Mathematics

Class - VIII

2019 - 20

State Council of Educational Research and Training
Chhattisgarh, Raipur

(For Free Distribution)

Publication Year - 2019

S.C.E.R.T. CHHATTISGARH, RAIPUR

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Published by

State Council of Educational Research & Training Chhattisgarh, Raipur

Printed by

Chhattisgarh Textbook Corporation, Raipur

Printing Press

Preface

The aim of mathematical studies is not only to provide a knowledge of mathematics, but also to develop an understanding which will enable the students to enjoy maths. They will also be able to form relevant mathematical questions and solve them. They may also be able to transfer their aquired skill in their daily life conveniently.

By the time they reach class-VIII, children start experiencing the inherent power of maths. At this juncture, they are not only able to write the largest number separately, but also to calculate the compound interest with the help of power, and to recognize the special features of Geometrical figures or shapes. They begin to understand the multiplication of Algebraic numbers (variables) and to understand the concept of a substance occupying place. The most important objective of Mathematics is to develop the skill of understanding and reasoning and to elaborate them.

This book has been prepared keeping in view the achievement level of the students and the objectives of Mathematics. But no book can be completely self-sufficient in itself, hence your suggestions are invited to make it more interesting and perceptible. Your suggestions will be beneficial to all the students of the state.

We are grateful to all the learned scholars of government and non-government institutions whose valuable guidance and co-operation we have always received. We are particularly thankful to Vidya Bhawan, Udaypur, whose contribution in preparing this book is very valuable.

The National Council of Educational Research and Training (NCERT) sets some clear and measurable goals for class 1 to 8th. They are known as Learning outcomes'.

We have made some necessary changes in this textbook in reference with Learning out comes'. Some new contents have been added and some chapters have been transferred from one class to another. Do not let the teachers and the students get confused.

Director
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Chapter—1

SQUARE & CUBE

Square Numbers

In the illustration given below each row (horizontal) and column (vertical) show equal number of dots. Each row and column has five dots, can you count the total number of dots?

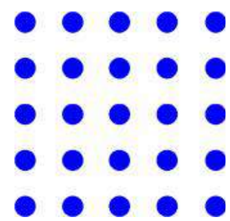


Fig.1

Now use two-two, three- three, four –four , eight-eight dots in rows and columns(equal numbers)and make some more square patterns and then complete the table given below -

Table 1.1

S.No.	No. of dots in each row or column	Total number of dots in the pattern made
1.	5	25
2.	2	-------
3.	-------	9
4.	4	-------
5.	-------	-------
6.	-------	-------

Look at the last column in the above table. All the numbers indicate that they can be obtained by multiplying a number with the same number e.g. 25= 5x5, 4=2x2, 9=3x3. So, all these number like 1, 4, 9,16, 25 …….etc. are (Perfect Square Numbers) Now can you write five more perfect Square Numbers?

We have ourselves made these numbers Perfect Square Numbers. If any number is given to you, how will you find out whether it is a Perfect Square Number or not?

Recognizing a Perfect Square Number

You have seen that you can arrange 9 dots in 3 rows of 3-3 dots each, 16 dots in 4 rows of 4-4 dots each, but when we have 10, 11 or 12 dots, we won't be able to arrange them in such a manner where the number of dots in each row and the number of rows be equal. You can verify such situations for small number of dots. If the number of dots are 109 or 784 or even larger, it would certainly be difficult to verify them using the above pattern. So, the Perfect Square Numbers can be verified by another method, which is the method of prime factorisation.

The Prime factor and the identification of the Perfect Square Number

For a Perfect Square Number, the number of dots in each row and the number of rows are equal e.g. the Perfect Square Number like 6×6, 5×5, 3×3, 7×7, etc.

For any number, where the multiple factors are made of complete pairs, would be Perfect Square Numbers. To get this, we would first break the number into its multiple factors and then make pairs

The Prime Factorisation Method

In this method, first the prime multiple factors of the given number is obtained and then pairs are made. The numbers in which all the prime multiple factors get in pairs, is recognized as a Perfect Square Number.
For example.

2	144
2	72
2	36
2	18
3	9
3	3
	1

(1) Take a number 144
The prime multiple factors of 144 are
$\underline{2\times2} \times \underline{2\times2} \times \underline{3\times3}$
Here, pair of prime muliples of 144 are formed.

2	252
2	126
3	63
3	21
7	7
	1

(2) Take a number 252.
The prime multiple factors of 252 are $\underline{2\times2} \times \underline{3\times3} \times 7$
For this number, the factor 7 does not have any pair,
Therefore, 252 is not a Perfect Square Number.

Activity 1.

Now find out the prime multiple factors for the numbers given in the Table and fill in the blanks given below.

Table 1.2

S.No.	Number	Prime factor	Are all the prime multiple factors in pairs	Whether the number is a Perfect Square or not
1.	16	$\underline{2 \times 2} \times \underline{2 \times 2}$	**Yes**	**Perfect Square**
2.	32	$\underline{2 \times 2} \times \underline{2 \times 2} \times 2$	**No**	**No**
3.	36			
4.	48			
5.	40			
6.	49			
7.	56			
8.	64			

Now verify, whether the numbers given below are Perfect Square Number or not.

(i) 164 (ii) 256 (iii) 81 (iv) 120 (v) 576
(vi) 205 (vii) 625 (viii) 324 (ix) 216 (x) 196

Some characteristics of Perfect Square Numbers

Identifying a Perfect Square by the unit number -

Carefully observe all the Perfect Square Numbers like 4,9,16, 25, 81, 100, 169, 324,256,625 etc. Is there any Perfect Square Number that has the digit 2,3,7or 8 in the unit's place? Take some more numbers and check the unit's place. What conclusion do you draw? Let's observe the table below.

Table 1.3

Number	Perfect Square	Numbers	Perfect Square Numbers
1	1	2	4
3	9	4	16
5	25	6	36
7	49	8	64
9	81	10	100
11	121	12	144
.....	.....	.....	.....
.....	.....	.....	.....

You can still take the table forward. What do you notice about the perfect squares of the odd and even numbers in the table?
Put a √ mark in the statements that follow:

(1) The Perfect Squares of odd numbers are : Odd/ Even

(2) The Perfect Squares of even numbers are : Odd/ Even

Another Interesting Observation

Look at the figure given below-

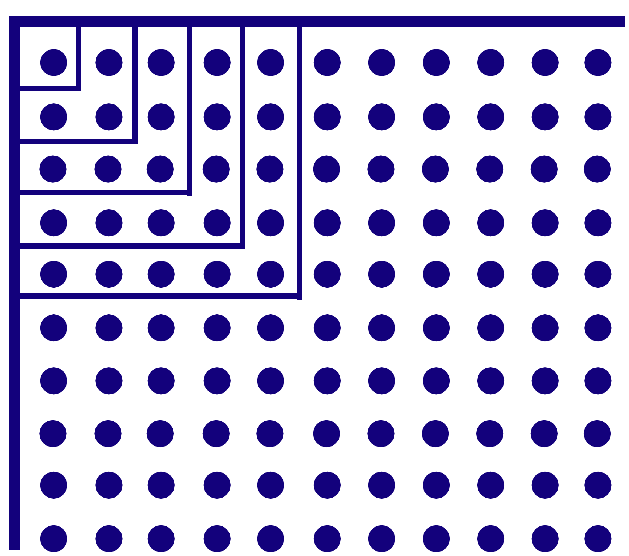


Figure 2

In the figure, starting from one end several squares have been drawn. Some parts of these squares have automatically been included in the new squares. If we add the total number of dots included in each part separately, we will see that the number of dots in the squares would be as follows.

1st square = 1	=	1	=	1^2
2nd square = 1 + 3	=	4	=	2^2
3rd square = 1 + 3 + 5	=	9	=	3^2
4th square = 1 + 3 + 5 + 7	=	16	=	4^2
5th square = 1 + 3 + 5 + 7+9	=	25	=	5^2
6th square = 1 + 3 + ...	=	36	=	6^2
7th square = ...	=	....	=	....
8th square = ...	=	....	=	....

What do you notice? We can see that whichever square is taken into consideration, the total number of dots in that square is a perfect square number. Can you tell what will be the number of dots in the eight & tenth squares?

We will notice that the dots included in the first, second, third …. and other squares are in the following manner :

1st square	=	first odd number	= 1^2
2nd square	=	sum of first two odd number	= 2^2
3rd square	=	sum of first three odd number	= 3^2

This continues, for example: the sum of the first 8 odd numbers is equal to 8^2. This holds true for as many numbers as you would work to check.

Hence we conclude that 'The square of any natural number 'n' is equal to the sum of the first 'n' consecutive odd number.

Some more interesting patterns:

Observe the square numbers of 1, 11, 111 ......

1^2	=	1
11^2	=	1 2 1
111^2	=	1 2 3 2 1
1111^2	=	1 2 3 4 3 2 1
11111^2	=	....
111111^2	=	....

Activity 2.

Ask your friend to say any two consecutive numbers. Add those numbers orally and write the sum in your notebook. Now ask your friend to find out the squares of the two numbers and subtract the smaller square number from the greater square number. After this, show the number you wrote in your notebook to your friend. Aren't the numbers the same?

How did that happen?

Can you guess how this will work out. Look at the following patterns:

$4^2 - 3^2 = 16 - 9 = 7 = 4 + 3$,

$9^2 - 8^2 = 81 - 64 = 17 = 9 + 8$,

$13^2 - 12^2 = 169 - 144 = 25 = 13 + 12$

Now look at the examples below:

$3^2 + 4^2 = 9+16= 25 = 5^2$, $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

$5^2 +12^2= 25+144 =169=13^2$ i.e. $5^2 + 12^2 = 13^2$

Can you find out more such examples? You will find that in every example there is a triplet of numbers and each triplet has some special numbers. The square of the larger number is the sum of the squares of remaining two numbers. Such numbers are known as Pythagoral Triplets. Triplets means a set of three numbers, like

(3, 4, 5), (6, 8, 10), or (5, 12, 13) are Pythagoral triplets.

Example 1. Verify whether (9, 40, 41) is a pythagoral triplet?

Solution : Here $9^2 + 40^2 = 81 + 1600 = 1681$

And $41^2 = 1681$

Therefore $9^2 + 40^2 = 41^2$. So, (9, 40, 41) is a Pythagoral Triplet.

PRACTICE - 1

Verify whether the sets given below are Pythagoral Triplet or not?

(i) (5, 12, 13) (ii) (8, 15, 17)

(iii) (10, 15, 25) (iv) (4, 7, 11)

NOTE : Our country knew about this triplet relationship much earlier. It is believed that in 600 BC an Indian mathematician, Bodhayan expressed it in an expanded form and explained it through multi digit numbers .

Making Perfect Squares From Numbers

As you have seen in the example on page 2 that in the multiple factors of 252, 2 and 3 had pairs but 7 didn't have any pair. If we divide or multiply the number by 7, we would be able to get pairs of all the multiple factors. This means 7 is the least number which when divided or multiplied by the number 252 will make the product or quotient, a perfect square. Now let us understand this by some more examples:

Example 2. Find out the least number which when multiplied by 720 will make a perfect square number.

Solution : $720 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

Out of the prime multiple factors of 720,5 did not make any pair. Therefore the perfect square number would be that in which 5 will also get a pair and for this we will have to multiply 720 by 5. Therefore 5 is the least number that will give a perfect square when multiplied with 720.

Example 3. Find out the least number which when divided by 140 will make a quotient a perfect square.

Solution : $140 = \underline{2 \times 2} \times 5 \times 7$

Out of the prime multiple factors of 140, 5 and 7 did not have pairs. If 140 is divided by 35 (5×7), then the quotient will be a perfect square.

Example 4. Find out the lowest perfect square that is completely divisible by 6 and 8.

Solution : The lowest common multiple of 6 and 8 = 24

The prime factor of 24 $= \underline{2 \times 2} \times 2 \times 3$

Now we see that 2 and 3 are the multiple factors of 24 that do not have pairs. If we multiply 24 by (2 x 3) = 6, it will be a perfect square .That would get divided by both 6 and 8. Therefore, the desired number is 24 x 6 = 144.

Try These Also

$(10)^2 = 100$, $(300)^2 = 90000$, $(5000)^2 = 25000000$

The square of 10 has two zeroes, the square of 300 has four zeroes, and that of 5000 has six zeroes. Can you think of a number the square of which has zero in the unit's place or has zeroes in all three places – units, tens and hundreds.

PRACTICE - 2

(i) Find out the smallest number which when multiplied by 200, so that the product becomes a perfect square.

(ii) Find out the smallest number which when multiplied by 180, so that the product becomes a perfect square.

(iii) Find out the smallest number which when divided by 2352 make a quotient a perfect square.

EXERCISE - 1.1

Q.1 Pairs the multiple factors of the following numbers and say whether they are perfect squares or not?

(i)	164	(ii)	121	(iii)	289
(iv)	729	(v)	1100		

Q.2 Give reasons why the given numbers are not perfect squares.

(i)	12000	(ii)	1227	(iii)	790				
(iv)	1482	(v)	165000	(vi)	15050	(vii)	1078	(viii)	8123

Q.3 Find out the numbers whose squares are even numbers and whose squares are odd numbers?

(i)	14	(ii)	277	(iii)	179
(iv)	205	(v)	608	(vi)	11288
(vii)	1079	(viii)	4010	(ix)	1225

Q.4 Look at the given pattern and fill in the blanks:-

11^2	=	121
101^2	=	10201
1001^2	=	1002001
10001^2	=	-----------
100001^2	=	------------
-----------	=	1000002000001

Cube Numbers

Till now we have thought about square numbers. When a number is multiplied by the same number, the product obtained is known as the square of that number. Now if the product is again multiplied by the original number, the number obtained would be the cube of that number.

e.g. $2 \times 2 \times 2 = 8$ or $2^3 = 8$

$7 \times 7 \times 7 = 343$ or $7^3 = 343$

Here 8 and 343 are the cubes of 2 and 7 respectively.

Observe the table below and fill in the blanks:-

Table 1.4

Number	**Multiplied thrice**	**Exponential form**	**Cube number**
1	$1 \times 1 \times 1$	1^3	1
2	$2 \times 2 \times 2$	2^3	8
3	$3 \times 3 \times 3$	------	------
4	---------	------	------
5	---------	------	------
6	---------	------	------
7	---------	------	------
8	---------	------	------
9	---------	------	------
10	---------	------	------

In the above table, the numbers obtained i.e. 1, 8, 27……etc are cube numbers of integer numbers, 1, 2, 3 etc.

such numbers are known as **Perfect cubes**.

Think about the even numbers and their cubes given in the tables, what do you conclude ? Are cubes of odd numbers also odd? Are cubes of even number also even?

How will you recognise whether a given number is cube or not? To recognise Square numbers ,we had made pairs of the prime multiple factors .Those numbers that made perfect squares had complete pairs made.

We can extend the same method for cubes. We can write 8 as 2 × 2 × 2. On finding the prime multiple factors, we notice that 2 has been multiplied thrice and after a triplet there is no other multiple factor left. Let us now take a look at the number 27. Here 3 is multiplied thrice and can make a triplet. If we take 24 as the number then we shall have 24 = 2 × 2 × 2 × 3 i.e. 2 come thrice and form a triplet but 3 remains without any pair. This means 24 is not a cube number.

Thus to identify a perfect cube number ,we can conclude that if the prime multiple factors get arranged in the form of triplets of that number then the Number is perfect cube, otherwise not.

Let us take few more examples:

Example 5. Is 216 a perfect cube number?

Solution : 216 $= \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$ (on finding in divisible multiple factors)

$= 2^3 \times 3^3$ (all the factors get arranged in triplets)

$= (2 \times 3)^3$

$= 6^3$

So here, 216 can be represented as (6^3) the cube of 6.

Example 6. Say whether the number 23625 is a perfect cube?

Solution : $23625 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} \times 7$ (on finding the prime multiple factors)
Here 23625 get arranged into triplets of 3 and 5 but not so for the multiple factor 7. Therefore 23625 is not a perfect cube.

Example 7. Find the smallest number that when multiplied by 68600 given a perfect cube?

Solution : $68600 = \underline{2 \times 2 \times 2} \times 5 \times 5 \times \underline{7 \times 7 \times 7}$

Here in the multiple factors of 68600 , 2 and 7 have made triplets but 5 is pair and the number 68600 will have to be multiplied by 5 to make it a perfect cube.

Example 8. Find the smallest number which when divided by 408375, gives the quotient as a perfect cube.

Solution : $408375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} \times 11 \times 11$

Here, 408375 has multiple factor that has triplets for 3 and 5 not for 11. Therefore, if 408375 is divided by 11×11= 121, then the quotient would be a perfect cube.

EXERCISE - 1.2

Q 1. Identify the perfect cubes in the given numbers.

(i)	125	(ii)	800	(iii)	729
(iv)	2744	(v)	22000	(vi)	832

Q 2. Find out that smallest number which when multiplied by 256 will make the product a perfect cube.

Q 3. Find out the smallest number which when multiplied by 1352 will make the product a perfect cube.

Q 4. Find out that smallest number which when multiplied by 8019will make the quotient a perfect cube.

Q 5. Find out the smallest number which when multiplied by the numbers which are not cube in Question-1 give cube numbers.

Square Root

In the beginning of this chapter, we have learnt about perfect square numbers. Let's revise it through an activity once again:

Activity 3.

Table 1.5

S.No.	Number	Prime multiple factor	The number forwhich it is a square
1.	16	$\underline{2 \times 2} \times \underline{2 \times 2}$	$2 \times 2 = 4$
2.	25	$\underline{5 \times 5}$	5
3.	36		
4.	49		
5.	64		
6.	100		
7.	144		
8.	196		

In the above activity you have seen that the square of 4 is 16, the square of 5 is 25 and that of 8 is 64. This can be also stated as follows: the square root of 16 is 4, the square root of 25 is 5and it is written as; the square root of 16 = $\sqrt{16} = 4$

The square root of 25 = $\sqrt{25} = 5$ (We indicate the square root by the symbol "$\sqrt{\ }$").

You must have noticed that the square of any natural number "n" is equal to the sum of the "n" initial consecutive odd numbers.

For example: $5^2 = 1 + 3 + 5 + 7 + 9 = 25$

Just as five (initial) consecutive odd numbers have been added to obtain the square of 5 (25); can we get the square root of 25 by subtracting the consecutive odd numbers from 25? Let's find out :

$25 - 1 = 24,\ \ 24 - 3 = 21,\ \ 21 - 5 = 16$

$16 - 7 = 9,\ \ \ 9 - 9 = 0,$

Here on subtracting the first five odd numbers from 25, the reminder obtained is zero (0). This means, the square root of 25 is 5 i.e. $\sqrt{25} = 5$.

Try to verify this process for some more perfect squares. You will find that **the number of initial odd numbers that need to be subtracted from a perfect square number to obtain zero, that number itself is the square root of the perfect square number.**

Can we verify perfect square number by this process?

You will find if the remainder on subtraction is not zero, the number is not a perfect square.

PRACTICE - 3

Orally state the square root of the following numbers:

(i) 25 (ii) 49 (iii) 64 (iv) 81 (v) 121 (vi) 144

We can find out the square root of some numbers orally but not for all. Let us discuss the method of finding out square roots.

Square root by prime multiple factors

To find out the square roots of numbers by this method we first, determine the prime multiple factors for the number given. Then we make pairs of these and taking one number from each of the pairs, multiply them to get the square root.

Example 9. Find out the square root of 441.

3	441
3	147
7	49
7	7
	1

Solution : $441 = \underline{3 \times 3} \times \underline{7 \times 7}$

∴ $\sqrt{441} = \sqrt{\underline{3 \times 3} \times \underline{7 \times 7}}$

$= 3 \times 7$ (taking one number from each pair)

$= 21$

Example 10. Find out the square root of 1296 .

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

Solution : $1296 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

∴ $\sqrt{1296} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

$= 2 \times 2 \times 3 \times 3$

$= 36$

PRACTICE - 4

Find out the square roots of the following numbers by the prime factorisation method.

(i) 289 (ii) 625 (iii) 900 (iv) 361 (v) 1764

Example 11. If the area of a square figure is 2025 square cm, find out the length of one side of the figure.

Solution : The area of the square figure = $(\text{side})^2 = 2025\ \text{cm}^2$.

∴ One side of the figure $= \sqrt{2025}$

$= \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}}$

$= 3 \times 3 \times 5$

$= 45$ cm.

Example 12. A person planted 11025 mango saplings in his garden in a way that the number of plants in each row is equal to number of rows. Find the number of rows.

Solution : Let the number of rows in the garden be 'X'

Since the total number of plants $= X \times X = X^2$

$X^2 = 11025$ or $X = \sqrt{11025}$

$= \sqrt{3 \times 3 \times 5 \times 5 \times 7 \times 7}$

$= 3 \times 5 \times 7 = 105$

Therefore, the number of rows in the garden $= 105$

EXERCISE - 1.3

1. Find out the square root of the given numbers by prime factorisation method.

(i) 361	(ii) 400	(iii) 784
(iv) 1024	(v) 2304	(vi) 7056

2. A group of boys bought 256 mangoes and distributed it among themselves. If each boy got the number of mangoes equal to the number of boys in the group, find out the number of boys in the group

Finding Square Root through Division Method.

Till now we have learnt to find out square root of perfect square numbers. In mathematics there is an interesting method by which we can find the square root of perfect square numbers and also the square root of those numbers which are not perfect square.

You know that the square root of one digit and two digit perfect square numbers are one digit numbers. Using their tables you can easily understand this.

Such as : 1 x 1 = 1, So 1 is the square root of 1

3 x 3 = 9, So 3 is the square root of 9

9 x 9 = 81, So 9 is the square root of 81

100 is the perfect square number after 81 which is a three digit number. Its square root is 10 which is a two digit number. (Do you find any pattern in digits of a number and its square root?) How will you find the square root of largest three digit number? Let's understand this by an example.

Example 13. Find the square root of 625?

Step 1. Starting from the unit place of the number 625 make pairs of numbers. A small horizontal line can be drawn to make pairs above the numbers. In this case only one pair 25 will be formed and the number 6 will remain alone.

$\overline{6}\ \overline{25}$

Step 2. Put 625 inside the division sign. Now find the largest divisor whose square cannot be greater than 6.

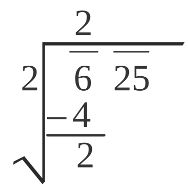
Here it is 2,

($\because$ 2 x 2 = 4 $<$ 6, 3 x 3 = 9 $>$6)

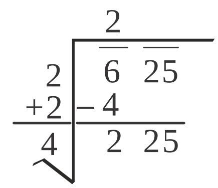
quotient

divisor $\overline{6}\ \overline{25}$ dividend

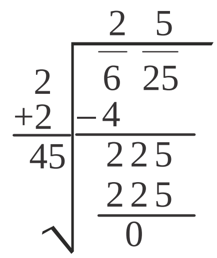
Step 3. Now placing the number 2 in divisor and quotient and subtracting their product 4 after placing it below the number 6. The Remainder 2 will come.



Step 4. Addition of the same number 2 to the divisor will result to number 4. Now write it below. After that shift the pair 25 besides the result 2 obtained from the step3. Here the new dividend will become 225.



Step 5. Now we have to put such number after 4 in the divisor and after 2 in the quotient that its product with the new divisor should not be exceed 225. if we put 3 in the divisor after 4 the new divisor will be 43.



$\therefore$ 43 x 3 = 129 < 225

Successively placing 4 and 5 in the divisor we see –

44 x 4 = 176 < 225

and 45 x 5 = 225 = 225

It is clear that taking 5. in the quotient is appropriate. Subtract the product 225 from the new dividend 225. 0 will be the Remainder. So the final quotient 25 will be the square root of 625.

i.e. $\sqrt{625} = 25$

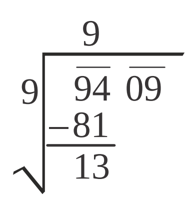
Example 14. Find the square root of 9409.

Solution –

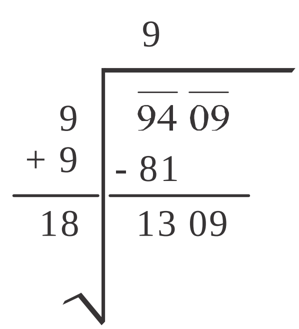
Step 1. We have given a number of 4 digit, Starting from the unit place of the number 9409 make pairs of two digit numbers. Two pairs 94 and 09 will be formed. Put them inside the division sign.

$\sqrt{\overline{94}\ \overline{09}}$

Step 2. Considering the second pair 94 as a dividend find the largest divisor whose square should not be greater than 94. It is obvious that the divisor is 9. Now placing the number 9 in divisor and quotient and subtracting their product 81 after placing it below the number 94.The Remainder 13 will come.



Step 3. Adding the same number 9 to the divisor. Write the sum 18 below it. Now write down the pair number 09 beside the remainder 13. the new dividend will be 1309. Now as shown in the previous example, what should be written beside the divisor 18 so that it's product with the new divisor become 1309, nearer or lesser.



Here we see that the divisor is a 3 digit number and the dividend is a 4 digit number. If we remove the unit place digit of both the divisor and the dividend then the divisor remains 18 and the dividend remains 130. Now it can be easily seen that 18 x 7 = 126 which is less then 130.

So we can put 7 in the divisor and the quotient.

187 x 7 = 1309

Subtracting this product 1309 from the new dividend 1309. The Remainder 0 will come. The final quotient 97 will be the square root of 9409.

	97
9	$\overline{94}\ \overline{09}$
+ 9	- 81
187	13 09
	-13 09
	0

From both the above examples we have got square roots of perfect square numbers. Now let's take an example which is not a perfect square. In such cases the square root gives digits after decimal point.

Example 15. Find the square root of 8772.

Solution – You know that 8772 can be written as 8772.0000 As in the last two examples we made pairs of 2 digit numbers starting from the unit place , the same procedure will be applied here. Digits in the unit and ten's place will make a pair and digits in the hundred and thousand's place will make a pair. In the right side of decimal points digits in the tenth and hundredth place put together to make a pair and so on.

We will write the number

8772.0000 as $\overline{87}\ \overline{72}.\ \overline{00}\ \overline{00}$ let's find out the square root of 8772 as before –

After the remainder 123 bring down the first pair of zeroes which are after the decimal points. Now put the decimal point before the number to be write in the quotient. Do the process of division in the similar way.

	93·65
9	$\overline{87}\ \overline{72}\cdot\overline{00}\ \overline{00}$
+9	–81
183	672
+3	–549
1866	12300
+6	–11196
18725	110400
	–93625
	16775

If we want square root upto two decimal places only then we can stop our procedure over here. If we have to proceed further, then put a pair of zero each time after remainder and proceed it and get new quotient. So the square root of 8772 will be approximately 93.65 i.e. approximate.

EXERCISE - 1.4

Q. 1. Find out the square root of following by division method.

(i) 529 (ii) 1369 (iii) 1024 (iv) 5776

(v) 900 (vi) 7921 (vii) 50625 (viii) 363609

Q. 2. In a cinema hall the owner wants to organize the chair in this way that the number of rows and columns of seats should be equal. If there are total 1849 seats then find out the number of rows and column.

Q. 3. The area of a square garden is 1444 square meter, so find out the length and breadth of that garden.

Example 16. Determine the square root of 51.84

Solution –

	7. 2
7	$\overline{51}.\overline{84}$
+ 7	- 49
14 2	02 84
	- 2 84
	0 00

$$\sqrt{51.84} = 7.2$$

Example 17. Determine the square root of 23.1 up to two places of decimal.

Solution –

	4. 80
4	$\overline{23}.\overline{10}\ \overline{00}\ \overline{00}$
+ 4	– 16
88	07 10
8	– 7 04
960	6 00
	–0 00
	6 00

$$\sqrt{23.1} = 4.80$$

Example 18. Determine the square root of 2 up to three places of decimal.

Solution –

Quotient: 1.41 4

Divisor	Dividend ($2.\overline{00}\;\overline{00}\;\overline{00}$)
1	2.00 00 00
+ 1	– 1
2 4	1 00
+ 4	– 96
2 81	04 00
+ 1	– 2 81
2824	1 19 00
	– 1 12 96
	0 06 04

EXERCISE - 1·5

Q. 1. Find out the squares root of following numbers.

(i) 7·29 (ii) 16·81 (iii) 9·3025

Q. 2. Find out the squares root of following up to two places of decimal.

(i) 0.9 (ii) 5 (iii) 7

Cube root

To find square root of a number we were making pair of two same number from the prime factors of that number. After that we were tarring one number from each pair and multiply them. To find cube root of any multiply, we processed this method . For cube root we trio the prime factors of that number and multiply one-one number from each trio. Obtained number will our required cube root.

Example 19. Determine cube root of 512.

Solution –

$$512 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}$$

$$\sqrt[3]{512} = 2\times2\times2$$

$$\sqrt[3]{512} = 8$$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 20. Determine cube root of 91125.

Hint - We see that the number has 5 in its unit place. So the number is completely divisible by 5.

5	91,125
5	18,225
5	3,645
3	729
3	243
3	81
3	27
3	9
3	3
	1

Solution –

$$91{,}125 = \underline{5\times5\times5}\times\underline{3\times3\times3}\times\underline{3\times3\times3}$$

$$\sqrt[3]{91{,}125} = 5\times3\times3$$

$$\sqrt[3]{91{,}125} = 45$$

EXERCISE - 1.6

Determine the cube root of following.

(i) 125 (ii) 343 (iii) 1331
(vi) 2197 (v) 9261 (vi) 166375
(vii) 4913 (viii) 42875

WE HAVE LEARNT

1. If "n" is a number, then n x n or n^2 will be known as its square and n x n x n or n^3 will be called its cube.
2. Those numbers whose unit place have numbers like 2,3,7or 8 cannot be perfect square numbers.
3. If a perfect square number ends in an even number of zeroes, then they would also be perfect squares.
4. The squares and cubes of even numbers are always **even numbers** and squares and cubes of odd numbers are always **odd numbers**.
5. The square of any natural number "n" is the sum of the initial consecutive odd numbers .
6. If three numbers are in such a sequence that the square of the greater number is equal to the sum of the square of the remaining two numbers , then the numbers are known as Pythagoral Triplets e.g. $3^2 + 4^2 = 5^2$ therefore (3,4,5) make a Pythagoral Triplet.
7. Square root is represented by the symbol "$\sqrt{\ }$". This is known as the symbol of under root or the square root of the number. The number written under this symbol is determined .

Chapter—2

EXPONENT

Exponents of Integers

Till now we have talked about exponents of natural numbers, but Fatima was thinking how she would solve the problems related to exponents of negative numbers. She thought why not write a negative number in place of the positive one and solve it for any exponent.

$(-1)^2 = (-1) \times (-1) = 1$

$(-1)^3 = (-1) \times (-1) \times (-1)$

$= \{(-1) \times (-1)\} \times (-1) = 1 \times (-1) = -1$

$(-1)^4 = (-1) \times (-1) \times (-1) \times (-1)$

$= \{(-1) \times (-1)\} \times \{(-1) \times (-1)\}$

$= 1 \times 1 = 1$

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1)$

$= \{(-1) \times (-1)\} \times \{(-1) \times (-1)\} \times (-1)$

$= 1 \times 1 \times (-1) = -1$

Kamli saw this and said, "When the exponent of (-1) is an even number, then its value is 1 and when the exponent of (-1) is an odd number, then its value is -1."

Therefore,

$(-1)^{\text{Even No.}} = 1$ and **$(-1)^{\text{Odd No.}} = -1$**

Thus Fatima & Kamli could understand that $(-1)^{25} = -1$, $(-1)^{50} = 1$

$(-1)^{143} = -1$, $(-1)^{144} = 1$ and so on.

Now think about the following :

$(-5) = (-1) \times 5$

$(-5)^4 = \{(-1) \times 5\}^4$

$= (-1)^4 \times 5^4$ [As $(a \times b)^m = a^m \times b^m$]

$= 1 \times 5^4$

$= 5^4$

$(-27)^{13} = \{(-1) \times 27\}^{13}$

$= (-1)^{13} \times 27^{13}$

$= (-1) \times 27^{13}$ $[\because (-1)^{\text{odd no.}} = -1]$

$= -27^{13}$

$(-m)^{16} = \{(-1) \times m\}^{16}$

$= (-1)^{16} \times m^{16}$ $[\because (-1)^{\text{even no.}} = 1]$

$= m^{16}$

Now think over and say which of this exponent numbers are positive and which of these are negative: $(-35)^{12}$, $(-149)^{23}$, $(-m)^{37}$, $(-m)^{100}$, $(-11)^{111}$. Can you draw any conclusion from these examination ?

You will find that if 'a' and 'm' are natural numbers, then

$(-a)^m = \{(-1) \times a\}^m = (-1)^m \times a^m$

This means

So $(-a)^m$ is positive or a negative it depends on $(-1)^m$.

Examples 1. Simplify the following :

(i) $(-5)^4 \times (-5)^7$

(ii) $(-4)^2 \times (-4)^6 \times (+4)^{17}$

(iii) $(-9)^8 \div (-9)^2$

(iv) $(-x)^7 \div (-x)^4$

Solutions:

(i) $(-5)^4 \times (-5)^7 = [(-1) \times 5]^4 \times [(-1) \times 5]^7$

$= [(-1)^4 \times 5^4] \times [(-1)^7 \times 5^7]$

$= 1 \times 5^4 \times (-1) \times 5^7$

$= -1 \times 5^{4+7} = -5^{11}$ $[\because a^m \times a^n = a^{m+n}]$

(ii) $(-4)^2 \times (-4)^6 \times (+4)^{17} = [(-1) \times (4)]^2 \times [(-1) \times (4)]^6 \times [(+1) \times (4)]^{17}$

$= (-1)^2 \times (4)^2 \times (-1)^6 \times (4)^6 \times (4)^{17}$

$= 1 \times 4^2 \times 1 \times 4^6 \times (4)^{17}$

$= 4^{2+6+17}$

$= 4^{25}$ $[\because a^{\ell} \times a^m \times a^n = a^{\ell+m+n}]$

(iii) $(-9)^8 \div (-9)^2 = \frac{(-9)^8}{(-9)^2} = \frac{\{(-1)\times 9\}^8}{\{(-1)\times 9\}^2}$

$= \frac{(-1)^8 \times 9^8}{(-1)^2 \times 9^2} = \frac{1\times 9^8}{1\times 9^2} = \frac{9^8}{9^2}$

$= 9^{8-2} = 9^6$ $[\because a^m \div a^n = a^{m-n}]$

(iv) $(-x)^7 \div (-x)^4 = \frac{(-x)^7}{(-x)^4} = \frac{\{(-1)\times x\}^7}{\{(-1)\times x\}^4}$

$= \frac{(-1)^7 \times x^7}{(-1)^4 \times x^4} = \frac{-1\times x^7}{1\times x^4}$

$= (-1) \times x^{7-4} = -x^3$ $[\because a^m \div a^n = a^{m-n}]$

EXERCISE - 2.1

1. Simplify the following:

(a) $(-5)^3$ (b) $(-4)^5$ (c) $(-2)^6$ (d) $(-3)^6$

2. Write the following in the form of exponents:

(a) $5^4 \times (-5)^2$ (b) $15 \times (-15)^{25}$

(c) $12^5 \div (-12)^3$ (d) $(-p)^{14} \div (-p)^7$

3. Verify the given statements by solving both the sides:

(a) $(-2)^4 \times (-2)^2 = (-2)^8 \div (-2)^2$

(b) $(-3)^2 \times (-3)^{-6} = \frac{1}{(3^2)^2}$

(c) $(-7)^{32} \div (-7)^{32} = 1$

Exponents of rational numbers

Razia thought that we have thought about the exponents of natural numbers & integers, but what would happen if we have rational number instead of these?

Let us find out the answers of this question .

Think about some exponents to rational numbers.

(1) $\left(\frac{5}{7}\right)^4 = \frac{5}{7}\times\frac{5}{7}\times\frac{5}{7}\times\frac{5}{7}$

$= \frac{5\times5\times5\times5}{7\times7\times7\times7} = \frac{5^4}{7^4}$

(2) $\left(-\frac{3}{11}\right)^5 = \left\{(-1)\times\left(\frac{3}{11}\right)\right\}^5 = (-1)^5\times\left(\frac{3}{11}\right)^5$

$= (-1)\times\frac{3}{11}\times\frac{3}{11}\times\frac{3}{11}\times\frac{3}{11}\times\frac{3}{11}$ [$(-1)^5 = -1$]

$= -\frac{3^5}{11^5}$

(3) $\left(-\frac{4}{3}\right)^6 = (-1)^6\times\left(\frac{4}{3}\right)^6$

$= \frac{4}{3}\times\frac{4}{3}\times\ldots\ldots 6$ times [$(-1)^6 = 1$]

$= \frac{4^6}{3^6}$

Therefore if you have a rational number $\left(\frac{5}{4}\right)^m$ then

$$\left(\frac{5}{4}\right)^m = \frac{5}{4}\times\frac{5}{4}\times\frac{5}{4}\times----(m \text{ times})$$

$$= \frac{5\times5\times\ldots\ldots m \text{ times}}{4\times4\times\ldots\ldots m \text{ times}} = \frac{5^m}{4^m}$$

Now expand this and see: $\left(\frac{3}{2}\right)^3, \left(\frac{9}{4}\right)^5, \left(-\frac{4}{7}\right)^6, \left(-\frac{2}{5}\right)^3, \left(\frac{2}{3}\right)^p$

If the exponent of a rational number $\frac{p}{q}$ (where $q \neq 0$) is m, then

$$\left(\frac{p}{q}\right)^m = \frac{p\times p\times p\times ....m\ \text{times}}{q\times q\times q\times ....m\ \text{times}}$$

$$\frac{p\times p\times p\times ....m\ \text{times}}{q\times q\times q\times ....m\ \text{times}} = \frac{p^m}{q^m}$$

Therefore $\boxed{\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}}$ (where both p and q are integers and $q \neq 0$

Now if the exponent of the rational number is negative, what will be the situation? Consider the following examples:

$$\left(\frac{5}{4}\right)^{-2} = \left(\frac{5^{-2}}{4^{-2}}\right) = \frac{1/5^2}{1/4^2} = \frac{4^2}{5^2} = \left(\frac{4}{5}\right)^2 \qquad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ and } a^{-m} = \frac{1}{a^m}\right]$$

$$\left(\frac{3}{7}\right)^{-4} = \frac{3^{-4}}{7^{-4}} = \frac{1/3^4}{1/7^4} = \frac{7^4}{3^4} = \left(\frac{7}{3}\right)^4$$

$$\left(\frac{2}{5}\right)^{-m} = \frac{2^{-m}}{5^{-m}} = \frac{1/2^m}{1/5^m} = \frac{5^m}{2^m} = \left(\frac{5}{2}\right)^m$$

PRACTICE - 1

Now try to solve the following yourself :

$$\left(\frac{7}{5}\right)^{-5}, \left(\frac{14}{13}\right)^{-9}, \left(\frac{15}{6}\right)^{-4}, \left(\frac{113}{53}\right)^{-11}, \left(\frac{5}{7}\right)^{-7}$$

Consider again

$$\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$$

Therefore,it is clear that:

$\boxed{\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m}$ where both a and b are integers and $a \neq 0, b \neq 0$.

Examples 2.

1. $\left(\frac{5}{7}\right)^4 \times \left(\frac{7}{5}\right)^2 = \left(\frac{5}{7}\right)^4 \times \left(\frac{5}{7}\right)^{-2} = \left(\frac{5}{7}\right)^{4+(-2)}$ $\left[\because \left(\frac{a}{b}\right)^m - \left(\frac{b}{a}\right)^{-m}\right]$

$= \left(\frac{5}{7}\right)^2 = \frac{5^2}{7^2} = \frac{25}{49}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

2. $\left(-\frac{2}{9}\right)^{-4} \times \left(\frac{9}{2}\right)^2 = \left(-\frac{9}{2}\right)^4 \times \left(\frac{9}{2}\right)^2$

$= (-1)^4 \times \left(\frac{9}{2}\right)^4 \times \left(\frac{9}{2}\right)^2$

$= 1 \times \left(\frac{9}{2}\right)^{4+2}$

$= \left(\frac{9}{2}\right)^6 = \frac{531441}{64}$

3. Express $-\frac{36}{49}$ in the form of exponents.

$-\frac{36}{49} = (-1) \times \frac{36}{49}$

$= (-1) \times \left(\frac{6}{7}\right)^2 = -\left(\frac{6}{7}\right)^2$

EXERCISE - 2.2

1. Simplify the following:

(a) $\left(\frac{2}{7}\right)^3 \times \left(\frac{1}{2}\right)^3$ (b) $\left(\frac{4}{5}\right)^4 \times \left(\frac{5}{4}\right)^2$

(c) $(-5)^3 \div \left(-\frac{1}{5}\right)^2$ (d) $\left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^{-5}$

2. Express in the form of exponents :

(a) $-\frac{25}{49}$ (b) $\frac{27}{125}$ (c) $\frac{729}{64}$

3. Prove :

(a) $\left(\frac{5}{7}\right)^7 \times \left(\frac{7}{5}\right)^7 - \left(\frac{3}{19}\right)^2 \times \left(\frac{19}{3}\right)^2 = 0$

(b) $\left(\frac{p}{q}\right)^m \times \left(\frac{p}{q}\right)^m \times \left(\frac{q}{p}\right)^m = \left(\frac{q}{p}\right)^{-m}$

(c) $\left(\frac{25}{16}\right)^{-4} = \left(\frac{16}{25}\right)^4$

4. Write true or false :

(a) $\left(\frac{-5}{4}\right)^{65} = \frac{-5^{65}}{4^{65}}$

(b) $\left(\frac{-32}{19}\right)^{150} = \frac{32^{150}}{19^{150}}$

(c) $(25 \times 3)^5 = 25 \times 3^5$

(d) $\left(\frac{27}{16}\right)^{-15} = \frac{27^{15}}{16^{15}}$

We have learnt

1. $(-1)^{\text{Even No.}} = 1$ and $(-1)^{\text{Odd No.}} = -1$

2. If $\frac{p}{q}$ is a rational number then $\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}$

3. If $\frac{a}{b}$ is a rational number then $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Chapter—3

PARALLEL LINES

Parallel Lines

In your previous class, you have studied about parallel lines. These are two straight lines on the same plane, the perpendicular distance between them is always the same. Extended in both directions, these lines never cross or intersect each other.

The two opposite sides of a square or rectangle, the edges of a black board, the railway track etc. are examples of parallel lines. Think about some more such examples of parallel lines and write them in your notebook.

Distance between two parallel lines:

The perpendicular distance between two parallel lines is always the same. To know this, we draw a perpendicular from any point on one line to the other line. The length of the perpendicular thus obtained is distance between the two lines.

Activity 1.

Draw two parallel lines on your notebook. Measure the distance between these two lines at different point and complete the table below.

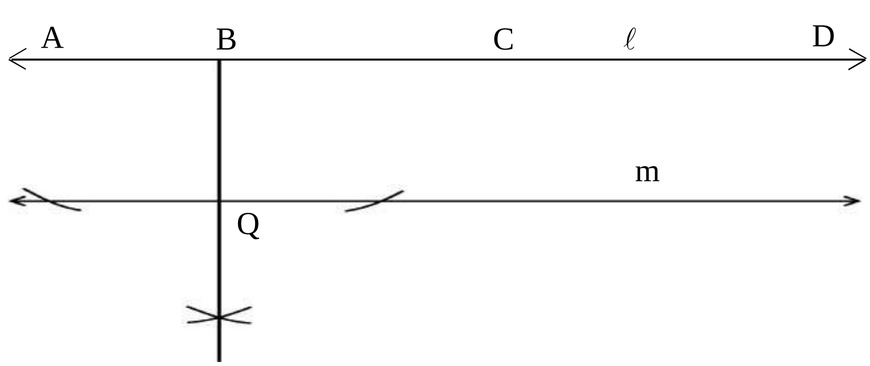


Table 3.1

S.No.	Points marked on line ℓ	Draw perpendicular from line to line making at point	Distance (in cm.)
1	A	....	....
2	B	Q.........	BQ =......
3	C	....	....
4	D	....	....

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Is the distance between the two lines are the same in all the cases?

....

To draw a parallel line at a given distance with respect to a given line.

To draw a line m parallel to line ℓ at a distance of 3cm.

Steps of construction

1. Take any point P on the given line ℓ. (Fig 3.2)

Fig 3.2

2. Draw a perpendicular $PQ \perp \ell$ (Fig 3.3) on point P.

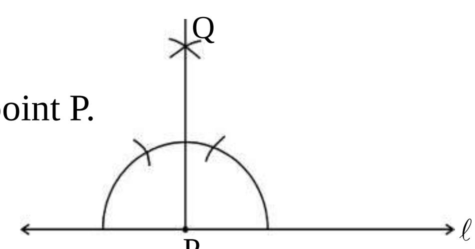

Fig 3.3

3. Taking point P as the centre, draw an arc of radius 3cm. on PQ which cuts PQ at R (Fig 3.4)

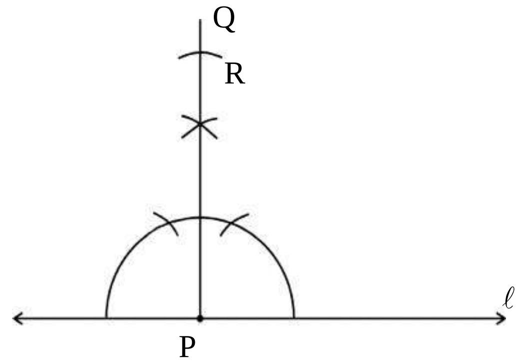

Fig 3.4

4. Draw RS⊥PR at point R and extend RS as line m . (Fig 3.5)

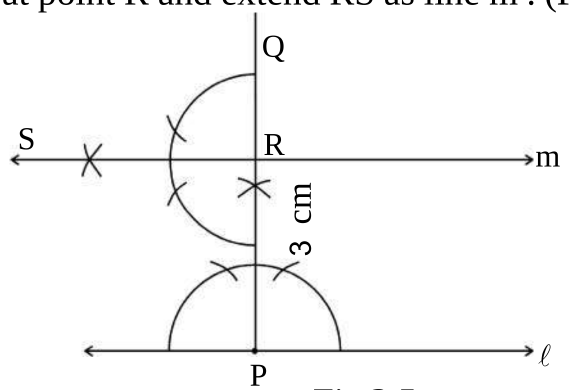

Fig 3.5

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Thus line m, is a parallel line to ℓ at a distance of 3cm.

