)} \\ &= \frac{2x^2+6x+5}{(x+2)(x+1)}\end{aligned}$$

Therefore, $(fog)(x)=\boxed{\frac{2x^2+6x+5}{(x+2)(x+1)}}$.

The domain of $fog$ is, $\boxed{\{x \mid x \ne -1,-2\}}$

(b) Find the function $(gof)(x)$.

$$\begin{aligned}(gof)(x) &= g(f(x)) \\ &= g\left(x+\frac{1}{x}\right) \qquad \text{Since } f(x)=x+\frac{1}{x} \\ &= \frac{x+\frac{1}{x}+1}{x+\frac{1}{x}+2} \qquad \text{Since } g(x)=\frac{x+1}{x+2} \\ &= \frac{\frac{x^2+1+x}{x}}{\frac{x^2+1+2x}{x}}\end{aligned}$$

The above is simplified to,

$$= \frac{x^2+x+1}{x^2+2x+1}$$
$$= \frac{x^2+x+1}{(x+1)^2}$$

Therefore, $(gof)(x) = \boxed{\frac{x^2+x+1}{(x+1)^2}}$.

The domain of $gof$ is, $\boxed{\{x \mid x \neq 0, -1\}}$

(c) Find the function $(fof)(x)$.

$$\begin{aligned}(fof)(x) &= f(f(x)) \\ &= f\left(x+\frac{1}{x}\right) \quad \text{Since } f(x) = x+\frac{1}{x} \\ &= x+\frac{1}{x}+\frac{1}{x+\frac{1}{x}} \\ &= \frac{x^2+1}{x}+\frac{1}{\frac{x^2+1}{x}}\end{aligned}$$

The above is simplified to,

$$= \frac{x^2+1}{x}+\frac{x}{x^2+1}$$
$$= \frac{(x^2+1)^2+x^2}{x(x^2+1)}$$
$$= \frac{x^4+2x^2+1+x^2}{x(x^2+1)}$$
$$= \frac{x^4+3x^2+1}{x(x^2+1)}$$

Therefore, $(fof)(x) = \boxed{\frac{x^4+3x^2+1}{x(x^2+1)}}$.

The domain of $fof$ is, $\boxed{\{x \mid x \neq 0\}}$.

(d) Find the function $(gog)(x)$.

$$\begin{aligned}(gog)(x) &= g(g(x)) \\ &= g\left(\frac{x+1}{x+2}\right) \quad \text{Since } g(x) = \frac{x+1}{x+2} \\ &= \frac{\left(\frac{x+1}{x+2}\right)+1}{\left(\frac{x+1}{x+2}\right)+2} \\ &= \frac{\frac{x+1+x+2}{x+2}}{\frac{x+1+2(x+2)}{x+2}}\end{aligned}$$

The above is simplified to,

$$= \frac{2x+3}{x+1+2x+4}$$
$$= \frac{2x+3}{3x+5}$$

Therefore, $(gog)(x) = \boxed{\frac{2x+3}{3x+5}}$.

The domain of $gog$ is, $\boxed{\left\{x \mid x \neq -2, -\frac{5}{3}\right\}}$

## Chapter 1 Functions and Limits Exercise 1.3 36E

Consider the functions:

$f(x) = \frac{x}{1+x}, g(x) = \sin 2x$.

(a) Find the function $(fog)(x)$.

$$\begin{aligned}(fog)(x) &= f(g(x)) \\ &= f(\sin 2x) && \text{Since } g(x) = \sin 2x \\ &= \frac{\sin 2x}{1+\sin 2x} && \text{Since } f(x) = \frac{x}{1+x}\end{aligned}$$

Therefore, $(fog)(x) = \boxed{\frac{\sin 2x}{1+\sin 2x}}$.

The domain of $fog$ is, $\boxed{\left\{x \mid x \neq \frac{1}{4}(4n\pi - \pi), \frac{1}{4}(4n\pi + 3\pi), n \in Z\right\}}$.

(b) Find the function $(gof)(x)$.

$$\begin{aligned}(gof)(x) &= g(f(x)) \\ &= g\left(\frac{x}{1+x}\right) && \text{Since } f(x) = \sin 2x \\ &= \sin 2\left(\frac{x}{1+x}\right) && \text{Since } g(x) = \sin 2x\end{aligned}$$

Therefore, $(gof)(x) = \boxed{\sin 2\left(\frac{x}{1+x}\right)}$.

The domain of $gof$ is, $\boxed{\{x \mid x \neq 1\}}$.

(c) Find the function $(fof)(x)$.

$$\begin{aligned}(fof)(x) &= f(f(x)) \\ &= f\left(\frac{x}{1+x}\right) && \text{Since } f(x) = \frac{x}{1+x} \\ &= \frac{\frac{x}{1+x}}{1+\frac{x}{1+x}} \\ &= \frac{\frac{x}{1+x}}{\frac{1+x+x}{1+x}}\end{aligned}$$

The above is simplified to,

$$= \frac{\frac{x}{1+x}}{\frac{1+2x}{1+x}}$$

$$= \frac{x}{1+2x}$$

Therefore, $(fof)(x) = \boxed{\frac{x}{1+2x}}$.

The domain of $fof$ is, $\boxed{\left\{x \mid x \neq -1, -\frac{1}{2}\right\}}$.

(d) Find the function $(gog)(x)$.

$$\begin{aligned}(gog)(x) &= g(g(x)) \\ &= g(\sin 2x) \qquad \text{Since } g(x) = \sin 2x \\ &= \sin 2(\sin 2x)\end{aligned}$$

Therefore, $(gog)(x) = \boxed{\sin 2(\sin 2x)}$.

The domain of $gog$ is, $\boxed{(-\infty, \infty)}$.

