Probability

Sample Space and Events of Experiments

Sample Space

- The set of all possible outcomes of a random experiment is called the s**ample space** associated with the experiment. Sample space is denoted by the symbol **S**.

- E.g., consider the experiment of tossing a coin. In this experiment, there are two possible outcomes—a head or a tail. Thus, the sample space of this experiment is **S** = {H, T}.

- Each element of the sample space is called a **sample point**. In other words, we can say, each outcome of the random experiment is called a **sample point.**

Event and Its Types

- Any subset E of a sample space S is called an **event**.

- E.g., if we throw a die twice, then the sample space so obtained is given by
  S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
  If we consider set E as the outcomes in which both the throws show the same number, then it is given by
  E = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}.
  Since E is a subset of S, E is called an event of S. In this case, the event E is defined as, "Same number appeared in both throws".

- An event E of a sample space S is said to have occurred if the outcome $\omega \in E$. On the other hand, if $\omega \notin E$, then we say that the event E has not occurred.

- Consider the experiment of throwing a die twice. Let E and F denote respectively the events "the sum of the appeared numbers is 8" and "the sum of the appeared numbers is 12".

  Then, E = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

  And F = {(6, 6)}

  Now, if in a trial, we get the outcome as (3, 5), then we can say that the event E has occurred and the event F has not occurred because (3, 5) ∈ E and (3, 5) ∉ F.

- Events can be classified into various types on the basis of the elements they have.

- The event of a sample space S which contains no sample point is called an **impossible event**. It is denoted by an empty set Φ.
  E.g., the event "The sum of the appeared numbers on the twice throw of a die is less than 2" is an impossible event since there is no outcome related to this event.

- The event of a sample space which contains all the sample points is called a **sure event**. The entire sample space S is called a sure event.

- An event containing only one sample point is called a **simple event** or an **elementary event**. In fact, a sample space S containing *n* elements has *n* simple events.
  On throwing a die two times, we get 36 outcomes. So, there are 36 simple events. They can be written as follows:
  $E_1 = \{(1, 1)\}$, $E_2 = \{(1, 2)\}$ ….. $E_7 = \{(2, 1)\}$, $E_8 = \{(2, 2)\}$ ….. $E_{36} = \{(6, 6)\}$

- An event containing more than one sample point is called a **compound event**.
  On throwing a die two times, let E denote the event "The sum of the appeared numbers is a multiple of 3", and then we have
  E = {(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6)}.
  Here, the event E contains 12 sample points. So, E is a compound event.

Solved Examples

Example 1: A boy has two bags B_1 and B_2, a coin and a die. Bag B_1 contains 4 equal-sized red balls and bag B_2 contains 5 equal-sized green balls. He takes a ball from bag B_1. Then, he tosses the coin. Thereafter, he takes a ball from bag B_2. Finally, he throws the die. Write the sample space for this experiment. Find how many sample points are there in the sample space.

Solution:

Let us denote a red ball as R and a green ball as G.

It is given that the two bags B_1 and B_2 contain 4 red balls and 5 green balls respectively.

We have the following observations while the boy performs the given experiment.

- When the boy takes out a ball from bag B_1, it will always be a red ball (R).

- When the boy tosses the coin, it can be either a head (H) or a tail (T).

- When the boy takes out a ball from bag B_2, it will always be a green ball (G).

- When the boy throws the die, any of the numbers 1, 2, 3, 4, 5 and 6 can appear on the die.

Hence, the sample space (S) of the experiment is given as

S = {RHG1, RHG2, RHG3, RHG4, RHG5, RHG6, RTG1, RTG2, RTG3, RTG4, RTG5, RTG6}

Clearly, the sample space S contains 12 sample points.

Example 2: With respect to the sample space "All the months of the year 2008", the events E, F, G and H are defined as

E: "Month/Months having 31 days"

F: "Month/Months having 28 days"

G: "Month/Months having 25 to 35 days"

H: "Month/Months having 29 days"

Classify the above events as impossible, sure, simple or compound.