Note : Parallel lines at a given distance can be drawn with the help of set square also.

Some characteristic features of parallel lines

The perpendicular drawn at two points on the same line are parallel.

Activity 2.

Draw a line ℓ and take any two points A & B on it draw perpendicular AM from point A and Perpendicular BN from point B on line ℓ.

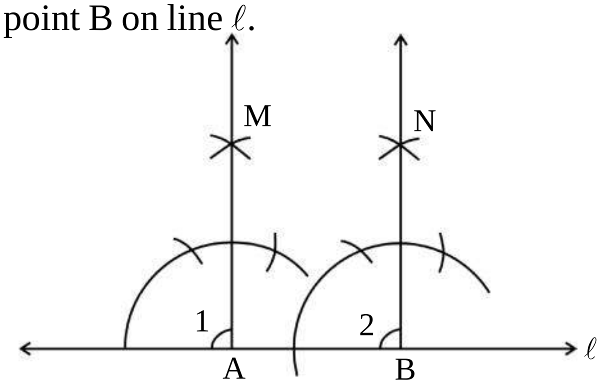


Fig 3.6

Ask your friends to draw two perpendicular on line in their notebooks and fill up the given table by measuring the adjacent angles in their figures.

Table 3.2

S.No	Name	Ď1	Ď2	Is Ď1 = Ď2 ?
1.	Mohan	------	------	------------
2.	------------	------	------	------------
3.	------------	------	------	------------
4.	------------	------	------	------------

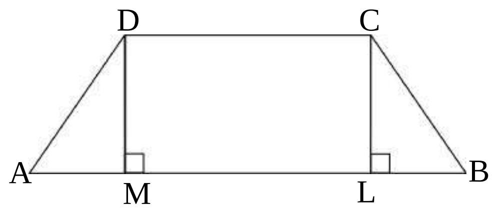
We see that in each situation , $\angle 1 = \angle 2$, since these are corresponding angles, therefore lines AM and BN are parallel lines. Thus on the same plane, the perpendiculars drawn on two points of a line are parallel to each other.

Practise - 1

1. Draw a line and construct a line parallel to it at a distance of 3 cm.

2. Draw a line and construct a line parallel to it at a distance of 4.3 cm. How may maximum number of lines like this can be drawn parallel to a line?

3. ABCD is a parallelogram where AB|| CD, CL⊥AB and DM⊥AB. Is CL||DM ? what kind of a quadrilateral is DMLC? What kind of triangles are ΔADM and ΔLCB.



Two lines parallel to a given line are parallel to each other.

Activity 3.

In the figures below m and n are lines parallel to line ℓ and t is an inclined line intersecting these lines. Now, measure the angle in the figures and complete the table that follows.

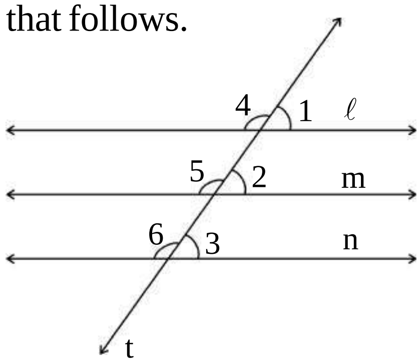


Fig 3.7

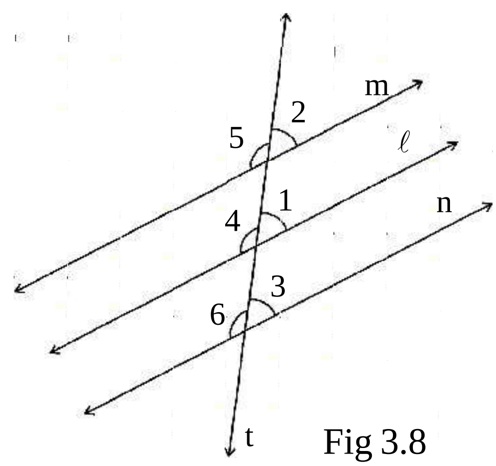


Fig 3.8

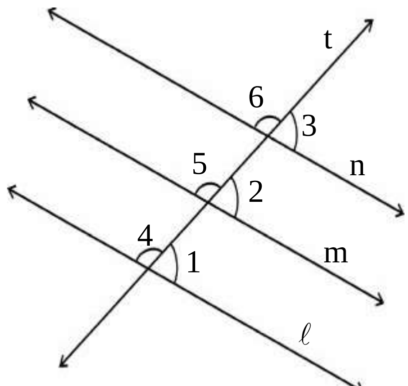


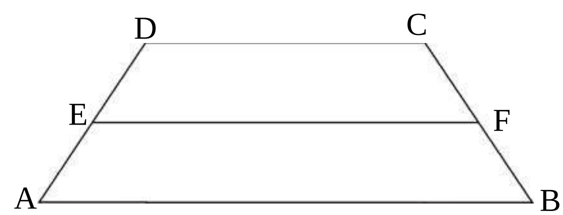
Fig 3.9

Table 3.3

Fig. No.	**Measure of angles (in degree)**							
	Ď1	Ď2	Ď3	Ď4 =	Ď5	Ď6	**Is** Ď2=Ď3?	**Is** Ď5=Ď6?
3.7								
3.8								
3.9								

In the figure, we can see that $\angle 2 = \angle 3$, and $\angle 5 = \angle 6$, but these are corresponding angles. Therefore lines *m* and *n* are parallel to each other i.e. lines drawn parallel to a given line are all parallel to each other.

Practise - 2



1. In the figure ABCD is a trapezium. Where AB||DC segment EF||AB and E and F are points on AD and BC respectively. Is EF||DC, if yes then why?
2. How many trapezium are there in this figure? Name them.

Intercepts

When two straight lines are intersected by any inclined line, then the intersected part between the two straight lines is known as the intersecting segement. In the figure AB line ℓ & *m* are intersected by the line *n* and the intersected part AB is known as the intercept because it is inside the space between the two lines.

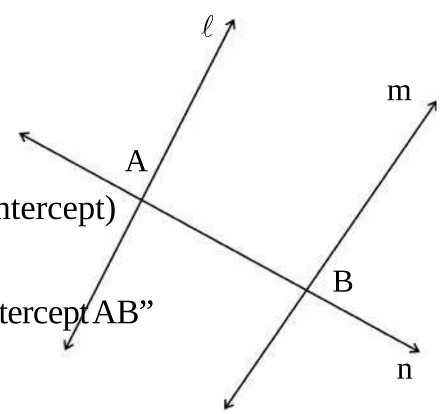


Fig 3.10

It is not necessary that the two lines ℓ & ***m*** are parallel to each other.

Practise - 3

1. In the given figures identity the intercepts and fill in the blanks.

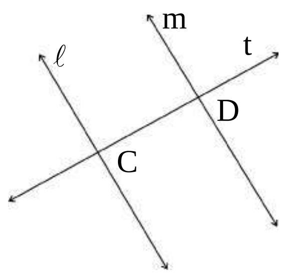


Fig 3.11

intercept ———————————

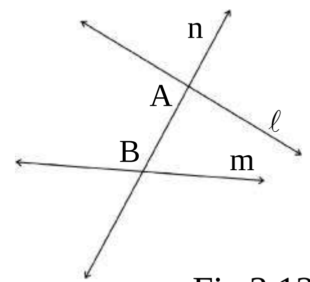


Fig 3.12

intercept ——————————

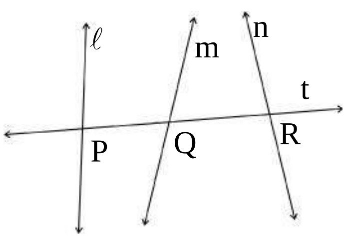


Fig 3.13

intercept ———————————

Parallel lines and equal intercepts

Activity 4.

In the given figures the line P contains there points A,B,C in a way that AB=BC. Three lines ℓ, m & n have been drawn passing through these three points intersecting there. Three lines are inclined by line *t* which intersect the lines ℓ, m, and n at points D, E and F respectively.

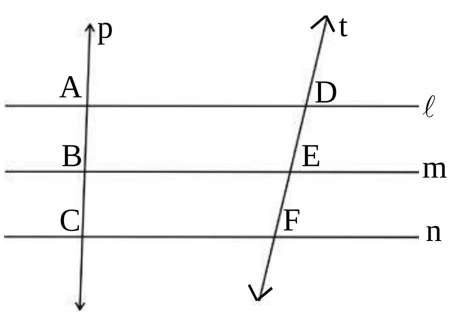


Fig 3.14

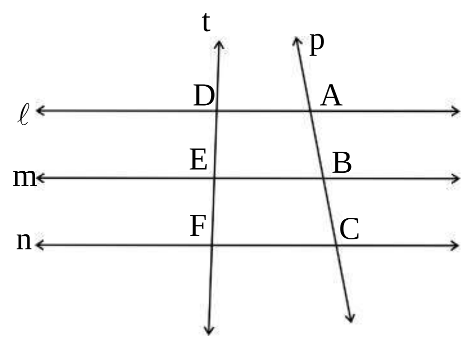


Fig 3.15

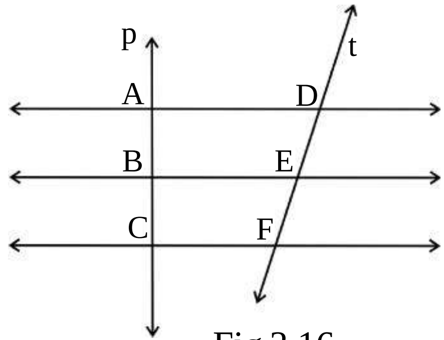


Fig 3.16

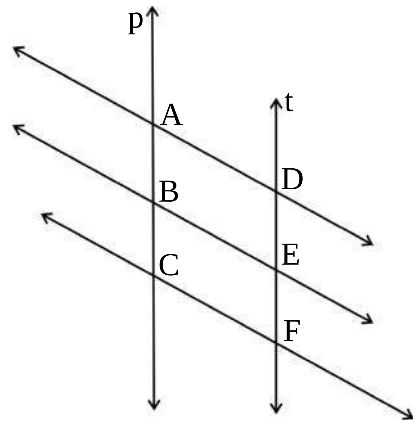


Fig 3.17

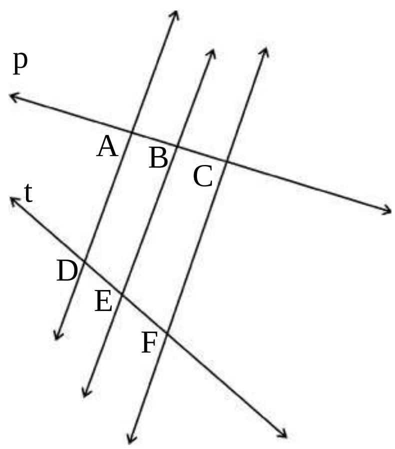


Fig 3.18

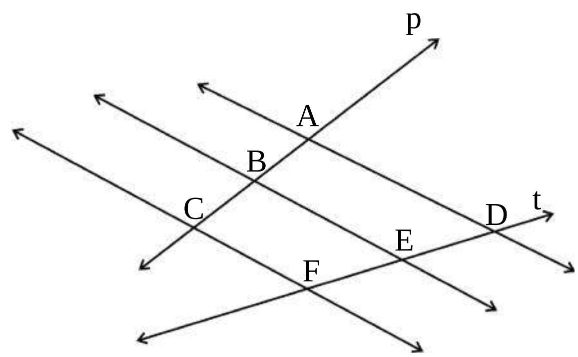


Fig 3.19

Table 3.4

S.No	DE	EF	Is DE = EF ?
3.14			
3.15			
3.16			
3.17			
3.18			
3.19			

In the above activity, you have seen that in every situation DE = EF.

Therefore, we can conclude that if a transversal line intersecting **"three parallel lines produces equal intercept then the other transversal will also produce equal intercept"**.

Activity 5.

In the given figures $\ell \parallel m \parallel n$ and the intersecting lines 's' and 't' crosses these parallel lines at A, B, C and D, E, F. Use scale to measure the sections given in the table and complete the information asked for.

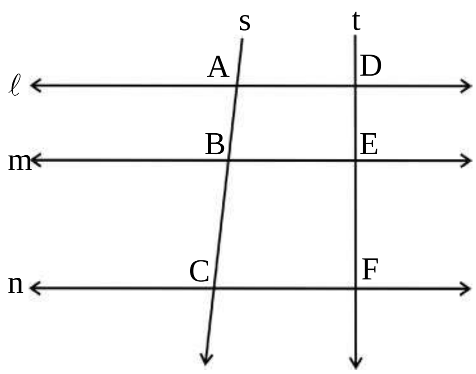


Fig 3.20

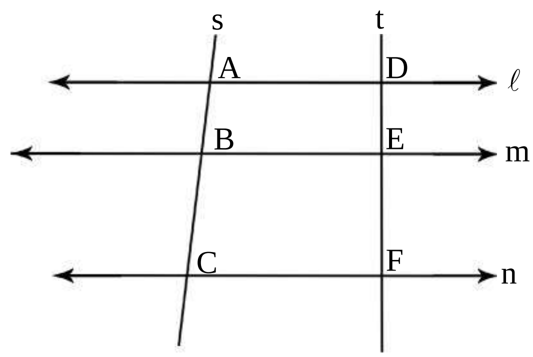


Fig 3.21

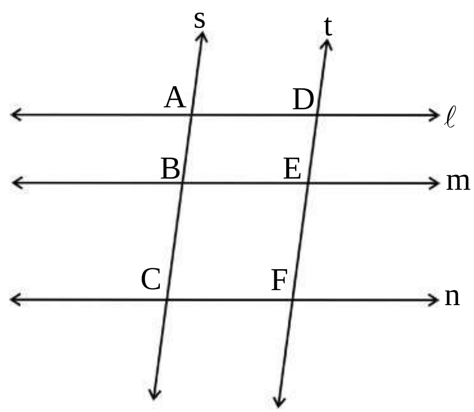


Fig 3.22

Table 3.5

S.No.	Fig. No.	AB	BC	$\frac{AB}{BC}$	DE	EF	$\frac{DE}{EF}$	Is $\frac{AB}{BC} = \frac{DE}{EF}$?
1.	3.20							
2.	3.21							
3.	3.22							

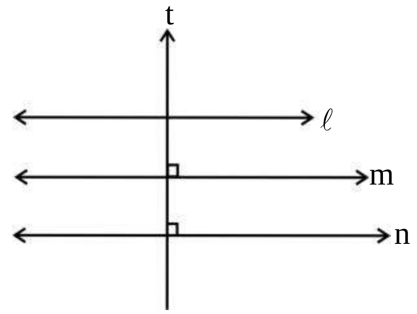
In the above activity in all the situations, we get $\frac{AB}{BC} = \frac{DE}{EF}$.

Therefore **if three parallel lines make equal intercepts on one transversal, then they make equal intercepts on any other transversal as well**.

EXERCISE - 3.1

1. Draw a line segment of 6 cm and taking any point P, draw a parallel line at a distance of 2.5cm .

2. In the given figure $\ell \parallel m$, $t \perp m$ and $t \perp n$, then

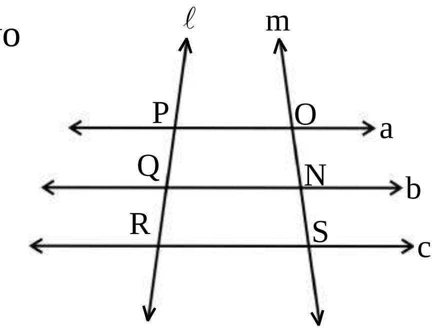


   (i) Is $m \parallel n$? Why?

   (ii) Is $\ell \parallel n$? Why?

   (iii) Is $t \perp \ell$? Why?

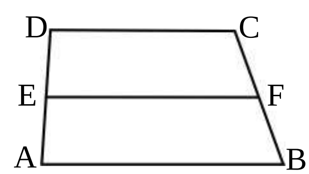
3. In the given figure $a \parallel b \parallel c$ and ℓ & *m* are two transversals. If PQ = QR, is ON= NS ? Why?



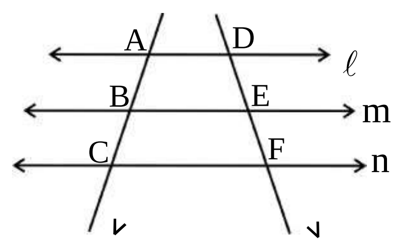
4. In the given figure $AB \parallel DC$, $EF \parallel AB$ and E is the midpoint for the line segment AD, then

   (i) Is $AB \parallel EF \parallel DC$? Why ?

   (ii) Is F, the midpoint for the line segment CB? Why?



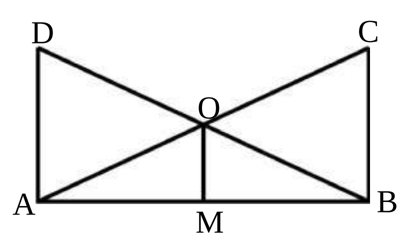
5. In the given figure $\ell \parallel m \parallel n$, will the ratio of the intercepts be equal ?



6. If line segments DA,CB and OM perpendicular on segment AB, where O is the intersecting points for line segments AC and DB, If OA =2.4 cm and OC =3.6 cm, then find out.

   (i) $\dfrac{AM}{BM}$

   (ii) If BO =3cm, find the value of DO.



Line drawn parallel to one side of a triangle

Activity 6.

In the given triangle ABC, DE|| BC that intersects AB on point D and AC on point E. with the help of the figures below measure the features asked for the table and complete it.

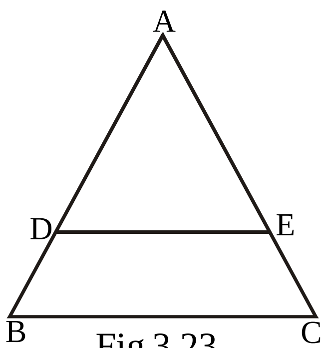


Fig 3.23

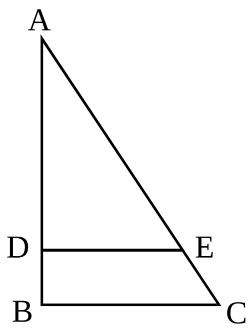


Fig 3.24

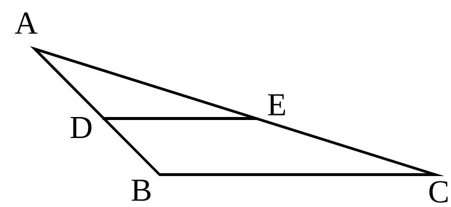


Fig 3.25

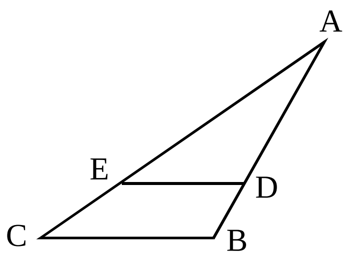


Fig 3.26

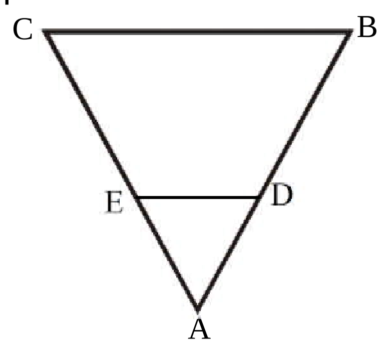


Fig 3.27

Table 3.6

S.No.	Fig. No	AD	DB	$\frac{AD}{DB}$	AE	EC	$\frac{AE}{EC}$	Is $\frac{AD}{DB} = \frac{AE}{EC}$?
1.	3.23							
2.	3.24							
3.	3.25							
4.	3.26							
5.	3.27							

In the above activity, every situation gives us sufficient proof for $\frac{AD}{DB} = \frac{AE}{EC}$.

Therefore a line drawn parallel to one side of a triangle divides the other two sides in equal ratio.

Relationship between the line joining the midpoint of two sides of a triangle & its third side.

Activity 7.

In the given triangles ΔABC ,the midpoints of AB and AC are D and E respectively. Measure the angels formed on DE and at points B & C and complete the table below.

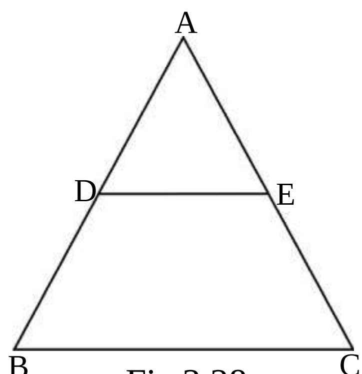


Fig 3.28

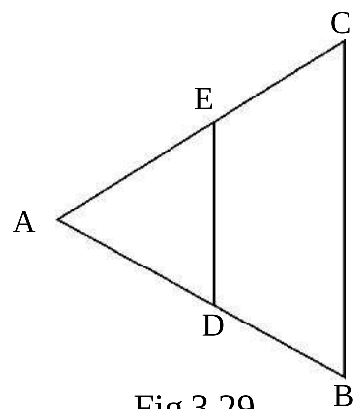


Fig 3.29

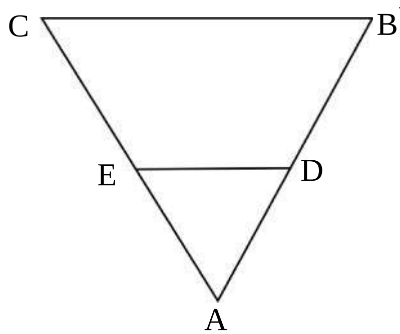


Fig 3.30

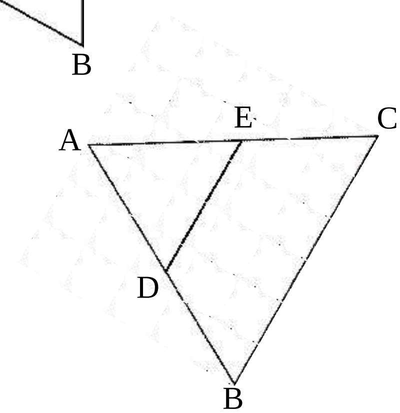


Fig 3.31

Table 3.7

S.No	Fig No	∠ADE	∠B	Is ∠ADE = ∠B?	∠AED	∠C	Is ∠AED = ∠C?
1.	3.28						
2.	3.29						
3.	3.30						
4.	3.31						

In the above activity you find that ∠ADE = ∠B and ∠AED = ∠C. Observe the angles carefully again ?What are the names of these angles?

When corresponding angles are equal, what is the relationship between the lines?

Therefore,

The line joining the midpoint of two sides of a triangle is parallel to its third side.

Division of a line segment into equal parts

Salma said to Ashok, "Can you divide a 5 cm. line into three equal parts?

Ashok said " Why not?" He divided 5 by 3 and 5/3 =1.66 cm was obtained But when he started measure 1.66 cm on the scale, he could not do so, he took two divisions of 1.6 cm each and the third part that remained was 1.8 cm.

Salma said, oh it is not possible to divide a line segment into desired divisions with the help of a scale. They thought there must be some way to divide a line segment into as many parts as you like without measuring them.

Let's find out how we can use parallel lines to divide a line segment into many equal parts without using a scale.

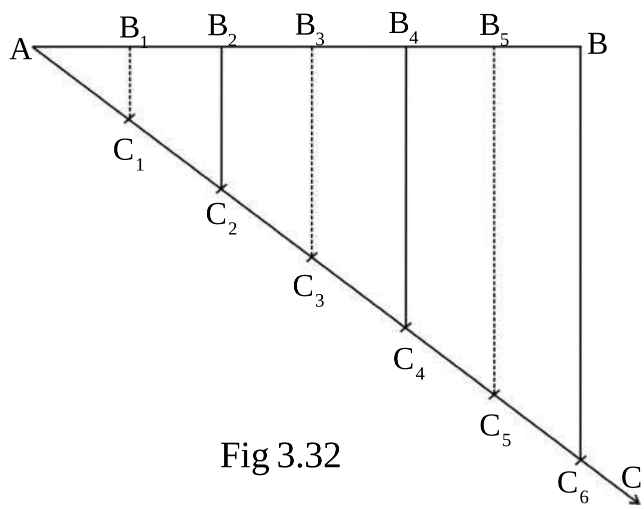


Fig 3.32

Example 1. To divide line segment AB into 6 equal parts.

Steps of the construction :

1. Draw a ray AC on point A of line segment AB, making an acute angle.

2. Now cut 6 equal parts from point A of ray AC as AC_1, C_1C_2, C_2C_3,...,C_5C_6 by using rounder.

3. Join C_6 and B and draw parallel lines in the reverse order C_5, C_4,....C_1 that meet at points $B_5B_4B_3$...B_1 on AB.

Thus segment AB gets divided into 6 equal parts as AB_1, B_1B_2, B_2B_3,B_3B_4, B_4B_5,B_5B, that make the 6 equal parts.

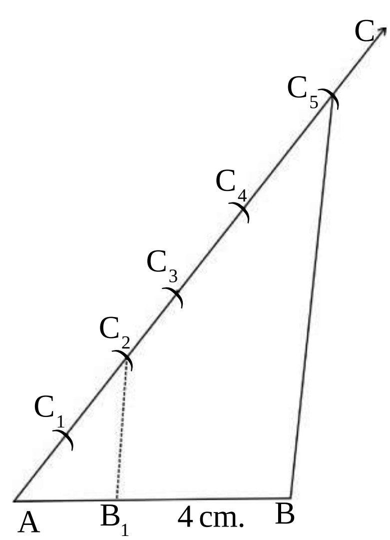


Fig 3.33

Example 2. Take a line segment of 4 cm and divide it in the ratio of 2:3 cm.

Steps of the construction :

1. Draw a line segment of 4cm. & then a ray AC that makes an acute angle on line AB.

2. Draw equidistance arcs AC_1, C_1C_2, ...C_4C_5 of the same radius on ray AC with the help of compass mark 5 equal parts in it that is the sum of the ratio (2+3=5)in cm.

3. Now join BC_5 and draw a line parallel to BC_5 at point C_2 that intersects line segments AB on B_1 Thus the line segments of desired ratio AB_1 and B_1B were obtained, which mean $AB_1 : B_1B = 2 : 3$

Thus taking different line segments of different measures and divide them in desired ratios and ask your friends to do the same.

<u>Another method to divide a line into equal parts</u> –

Harish was enjoying the division of segments but sometimes, he was facing problems in having the parallel lines. Let us look at another method of dividing a line segments into equal parts. This method is also very easy.

Example 3. Draw line segment AB and divide into four equal parts.

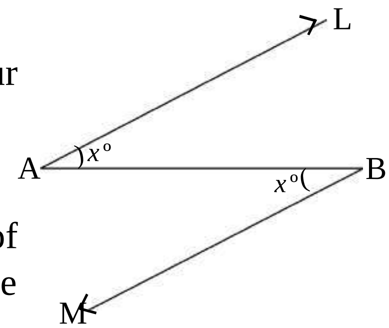


Fig 3.34

Construction 1. Draw a line AB and draw acute angles of equal measure at both its ends in the opposite directions.

Remember that the angles should be equal and acute.

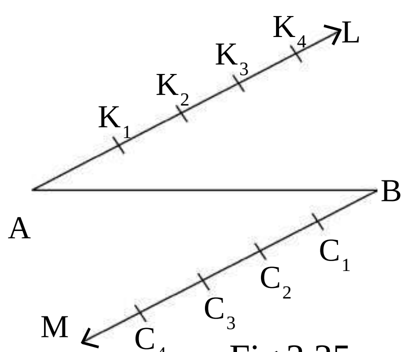


Fig 3.35

Construction 2. With the help of compass cut 4 - 4 arcs of equal radius on both the rays AL& BM .

Construction 3. Join the last point K_4 & C_4 to points B & A respectively.

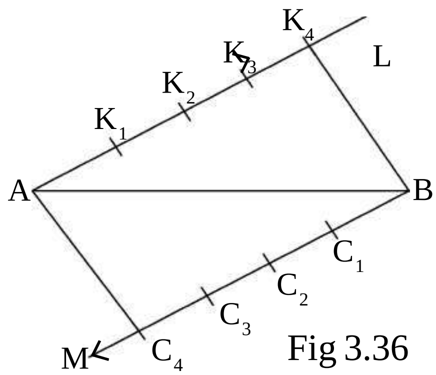


Fig 3.36

Construction 4. Now join K_3 to C_1 and K_2 to C_2 & K_1 to C_3 .

Thus we get Three points D_1, D_2, D_3 on AB which divides the segment into four parts equally.

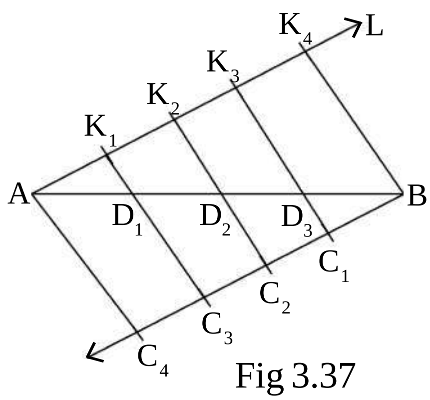
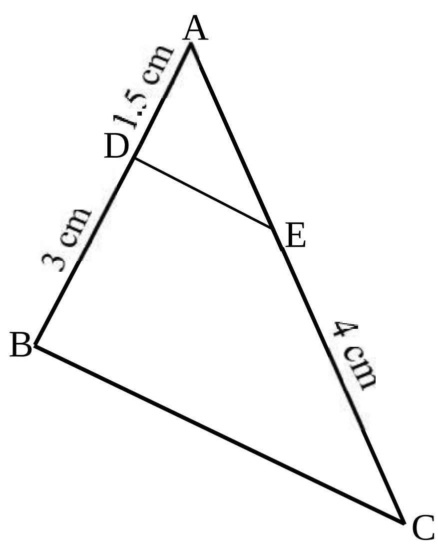


Fig 3.37

EXERCISE - 3.2

1. In the given figure DE|| BC, if AD=1.5 cm, DB= 3cm & EC =4 cm then find out the measure of AE.



2. Draw a line segment AB of 7.5 cm and divide it into 3 equal parts. Measure the length of each part.

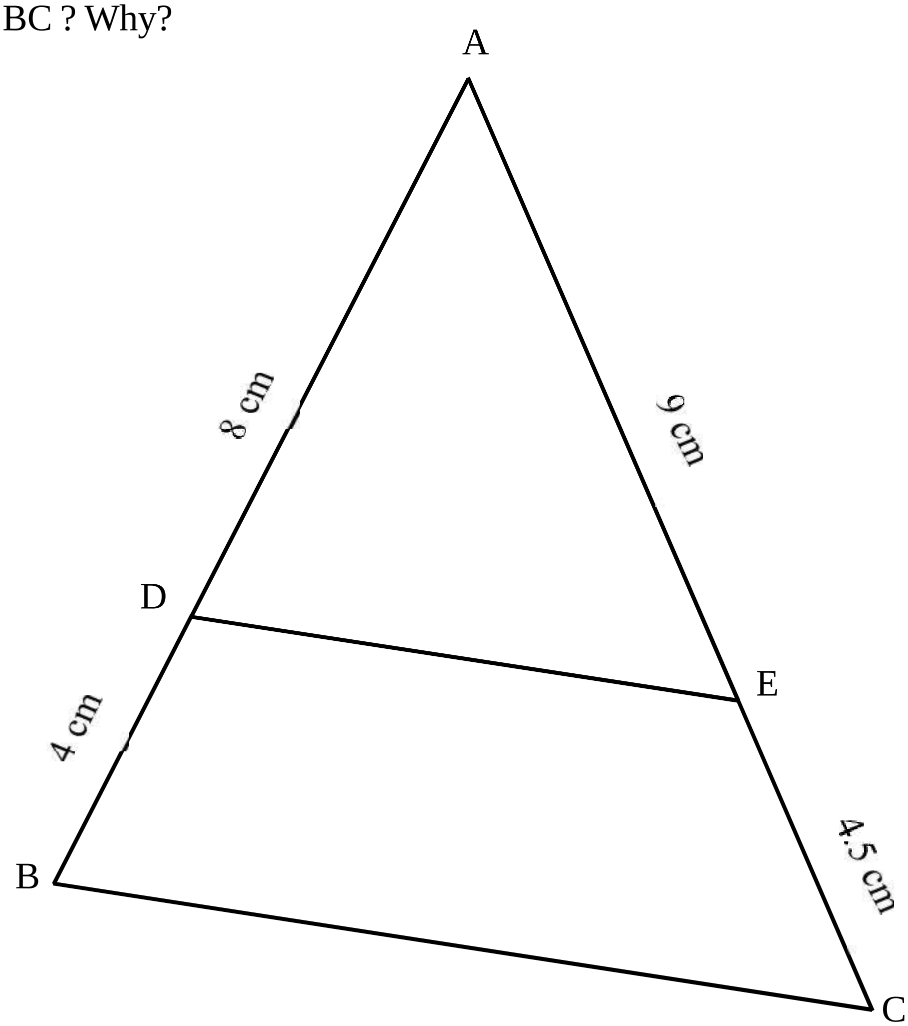
3. Draw a line segment of 8.4 cm & divide it into seven equal parts. Measure the length of each part.

4. Divide a line segment of 10 cm into ratio of 2:3.

5. Draw a line segment AB of 7cm. Find a point P on this in such a way that

$$AP = \frac{2}{5}AB.$$

6. In the given figure AD = 8cm BD=4 cm and AE =9cm EC=4.5 cm. Is DE|| BC ? Why?



WE HAVE LEARNT

1. The perpendicular distance between two parallel lines is always equal .
2. All the lines drawn parallel to a line are parallel to each other.
3. The Perpendicular drawn on different points on the same line are parallel to each other.
4. When a transversal line intersects three parallel lines to produce equal intercepts then other intersecting lines would also produce equal intercepts.
5. The ratio of intercepts on parallel lines by two intersecting lines are equal.
6. The line parallel to one side of a triangle divides the other sides in equal ratio.
7. The line joining the midpoints of the two sides of a triangle is parallel to its third side.

Chapter—4

MULTIPLICATION & DIVISION OF ALGEBRAIC EXPRESSIONS

You are already acquainted with addition and subtraction of algebraic expressions. In the process of addition & subtraction integer numbers are added & subtracted respectively and the algebriac remains the same. Similarly in class VII you have studied – On multiplying any two algebraic expressions, their constants are multiplied with the constants and the variables are multiplied with the variables.

Activity 1.

In the table given below, algebraic expressions as the two variables and their products are shown. Some blank spaces are given in the table. Fill in the blanks with correct values.

Table 4.1

S.No	First Expressions	Second Expressions	1st Expressions x 2nd Expressions	2nd Expressions x 1st Expressions	Product
01	-3	a	-3.a	a.(-3)	-3a
02	x	5	x.5	5.x	5x
03	2a	3a	2a.3a	3a.2a	$6a^2$
04	7x	-4y	--------------	--------------	-------------
05	-5xy	2x	--------------	--------------	-------------
06	4a2	------------	--------------	--------------	$-12a^2b$
07	$-7a^2b^2$	8ab	--------------	--------------	-------------

Here we can see in table the product value remains the same even if the place of the expression are exchanged. From this which rule about multiplication does this observation satisfy?

Let us look at a few more examples :

1. $3x \cdot 5x = (3 \cdot 5)x \cdot x = 15x^2$

2. $(-4x)6y = (-4 \times 6)x.y = -24xy$

3. $(-ab)5b^2 = (-1 \times 5)ab.b^2 = -5a.b.b^2 = -5ab^3$

Thus, we can see that when the base is the same, the exponents get added according to the rule of exponential numbers.

While adding a algebriac expression you have seen that the factors get added to each other. For example : $x+x=(1+1)x=2x$ (here is the multiple factor of x)

Similarly 2x is obtained by adding x to himself twice.

Thus $\quad x+x+x \quad =3x$

$\quad x+x+x+x=4x$

So the multiple factor of x becomes the numbers that is the number of times it is added and hence in 2x, 2 is the product factor and x is the variable.

Therefore, for different values of 'x' the value of 2x will be different.

If $x=3,$ than $2x=2.(3)=06$

If $x=-5,$ than $2x=x.(-5)=-10$

If $x=0,$ than $2x=2.(0)=0$

If $x=\frac{3}{8},$ than $2x\frac{3}{8}=\frac{3}{4}$

Fill in the blanks in the table below :-

x		$2x$
1	→	2
3	→	-----
2	→	-----
8	→	-----
7	→	-----

some time while calculating it goes wrong If $x=5,$ thus $2x$ will not be 25, $2x\neq 25$ but $2x=2\times5=10$ is correct.

One day the teacher asked Neeraj in the class,

Teacher : "What is your age?"

Neraj : I am 13 years old.

Teacher : What will be your age after 2 years ?

Neraj : After 2 years my age will be 13 + 2 = 15 years.

Teacher : How old are you, Jeetendra ?

Jeetendra : I am nearly 12 years old.

Teacher : What will be your age after 2 years ?

Jeetendra : After 2 years, I will be 12+2 = 14 years old.

Teacher : If a person is x year old, then after 2 years. How old would he be ?

Manisha answered that after 2 years, the person would be $(x+2)$ years old.

If we keep the value of x different, then the value of $(x+2)$ will also be different.

If $x=3$, then $x+2=3+2=05$ years

If $x=8$, then $x+2=8+2=10$ years

If $x=5$, then $x+2=5+2=07$ years

Fill in the blanks in the table below -

$2 + x = x + 2$

1	→	3
7	→	-----
12	→	-----
20	→	-----
31	→	-----

Thus, we can see that 2 x represent twice of x, while $2 + x$ means a value that is 2 more than x.

If $x=0$, then $2x=2\times 0=0$

If $x=0$, then $2+x=2+0=2$

Therefore $2x \neq 2+x$

Multiplying monomial expression with polynomial expression.

In class VII, we have learnt how to multiply -

Any monomial algebraic expression to binomial algebraic expression.

Let us revise the multiplication of monomial expression with binomial expressions.

Once again by activity.

Activity - 2

In the table given below the product of monomial expressions, with binomial expressions is shown. Below are given some blank spaces fill these up.

Table 4.2

S.No.	Monomial Expression	Binomial Expression	Monomial×binomial Expression	Product
1.	x	a + b	x(a + b)	ax + bx
2.	−4y	3a + b	----------	----------
3.	xy	7 + 8x	xy(7 + 8x)	----------
4.	$2t^2$	$3r^2 - 55$	----------	----------
5.	$\frac{1}{2}m$	$m^3 + \frac{3}{2}n$	----------	----------
6.	4a	$5x - \frac{1}{2}y$	----------	----------

Similarly we can multiply any monomial expression by polynomial expression.

Or a (b + c + d) = ab + ac + ad

(b + c + d) a = ba + ca + da

Similarly a (b + c + d + e) = ab + ac + ad + ae

Or (b + c + d + e) a = ba + ca + da + ea

Example 1. 2a (a + 2b + 5c) = 2a· a + 2a · 2b + 2a · 5c

= $2a^2$ + 4 ab + 10ac

Example 2. (2q + r + 3s - t) p = 2q · p + r · p + 3s · p + t p

= 2 pq + pr + 3 ps + pt

Example 3. $(xy + 2y^2z + x^2)\ yz^2$ = $xy \cdot yz^2 + 2y^2z \cdot yz^2 + x^2 \cdot yz^2$

= $xy^2z^2 + 2\ y^3z^3 + x^2\ yz^2$

Activity 3

Fill in the blanks -

Table 4.3

S.No.	Multiplicatoin of algebraic expression	Process of multiply	Product
1.	(2a +b + c) 5d	2a × 5d + b × 5d + c × 5d	10ad + 5 bd + 5 cd
2.	$7a^2$ (b + 2d - t)	....	....
3.	····· $(x^2 + xy + z)$	$p \times x^2 + p \times xy + p \times z$	$p\ x^2$ + ····················
4.	–5 m (....+....+ b)	....	$-5m^2 - 10mn - 5mb$
5.	$7p^2m\ (m + n^3 + p)$	....	....

Let us think of multiplication of two binomial expression -

Multiplication of two binomial expressions

Multiplication of two binomial expression is equal to the sum of product of two monomial with polynomial expression.

$$(a+b)(c+d) = a(c+d)+b(c+d)$$

$$= (ac+ad)+(bc+bd)$$

$$= ac+ad+bc+bd$$

we can also solve this in the following way

$$(a+b)(c+d)$$
$$= (a+b)c+(a+b)d$$
$$= ac+bc+ad+bd$$

In this process the product using the distribution property of multiplication over addition used twice.

Example - 4

Multiply $(5x+3y)$ and $(4x+5y)$ to each other.

Solution :

$$(5x+3y)(4x+5y) = 5x(4x+5y)+3y(4x+5y)$$

[using $(a+b)(c+d) = a(c+d)+b(c+d)$]

$$= 5x.4x+5x.5y+3y.4x+3y.5y$$

[using $a(b+c) = ab+ac$]

$$= 20x^2+25xy+12yx+15y^2$$

$$= 20x^2+37xy+15y^2$$

This can also be solved in the following manner :

$$(5x+3y)(4x+5y) = 5x+3y.4x+(5x+3y).5y$$

[using $(a+b)(c+d) = (a+b)c+(a+b)d$]

$$= 5x.4x+3y.4x+5x.5y+3y.5y$$

[using $(a+b).c = ac+bc$]

$$= 20x^2+12yx+25xy+15y^2$$

$$= 20x^2+37xy+15y^2$$

Example - 5

Multiply $(3s^2+2t)$ and $(2r^2+5t)$

Solution :

$$(3s^2+2t)(2r^2+5t)=3s^2.(2r^2+5t)+2t.(2r^2+5t)$$

[using $(a+b)(c+d)=a(c+d)+b(c+d)$]

$$=3s^2.2r^2+3s^25t+2t.2r^2+2t.st$$

[using $a(b+c)=ab+ac$]

$$=6s^2r^2+15s^2t+4tr^2+10t^2$$

Example - 6

Multiply $(5x+3y)$ to $(x+y)$ and verify the product for $x=3,\ y=-2$

Solution : $(5x+3y)(x+y)\ =5x(x+y)+3y(x+y)$

$$=5x.x+5x.y+3y.x\quad 3y.y$$

$$=5x^2+5xy+3xy+3y^2$$

$$(5x+3y)(x+y)\ =5x^2+8xy+3y^2$$

Verification L.H.S. $=(5x+3y)(x+y)$

$$=\{5(3)+3(-2)\}(3-2) \qquad \text{(when } x=3,\ y=-2\text{)}$$

$$=(15-6)(1)$$

$$=9\times1=9$$

R.H.S. $=\ 5x^2+8xy+3y^2$

$$=5(3)^2+8(3)(-2)+3(-2)^2$$

$$=5(9)-48+3(4)$$

$$=45-48+12$$

$$=45-48+12$$

$$=57-48=9$$

It is clear that

L.H.S. = R.H.S.

Therfore the product is correct.

Activity -4

With the process of multiplication fill in the blanks in the table given below -

Table 4.4

Multiplication of two algebraic expressions	Process of Multiplication		Product obtained
	Using the distributive law	Using the distributive law the second time	
1. (a + b) (c + d)	a (c + d) + b (c + d) or (a+b)c + (a+b) d	ac + ad +bc +bd or ac + bc+ad+bd	ac+ad+bc+bd or ac+bc+ad+bd
(a) (4x+5y) (2x +3y)	4x(2x+3y)+5y (2x+3y)	4x×2x+4x×3y+5y×2x+5y×3y	$8x^2+22xy+15y^2$
(b) $(5x^2+2s)$ (2t +5)	....	....	....
(c) $(2r^2+5s^3)$ (r^2+t^3)	....	....	....
2. (a + b) (c – d)	a (c – d) + b (c – d)	ac – ad +bc – bd	ac–ad+bc–bd
(a) (b+2c) (3b – c)	....	....	....
(b) (5x+3y) $(2y^2 - z)$	....	....	....
3. (a – b) (c + d)	a (c + d) – b (c + d)	ac + ad – bc – bd	ac+ad–bc–bd
(a) (2x–3y) (3x+z)	....	....	....
(b) (5p–2q) (3x+4s)	....	....	....
4. (a – b) (c – d)	a (c – d) – b (c – d)	ac – ad – bc+bd	ac–ad–bc+bd
(a) (2s–3p) (4x–5t)	....	....	....
(b) (x^2+xy) (y^2-z) (y^2-z)	....	....	....

Exercise 4.1

Q. 1. Multiply the given expressions to each other

(i) $(2x+7)(3x+2)$ (ii) $(3x-5)(2x+9)$ (iii) $(7x-6)(15x-2)$

(iv) $\left(\frac{1}{2}x+5y\right)\left(3x-\frac{6}{5}y\right)$ (v) $(x+5y)(7x-y)$

Q. 2. Find the values

(i) $(x+y)(2y+3x)+(3x+y)(y+2x)$ (ii) $\left(2x+\frac{1}{2}\right)\left(\frac{3x}{2}-\frac{1}{4}\right)$

(iii) $(x^2+y^2)(3x-5y)$ (iv) $(a+b)(a+b)$

Q. 3. Multiply $(x+y)$ and $(3x+4y)$ to each other & verify the product for the following values :

(i) $x=2, \quad y=-1$, (ii) $x=1, \quad y=0$

DIVISION OF ALGEBRAIC EXPRESSIONS

You know how to multiply a whole number and another whole number & also how to divide them. Let us see some examples ?