## Chapter 1 Functions and Limits Exercise 1.3 37E

Given functions

$$f(x) = 3x - 2$$
$$g(x) = \sin x$$
$$h(x) = x^2$$

Therefore

$$(f \circ g \circ h)(x) = f(g(h(x)))$$
$$\Rightarrow (f \circ g \circ h)(x) = f(g(x^2))$$
$$\Rightarrow (f \circ g \circ h)(x) = f(\sin(x^2))$$
$$\Rightarrow (f \circ g \circ h)(x) = 3\sin(x^2) - 2$$

## Chapter 1 Functions and Limits Exercise 1.3 38E

Given functions are

$$f(x) = |x - 4|$$
$$g(x) = 2^x$$
$$h(x) = \sqrt{x}$$

Therefore

$$(f \circ g \circ h)(x) = f(g(h(x)))$$
$$\Rightarrow (f \circ g \circ h)(x) = f(g(\sqrt{x}))$$
$$\Rightarrow (f \circ g \circ h)(x) = f(2^{\sqrt{x}})$$
$$\Rightarrow (f \circ g \circ h)(x) = |2^{\sqrt{x}} - 4|$$

## Chapter 1 Functions and Limits Exercise 1.3 39E

Consider the functions:

$f(x)=\sqrt{x-3}, g(x)=x^2, h(x)=x^3+2$.

Find the function $(fogoh)(x)$.

$$\begin{aligned}(fogoh)(x) &= fog(h(x)) \\ &= f(g(h(x))) \\ &= f(g(x^3+2)) && \text{Since } h(x)=x^3+2 \\ &= f((x^3+2)^2) && \text{Since } g(x)=x^2\end{aligned}$$

The above is simplified to,

$$\begin{aligned}&= f(x^6+4x^3+4) \\ &= \sqrt{x^6+4x^3+4-3} && \text{Since } f(x)=\sqrt{x-3} \\ &= \sqrt{x^6+4x^3+1}\end{aligned}$$

Therefore, $(fogoh)(x)=\boxed{\sqrt{x^6+4x^3+1}}$.

## Chapter 1 Functions and Limits Exercise 1.3 40E

Consider the functions:

$f(x)=\tan x, g(x)=\frac{x}{x-1}, h(x)=\sqrt[3]{x}$.

Find the function $(fogoh)(x)$.

$$\begin{aligned}(fogoh)(x) &= fog(h(x)) \\ &= f(g(h(x))) \\ &= f(g(\sqrt[3]{x})) && \text{Since } h(x)=\sqrt[3]{x} \\ &= f\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right) && \text{Since } g(x)=\frac{x}{x-1}\end{aligned}$$

$$= \tan\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right) \qquad \text{Since } f(x)=\tan x$$

Therefore, $(fogoh)(x)=\boxed{\tan\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)}$.

## Chapter 1 Functions and Limits Exercise 1.3 41E

Given function

$F(x)=(2x+x^2)^4$

The formula for F says: Take power 4 of $(2x+x^2)$

So we let

$g(x)=2x+x^2$

$f(x)=x^4$

Now we check that

$(f\circ g)(x)=f(g(x))$

$\Rightarrow (f\circ g)(x)=f(2x+x^2)$

$\Rightarrow (f\circ g)(x)=(2x+x^2)^4$

$\Rightarrow (f\circ g)(x)=F(x)$

## Chapter 1 Functions and Limits Exercise 1.3 42E

Given function
$F(x) = \cos^2 x$

The formula for F says: First take cosine of $x$ and then square it.

So we let
$g(x) = \cos x$
$f(x) = x^2$

Now we check that
$(f \circ g)(x) = f(g(x))$
$\Rightarrow (f \circ g)(x) = f(\cos x)$
$\Rightarrow (f \circ g)(x) = \cos^2 x$
$\Rightarrow (f \circ g)(x) = F(x)$

## Chapter 1 Functions and Limits Exercise 1.3 43E

Consider:

$$F(x) = \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}.$$

Express the function $F$ in the form of $f o g$.

Since $F(x) = \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$, let $g(x) = \sqrt[3]{x}$, $f(x) = \frac{x}{1+x}$.

Now,

$$(f o g)(x) = f(g(x))$$
$$= f(\sqrt[3]{x}) \quad \text{Since } g(x) = \sqrt[3]{x}$$
$$= \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}} \quad \text{Since } f(x) = \frac{x}{1+x}$$
$$= F(x)$$

Therefore, $f(x) = \boxed{\frac{x}{1+x}}, g(x) = \boxed{\sqrt[3]{x}}$.

## Chapter 1 Functions and Limits Exercise 1.3 44E

Consider:

$$G(x) = \sqrt[3]{\frac{x}{1+x}}.$$

Express the function $F$ in the form of $f o g$.

Since $G(x) = \sqrt[3]{\frac{x}{1+x}}$, let $g(x) = \frac{x}{1+x}$, $f(x) = \sqrt[3]{x}$.

Now,

$$(f o g)(x) = f(g(x))$$
$$= f\left(\frac{x}{1+x}\right) \quad \text{Since } g(x) = \frac{x}{1+x}$$
$$= \sqrt[3]{\frac{x}{1+x}} \quad \text{Since } f(x) = \sqrt[3]{x}$$
$$= F(x)$$

Therefore, $f(x) = \boxed{\sqrt[3]{x}}, g(x) = \boxed{\frac{x}{1+x}}$.

## Chapter 1 Functions and Limits Exercise 1.3 45E

Given function

$v(t) = \sec(t^2)\tan(t^2)$

The formula for $v$ says: First take square of $t$ and then find the product of secant and tangent of the result.

So we let
$g(t) = t^2$
$f(t) = \sec t \tan t$

Now we check that
$(f \circ g)(t) = f(g(t))$
$\Rightarrow (f \circ g)(t) = f(t^2)$
$\Rightarrow (f \circ g)(t) = \sec(t^2)\tan(t^2)$
$\Rightarrow (f \circ g)(t) = v(t)$

## Chapter 1 Functions and Limits Exercise 1.3 46E

(A) By using the given table

$g(1) = 6$

$f(g(1)) = f(6)$

$= \boxed{5}$

(B)

$f(1) = 3$

$g(f(1)) = g(3)$

$= \boxed{2}$

(C)

$f(f(1)) = f(3)$

$= \boxed{4}$ $\quad [f(1) = 3]$

(D)

$g(g(1)) = g(6)$

$= \boxed{3}$ $\quad [g(1) = 6]$

(E)

$(g \circ f)(3) = g(f(3))$

Since $f(3) = 4$

So $g(f(3)) = g(4)$

$= \boxed{1}$

(F)

$(f \circ g)(6) = f(g(6))$

$= f(3)$

$= \boxed{4}$

## Chapter 1 Functions and Limits Exercise 1.3 47E

Given function

$R(x) = \sqrt{\sqrt{x} - 1}$

The formula for R says: First take square root of $x$, then subtract 1 from the result and finally take the square root of the result.