Solution:

We have the sample space as

S = {January, February, March, April, May, June, July, August, September, October, November,

December}

And E = {January, March, May, July, August, October, December}

2008 is a leap year. So, the number of days in the month of February is 29.

∴H = {February}

And F = Φ

G = {January, February, March, April, May, June, July, August, September, October, November,

December}

With respect to the given sample space:

1. F is an impossible event.

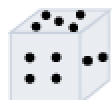
2. G is a sure event.

3. H is a simple event.

4. Both E and G are compound events.

Finding Probability Using Complement of a Known Event

Consider the experiment of throwing a dice. Any of the numbers 1, 2, 3, 4, 5, or 6 can come up on the upper face of the dice. We can easily find the probability of getting a number 5 on the upper face of the dice?



Mathematically, probability of any event E can be defined as follows.

$$P(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

Here, S represents the sample space and $n(S)$ represents the number of outcomes in the sample space.

For this experiment, we have

Sample space (S) = {1, 2, 3, 4, 5, 6}. Thus, S is a finite set.

So, we can say that the possible outcomes of this experiment are 1, 2, 3, 4, 5, and 6.

∴ Number of all possible outcomes = 6

Number of favourable outcomes of getting the number 5 = 1

∴ Probability (getting 5) $= \frac{1}{6}$

Similarly, we can find the probability of getting other numbers also.

P (getting 1) $= \frac{1}{6}$, P (getting 2) $= \frac{1}{6}$, P (getting 3) $= \frac{1}{6}$, P (getting 4) $= \frac{1}{6}$ and

P (getting 6) $= \frac{1}{6}$

Let us add the probability of each separate observation.

This will give us the sum of the probabilities of all possible outcomes.

P (getting 1) + P (getting 2) + P (getting 3) + P (getting 4) + P (getting 5) + P (getting 6) = $\frac{1}{6}$ + $\frac{1}{6}$ + $\frac{1}{6}$ + $\frac{1}{6}$ + $\frac{1}{6}$ + $\frac{1}{6}$ = 1

∴ "**Sum of the probabilities of all elementary events is 1**".

Now, let us find the probability of **not** getting 5 on the upper face.

The outcomes favourable to this event are 1, 2, 3, 4, and 6.

∴ Number of favourable outcomes = 5

∴ P (not getting 5) $= \frac{5}{6}$

We can also see that P (getting 5) + P (not getting 5) $= \frac{1}{6} + \frac{5}{6} = 1$

∴ "**Sum of probabilities of occurrence and non occurrence of an event is 1**".

i.e. **If *E* is the event, then *P* (*E*) + *P* (not *E*) = 1** … (1)

or we can write ***P(E) = 1 − P* (not *E*)**

Here, the events of getting a number 5 and not getting 5 are complements of each other as we cannot find an observation which is common to the two observations.

Thus, event **not *E*** is the complement of event *E.* Complement of event *E* is denoted by $\overline{E}$ or *E*'.

Using equation (1), we can write

$P(E) + P(\overline{E}) = 1$

or

$$P(\bar{E}) = 1 - P(E)$$

This is a very important property about the probability of complement of an event and it is stated as follows:

If E is an event of finite sample space S, then $P(\bar{E}) = 1 - P(E)$ where $\bar{E}$ is the complement of event E.

Now, let us prove this property algebraically.

Proof:

We have,

$E \cup \bar{E} = S$ and $E \cap \bar{E} = \phi$

$\Rightarrow n(E \cup \bar{E}) = n(S)$ and $n(E \cap \bar{E}) = n(\phi)$

$\Rightarrow n(E \cup \bar{E}) = n(S)$ and $n(E \cap \bar{E}) = 0$ ...(1)

Now,

$n(E \cup \bar{E}) = n(S)$

$\Rightarrow n(E) + n(\bar{E}) - n(E \cap \bar{E}) = n(S)$

$\Rightarrow n(E) + n(\bar{E}) - 0 = n(S)$ [Using (1)]

$\Rightarrow n(\bar{E}) = n(S) - n(E)$

On dividing both sides by $n(S)$, we get

$$\frac{n(\bar{E})}{n(S)} = \frac{n(S)}{n(S)} - \frac{n(E)}{n(S)}$$

$\Rightarrow$**$P(\bar{E}) = 1 - P(E)$**

Hence proved.