1. If $6\times8=48$, then $48\div8=6$ and $48\div6=8$
2. If $-15\times3-45$ then $-45\div-15=3$ and $-45\div3=-15$
3. If $m\times n=mn$ then $mn\div m=n$ and $mn\div n=n$

Activity - 5

Fill in the blanks in the following table -

Table 4.5

S.No	1st Number x 2nd Number	Product of the two numbers	Exhibiting the process of Division	
			1st Expressions	2nd Expressions
01	$3x\times4y$	$12xy$	$12xy\div3x=4y$	$12xy\div4y=3x$
02	$2x(-7x)$	$-14x^2$	--------------	--------------
03	$m\times4n$	$4mn$	--------------	--------------
04	$18a^2\times2b^2$	--------------	--------------	-------------
05	$13p^2\times7pq$	$91p^3q$	--------------	--------------

Thus we note that on multiplying $3x$ by $4y$ we get $12xy$ and on dividing $12xy$ by $3x$, we get $4y$ and if $12xy$ is divided by $4y$, we would get $3x$. Hence multiplication and division process are opposite to each other.

Division of a monomial expression by another monomial expression -

Let us know how to divide a monomial expression by another monomial expression.

Example - 7

Divide $18x^2y$ by $6xy$

Solution :

Here $18x^2y\div6xy=\dfrac{18x^2y}{6xy}$

$$=\frac{18}{6}\times\frac{x^2}{x}\times\frac{y}{y}=3\times\frac{x\times \cancel{x}}{\cancel{x}}\times\frac{\cancel{y}}{\cancel{y}}$$

$=3x$

Example - 8

Divide $-35mn^2p$ by $7np$

Solution :

$$-35mn^2p \div 7np$$

$$= \frac{-35mn^2p}{7np}$$

$$= \frac{-35}{7} \times \frac{m}{1} \times \frac{n^2}{n} \times \frac{p}{p}$$

$$= -5 \times m \times \frac{n \times \cancel{n}}{\cancel{n}} \times \frac{\cancel{p}}{\cancel{p}}$$

$$= -5mn$$

So, you have observed that the process of division is taken up in the following steps.

1. If the sign of the divisor and the dividend are same, the sign of the quotient is positive.
2. If the sign of the divisor and the dividend are different the quotient will have a negative sign.
3. The multiple factor of the dividend is divided by the multiple factor of the division.
4. To find the value of the exponent of any variable in the quotient the exponential law $a^m \div a^n = a^{m-n}$ is used. Let us take the following example :

Example - 9

Divide $-25a^3b^2c$ by $-5ab^2c$

Solution :

Hence $\left(-25a^3b^2c\right) \div \left(-5ab^2c\right)$

$$= \frac{-25a^3b^2c}{-5ab^2c}$$

$$= \frac{-25}{-5} \times \frac{a^3}{a} \times \frac{b^2}{b^2} \times \frac{c}{c}$$

$$= 5 \times a^{3-1} \times b^{2-2} \times c^{1-1} \qquad \left(\because a^m \div a^n = a^{m-n}\right)$$

$$= 5a^2b^0c^0 \qquad (\text{since } b^0 = 1,\ c^0 = 1)$$

$$= 5a^2$$

Division of a polynomial expression by a monomial expression -

You have known how to divide a monomial expression by another monomial expression. Now let us see the division of polynomial expression by a monomial expression.

Example - 10

Divide $16m^2 + 4mn - 12mn^2 \; by \; 2m$

Solution :

$16m^2 + 4mn - 12mn^2 \div 2m$

or

$$\frac{16m^2 + 4mn - 12mn^2}{2m}$$

$$= \frac{16m^2}{2m} + \frac{4mn}{2m} - \frac{12mn^2}{2m}$$

$$= 8m^{2-1} + 2m^{1-1}n - 6m^{1-1}n^2$$

$$= 8m + 2n - 6n^2$$

Here the polynomial has been changed in to monomial expression to continue the process of division.

Exerciese 4.2

1. Find the values.

(i) $\left(18x^2y^2\right) \div \left(-6xy\right)$

(ii) $\left(-15x^3y^2z\right) \div \left(-5x^2yz\right)$

(iii) $\left(-x^5y^7\right) \div \left(-x^4y^5\right)$

(iv) $32a^4b^2c \div \left(-8abc\right)$

(v) $\left(28a^4b^6c^8\right) \div \left(-7a^2b^4c^6\right)$

2. Divide

(i) $2x^4 - 6x^3 + 4x^2 \; by \; 2x^2$

(ii) $5a^4b^3 - 10a^3b^2 - 15a^2b^2 \; by \; 5a^2b^2$

(iii) $27a^4 - 36a^2 \; by \; -9a$

(iv) $x^4 + 2x^3 + 2x^2 \; by \; 4x^2$

(v) $a^2 + ab + ac \; by \; a$

Division of a polynomial by binomial -

You now know how to divide a polynomial by a monomial expression. Now let us see this.

Example - 11

Divide $18a^2 + 12a + 27a^3 + 8 \; by \; 3a + 2$

Solution :

Write done the polynomial in the decreasing order of its exponents.

Example : $27a^3+18a^2+12a+8$

Step 1- Here the frist dividend is $27a^3$. In the beginning this is divided by the first part of the divisor that is $3a$

$$\frac{27a^3}{3a}=9a^2 \qquad 3a+2\overline{)27a^3+18a^2+12a+8}$$

and $9a^2$ is written as the quotient.

Step 2- $9a^2$ is multipled to the complete divisor.

$$9a^2(3a+2)=27a^3+18a^2$$

$$\begin{array}{r l} & 9a^2 \\ 3a+2) & \overline{27a^3+18a^2+12a+8} \\ & \pm 27a^3 \pm 18a^2 \end{array}$$

$27a^3+18a^2$ is written under the similar factors of the dividend and is subtracted. That is the sign of the lower expression is changed.

Step 3- After subtraction the remaining number is written below.

$$\begin{array}{r l} & 9a^2 \\ 3a+2) & \overline{27a^3+18a^2+12a+8} \\ & \underline{\pm 27a^3 \pm 18a^2} \\ & \qquad 12a+8 \end{array}$$

Step 4- The intial part of the remaining part of the dividend $12a$ is divided by the first part of the divsor, $3a$. so, $12a\div 3a=4$

+ 4 is to be written in the quotient and + 4 is again multipled to the whole divisor.

Therefore $(3a+2)\times 4 = 12a+8$

$$\begin{array}{r l} & 9a^2+4 \\ 3a+2) & \overline{27a^3+18a^2+12a+8} \\ & \underline{\pm 27a^3 \pm 18a^2} \\ & \qquad 12a+8 \end{array}$$

Step 5- Similar factors are written under each other of the divididend 12a+8 and is subtacted.

$$\begin{array}{r l} & 9a^2+4 \\ 3a+2) & \overline{27a^3+18a^2+12a+8} \\ & \underline{\pm 27a^3 \pm 18a^2} \\ & \qquad \pm 12a+8 \\ & \qquad \underline{\pm 12a \pm 8} \\ & \qquad\quad 0 \end{array}$$

Step 6- Subtraction would give the remainder as zero.

$$\begin{array}{r|l} & 9a^2+4 \\ 3a+2 & 27a^3+18a^2+12a+8 \\ & 27a^3+18a^2 \\ \hline & 0 \quad + \quad 0 \quad +12a+8 \\ & \pm 12a \pm 8 \\ \hline & 0 \quad 0 \\ \hline \end{array}$$

Step 7- Thus, desired quotient = $9a^2 + 4$

You already know that when a number is completely divided by another number and the remainder is zero, then the second number is know as the multiple factor of the first number.

Here $27a^3+18a^2+12a+8$ divided by ($3a$ + 2) gets completely divded and the remainder is zero, therefore ($3a$ +2) is a multiple factor of $27a^3+18a^2+12a+8$

Let us now take another example :

Example - 12

Divide $-12x^3-8x^2-5x+10$ *by* $(2x-3)$

Solution :

$$\begin{array}{r|l} 2x-3 & -12x^3-8x^2-5x+10 \;(-6x^2-13x-22 \\ & \mp 12x^3 \pm 18x^2 \\ \hline & -26x^2-5x+10 \\ & \mp 26x^2 \pm 39x \\ \hline & -44x+10 \\ & \mp 44x \pm 66 \\ \hline & -55 \\ \hline \end{array}$$

Here also, the divison has been done as in the earlier example, but the remander is -55, not zero, So we might say that (2x - 3) is not a multiple factor of the polynomial

$-12x^3-8x^2-5x+10$.

Example - 13

Divide $8q^3+2q-8q^2-1$ *by* $4q+2$

Solution :

Here the exponents of are not in descending order of q, so first the expression is written in the descending order of the exponents of q.

Thus $8q^3-8q^2+2q-1$

$$
\begin{array}{r}
2q^2-3q+2 \\
4q+2\overline{)8q^3-8q^2+2q-1} \\
\underline{\pm 8q^3 \pm 4q^2 \qquad\qquad} \\
-12q^2+2q-1 \\
\underline{\mp 12q^2 \mp 6q \qquad} \\
8q-1 \\
\underline{\pm 8q \pm 4} \\
-5
\end{array}
$$

Here also the division has been done like the previous solutions. The steps of division are continued until the exponents of the algebraic variable of the remainder does not become less than exponents of the algebraic variable of the divisor.

Verification

Dividend = divisor x quotient + remainder in this question.

Dividend = $8q^3-8q^2+2q-1$

Divisor = $4q+2$

Quotient = $2q^2-3q+2$

Remander = -5

Right hand side = divisor x quotient + remainder

= $(4q+2)(2q^2-3q+2)+(-5)$

= $4q(2q^2-3q+2)+2(2q^2-3q+2)-5$

= $8q^3-12q^2+8q+4q^2-6q+4-5$

= $8q^3-8q^2+2q-1$

= Left hand side

This means :

Dividend = divisor x quotient + remainder

Therefore obtained quotient = $2q^2-3q+2$

remainder = -5, is correct

Exercise 4.3

Q. 1. Write the given polynomial in the decreasing order of the variables.

(i) $15x^2 + 3x + 8x^4 - 4x^3 - 15$

(ii) $12m^5 - 9m^3 + 16 - 6m^2 + 8m$

(iii) $9m^4 - 16m^2 - 4m + 16 - m^3$

(iv) $4 - 8y^3 + 12y^4 - 6y^2$

Q. 2. Divide & say whether the divisor are multiple factors of the dividend.

(i) $x^2 - 11x + 30\ by\ (x-5)$

(ii) $x^2 + 20x + 91\ by\ (x+7)$

(iii) $x^2 - 5x - 6\ by\ (x-6)$

(iv) $x^3 - 5x^2 - 2x + 24\ by\ (x-4)$

(v) $a^2 + 2ab + b^2\ by\ (a+b)$

Q. 3. Divide and prove that the divisor and the dividend are not multiple factors. Write down the quotient and the remainder for the following expressions.

(i) $x^3 + 2x^2 + 3x + 4\ by\ (x-1)$

(ii) $-12 + 3x^2 - 4x + x^3\ by\ (x+5)$

(iii) $4x^4 - 2x^3 - 10x + 13x - 6\ by\ (2x+3)$

(iv) $8x^3 - 6x^2 + 10x + 15\ by\ (4x+1)$

Q. 4. Divide and verify :

Dividend = Divisor x quotient + remainder.

(i) $m^2 - 3m + 7\ by\ m - 2$

(ii) $a^3 - 2a^2 + a + 2\ by\ a + 2$

(iii) $9x^3 + 15x^2 - 5x + 3\ by\ 3x + 1$

(iv) $2x^3 + 3x^2 + 7x + 15\ by\ x^2 + 4$

We Have Learnt

1. Before multiplying two monomial expressions, we multiply factors first and then multiply its variables.
2. To multiply a monomial expression to a binomial expression, the monomial expression is multiplied to each term of the binomial expression and the products are added. Thus the distributive law is used.
3. While multiplying the variable exponential law is followed.
4. To multiply two binomial expressions to each other, the distribution law is used twice e.g.

$$(a+b)(c+d) = a(c+d)+b(c+d)$$

$$= ac+ad+bc+bd$$

5. While multiplying if the sign of the algebraic expression are same, then the sign of the product is also positive and when the signs are dissimilar the product is negative.
6. The process of division is continue till the exponent of the divisor does not become less than the exponent of the remainder for the algebraic expression.
7. To divide a polynomial by a monomial, it is convenient to divide each term of the polynomial by the monomial.

Chapter—5

CIRCLE AND ITS ELEMENTS

Anu had studied in the chapter on 'Circles' that the distance of any point on the circle from its centre is the same and this distance is known as the radius. She also knows how to make circles of different measures. She loves making a variety of new designs with the help of circles drawn by bangles of different sizes.

One day one of her bangles broke. She put the pieces in the right places to rejoin the bangle and could make the outline of a circle with the help of a pencil. Anu was thinking. "If bangle can break into many pieces, can the circle be also broken into many pieces ?

What do you think ? Perhaps, you are thinking that if a bangle can be broken into pieces why not a circle ?

Let us find answer to these questions :

In class VI, you have studied that a straight line joining any two points through the centre of a circle is known as its diameter.

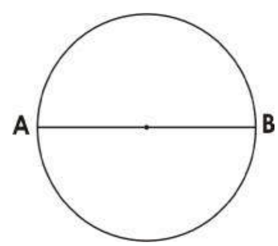


Fig. 5

Activity 1

Make circle of any radius on the paper and mark the centre of the circle, Now draw a diameter in the circle. Cut the circle along with its diameter, Are the two parts equal ? Will any circle cut along the diameter get divided into two equal parts ?

Is a circle thus divided into two equal parts called a semicircle ?

Activity 2

On the circles given below, a few points have been marked. If you join them, which line segment would divide the circle into two semicircles and why ?

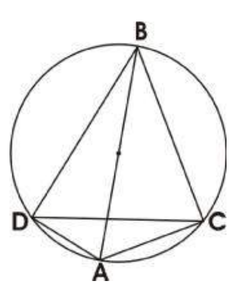


Fig. 5.1 (i)

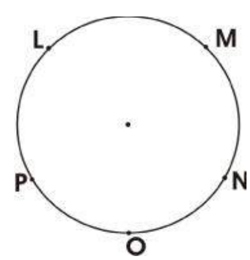


Fig. 5.1 (ii)

In the figure 5.1 (i) AB is the diameter and AC, AD, DC, BD and BC are the chords of the circle. Each chord divides the circle into two segments. These parts of the circumference of the circle are known as arch. In fig 5.1 (ii) ON, NM, ML, PO etc. are archs.

In 5.1 (iii) if an arch is named as xy it is not clear which arch is the upper one or the lower one. Therefore we need to indicate arch by three points e.g. X L Y or $\overset{\frown}{XLY}$ arc XMY or measure $\overset{\frown}{XMY}$, are

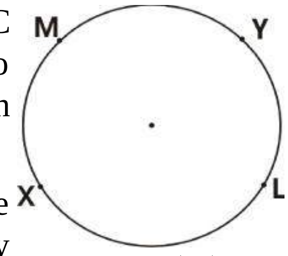


Fig.5.1 (iii)

XLY is lesser then the semicircle. Therefore it is know as a minor are. Similar $\overset{\frown}{XMY}$ measures greater than the semicircle, hence is known as a greater or major arch.

Activity 3

Identify the minor and major arch in the given examples :

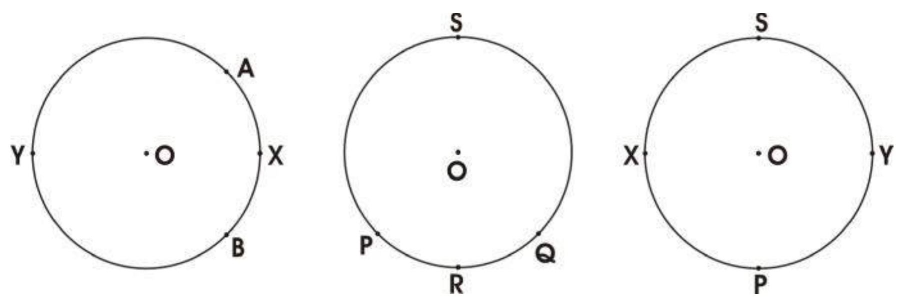


Fig. 5.2 (i) Fig. 5.2 (ii) Fig. 5.2 (iii)

Can you say whether an arch is major or minor by just taking a look at it ? Pankaj thought that in figure 5.2(i) $\overset{\frown}{AXB}$ is a minor are and $\overset{\frown}{AYB}$ is major arc, similarly in fig 5.2(ii) $\overset{\frown}{PRQ}$ is a minor arche and $\overset{\frown}{PSQ}$ is a greater arch. But it was difficult to say for fig. 5.2(iii) which arcs out of $\overset{\frown}{XPY}$ and $\overset{\frown}{XQY}$ was major and which was minor.

Just then Rakesh Joined point A and B to the centre of the circle O in figure (i) and found that $\angle AOB$ was formed towards AXB is smaller than the $\angle AOB$ is angle formed towards AYB. Now you also join points P & Q in fig. 5.2(ii) with center measure the angles and verify whether the conclusions drawn by Rakesh are true ?

Anu, found that in fig. 5.2(iii) XOY is a straight line. Therefore . Therefore the angles subtended by XPY & XQY are equal and hence both the archs are equal.

Rakesh said, the arch towards which the angle at the centre would be less than 180^0 would be a minor arc and the arch towards which the angle at the centre is more than 180^0, would be the major arch.

Now you have begun to recognize the arc, let us take up some activities related to these :

Given below is a circle. How many arcs can you draw in this circle ? Suresh was thinking that a circle is made up of numerous points and the space point between any two points should be an arc. Therefore any circle would be made up of numerous arcs suresh was thinking right. A circle can have several arcs such as there are numerous points in a circle in the same way by joining any two points we can also draw numerous chord.

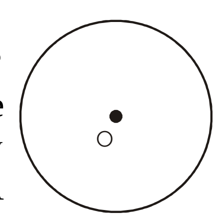


Fig. 5.3

Rakesh made a circle and marked the dots or points on the circle as shown in the diagram and then pointed A & C to get the chord AC Shresh saw this and said , "Chord AC divides the circle into two arcs. The minor arc is AC and again AC makes a Major arc as well. The minor arc AC is written as $\overset{\frown}{ABC}$ because the point B is included in this arc where as the greater arc AC has many points. How do we write this? Let us think about it.

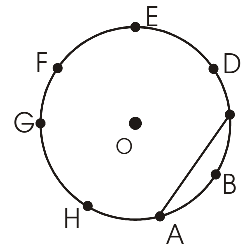


Fig. 5.4

Would you like to think yourself ?

In the major arc AC, all the points within that will be included in it AHGFEDC. This would be written like $\overset{\frown}{ABC}$ as $\overset{\frown}{AHC}$, $\overset{\frown}{AGC}$, or -------- or $\overset{\frown}{ADC}$ etc. that represents the arc or part of the circle.

Angle made by the arc on the circle

ABC is a triangle in fig. 5.5 where all the three vertices A, B, C arc on the circle. Here $\overset{\frown}{BXC}$ makes an angle $\angle BAC$ at point A, can you recognize the angle subtended by $\overset{\frown}{CYA}$ and $\overset{\frown}{AZB}$ on the circle ? Recognize the angles and name them.

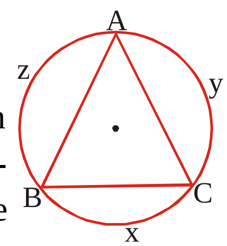


Fig. 5.5

Activity 4

In the quadrilateral ABCD given below the vertices A, B, C and D are on the circle. Point out the arcs and their respective angles subjoined on the circumference of the circle by them.

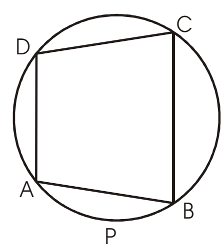
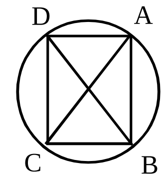


Fig. 5.6

S.No	Points between which arcs are formed	Name of the minor arc	Angle made of the minor arc m the circule	Name of the Major arc	angle made by the major arc on the circle
01.	A and B	A PB	no angle	A D B or A C B	no angle
02.	A and C				
03.					
04.					
05.					
06.					

Exercise 5.1

1- Find out the answers of the question related to the diagram given below.



(i) Indentify the arc $\overset{\frown}{ABC}$

(ii) Indentify the arc $\overset{\frown}{BCD}$

(iii) Name the minor angles formed by minor arc $\overset{\frown}{AB}$

(iv) On which arc is $\angle ACB$ formed ?

(v) On which arc is $\angle CBA$ formed ?

(vi) On which arc is $\angle CBD$ & $\angle CAD$ formed ?

(vii) Write the names of the angles formed on point D by the arcs ?

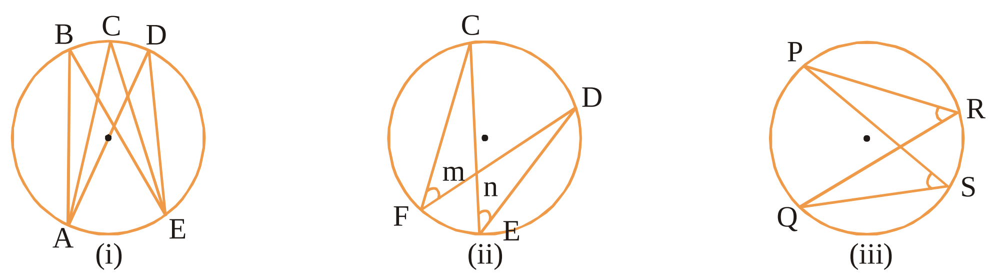
You have now learnt to recognize all the angles formed on circle by an arc. Let us examine the relationship between these angles.

Qualities of an arc

Activity 5

In the given figure, several angles are formed by a single arc on the circle by measuring the angles.

Complete the table given below.



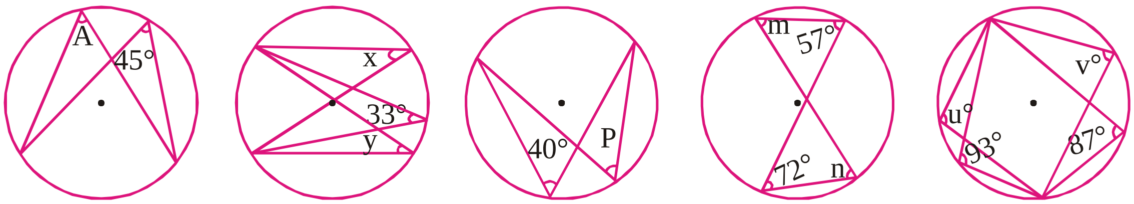
S.No.	Fig No.	Name of arc	angles an their measurment formed by the arc on the circle		
			1	2	3
01.					
02.					
03.					

By doing the above activities, Shelley observed that the angle formed opposite side of the arc are of same measures which means that all the angles formed by a single arc in a circle are equal.

You also consider some circle, divide them into arcs and draw some angles formed from the arc. Let your friends measure them and find out whether all the angles made by an arc are equal.

Exercise 5.2

In the following figuers observe the angles shown in english latters and determine their values without using the protactor.



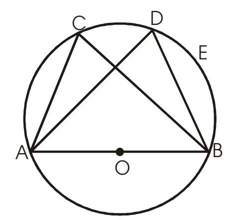
(i) (ii) (iii) (iv) (v)

a =---------- x =---------- p =---------- m =---------- u =----------

y =---------- n =---------- v =----------

Activity 6

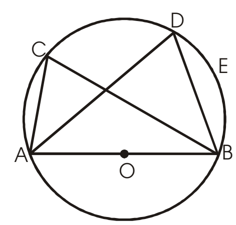
Draw a diamter AOB for the circle with centre O. Take the point, C,D,E in the semicircle above the diameter and make $\angle ACB$, $\angle ADB$ and $\angle AEB$. Measure the angles with the help of a protactor and write down the values.



$\angle ACB =$............. $\angle ADB =$............. $\angle AEB =$.............

What conclusions do you draw from the measurment angles.

Similarly, take the points F & G from the lower senicircle and make angle and Measure these angles.

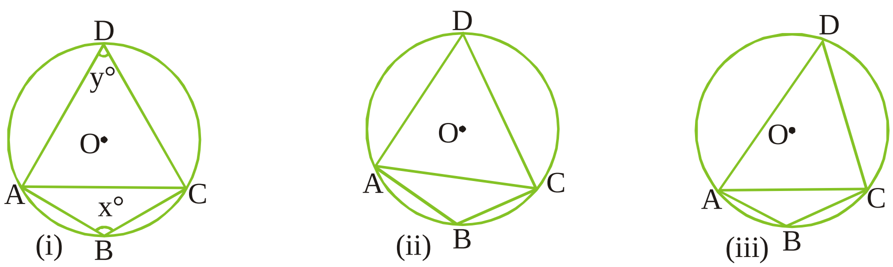


After observation you would find that all the angles are of 90^0. which means in a s<u>*emicircle all the angles are right angles.*</u> ***<u>Two points situated on the circle subtending angles from minor and major arcs.</u>***

t

Activity 7

Make a circle taking the centre O, Take four points A,B,C and D on the circle, in such a way that minor arc ABC and major are ADC is formed. Angle of minnor arc who make remaining part of the circle is $\angle ADC$ and angle of relative major arc who make remaining part of circle is $\angle ABC$. Measur them and fill in the following table-



S.No.	angle made by the minor arc on any point of remainy part of the circle x^0	corresponding angles to the greater part of the circle (Remaining part of the circle) y^0	$x^0 + y^0$
01.	------------------------	--------------------------------------	-----------
02.	------------------------	--------------------------------------	-----------
03.	------------------------	--------------------------------------	-----------

After completing the table, you will find that the sum of the angles subtended from a segment of the circle on both sides of the chord is 180^0.

Exercise 5.3

Fill in the blanks-

1- The angle formed by the minor/smaller part of the circle on any point of is a .... (acute angle/ obtuse angle)

2- The angle formed by the major/greater part of a circle on any point of is a .... (acute angle/ obtuse angle)

3- In a circle the sum of the corresponding angles formed by the smallar segment and the greater seqment of the circle is .... (180^0,270^0,360^0)

Example : 1

Look at the fig. 5.10 the centre of this circle is O. if $\angle C = 55^0$ then $\angle ABC = ?$

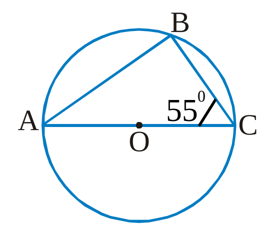


Fig. 5.10

Solution :

AC is the diamter (It passes through O)

Therefore $\angle ABC$ is an angle from the semicircle.

Which is a right angled. That is, $\angle B \ \ or \ \angle ABC = 90^\circ$

Therefore $\angle A + \angle B + \angle C = 180^0$ (Where the sum of three angles of a triangle is equal to 180^0)

$\Rightarrow \angle A + 90^0 + 55^0 = 180^0$

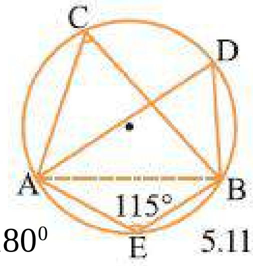
$\Rightarrow \angle A + 145^0 = 180^0$

$\Rightarrow \angle A = 180^0 - 145^0$

$\Rightarrow \angle A = 35^0$

Example : 2

In the given picture $\angle AEB = 115^0$ then find the values of $\angle ACB$ & $\angle ADB$?



Solution :

Given that : $\angle AEB = 115^0$

To find : $\angle ACB$ & $\angle ADB$

Since the sum of the angles subtended on both sides of a chord is 180^0

$$\therefore \angle AEB + \angle ACB = 180^0$$

$$\Rightarrow 115^0 + \angle ACB = 180^0$$

$$\Rightarrow \angle ACB = 180^0 - 115^0$$

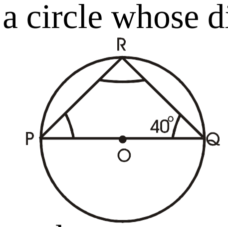
$$\Rightarrow \angle ACB = 65^0$$

We know that the angles of an arc on the same segment of the circle are equal,

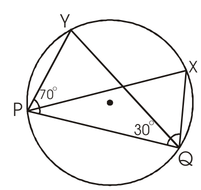
Hence $\angle ACB = \angle ADB = 65^0$

Exercise 5.4

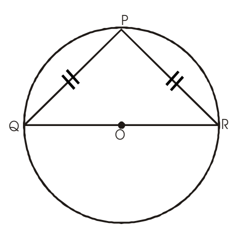
1- In the given fig. O is the centre of a circle whose diameter is PQ find the measures of $\angle PRQ$ & $\angle QPR$



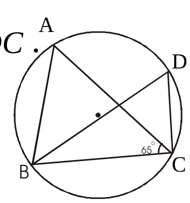
2- In the given fig. angles in the segment of the circle PQXY arc $\angle YPQ = 70^0$ & $\angle YQP = 30^0$ find out meaurement of $\angle PYQ$ & $\angle PXQ$



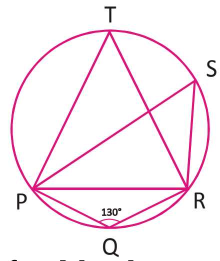
3- In the given fig. gave PQ = PR and the centre of the circle is O, then find the three angles of the triangle PQR.

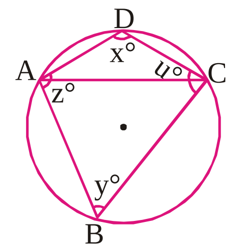


4- In the given fig. AB = AC and $\angle ACB = 65^0$ find out the measure of $\angle BDC$.



5- In the given fig. find out the angle measures for $\angle PTR$ & $\angle PSR$ where in $\angle PQR = 130^0$.





6- Take circles of different radius and make fig. like the ones & shown in the given picture. Then measure the angles and complete the table given below :

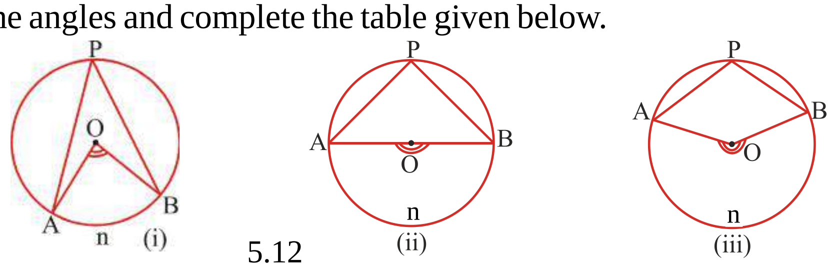
S.No.	x^0	y^0	$x^0 + y^0$	z^0	u^0	$z^0 + u^0$
1.	110°	70°	180°	100°	80°	180°

Relaionship between the angles formed by an arc on the remaining segment of a circle and its centre.

You have already recognised the angle subtended at the centre O by arc $\overset{\frown}{ANB}$. You have also learnt to make an $\angle AOB$ on remaing part of the segment of a circle at point p.

Activity 8

Make $\angle AOB$ at the centre of the circle from arc AB and an angle $\angle APB$ on the remaining arc of the same circle in the way shown in the diagram in your notebooks. Now measure the angles and complete the table given below.



5.12

Fig. No.	Meansure of $\angle AOB$	Meansure of $\angle APB$	Meansure of $2\angle APB$	If $\angle AOB = 2\angle APB$
5.12(i)				
5.12(ii)				
5.12(iii)				

What did you see?

$\angle AOB$ & $2\angle APB$ are equal or nearly equal.

Therefore, $\angle AOB = 2\angle APB$ ....(1)

So angle formed by AB at the centre = 2 x (angle formed by the remaining arc of the circle)

Therefore we conclude that.

In a circle the angle at the centre subtended from an arc is double or thrice the angle subtended at the remaining arc by equation (1)

$$\angle APB = \frac{1}{2} \times AOB = \frac{1}{2} \times m \widehat{AB}$$ (here $m\widehat{AB}$, is a partial measurement of arc$\widehat{AB}$)

In other words,

"The angle inseribed by an arc on any point on the remaining segment of the cricle is half the angle subtended at the centre by the same arc."

Practice

Complete the blanks in the given table :

Central angle of the arc or in degree mea-surement	The angle formed by the arc at a point on the remaining segment of the circle	How is the arc ? Minor/Major/semi Circular
150^0	75^0 (Why ?)	Minor arc (why ?)
220^0	------------------	Major arc
------------------	90^0	Semicercular
------------------	------------------	Major arc
------------------	------------------	Minor arc

Example 3

The measure of an arc of a circle is 132^0. Find the angle $\angle ACB$ intercepted by the same arc at any point C on the remaining segment of the circle.

Solution :

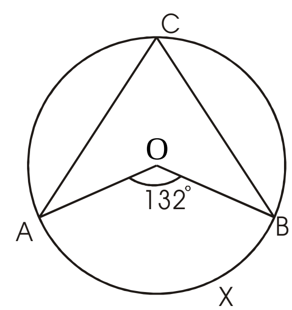


Fig. 5.13

$\angle AOB = 132^0$ (given)

Since $\angle AOB = 2 \times \angle ACB$

$\Rightarrow \angle ACB = \frac{1}{2} \times \angle AOB$

$$\Rightarrow \quad \angle ACB = \frac{1}{2} \times 132^0$$

$$\Rightarrow \quad \angle ACB = 66^0$$

Example 4

The centre of the exterior circle of the equilateral triangle ABC is O. find the measurement of $\angle BOC$.

Solution :

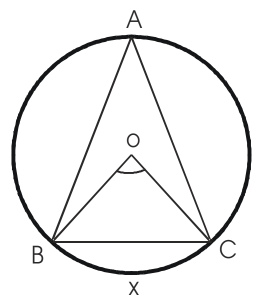


Fig. 5.14

ΔABC is a equilateral triangle

Therefore $\angle A = \angle B = \angle C = 60^0$

$\because$ the central angle of $\overset{\frown}{BXC} = \angle BOC$

$\because \quad \angle BOC = 2\angle BAC$

$\Rightarrow \quad \angle BOC = 2 \times 60^0 = 120^0$

therefore $\angle BOC = 120^0$

In this example joining OA will give us $\angle AOC$ & $\angle AOB$. Could you find their measurcs?

Equal arcs, corresponding chords and the measures of arcs in dgrees.

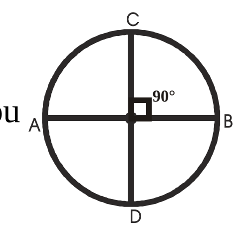


Fig. 5.15

Activity 9

Make a circle of radius 4cm on a piece of white paper with the centre O, Draw its diameter AB. Now draw another diameter CD that makes a 90^0 angle with respect to AB i.e. perpeidicular to AB.

You know that every circle is symmetrical with respect to its dimeter and therefore diameters AB and CD make two right angles and divide the circle into four equal parts.

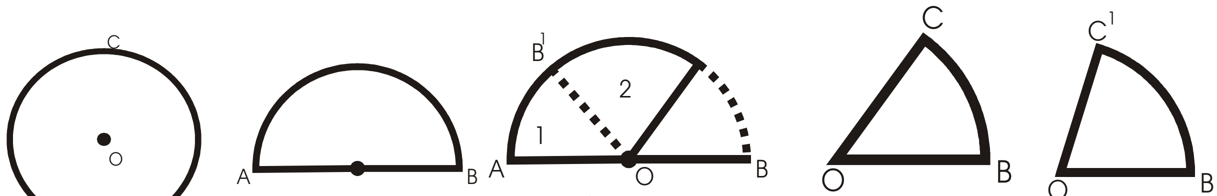
If the papers is folded twice along with AB and CD each quarter willcover the other quarter completely i.e. superpose each other. Here arc AD, DB, BC and CA superpose each other completely. The angle at the centre for these arcs is 90^0.

Therefore minor $\overset{\frown}{AD}$ = minor $\overset{\frown}{DB}$ = minor $\overset{\frown}{BC}$ = minor $\overset{\frown}{CA}$.

So, $m(\overset{\frown}{AD}) = m(\overset{\frown}{DB}) = m(\overset{\frown}{BC}) = m(\overset{\frown}{CA}) = 90^0$.

Activity 10

Make a circle with centre O on a piece of paper. Draw diamter AB. Fold it along the diameter. Fold it at the centre on the radius OC. Cut out this part along OC first and then along OB. Thus we get two piece OBC and OB'C'. Both the segments of a radius and a rc of equal length.



5.16

Now the length of the arc BC = length of arc B 'C' and $\angle COB = \angle C'OB'$

Result

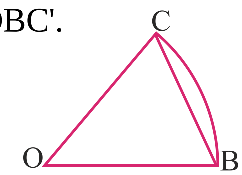
In a circle arcs of equal length foremed equal angles at the centre. The contrary of this is also true i.e. in a circle. The arcs that formed equal angles at the centre arc also equal. In the above activity join BC and BC'.

Now see, the two radius segments OBC and OBC'.

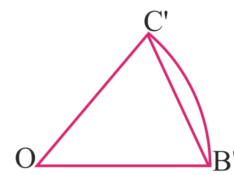
Here $\angle COB = \angle C'OB'$

arc BC=arc = B'C'

chord BC = chord B'C'



5.17

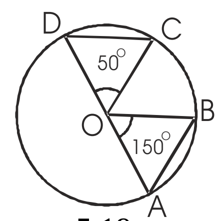


5.18

Therefore the chords related to arcs of equal length arc equal & converely, the angles formed by equal chords at the centre of the circle are equal.

Example 5

In Fig. 5.19, there are two segments of radius AOB and COD and $\angle AOB = \angle COD = 50^0$. AB = 2.5i.e. find the length of CD



5.19

Solution :

Since the equal angles formed at the centre belong to two equal chords.

Therefore Chord AB = Chord CD

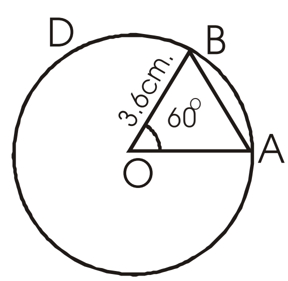
Chord AB = 2.5cm.

$\therefore$ Chord CD = 2.5cm.

Example 6

Fig. 5.20 shows a circle with radius 3.6cm. and $\angle AOB = 60^0$. Find the length of chord AB and $\angle OAB$ & $\angle OBA$

Solution :



5.20

Given, radius = 3.6cm...

In ΔOAB, $OA = OB = 3.6cm.$ (radius)

$\therefore$ The angles opposite to these sides would be equal.

$\therefore$ $\angle OBA = \angle OAB = x^0$ (Supposed)

In $\Delta\ OAB$, the sum of the angles = 180^0.

$\Rightarrow \quad \angle BOA + \angle OBA + \angle OAB = 180^0$

$\Rightarrow \quad 60^0 + x^0 + x^0 = 180^0$

$\Rightarrow \quad 60^0 + 2x^0 = 180^0$

$\Rightarrow \quad 2x^0 = 180^0 - 60^0$

$\Rightarrow \quad 2x^0 = 120^0$

$$x^0 = \frac{120^0}{2} \quad = 60^0$$

$\therefore \ \Delta\, OAB$, has three angles of 60^0 each.

$\Rightarrow \quad \Delta\, OAB$ is a equilateral tringle.

$\therefore$ AB = OB = OA = 3.6cm.

$\therefore$ Chord AB = 3.6cm., $\angle OAB = 60^0, \ \angle OBA = 60^0$

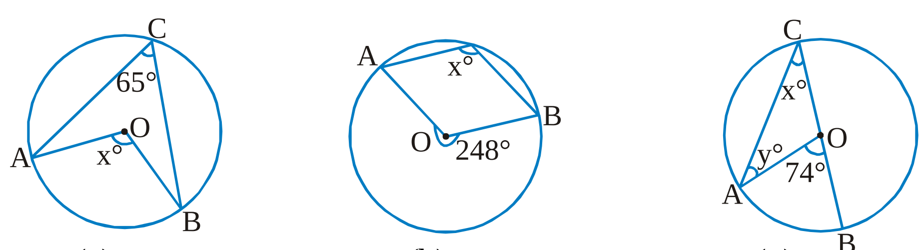
All the angles in OAB are $60^\circ, 60^\circ, 60^\circ$

AB= OB = OA = 3.6cms.

So, AB = 3.6cm. , $\angle OAB = 60^0, \ \angle OBA = 60^0$

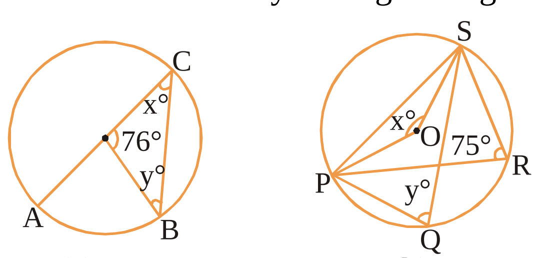
Exercise 5.5

Q. 1 . . . . x and y in the figures given below.



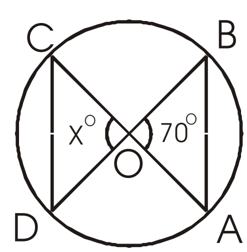
(a) (b) (c)

Q. 2 Find the values of x and y in the given figures.

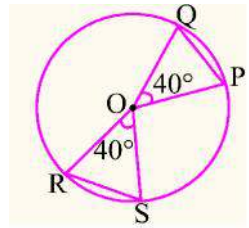


(a) (b)

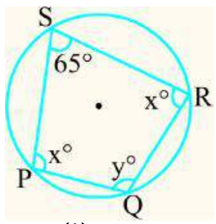
Q. 3 In the figure AB =CD, if $\angle AOB = 70^0$, find out the measurement of $\angle COD$.



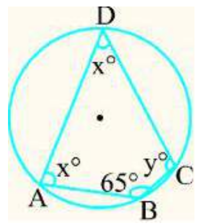
Q. 4 In the figure, if RS = 3.2cm. What will be the length of PQ?



Q. 5 Find the values of x and y in the given figures.



(i)



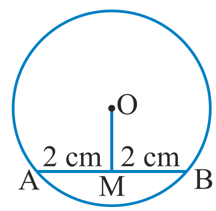
(ii)

CHORD- You have learnt before that the line segment obtained by joining any two points on a circle is known as a chord and the largest chord is the diameter of the circle. Let us know some more characteristics of chords.

Perpendicular drawn from the centre of a circle to the chord.

Activity 11

Draw a circle with centre O Draw a chord AB in this circle. Now draw OM $\perp$ AB in such a way that M is on chord AB.



5.21

Repeat this activity with circles of different radius.

Name those figures also like we have done in fig. 5.21. Number the circles in 1, 2, 3, ... etc.

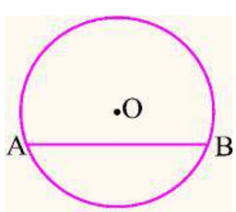
Now measure AM and MB in each condition and complete the following table :

Circle	AM	AB	Is AM = MB ?
1.			
2.			
3.			

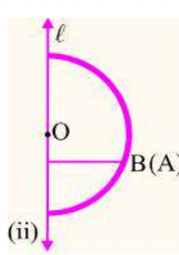
You will find that in each condition AM = MB therefore AM= MB.

Activity 12

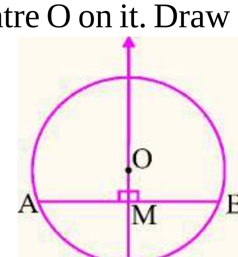
Take a Piece of thick paper and draw a circle with centre O on it. Draw chord AB on it.



(i)



(ii)



(iii)

5.22

Now fold the sheet (circle) in such a way that point A falls on point B and press the fold to get a crease along line *l*. We observe that the fold along line *l* passes though the centre O and the both part of the circle completely cover each other.

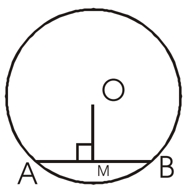
Now open the fold and mark the intersecting point of line ℓ and chord AB as M.

Measure $\angle OMA$ & $\angle OMB$. Both would be of 90^0. Since M is the mid point of AB. Therefore, AM=BM, Activty 11 and 12 make it clear that.

The perpendicular from the centre of circle to a chord biscets the chord.

Activity 13

Draw a circle with centre O. Also draw a chord AB. bisect AB at a point M and join O & M. Repeat this activity for different circle and number the circle as 1, 2, 3, ... Name the figures in similar way. Measure $\angle OMA$ in every circle and complete the table below :



5.23

Circle	$\angle OMA$	$\angle OMB$	Is $\angle OMA = \angle OMB$?
1.			
2.			
3.			
4.			

You will find that in every case $\angle OMA = \angle OMB$ (approx. 90^0) is obtained.

Since $\angle OMA$ & $\angle OMB$ are both angles drawn at the midpoint of chord AB, so their sum would be 180^0.

$\Rightarrow \quad \angle OMA + \angle OMB = 180^0$ (since $\angle OMA = \angle OMB$)

$\Rightarrow \quad 2 \times \angle OMA = 180^0$

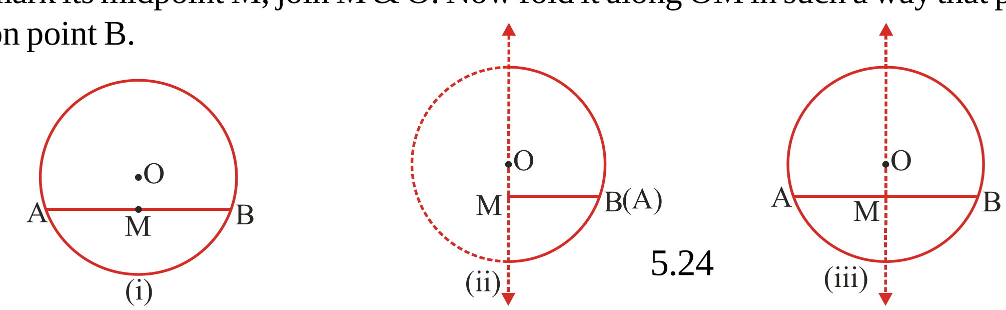
$\Rightarrow \quad \angle OMA = \frac{1}{2} \times 180^0 = 90^0$

$\Rightarrow \quad \angle OMA = \angle OMB = 90^0$

$\Rightarrow$ Implies $OM \perp AB$

Activity 14

Take a piece of paper and draw a circle with centre O, Draw a chord AB on the circle & mark its midpoint M, join M & O. Now fold it along OM in such a way that point A falls on point B.