So we let
$h(x) = \sqrt{x}$
$g(x) = x - 1$
$f(x) = \sqrt{x}$

Now we check that

$(f \circ g \circ h)(x) = f(g(h(x)))$

$\Rightarrow (f \circ g \circ h)(x) = f(g(\sqrt{x}))$

$\Rightarrow (f \circ g \circ h)(x) = f(\sqrt{x} - 1)$

$\Rightarrow (f \circ g \circ h)(x) = \sqrt{\sqrt{x} - 1}$

$\Rightarrow (f \circ g \circ h)(x) = R(x)$

## Chapter 1 Functions and Limits Exercise 1.3 48E

Here $u(t) = \dfrac{\tan t}{1 + \tan t}$

Let $f(t) = \dfrac{t}{1+t}$ and $g(t) = \tan t$

$u(t) = \dfrac{g(t)}{1 + g(t)}$

$= (f \circ g)(t)$

## Chapter 1 Functions and Limits Exercise 1.3 49E

Here $H(x) = \sec^4(\sqrt{x})$

Let $f(x) = x^4$, $g(x) = \sec x$, $h(x) = \sqrt{x}$

$H(x) = (f \circ g \circ h)(x)$

## Chapter 1 Functions and Limits Exercise 1.3 50E

(A) By using the given table

$g(1) = 6$

$f(g(1)) = f(6) = \boxed{5}$

(B)

$f(1) = 3$

$g(f(1)) = g(3) = \boxed{2}$

(C)

$f(f(1)) = f(3) = \boxed{4}$ since $[f(1) = 3]$

(D)

$g(g(1)) = g(6) = \boxed{3}$ since $[g(1) = 6]$

(E)

$(g \circ f)(3) = g(f(3))$

Since $f(3) = 4$

So $g(f(3)) = g(4) = \boxed{1}$

(F)

$(f \circ g)(6) = f(g(6)) = f(3) = \boxed{4}$

## Chapter 1 Functions and Limits Exercise 1.3 51E

(A)

After viewing the given graph we have

$g(2) = 5$

So $f(g(2)) = f(5) = 4$

$\boxed{\text{Ans} = 4}$

(B)

We have from the graph $f(0) = 0$

So $g(f(0)) = g(0) = 3$

$\Rightarrow \boxed{\text{Ans} = 3}$

(C)

$(f \circ g)(0) = f(g(0)) = f(3)$

From graph we have $f(3) = 0$

$\boxed{\text{Ans} = 0}$

(D)

$(g \circ f)(6) = g(f(6))$

Where $f(6) = 6$

So $g(f(6)) = g(6)$ but for value 6, g is not defined

So $(g \circ f)(6)$ is not defined

Means $\boxed{(g \circ f)(6) \text{ is undefined}}$

(E)

$(g \circ g)(-2) = g(g(-2))$ here $g(-2) = 1$

Then $g(g(-2)) = g(1) = 4$

$\boxed{\text{Ans} = 4}$

(F)

$(f \circ f)(4) = f(f(4))$

Here from the graph we have $f(4) = 2$

So $f(f(4)) = f(2) = -2$

$\boxed{\text{Ans} = -2}$

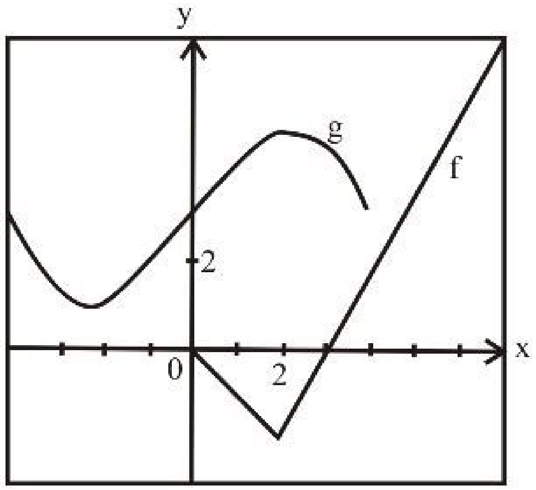
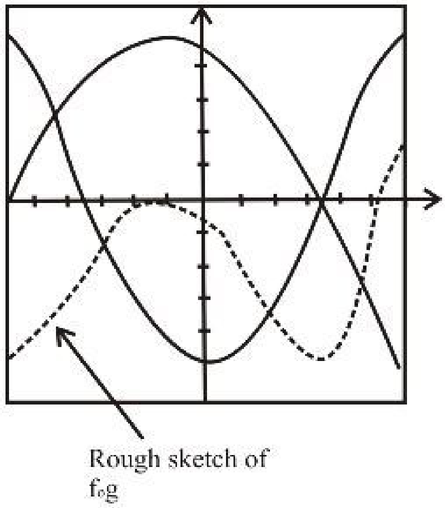



Fig 1
(Given figure in the problem)

## Chapter 1 Functions and Limits Exercise 1.3 52E



We have to find out the values of $f(g(x))$ with the help of given graph for

$x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5,$

For $x = -5 \Rightarrow f(g(x)) = f(g(-5))$

$= f(-0.2)$

Here $g(-5) = -0.2$

And $f(-0.2) = -4$

*Ans* $\boxed{f(g(-5)) = -4}$

For $x = -4 \Rightarrow f(g(-4)) = f(1.2)$

Here $g(-4) = 1.2$

And $f(1.2) = -3.3$

$\Rightarrow \boxed{f(g(-4)) = -3.3}$

For $x = -3$

$\Rightarrow f(g(-3)) = f(2.2)$

$\Rightarrow f(2.2) = \boxed{-1.7}$

For $x = -2$

$\Rightarrow f(g(-2)) = f(2.8)$

$\Rightarrow f(2.8) = \boxed{-0.5}$

For $x = -1$

$\Rightarrow f(g(-1)) = f(3) = \boxed{-0.2}$ (Near 0)