Let us solve some examples based on this concept.

Example 1: One card is drawn from a well shuffled deck. What is the probability that the card will be

(i) a king?

(ii) not a king?

Solution:

Let *E* be the event 'the card is a king' and *F* be the event 'the card is not a king'.

(i) Since there are 4 kings in a deck.

∴ Number of outcomes favourable to $E = 4$

Number of possible outcomes = 52

∴ $P(E) = \frac{4}{52} = \frac{1}{13}$

2. Here, the events E and F are complements of each other.

∴ $P(E) + P(F) = 1$

$P(F) = 1 - \frac{1}{13}$

$= \frac{12}{13}$

Example 2: If the probability of an event A is 0.12 and B is 0.88 and they belong to the same set of observations, then show that A and B are complementary events.

Solution:

It is given that $P(A) = 0.12$ and $P(B) = 0.88$

Now, $P(A) + P(B) = 0.12 + 0.88 = 1$

∴ The events A and B are complementary events.

Example 3: Savita and Babita are playing badminton. The probability of Savita winning the match is 0.52. What is the probability of Babita winning the match?

Solution:

Let E be the event 'Savita winning the match' and F be the event 'Babita wining the match'.

It is given that $P(E) = 0.52$

Here, E and F are complementary events because if Babita wins the match, Savita will surely lose the match and vice versa.

$\therefore P(E) + P(F) = 1$

$0.52 + P(F) = 1$

$P(F) = 1 - 0.52 = 0.48$

Thus, the probability of Babita winning the match is 0.48.

Example 4: In a box, there are 2 red, 5 blue, and 7 black marbles. One marble is drawn from the box at random. What is the probability that the marble drawn will be (i) red (ii) blue (iii) black (iv) not blue?

Solution:

Since the marble is drawn at random, all the marbles are equally likely to be drawn.

Total number of marbles = 2 + 5 + 7 = 14

Let A be the event 'the marble is red', B be the event 'the marble is blue' and C be the event 'the marble is black.

(i) Number of outcomes favourable to event A = 2

$\therefore P(A) = \frac{2}{14} = \frac{1}{7}$

(ii) Number of outcomes favourable to event B = 5

$\therefore P(B) = \frac{5}{14}$

(iii) Number of outcomes favourable to event C = 7

$\therefore\ P(C) = \frac{7}{14} = \frac{1}{2}$

(iv) We have, $P(B) = \frac{5}{14}$

The event of drawing a marble which is not blue is the complement of event B.

$\therefore\ P(\overline{B}) = 1 - P(B) = 1 - \frac{5}{14} = \frac{9}{14}$

Thus, the probability of drawing a marble which is not blue is $\frac{9}{14}$.

Algebra of Events & Mutually Exclusive and Exhaustive Events

Algebra of Events

- Algebra of events in a sample space is the same as the algebra of sets in a universal set.
- The sample space S is considered as the universal set.
- The events of the sample space are considered as the subsets of the universal set.
- The **complement of an event** A (also called **not A**) in a sample space S is denoted by **A'** and defined as A' = S − A = {ω: ω ∈ S and ω ∉ A}
- If we throw a coin and then throw a die, then the sample space (S) is
  S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}
  Let us define an event A = {H1, H2, H3, T2, T4, T5, T6}, then A' is given by
  A' = S − A = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6} − {H1, H2, H3, T2, T4, T5, T6}
  = {H4, H5, H6, T1, T3}
- The **union of two events** A and B, i.e., A ∪ B denotes the event "A or B". It consists of all the sample points which are in A or in B or in both. Mathematically, it is defined as A ∪ B = {ω: ω ∈ A or ω ∈ B}
- Consider the experiment of throwing a coin and then tossing a die. Let us define two events A and B as
  A: "First observation is a tail and the second observation is a number that is a multiple of 3"
  B: "Second observation is a number more than or equal to 4"
  Then, we have
  A = {T3, T6}