5.24

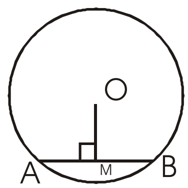
Now open the fold and you will find that $\angle OMB$ falls $\angle OMA$. Therefore, $\angle OMA = \angle OMB = 90^0$. Therefore, $OM \perp AB$.

It is clear from activities 13 and 14 that :

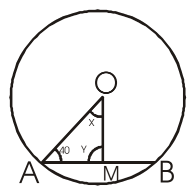
In a circle the line joining the centre to the midpoint of a chord is perpendicular to the chord.

Exercise 5.6

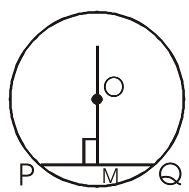
Q. 1 In the figure given, $OM \perp AB$. If AM = 3.5cm. find the values of BM and AB.



Q. 2 In the figure, M is the midpoint of chord AB. Find the values of *x* and y.



Q. 3 In the figure, $OM \perp PQ$. If $PQ = 8$ cm. then find the values of PM & MQ. Is PM =MQ?



Fill in the blanks _

Q. 4 a. The perpendicular from the centre of a circle to a chord .... the chord.

b. In a circle the line joining the centre to the midpoint of chord is...... the chord.

We have learnt

1. In a circle, the angles formed by an arc on the remaining segment of the circle arc equal.
2. The sum of the angles on the segments of the circle on both sides of a chord is 180^0.
3. The angle formed by an arc of a circle on the remaining segment of the circle is half of the angle formed by the arc at the centre of the circle.
4. In a circle, arcs of equal length formed angles at the centre which arc equal.
5. The perpendiicular from the centre of a circle to a chord bisect the chord.
6. In a circle, a line joining the midpoint of chord to its center, is perpendicular to the chord.

Chapter—6

STATISTICS

Mean

Radha enjoys feeding the cattle and giving them water to drink. She fills water for the cattle and also keeps record of the fact that from 8 to 11 in the morning, how many cows drink water. The record she has for the last week is as follows :

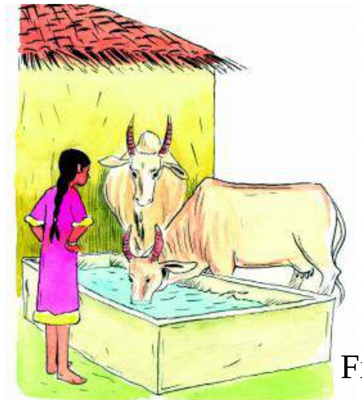


Fig. 6.1

Monday -12, Tuesday - 15, Wednesday - 13, Thusday - 11, Friday -13, Saturday - 13, Sunday - 14.

Can you tell how many cows drink water on average everyday ?

Cricket player A in 10 innings had scored 60, 70, 15, 90, 72, 45, 11, 77, 125, 200 runs respectively. Similarly player B made 220, 110, 70, 37, 15 and 07 runs in 06 innings. Can you say which player had a better performance ?

We can easily compare such comparison with the help of average ? We use average in several centexts in our everday life. For example :

1. The average age of students studying in your class is 14 years.
2. The average duration of sleep at night for you is 8 hours.
3. The average rate of our daily newspaper is Rs. 2.50.
4. The average attendance of students in the class is 45.
5. This year Raipur received rains below average.

The above examples show that the average age of sudents in the class is 14 years. On the average duration of sleep at night is 8 hours. It is neither the maximum limit nor the minimum limit.

In fact average is attained by dividing the sum of the scores in a given data by the number of scores. This is also known as the mean. This is indicated by M.

$$Therefore\ Average\ or\ mean\ (M) = \frac{sum\ of\ Scores}{No.\ of\ Scores}$$

Now we can easily find the number of cattle that Radha served water to drink.

$$Average = \frac{12+15+13+11+13+12+15}{7} = \frac{91}{7} = 13$$

Therefore on an average 13 cattle drink water that Radha served everyday.

Now you can yourself find out which cricket player had a better performance.

Activity 1

Find out the average age of the members of your family.

Activity 2

Find out the average of the scores obtained by you in all the subjects in your halfyearly exams.

Example 1

In a Fruit shop apples have been kept in five baskets. Containing 46 kg, 21kg, 18kg, 25kg, and 35kg apples. Find out the means.

$$Mean(M) = \frac{Sum\ of\ the\ scores}{No.\ of\ scores}$$

$$(M) = \frac{46+21+18+25+35}{5}$$

$$= \frac{145}{5} = 29kg$$

Example 2

Find out the mean of the first 10 natural numbers

Solution :

The first ten natural numbers are :-

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

$$Mean(M) = \frac{Sum\ of\ the\ scores}{No.\ of\ scores}$$

$$M = \frac{1+2+3+4+5+6+7+8+9+10}{10}$$

$$= \frac{55}{10} = 5.5$$

MODE

The school decided to take 30 students of class VIII for an excursion during the Deepawali vacations. The headmaster instructed the students to select a place. out of Sirpur, Ratanpur, Jagdalpur and Ambikapur. Some students wanted to go to Sirpur while others thought about Jagdalpur. Since the place could not be decided, the class teacher wrote the names of all the four places on the blackboard and asked the students to raise their hands for each option. He put tally marks in front of each name which was as follows :

Table 6.1

Places to be Visited	Tally Marks	No. of Students
Sirpur	~~IIII~~ II	07
Jagdalpur	~~IIII~~ ~~IIII~~ III	13
Ratanpur	~~IIII~~	05
Amtikapur	~~IIII~~	05

After making the table, the class teacher said that the maximum number of student i.e. 13 of the the students want to go to Jagdalpur, so we should proceed towards Jagdapur.

In our daily life, we have many such situations, where selection is made in this manner for example, mostly the size of shirts that people wear are 38 or 40. Therefore we easily get the sizes 38 or 40 in the readymade stores. Generallly shops seldom keep the smaller or bigger sizes beause they are less in demand and the manufacture depends on the maximum demand in the market.

This basis of selection is called the mode.

So, mode is that value in a given data which has been repeated maximum number of times. This is indicated by M_O.

Example 3

In a football Team the sizes of the shoes put on the eleven players are as follows :

6, 4, 5, 6, 7, 7, 6, 5, 6, 7, 8. Find the mode.

Solution :

On writing the given scores in ascending order we get :

4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 8

Clearly, the score 6. appears the maximum number of times (4 times). Therefore the mode of this score would be 6.

$M_O = 6$

Median

Example 4

In a class of 15 students received the following marks in Mathematics out of 100 :-

15, 35, 16, 25, 45, 76, 90, 99, 50, 16, 57, 60, 86, 17, 95. How many students have scored more than 50% marks ? This is not very clear by the marks. Let us arrange the marks in ascending order :

15, 16, 16, 17, 25, 35, 45, 50, 57, 60, 76, 86, 90, 95, 99.

Now, we can see that 07 students have got more than 50% marks. We can also see that 7 students have received less than 50 marks.

Example 5

The number of chapatis that 11 people eat in a day are as follows :

3, 7, 9, 8, 6, 5, 4, 2, 12, 10, 11

Find the mean of these numbers Mary quickly wrote it in her notebook and said that the mean is 7. Can you find out how many people ate more than 7 ? Radha calculated it and said that 5 people ate less than 7 chapatis while Aslam reported that 5 people ate more than 7 chapatis.

In the example (4) 7 students have scored above 50% marks while 7 students have got less than 50% while in example (5) also the number of people who ate less than 7 and more than 7 chapaties are equal i.e. 5.

Therefore, we can say that putting in order, the number 50 (in Example 4) and 5 (in Example 5) are the number that occure in the middle. This number is know as the median.

This means, when the scores are arranged in descending or ascending order, the value that occures in the middle is known as the Median. It is denoted by M_d.

A. Finding the Median when the number of scores N is odd.

When the number of scores in a given data is odd., then first we write them in an ascending order and find the value of the $M_d = \left(\frac{N+1}{2}\right)$ to score. This number obtained is the median.

Therefore $Median\ \ M_d = \left(\frac{N+1}{2}\right)$ th item

Example 6

Find the median of the given data :

3, 5, 10, 9, 8, 14, 6, 12, 13, 11, 7

Solution :

On writing the scores in an ascending order :

3, 5, 6, 7, 8, (9) 10, 11, 12, 13, 14   (here the total numbers of item is 11 i.e. odd)

$$M_d = \left(\frac{N+1}{2}\right) \text{th item value}$$

$$= \frac{11+1}{2} \text{ th item} = 6\text{th item's value}$$

$$\therefore \quad M_d = 9$$

B. Finding the Median when the number of scores N is even.

When the number of scores in a given data is even, then arranging in ascending or descending order shows two items in the middle position.

In such a state, the mean of these two items are taken to find the median. This means,

$$M_d = \frac{\left[\left(\frac{N}{2}\right)^{th} item + \left(\frac{N}{2}+1\right)^{th} item\right]}{2}$$

Example 7

Find the median of the given descending numbers :

5, 9, 4, 6, 12, 8

Solution :

On arranging the scores in ascending order, we get :

4, 5, 6, 8, 9, 12

Here N = 6 (even number)

$$\text{Median } M_d = \frac{\left[\left(\frac{N}{2}\right)^{th} item + \left(\frac{N}{2}+1\right)^{th} item\right]}{2}$$

$$M_d = \frac{\left(\frac{6}{2}\right)^{th} item + \left(\frac{6}{2}+1\right)^{th} item}{2}$$

$$= \frac{(Value\ of\ the\ 3rd\ item + Value\ of\ the\ 4th\ item)}{2}$$

$$= \frac{6+8}{2} = \frac{14}{2}$$

$$M_d = 7$$

Exercise 6.1

Q.1 Find the mean : 81, 74, 69, 73, 91, 55, 61.

Q. 2. Find the mean of the even number between 50 and 70.

Q. 3 Find the median : 4, 5, 10, 6, 7, 14, 9, 15.

Q. 4 The weight of 11 students (in kgs) in a class ar e as follows : 25, 27, 29, 32, 30, 28, 26, 31, 35, 41, 34. Find its median.

Q. 5 In a Science quiz competition, one student of class VIII received the following marks : 83, 61, 48, 73, 76, 52, 67, 61, 79.

Find the median of the above marks.

Q. 6 Find the mode of the given data:

7, 5, 99, 3, 1, 9, 7, 5, 3

1, 1, 9, 7, 7, 5, 5, 5, 3, 1

5, 3, 5, 1, 5, 7, 7, 9, 9, 1

Q. 7 Find the mode of the given distribution :

5, 3, 2, 2, 4, 5, 3, 3, 4, 3, 5, 3.

Q. 8 Find the mean of the first five odd natural numbers.

Q. 9 The mean of the number 8, 5, x, 6, 10, 5 is 7. Find the value of x.

Pie Chart

Activity 1

The area of forest in 5 districts of a state A, B, C, D, E have been represented in a circular digram as in fig. 6.2.

If it is considerd that the district where the area under forest is the maximum gets the maxiuman rainfall then can you say :

1. Which district gets the maximum rainfall ?
2. Which district gets the minimum rainfall ?

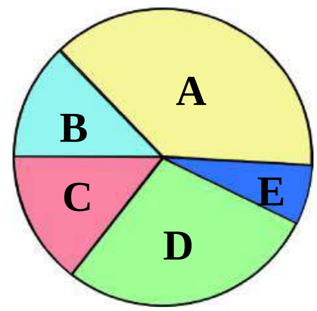


Fig. 6.2

Activity 2

In an Asseimbly election 4 candidates A, B, C & D faced the election. The votes received by them have been represented in the circular diagram.

Look at the diagram and answer.

1. Which candidate got the maximum number of votes ?
2. Which candidate got the least votes ?

How did you conclude this ?

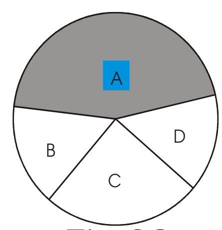


Fig. 6.3

You know that the sum of angles subtended at the centr e of a circle is 360^0 the area of votes obtained by candidate A subtends the largest angle at the centre. Similarly, the area of votes obtained by candidate D subtends the smallest angle at the centre. Hence, the conclusions you arrive at.

Example 8

The following are the number of students studying in classes 6 to 10 of a school in Jashpur. Show the following data in a diagramatic representation as a pi-chart.

Class	6	7	8	9	10
No. of Students	216	180	150	110	64

Solution :

To make a pi-chart we first add the number of students in all the classes and find out the value of angle subtended at the centre by the number of student in each class. Thus:

= 216 + 180 + 150 + 110 + 64 = 720

Hence, for 720 students the angle subtended at the centre of the circle is 360^0.

$\therefore$ For 1 student the angle subtended at the centre would be $\frac{360^0}{720}$

$\therefore$ For 216 students the angle subtended at the centre $=\frac{360}{720}\times 216$

$\therefore$ The angle subtended for students of class VI $=\frac{360}{720}\times 216 = 108^0$

The angle subtended for students of class VII $=\frac{360}{720}\times 180 = 90^0$

The angle subtended for students of Class VIII

$=\frac{360}{720}\times 150 = 75^0$

The angle subtended for students of class IX $=\frac{360}{720}\times 110 = 55^0$

The angle subtended for students of class X $=\frac{360}{720}\times 64 = 32^0$

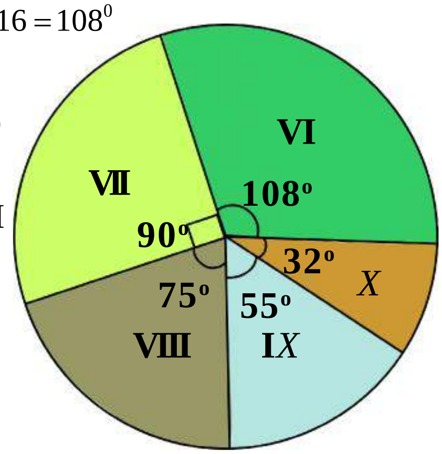


Fig. 6.4

Now when you know the measurement of angles you can take a circle of any radius and depict these measuremets by segtments of r adius for each class as shown in the diagram.

Example 9

The percentage of interest of 100 students, in class VIII in different games are as follows:

Name of the Game	Interest in the Game (%)	Angle at the centre
Cricket	65	$\frac{65}{100}\times 360^0 = 234^0$
Football	15	$\frac{15}{100}\times 360^0 = 54^0$
Hockey	10	$\frac{10}{100}\times 360^0 = 36^0$
Handball	03	$\frac{3}{100}\times 360^0 = 11^0$ (Approx)
Volleyball	07	$\frac{7}{100}\times 360^0 = 25^0$ (Approx)
Total No. of Student	100	Total angle the centre = 360^0

Graphic representation

In the above examples the data have been represented in a circle.

When the data is represented by segments of radius in a circle, it is known as the pi-chart (Circular diagram or pi-graph).

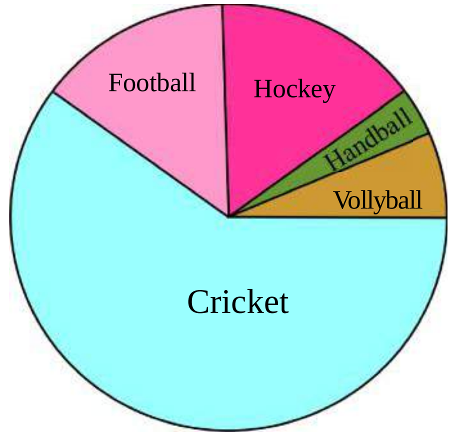


Fig.6.5

Activity 3

Make a pi-chart of the scores obtained by you in different subjects in class VII.

Example 10

A farmer represented the crops grown in his farm last year in a pi-chart. If the total production of crops in 720 quintals, find the quantity of each crop produced.

The total producton of crops = 720 qunitals.

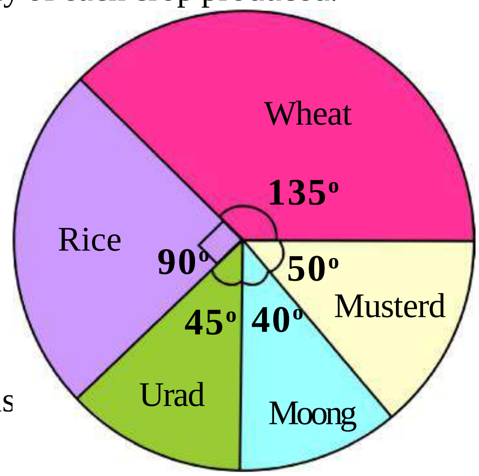


Fig. 6.6

$\therefore \quad 360^0 = 720$ qunitals

$\therefore \quad 1^0 = \frac{1}{360^0} \times 720$ quintals

$\therefore \quad 135^0 = \frac{135^0}{360^0} \times 720$ quintals

Production of Wheat $= \frac{720}{360^0} \times 135^0 = 270$ quintals

Production of Rice $= \frac{720}{360^0} \times 90^0 = 180$ quintals

Production of Urad $= \frac{720}{360^0} \times 45^0 = 90$ quintals

Production of Moong $= \frac{720}{360^0} \times 40^0 = 80$ quintals

Production of Mustard $= \frac{720}{360^0} \times 50^0 = 100$ quintals

Excercise 6.2

Q.1 The marks obtained by Geeta in the monthly test exams in mathematies for 6 months are as follows:

Months	July	August	September	October	Nov.	Dec.
Marks obtained (Out of 100)	40	45	65	35	55	60

Construct a pi-chart with the help of the above data.

Q. 2 The monthy income of a family Rs. 12,000=00. The expendicture of the family in the month is as follows. Make a pi-chart of the given data.

S.No.	Item	Expenditure in Rs.
01.	House rent	1500=00
02.	Food	6000=00
03.	Education	2200=00
04.	Entertainment	800=00
05.	Health	1500=00

Probability

Today as the school was going to be over, it started raining very heavily. The students were worried how to go home in such heavy rains. Just then meena said to Anu, there was no possibility of such rain in the month of October. Anu said, no, we cannot say that there was no possibility of rains but there was less possibility. The possibility of rains is always stonger in July. Even otherwise, the possiblity of rains in October or April is always very less.

In our daily life, probability is used in many situations. For example, who will win among the teams participating in a game is never known but the better team has a greater possibility of winning. Some situations are given below, write whether the possibility of their happening is more or less :

1. The possibility of suffering from polio after being vaccinated.
2. The possibility of lung cancar due to smoking.
3. The possibility of rains in a place where there are more trees.
4. The possibility of meeting with an accident when one is driving slow.
5. The possibility of seeing snakes in the rainy season.

All the above possibility can be understood with simple assumption on the basis of available data, for example, data indicate that the possibility of polio attacks are more prominent when the individual has not been vaccinated, similarly generally the possibility of accident is always more when people drive very fast. Let us try to presume possibilities.

Activity 4

Take two boxes and write A & B on them. Take 25 pieces of paper of equal size. Write 'x' on 10 picees of paper and 'y' on 15 of them. Now fold all the pieces in the same manner and keep in two separate heaps. Take 5 pieces from the heap of papers marked 'x' and 5 pieces from the heap of papers marked 'y' & put them into one box and then put 5 pieces from the 'x' marked ones and 10 pieces from the papers marked 'y' in another box.

Shake the boxes to mix the pieces of papers well after you have put them in the boxes. Now askone of your friends to close the eyes and take out one piece of paper from each box let him note down whether he has got a piece of paper marked x or y, fold it back as it was and put them in the box again. Now shake the box again to mix the papers all over well and ask your other friends also to repeat the activity. Then comptete the table given below.

S.No.	Name	Letter on the piece of paper fox A	Letter on the piece of paper from box B
1.			
2.			
3.			
4.			
		Total no. of *x* from box A=	Total no. of *x* from box B =
		Total no. of y from box A=	Total no. of *x* from box B =

Now look at the table and say :

from which box the probability of getting letter '*x*' would be more and why ?

While showing the problem, Suresh said, "Therefore 15 pieces of paper in box B out of which '*x*' is written on 5 pieces and y on 10 pieces. Since y is written on more pieces of paper, therefore the probability of getting letter y is more.

Rani asked, is the probability of getting letter '*x*' and 'y' from box 'A' would be equal? Think about rani's question and write its answer in your notebooks with proper reason.

Activity 5

In the pricture there are boxes with black (B) and white (W) balls. If we are asked to take out of one ball from the boxes without seeing, then answer the questions given below -

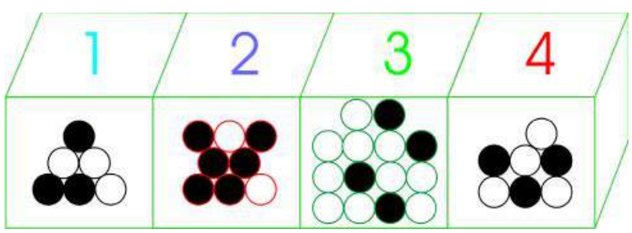


Fig. 6.7

1. From which box is the possibility of getting a white ball maximum.?
2. From which box is the possibility of getting a black ball maximum ?

With the help of the table made by Rani, Suresh drew the following conclusions :

1. The total number of balls in box 1 is 6 out of which 3 balls are white and 03 are black, therefore the pobability of getting both the colours are equal.
2. The total no of balls in box 2 is 8, of which 02 are white balls and 6 balls are black, hence the probability of getting black balls is greater.
3. In box 3, the total no. of balls is 14 out of which 10 balls are white and 04 balls are black, therefore the probability of getting white balls is more.

4. In box 4 out of total 7 balls, 04 balls are white and 03 balls are black, therefore the probability of getting white balls is greater.

The answer that Suresh has thought is right, but he is not able to understand that when box 3 has 10 white balls out of total 14 balls and box no 4 has 04 white balls out of total 07 balls, then from which box is the probability of getting a white ball is more ?

Mary suggested, How it would be, if we consider in the form of a fraction.

$$10 \text{ out of } 14 = \frac{10}{14}$$

$$\text{and } 4 \text{ out of } 7 = \frac{4}{7}$$

On comparing the two fractions, we will have -

$$\frac{10}{14} = \frac{10}{14} \text{ and } \frac{7}{4} = \frac{4\times2}{7\times2} = \frac{8}{14} \text{ (equalising demonstrations)}$$

Therefore out of $\frac{10}{14}$ and $\frac{8}{14}$, $\frac{10}{14}$ is greater. Hence out of 14 balls, the probability of getting white balls would be more when 10 balls are drawn out as compared to drawing out 04 white balls out of 07 balls.

Taking a decision by tossing a coin

You might have seen that in the beginning of a cricket match, the captains of the two teams decide to ball or bat on the basis of the toss when it is in favour of the team.

Can you think of some examples where you can take decision by tossing a cion ?

Activity 6

Let us perform an activity and see if we can do so - Take a coin and toss it with all your Friends one by one, and observe whether it shows head or tail as it falls on the ground every time. Note the occurances in the given table.

Table 6.5

S.No.	Name of the Student	What did you get ? Head or tail
1.		
2.		
3.		
4.		

Look at the table and say :-

1. Do head & tail appear one after another.
2. Is the number of heads and tails nearly equal ?
3. Which is more probable when a coin is tossed head or tail ?

You must have noticed that a coin has two sides -

A head and a tail. These out of two sides the probability of occurance of head is 1 out 2 or ½. Similarly out of 2 sides, the probability of occurance of tail is 1 out of 2 or ½.

In a box, if there are 3 balls of red, yellow and white colours respectively and we take out any one ball while closing our eyes, then the probability of the ball being red would be $\frac{1}{3}$ because out 3 one ball is red. Similarly, the probability of the ball being yellow would be $\frac{1}{3}$, when only one ball out of 3 is yellow and the probability of the ball being white would also be $\frac{1}{3}$. So, be now you must have understood that probability can also be measured.

Thus, the possibility of occurance of an event is measured as probability.

Let us know more about probability with a few more examples :

Example 11

If you are asked to take out a spade out of a pack of cards then what will be the probability of getting a spade card ?

Solution :

Since the pack of cards has 52 cards, out of which 13 are spades.

Hence, the probability of finding a spade is 13 out of 52 i.e. $\frac{13}{52} = \frac{1}{4}$.

Example 12

Find out the probability of getting the number 3, a dice head when it is tossed once.

Solution :

A dice has 6 faces which includes dots 1, 2, 3, 4, 5 and 6.

Therefore, the probability of digit 3 appearing on the head = 1 out of $6 = \frac{1}{6}$.

(Since only one face out of the six faces on the dice has three dots on it.)

Example 13

In a bag there are three white, five red and eight black balls. What is the probabnility of taking out a red ball out of it ?

Solution :

Total no. of balls in the bag = 3 white + 5 red + 8 black = 16 balls.

The probability of taking out a red ball out of the 16 balls would be 5 out of 16 because the bag contains 5 red balls. Therefore, The probability of one red ball = $\frac{5}{16}$

Example 14

One is asked to draw out one card out of a pack of cards. Find out the probability of that card being a King.

Solution :

Total no. of cards in the pack = 52.

The no of King cards in the pack = 04.

When we draw out of the pack any one out of the 4 Kings cards one would come out.

Therefore, the probability of getting a King cards.

= 4 out of 52

$= \frac{4}{52} = \frac{1}{13}$

Excercise 6.3

Q.1 What will be the probability of drawing a card of diamond from a pack of cards ?

Q. 2 A bag has 6 white, 11 red & 7 blue balls. Find out the probability of drawing a white ball out of that bag ?

Q. 3 A horse race competition has five competitors. Find out their possibility of winning the race.

Q. 4 In a basket there are 10 apples, 8 pomegranates (Anar) and 12 guavas. What will be the probability of taking out apples from the basket ?

Q. 5 Find out the probability of appearance of an even number when a coin is tossed .

Q. 6 On tossing a coin, what will be the possibabilty of appearance of head and the appearance of tail for that coin?

WE HAVE LEARNT

1. Average (Mean) is one unique number, that represents a group of scores or data.
2. $$Average = \frac{The\ sum\ of\ all\ scores}{Total\ no.\ of\ scores}$$
3. While finding out the median the scores are arranged in ascending order.
4. The medan is the number in the middle of the scores arranged in ascending order.
5. (a) $M_d = \left(\frac{N+1}{2}\right)^{th}$ item (when N is an odd No.)

   (b) $M_d = \frac{\left[\left(\frac{N}{2}\right)^{th} item + \left(\frac{N}{2}+1\right)^{th} item\right]}{2}$ (when N is an even No.)
6. The mode is the number that has the highest frequency in the scores.
7. Probability is the possibility of occourance of any event.

Chapter—7

DIRECT AND INVERSE VARIATION

Introduction

Sometimes we hear statements like in a year of heavy rainfall the water level in ponds and wells increases. Consumption of water increases as the population grows. As the number of ponds decreases, the capacity to store water decreases. With the fall in electricity production the quantity of goods produced in factories decrease. As the rainfall has been more water level in the pond has increased.

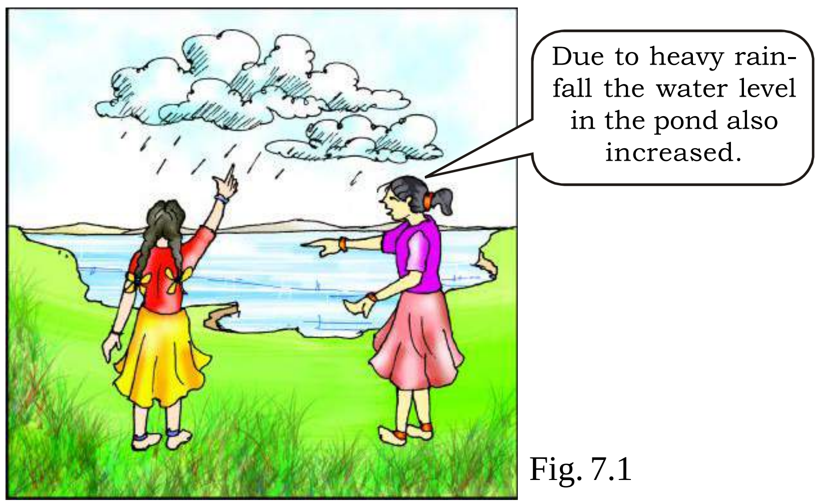


Fig. 7.1

If we consider the above statements we notice that the two quantities depend on one another. Change in the value of one also changes the value of the other. Thus for two related quantities, the change in the value of the second when the value of the first quantity changes is called variation. When on the increase or decrease in one value, the value of other also increases or decreases, then such a variation is called direct variation.

Think and write such examples of changes from your surrounding.

Raju wrote that if farm land is larger then the quantity of crop produced would be more i.e. if 1 acre land produced 24 bags rice then 3 acres of land would produce 72 bags rice.

Sudha wrote a relation between the quantity and the cost of a crop. She said if price of the 1 kg rice is Rs. 9 then the price of 2 kg rice will Rs. 18 and price of 5 kg rice will Rs. 45".

After reading these examples Mary said, "But this does not happen every time. It is not necessary that on increasing the value of one, the value of other increases in the same ratio. It also does not happen that on a decrease in the value of one, leads to a decrease in the value of the other in the same ratio. Some time we may have situations in which the increase in the value of one quantity leads to a decrease in the value of the other or on the decrease in the value of one results in increase of the other". For example – unemployment decreases with increase in industries. The water level in ponds decreases with increase in temperature. Hiring more

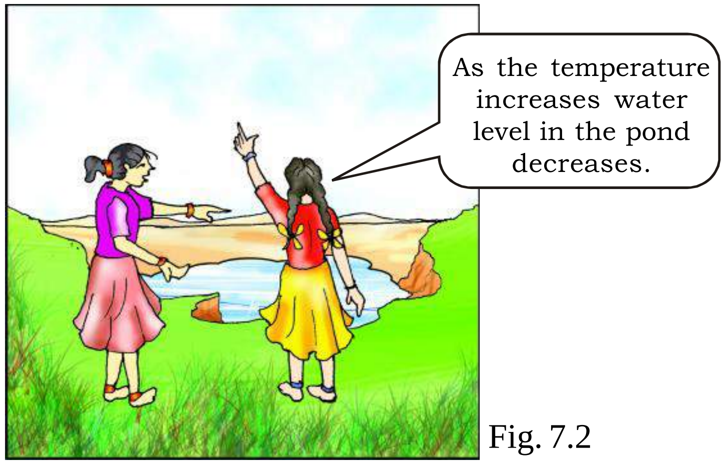


Fig. 7.2

labourers reduces the amount of time required for completion of the task. Such relations are not direct variations. What will these relations be called?

As the temperature increases the water level in the pond keeps falling.

Raju was listening carefully to Mary, he said, "In direct variation, if the value of one increases, the value of the other increases or if the value of one decreases the value of the other also decreases. But in a situation where when one quantity increase the other decrease or when on decreasing one quantity the other increases, we have inverse variation. This is the opposite of direct variation.

Raju was right in his thinking. The opposite of direct variation is called inverse variation. Let us first understand direct variation through some more examples:-

Direct Variation

Example 1: The price of 5 pens is Rs. 20. What will be the price of 10, 15, 20, 25 and 30 pens of the same type?

Solution: If the numbers of pens is denoted by x and their corresponding price by y then we can solve such questions by unitary method and make the following table.

No. of pens x	5	10	15	20	25	30
Price of pens y(Rs.)	20	40	60	80	100	120
Ratio $\frac{x}{y}$	$\frac{5}{20}$	$\frac{10}{40}$	$\frac{15}{60}$	$\frac{20}{80}$	$\frac{25}{100}$	$\frac{30}{120}$
Ratio $\frac{x}{y}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

From the above table it is clear that as the number of pens increases, their corresponding price also increases. In each case the ratio between number of pens and their price remains the same ($\frac{1}{4}$). An increase of this kind where the price increases in direct proportion to the number of pens is called **direct variation**.

Activity 1

In the following tables, number of balls along with their prices are given. On the basis of this fill the blank boxes in the table and answer the questions given below the table.

No. of balls x	10	6	4	3	2	1
Price (Rs.) y	30			9		3
$\frac{x}{y}$	$\frac{10}{30}$			$\frac{3}{9}$		$\frac{1}{3}$
Ratio $\frac{x}{y}$	$\frac{1}{3}$			$\frac{1}{3}$		$\frac{1}{3}$

(1) What is the price of 6 balls?

(2) What is the price of 4 balls?

(3) What is the price of 2 balls?

(4) What is the nature of variation between the number of balls and their price?

The decrease in the number of balls is resulting reducing the corresponding price by a constant ratio i.e. number of balls is changing together with the corresponding price and their ratio is the same $\frac{1}{3}$. Therefore the relation between them is called direct variation.

Activity 2

The value of y for each x is given in the following table. Which of the following values have direct variation? Identify and write.

x	4	9	12	15	20	7	13
y	28	63	72	105	100	49	91
$\frac{x}{y}$	----	----	----	----	----	----	----

Write down five examples of direct variation related to your daily life. If two variables have direct variation the ratio between them is always equal. The ratio is a constant or non variable quantity.

Or if x and y are in direct variation then $\frac{x}{y} = k$ (constant)

If the ratio between x_1, y_1 is same as the ratio between x_2, y_2 then $\frac{x_1}{y_1} = \frac{x_2}{y_2} = k$

Activity 3

If x and y are in direct variation then fill the blanks in the following table:

S.No.	x	y	$\frac{x}{y}$
(1)	3	18	---
(2)	25	---	$\frac{1}{6}$
(3)	9	---	$\frac{1}{6}$

S.No.	x	y	$\frac{x}{y}$
(4)	---	24	$\frac{1}{6}$
(5)	7	42	---
(6)	---	66	$\frac{1}{6}$

Example 2: The price of 3 kg of wheat is Rs. 36 .Find the price of 18 kg of wheat.

Solution: Since on increasing the amount of wheat its corresponding price will also increase. This relation is of a direct variation. Let the price of 18 kg of wheat be Rs. x. It can be written in the form of the following table

Quantity wheat (kg)	3	18
Price (Rs)	36	x

Here ratio between 18 and x would be the same as between 3 and 36.

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{3}{36} = \frac{18}{x}$$

Or $3 \times x = 18 \times 36$ (cross multiplying)

Or $x = \frac{18 \times 36}{3}$

Or x = Rs. 216

Thus the price of 18 kg of wheat is Rs. 216

Example 3: The distance covered by a train in 2 hours is 120 km. Find the distance that the train will cover in 5 hours with the same speed?

Solution: Since the distance covered will increase with the increase in time. Therefore here the relation is of direct variation. Let the distance covered in 5 hours be x km. It can be written in the form of the following table:

Time (in hours)	2	5
Distance covered (in km)	120	x

Here the ratio between 5 and x is equal to the ratio between 2 and 120.

Or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$\therefore \quad \frac{2}{120} = \frac{5}{x}$

Or $2 \times x = 5 \times 120$ (cross multiplying)

Or $x = \frac{5 \times 120}{2}$

$\therefore$ x = 300 km

Therefore, the distance covered by train in 5 hours is 300 km.

Example 4: A man gets Rs. 32 by working for 4 hours, what will he get by working for 7 hours?

Solution: Since when you work for more time the amount of labor is more. Here the relation is of direct variation. Let the man gets Rs x for working 7 hours.

Then table will be –

Time (in hours)	4	7
Wages (Rs.)	32	x

Here the ratio between 7 and x is the same as that between to 4 and 32.

Or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$\therefore \quad \frac{4}{32} = \frac{7}{x}$

Or $4 \times x = 7 \times 32$ (cross multiplying)

Or $x = \frac{7 \times 32}{4}$

$x = 56$

Or the man will get Rs. 56 for working 7 hours.

Example 5: If the weight of 6 letters is 45 grams than how many letters would have a weight of $1\frac{1}{2}$ kg?

Solution: Since, on increasing the number of letters their weight will also increase in the same ratio. Therefore the relation is of direct variation. Let the weight of x .... 1½ kg or 1500 gm.

(Here weight is changed into same units (grams))

Table is as follows:

No. of letters	6	x
Weight (in gm)	45	1500

Here ratio between x and 1500 is same as 6 and 45.

Or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$\therefore \quad \frac{6}{45} = \frac{x}{1500}$

Or $6 \times 1500 = 45 \times x$ (cross multiplying)

Or $x = \frac{6 \times 1500}{45}$ or $\frac{6 \times 1500}{45} = x$

$\therefore$ x = 200 letters

Or the weight of 200 letters is $1\frac{1}{2}$ kg.

Exercise 7.1

Q1. In the following tables are x and y are in direct variation. Find the value of the constant ratio also.

Table I.

x	7	9	13	21	25	30	41
y	21	27	39	63	75	90	123

Table II.

x	2.5	7.5	11	17.5	19
y	2.5	7.5	11	17.5	19

Table III.

x	5	6	7	8	9	11
y	25	24	35	40	50	66

Table IV.

x	1	2	3	4	5
y	2	1	2/3	½	2/5

Q2. Fill in the blanks in the following direct variation table:-

No. of workers	1	2	……..	4	5	……..
Wages (Rs.)	50	……..	150	200	……..	300

Q3. In following table x & y are in direct variation. Find the value of constant ratio k.

x	2	4	8	16	32
y	14	28	56	112	224
$k = \frac{x}{y}$	-------	-------	-------	-------	---------

Q4: A car runs 600 km in 3 hours. How far will it travel in 5 hours?

Q. 5: Which of the following quantities are in direct variation?

(i) Number of objects and their prices.

(ii) Number of Mathematics books of class 7^{th} and their prices.

(iii) Area of a field and its price

(iv) Quantity of milk (in lt.) and its price.

(v) Number of labourers and the total number of days in which the work is completed.

(vi) Speed and time when the distance travelled remains the same.

Q6: Price of 15 tickets (of equal price) is Rs.18. How many tickets of the same price can be bought in Rs. 72?

Q 7: A car covers 432 km in 48 lt of petrol. What distance will it travel in 20 lt. of petro1?

Q8. The cost of 2 dozen oranges is Rs. 48. Find the cost of 108 oranges?

Q9: A machine prints 200 pages in 5 min. How much time will it take to print 2 x 10^3 pages?

Q10. A cyclist covers 12km in 3 hours. How much time will be taken by him to cover 20 km?

Q11. The wages of 25 labourers is Rs. 1250. What is the amount of wages for 40 labourers?

Q12. A labourers gets Rs. 806 for working 13 days. How many days would he have worked if he was paid Rs. 1798?

Q13. Namrata takes 1225 steps to cover a distance of 100m. How much distance will she cover in 2835 steps?

Q14. A dealer gets a commission of Rs. 73 for selling items worth Rs.1000/-. How much commission will he get on selling items of Rs. 100?

Q15. The thickness of 500 sheets of paper is 3.5 cm, find the thickness of 275 sheets.

Q16. A man can read 180 words in 1 min. How much time will it take him to read 768 words?

Q17. Sunita types 1080 words in 1 hour. Find her rate of typing per minute.

Q18. 25 labourers make a 7.5 km long road in one week. How many labourers will make a road of 10.2 km in 1 week?

Q19. The weight of 10 bags of cement is 4.5 quintal. What will be the weight of 35 such bags?

Q20. The speed of a car is 60 km per hour. The distance traveled and corresponding time taken are in direct variation (in the following table). Match the correct distance and time in the columns.

Distance (KM)		Time (in hours)
(i)	120	(a) 3
(ii)	180	(b) 2
(iii)	210	(c) 4
(iv)	240	(d) 3½

INVERSE VARIATION

In our daily life, we see sometimes that on increasing a quantity another quantity starts decreasing in a constant ratio or with a decrease in the first quantity the second quantity starts to increase in a constant ratio. Such proportional relations are called inverse ratio.

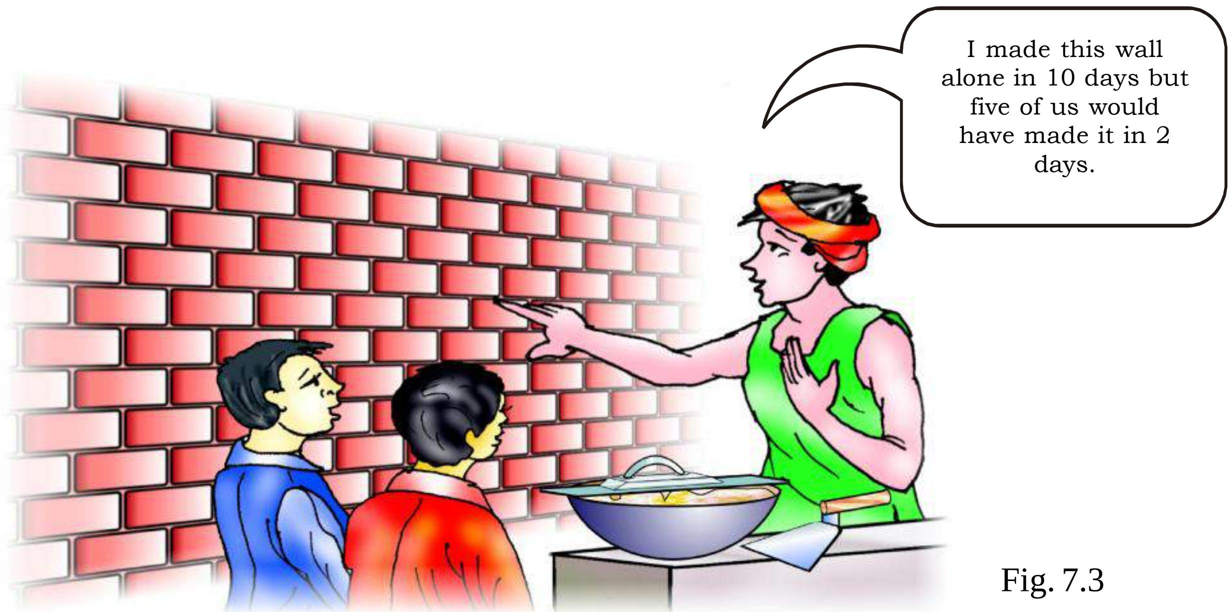


Fig. 7.3

I made this wall alone in 10 days but five of us would have made it in 2 days.

Let us see an example:-

The number of labourers and the days required by them to spread sand on the road is given below:

No. of labourers (x)	5	10	15	20	30
No. of days (y)	60	30	20	15	10

In above table the number of labourers (x) and number of days (y) are given. Can you find a relation between each x and each y? A relation that is constant for all values x and corresponding y.

Ashu considered the examples and thought that doubling the number of labourer the number of days became half and when number of labourers were tripled the numbers of days became $\frac{1}{3}$rd. In the same way if the number of labourers is made 10 times then the number of days will become $\frac{1}{10}$. If we multiply the values of x and y, we will get a constant. In direct relation as $\frac{x}{y}$ or x/y is constant similarly here x × y or $\frac{x}{1/y}$ or $x : \frac{1}{y}$ is a constant. This is the inverse of direct ratio therefore it is called inverse ratio.

Do you agree with Ashu?

Here we find that the manner in which the number of labourers increases, the number of days decreases in the opposite ratio and the number of days increases in the opposite ratio to the manner in which number of labourers decrease. Such a ratio is called opposite ratio or inverse ratio. In the above example the number of labourers is in inverse ratio with the number of days taken i.e. the variation between both quantities is an inverse ratio.

Activity 4

A passenger train covers a distance in 4 hours with a speed of 12 km/hour.
Answer the following:
(i) If the speed is increased to 24 km/hr, how much time will it take to cover the same distance?
(ii) If the speed is increased to 36 km/hr, how much time will it take to cover the same distance?
Also fill up the following table.

Speed (km/hr) on increasing				
Time (in hours)				

Result: If the speed is increased, time taken is ....

Speed (km/hr)	48	32	16	6
Time (in hours)	1	-----	-----	-----

Result: If the speed is decreased, time taken is ....

Construct five example of inverse ratio from your daily life.

Let us discuss one more example:

A book can be finished in 15 days, if 16 pages are read every day. If 8 pages are read in a day, how many days will it require to finish the book? If 12, 15, 24 pages are read in a day, how many days will it require to finish the book in each case? If the number of pages read each day is denoted by x and the number of days taken to finish the book are denoted by y, the answer to the questions can be written in the following table:

No. of pages read each day (x)	16	8	12	15	24
No. of days (y)	15	30	20	16	10

$\frac{1}{y}$ $\quad \frac{1}{15}$ $\quad \frac{1}{30}$ $\quad \frac{1}{20}$ $\quad \frac{1}{16}$ $\quad \frac{1}{10}$

$x:\frac{1}{y} = \frac{x}{1/y}$ $\quad \frac{16}{1/15}$ $\quad \frac{8}{1/30}$ $\quad \frac{12}{1/20}$ $\quad \frac{15}{1/16}$ $\quad \frac{24}{1/10}$

x × y in standard form	16×15 =240	8×30 =240	12×20 =240	15×16 =240	24×10 =240

$$x:\frac{1}{y} = x \Big/ \frac{1}{y} = xy = 240 = \text{k (say)}$$

Here numbers of pages read per day are in inverse ratio to the number of days taken. The inverse relation between the numbers of pages read per day and numbers of days taken gives a constant value for the product each time. In other words, we can say that the product of the number of pages read per day x and the corresponding number of days taken y is a fixed quantity i.e. xy=k

In general for any values x_1 and x_2 of number of pages read per day and corresponding values y_1 and y_2 of the days taken we conclude. $x_1y_1=x_2y_2$

Conclusion: We find that if the relation between two variable quantities is such that on increasing one quantity the other starts decreasing or on decreasing one quantity the other starts increasing and the product of both quantities remains constant then the relation between them is called inverse variation. Mathematically, we can write that if x and y are in inverse variation then $\boxed{xy=k}$

If x has two values x_1, x_2 and the corresponding y has y_1 and y_2 then $\mathbf{x_1y_1=x_2y_2}$

Activity 5

In which of the following tables x and y are in inverse variation-

(i)

x	6	2	3	18
y	3	9	6	1

(ii)

x	40	20	16	10	2.5
y	2	4	5	8	32

(iii)

x	10	5	2	4
y	3	6	15	8

(iv)

x	9	10	12	15
y	5	4.5	3.75	3

Activity 6

If x and y are in inverse variation, fill in the blanks in the following table.