For $x = 0$

$\Rightarrow f(g(0)) = f(2.8) = \boxed{-0.5}$

For $x = 1$

$\Rightarrow f(g(1)) = f(2.2) = \boxed{-1.7}$

For $x = 2$

$\Rightarrow f(g(2)) = f(1.2) = \boxed{-3.3}$

For $x = 3$

$\Rightarrow f(g(3)) = f(-0.2) = \boxed{-4}$

For $x = 4$

$\Rightarrow f(g(4)) = f(-1.9) = \boxed{-2.2}$

For $x = 5$

$\Rightarrow f(g(5)) = f(-4.1) = \boxed{-1.9}$

## Chapter 1 Functions and Limits Exercise 1.3 53E

(A)

Let stone covers the distance (r) in time t second

Speed of stone is 60 cm/s

It means in 1 second, it covers the distance = 60 cm. therefore in t second it will cover the distance = 60t cm

Hence we have r = 60t

r is the function of t so we can write $\boxed{r(t) = 60t}$ cm

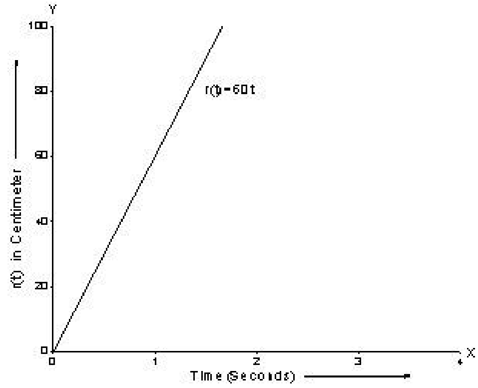


Fig.1

(B)

Area of this circle $= \pi(\text{radius})^2$

We have $r(t) = 60t$

So area $\Rightarrow (A \circ r)(t) = A(r(t))$

$= \pi(60t)^2$

$\Rightarrow \boxed{A = 3600\pi t^2}$

Hence the area of the circle is the function of time.

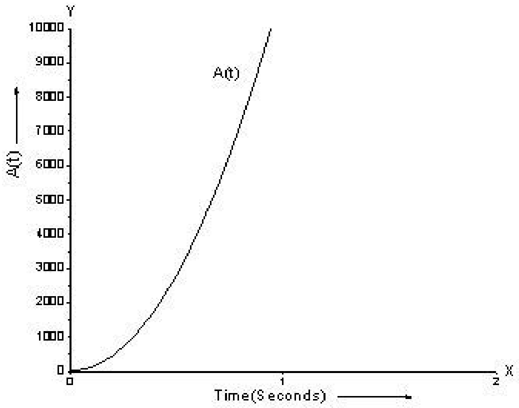


Fig.2

## Chapter 1 Functions and Limits Exercise 1.3 54E

Suppose that, a spherical balloon is being inflated and the radius of the balloon is increasing at a rate of $2\,\text{cm/s}$.

That is,

$$\frac{dr}{dt} = 2\,\text{cm/s} \quad \ldots\ldots (1)$$

(a)

Its need to express the radius $r$ of the balloon as a function of the time $t$ (in seconds)

On taking integration on both sides of differential equation (1) with respect to $t$, we get

$$\int \frac{dr}{dt}dt = \int 2\,dt$$

$r(t) = 2t + c$ Here $c$ is constant of integration.

Initially radius of the balloon is zero, that is $r(0) = 0$

Use the condition $r(0) = 0$ in $r(t) = 2t + c$ to find the constant $c$

$r(0) = 0 \Rightarrow 2(0) + c = 0$

$\Rightarrow c = 0$

Hence, radius $r$ of the balloon as a function of the time $t$ (in seconds) is

$r(t) = \boxed{2t}$

(b)

Its need to find the composite function $V \circ r$

Here $V$ is the volume of the balloon as a function of the radius.

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, here r is the radius of the sphere.

From part (a), radius for this problem is a function of time is $r(t) = 2t$

Thus $V(r) = \frac{4}{3}\pi r^3$ and $r(t) = 2t$

**Composite function $f \circ g$ :**

Given two functions $f$ and $g$, the composite function $f \circ g$ (also called the composition $f$ and $g$) is defined by

$(f \circ g)(x) = f(g(x))$

By the definition of **composite function**, we have

$(V \circ r)(t) = V(r(t))$

$= V(2t)$ Use $r(t) = 2t$ from part (a)

$= \frac{4}{3}\pi(2t)^3$ Replace $r$ by $2t$ in $V(r) = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(2^3)t^3$

$= \frac{32}{3}\pi t^3$

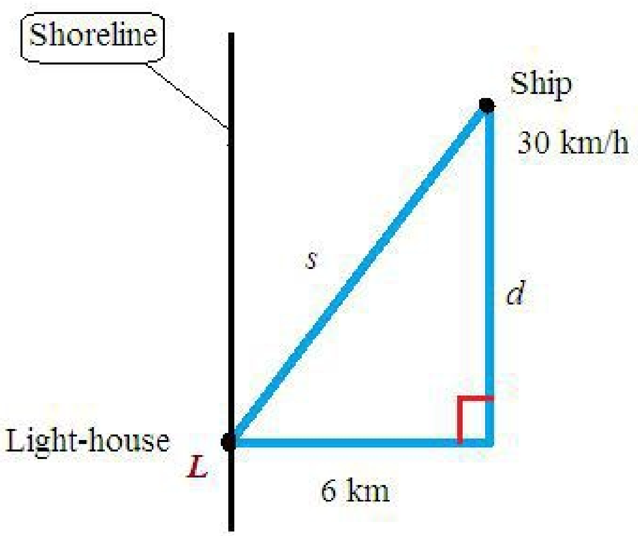
The resulting function $\boxed{V(t) = \frac{32}{3}\pi t^3}$ expresses the volume of the balloon, in $\text{cm}^3$, as a function of time, in seconds.