B = {H4, H5, H6, T4, T5, T6}
∴ A ∪ B = {T3, T6} ∪ {H4, H5, H6, T4, T5, T6} = {H4, H5, H6, T3, T4, T5, T6}

- The **intersection of two events** A and B, i.e., A ∩ B denotes the event "A and B". It consists of all the sample points which are common in A and B. Mathematically, it is defined as A ∩ B = {ω: ω ∈ A and ω ∈ B}

- Consider the experiment of throwing a coin and then tossing a die. Let us define two events A and B as
  A: "First observation is a tail and the second observation is a number that is a multiple of 3"
  B: "Second observation is a number more than or equal to 4"
  Then, we have
  A = {T3, T6}
  B = {H4, H5, H6, T4, T5, T6}
  ∴ A ∩ B = {T3, T6} ∩ {H4, H5, H6, T4, T5, T6} = {T6}

- The set A − B denotes the event "**A, but not B**". It consists of the sample points which are in A, but not in B. Mathematically, it is defined as A − B = {ω: ω ∈ A and ω ∉ B}

- Consider the experiment of throwing a coin and then tossing a die. Let us define two events A and B as
  A: "First observation is a head and the second observation is an even number"
  B: "Second observation obtained is a number that is a multiple of 3"
  Then, we have
  A = {H2, H4, H6}
  B = {H3, H6, T3, T6}
  ∴ A − B = {H2, H4, H6} − {H3, H6, T3, T6} = {H2, H4}
  And B − A = {H3, H6, T3, T6} − {H2, H4, H6} = {H3, T3, T6}

- The other ways to find A − B are

  1. A − B = A ∩ B'
  2. A − B = A − (A ∩ B)

- Two important formulae are

- A' ∪ B' = (A ∩ B)' = S − (A ∩ B)

- A' ∩ B' = (A ∪ B)' = S − (A ∪ B)

Mutually Exclusive and Exhaustive Events

- Let E_1, E_2, E_3 …. E_n be *n* events in a sample space S.

- If $E_i \cap E_j = \Phi$ for $i \neq j$, where $i, j = 0, 1, 2 \ldots n$, then $E_1, E_2, E_3 \ldots E_n$ are called **mutually exclusive events**.

- If $\bigcup_{i=1}^{n} E_i = S$, then $E_1, E_2, E_3 \ldots E_n$ are called **exhaustive events**.

- All the simple events of a sample space are mutually exclusive and exhaustive events.

Solved Examples

Example 1: A die is thrown and then a slip is drawn out from four slips numbered 2, 5, 8 and 9. Describe the following events:

A: "Both appeared numbers are even"

B: "The sum is a prime number"

C: "The sum is divisible by 3"

D: "First appeared number is odd and the second appeared number is even"

Also, find $[A - (B \cup C)] \cap D'$.

Solution: The sample space S of the given experiment is:

S = {(1, 2), (1, 5), (1, 8), (1, 9), (2, 2), (2, 5), (2, 8), (2, 9), (3, 2), (3, 5), (3, 8), (3, 9), (4, 2),

(4, 5), (4, 8), (4, 9), (5, 2), (5, 5), (5, 8), (5, 9), (6, 2), (6, 5), (6, 8), (6, 9)}

We have the events A, B, C, D and E as follows:

A = {(2, 2), (2, 8), (4, 2), (4, 8), (6, 2), (6, 8)}

B = {(1, 2), (2, 5), (2, 9), (3, 2), (3, 8), (4, 9), (5, 2), (5, 8), (6, 5)}

C = {(1, 2), (1, 5), (1, 8), (3, 9), (4, 2), (4, 5), (4, 8), (6, 9)}

D = {(1, 2), (1, 8), (3, 2), (3, 8), (5, 2), (5, 8)}

Now, we have

B ∪ C

= {(1, 2), (2, 5), (2, 9), (3, 2), (3, 8), (4, 9), (5, 2), (5, 8), (6, 5)} ∪ {(1, 2), (1, 5), (1, 8), (3, 9), (4, 2), (4, 5), (4, 8), (6, 9)}