(i)

x	9	18	20	---	30
y	4	---	---	1.5	----

(ii)

x	16	8	--	---
y	3	---	12	24

(iii)

x	20	50	25	----
y	---	4	----	5

Let us see some more examples of inverse variation:-

Example 6: 12 labours can build a wall in 10 days. How many days will 20 labours take to build the same wall?

Solution: As by increasing the number of labourers, the time taken to complete the work will decrease. Hence this is a case of inverse variation.

Let 20 labours make the wall in y days. The table will be made as follows:-

No. of labourers (x)	12	20
No. of days (y)	10	y

For inverse variation

$$x_1y_1 = x_2y_2$$

$$\therefore\ 12\times10 = 20\times y$$

$$\text{or}\quad \frac{12\times10}{20} = y$$

$$\text{or}\quad 6 = y \qquad \text{or} \qquad y = 6$$

$\therefore$ 20 labours will complete the wall in 6 days.

Example7. A hostel has 24 days food for 200 students.

If 100 more students join that hostel, then how many days will the food last?

Solution: The joining of 100 students in the hostel, means that the total number of students = 200 + 100 = 300.

Since, quantity of food is the same, the increase in the numbers of students means the food will be consumed faster therefore this is of an inverse variation.

Let the food be finished in y days.

The table is as follows:-

No. of students (x)	200	300
No. of days (y)	24	y

In inverse variation

$$x_1y_1 = x_2y_2$$

$\therefore$ $200 \times 24 = 300 \times y$

or $\frac{200 \times 24}{300} = y$

or $16 = y$ or $y = 16$

$\therefore$ The food will finish in 16 days.

Example8. Shallu goes to school on a cycle with an average speed of 12 km/hr. She reaches the school in 20 min. If she wants to reach the school in 15 min, what should be her average speed?

Solution: Since to reach school in less time the speed has to increase, therefore this is a case of inverse variation. Let the average speed be x km/hr. The table is :-

Sped (km/h) (x)	12	x
Time (in min y)	20	15

In inverse variation

$$x_1y_1 = x_2y_2$$

$\therefore$ $12 \times 20 = x \times 15$

or $\frac{12 \times 20}{15} = x$

or $16 = x$

$\therefore$ $x = 16$ Km/hr

Therefore the average speed of Shallu should be 16 km/hr to reach the school in 15 min.

Exercise 7.2

1. If x and y are in inverse ratio than fill in the table appropriately.

x	8	6	4	-----	36
y	9	12	-----	10	----

2. Fill in the blanks in variation table.

Speed (in km /hr)	4	8	----	-----	64
Time taken (in min)	-----	40	20	10	------

3. 10 labours finish a task in 2 days. How many days will 2 labours take to finish the same task?

4. 45 labours finish the work in 27 days. How many labourers will complete the same work in 15 days?

5. A bus takes 6 hours to cover a certain distance with a speed of 30 km/hr. What speed the bus should have to cover the same distance in 4 hours?

6. 40 horses eat 1 quintal of grain in 7 days. How many horses will eat the same quantity of grain in 28 days?

7. A hostel has 15 days of food for 300 students. If 200 students leave the hostel due to vacations then for how many days will the food last?

8. A military camp has 25 days of food for 700 soldiers. Due to the arrival of some more soldiers the food finishes in 20 days. Find the number of new soldiers that arrived in the camp.

9. A man completes a book in 15 days by reading 8 pages per day. If he reads 12 pages daily, then how many days will he take to finish the book?

10. A military camp has 21 days of ration for 105 soldiers. If 42 more soldiers arrive in the camp, then how many days will the ration last?

11. Which are the inverse variation in the following:-

    (i) Number of books bought and the cost of each book.

(ii) Distance traveled by the bus and the cost of petrol consumed.

(iii) Time taken by a cycle to cover a fixed distance with its speed.

(iv) The number of men employed to make a bridge and the time taken to make the bridge.

(v) Number of students and the weight of sweets distributed among them if 40 kg sweets are to be distributed.

(vi) Wages and hours of work

(vii) Number of objects and their total price.

12. On 26th January 100 gm sweets are distributed per student among 800 students. If the same amount of sweets is to be distributed among 1000 students, how many grams of sweets will each student get?

13. If a tap takes 1 hr in filling 640 lt. water, then a water tank is filled in 10hours. If the same tank was filled by another tap in 8 hours, how many liters will be filled in 1 hour by this tap?

We have learnt

1. When two variable quantities are related to each other such that on increasing (or decreasing) the value of one variable, the value of the other increases (or decreases) in the same ratio, then they are in direct variation.

2. When two variables quantities x and y are in direct variation then their ratio is a constant or $\boxed{\frac{x}{y} = k}$

3. If two quantities are in direct ratio, and one quantity has values x_1, x_2 and the corresponding values for the other quantity are y_1 and y_2 respectively. Then $\boxed{\frac{x_1}{y_1} = \frac{x_2}{y_2}}$

4. If two variable quantities are related to each other such that on increasing (or decreasing) the value of one variable, the value of the second variable quantity

decreases (or increases) in the same ratio, then the two quantities are in inverse ratio.

5. When two quantities are in inverse ratio then their product is a constant i.e. $xy = k$ where k is a constant.

6. If two quantities are in inverse ratio, and one quantity has values x_1, x_2 with the corresponding values for the second quantity y_1 & y_2 respectively then, $x_1 \times y_1 = x_2 \times y_2$

Chapter—8

FACTORS AND FACTORIZATION OF ALGEBRAIC EXPRESSIONS

Introduction

The teacher was distributing toffees to the students of class VIII on the occassion of Republic Day. All the students are careful about the fact that none of their friends got any extra toffee. They wanted the toffees to be equally distributed among themselves. Lata started thinking that her class had 10 students and if every student got 6 toffees no toffee would be left. Had there been 15 students in her class, each one would have got 4 toffees. Lata took out her notebook and started wrting down the calculation. To how many students can 60 toffees be equally distributed so that no toffee is left and in how many ways? She found that 60 toffees can be distributed in such a way that no toffee would be left.

Rama was watching Lata's activities. She told Lata that all the numbers that she had written in her notebook are completely divisible to 60. Therefore, all these numbers are factors of 60 and are also the factors possible for 60.

How many multiple factor ?

Umesh aksed lata whether factors for the algebraic expression 5 ab can also be determined in a similar manner ? Lata wrote in her notebook and showed that 5, a and b are completely divisible to 5 ab. Rama was listening to the conversation and she remembered that they have already studied this rule that every number is completely divisible by 1 and by it self, so 1and 5ab willl also be divisible factor along with 5a and 5b that would completety divide 5ab Thus :

The multiple factrors of 5ab = 1, 5, a, b, 5a, 5b and 5ab. Rekha said, "You have left one multiple factor out Everyone started thinking. Then Ramesh said, "Yes, ab willalso be one, multiple factor. Thus, the factors for 5ab are 1, 5, a, b, ab, 5a, 5b and 5ab.

Rama, suggested, "Let us play a game of finding out factors, Rohit wrote the factors of $12x^2$ in his notebook and found that the factors of $12x^2$ = 1, 2, 3, 4, 6, $2x^2$, $3x^2$, $4x^2$, $6x^2$ and $12x^2$. But Rama told Rohit that all the factors of $12x^2$ have not been written yet. Rama first wrote the factors for the factors and then for all the algebraic expression to get the complete factors of $12x^2$ like this :

Factors of 12 = 1, 2, 3, 4, 6 and 12

Factors of x^2 = 1, x, x^2

Therefore, total factors for $12x^2$ = 1, 2, 3, 4, 6, 12, x, $2x$, $3x$, $4x$, $6x$, $12x$, x^2, $2x^2$, $3x^2$, $4x^2$, $6x^2$, and $12x^2$.

How to recognize all the factors ?

Rohit said, "alright, but there must be some way to write many factors for a monomial algebraic expression that verifies that all the factors have been actually written". Lata said, "Let us go and ask our Maths teacher about it and also get the factors of $12x^2$ that Rama has written down cheked by her."

The teacher said, "you have rightly written all the factors of $12x^2$. Very good. Now learn to write all the factors of such algebraic expressions, we write all the factors for the constant in horizontal rows and the factors of the variable in vertical columns. Thus we shall get a multiplicative table when we fill up this table, we get all the factors of a monomial." The teacher made a table like thus :

Table 8.1

x	1	2	3	4	6	12
1	1	2	3	4	6	12
x	x	$2x$	$3x$	$4x$	$6x$	$12x$
x^2	x^2	$2x^2$	$3x^2$	$4x^2$	$6x^2$	$12x^2$

The teacher asked the students whether all the expressions mentioned in the table are completely divisible to $12x^2$? Verify yourself whether the table contains all the factors that Rama wrote for $12x^2$.

Some more example

The teacher asked Lata to findout the factors for $10ab^2$ in the above mentioned manner. Lata made the table for $10ab^2$ on the black board and asked all the students to verify it.

The factors of 10 = 1, 2, 5, 10.

Factors of ab^2 = 1, a, b, b^2, ab, ab^2.

Therefore $10ab^2$ will have the following possible factors :

Table 8.2

x	1	2	5	10
1	1	2	5	10
a	a	2a	5a	10a
b	b	2b	5b	10b
b^2	b^2	$2b^2$	$5b^2$	$10b^2$
ab	ab	2ab	5ab	10ab
ab^2	ab^2	$2ab^2$	$5ab^2$	$10ab^2$

All the expression in the table are completely divisible to $10ab^2$ and the remainder is zero. Now the students got one method for finding the factors of monomials.

Activity 1

Write the factors of the algebraic expressions given below in your notebook.

$8x$, $4a^2$, $6ab$, xy^2, $3x^2y$, $6y^2$

Indetify the similar common factors

Rohit wrote the factors of 6ab the teacher wrote them on the blackboard :

6ab = 1, 2, 3, 6, a, 2a, 3a, 6a, b, 2b, 3b, 6b, ab, 2ab, 3ab, 6ab.

The teacter wrote the factors of $4a^2$ that Rajesh found out on the blackboard below the factors of 6ab.. Hence the factors of

$4a^2 = 1, 2, 4, a, 2a, 4a, a^2, 2a^2, 4a^2$.

Now, the teacher asked the class whether there was any similarity between the factors of both the algabraic expressions. Rama said that the total number of factor for 6ab and $4a^2$ are different, but some factors are similar in both the expressions. There are common factors 1, 2, a, $2a$.

The teacher said, "right, and among these 2a is the largest or greatest common factor. Since you know that the greatest common factor is the Highest common factor, 2a will be known as the highest common factor for 6ab and $4a^2$.

The Highest common factor for two or more than two monomials is that greatest algebraic expression that is completely divisible to each of the given algebraic expressions.

Now the teacher wrote all the factors for $3x^2y$ and $6y^2$ on the blackboard and asked Praveen to identify the common factors.

The factors of $3x^2y$ = 1, 3, x, $3x$, x^2, $3x^2$, y, 3y, xy, $3xy$, x^2y, $3x^2y$.

The factors of $6y^2$ = 1, 2, 3, 6, y, 2y, 3y, 6y, y^2, $2y^2$, $3y^2$, $6y^2$.

Praveen wrote the common multiples as : 1, 3, y, 3y.

And everyone could see that the highest common factors for expression one was 3y.

Another Method

The students found this method a bit lenghtly. So Rama asked the teacher if there is any other short method to find the Highest common factor for two or more than two monomials ? The teacher said, well, look at this : Suppose we need to find the Highest common factor for $6x^2y$ and $8xy^2$. First we find the highest common factors (H.C.F.) for the coffcients 6 and 8.

The H.C.F. for Coefficients 6 & 8 = 2

The H.C.F. for x and x^2 = x (Item with the lowest exponent for x)

The H.C.F. for y and y^2 = y (Item with the lowest exponent for y)

Therefore $6x^2$ and $8xy^2$ = $2xy$ (the product of all the H.C.F. found above)

Example 1

Find the H.C.F. of $12\,s^3t^2u^3$ and $6s^4t\,u^2$.

Solution

H.C.F. for 12 and 16 = 4

H.C. F. for s^3 and s^4 = s^3

H.C. F. for t^2 and t = t

H.C. F. for u^3 and u^2 = u^2

Therefore the H. C. F. for $12s^3 t^2 u^3$ and $16s^4 t u^2$ = $4s^3 t u^2$.

Example 2

Find the H.C.F. of $20 a^2b$, ab^3c.

Solution

Here the coeffecient for the given algebraic expressions are 20 and 1 respectively.

Thus, H. C. F. for 20 and 1 = 1

H. C. F. for a^2 and a = a

H. C. F. for b and b^3 = b

Here c occures only in the second expression and not in the first.

Therefore, H. C. F. for $20 a^2b$ and $ab^3c = 1.\ a.\ b$ = ab.

Exercise 8.1

1. Find out all the factors for the given algebraic expressions :

   (i) $5t^2$ (ii) $7xy$ (iii) $14l^2m$ (iv) $39\ lmn$.

2. Write down all the factors for the given algebraic expriession and find out their H.C.F.

   (i) $5s$, $2s^2$ (ii) $9m^2$, $3t$ (iii) $6a^2$, $8ab$ (iv) $7m^2$, $6m$

3. Find out the highest common factors for the following :

   (i) $6m^2l, 12ml$ (ii) $24a^2bc, 20bc^2$ (iii) $xy^3z, 10x^2y$ (iv) $14x^3y, 21xz^5$

   (v) $22p^2q^2r, 33pq^2r^2$ (vi) $3xy, 23x^2z$ (vii) 6pqr, 23 xyz

Factors of binomials

You have lerant how to find the factors monmials Now on the basis of your experienes can you find the factors of binomials ? For example, If thrice the number of boys in a class is added to thrice the number of girls, what will be the sum ? will this sum be equal to 3 times the sum of total number of boys and girls ? Verify the answer by taking any number of boys and girls.

Suppose the number of boys is 15 and the number of girls is 18. Thrice the number of boys is 45 and thrice the number of girls is 54. Thus the total becomes 99. While the original number of boys and girls together is 33. And you can see that 99 is 3 times 33.

This means , if the number of boys be taken as x and the number of girls be taken as y, then thrice the number of boys would be 3x and thrice the number of girls would be 3y.

The sum of both would be $3x+3y$. The total number of boys and girls would be $x+y$ and thrice this sum would be $3(x+y)$.

Also, $3(x+y)=3x+3y$

Here in $3x$ & $3y$, 3 is common.

Let us check this for the expression $9+3y$.

Therefore $9+3y=3\times3+3\times y$

$$=3(3+y)$$

Thus, the factors for the expression $3x+3y$ above are $3, (x+y)$ and $3x+3y$ but 3 and $(x+y)$ are such factors whose product is equal to $3x+3y$. Similarly, the factors for 9+3y are 3, (3+y) and 9+3y , but 3 and 3+y are two such factors whose product is (9 + 3y) again.

Do this also

Can you write down the two factors of 12 + 18y as products ?

Here the factors for 12 & 18y, 2, 3 & 6 are common factors.

1. Taking 2 common $12+18y=2\times6+2\times9y=2(6+9y)$. But here 3 can again be found as a common factor from $6+9y$, Therefore

$$12+18y=2\{3(2+3y)\}$$

OR

$$12+18y=6(2+3y)$$

2. If in $12+18y$, 3 is the common factor, then

$$12+18y=3(4+6y) \text{ (Here again 2 is common in } 4+6y\text{)}$$

$$\therefore \quad 12+18y=3\{2(2+3y)\}$$

$$=6(2+3y)$$

Here out of 2, 3, 6 as common factors for $12+18y$, 6 is the highest common factor.

Activity 2

Find out the Highest common factor for the binomials given below and complete the table as shown in the example.

S.No.	Binomial	Writing both the items separately	The gretest common factor between two terms	On writing the binomial as the product of the common factors
1.	$36x+27y$	$36x$ & $27y$	9	$9(4x+3y)$
2.	$33y^2-11xy$			
3.	$15xz-90x^2$			
4.	$8ab+9ac$			

Make such problems yourself and ask your friends to solve them.

Factorization

To write binomials and polynomials as the product of its common factors, we write the greatest common factor (H.C.F.) for each of the expression in the polynomial outside the brackets. The written expression of any algebraic expression as the product of its factors is known as factorization.

For the highest common factor for $2ab+2ac=2a$

$$2ab+2ac=2a\times b+2a\times c=2a(b+c)$$

Thus, On factorizing $2ab+2ac$ we get $2a$ and $(b+c)$ whose product would be $2ab+2ac$.

Example 3

Factorize $4x^2y^2-18xy$

Solution

Here the highest common factor for $4x^2y^2-18xy=2xy\times 2xy-2xy\times 9$.

$$=2xy(2xy-9)$$

Example 4

Factorize the experssion $6ab^2+9a^2b^3+12a^2b^2$

Solution

Here the highest common factor

$$\therefore \quad 6ab^2+9a^2b^3+12a^2b^2=3ab^2\times(2+3ab+3ab^2\times 4a)$$

$$=3ab^2(2+3ab+4a)$$

Factorization of polynomials

Rama had learnt Factorization of binomials quite well. She was thinking how the Factorization of an algebraic expression with many would be determined ?

Do you have an answer to Rama's question ? The teacher explained : To factorize such algebraic expression, we make groups of the compoments. After making suitable groups of the algebraic expressions, we find the common factor. Then, we write them as the product of the factors.

For example : Factorize $ax+by+ay+bx$

Here it would be convenient to put the components a and the components with b in separate group. On writing them together we get :

$$=ax+ay+bx+by$$

$$=a(x+y)+b(x+y)$$ (Here $x+y$ is common in both the items)

$$=(x+y)(a+b)$$

The expression can be Factorized by writing the multiple of items containing x and y separately. Do this yourself. Did you get the same answer ?

Example 5

Facorize $2x^2-6y+4x^2y-12y^2$

Solution

In $2x^2-6y+4x^2y-12y^2$, it would be easy to take $2x^2$ & $4x^2y$ together.

Here, $2x^2+4x^2y$ have $2x^2$ common

$$2x^2+4x^2y=2x^2(1+2y)$$

similarly in $-6y-12y^2$ taking $-6y$ as common we get $-6y-12y=-6y(1+2y)$ and $(1+2y)$ is common is both the expressions.

Therfore the facrors are $(1+2y)(2x^2-6y)$.

But can we have factors of $2x^2-6y$ also.

Here 2 is common. Therefore its factors are 2 and (x^2-3y)

Therefore

$$2x^2-6y+4x^2y-12y^2$$

$$=2(x^2-3y)(1+2y)$$

Example 6

Factorize $2xy + y + 4x + 2$

Solution

$$2xy + y + 4x + 2$$

$$= y(2x+1) + 2(2x+1)$$

$$= (2x+1)(y+2) \text{ [common factor } (2x+1)\text{]}$$

Now solve this question by taking the first and third component and the second & fourth component together.

Exercise 8.2

Q.1. Fill in the blanks :

i. $x^2 + 5x^3 =$ __________ $(1+5x)$

ii. $10a^2 - 12b^2 = 2($____$- 6b^2)$

iii. $27ab^2 + 18abc = 9ab(3b +$ ____$)$

iv. $16xz - 9z^2 - z = ($____ $-$ ____$)$

v. $12ab^2c + 8abc^2 - 10a^2c = 2ac($____ $+$ ____ $-$ ____$)$

Q.2. Factorize

i. $4ax + 6a^2y$

ii. $a^5y + ab^3$

iii. $pq^2r - 2q^2t$

iv. $-5lm^2 - 10l^2mn$

v. $5m^2 - 5n^2$

Q.3. Factorizing the group method

i. $2x^2y + 6xy^2 + 4x + 12y$

ii. $5m^2n - 10mn^2 + 12m - 24n$

iii. $6x^3 + 8x^2 + 9xy + 12y$

iv. $15x^4 + 10x^2y^2 + 12x^2y + 8y^3$

v. $x(x+3) + 8(x+3)$

vi. $3x(x-4)-5(x-4)$

vii. $2m(l-m)+3(l-m)$

Q.4. Solve the following :

i. $x(1-3y^2) =$ ____ $-$ ____

ii. $-17x^2(3x-9) =$ ____ $+$ ____

iii. $2a^2(3a-4a^2) =$ ____ $-$ ____

iv. $9m(m-n) =$ ____ $-$ ____

v. $9t^2(t-7t^3) =$ ____ $-$ ____

We have learnt

1. If an algebraic expression is written as a product of two or more algebraic expressions, then the expressions obtained are known as the factors of the given expression and the way of wrting any algebraic expression in this manner is called factorization.
2. The factorization of any two binomials is done by taking out the common comment of its highest common factors.
3. The H.C.F. of algebraic expressions is the greatest divisor of these expressions.
4. The factorization of algebraic expressions with more than 3 components is done by group method.

Chapter—9

IDENTITIES

Indetities

While solving the exercises related to multiplication of binomials. Fatima thought that if two binomials have both the first and second components similar (not different), then what would happen ? On multiplication, shall we get a new result ? for example, if we multiply (x +y) with (x + y) :

$$(x+y)(x+y) = x(x+y)+y(x+y)$$

$$= x^2 + xy + yx + y^2 \quad (\because xy = yx)$$

$$= x^2 + 2xy + y^2$$

Anu also found it interesting and she also started multiplying expressions like factions.

$$(r+s)(r+s) = r(r+s)+s(r+s)$$

$$= r^2 + rs + sr + s^2 \quad (\because rs = sr)$$

$$= r^2 + 2rs + s^2$$

Fatima & Anu got similar kinds of result. Can you now multiply the binomials given below and see what kind of results you get ?

1. $(p+q)(p+q)$ = ...
2. $(u+r)(u+r)$ = ...
3. $(m+n)(m+n)$ = ...
4. $(r+w)(r+w)$ = ...

What conclusions do you get on multiplying similar binomials ? Write them you have seen that similar binomials when multipled give product like this :

(first component)^2 + 2 x (first component) x (second component) + (Second Component)^2.

Which means if a & b are taken as components :

$(a+b)(a+b) = a^2 + 2ab + b^2$ but $(a+b)(a+b) = (a+b)^2$

Therefore, $(a+b)^2 = a^2 + 2ab + b^2$

Thus, this speicial multiplication relationship would be true for any value of *a* and b. This is known as an identity and it gives the first indentity.

Formula (1) $(\boldsymbol{a}+\boldsymbol{b})^2 = \boldsymbol{a}^2 + 2\boldsymbol{ab} + \boldsymbol{b}^2$

i.e. [(1st component) + (2nd component)]^2 = (1st component)^2 + 2x(1st component) *x* (2nd component) + (2nd component)^2.

This can be explained in the following manner also : -

x	a	b
a	a^2	ab
b	ab	b^2

$(a+b)^2 = a^2 + ab + ab + a^2$

$(a+b)^2 = a^2 + 2ab + b^2$

Verify the rule by taking variables other than *a* & b.

Example - 1

Solve $(3x+4y)^2$ with the help of formula.

Solution

We know that $(a+b)^2 = a^2 + 2ab + b^2$

On comparing $(3x+4y)^2$ with $(a+b)^2$, we have :

$a = 3x,\ b = 4y$

Therefore $(3x+4y)^2 = (3x)^2 + 2(3x)(4y) + (4y)^2$

$= 9x^2 + 24xy + 16y^2$

Example - 2

Solve $(2a+3)^2$ with the help of formula.

Solution

We know that $(x+y)^2 = x^2 + 2xy + y^2$

On comparing $(2a+3)$ with $(x+y)^2$, we have

$x = 2a,\ y = 3$

$$(2a+3)^2 = (2a)^2 + 2(2a)(3) + (3)^2$$

$$= 4a^2 + 12a + 9$$

Example - 3

Use Identity to solve $\left(\frac{1}{2}p+2q\right)^2$

Solution

On comparing $\left(\frac{1}{2}p+2q\right)^2$ with $(a+b)^2$

We have, $a = \frac{1}{2}p$ and $b = 2q$

$$\therefore \quad \left(\frac{1}{2}p+2q\right)^2 = \left(\frac{1}{2}p\right)^2 + 2\left(\frac{1}{2}p\right)(2q) + (2q)^2$$

$$= \frac{1}{4}p^2 + 2pq + 4q^2.$$

Example - 4

Find the value of $(101)^2$

Solution

Here $(101)^2 = (100+1)^2$

Now $(a+b)^2 = a^2 + 2ab + b^2$

On Comparison : $a = 100$ & $b = 1$

Therefore $(101)^2 = (100+1)^2$

$$= (100)^2 + 2(100)(1) + (1)^2$$

$$\Rightarrow \quad (101)^2 = 10000 + 200 + 1$$

$$= 10201$$

Now, if we write $a - b$

Instead of a + b what will happen ?

x	100	1
100		
1		

In a table we can see :

Thus $(a-b)^2 = (a-b)(a-b)$

$= a(a-b) - b(a-b)$

$= a^2 - ab - ba + b^2$

$= a^2 - 2ab + b^2 \quad (\because ab = ba)$

x	a	$-b$
a	a^2	$-ab$
$-b$	$-ba$	$(-b)^2$

This Identify formula (2)

$$\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$$

[(1st componant) - (2nd componant)]^2 = (1st componant)^2 - 2 (1st componant) × (2nd componant) + (2nd componant)^2

Will the identity be true of any other binomial like $(p-q)$ *OR* $(r-s)$ be taken instead of $(a-b)$? Verify.

Example - 5

Solve $(3x-8y)^2$ with the help of identity.

Solution

We know that $(a-b)^2 = a^2 - 2ab + b^2$

On compaing $(3x-8y)^2$ with $(a-b)^2$

$a = 3x, \; b = 8y$

$\therefore \quad (3x-8y)^2 = (3x^2) - 2(3x)(8y) + (8y)^2$

$= 9x^2 - 48xy + 64y^2$

Example - 6

Solve $\left(x - \frac{2}{5}\right)^2$ with the help of identify

Solution

On comparing $\left(x - \frac{2}{5}\right)^2$ with $(a-b)^2$, $a = x, \; b = \frac{2}{5}$

Thus $\left(x - \frac{2}{5}\right)^2 = x^2 - 2(x)\left(\frac{2}{5}\right) + \left(\frac{2}{5}\right)^2$

$$= x^2 - \frac{4x}{5} + \frac{4}{25}$$

Example - 7

Use identity formula to solve $(2x-3y)^2$. Also verify the answer for $x=4$ and $y=2$

Solution

$(2x-3y)^2$ comparing with $(a-b)^2$ give $a=2x,\ b=3y$.

$$\left[\because (a-b)^2\ a^2 - 2ab + b^2\right]$$

Thus $\quad (2x-3y)^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$

$$= 4x^2 - 12xy + 9y^2$$

Verfication

L.H.S. $\quad = (2x-3y)^2 = (2\times4-3\times2)^2 = (8-6)^2 = (2)^2 = 4$

R.H.S. $\quad = 4x^2 - 12xy + 9y^2 = 4(4)^2 - 12(4)(2) + 9(2)^2$

$$= 64 - 96 + 36 = 100 - 96 = 4$$

Hence, it is clear that L.H.S. = R.H.S.

For $x=4$ and $y=2$, then is true

Example - 8

Find the value of 98^2 with the help of Identity formula.

Solution

Here $(98)^2 = (100-2)^2$

Now $\quad (a-b)^2$ compared to 98^2, we have

$a=100,\quad b=2$

$$98^2 = (100-2)^2 = 100^2 - 2\times100\times2 + 2^2$$

$$= 10000 - 400 + 4$$

$$= 9604$$

Now let us find out what happens with the product if the binomials being multipled have the same components but different signs for the first & second expressions.

For Example : If we multiply $(a+b)$ with $(a-b)$

$$(a+b)(a-b)=a(a-b)+b(a-b) \text{ (by distributive law)}$$

$$=a^2-ab+ab-b^2 \quad (\because a\times b=b\times a)$$

$$(a+b)(a-b)=a^2-b^2$$

Hence **Identify formula (3) $= (\mathbf{a+b})(\mathbf{a-b}) = \mathbf{a^2 - b^2}$**

which means

(1st componant + 2nd componant) (1st comp. - 2nd comp) = (1st comp.)^2-(2nd comp.)^2

Table

x	a	b
a	a^2	ab
$-b$	$-ab$	$-b^2$

$$(a+b)(a-b)=a^2+ab-ab-b^2$$

$$=a^2-b^2$$

Now if you multiply $(p+q)$ by $(p-q)$ or $(r+s)$ by $(r-s)$, will you get similar result ? verify.

Example - 9

Solve $(7x+2y)\times(7x-2y)$ by identity.

Solution

On comparing $(7x+2y)(7x-2y)$ with $(a+b)(a-b)$ we have $a=7x,\ \ b=2y$

$$\because (a+b)(a-b)=a^2-b^2$$

Therefore, $(7x+2y)(7x-2y)=(7x)^2-(2y)^2$

$$=49x^2-4y^2$$

Example - 10

Solve $\left(\frac{x}{3}+\frac{y}{5}\right)\left(\frac{x}{3}-\frac{y}{5}\right)$ with the help of identity.

Solution

$\left(\frac{x}{3}+\frac{y}{5}\right)\left(\frac{x}{3}-\frac{y}{5}\right)$ compared to $(a+b)(a-b)$, $a=\frac{x}{3},\ \ b=\frac{y}{5}$

$$\left(\frac{x}{3}+\frac{y}{5}\right)\left(\frac{x}{3}-\frac{y}{5}\right)=\left(\frac{x}{3}\right)^2-\left(\frac{y}{5}\right)^2$$

$$=\frac{x^2}{9}-\frac{y^2}{25}$$

Example - 11

Find the value of 52×48 using identity.

Solution

$$52 \times 48 = (50+2)(50-2)$$

Now on comparing

$$(a+b)(a-b) = a^2 - b^2, \quad a = 50, \quad b = 2$$

$$(50+2)(50-2) = 50^2 - 2^2$$

$$= 2500 - 4$$

$$= 2496$$

Exercise 9.1

1. find the product of the following by using the appropriate identity formula.

   (i) $(2a+3)(2a+3)$ (ii) $\left(\frac{2}{5}m+\frac{3}{4}\right)\left(\frac{2}{5}m+\frac{3}{4}\right)$

2. Find the product with the help of identity rule.

   (i) $(x-5)(x-5)$ (ii) $\left(\frac{3}{2}x-\frac{4}{5}y\right)\left(\frac{3}{2}x-\frac{4}{5}y\right)$

   (iii) $\left(2a-\frac{1}{2}\right)\left(2a-\frac{1}{2}\right)$ (iv) $(x^2-y^2)(x^2-y^2)$

3. Find the values of the following using identity method.

   (i) $(4x+5)(4x-5)$ (ii) $\left(\frac{x}{2}+\frac{y}{3}\right)\left(\frac{x}{2}-\frac{y}{3}\right)$

   (iii) $(-a^2+b^2)(a^2+b^2)$ (iv) $(x^3+y^3)(x^3-y^3)$

4. Solve using the identity method.

   (i) $(2a+5)^2$ (ii) $\left(\frac{2}{3}m^2+\frac{5}{6}n^2\right)^2$

   (iii) $(-8x^3+5y^3)^2$

5. Use the identity method to find the value of the problems given below :

(i) $(41)^2$ (ii) $(69)^2$

(iii) $(97)^2$ (iv) $(84)^2$

6. Use the identity method to find the value of :

(i) 105×95 (ii) 92×88

(iii) 503×497

7. Find the value of 'x' when :

(i) $6x=(28)^2-(22)^2$ (ii) $3x=(17)^2-(14)^2$

(iii) $19x=3^2-16^2$ (iv) $43x=28^2-15^2$

8. Solve $(3x-5y)^2$ and verify the answers for $x=4$ and $y=2$.

9. Solve $\left(\frac{x}{3}+\frac{y}{5}\right)^2$ and verify the answers for $x=9$ and $y=12$.

Observe these also :

1. $(7-5)\times(7-5)=2\times2=4$

$(5-7)\times(5-7)=(-2)\times(-2)=4$

2. $(8-4)\times(8-4)=4\times4=4\times4=16$

$(4-8)\times(4-8)=-4\times-4=16$

Since $(x-y)(x-y)=(y-x)(y-x)$

Take any two numbers and try to find out whether.

$(x-y)^2=(y-x)^2$?

Factorization with the help of identities.

You have studied about identities in the chapter on multiple factors of algebraic expressions. Identities are statements that are true for all variables. So, you can use identities for factorization too.

1. $\mathbf{a^2+2ab+b^2=(a+b)^2}$

$\mathbf{=(a+b)(a+b)}$

2. $\mathbf{a^2 - 2ab + b^2 = (a - b)^2}$

$$\mathbf{= (a - b)(a - b)}$$

3. $\mathbf{a^2 - b^2 \quad = (a + b)(a - b)}$

Let us find out how are these identities useful for factorization.

Example - 12

Factorize $9x^2 + 24xy + 16y^2$

Solution

Here the 1st comp & the 3rd comp. are the squares of 3x and 4y respectively and the middle component is equal to double the product of 3x and 4y.

Therefore $9x^2 + 24xy + 16y^2 \quad = (3x)^2 + 2(3x)(4y) + (4y)^2$

$$\left[\because a^2 + 2ab + b^2 = (a+b)^2\right] = (3x+4y)^2$$

$$= (3x+4y)(3x+4y)$$

Example - 13

Factorize $p^2 - 4pq + 4q^2$

Solution

Here the first and the third comp. are squares of p and 2q respectively and the medial component is negative as well as two times the product of p and 2q.

Therefore $p^2 - 4pq + 4q^2 \quad = (p)^2 - 2(p)(2q) + (2q)^2$

$$\left[\because a^2 - 2ab + b^2 = (a-b)^2\right] = (p-2q)^2$$

$$= (p-2q)(p-2q)$$

We use identity (m) when these are two components in the algebraic expresisons, both are in the form of complete sqares and have a negative sign in between.

Example - 14

Factorize $16m^2 - 25n^2$

Solution

Here the first component is the square of 4m. and the 2nd component is the square of

5n. There is minus sign in between.

Therefore $16m^2 - 25n^2 = (4m)^2 - (5n)^2$

$$\left[\because a^2 - b^2 = (a+b)(a-b)\right] = (4m+5n)(4m-5n)$$

Example - 15

Factorize $(4x-3y)^2 - 100$

Solution

Here the 1st component is a square of $(4x-3y)^2$ and the second component is a square of 10.

Therefore $(4x-3y)^2 - 100 = (4x-3y)^2 - (10)^2$

$$= (4x-3y+10)(4x-3y-10) \left[\because a^2 - b^2 = (a+b)(a-b)\right]$$

Excercise 9.2

Q.1. Factorize using identities.

i. $4x^2 + 20xy + 25y^2$

ii. $25a^2 + 70ab + 49b^2$

iii. $9x^2 + 6x + 1$

iv. $1 + 18a + 81a^2$

v. $p^2 + p + \frac{1}{4}$

vi. $36a^2 + 132ab + 121b^2$

Q. 2. Use Identities to factorize -

i. $a^2 - 10ab + 25b^2$

ii. $16x^2 - 104x + 169$

iii. $121x^2 - 88xy + 16y^2$

iv. $x^2 - 30x + 225$

v. $36a^2 - 12a + 1$

Q. 3. Use identities to factorize.

i. $25a^2 - 49b^2$

ii. $9x^2 - 121y^2$

iii. $64a^2 - 1$

iv. $1 - 16b^2$

v. $\frac{16}{25}m^2 - \frac{4}{9}n^2$

Q. 4. Factorize using identities.

i. $(x+4y)^2-49$

ii. $100-(2a+3b)^2$

iii. $(4x^2+20xy+25y^2)-36$

iv. $9x^2-(4x-5y)^2$

v. x^2y^2-16

Q. 5. Fill in the blanks :

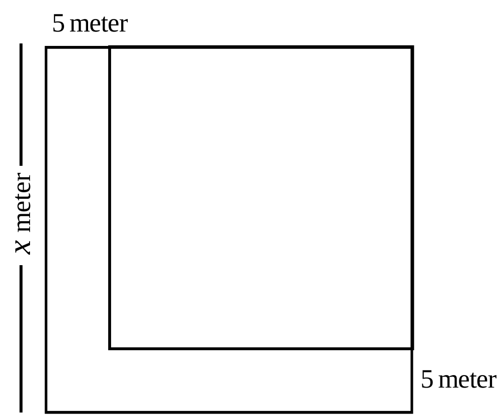
(i) $x^2-25y^2=(x+5y)(..........)$

(ii) $x^2+6x+9=(x+......)(......+3)$

(iii) $4x^2-28x+49=(2x-......)(2x-......)$

(iv) $(a+b)^2-4=(a+b+2)(..........)$

Q. 6 - Ram has a square field. The length of side of the square is x m. He left 5m wide space along length and width of the field for path. What is the remaining area.



Q. 7 - In a temple, the number of flowers offering to an idol is equal to the total numbers of idols in the temple. Two more idol will established over there. Now, also priest offer the equal number of flowers to the equal number of idol but 24 more flowers are needed. Determine the previous number of idols in temple.

Q. 8 - Area of the rectangular field is ($x^2 – 25$)m^2. If the length of the field is (x + 5), find the width.

Q. 9 - The width and length of the rectangular mirror are consequent numbers if measured in feet. The difference between square of these numbers is 05. Find the length and width of the mirror.

We have learnt

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)(a-b) = a^2 - b^2$
4. We also use identities to factorize algebraic expression according to our need.

Chapter—10

POLYGON

Rani made five diagrams with scale and pencil.

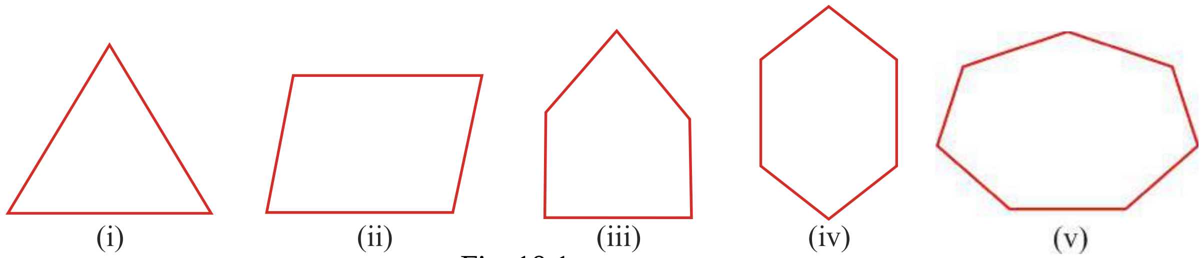


Fig. 10.1

She asked her friends Mary, “Do you recognise these figures ?” Mary said, “In the last class we have learnt that we name figure on the basis of the number of arms or sides.”

In figure 10.1(i), the figure obtained by three sides a tringle; figure 10.1(ii) is made by four sides and is a Quadrilateral figure 10.1(iii) is made up of five sides and is a pentagon".

Rani said to Mary, "Right, you have correctly identified the figures but what name will you give for figure no. 10.1 (iv) and figure 10.1(v) ? Rani said, “You know, fig. no. 10.1(iv) is known as a hexagon and fig. no. 10.1(v) is called a septagon”. All these closed fig. are made up of many sides, therefore closed fig. with three or more than three sides are known as polygons.

Activity 1

Look of the figure below carefully and fill in the blanks.

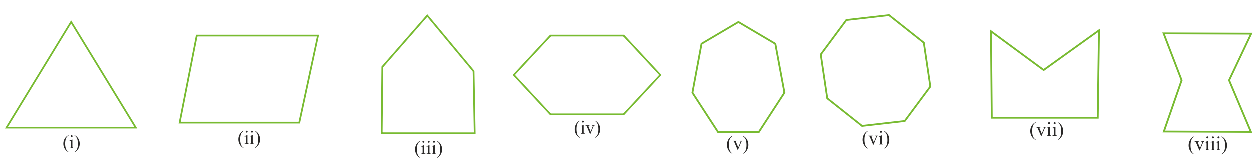


Fig. 10.2

Table 10.1

S. No.	Fig. No.	Name of the fig.	No. of Vertices	No of sides	No. of angles
1.	10.2(i)	triangle	3	3	3
2.	10.2(ii)				
3.	10.2(iii)				
4.	10.2(iv)				
5.	10.2(v)				
6.	10.2(vi)				
7.	10.2(vii)				
8.	10.2(viii)				

In this activity, we saw that the ploygons (iii) and (vii) are both pentagons and fig. no. (iv) and (viii) are both hexagons, but is there any difference between the polygons (iii) and (vii) and in fig. (iv) and (viii) ?

Think about it, discuss with your friends and your teachers and write down the conclusions.

In fig. (vii) and (viii) some vertices are projected inwards whereas in all other fig. all the vertices are projected outwards, Thus we get two types of polygons.

1- Convex ploygons

Those ploygons whose vertex are projected outwards and each internal angle is less than 180^0 are called convex polygons like.

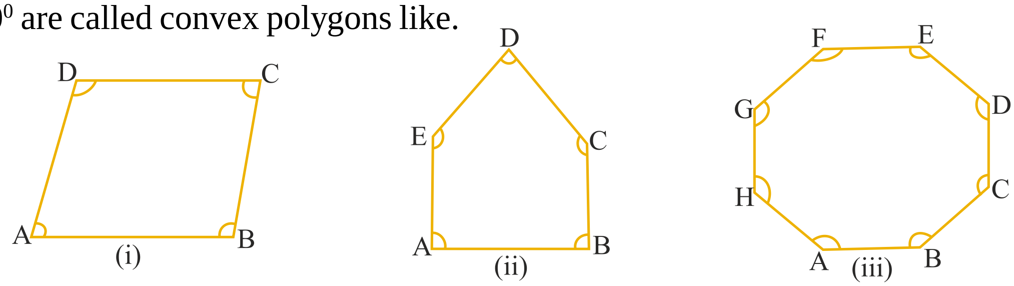


fig. 10.3

2- Concave ploygon

Polygons in which at least one vertex is projected inwards and at least one angle is more than 180° are known as concave polygons.

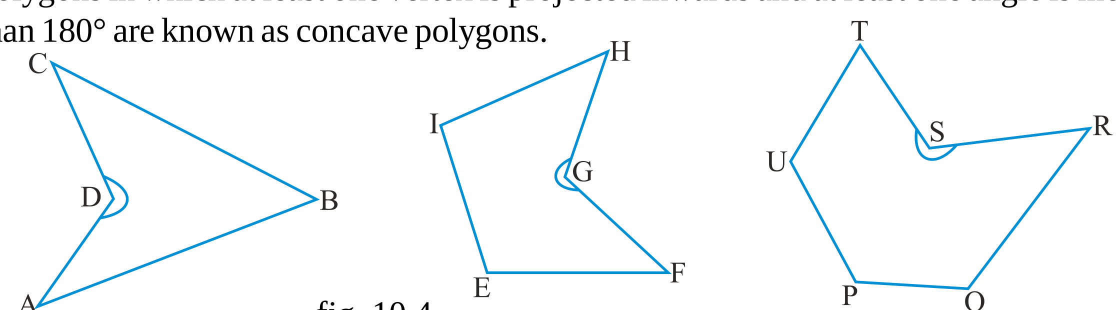


fig. 10.4

What conclusions can we draw from activity (i) observe than draw conclusions.

After observation we get the result that in any polygon the number of vertices, the number of arms or sides and the number of angles are equal to each other.

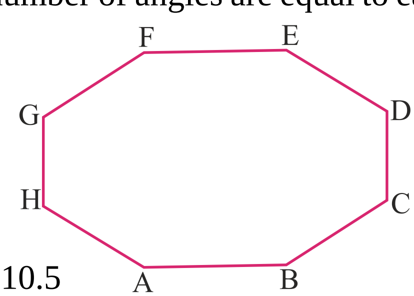


fig. 10.5

The polygons in fig 10.5 is a octagon. Try to identify the number of arms and the diagonals that can be drawn from vertex A with respect to the vertices.

Complete the following table -

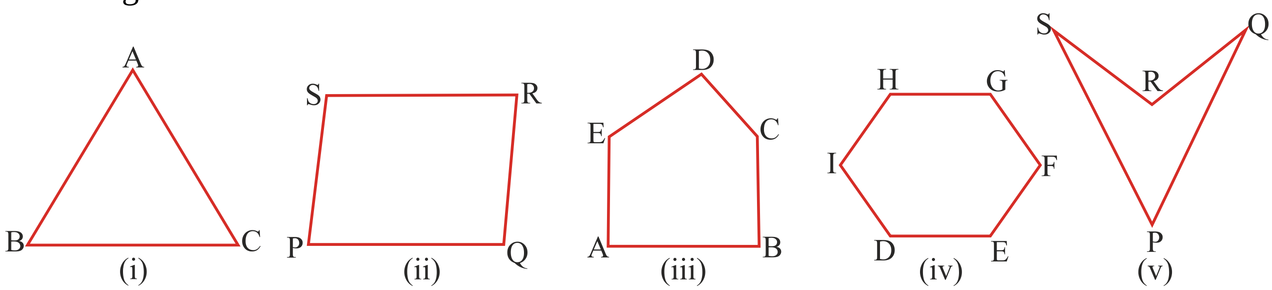
Table 10.2

S.No.	The Name of the vetex to which other vertices have been joined	Name of the line segments	
		arms/sides	diagonals
1.	A	AB, AH	AC, AD, AE, AF, AG
2.	B	BC, BA	BD, BE, BF, BG, BH
3.	C	--------	------------------------

On the basis of this activity write the definition of diagonal of a polygon :

Activity 2

In the fig. given below, draw diagonals from any one vertex and write the numbers of diagonals drawn in the blanks.