## Chapter 1 Functions and Limits Exercise 1.3 55E

Consider a ship is moving at a speed of 30 $\text{km/h}$ parallel to a straight shoreline.

The ship is 6 km from shore and passes a light-house at noon.

The following is the figure:



(a)

Suppose the ship is straight to the light house *L* at noon.

From the above figure, If *d* is the distance traveled by the ship after noon, and *s* is the distance between the light house and the ship, it is observed that *s*, *d* and 6 km are the sides of a right triangle.

So, by the Pythagorean theorem that states " $\text{base}^2 + \text{height}^2 = \text{hypotenuse}^2$ " gives

$$6^2 + d^2 = s^2$$

Then,

$$s = \sqrt{d^2 + 6^2}$$
$$s = \sqrt{d^2 + 36}$$

So, the distance between the light house *L* and the ship as a function of *d*, that is,

$$s = f(d)$$

Thus, $\boxed{f(d) = \sqrt{d^2 + 36}}$

(b)

The distance formula is,

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Here, distance = *d* km., speed = 30 km./h

So, $d = 30t$ km.

Now write the distance traveled by the ship as a function of time, namely $d = g(t)$

Thus, $\boxed{g(t) = 30t}$.

(c)

Observe that *f* is the function of distance between light house and the ship and *g* is the function of the distance traveled in the time *t*.

Here, $g(t) = 30t$ since $d = g(t)$ and $d = 30t$.

Also $f(d) = \sqrt{36 + d^2}$

Thus,

$$(f \circ g)(t) = f(g(t))$$
$$= f(30t)$$

$$= \sqrt{(30t)^2 + 36}$$
$$= \sqrt{900t^2 + 36}$$

Therefore, $(f \circ g)(t) = \boxed{\sqrt{900t^2 + 36}}$.

Here, the function $f \circ g$ represents the location of the ship from the light house at the time '*t* 'after noon.

## Chapter 1 Functions and Limits Exercise 1.3 56E

Suppose that, an airplane is flying at a speed of $350\,\text{mi/h}$ at an altitude of one mile and passes directly over a radar station at time $t = 0$

Let $d$ be the horizontal distance in miles that the plane flown at time $t$

Then

$$\frac{d}{dt}(d) = 350\,\text{mi/h} \text{ (By the data) ...... (1)}$$

(a)

Its need to find the horizontal distance $d$ (in miles) that the plane flown as a function of $t$

On taking integration on both sides of differential equation (1) with respect to $t$, we get

$$\int \frac{d}{dt}(d)\,dt = \int 350\,dt$$

$d(t) = 350t + c$ Here $c$ is constant of integration.

When the plane is over the radar station, the time and distance traveled be 0.

That is, $d(0) = 0$

Use the condition $d(0) = 0$ in $d(t) = 350t + c$ to find the constant $c$

$d(0) = 0 \;\Rightarrow\; 350(0) + c = 0$

$\Rightarrow c = 0$

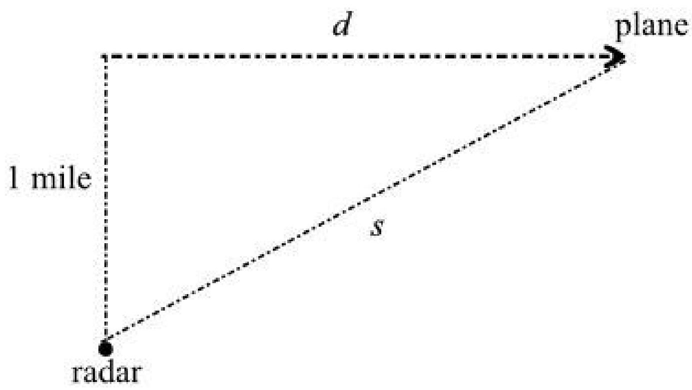
Hence, the horizontal distance $d$ (in miles) that the plane flown as a function of $t$ is

$d(t) = \boxed{350t}$

(b)

Its need to express the distance the distance $s$ between the plane and the radar station as a function of $d$

We construct a diagram displaying the plane distance, $s$, from the radar station:




The diagram shows that the plane was 1 mile above the radar when flying directly overhead. The distance it has flown since then is $d$.

The triangle created is a right triangle whose side-lengths obey the Pythagorean Theorem:

$c^2 = a^2 + b^2$

where $c$ is the hypotenuse length. The hypotenuse is the side opposite the right angle.

Substituting into the Pythagorean Theorem, and solving for $s$, we get:

$a^2 + b^2 = c^2$

$1^2 + d^2 = s^2$ ( $a = 1$, $b = 1$, $c = 1$)

$\sqrt{1 + d^2} = s$ (Take square root on both sides)

We only consider the positive square root here because $s$ is a measure of distance.

Therefore, the function expressing distance between the plane and radar is

$\boxed{s(d) = \sqrt{1 + d^2}}$

(c)

Its need to express $s$ as a function of $t$ by using composition

From part (a), we have $d(t)=350t$

From part (b), we have $s(d)=\sqrt{1+d^2}$

To express $s$ as a function of $t$, we need to compute the composition $(s\circ d)(t)$

**Composite function** $f\circ g$ **:**

Given two functions $f$ and $g$, the composite function $f\circ g$ (also called the composition $f$ and $g$) is defined by

$$(f\circ g)(x)=f(g(x))$$

By the definition of composite function

$$(s\circ d)(t)=s(d(t))$$

$=s(350t)$ Use $d(t)=350t$ from part (a)

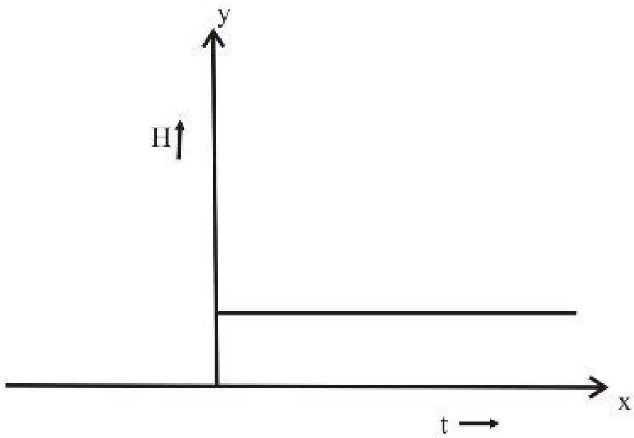
$=\sqrt{1+(350t)^2}$ Replace $d$ by $350t$ in $s(d)=\sqrt{1+d^2}$