= {1, 2), (1, 5), (1, 8), (2, 5), (2, 9), (3, 2), (3, 8), (3, 9), (4, 2), (4, 5), (4, 8), (4, 9), (5, 2), (5, 8), (6, 5), (6, 9)}

So, A − (B ∪ C) = {(2, 2), (2, 8), (4, 2), (4, 8), (6, 2), (6, 8)} − {1, 2), (1, 5), (1, 8), (2, 5), (2, 9), (3, 2), (3, 8), (3, 9), (4, 2), (4, 5), (4, 8), (4, 9), (5, 2), (5, 8), (6, 5), (6, 9)}

= {(2, 2), (2, 8), (6, 2), (6, 8)}

D’ = S − D

= {(1, 2), (1, 5), (1, 8), (1, 9), (2, 2), (2, 5), (2, 8), (2, 9), (3, 2), (3, 5), (3, 8), (3, 9), (4, 2), (4, 5), (4, 8), (4, 9), (5, 2), (5, 5), (5, 8), (5, 9), (6, 2), (6, 5), (6, 8), (6, 9)} − {(1, 2), (1, 8), (3, 2), (3, 8), (5, 2), (5, 8)}

= {(1, 5), (1, 9), (2, 2), (2, 5), (2, 8), (2, 9), (3, 5), (3, 9), (4, 2), (4, 5), (4, 8), (4, 9), (5, 5), (5, 9), (6, 2), (6, 5), (6, 8), (6, 9)}

∴ [A − (B ∪ C)] ∩ D’

= {(2, 2), (2, 8), (6, 2), (6, 8)} ∩ {(1, 5), (1, 9), (2, 2), (2, 5), (2, 8), (2, 9), (3, 5), (3, 9), (4, 2), (4, 5), (4, 8), (4, 9), (5, 5), (5, 9), (6, 2), (6, 5) (6, 8), (6, 9)}

= {(2, 2), (2, 8), (6, 2), (6, 8)}

Example 2: The sample space S of an experiment is defined by S = {(x, y): x, y ∈ N, $x < 5$, $y \leq 3$} and the events A, B, C, D, E, F, G and H of S are defined by

A: “$x + y \geq 5$”

B: “$x^y \leq 3$”

C: “$xy = 4$”

D: “x is odd”

E: “xy is even, where $y = 2n + 1$, n is a whole number”

F: “$xy = 8$”

G: "$y^x = 4$"

Describe the events A, B, C, D, E, F and G. On the basis of these events, answer the following:

1. Find the events which are exhaustive, but not mutually exclusive.

2. Find the events which are both mutually exclusive and exhaustive.

Solution: We have

S = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)}

A = {(2, 3), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)}

B = {(1, 1), (1, 2), (1, 3), (2, 1), (3, 1)}

C = {(2, 2), (4, 1)}

D = {(1, 1), (1, 2), (1, 3), (3, 1), (3, 2), (3, 3)}

E = {(2, 1), (2, 3), (4, 1), (4, 3)}

F = {(4, 2)}

G = {(2, 2)}

1. It can be observed that
A ∩ B = {(2, 3), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)} ∩ {(1, 1), (1, 2), (1, 3), (2, 1), (3, 1)}
= Φ
A ∩ C = {(2, 3), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)} ∩ {(2, 2), (4, 1)} = {(4, 1) ≠ Φ
So, A, B and C are not mutually exclusive events.
Now,
A ∪ B ∪ C
= {(2, 3), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)} ∪ {(1, 1), (1, 2), (1, 3), (2, 1), (3, 1)} ∪ {(2, 2), (4, 1)}
= {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)}
= S.
So, A, B and C are exhaustive events.
Thus, A, B and C are the events which are exhaustive, but not mutually exclusive.