________ ________ ________ ________ ________

fig. 10.6

Activity 3

Below are given some quadrilaterals, draw the diagonals and find the answers of the following questions :

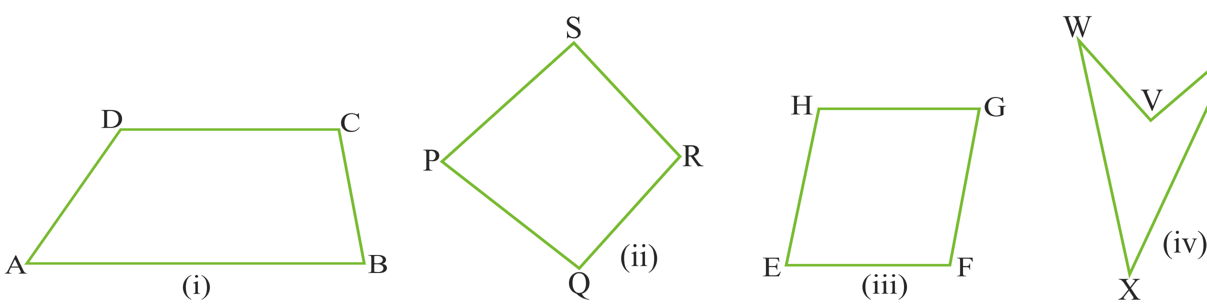


fig. 10.7

1. In how many triangles did the quadrilateral get divided ?
2. Is the sum of the internal angles of the two triangles equal to the sum of internal angles of the quadrilateral ?
3. What is the value of the sum of internal angles of a triangle ?
4. What is the sum of internal angles of a quadrilateral ?

Activity 4

In the fig. given below, diagonals have been drawn.

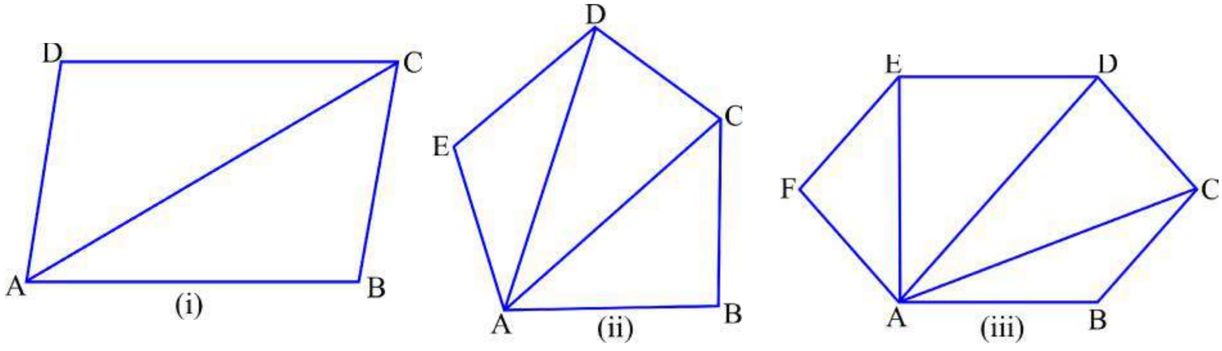


Fig. 10.8

Look at the fig. carefully complete the table given below.

Table 10.3

S.No.	Name of Polygons	No. of sides of a polygons	No of triangles obtained in a polygon	Sum of the internal angles of a polygon= No. of tringles x 180^0	Sum of internal angles
1.	quadrilateral	04	02 or (4 - 2)	2 x 180^0	360^0
2.	pentagon	05	03 or (5 - 2)	3 x 180^0	540^0
3.	hexagon	--------------	--------------	----------------------	-----------
4.	heptagon	--------------	--------------	----------------------	-----------
5.	octagon	--------------	--------------	----------------------	-----------

While doing the activity 4, Mary said that the number of triangles in a polygon is two less than the number of sides.

So, if there are n number of sides in a polygon, then the number of triangles in the polygon would be (n - 2).

Rani said, for (n - 2) triangles, the sum internal angles of a polygon would be (n -2) x 180^0.

Therefore

The sum of internal angles in a polygon with n sides = (n - 2) x 180^0.

Example - 1

If the number of sides in a ploygon is 15, find the sum of its intenal angles.

Solution

The sum of internal angles of a polygon = (n - 2) ×180° where 'n' is the no of sides of the polygons

= (15 - 2) × 180°

= 13 × 180° = 2340° Ans.

Regular and Irregular polygons

You know that the triangle in which all the sides are equal is called an equilateral triangle when each angle in a quadrilateral is of 90° and all the sides are equal, what is that fig. known as ? Can the sides and angles be equal in other polygons also ?

When a polygon has equal sides and its angles are equal, it is known as a equilateral polygon or a regular polygon.

When a pentagon has equal sides and angles it is known as an equipentagon. Will the internal angles of an equipentagon also be equal? Is this a Regular polygons?

Those polygons in which the measures of sides and angles are different are known as irregular polygons.

Different kinds of quadrilateral on the basis of characteristics.

In class VII you have studied about kinds of quadrilaterals, sides of a quadrilateral, External angles, adjacent angles opposite angles and the sum of internal angles.

Let us try to identify the different types of quadrilaterals through an activity.

Activity 5

Complete the table as per the first example .

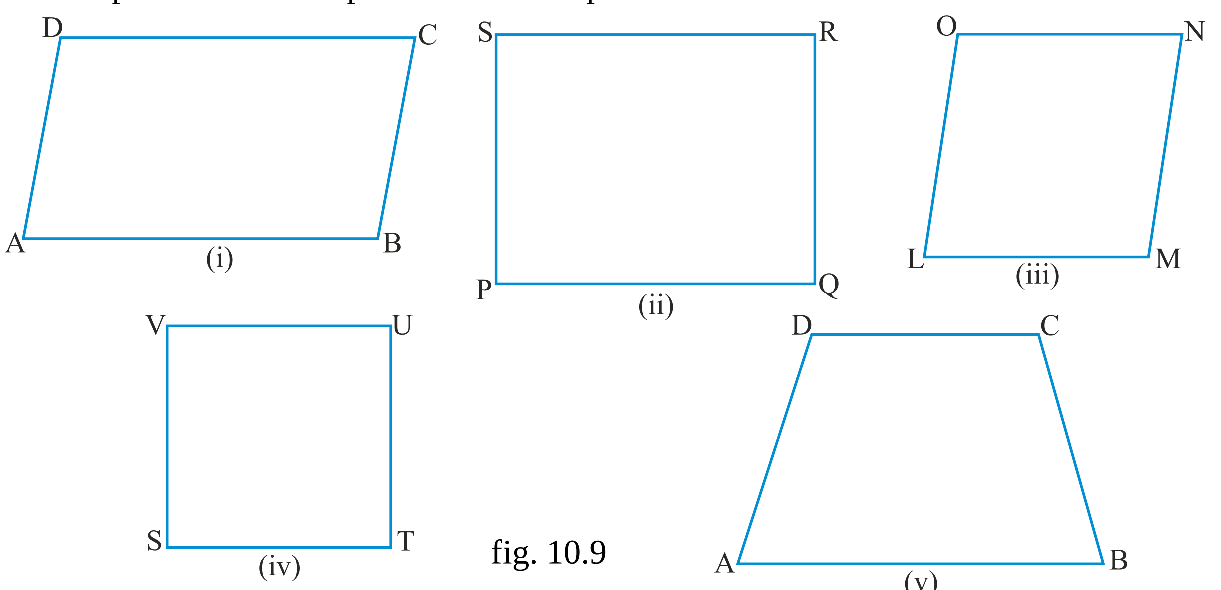


fig. 10.9

Table 10.4

Fig.	Name of Parallel sides	Name of equal sides	Type of quadrilateral
10.9(i)	AB//CD, BC//AD	AB = CD, BC = AD	Parallelogram
10.9(ii)			
10.9(iii)			
10.9(iv)			

Activity 6

Given below are three fig. of quadrilateral. Fill in the blanks by measuring the fig. as shown in the example.

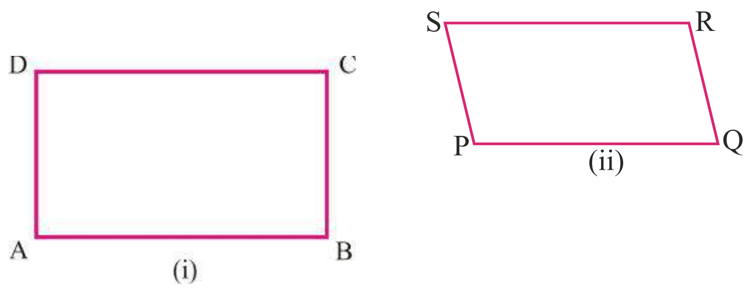


fig. 10.10

Table 10.5

S. No.	Name of Parallelogram	Measures of the sides (In cms.)	Measures of angles (in degrees)
1.	Quadrilateral ABCD	AB=----cm., BC=......cm. CD=----cm., DA=......cm	$\angle A=$ ___ $\angle B=$ ___ $\angle C=$ ___ $\angle D=$ ___
2.	Quadrilateral PQRS	PQ=----cm., QR=......cm RS=----cm., SP =......cm	$\angle P=$ ___ $\angle Q=$ ___ $\angle R=$ ___ $\angle S=$ ___
3.	Quadrilateral EFGH	EF=----cm., FG=......cm GH=----cm., HE =......cm	$\angle E=$ ___ $\angle F=$ ___ $\angle G=$ ___ $\angle H=$ ___

On the basis of the results obtained in the fig. you have seen that in a parallelogram, the parallel sides and the opposite angles are equal in measurement.

So we can conclude that.

The measurement of opposite sides and opposite angles of a parallelogram are equal.

Practice - 1

1. Draw any two parallelograms and write the measurement of the opposite sides and opposite angles. Are they equal ?

Example - 2

ABCD is a parallelogram in which $\angle c = 75^0$. Find the other angles.

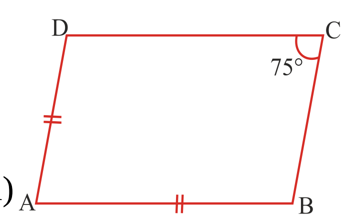


Fig. 10.11

Solution

Given : $\angle c = 75^0$

Therefore $\angle A = 75^0$ (because opposite angle are equal)

Since $\angle A + \angle B + \angle C + \angle D = 360^0$ (why ?)

$$\Rightarrow \quad 75^0 + 75^0 + \angle B + \angle D = 360^0$$

$$\Rightarrow \quad 150^0 + \angle B + \angle D = 360^0$$

$$\Rightarrow \quad \angle B + \angle D = 360^0 - 150^0$$

$$\Rightarrow \quad \angle B + \angle D = 210^0$$

$$\Rightarrow \quad \angle B + \angle B = 210^0 \qquad [\therefore \angle D = \angle B]$$

$$\Rightarrow \quad \angle B = \frac{210^0}{2}$$

or

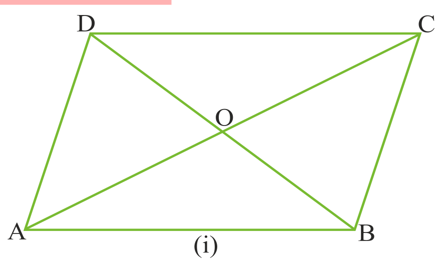
$$\angle B = 105^0$$

Thus, $\angle D = 105^0$

Characteristics of Diagonals drawn in a parallelogram.

You have already learnt how to draw diagonals in a parallelogram. You also known that the diagonals drawn in a quadritateral intersect each other. Let us examine the features of the diagonals of a quadrilateral.

Activity 7



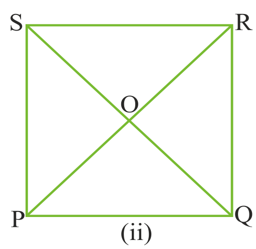


fig. 10.12

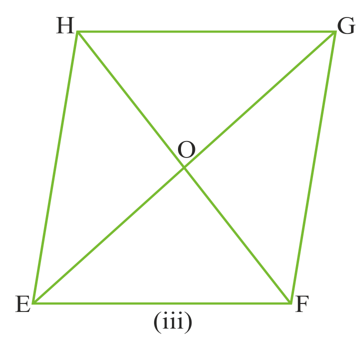


Table 10.6

S.No	Name of the parallelogram	Measurement of the line segments in cm.	Is the point of intersection 'O' the midpoint of the diagonals?
1.	Parallelogram ABCD	AO =___ cm. OC=___cm OB =___ cm. OD=___cm	
2.	Parallelogram PQRS	OP =___ cm. OR=___cm OQ =___ cm. OS=___cm	
3.	Parallelogram EFGH	OE =___ cm. OG=___cm OF =___ cm. OH=___cm	

Conclusion

Draw many such parallelograms and examine your conclusions whether the diagonals of the parallelogram bisect each other.

Give reasons for the questions below -

1. Is every square a parallelogram ?

----

2. Is every rectangle a parallelogram ?

----

3. Is every quadrilateral a parallelogram ?

----

4. Then in all the above cases do the diagonals bisect each other ?

Practice - 2

Draw a quadrilateral is which the diagonals bisect each other.

Activity 8

Given below are some figures of rectangle and squares. Measures the diagonals in these figures and fill in the blanks.

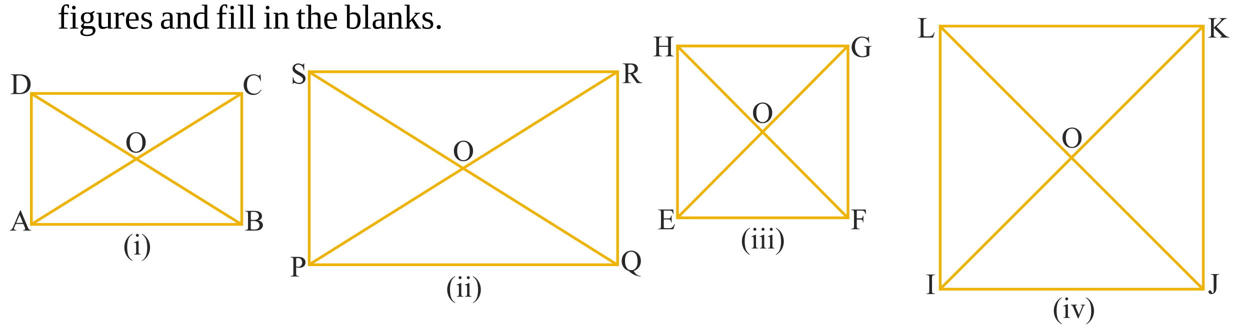


Fig. 10.13

Table : 10.7

Fig No.	Name of the quadrilateral	Measurement of diagonals (in cms.)	Mesurement of line segments (in cms)	Are diagonal equal to each other Yes or No	The diagonals bisect each other Yes or No
10.13(i)	Rectangle ABCD	AC=.....BD=....	OA=.....OC=.... OB=.....OD=....		
10.13(ii)	Rectangle PQRS	PR=.....QS=....	OP=.....OR=.... OQ=.....OS=....		
10.13(iii)	Square EFGH	EF=.....FH=....	OE=.....OG=.... OF=.....OH=....		
10.13(iv)	Square IJKL	JK=.....JL=....	OI=.....OJ=.... OJ=.....OL=....		

In every situation, we conclude that " The length of the diagonals in a square or a rectangle are equal to each other and they bisect each other."

Practice - 3

Draw different measurement of squares and rectangles with their diagonals and verify their characteristics.

Activity 9

Below are given some squares and rectagles. Draw line segments to join their diagonals and measure the angles at their points of inter section.

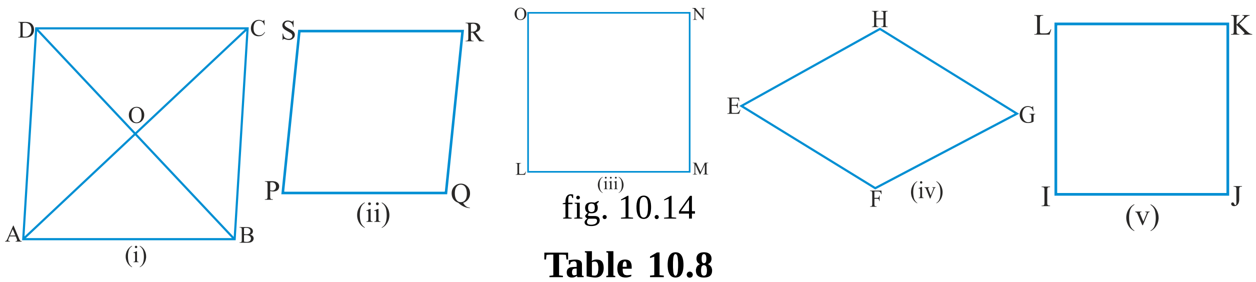


fig. 10.14

Table 10.8

S. No.	Figure No	Name of the quadrilateral	Measurement of the angle at the point of inter section of the diagonals
1.	10.14(i)	parallelogram ABCD	$\angle AOB =$ ____ $\angle BOC =$ ____ $\angle COD$ ____ $\angle DOA =$ ____
2.	10.14(ii)		
3.	10.14(iii)		
4.	10.14(iv)		
5.	10.14(v)		

What is the value of the angles at the point of intersection of the diagonals ?

Thus, we find that "The diagonals of parallelograms and squares intersect each other at right angles".

Table 10.9

S. No.	Type of quadrilateral	Arms/sides	Angles	Diagonals
1.	Parallelogram	Opposite sides are parallel and equal.	Opposite angles are equal.	Diagonals bisect each other at the point of intersection.
2.	Rectangle	Opposite sides are parallel and equal.	Opposite angles are equal. (each angle measures 90^0)	Diagonals are equal and bisect each other at the point of intersection

3.	Square	All sides are equal opposite sides are parallel	Opposite angles are equal (each angle measures 90°)	Diagonals bisect each other at right angles & the diagonals are also equal to each other
4.	Rhombus	All sides are equal opposite sides are parallel	Opposite angles are equal.	Diagonals bisect each other at right angles.

Exercise 10.1

1. Each question has four options. Choose the correct alternative answer.

   a. The two diagonals are equal in

      (i) a parallelogram (ii) quadrilateral

      (iii) rectangle (iv) trapezium

   b. In which quadrilateral are all angles equal to each other ?

      (i) parallelogram (ii) trapezium

      (iii) rectangle (iv) rhombus

   c. If one of the angles of a parallelogram is 60^0 then value of opposite angle are:

      (i) 60^0 (ii) 120^0

      (iii) 90^0 (iv) 180^0

   d. The adjacent sides of a parallelogram are of 8cms. and 6 cms. respectively. What would be its perimeter ?

      (i) 14cm. (ii) 28cm.

      (iii) 56cm. (iv) 60cm.

2. Fill in the blanks

   (1) A quadrilateral where the diagonals are equal and intersect each other at right angle, is known as a ....

   (2) The quadrilateral in which only one pair of opposite sides are parallel is known as an ....

   (3) If a polygon has 6 vertices, then the number of sides in the polygon would be ....

   (4) If the number of sides in a polygon is "n" then the number of diagonals that can be drawn from one of the vertices would be ....

3. The four vertices of a square piece of paper have been cut off with the help of a pair of seissiors. What kind of polygonic figure has been obtained ? Discuss with your friends.

4. The sides of a polygon are 8 in numbers what would be the sum of its internal angles?

5. If one of the angles of a parallelogram is 120^0, find the measurement of the other angles?

6- The adjacent sides of a parallelogram are in the ratio 3 : 4, If the perimeter of a parallelogram is 84cm, find out its sides.

7- Which of these statments are not true. Correct them and rewrite.

1. All squares are rhombus.
2. The diagonals of a rectangle are equal.
3. All rectagles are not parallalograms.
4. The diagonals of a parallelogram bisect each other at right angles.
5. All squares are rectangles.

Symmetry in polygons

You have studied about quadrilaterals and are now well aquainted with four sided figures. You have learnt in class VII that if any figure can be folded into two in such a way that half completely covers the other half, then the line along which this figure is folded is known as the line of synmetry. For example : Cut square from a piece of paper and if you fold it along the dotted lines as shown in fig. 10, you can get four lines of symmetry.

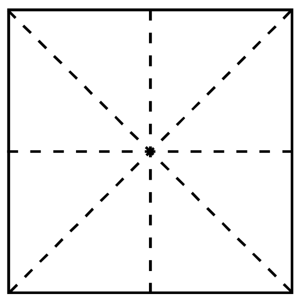
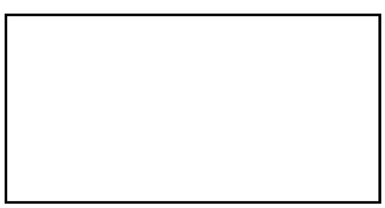
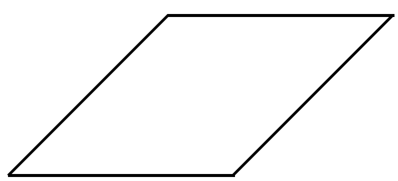


fig. 10

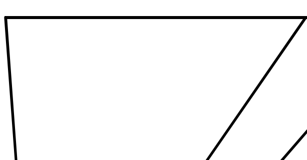
Similary, cut papers like the figures given below and try to find the lines of symmetry for these figures. With a pencil, now draw dotted lines in the book for these figures. Also write the number of such lines you have drawn in each figure.



Rectangle



trapezium



rhombus

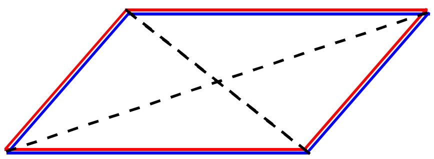
Parallelogram

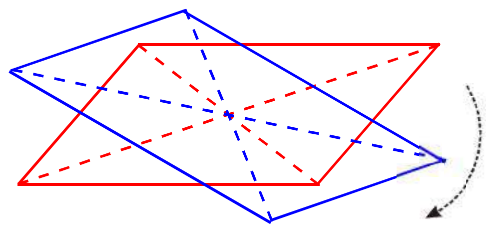
Rhombus with two equal sides

A parallelogram which is neither a square nor a rectangle and has no line of symmetry but still it seems that there is some kind of a equilibrium in these figures also, Let us find out.

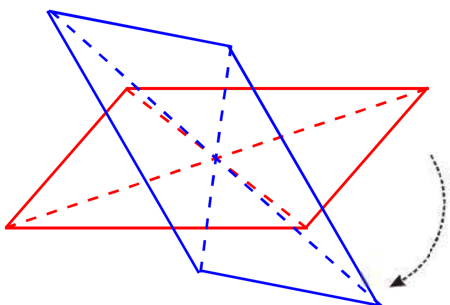
Fold a piece of paper in two equal part and make a parallelogram. Cut the paper into two equal shapes in such a way that two parallelograms of the same size are obtained.

Draw diagonals in the two parallelograms to get their intersecting point. On the point of intersection intersect a pin and rotate the apparatus in such a way that the upper parallelograms rotates on the lower one. Notice at which points does the upper parallelogram completely overlaps the lower figure.

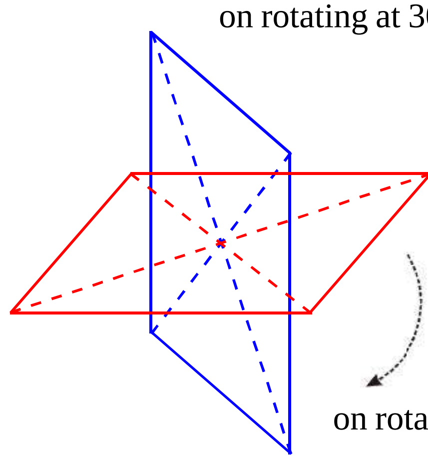




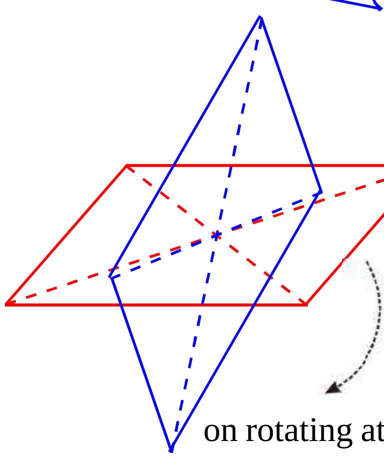
on rotating at 30°



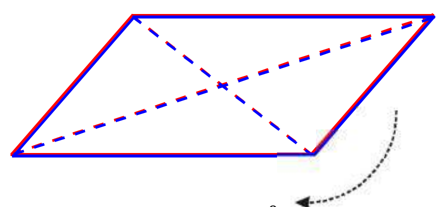
on rotating at 60°



on rotating at 90°

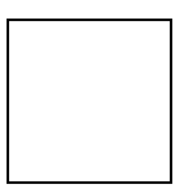


on rotating at 120^0

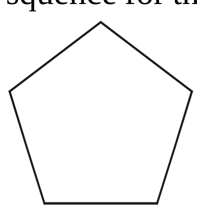


on rotating at 180^0

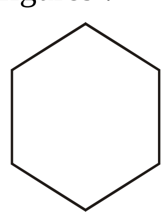
Thus, we see that first at 180^0 angle and later at 360^0 angle, the parallelogram on top rotated on the lower one completely overlaps the parallelogram. The symmetry of this kind is known as rotating symmetry and the point at which the figures are rotated is known as the centre of rotation. On completing a full rotation around the centre, the number of times the figures overlap each other is called the rotational sequence. Can you find out the rotational squence for the figures ?



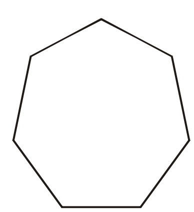
Square



Pentagon



Hexagon



Septagon

Activity 10

Below are given some figures complete the table by determining the rotational sequence and axis of symmetry.

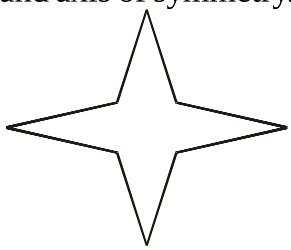


fig 10 (i)

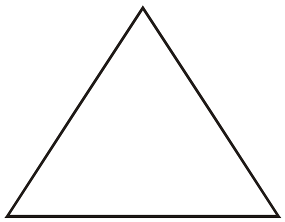


fig. 10(ii)

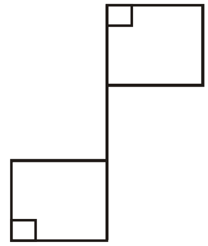


fig. 10(iii)

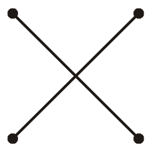


fig. 10(iv)

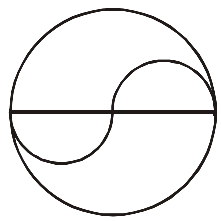


fig. 10 (v)

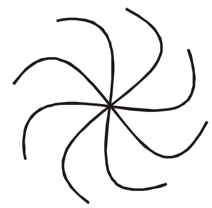


fig. 10 (vi)

Fig. No.	Rotational squence	No. of symmetrical axes
fig 10 (i)		
fig 10 (ii)		
fig 10 (iii)		
fig 10 (v)		
fig 10 (v)		
fig 10 (vi)		

Is there any relationship between the rotational sequence and the number of symmetrical axes ? What conclusions do you draw from this ?

Practice

1. Where do you find examples of rotational symmetry in your daily life ? Give any five examples.
2. Can you think of any kind of symmetry ? Think and discuss with your friends.

<u>Exercise 10.2</u>

Q. 1- Three angles of a quadrilateral are 80^0, 110^0 and 120^0respectively. Determine the fourth angle.

Q. 2- If three angles of any quadrilateral are equal and fourth angle is 75^0 then determine the measure of each of the three equal angles.

Q. 3- Ratio of the adjacent angles of a parallelogram is 2 : 3 . Determine the measure of each angle.

Q. 4- One angle of a quadrilateral is a right angle and remaining (three) angle are in the ratio of 2 : 3 : 4 determine the measure of each of them.

Q. 5- Angles of a quadrilateral are in the ratio of 1 : 2 : 3 : 4, determine the measures of all the angles.

Q. 6 - Adjacent angles of a parallelogram are x^0 and $(x + 20)^0$ then determine the measure of each angle.

We have learnt

1. A figure circumscribed by three or more sides is called a polygon.
2. When all the arms or sides of a polygon are equal, then it is known as a regular polygonal area or a equi-polygonal area.
3. Polygons in which the lengths of the sides are different are known as irregular polygonal areas.
4. The measures of each angle of a equi-polygonal area or a regular polygon are also equal.
5. If the number of sides in a polygon is ‘n’ then on joining all the opposite vertices from one vertex, the area gets divided into (n-2) triangles.
6. The value of the sum of all the internal angles of a prolygonal area is $(n-2)\times 180°$.
7. The diagonals of a parallelogram, rectangle, square and rhombus bisect each other at the point of intersection.
8. The two diagonals of a square and a rectangle are equal to one other.
9. The diagonals of a square and a rhombus bisect each other at right angles.
10. The polygonal area in which each angle is less than 180° is known as a convex polygon.
11. The polygonal area in which at least one internal angle is more than 180° is known as a concave polygonal area.

Chapter—11

CONSTRUCTION OF QUADRILATERAL

Kamla, knows how to construct a triangle with the help of given measurement. She also knows about quadrilateral. She wants to construct a quadrilateral. Then she thinks of weather quadrilateral is constructed with the given four sides only?

Kamla took a broom stick and divided into 4 parts measuring 3cm, 4cm, 5 cm and 13cm and wanted to construct a quadrilateral but she could not. Can you construct a quadrilateral with the given measurement sticks.?

If the quadrilateral is not formed then why?
Give reason.

Now you also think your measurements to construct a quadrilateral and take four broom stick to construct it.

Given below in the table some children measurement and their result. Further you also think some measurement and give your result.

Table 11.1

S.No.	**Name**	Measuremnt of sides (cm)				Resulet
		First	Second	Third	Fourth	
1.	Mayank	3	4	5	6	Quadrilateral formed
2.	Neeraj	3	3	4	10	Quadrilateral not formed
3.	Namarata	2	4	5	7	Quadrilateral formed
4.	Raziya	3	5	6	15	Quadrilateral not formed
5.	Gurpreet	4	6	7	8	Quadrilateral formed
6.	--------	-----	-----	-----	-----	-------------------

Read the table carefully and think why the quadrilateral are not formed with the measurement of neeraj and Raziya.

Conditions for forming a quadrilateral

A quadrilateral is formed only when the sum of the three given sides is greater than the fourth side.

Hamid has found that when the sides are bent the shape of the quadrilateral changes and in this way we can form more quadrilaterals.

Then by knowing the four sides of a quadrilateral we cannot form a particular quadrilateral. But this is not true as we shall explain below.

Activity -1

Take four strips of suitable lengths, with the holes at the ends. Join the strips with pins at the ends to form a quadrilateral. Now try to change the shape of the quadrilateral by pressing any two opposite corners.

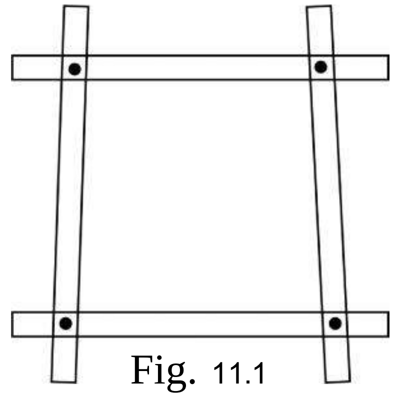


Fig. 11.1

You find that you can easily change it. Thus it is possible to have two different quadrilaterals with the same four sides, so that even if we are able to construct a quadrilateral with given four sides it is not unique.

Now, take one more strip and join it to the quadrilateral, in the form of a diagonal.

Try to change the shape of a quadrilateral.

Now you cannot do it.

This shows that in the case of a quadrilateral it is necessary to have at least the measurement of five parts (i e four sides and one diagonal) to be able to construct it specifically.

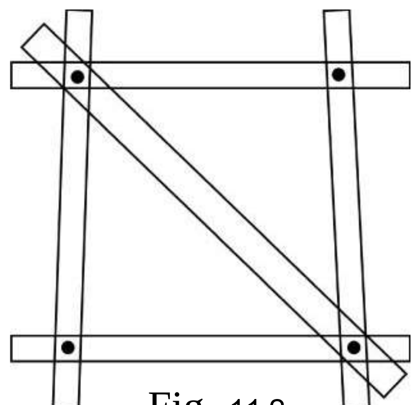


Fig. 11.2

Anu was thinking that a quadrilateral formed with four sides, if the croners are pressed many more quadrilaterals are formed but if any one angle is fixed, then two sides will also be fixed and only one quadrilateral is formed.

Let us think of some other situations in forming a quadrilateral.

Activity -2

Draw a triangle with appropriable a measurement.

[After forming a triangle we get three vertices. Since a quadrilateral has four vertices therefore for making a quadrilateral one more vertex is required. Now think that for making one more vertex how many measurement are required.]

Definitely two measurement are required for fourth vertex.

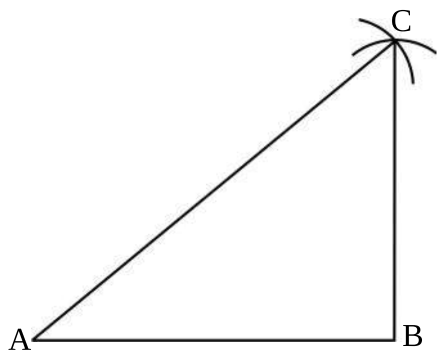


Fig. 11.3

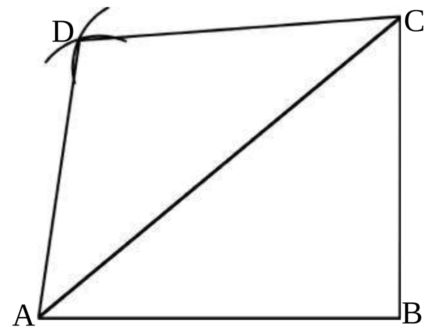


Fig 11.4

In this way we can say to construct a quadrilateral five parts are required.

We will study about some easy ways of construction of quadrilateral.

(i) If a quadrilateral has four sides and one diagonal are given.

(ii) If a quadrilateral has three sides and two diagonals are given.

(iii) If a quadrilateral has four sides and one angle are given.

(iv) If a quadrilateral has three sides and two internal angles are given.

(v) If a quadrilateral has two adjacent sides and three angle are given.

(vi) Construction of Special type of quadrilateral.

Why do we draw arc?

Activity-3

Draw a line segment AB = 6cm. Now try to find out point C in such a way that it is at a distance of 3.7cm from A and 4.2 cm form B.

Can you find out point C with the help of scale?

You will find that with the help of the scale it is difficult to find a particular point

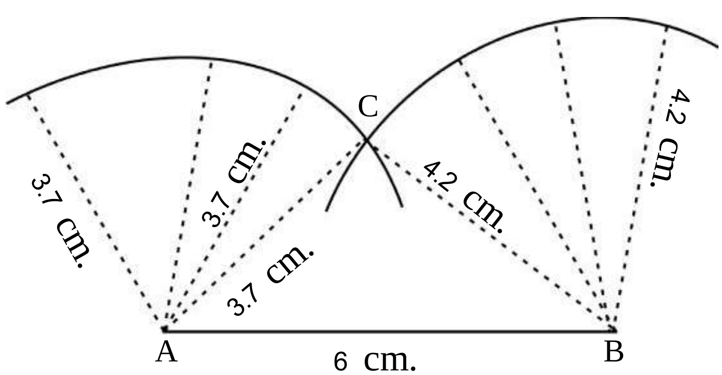


Fig. 11.5

let us draw an arc taking 'A' As center and of radius 3.7 cm (Now try to measure from the different points of Arc to point 'A'. Are they equal?

The distance from point 'A' to all the points on the arc are equal because they all are the radius of the circle and in one circle all the radius are equal. In this way, many points will be there from point A at distance of 3.7 cm.

In this way taking B as center and radius 4.2 cm draw an Arc. In this arc also the distance from point B to all the points on arc are same. Therefore many points will be there from point B at a distance of 4.2 cm.

The two arc which cut at a points is 'C' which is at a distance of 3.7cm from A and 4.2 cm from B.

Thus it is easy to determine a point by drawing arcs from two different points from a definite distance.

I. Construction of a quadrilateral when four sides and one diagonal are given.

Example -1

Construct a quadrilateral ABCD in which AB = 5cm, BC= 4cm, CD= 4.5cm, AD = 3.5cm and AC= 6cm.

Solution :

At first draw a rough sketch of the figure without measuring it keeping in mind the given question and write their measurements-

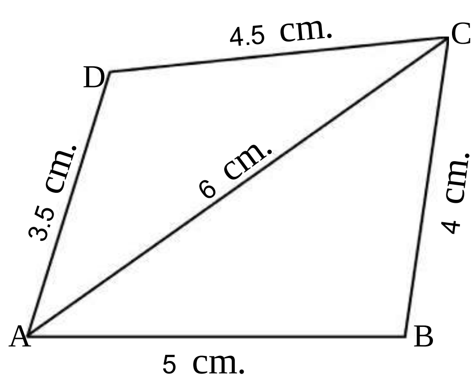


Fig. 11.6

Steps of construction:-

1. Draw a line segment AB= 5cm

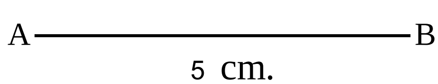
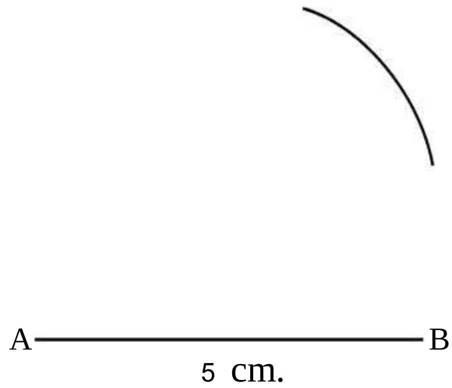


Fig. 11.7(i)

2. Taking 'A' as center and of radius 6cm draw an arc above the line AB.



3. Taking 'B' as center and of radius 4cm draw another are which cut the previous at a point and mark it as point C.

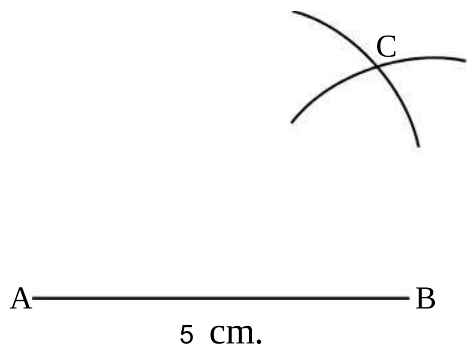


Fig. 11.7(iii)

4. Join AC and BC with help of scale (ruler)

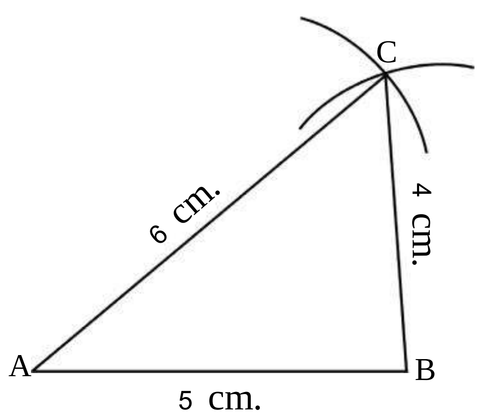
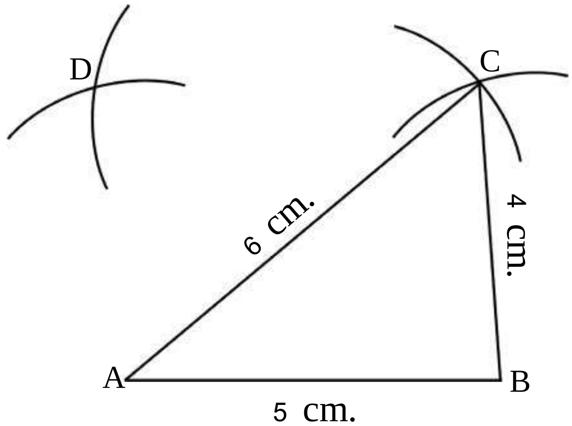
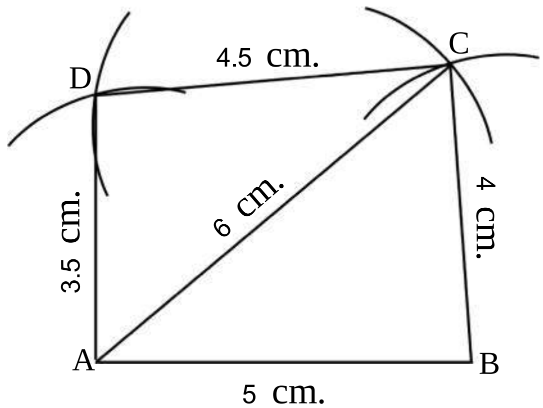


Fig. 11.
7(iv)

5. Now taking 'A' and 'C' as center and of radius 3.5cm and 4.5cm draw two arcs which meat at point D.



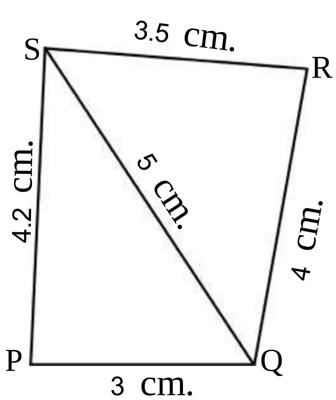
6. Join AD and CD with the help of scale (ruler)



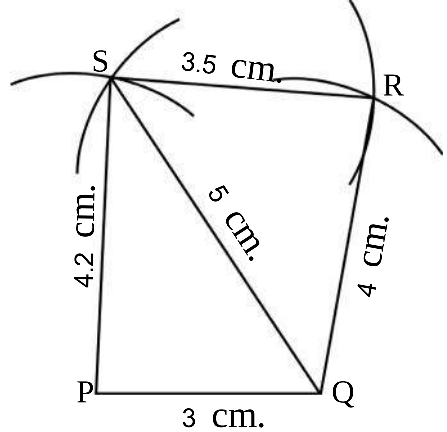
There fore ABCD is the required quadrilateral.

Example -2

Construct a quadrilateral PQRS in which PQ= 3cm, OR= 4cm, RS= 3.5cm, PS= 4.2cm and QS= 5cm?



Rough Fig.



Required Fig.

Fig. 11.8

Steps of construction

1. Draw line segment PQ = 3cm
2. Taking Q as center and of radius 5 cm draw an arc.
3. Taking P as center and of radius 4.2 cm draw an arc which cut the previous are at a point S.
4. Join PS and QS with help of scale.
5. Taking Q and S as center and of radius 4cm and 3.5 cm respectively draw two arcs which cut at a point 'R'.
6. Join QR and SR with the help of scale.

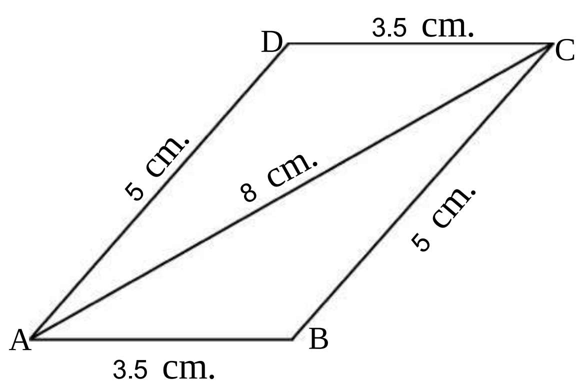
- There fore PQRS is the required quadrilateral.

Example -3

Construct a parallelogram ABCD in which AB= 3.5cm BC= 5cm and diagonal AC=8cm?

Solution :

First of all with the given measurement draw a rough figure. With the help of given measurement. Can you draw a parallelogram? If not why?



Rough Fig. 11.9

We know that opposite sides of parallelogram are equal.

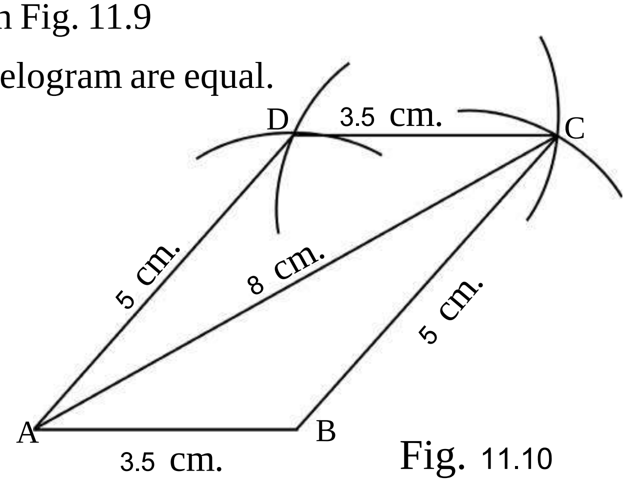


Fig. 11.10

Steps of construction :

1. Draw a line segment AB = 3.5cm
2. Taking 'A' as center and of radius 8cm draw an arc.
3. Taking 'B' as center and of radius 5cm draw an arc on the previous arc which cuts at point 'C'.
4. Join AC and BC with the help of scale.
5. Taking A and C as center and of radius 5cm and 3.5 cm respectively draw two arcs which cut at point D.
6. Join AD and D C. ABCD is the required parallelogram.