Hence, the resulting function for the distance $s$ as a function of time, $t$, is

$$\boxed{s(t)=\sqrt{1+(350t)^2}}$$

### Chapter 1 Functions and Limits Exercise 1.3 57E

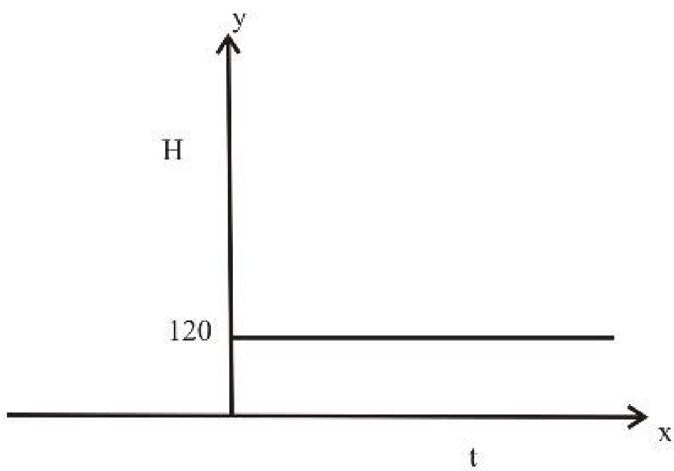
(A)




Here if t < 0 then $H(t)=0$ and $t\geq 0$ then $H(t)=1$

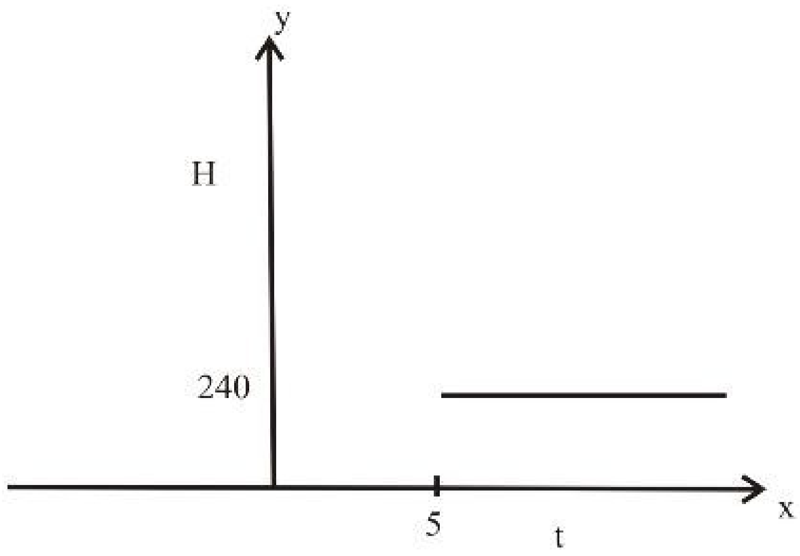
So we get a straight line parallel to x-axis in the positive direction.

(B)




Here $V(t)=120H(t)$

(C)




The formula is $V(t) = 240[H(t-5)]$

## Chapter 1 Functions and Limits Exercise 1.3 58E

(a)

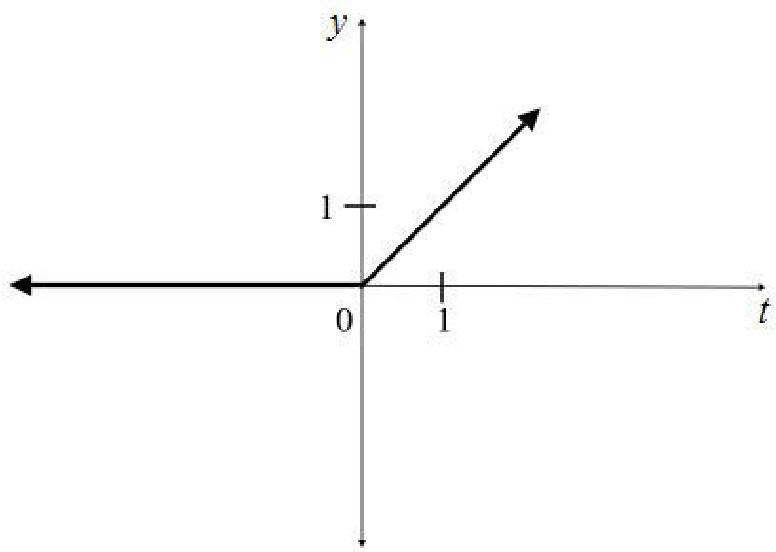
The Heaviside function is a piece-wise function because it is defined by different formulas in different parts of its domain. The Heaviside function $H$ is defined by

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases} \quad \ldots\ldots (1)$$

The ramp function $y = tH(t)$ is therefore defined as:

$$y(t) = \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } t \geq 0 \end{cases}$$

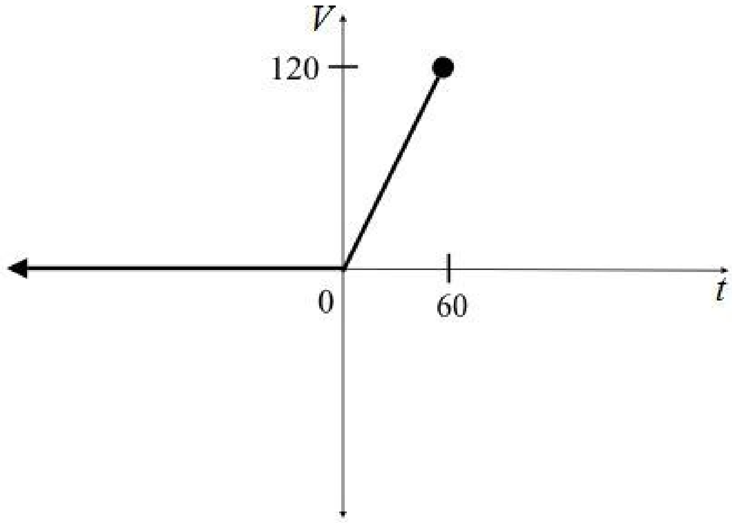
We plot $y$ on the vertical axis and $t$ on the horizontal axis. The domain $t < 0$ is to the left of the vertical axis, where $y$ is 0. To the right of the vertical axis, $y = t$, which is a line with a slope of 1:



(b)

If the voltage is turned on at $t=0$, it will gradually increase to 120 volts over 60 seconds. Therefore, the graph should be the ramp function, which shows gradual increase.