2. It can be observed that
D ∩ E = {(1, 1), (1, 2), (1, 3), (3, 1), (3, 2), (3, 3)} ∩ {(2, 1), (2, 3), (4, 1), (4, 3)} = Φ
D ∩ F = {(1, 1), (1, 2), (1, 3), (3, 1), (3, 2), (3, 3)} ∩ {(4, 2)} = Φ
D ∩ G = {(1, 1), (1, 2), (1, 3), (3, 1), (3, 2), (3, 3)} ∩ {(2, 2)} = Φ
E ∩ F = {(2, 1), (2, 3), (4, 1), (4, 3)} ∩ {(4, 2)} = Φ

$E \cap G = \{(2, 1), (2, 3), (4, 1), (4, 3)\} \cap \{(2, 2)\} = \Phi$
$F \cap G = \{(4, 2)\} \cap \{(2, 2)\} = \Phi$
So, the events D, E, F and G are mutually exclusive.

Now,
$D \cup E \cup F \cup G$
$= \{(1, 1), (1, 2), (1, 3), (3, 1), (3, 2), (3, 3)\} \cup \{(2, 1), (2, 3), (4, 1), (4, 3)\} \cup \{(4, 2)\} \cup \{(2, 2)\}$
$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)\}$
$= S$
So, the events D, E, F and G are exhaustive events.
Thus, D, E, F and G are the events which are both mutually exclusive and exhaustive.

Probability of an Event

Axiomatic Approach to Probability

- Axiomatic approach is one of the ways of describing the probability of an event. In this approach, some rules or axioms are used for doing so.

- Let S be the sample space of a random experiment. The **probability P** is a real-valued function whose domain is the power set of S and range is the interval [0, 1], satisfying the following axioms:

  i. For any event E, $P(E) \geq 0$
  ii. $P(S) = 1$
  iii. If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$

- $P(\Phi) = 0$ and it can be proved using axiom (iii) as follows:
  If we take $F = \Phi$, then
  $P(E \cup \Phi) = P(E) + P(\Phi)$
  $\Rightarrow P(E) = P(E) + P(\Phi)$ $(E \cup \Phi = E)$
  $\Rightarrow P(\Phi) = 0$

- Let S be the sample space of a random experiment such that $S = \{\omega_1, \omega_2 \ldots \omega_n\}$.
  By the axiomatic definition of probability, it follows that

- $0 \leq P(\omega_1) \leq 1$
  Probability of each elementary event lies in the range [0, 1].

  E.g., We cannot have a sample space, $S = \{\omega_1, \omega_2\}$, such that $P(\omega_1) = \frac{-2}{3}$ and $P(\omega_2) = \frac{5}{3}$, since $P(\omega_1) < 0$.

- $P(\omega_1) + P(\omega_2)$ …. $P(\omega_n) = 1$
  The sum of the probabilities of all the elementary events is always 1.

  E.g., We cannot have a sample space, $S = \{\omega_1, \omega_1 …. \omega_5\}$, such that $P(\omega_1) = \frac{1}{4}$, $P(\omega_2) = \frac{2}{9}$, $P(\omega_3) = \frac{3}{5}$, $P(\omega_4) = \frac{2}{3}$ and $P(\omega_8) = \frac{5}{6}$. Since $P(\omega_1) + P(\omega_2) + P(\omega_3) + P(\omega_4) + P(\omega_5) = \frac{1}{4}+\frac{2}{9}+\frac{3}{5}+\frac{2}{3}+\frac{5}{6}=\frac{463}{180}\neq 1$.