Practice -1

Solve these

1. Construct a Rhombus ABCD in which AB = 4cm and AC= 7cm
2. Construct a parallelogram PQRS in which PQ= 6cm QR= 8cm and PR= 10cm.

Exercise 11.1

1. Construct a quadrilateral ABCD in which AB= 4cm, BC= 3.1 cm, CD= 3cm, AD= 4.2cm, and AC= 5cm? measure the diagonal BD?
2. Construct a quadrilateral PQRS in which PQ= 3cm, QR= QS = 5cm, PS= 4cm and SR = 4cm measure RP.
3. Construct a quadrilateral MNOP in which MN = 4.3 cm, NO = 3.9cm, OP= 5cm, MP=3.7 and MO= 5.6cm?
4. Construct a Rhombus PQRS in which PQ = 5cm and a diagonal PR= 8cm.
5. Construct a Rectangle MNOP in which MN=3cm, NO= 4cm, and MO= 5cm.

II Quadrilateral with three given sides and two diagonal:

Example -4

Construct a quadrilateral ABCD in which AB= 4cm, AD = 3.5cm, DC= 5cm Diagonal AC= 7cm and diagonal BD = 6cm.

Solution : Draw a rough sketch of the figure with out measuring it keeping in mind the given question on and write their measurements-

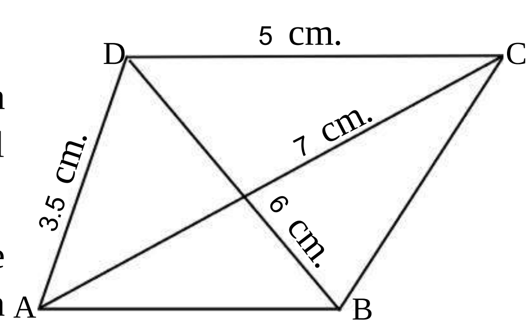


Fig. 11.11

Rough Fig.

Steps of construction :

1. Draw a line segment AB = 4cm.

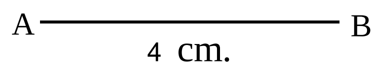


Fig. 11.12(i)

2. Taking B as center and of radius 6cm draw an Arc.

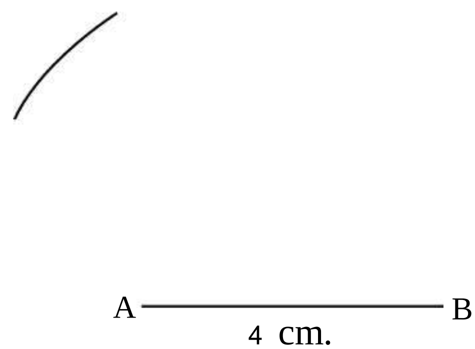


Fig. 11.12(ii)

3. Taking A as center and of radius 3.5 cm draw another arc which cuts the previous arc at D.

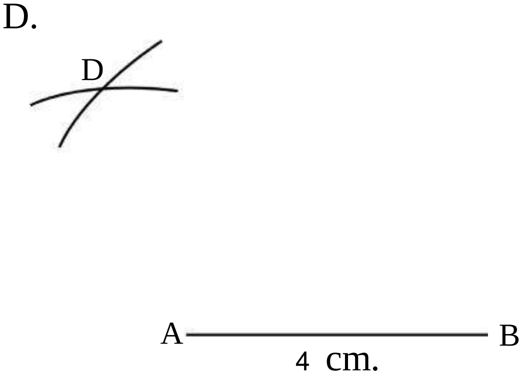


Fig. 11.12(iii)

4. Join AD and BD with help of scale.

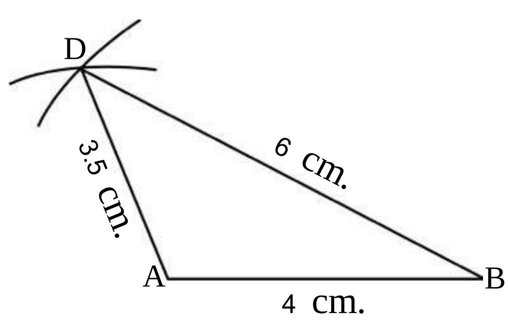
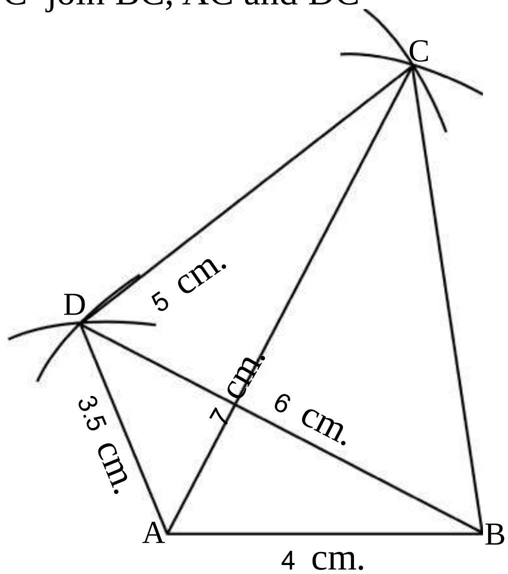


Fig. 11.12(iv)

5. Again taking D & A as center and of radius 5cm and 7cm respectively draw Arcs which cuts at ‘C’ join BC, AC and DC



Required Fig.

Fig. 11.12(v)

Therefore ABCD is the required quadrilateral.

Practice -2

Solve these:

Construct a rectangle ABCD in which AB= 5cm and AC = 8cm.

Exercise 11.12

1. Construct a quadrilateral ABCD in which AB = 4cm BC= 3cm, AD = 3.5 cm diagonal AC = 5cm and diagonal BD = 4cm measure the side CD.
2. Construct a quadrilateral PQRS in which PQ = 3cm QR = 4cm, RS= 3.8cm diagonal RP = 4.5 cm and diagonal QS= 5cm. Measure the side PS?
3. Construct a quadrilateral PQRS in which PQ = 4.2 cm QR= 3.6, PS = 4.5 cm diagonal PR= 6cm and diagonal QS = 5.7 cm.
4. Construct a quadrilateral ABCD in which AB= 3.5cm, BC= 3.1cm, CD =2.9. AC= 4.9 cm.
5. Construct a rectangle PQRS in which PQ= 4.5cm and diagonal PS = 6.5cm.

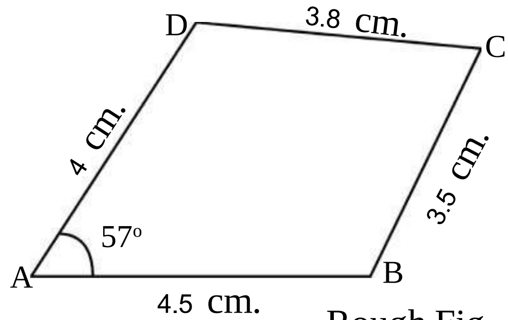
III. Construction of a quadrilateral, when four sides and one angle are given.

Example 5

Construct a quadrilateral ABCD in which AB=4.5cm, BC= 3.5cm, CD= 3.8cm, AD= 4cm and ∠A =57°.

Solution

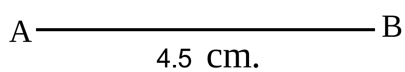
Keeping the question in mind draw a rough figure and write their given measurement.



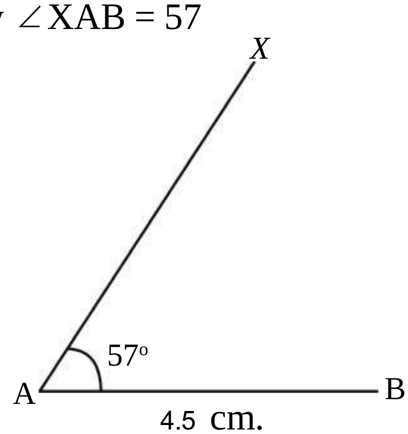
Rough Fig.

Steps of construction ;

1. Draw a line segment AB= 4.5 cm



2. With the help of protector draw ∠XAB = 57



3. Taking A as center and of radius 4cm, draw an arc line AX and mark it as point D.

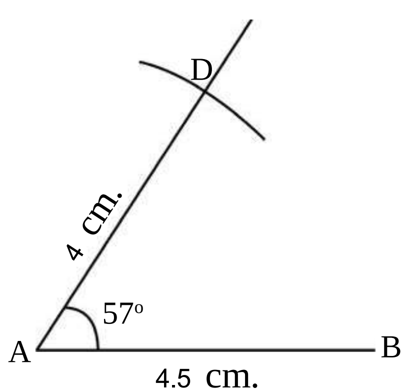


Fig. 11.14 (iii)

4. Taking D and B as center and of radius 3.8cm and 3.5cm draw two Arcs which meet at point C.

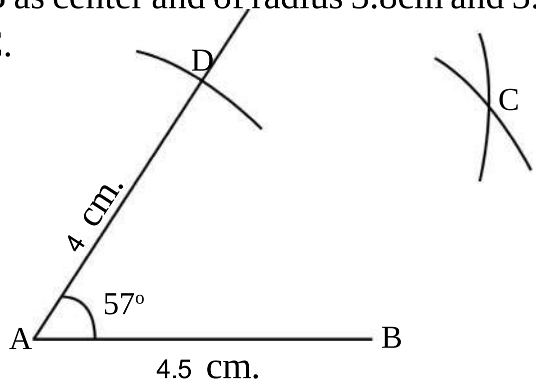


Fig. 11.14 (iv)

5. Join BC and DC with the help of scale.

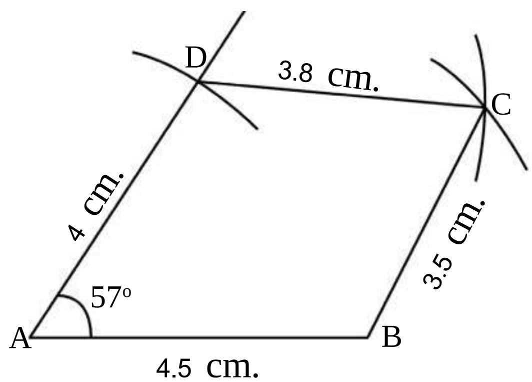


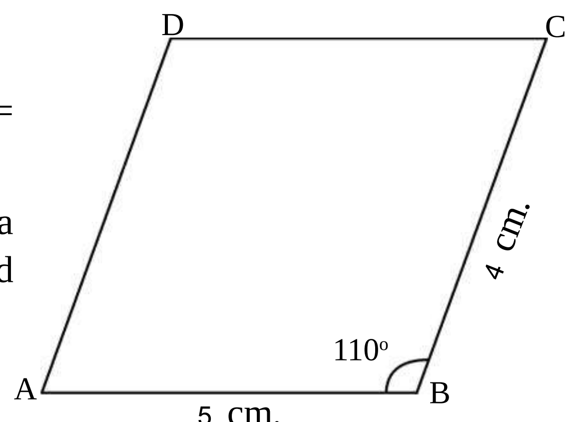
Fig. 11.14 (v)
Required figure

Therefore ABCD is the required quadrilateral.

Example -6

Construct a parallelogram ABCD in which AB = 5cm, BC= 4cm and ∠B =100°

Solution : Keeping the question in mind draw a rough sketch in which AB= 5cm, BC= 4cm and ∠B= 110°



Rough Fig. A
Fig. 11.15(i)

Here only two sides and one angle are given, but for constructing any quadrilateral five parts are required.

Since opposite sides of a parallelogram are equal. There fore we can find four sides and one angle.

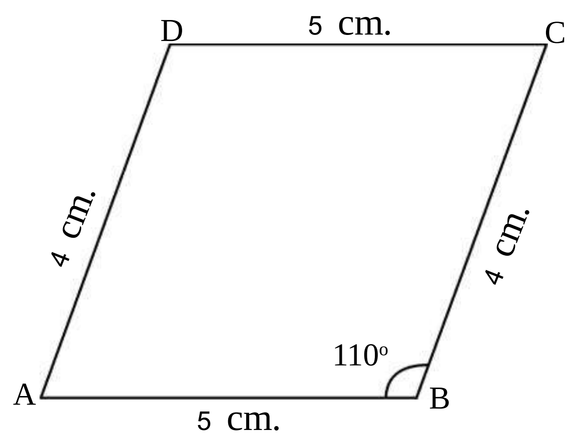


Fig. 11.15(ii) Rough Fig. B

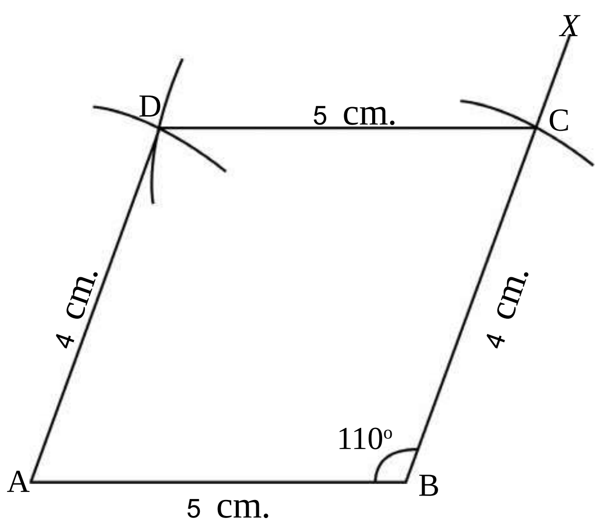


Fig. 11.16

Steps of construction :

1. Draw a line segment AB= 5cm
2. Taking B as center draw ∠ABX= 110
3. Again taking B as center and of radius 4cm draw an Arc on ray BX and mark it as point C.
4. Taking A and C as center and of radius 4cm and 5cm respectively draw two Arc which meet at point D.
5. Join AD and DC
6. Therefore ABCD is the required parallelogram.

Practice – 3

Do this also

1. Construct a Rhombus in which ABCD.In Which AB = 5cm and ∠A =70°.
2. Construct a rectangle PQSR in which PQ= 4cm and QR = 3cm.
3. Construct a square LMNO in which LM = 2.8cm

Exercise 11.3

1. Construct a quadrilateral ABCD in which AB = 5.4cm, BC = 4.8 cm, CD=AD= 5cm and $\angle A=120^0$
2. Construct a quadrilateral ABCD in which AB= 3cm, BC=4cm, CD= 3.5cm, DA= 4.2cm and $\angle A=60^0$
3. Construct a parallelogram PQRS in which PQ= 5.4cm OR= 3.8cm and $\angle P=75°$
4. Construct a Rhombus STUV in which ST = 4cm and $\angle S=60^0$
5. Construct a square EFGH in which EF= 3.5cm.
6. Construct a rectangle MNOP in which MN=6cm and NO= 8cm

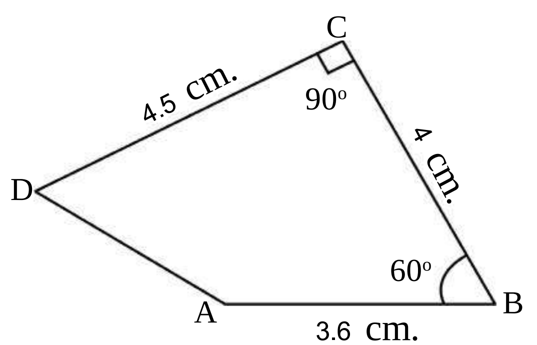
IV. **Construction of quadrilateral when three sides and two internal angle are given -**

Example 7

Construct a quadrilateral ABCD in which AB= 3.6cm, BC= 4cm, CD= 4.5cm, $\angle B=60^0$ and $\angle C= 90^0$

Solution :

Keeping the question in mind, draw a rough figure and write the measurement along their sides and angles.



Rough Fig.

Fig. 11.17

Steps of construction

1. Draw a line segment AB = 3.6 cm
2. Taking B as center $\angle ABX = 60^0$
3. Taking B as center and of radius 4cm cut an arc on ray $\overrightarrow{BX}$ and mark it as C.
4. Taking C as center draw $\angle BCY=90^0$

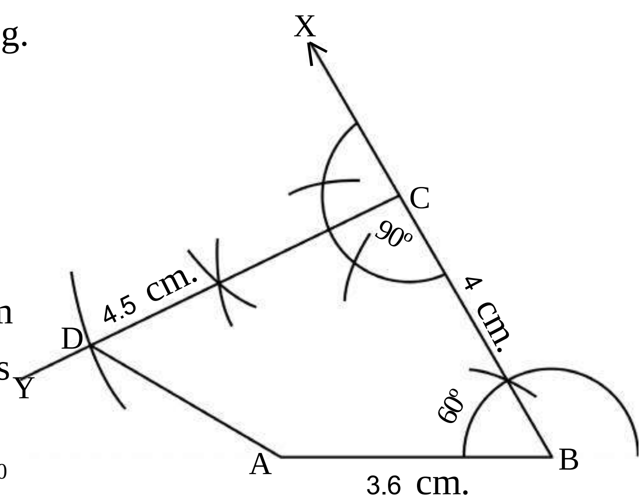


Fig. 11.18

5. Again taking C as center and of radius 4.5cm cut an arc on ray $\overrightarrow{CY}$ and mark it a D.

6. Join D to A.

ABCD is the required quadrilateral.

Exercise 11.4

1. Construct a quadrilateral ABCD in which AB=4.5cm, BC= 3.5, AD=5cm, $\angle A=60^0$ and $\angle B=110^0$. Measure side CD?

2. Construct a quadrilateral PQRS in which PQ = 3.5cm QR=2.5cm RS=4.1cm, $\angle Q=75°$ and $\angle R=120°$. Measure the side PS?

3. Construct a quadrilateral EFGH in which EF = 3cm, HE= 5cm FG= 7cm, $\angle E= 90^0$ and $\angle H= 120^0$. Measure the side GH.

IV. **Construction of a quadrilateral when two adjacent sides and three angle are given.**

Example 8

Construct a quadrilateral ABCD in which AB = 5cm, BC= 4.5 cm $\angle A= 65^0$, $\angle B =120^0$ and $\angle C=75$.

Solution :

According to the given measurements draw a rough sketch and write the measurement along their sides and angles-

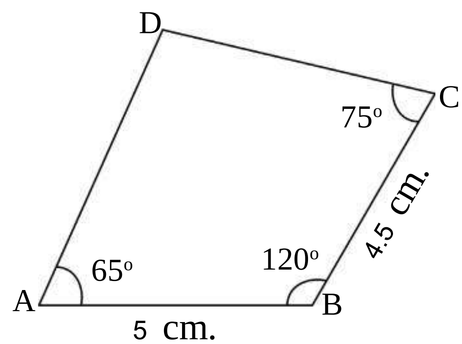


Fig. 11.19 Rough Fig.

Steps of construction -

1. Draw a line segment AB= 5cm

2. Taking Aas center draw $\angle BAX = 65^0$

3. Taking B as center draw $\angle ABY = 120^0$

4. Taking B as center and of radius 4.5cm draw an arc on ray $\overrightarrow{BY}$ and mark it as C.

5. Taking C as center draw $\angle BCZ = 75^0$ which cuts the line BX and mark it as D.

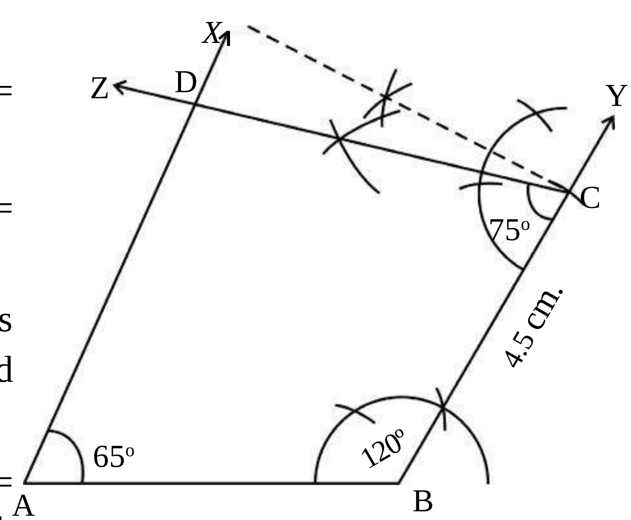


Fig. 12.20

There fore ABCD is the required figure

Example – 9

Construct a quadrilateral ABCD in which AB = 4cm, BC= 3.6cm, ∠A=60^0, ∠C=90^0 and ∠D=100^0.

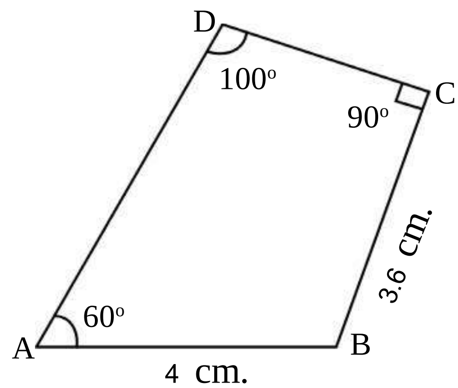


Fig. 11.21

Solution:

Keeping the question in mind draw a rough sketch of the figure and write their measurement along their sides and angle. From the rough figure it is very clear that unless we know ∠B. It is not possible to construct a quadrilateral ABCD.

We know that sum of four angels of a quadrilateral is 360^0 construct of a quadrilateral is possible only when sum of the three angle is less than 360.

Forth angle ∠B= 360^0 - (60°+100°+90°)
= 360^0-250^0
=110^0

Steps of construction :

1. Draw a line segment AB= 4cm
2. Taking A as center draw ∠XAB = 60^0
3. Taking B as center draw ∠ABY = 110^0
4. Again taking B as center and of radius 3.6cm cut an arc on ray $\overrightarrow{BY}$ and mark it as C.
5. Taking C as center draw ∠BCZ = 90^0 in such a way that it cut ray AX at point D therefore ABCD is the required quadrilateral.

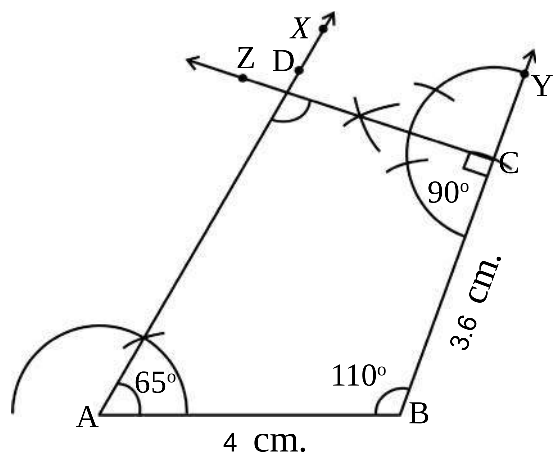


Fig. 11.22

We have learnt the construction of quadrilateral in different situations in which five measurement were given. If in a quadrilateral one side and four angle are given (In this also five parts are given). In this situation also can we construct a quadrilateral? Try to do it.

Now Hamid has a question that on which point we have to mark an angle for marking fourth side. Can you tell that at which point of ray $\overrightarrow{BX}$ ∠C will be drawn? Or at which point of ray $\overrightarrow{AY}$ ∠D will be drawn?

Think and discuses with your friends and teacher.

After discussion and activity you might have got that if we make $\angle$C at any point of ray $\overrightarrow{BX}$, or $\angle$D at any point of ray $\overrightarrow{AY}$. We get the required quadrilateral. In the same way we can make many quadrilateral but not a unique one.

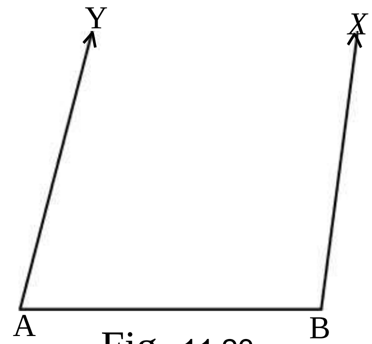


Fig. 11.23

Exercise 11.5

1. Construct a quadrilateral ABCD in which AB= 3.5cm BC=4cm $\angle$A = 105^0, $\angle$B=90^0 and $\angle$C=75^0?
2. Construct a quadrilateral PQRS in which PQ= 4.5cm QR= 5cm $\angle$P=100^0, $\angle$R= 75^0 and $\angle$S=110^0
3. Construct a quadrilateral ABCD in which AB=6cm BC= 3.5cm $\angle$A= 60^0, $\angle$B=110^0 and $\angle$D=90^0 measure side CD?

VI **Special type of quadrilateral and their construction :**

A. Construction of a parallelogram when one side and two diagonal are given.

Example 10

A. Construct a parallelogram ABCD in which AB= 5cm AC= 5.6 cm and BD=7.6cm.

Solution:

Keeping the question in mind draw a rough sketch of the figure and write the measurements along their sides.

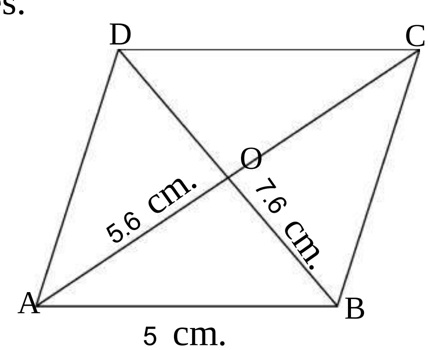


Fig. 11.24

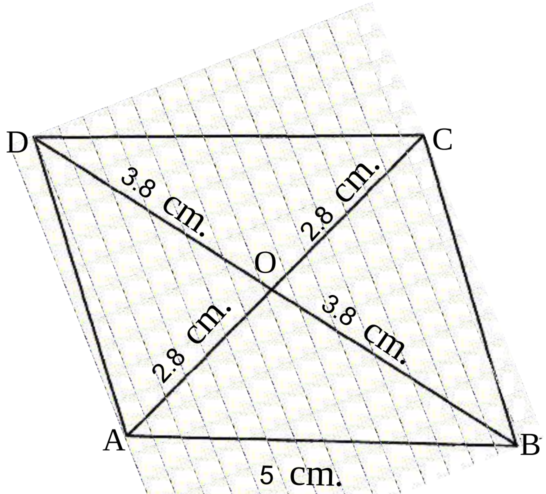


Fig. 11.25 Rough Fig.

From this can you construct a parallelogram? We know that in a parallelogram the diagonals bisect each other.

Therefore OB= OD = 7.6 ÷ 2 = 3.8cm

Steps of construction

1. Draw a line segment AB = 5cm

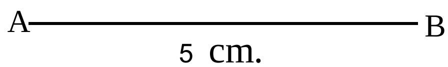


Fig. 11.26 (i)

2. Taking A as center and of radius 2.8 cm cut an arc above the line AB.

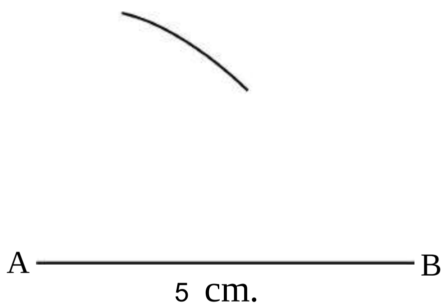
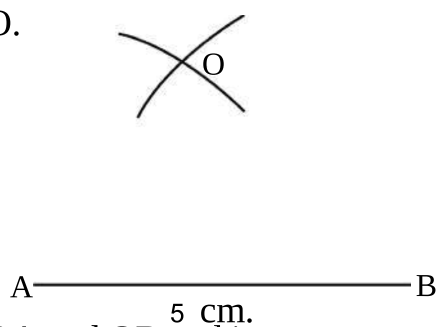


Fig. 11.26 (ii)

3. Now taking B as center and of radius 3.8 cm cut an arc on the pervious arc and mark it as O.



4. Join OA and OB and increase up to X and Y.

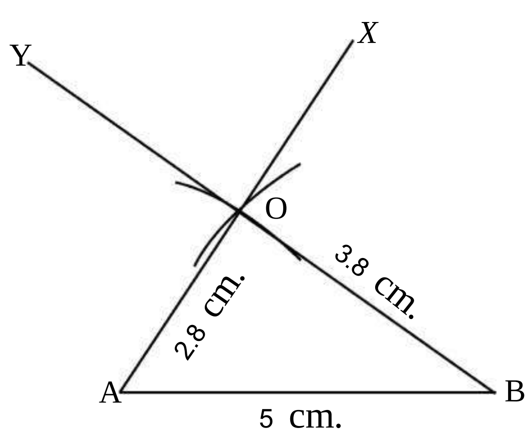


Fig. 11.26(iv)

5. Taking O as center and of radius 2.8 cm and 3.8cm respectively draw arc cutting ray $\overrightarrow{OX}$ and $\overrightarrow{OY}$ and mark it as C and D.

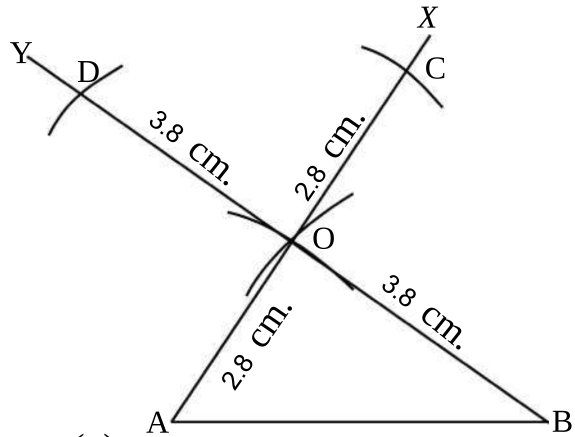


Fig. 11.26(v)

6. Join BC, AD and CD

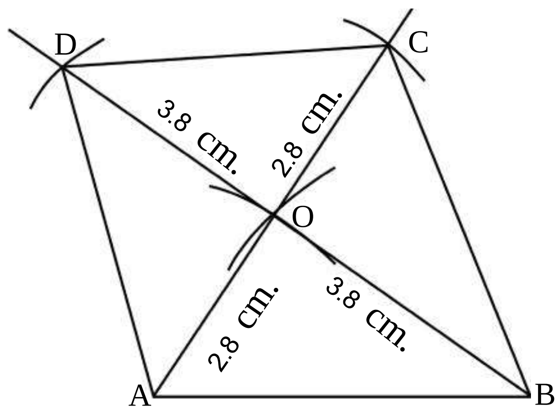


Fig. 11.26(vii)

Therefore ABCD is the required parallelogram.

B. **Construct a Rhombus when both the diagonals are given:-**

Example 11

Construct a Rhombus ABCD in which one diagonal BD= 8cm and the other diagonal AC = 6 cm.

Solution:

Keeping the question in mind draw a rough stack of the figure.

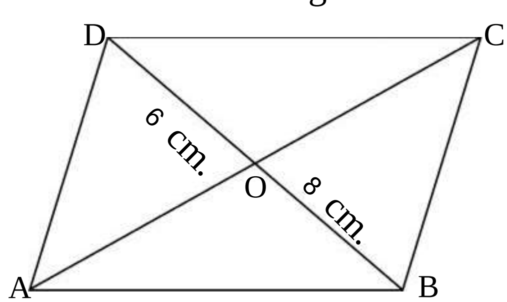


Fig. 11.27

Can you tell how can you construct a Rhombus.

We have learnt in the properties of quadrilateral that the diagonals bisect each other at right angle. On this basis we construct a Rhombus.

Steps of construction

1. Draw a line segment AC= 6cm

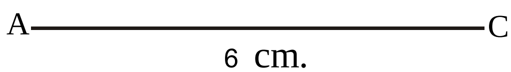
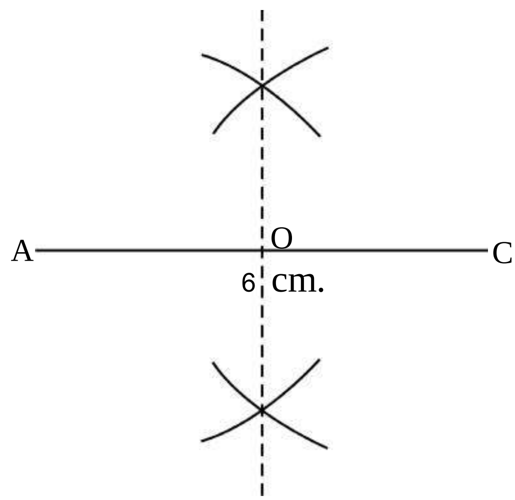
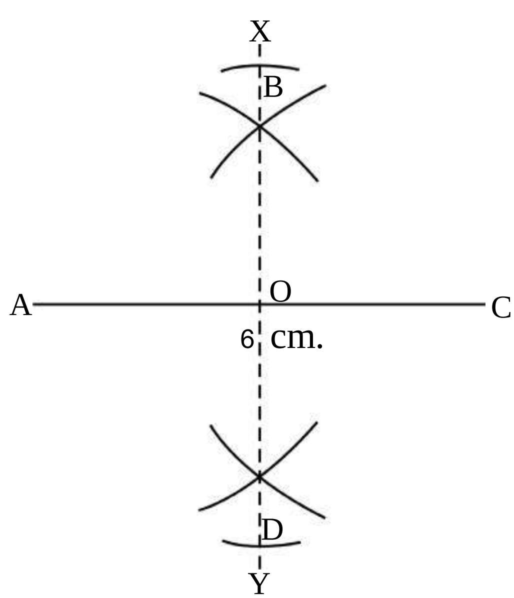


Fig. 11.28(i)

2. Draw a perpendicular bisector XY on AC which cuts AC at O.



3. Taking O as center and OD = OB = 8/2 = 4cm radius draw are on both sides.



4. Join AB, AD, BC, and CD

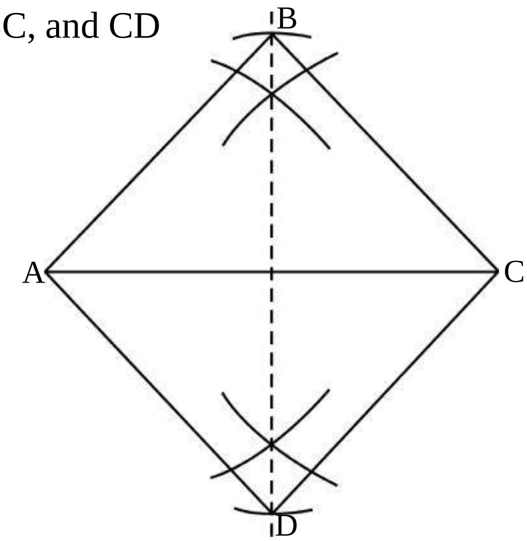


Fig. 11.28(iv)

Therefore ABCD is the required rhombus

Practice - 4

1. Construct a square in which one diagonal AC = 7cm?

Exercise 11.6

1. Construct a parallelogram ABCD in which AB=4cm one diagonal AC=6.2 cm and the other diagonal BD = 8.4cm?
2. Construct a Rhombus EFGH in which one diagonal EG= 7cm and the other diagonal FH= 8.4cm.
3. Construct a square PQRS in which one diagonal PR= 5cm.

We have learnt

1. To construct a special type of quadrilateral we should know at least the measurement of five parts in which maximum three angle are there.
2. It is useful to draw a rough sketch with the measurements marked before the costruction of a quadrilateral.
3. To construct a quadrilateral if
   (i) Four sides and one diagonal are given.
   (ii) Three sides and two diagonals are given.
   (iii) Four sides and one diagonal is given
   (iv) Three sides and two internal angle are given
   (v) Two adjacent sides and three angle are given.

4. If one side and four angles are given of a quadrilateral then we can not draw a unique quadrilateral.

5. Construction of a quadrilateral is possible only when it satisfies the following conditions.

   (i) Sum of three sides of a quadrilateral should be greater than fourth one.

   (ii) Sum of all the four angles of a quadrilateral is equal to 360.

Chapter—12

EQUATION

In your previous class, you have solved simple equaltions. Let us practice a few more simple equations.

S. No.	Equation	Solution of the Equation
01.	$3x + 9 = 12$	$x = 1$
02.	$3x + 5 = 8$	....
03.	$5x + 9 = 2x + 12$	....
04.	$6x + 18 = 24$	....
05.	$x + 3 = 4$	....

While solving the above equations shaily told to Anu, that all equations give x=1 as the answer. Equations are many, but the solutions is only one, why so ?

Look at the equations carefully, you will see that the second equation is obtained from the first equation. On subtracting 4 from both sides of the first equation, the second equation is obtained, so the solution for both the equations, is the same. Similarly, adding $2x$ to both sides of the first equation gives third equation. The 4^{th} & 5^{th} equations are obtained by multiplying the 1^{st} equation by 2 and dividing the first equation by 3 respectively.

Rahim said, "Since every time, we are performing similar kind of function with the same number on both sides of the equation, so we are getting the same result for the equation every time".

Therefore

1. On adding or subtracting the same number on both sides of a equation.

2. On multiplying or dividing a number other than zero (that is not zero) to both the sides of a equation.

We get the same answer or there is no change in the solution of the equation.

What will happen if both sides of a equation is multiplied or divided by zero ? Think and discuss with your friends and teacher.

Activity 1

Now make a equation in the table given below as per instructions.

Table 12.1

S.No.	Equation	Taking on both sides of the Equation	New Equation
01.	$5x + 6 = 1$	On adding 5	$5x + 11 = 6$
02.	$17x - 8 = 22$	On subtracting $4x$	
03.	$4x - 5 = 2x + 9$	On adding 6	
04.	$3x + 2 = 5$	On Multiplying 2	
05.	$6x + 15 = 9$	On Dividing 3	
06.	$2x - 7 = 11$	On adding $3x$	

You have already studied this type of equation in your previous class :

for example $\frac{x}{7} = 3$

To solve this equation, we need to delet 7 from the left hand side. For this we multiply 7 on both sides of the equation.

Or $\frac{x}{7} \times 7 = 3 \times 7$

$\Rightarrow \quad x = 21$

Now if the equation is $\frac{7}{x} = 1$, then how will you solve this ?

Here also, to remove x from the denominator we shall multiply x on both sides of the equation. Thus, the denominator can be deleted from equation with fractional algebric expressions. Solve the equations in the table on the next page according to the given instructions.

Table 12.2

S.No.	Equation	Writing on side of the equation as numerator and denominator	On Multiplying denominator on both sides	On writing the equation as a linear equation	By transposing the variables and constant	Solution
1.	$\frac{8x+5}{2x+7}-3=0$	$\frac{8x+5}{2x+7}=3$	$\frac{(8x+5)(2x+7)}{2x+7}$ $=3(2x+7)$	$8x+5=6x+21$		$x=8$
2.	$\frac{9x}{x+5}=4$					
3.	$\frac{2y+9}{3y+10}-3=0$					
4.	$\frac{3(x-3)}{x+4}=2$					

Actually, such question are in the from $\frac{ax+b}{cx+d}=k$ where a,b,c,d and k are integers and x a variable and $cx+d\neq 0$.

On solving $\frac{ax+b}{cx+d}=k$ type of equations.

The equation $\frac{6x+2}{4x+1}=2$ is a equation with one variable. In this equation the denominator also have a variable. On comparing with $\frac{ax+b}{cx+d}=k,\ a=6,\ b=2,$ $c=4,\ d=1\ and\ k=2$.

Example - 1

Solve the equation

$$\frac{2x+5}{3x+1}=\frac{3}{11}$$

In the equation

$$\frac{2x+5}{3x+1}=\frac{3}{11}$$

On multiplying $(3x+1)$ to both sides of the equation.

$$\frac{2x+5}{3x+1}\times(3x+1)=\frac{3}{11}\times(3x+1)$$

$$=2x+5=\frac{3}{11}(3x+1)$$ Step - 1

For deleting 11 from the denominator on left side, we multiply 11 to both sides of the equation.

$$11\times(2x+5)=\frac{3}{11}(3x+1)\times 11$$

$$11(2x+5)=3(3x+1)$$

$$=22x+55=9x+3$$ Step - 2

$\Rightarrow \quad 22x-9x=3-55$ (On solving the brackets)

$\Rightarrow \quad 13x=-52$

$$x=\frac{-52}{13}$$

$$x=-4$$

Verification

L.H.S.= $\dfrac{2x+5}{3x+1}$

$$=\frac{2(-4)+5}{3(-4)+1}$$

$$=\frac{-8+5}{-12+1}$$

$$=\frac{-3}{-11}$$

$=\dfrac{3}{11}$ R.H.S.

Therefore, $x=-4$ is the right solution for the given equation.

In the above example you can see that the denominator $(3x+1)$ of the L.H.S. in step 1 gets eliminated from its place and is multiplied to the numerator of R.H.S. This type of change is known as corss multiplication.

Such problem can also be solved by cross multiplication.

Example - 2

Solve the equation

$$\frac{5-7y}{2+4y}=\frac{-8}{7}$$

Solution: Given equation

$$\frac{5-7y}{2+4y}=\frac{-8}{7}$$

On cross multiplication

$$7\times(5-7y)=-8(2+4y) \quad \text{(On Solving the bracket)}$$

$$\Rightarrow \quad 35-49y=-16-32y$$

$$\Rightarrow \quad -49y+32y=-16-35 \quad \text{(by transposing)}$$

$$\Rightarrow \quad -17y=-51$$

$$\Rightarrow \quad y=\frac{-51}{-17}$$

$$\Rightarrow \quad y=3$$

Verify the answer yourself.

Example - 3

Solve the equation

$$\frac{y-(7-8y)}{9y-(3+4y)}=\frac{11}{7}$$

Solution: Given equation

$$\frac{y-(7-8y)}{9y-(3+4y)}=\frac{11}{7}$$

$$\Rightarrow \quad \frac{y-7+8y}{9y-3-4y}=\frac{11}{7}$$ (On solving bracket)

$$\Rightarrow \quad \frac{9y-7}{5y-3}=\frac{11}{7}$$

$$\Rightarrow \quad \frac{9y-7}{5y-3} \times \frac{11}{7}$$ (On cross multiplcation)

$$\Rightarrow \quad 7(9y-7)=11(5y-3)$$

$$\Rightarrow \quad 63y-49=55y-33$$ (On Solving the bracket)

$$\Rightarrow \quad 63y-55y=-33+49$$ (By transposing)

$$\Rightarrow \quad 8y=16$$

$$\Rightarrow \quad y=\frac{16}{8}$$

$$\Rightarrow \quad y=2$$

Verify the answer yourself.

Example - 4

Solve the equation

$$\frac{x+0.5}{0.3x}=20$$

Solution: Given equation

$$\frac{x+0.5}{0.3x}=20$$

$$\Rightarrow \quad \frac{x+0.5}{0.3x} \times \frac{20}{1}$$

$$\Rightarrow \quad 1(x+0.5)=20(0.3x)$$ (on cross multiplication)

$$\Rightarrow \quad x+0.5=6x$$

$$\Rightarrow \quad 6x=x+0.5$$

$$\Rightarrow \quad 6x-x=0.5$$ (By transposing)

$\Rightarrow \quad 5x = 0.5$

$\Rightarrow \quad 5x = \frac{5}{10}$

$\Rightarrow \quad x = \frac{5}{10} \times \frac{1}{5}$ (on changing side of *x*)

$\Rightarrow \quad x = \frac{1}{10}$

$\Rightarrow \quad x = 0.1$

Therefore the solution of the euqation is $x = 0.1$

Verify the answer yourself.

Exercise 12.1

Solve the following equations and verify your answers.

1. $\frac{4x+18}{5x} = 2$
2. $\frac{5x+2}{2x+3} = \frac{7}{5}$
3. $\frac{7m+6}{4m+2} = 2$
4. $\frac{x-3}{x+2} = \frac{-3}{7}$
5. $\frac{2y-5}{3y+1} = \frac{3}{13}$
6. $\frac{8-3y}{5y+2} = \frac{1}{6}$
7. $\frac{17-2k}{k-5} = -3$
8. $\frac{4x-(x+7)}{3x-(5x-9)} = \frac{2}{3}$
9. $\frac{1.5x+0.3}{3x} = \frac{3}{10}$

Application of Equations in daily life.

We often face problems related to known, unknown variables or quantities in everyday situation. If the relationships are changed in equations, we can easily solve such problems. Solutions to equation are generally solved through the following steps:

1. Read the question carefully and identify the known and unknown variables.
2. The unknown quantities are indicated by letters *x*, *y*, *z* etc.
3. Verbal problems are changed into mathematical statements in the form of equations.
4. Questions are solved to get the value of the unknown variable quanity.

5. The answer is verified.

Let us understand this process through some examples.

Example - 5

The sum of two numbers is 35 and their ratio is 1 : 4. Find out the numbers.

Solution: suppose the first number is x.

According to the first condition, the sum of the two numbers = 35.

$\Rightarrow$ x + 2nd number = 35

$\Rightarrow$ 2nd number = 35 – x

According to the second condition,

Ratio of the numbers = 1 : 4

Therefore, $\frac{x}{35-x} = \frac{1}{4}$

$\Rightarrow$ $\frac{x}{35-x} = \frac{1}{4}$ (On cross multiplication)

$\Rightarrow$ $x \times 4 = 1 \times (35 - x)$

$\Rightarrow$ $4x = 35 - x$

$\Rightarrow$ $4x + x = 35$ (On changing side for x)

$\Rightarrow$ $5x = 35$

$\Rightarrow$ $x = \frac{35}{5}$ (On changing side for x)

$\Rightarrow$ $x = 7$

$\therefore$ The first number is = 7

and the second number is = 35 – 7 = 28

Verification

1. The sum of the two numbers 7 + 28 = 35

2. The ratio of the two numbers $7:28 = \frac{7}{28} = \frac{1}{4}$

Therefore, our answer is correct.

Example - 6

The denominator of a rational number is greater than its numberator by 4. On adding 6 to the numerator and sutracting 3 from the denominator the fraction obtained is $\frac{3}{2}$. Find out the rational number.