The point $(60,120)$ must be on the graph because when $t=60$ seconds, $V=120$ volts.




The voltage function is a modified ramp function. The domain of the second part of the function is $0 \le t \le 60$ and the slope of this segment between $(0,0)$ and $(60,120)$ is:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{120-0}{60-0}$$

$$= 2$$

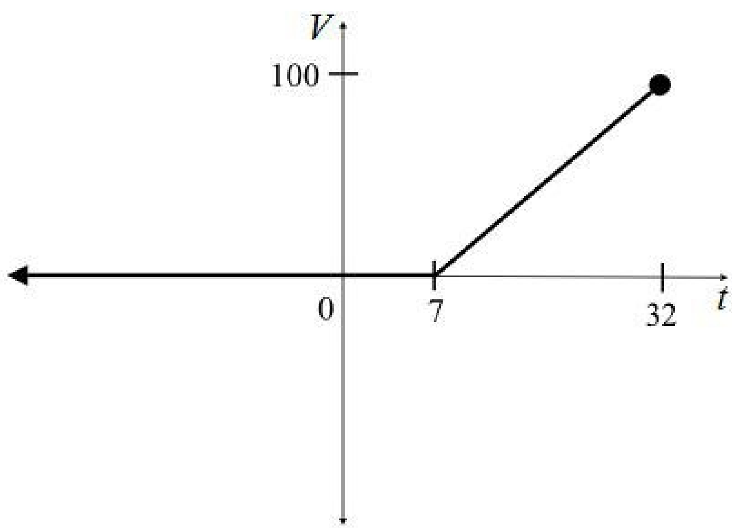
Therefore, the piecewise function for voltage would be

$$V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 2t & \text{if } 0 < t \le 60 \end{cases}$$

The function is a result of multiplying the Heaviside function, (1), by 2*t*: $\boxed{V(t) = 2tH(t)}$

(c)

If the voltage is turned on at $t=7$, it will gradually increase to 100 volts over 25 seconds. Therefore, the graph should be the ramp function, which shows gradual increase. The points $(7,0)$ and $(32,100)$ must be on the graph because when $t=0$ seconds, $V=0$ volts and 25 seconds later, when $t=32$ seconds, $V=100$ volts.



The voltage function is a modified ramp function. We note that since the voltage is turned on at 7 seconds, the ramp is shifted 7 units to the right.

To shift the graph of any function $f$ to the right by $c$ units, the equation would become $y = f(x-c)$ where $c > 0$.

Therefore, a shift of the ramp function to the right by 7 units would be $R(t-7)$.

This function is defined:

$$y(t-7) = \begin{cases} 0 & \text{if } t < 7 \\ t-7 & \text{if } t \geq 7 \end{cases}$$

The domain of the second part of the voltage function, $V(t)$, is $7 \leq t \leq 32$.

The slope of the second part of the voltage function between $(7,0)$ and $(32,100)$ is:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{100-0}{32-7}$$

$$= \frac{100}{25}$$

$$= 4$$

Therefore, the second part of the voltage function has a slope of 4.

Combining the shifted ramp function with the slope of 4 in the second part, we get:

$$V(t) = \begin{cases} 0 & \text{if } t < 7 \\ 4(t-7) & \text{if } 7 \leq t \leq 32 \end{cases}$$

Finally, we note that the function $V(t)$ is the result of multiplying the Heaviside function by $4(t-7)$ and restricting the domain to $t \leq 32$.

Hence, the above graph is defined:

$$\boxed{V(t) = 4(t-7)H(t),\ t \leq 32}$$

## Chapter 1 Functions and Limits Exercise 1.3 59E

Let $f$ and $g$ be linear functions with the following equations.

$f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$

Determine whether $f \circ g$ is also a linear function, if so, find the slope of its graph.

**Composite function $f \circ g$ :**

Given two functions $f$ and $g$, the composite function $f \circ g$ (also called the composition $f$ and $g$) is defined as follows:

$$(f \circ g)(x) = f(g(x))$$

By the definition of the composite function.

$$(f \circ g)(x) = f(g(x))$$

$= f(m_2 x + b_2)$ (Replace $g(x)$ with $m_2 x + b_2$)

$= m_1(m_2 x + b_2) + b_1$ (Replace $x$ by $m_2 x + b_2$ in $f(x) = m_1 x + b_1$)

$= m_1 m_2 x + m_1 b_2 + b_1$ (Distribute: $A(B+C) = AB + AC$)

Thus, $(f \circ g)(x) = m_1 m_2 x + (m_1 b_2 + b_1)$.

The resulting rule for the composition $(f \circ g)(x)$ is $\boxed{\text{linear}}$ because the variable $x$ has an exponent of 1 and its coefficient is non-zero.

The slope of the linear function $(f \circ g)(x) = m_1 m_2 x + (m_1 b_2 + b_1)$ is the coefficient of the $x$ term, which is $\boxed{m_1 m_2}$.

## Chapter 1 Functions and Limits Exercise 1.3 60E

Suppose that, $x$ dollars are invested at $4\%$ interest compounded annually, then the amount $A(x)$ of the investment after one year is given as follows:

$A(x) = 1.04x$

Its need to compute the compositions $A \circ A$, $A \circ A \circ A$ and $A \circ A \circ A \circ A$, and also needed to find a formula for the composition of $n$ copies of $A$.