- For any event A, $P(A) = \sum P(\omega_i), \omega_i \in A$

  Probability of Equally Likely Outcomes

- Let $S = \{\omega_1, \omega_1 …. \omega_n\}$ be the sample space associated with a random experiment. If the chance of occurrence of each simple event is the same, then it is said that the outcomes $\omega_1, \omega_1 …. \omega_n$ are equally likely outcomes. In such a case

  $P(\omega_1) = P(\omega_2) = …. = P(\omega_n) = \frac{1}{n}$

- Let S be a sample space containing *n* outcomes and E be an event containing *m* outcomes.
  $P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total possible outcomes}} = \frac{m}{n}$

  - **Probability of the Event "A or B"**
  - If A and B are two events associated with a random experiment, then the probability of the event "A or B", i.e., $P(A \cup B)$ is calculated by using the following formula:
    $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- For two mutually exclusive events A and B, $(A \cap B) = \Phi$, and we know that $P(\Phi) = 0$

  $\therefore P(A \cup B) = P(A) + P(B)$

  Probability of Event "not A"

- For an event A in a sample space S, the probability of the event "not A", i.e., $P(A')$ is calculated by using the formula

  $P(A') = 1 - P(A)$

Example 1: In grade XI of a school, each student is allowed to choose only one of the languages from French, German or English as the compulsory subject. If the respective probabilities of the students opting for French and German as the compulsory subject are 0.29 and 0.34, then find the probability of the students opting for English as the compulsory subject.

Solution:

Let E, F and G respectively denote the events of opting for English, French and German as the compulsory subject.

It is given that P (F) = 0.29 and P (G) = 0.34.

Here, the events E, F and G are mutually exclusive and exhaustive.

$\therefore P(E \cup F \cup G) = P(E) + P(F) + P(G)$

Since each student is allowed to opt for exactly one language, the event $E \cup F \cup G$ represents the sample space S. Also, we know that

$P(E \cup F \cup G) = P(S) = 1$

$\therefore P(E) + P(F) + P(G) = 1$

$\Rightarrow P(E) + 0.29 + 0.34 = 1$

$\Rightarrow P(E) = 1 - 0.63$

$\Rightarrow P(E) = 0.37$

Thus, the probability of the students opting for English as the compulsory subject is 0.37.

Example 2: From a deck of 52 cards, 2 red cards are removed and the remaining cards are well shuffled. If 8 cards are drawn from these well-shuffled cards, then find the probability that these 8 cards contain at least 2 black cards.

Solution: When two red cards are removed from a deck of 52 cards, then the remaining numbers of cards = 52 − 2 = 50

8 cards out of these 50 cards can be chosen in ${}^{50}C_8$ ways.

P (At least 2 black cards) = 1 − P (at most 1 black card)

P (At least 2 black cards) = 1 − [P (no black card) + P (1 black card)] … (1)

Out of 8 cards, if no black card is drawn, then 8 red cards should be drawn.

No black card out of 26 black cards can be drawn in $^{26}C_0 = 1$ way.

8 red cards out of 24 red cards can be drawn in $^{24}C_8$ ways.

So, no black card and 8 red cards can be drawn in $^{26}C_0 \times {}^{24}C_8$ ways.

∴P (no black card) = P (0 black card and 8 red cards) $= \frac{^{26}C_0 \times {}^{24}C_8}{^{50}C_8}$

Similarly, P (1 black card) = P (1 black card and 7 red cards) $= \frac{^{26}C_1 \times {}^{24}C_7}{^{50}C_8}$

Using equation (1), we obtain

P (At least 2 black cards) = 1 − $\frac{^{26}C_0 \times {}^{24}C_8 + {}^{26}C_1 \times {}^{24}C_7}{^{50}C_8}$

Example 3: When three identical coins are tossed, what is the probability of getting at least two heads?

Solution:

The outcome of tossing a coin is a head or a tail. This means that when a coin is tossed, the total number of outcomes = 2

Total outcomes of tossing three coins $= 2^3 = 8$

Thus, the sample space has 8 elements:
{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Out of these possible outcomes, 3 outcomes have 2 heads each and 1 outcome has all the three heads.

So, total outcomes of getting at least 2 heads = 4

Therefore, the probability of getting at least 2 heads is given by:

$$\frac{\text{Number of outcomes of getting at least 2 heads}}{\text{Total sample space}} = \frac{4}{8} = \frac{1}{2}$$