**Composite function $f \circ g$:**

Given two functions $f$ and $g$, the composite function $f \circ g$ (also called the composition $f$ and $g$) is defined as follows.

$(f \circ g)(x) = f(g(x))$

From the definition of composite function, proceed as follows:

$(A \circ A)(x) = A(A(x))$

$= A(1.04x)$ Use $A(x) = 1.04x$

$= 1.04(1.04x)$ Replace $x$ by $1.04x$ in $A(x) = 1.04x$

$= (1.04)^2 x$ Multiply

The resulting rule for the composition $(A \circ A)(x)$ is $\boxed{(1.04)^2 x}$.

Similarly, use the rule for the function $A(x)$ in the composition $(A \circ A \circ A)(x)$.

Also use the fact that $(A \circ A)(x) = (1.04)^2 x$.

From the definition of the composite function, proceed as follows:

$(A \circ A \circ A)(x) = A((A \circ A)(x))$

$= A\left((1.04)^2 x\right)$ (Replace $(A \circ A)(x)$ with $(1.04)^2 x$)

$= 1.04\left((1.04)^2 x\right)$ (Replace $x$ by $(1.04)^2 x$ in $A(x) = 1.04x$)

$= (1.04)^3 x$ (Multiply)

The resulting rule for the composition $(A \circ A \circ A)(x)$ is $\boxed{(1.04)^3 x}$.

Now, use the rules of $A(x)$ and $(A \circ A \circ A)(x)$ to find $(A \circ A \circ A \circ A)(x)$:

$(A \circ A \circ A \circ A)(x) = A((A \circ A \circ A)(x))$

$= A\left((1.04)^3 x\right)$ (Replace $(A \circ A \circ A)(x)$ with $(1.04)^3 x$)

$= 1.04\left((1.04)^3 x\right)$ (Replace $x$ by $(1.04)^3 x$ in $A(x) = 1.04x$)

$= (1.04)^4 x$ (Multiply)

The resulting rule for the composition $(A \circ A \circ A \circ A)(x)$ is $\boxed{(1.04)^4 x}$.

Since the interest is compounded annually, notice that the result of the composition amounts to reinvesting the money every year. This also calculates the amount of the investment after 2, 3 and 4 years.

Based on this pattern, if the task were to compose $A$ with itself $n$ times, the resultant formula would have been $\boxed{\underbrace{(A \circ A \circ \cdots \circ A)}_{n \text{ times}}(x) = (1.04)^n x}$.

## Chapter 1 Functions and Limits Exercise 1.3 61E

(A)

$g(x)=2x+1, \qquad h(x)=4x^2+4x+7$

Given that $f\circ g=h$

Hence $f(g(x))=4x^2+4x+7$

By braking 7 in (1 + 6) on right hand side

$f(g(x))=4x^2+4x+1+6$

[Since $(2x+1)^2=4x^2+4x+1$ and here $f(g(x))=4x^2+4x+7$. We have to make right hand side, the function of $(2x+1)$.]

So $f(g(x))=4x^2+4x+1+6$

$=(2x+1)^2+6$

$\Rightarrow f(g(x))=[g(x)]^2+6$

Clearly we have $\Rightarrow \boxed{f(x)=x^2+6}$

(B) $f(x)=3x+5, h(x)=3x^2+3x+2$ and $h=f\circ g$

$(f\circ g)(x)=f(g(x))$

$=h(x)$

$=3x^2+3x+2$

$\Rightarrow 3(g(x))+5=3x^2+3x+2+5-5$ (By adding and subtracting 5 from right hand side)

$=3x^2+3x-3+5$

$=3(x^2+x-1)+5$

Comparing both sides we have $\boxed{g(x)=x^2+x-1}$

### Chapter 1 Functions and Limits Exercise 1.3 62E

$f(x)=x+4, \quad h(x)=4x-1$

Condition is given as $g\circ f=h$

$(g\circ f)(x)=h(x)$

$\Rightarrow g(f(x))=4x-1$

$\Rightarrow g(x+4)=4x-1$

Replace x by x- 4 we get

$\Rightarrow g(x+4)=4x-1$

$\Rightarrow g(x-4+4)=4(x-4)-1$

$\Rightarrow g(x)=4x-17$

Then we have

$\boxed{g(x)=4x-17}$

### Chapter 1 Functions and Limits Exercise 1.3 63E

If g is an even function and $h=f\circ g$

then h is always an even function .We will see that by a simple example.

Let $g(x)=x^2$ (Even function)

Now Let $f(x)=x^3$ is a odd function

Then $f(g(x))=(fog)(x)=(x^2)^3=x^6$ which is also even

Again we take $f(x)=x^2$ as even function.

Then $(fog)(x)=f(g(x))=f(x)^2=(x^2)^2=x^4$ which is also even

So we conclude that if g is even function then $h=f\circ g$ will be always an even function.

Things to remember:-

Any function [(even function)] = even function.

| Any function over an even function will always be even |
|---|

### Chapter 1 Functions and Limits Exercise 1.3 64E

If g is an odd function and $h = f \circ g$

Then h can not be odd always.

It will depend on function f; it means if f is odd function then h will be odd and if f is even then h will be even .We will see this concept by an example

Let $g(x) = x^3$ (odd function)

Now let $f(x) = x^2$ (even function)

Then $(f \circ g)(x) = f(g(x))$

$$= f(x^3)$$

$$= (x^3)^2$$

$$= x^6 \Rightarrow \text{(even function)}$$

Again let $f(x) = x^3$ (odd function)

$$(f \circ g)(x) = f(g(x))$$

$$= f(x^3)$$

$$= (x^3)^3$$

$$= x^9 \Rightarrow \text{(Odd function)}$$

If f is neither even nor odd then h will be neither even nor odd

For example

$g(x) = x^3$ (Odd) and $f(x) = x^2 + x$

then $h(x) = (f \circ g)(x)$

$$= f(g(x))$$

$$= f(x^3)$$

$\Rightarrow h(x) = x^6 + x^3$ (Neither even nor odd)