Solution : Consider the numerator as x.

Then denominator $= x + 4$

$\therefore$ The rational number $= \frac{numerator}{denominator}$

$$= \frac{x}{x+4}$$

Now on adding 6 to the numerator.

$$= x+6$$

On subtracting 3 from the denominator

$$= (x+4)-3 = x+1$$

$\therefore$ The new rational number

$$= \frac{x+6}{x+1}$$

According to the condition, the new number becomes $\frac{3}{2}$, then $\frac{x+6}{x+1} = \frac{3}{2}$

then $\frac{x+6}{x+1} = \frac{3}{2}$ (on cross multiplication)

$\Rightarrow$ $3(x+1) = 2(x+6)$

$\Rightarrow$ $3x+3 = 2x+12$

$\Rightarrow$ $3x-2x = 12-3$ (on changing sides)

$\Rightarrow$ $x = 9$

Therefore, the required number is $\frac{x}{x+4} = \frac{9}{9+4} = \frac{9}{13}$

Verification

1. $13-9=4$, Therefore denominator is 4 less than its numerator.
2. On adding 6 to numerator $6+9=15$

   On subtracting 3 from denominator $13-3=10$.

   New number $\frac{15}{10}=\frac{3}{2}$

   $\therefore$ Answer is correct.

Example - 7

The sum of the digits of a two digit number is 12. On reversing the digits if the new number obtained becomes 18 more than the original number, findout the original number.

Solution: Suppose, the original number in the units place is *x*.

According to the question,

Units digit + tens digit = 12

$\Rightarrow$ *x* + ten's digit = 12

$\Rightarrow$ ten's digit = 12 – *x*

Therefore the original number

$=10\times$(ten's digit) + (unit's digit)

$=10\times(12-x)+x$

$=120-10x+x \quad =120-9x$

On reversing the digits, the digit in the units place will be in the ten's place and that in the ten's place will be in the units place.

Therefore, units place $=12-x$

ten's place $=x$

Hence the new number $=10\times x+(12-x)$

$=10x+12-x$

$=9x+12$

According to the condition, the new number becomes 18 more than the original.

Therefore the new number = original number $+18$

$\Rightarrow \quad 9x+12=120-9x+18$

$\Rightarrow \quad 9x+12=138-9x$

$\Rightarrow \quad 9x+9x=138-12$

$\Rightarrow \quad 18x=126$

$\Rightarrow \quad x=\frac{126}{18}$

or $\quad x=7$

Therefore, the original number

$= \quad 120-9x$

$=120-9\times 7$

$=120-63 \quad =57$

Verify the answer yourself.

Example - 8

Two numbers are in a ratio 3: 5 on subtracting 4 from each, the ratio becomes 5 : 9. Find the numbers.

Solution

Suppose the first number = $3x$

And the 2nd number = $5x$

On subtracting 4 from the 1st number = $3x-4$

Subtracting 4 from the 2nd number = $5x-4$

According to the condition, the ratio obtained after subtracing 4 from each number = $\frac{5}{9}$.

Therefore, $\quad \frac{3x-4}{5x-4}=\frac{5}{9}$

$\Rightarrow \quad \frac{3x-4}{5x-4} \times \frac{5}{9}$ (on cross multiplication)

$\Rightarrow \quad 9(3x-4)=5(5x-4)$

$\Rightarrow \quad 27x-36=25x-20$ (on solving bracket)

$\Rightarrow \quad 27x-25x=-20+36$ (on changing sides)

$\Rightarrow \quad 2x=16$

$\Rightarrow \quad x=\frac{16}{2}$

$\Rightarrow \quad x=8$

Therefore the first number $= 3x=3\times 8=24$

2nd number $= 5x=5\times 8=40$

Verification

1. Ratio of the two number

$$=\frac{24}{40}=\frac{3}{5}$$

2. On subtacting 4 from each the ratio is

$$=\frac{24-4}{40-4}=\frac{20}{36}$$

$$=\frac{5}{9}$$

Therefore the solution is correct.

Example - 9

The present age of the father is 20 years more than that of the son. After three years ratio of the age of the son and father becomes 19 : 39. Then find the present age of the son?

Solution :- Given equation

Suppose, the present age of son = x years

Then father age = $(x+20)$ years.

After 3 years, son's age would be $(x+3)$ years

After 3 years, father's age would be $(x+20+3)=(x+23)$ years.

According to the question,

After 3 years the ratio of the ages of son and father = 19 : 39

Thus $(x+3):(x+23)=19:39$

$\Rightarrow \quad \frac{x+3}{x+23} = \frac{19}{39}$ (on cross multiplication)

$\Rightarrow \quad (x+3)39 = 19(x+23)$

$\Rightarrow \quad 39x+117 = 19x+437$

$\Rightarrow \quad 39x-19x = 437-117$

$\Rightarrow \quad 20x = 320$

$\Rightarrow \quad x = \frac{320}{20}$ (on dividing both sides by 20)

$\Rightarrow \quad x = 16$

Therefore the present age of the son = 16 years

and the present age of the father = 16 + 20

= 36 years.

Verification

After 3 years the age of the son = 16 + 3 = 19 years.

After 3 years the age of the father = 36 + 3 = 39 years.

The ratio of the age of the son and father after 3 years

= 19 : 39

Hence the solution is correct.

Exercise 12.2

1. The sum of two numbers is 42. If second number is twice the first one, find the number.
2. Neeraj has mangoes three times that of Mayank If mayank is given 8 mangoes more than neeraj is given 6 mangoes more, the ratio of mangoes with mayank and Neeraj will become 1 : 2. Then how many mangoes does each of them have ?
3. Out of 2 equilateral triangles the side of the first triangle is 3 cms. more than

that of the second. The ratio of the perimeters of both the trangles is 5 : 2, then find out the sides of the triangles.

4. In a two digit number, the ten's place digit is three times the digit in units place. If the digits are reversed, the new number becomes 36 less than the original number. Find the number.

5. The sum of the digits of a two digit number is 7. On reversing the digits, the new number obtained is 9 more than the original number. Find the number.

6. The denominator of a rational number is 2 more than its numerator. If the numerator is increased 4 times, and 8 is added to the denominator, the new number would be $\frac{4}{3}$. Find the original number.

7. The ages of arun and Aakash are in the ratio of 7 : 5. After 6 years, their ages would be in the ratio of 5 : 4. Find their present ages.

8. Manisha's mother is thrice the age of Manisha. After 4 years, mother's age would be two and a half times that of Minisha. Find their present ages?

We have learnt

1. There is no difference in the value of the equation if
   (i) The same number is added to both sides.
   (ii) The same number is subtracted from both sides.
   (iii) Multipled by the same number other than zero.
   (iv) Divided by the same number other than zero.

2. If easy to solve $\frac{ax+b}{cx+d}=k$ type of equation by cross multiplicatin method.

3. $\frac{ax+b}{cx+d}=k$, a, b, c, d and k are integers and $cx+d \neq 0$, the equation is single variable or one unknown equation.

4. The solve a verbal problem, the unknown quantity is indicated by a variable and after changing it into an algebraic equation according to the question, the solution is obtained.

Chapter—13

APPLICATION OF PERCENTAGE

INTRODUCTION:

We have seen that in the calculation of interest we must know the rate of interest generally. This rate is considered equal to the interest on Rs. 100 in one year. We have solved several problems on rate of interest and percentage earlier we have also solved problems on profit & loss and have also calculated the rates in cost prices & selling prices. The more the rate, the higher the gain or loss or interest.

To repeat all this, let us solve some problems and revise some points once again.

1. If a student gets 396 out of 600, what is the percentage of marks obtained by him?

2. In a school 80% students passed out in 2004 if 12 students failed, find out the total number of students.

3. Rajani sold 8 calculators for Rs. 3500, if she gained Rs. 500, then what is the percentage of gain per calculator also find the cost price of per calculator.

4. Gamila purchased 50 card sheets for Rs. 400/- she made pictures on the card sheets after purchasing colour brush etc. for Rs. 100/- Out of each cardsheet she made 12 cards and sold them for Rs. 2 each How much profit did she get? Find the percentage of gain also.

5. A business man purchased 50 Kg of paddy for Rs. 1000/- and got it cleaned and packed in smaller packets of one kilogram, for this he spent Rs. 200. There was large incoming stock of paddy in the market and the price of paddy went down. So he could only sell the paddy at the rate of Rs. 21 per kilogram was he at loss or did he gain? Find out the rate percent also.

6. Mohan returned the borrowed amount after two years along with an interest of Rs. 500. If the rate of simple interest is 10% per annum, how much paddy did he borrow?

7. Rita gave someone Rs. 5000. After two years, how much amount will she get back if the rate of interest is 12% per annum.

8. Mohan returned Rs. 3024 to a merchant after 4 years. If the rate of interest is 11% per annum, how much money did he borrow?

We have found solutions to several such problems in class VI and VII.

Let us find solution for a new problem of this type.

Compound Interest :

Akhilesh borrowed in 10000 from the bank for some occasion in his family. He had to pay 10% interest per annum. This year he had a low income, he went to the bank to seek excuse for his debt and said that he would return Rs. 12000 next year. He was told that he would have to return Rs. 12100. Akhilesh pleaded that he shouldn't be charged for the delay and he will pay Rs. 12000 only.

The officer at the bank said that he was not being charged anything extra, but that was the interest over the interest that Akhilesh was not paying that year. Which meant that the annual interest on Rs. 10000 was Rs. 1000 since that interest was not being returned the next year his total principal amount turned to be 10000 + 1000 that is Rs. 11000. So, the coming year they would take an interest on Rs. 11000 Akhilesh said "And if I cant pay the amount next year you would take an interest on Rs. 12,100? This was something different for him. The people at the bank explained Akhilesh that generally we talk of simple interest only where the interest does not get added to the principal amount but when a person is given a loan, the loaner has to pay an interest over the interest also. This is known as Compound interest

Let us find out how much amount Akhilesh has to pay look at the table below :

TABLE 13.1

S.No.	Principal Amount	Rate	First year		Second year		Third year	
			Interest	Amount	Interest	Amount	Interest	Amount
1.	10,000	10%	1000	11000	1100	12100	1210	3310
2.	80,000	5%						
3.	5,000	10%						

Naturally, Compound interest is more than simple interest let us solve a few problems.

Example 1 : Find the compound interest on Rs. 1500 at 6% interest for 2 years find out the amount also.

Solution : According to the question.

For the first year P = Rs. 1500

R = 6%

T = 1 year

The interest for 1^{st} year $= \frac{P \times R \times T}{100}$

$= \frac{1500 \times 6 \times 1}{100}$

= Rs. 90

At the end of 1^{st} year the amount would be

= Principal + Interest

= 1500 + 90

= Rs. 1590

The amount at the end of the first year becomes the Principal amount for the next year.

So for the Second year.

P = Rs. 1590

R = 6%

T = 1 year

Hence interest for the Second year $= \frac{P \times R \times T}{100}$

$= \frac{1590 \times 6 \times 1}{100}$

$= \frac{159 \times 6}{10}$ = Rs. 95.40

Compound Interest = Interest for the First year + Interest for the Second year

= 90.00 + 95.40

= Rs. 185.40

Amount = Principal + Compound Interest

= Rs. 1500 + Rs. 185.40

= Rs. 1685.40

This is how we calculate compound interest. Thus we need to calculate the interest every year to calculate compound interest.

Example 2 :

Find the amount for Rs. 4000/- for 2 years at the rate of 8% per annum.

Solution : Here Principal P = Rs. 4000

Rate R = 8% or $\frac{8}{100}$

Time T = 2 years

We need to find out the interest of 1 year

Interest in 1st year $= \frac{P \times R \times T}{100}$

$= \frac{4000 \times 8 \times 1}{100} =$ Rs. 320

Amount at the end of 1st year

= Principal + Interest

= 4000 + 320

= Rs. 4320

This is the Principal for 2nd year

Interest in 2nd year $= \frac{P \times R \times T}{100}$

$= \frac{4320 \times 8 \times 1}{100} =$ Rs. 345.60

Amount at the end of 2nd year = Principal + Interest

= Rs. 4320 + Rs. 345.60

= Rs. 4665.60

Exercise 13.1

Q. 1. Find the compound interest for

(i) Rs. 4000 at the rate of 5% for 2 years.

(ii) Rs. 6000 at the rate of 10% for 3 years

(iii) Rs. 6250 at the rate of 8% for 2 years

Q. 2. Find the amount while interest is calculated every year.

(i) Rs. 7500 at the rate of 6% for 2 years

(ii) Rs. 2500 at the rate of 8% for 2 years

(iii) Rs. 5120 at the rate of 12½% for 2 years

Q. 3. A farmer wanted to buy a diesel pump and took a loan of Rs. 5500 from the rural bank at 4% annual compound interest. Calculate annually & find out how much money will the farmer return to the bank after 2 years?

Q. 4. Anuradha deposited Rs. 8000 in an institute at 5% interest per annum in Compound interest, find out the amount of money that she would get after a period of 3 years.

Compound Interest on quarterly and half yearly calculations

In banks and many other institutes, the calculation of interest is not only annual but also half yearly & some times quarterly also. This means that every six or three months the principal amount changes.

Example 3 :

Sakina deposited Rs. 4000 in the bank. Bank gives an interest at the rate of 5% per annum and the interest is calculated every 6 months. What will be the amount is her account after one year?

Solution :

If this interest is calculated annually then Sakina would get $\frac{4000 \times 5 \times 1}{100}$ that is Rs. 200 and there will be Rs. 4200 as her account. Since interest is calculated in 6 months so we hed to calculate the interest till 6 months and add principal amount to this.

Interest for first 6 months $= \frac{4000 \times 5}{100} \times \frac{6}{12} = 100$ (6 months could be written as 6/12 years)

Therefore after 6 months the Principal amount is Rs. 4100

Interest for second 6 months is

$$= \frac{4100 \times 5}{100} \times \frac{6}{12} = \text{ Rs. } 102.50$$

Thus after an year the amount in the account =

4000 + 100 + 102.50 Or

4100 + 102.50 = 4202.50

Obviously the calculation for 6 months we would get an excess interest of Rs. 2.50. The less or the time for which the interest is calculated, the faster will interest be calculated on the interest and the deposited or balance amount will keep on increasing.

EXERCISE 13.2

Q. 1. In a savings bank account the bank offers an annual compound interest of 5% If the interest is added to the principal amount every 6 months, then how much interest will Naresh get after 1 year If he deposited Rs. 1600/- in the bank.

Q. 2. Anamika deposited Rs. 24000/- at 10% annual interest for 1½ years. If the interest is calculated every 6 months, how much money will she get on maturity?

Q. 3. Find out the difference between the compound interest and simple interest for Rs. 7500 at the rate of 8% per annum for one year. Note that Compound interest is calculated every 6 months.

Q.4. What will be the compound interest on a principal of Rs. 8000 in 1 year when rate of interest is 5% per annum. Note that interest is calculated every 6 month.

Formula for Compound Interest:

Obviously as the number of years increase the calculation for compound interest will also become lengthy. So, we must make a formula.

For Principal amount we write P_1, for the rate of interest we write R and time is indicated by T. If the calculation of interest is annual, this the principal amount for the second year is P_2. Which is the sum of interest for the first year (I_1), and the principal amount P_1

$\therefore\ P_2 = P_1 + I_1$

Also, the interest for the 1st year $I_1 = \dfrac{P_1 \times R}{100}$ (Time T = 1 year)

This means, $P_2 = P_1 + I_1 = P_1 + \dfrac{P_1 \times R}{100} = P_1\left(1 + \dfrac{R}{100}\right)$

Interest for the second year $I_2 = P_2\dfrac{R}{100}$

The principal amount for the 3rd year $= P_2 + I_2 = P_1\left(1 + \dfrac{R}{100}\right) + P_2\dfrac{R}{100}$

$$= P_1\left(1 + \frac{R}{100}\right) + P_1\left(1 + \frac{R}{100}\right)\frac{R}{100}$$

$$= P_1\left(1 + \frac{R}{100}\right)^2$$

Thus if we more on like this the principal amount and the interest can be calculated.

Therefore the total amount at the end of the 3rd year

= Principal amount for the 4th year (P_4) $= P_1\left(1 + \dfrac{R}{100}\right)^3$

Total amount at the end of the 4th year = $P_1\left(1 + \dfrac{R}{100}\right)^4$

And similarly.

This means that if after "t" years we need to find the amount (Total amount), then we would indicate. This as $P_1\left(1 + \dfrac{R}{100}\right)^t$,Where P, is the basic principal amount.

Therefore the interest obtained in "t" years = the total amount after "t" years – principal amount $P_1\left(1+\frac{R}{100}\right)^t - P_1$

Therefore compound Interest in "t" years = $(C.I.) = P_1\left[\left(1+\frac{R}{100}\right)^t - 1\right]$

Example 4 :

Find out the compound interest for Rs. 800/- at the rate of 10% per annum for 2 years.

Solution : According to the question.

Principal (P)	=	800/- Rs.
Interest rate (R)	=	10% annum
Time (t)	=	2 years

Here the calculation of interest is annual therefore compound interest

$$C.I. = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

$$= 800 \times \left[\left(1+\frac{10}{100}\right)^2 - 1\right] = 800 \times \left[\left(1+\frac{1}{10}\right)^2 - 1\right]$$

$$= 800 \times \left[\left(\frac{11}{10}\right)^2 - 1\right] \qquad = 800 \times \left[\frac{121}{100} - 1\right]$$

$$= 800\left(\frac{121-100}{100}\right) = 800 \times \frac{21}{100} \qquad = \text{Rs. } 168$$

Example 5 :

If Jacob borrowed Rs. 80000/- as a loan to purchase a house from a housing society at the rate of 15% annual compound interest then how much amount will be returned by him after 3 years Find also the interest paid by him.

Solution : According to the question.

Principal (P) = 80000/- Rs.

Interest rate (R) = 15% annum

Time (t) = 3 years

Therefore to find amount $A = P\left(1+\frac{R}{100}\right)^t$

$$= 80000 \times \left(1+\frac{15}{100}\right)^3$$

$$= 80000 \times \left(1+\frac{3}{20}\right)^3 = 80000 \times \left(\frac{23}{20}\right)^3$$

$$= 80000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{23}{20} \quad = 80000 \times \frac{12167}{8000} = 10 \times 12167$$

= Rs. 121670

Jackob will have to return Rs. 121670/- after 3 years.

Therefore compound interest = Amount – Principal

= A – P

= 1,21,670 – 80,000

= Rs. 41670

In the above examples, compound interest has been calculated annually but it is not necessary that compound interest should always be calculated annually. Almost all banks calculate their interest half yearly or every six months. Some banking agencies also calculate their interest quarterly and keep adding the amount to the principal amount. It is remarkable that when time period is not mentioned, it is considered to be annual.

Therefore if interest is calculated every 6 months or half yearly, the time period is increases to double and the rate is reduced to half with the help of a formula we can calculate compound interest and the amount.

Let us understand this with the help of an example.

Example 6 :

Urvashi borrowed a loan of Rs. 2000/- at the rate of 20% annual interest. If interest is calculated half yearly, how much money will she return in after 1½ years? Also tell the amount of interest.

Solution : According to the question.

Principal amount (P) = Rs. 2000

Rate (R) = 20% annual

= 10% half yearly

Time (t) = 1½ years = 3 half yearly interest.

Therefore, $A = p\left(1+\frac{R}{100}\right)^t$

$= 2000\times\left(1+\frac{10}{100}\right)^3 = 2000\times\left(1+\frac{1}{10}\right)^3$

$= 2000\times\left(\frac{11}{10}\right)^3 = 2000\times\frac{1331}{1000}$

= Rs. 2662

C.I. = A – P

= 2662 - 2000 = 662

Example 7 :

Find out the amount which will be Rs. 13310 in 3 years at the rate of 10% per annum compound interest.

Solution : According to the question.

A = 13310

R = 10%

T = 3 years

$$A = P\times\left(1+\frac{R}{100}\right)^T$$

$$13310 = P\times\left(1+\frac{10}{100}\right)^3$$

or $$13310 = P\times\left(\frac{11\cancel{0}}{10\cancel{0}}\right)^3$$

or $$13310 = P \times \frac{11 \times 11 \times 11}{10 \times 10 \times 10}$$

or $$P = \frac{\overset{\overset{\overset{10}{\cancel{110}}}{\cancel{1210}}}{\cancel{13310}} \times 10 \times 10 \times 10}{\cancel{11} \times \cancel{11} \times \cancel{11}} = \text{Rs. } 10000$$

Therefore the principal amount is Rs. 10000/-

EXERCISE 13.3

Q. 1. Find out the compound interest and amount for the following :

(i) Principal = Rs. 6000, Time = 3 years, Rate 10% annual

(ii) Principal = Rs. 1600, Time = 2 years, Rate 5% annual

(iii) Principal = Rs. 8500, Time = 2 years, Rate 15% annual

(iv) Principal = Rs. 20000, Time = 3 years, Rate 5% annual

Q. 2. Salma took a loan of Rs. 625/- from the Mahila Samiti to purchase a sewing machine. If the rate of interest be 8% calculated annually, then how much money would salma return to the samiti after 2 years.

Q. 3. Find out the principal that becomes Rs. 5832/- in two years at the rate of 8% compound interest.

Q. 4. At what percentage of annual interest does Rs. 4000 become Rs. 5290/- in 2 years if interest be calculated annually.

Q. 5. At what rate does the compound interest for Rs. 1800 be Rs. 378 in 2 years if interest has been calculated annually.

Q. 6. Find out the difference between the simple interest and compound interest for Rs. 3200/- at the rate of 12% annually in 2 years.

Another application of percentage discount

Rehana went to buy a compass box with her mother. Her mother asked the shopkeeper “How much does this cost” The shopkeeper said “This is for Rs. 50/- but I will take Rs. 46/- only from you” Mother said “ give me some more discount for it” ? She persuaded the shopkeeper for sometime and the compass box was purchased for Rs. 42/-.

Rehana got her compass box, but did you understand what does discount mean? Discount or rebate is given on the base price (print value) of an item and after discount the item is sold at a lower price than that is fixed for it. Thus.

Discount or rebate = fixed price – selling price

Many a times the shopkeeper gives a considerable discount on a large purchase. Sometimes they give us a fixed discount only. The rate of discount is always calculated on the print value or fixed price of the item.

Example 8.

The printed value of a book is Rs. 40 and 12% discount is available find the discount & selling price of the book.

Solution :

Print value = Rs. 40, discount = 12%

Since on a print value of Rs. 100, discount in Rs. 12

$\therefore$ on a print value of Rs. 1, discount is $= \text{Rs.} \frac{12}{100}$

$\therefore$ on a print value of Rs. 40, discount will be $= \frac{12}{10\cancel{0}} \times 4\cancel{0} = \frac{48}{10}$

$\therefore$ Discount = Rs. 4.80

There selling price = Rs. 40 – Rs. 4.80 = Rs. 35.20

Example : 9

The print value of a table is Rs. 1250/- It is sold to a customer on Rs. 1100/- find out the percentage of discount given on the table.

Solution :

Print value = Rs. 1250/-

Selling value = Rs. 1100/-

Discount = 1250 – 1100 – 150/-

Discount on Rs. 1250 = 150/-

Discount on Rs. 1 $= \frac{150}{1250}$

Discount on Rs. 100 $= \dfrac{150 \times 100}{1250} = \dfrac{15\cancel{0} \times 4}{5\cancel{0}} = \dfrac{\overset{3}{\cancel{15}} \times 4}{\cancel{5}}$

This percentage of discount on the table is 12%.

Example 10 :

After a discount of 15% on the print value a shirt was sold for Rs. 442/- find the print value or print value of the shirt.

Solution :

Suppose the print value of the shirt = Rs. x

Discount = 15% of Rs. $x = x \times \dfrac{15}{100} =$ Rs. $\dfrac{3x}{20}$

Selling price = print value – discount

$$442 = x - \frac{3x}{20}$$

$$442 = \frac{20x - 3x}{20}$$

$$442 = \frac{17x}{20}$$

$$\frac{442 \times 20}{17} = x$$

$x = 520$

$\therefore$ Therefore the print value of the shirt is Rs. 520/-

Activity 1.

Now look at the table below & fill in the blanks given below :

Table 13.2

S.No.	Name of the student	Name of the book purchased	Print Value	Selling Price	Discount	Rate of discount
1	Rohit	Dictionary	Rs. 50	Rs. 40	Rs. 10	$\frac{10}{50}\times 100 = 20\%$
2	Alpana	Games in arithmetic	Rs. 60	Rs. 45	--------	$\frac{....}{60}\times 100 = 25\%$
3	Abida	Songs for Children	Rs. 45	--------	Rs. 30	----------------
4	Helen	Fast Mathematic	Rs. 60	Rs. 48	Rs. 12	----------------
5	Mahesh	Story book	--------	--------	Rs. 7.20	---- = 5%
6	Ahmed	Ramanujan	Rs. 72	--------	--------	---- = 10%
7	----------------	----------------	--------	--------	--------	----------------
8	----------------	----------------	--------	--------	--------	----------------

Example 11 :

A shopkeeper gave a off season 10% discount on sweaters to his customers in summer even then, 12.5% gain was in the shopkeepers pocket. At what price did the shopkeeper buy the sweaters if the print price on the sweaters is Rs. 500.

Solution :

Print value = Rs. 500

rate of Discount = 10%

Discount given = $\frac{500\times 10}{100}$ = Rs. 50

Selling price of the sweater = (500 – 50) = Rs. 450

Gain by the shopkeeper (in percentage) = Rs. 12.5

$$\text{Cost price} = \frac{450\times 100}{100+12.5} = \frac{450\times 100}{112.5} = 400$$

$$\left[\because \text{Cost Price} = \frac{\text{Selling Price}\times 100}{100+\%\text{ gain}} = \frac{\text{Selling Price}\times 100}{100-\%\text{ less}}\right]$$

Thus the cost price of a sweater = Rs. 400/-

TAX

You must have read and heard about taxes there are different kinds of taxes like income tax, sales tax, agriculture revenue tax, entertainment tax etc. Some taxes are collected by the central government and some others are collected by the state government. Some tax amounts are also paid to the Municipality or village, Panchayat why are taxes put, what are the uses of amount collected through taxes? You will study about these in your social studies book.

Example 12 :

Farmer Ramdeen has 25 acres of land If property or land tax is charged Rs. 15 per area annually, how much tax would Ramdeen pay for his land every year?

Solution :

Tax for per acre = Rs. 15

$\therefore$ Tax for 25 acres of land = 25×15 = Rs. 375/-

$\therefore$ Ramdeen would pay Rs. 375/- as land tax for his land.

Example 13 :

A motor cycle costs Rs. 42000 and 4% VAT or value added tax is charged on it. How much tax as VAT would be paid on the motor cycle?

Solution :

On Rs. 100 VAT is Rs. 4

$\therefore$ On Rs. 1 VAT is 4/100

$\therefore$ On Rs. 42000 VAT is $4/100 \times 42000$

= Rs. 1680 VAT would be paid

Example 14 :

A city has 5242 houses and if Rs. 2/- per house is deposited as property tax and Rs. 20/- as water tax every year, then calculate the amount deposited as tax for the city every year.

Solution :

According to the question :

Property / house tax = Rs. 2/- per house every year

Water tax = Rs. 20/- per house every year

Total property tax = Total houses x per house tax

= 5242×2

= Rs. 10484/-

Water tax = Total houses × per house water tax

= 5242×20

= Rs. 104840

Total amount Deposited as Tax = 10484 + 104840 = Rs. 115324

Example 15 :

A shopkeeper deposited Rs. 4500 as VAT after 6 months of sale. If VAT rate is 4% find out how much material of original amount did he sell?

Solution : Accroding to the question , Vat rate = 4%

When VAT is Rs. 4/- the original amount for the material = Rs. 100

When VAT is Rs. 1/- the original amount for the material =Rs. $\frac{100}{4}$

When VAT is Rs. 4500,

$$= \frac{100}{4} \times 4500 = \text{Rs.}1{,}12{,}500/-$$

Example 16 :

Rajia went to purchase medicines. She brought medicines of Rs. 625/- as print amount and paid an extra tax of Rs. 12.50 on it what is the percentage of the rate of tax?

Solution :

tax on Rs. 625/- = Rs. 12.50

tax on Rs. 1 $= \frac{12.50}{625}$

tax on Rs. 100 $= \frac{12.50}{625} \times \frac{100}{1}$

$$= \frac{1250}{625} \times \frac{\cancel{100}}{\cancel{100}}$$

$$= 2\%$$

Example 17 :

Suresh bought a Radio. He paid Rs. 780/- to the shopkeeper including 4% tax. Find the original price of the Radio.

Solution :

Let us suppose, the original price of the Radio Rs. 100/-

Rate of tax = 4%

Now the cost amount paid to the shopkeeper = 100 + 4 = Rs. 104

For Rs. 104, original price is Rs. 100/-

For is 780/- original price is Rs. $= \dfrac{780 \times 100}{104}$

= Rs. 750

Now you must have noticed that these questions are applications of unitary Method and percentage application only. The only thing you need to remember is what is the original price, the rate of tax and the money paid for tax and you can understand what is asked in the question & what is given. For example, in example 15 the amount paid as tax and the rate of tax is given and we have to find out the original amount and in example 16, the print amount and tax a given you need to find the rate of tax.

EXERCISE 13.4

Q. 1. Sarjus bought a bicycle that costs Rs. 1750/-. If the rate of sales tax on the cycle is 4% how much amount have Sarjues paid for the cycle?

Q. 2. The Gram Panchayat Gidhpuri near Mahanadi collects tax of Rs. 20 per cubic metre for the sand from the river bank. If a truck contains 5 cubic metre of sand, then how much tax would the Panchayat collects for 12 such trucks?

Q. 3. Anjali bought scents that cost Rs. 500 and ornaments that cost Rs. 800/- from a shop. The sale tax on scent is 16% and that on ornaments is 8% how much amount did Anjali in total pay to the shopkeeper.

Q.4. A farmer pay Rs. 4 per acre as land revenue to the government. If Ramdeen has 85 acres of agricultural land, how much land revenue would he pay to the government?

Q.5. Municipal Corporation of Sunderpur decided Rs. 8/- per square feet as development tax. If the land owned by Bhanuprakash measures 50 feet × 30 feet, how much development tax will be pay?

Q.6. Dinesh brings trucks of grains from other states for Rs. 37500. If 2.5% entry tax is payable on grains, how much entry tax would Dinesh pay?

Q. 7. Gram Panchayat Adrena charges house tax of Rs. 25/- per house and if there are 216 houses under the Panchayat area, how much income will the village Panchayat have through house tax?

Q. 8. The Indian government charges 11% production tax on tractor over the invested amount. If a tractor produced in a factory costs Rs. 120000, find out the production tax amount for per tractor?

We have learnt

1. When interest after a fixed time is added to the principal amount and then interest is calculated this kind of interest is called compound interest.

2. Compound Interest $\text{C.I} = P\left[\left(1+\frac{R}{100}\right)^{T}-1\right]$

3. Amount $A = P \times \left(1+\frac{R}{100}\right)^{T}$

4. When the interest is calculated half yearly time is double & rate becomes half.

5. Discount is always given on the print price or print value.

6. The tax put on the sale of any item is its sale tax.

7. If the selling price & the percentage of gain or loss is known then cost price would be $\text{Cost Price} = \frac{\text{Selling Price} \times 100}{100 + \%\text{ gain}} = \frac{\text{Selling Price} \times 100}{100 - \%\text{ loss}}$.

Chapter—14

MENSURATION - I

Akanksha and Ranu cut thick papers into different sizes of rectangles & parallelograms as shown below:

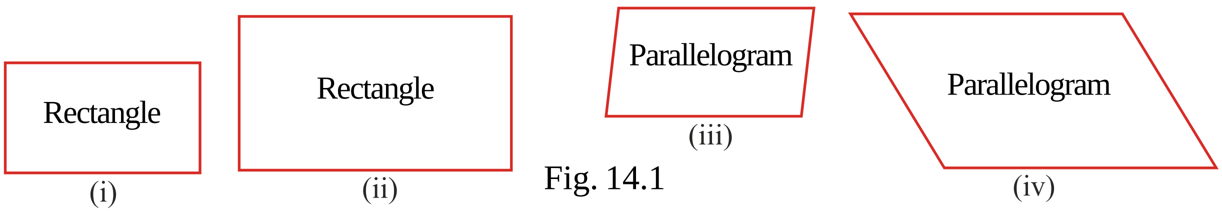
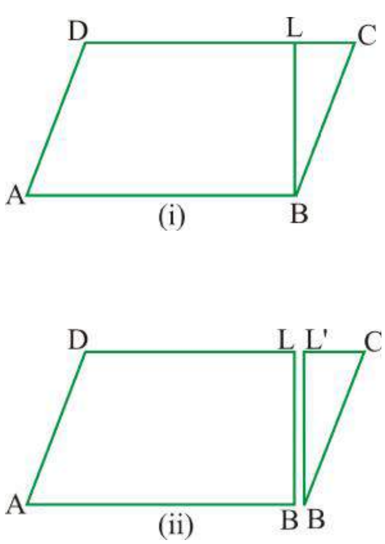


Fig. 14.1

Ranu asked Akaksha, to find out their areas. Akanksha multiplied the length and breadth of the rectangular pieces and found out the areas (Area of a rectangle = length x width) But she could not find the area of the parallelograms because she could not decide their length and breadths. Ranu said, "If we cut the pieces of parallelogram and make them rectangle, we will be able to get their areas, so they did the following activity.



(i) They took a figure of parallelogram and named it as ABCD. Considering AB as the base, draw a perpendicular on CD as BL. Then they could separate BLC and cut it out from the parallelogram figure 14.2.

(ii) When Ranu tried to join the triagular figure, the figure 14.2 (iii) was obtained After this, Akansha attached it to the figure and got the shape of fig. 14.2 (iv).

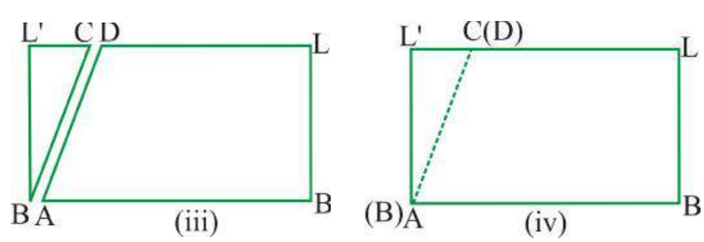


Fig. 14.2

Thus they got the shape of a rectangle. Akansha said figure 14.2(1) and figure 14.2 (iv) have equal areas, become fig. 14.2 (iv) is a changed form of fig. 14.2(1).

Rectangle ABLL' = AB × BL = Base of the parallelogram ABCD × height.

Therefore, area of the parallelogram ABCD = base × height

Thus Ranu & Akanksha found out area of parellegoram. So,

(i) Area of a parallegoram = *base* × *height*

(ii) Base for a parallelogram $= \frac{area}{height}$

(iii) Height of a parallelogram $= \frac{area}{base}$

Example 1 :

Find out the area of a parallelogram where base is 15 cm and height is 5cm.

Solution :

According to the question.

Base = 15 cm, height = 5 cm.

∴ Area of the parallelogram = base x height

= 15 cm x 5 cm

= 75 cm^2

Example 2 :

Find out the base of the parallelogram where area is 240 cm^2 and height is 8 cm.

Solution : We knew that base of a parallelogram $= \frac{\text{Area}}{\text{height}}$

area = 240 Cm^2

height = 8 cm

Base $= \frac{240cm^2}{8cm}$

= 30 cm.

Exercise 14.1

Q. 1. Find out the area of a parallelogram whose base and height are as follows:

(i) Base = 15 cm, Perpendicular (apex) = 10 cm

(ii) Base = 90 cm ,Perpendicular (apex) = 8 cm

(iii) Base = 120 cm, Perpendicular (apex) = 15 cm

Q.2. Find the area of the parallelogram where base is 26.5 cm and (apex) perpendicular is 7 cm.

Q.3 Find out the base of the parallelogram, whose area is 390 cm^2 and the perpendicular (apex) in 26 cm.

Q. 4. Find out the Perpendicular (apex) of parallelogram with area is 1200 m^2 and base is 60 m.

Let us do this activity:

Draw a triangle ABC taking A as centre and of BC as length and C as centre and of BA as length raw two arcs which intersects each other. Now join point A & C and name it by D. Thus parallelogram ABCD is obtained because AB = DC & AD = BC.

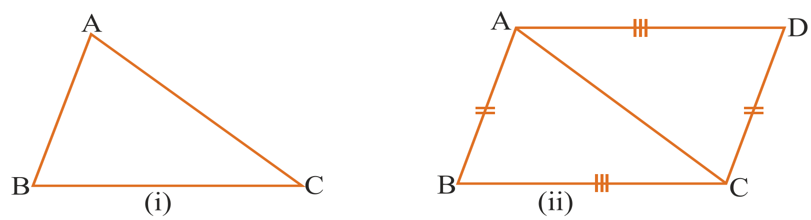


Fig.14.3

Activity

Cut a rectangular piece of hard board ABCD along its diagonal AC with the help of scissor.

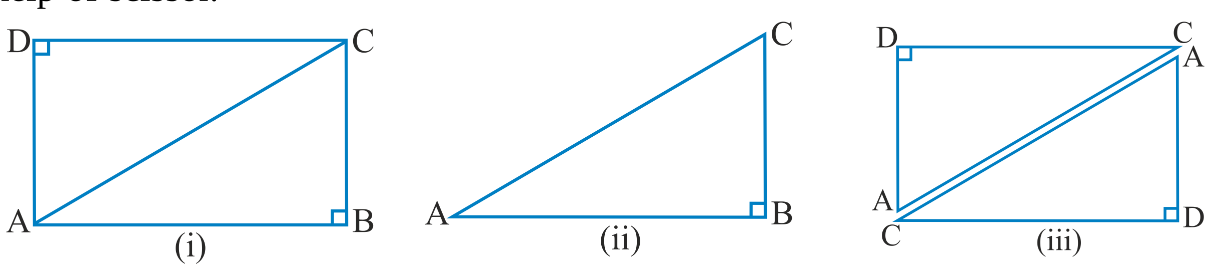


Fig.14.4

Thus, two triangles ABC and ADC were formed. Place the two triangles ABC on ADC on each other. Do they overlap each other completely? You will find that the two triangles are congruent and their areas are equal.

$\therefore$ Area of Δ ABC + area of Δ ADC = area of rectangle ABCD

Area of Δ ABC = area of Δ ADC

2 (Area of Δ ABC) = AB $\times$ BC

Area of Δ ABC = ½ × AB × BC

Practice 1

Make a parallelogram on a piece of hard board. Cut it out. Cut it along one of the diagonals. You will get two triangles. The areas of the two triangles are equal. Place them on each other & observe.

Area of a triangle:

We can construct a parallelogram by joining two triangles of equal measurement. On drawing a diagonal in a parallelogram we get two triangles of equal measurement. So when we draw a diagonal AC in the parallelogram ABCD we get two triangles ABC and ADC that are congruent. Their areas are also equal.

Therefore, the area of parallelogram ABCD = Area of Δ ABC + Area of Δ ADC

= 2 (Area of Δ ABC)

∴ Area of Δ ABC = ½ area of parallelogram gm ABCD

= ½ × base × height

= ½ × BC × AL

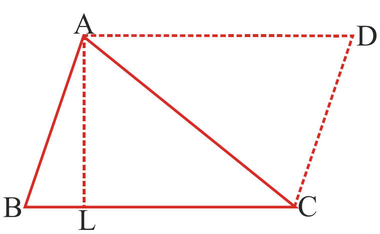


Fig.14.5

Therefore : **Area of a triangle A = ½ × b × h**

Where b = base of the triangle and h = height of the triangle

Remember : *The area of a triangle between two parallel lines is half the area of a parallelogram with the same base and height.*

Example 3 :

Find out the area of a triangle with base 28 cm and height 6 cm.

Solution :

According to the question.

Base of a triangle b = 28 cm

Height h = 6 cm.

So, the area of a triangle A = ½ × b × h

= ½ × 28 × 6 cm = 84 cm^2

Example 4 :

Find out of a triangle where base is 80 cm and area is 0 . 08 square m.

Solution:

Since base is given in Cms, the area also should be converted to cm.

Thus 1 $meter^2$ = 1 metre × 1 metre

= 100 cm × 100 cm

= 10000 cm^2 ($\because$ 1 m = 100 cm)

Therefore .08 $metre^2$ = 0.08 × 10000 Cm^2

= 800 Cm^2

Now the area of triangle A = ½ × b × h

$$\text{Height of a triangle} = h = \frac{2A}{b} = \frac{2 \times 800}{80}$$

$\Rightarrow$ h = 20 cm

Exercise 14.2

Q. 1. Find out the area of a triangle with base 12 cm and corresponding height is of 7 cm.

Q. 2 Find out the area of a triangle with base 25 cm and a Perpendicular from apex length is of 1.5 cm.

Q. 3. Find out the perpendicular (apex) length of a triangle where base is 6.5 cm and the area is 26 cm^2.

Q. 4. Find out the area of a triangle where base is 120 dm and height is 75 dm.

Area of a rhombus :

A rhombus is a type of parallelogram and so if its base and height are known, we can find its area, If the base of the fig. be ‘b’ and height be ‘h’ then area A = b x h

ABCD is a rhombus and d_1 and d_2 are its diagonals since they intersect each other at right angles, therefore, the four right angled triangles will have perpendicular sides $\frac{d_1}{2}$ and $\frac{d_2}{2}$.

Area of the rhombus = 4 x area of a right angled triangle

$= 4 \times \frac{1}{2}$ base × height

$= 4 \times \frac{1}{2}\left(\frac{1}{2}d_1\right)\times\left(\frac{1}{2}d_2\right)$

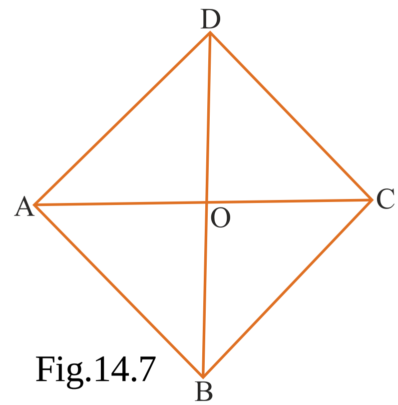


Fig.14.7

Area of a rhombus	**$= \frac{1}{2} \times d_1 \times d_2$**

Area of the rhombus	**$= \frac{1}{2} \times 1^{st}$ diagonal $\times 2^{nd}$ diagonal**

Area of a Trapezium:

Trapezium is a quadrilateral whose two opposite sides are parallel to each other. A trapezium ABCD is shown in fig. 14.8. Side AB and DC are parallel. The perpendicular distance from the parallel sides have been shown as AM and CL.

If a diagonal AC of this triangle be drawn, we get two triangles ABC and ACD from this trapezium.

Therefore, the area of a trapezium ABCD = Area of Δ ABC + Area of Δ ACD

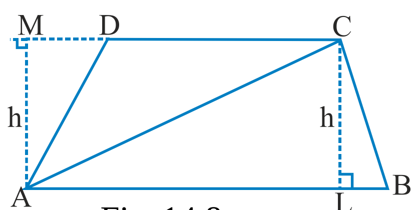


Fig. 14.8

Area of a trapezium ABCD =

$= \frac{1}{2} \times AB \times CL + \frac{1}{2} \times DC \times AM$

Since CL and AM indicate the height of the trapezium, therefore this will be equal so, we consider this as 'h'.

Thus the Area of a Trapezium = $\frac{1}{2}$ AB × h + $\frac{1}{2}$ DC × h

If AB = b_1, and DC = b_2 then

Area of a Trapezium $= \frac{1}{2} b_1 \times h + \frac{1}{2} b_2 \times h$

$= \frac{1}{2} (b_1 + b_2) \times h$

= ½ × (Sum of parallel sides)× (Distance between parallel sides)

= ½ × (sum of parallel sides) × height

Area of a trapezium = ½ × (Sum of parallel sides) × height

or **Area of a Trapezium = ½ (b_1 + b_2) × h**

Example 6 :

The sides of a rhombus are 7cm. and its height is 32 cm. find its area.

Solution :

According to the question

Base = 7 cm, height = 3.2 cm

Area of a rhombus = base × height

= 7 × 3.2 cm^2

= 22.4 cm^2

Example 7 :

The first diagonal of a rhombus is of 10 cm and the second diagonal of a rhombus is of 12 cm. find its area.

Solution :

According to the question

First diagonal of the rhombus = 10 cm

Second diagonal of the rhombus = 12 cm

Area of a rhombus = ½ × (1^{st} diagonal) × (2^{nd} diagonal)

= ½ × 10 × 12 cm

= 60 square cm

Example 8 :

The parallel sides of a trapezium are 25 m and 20 m respectively. The distance between the two parallel sides is 8 m. Find its area.

Solution :

Given :- b_1 = 25 m, b_2 = 20 m, h = 8 m

Therefore, Area of a trapezium A $= \frac{1}{2} \times h \times (b_1 + b_2)$

$= \frac{1}{2} \times 8 \times (25 + 20)$

$= \frac{1}{2} \times 8 \times (45)$

$=$ 180 Square metres Ans.

Example 9 :

Area of a trapezium is 140 cm^2, if the lenght of one of its parallel sides is 25 cm and height is 7cm. then find out the length of the second parallel side.

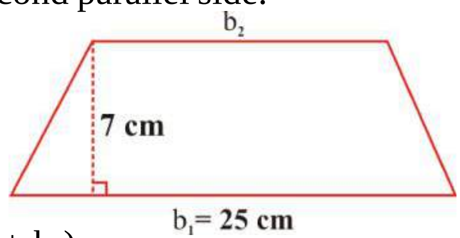


Fig.14.9

Solution :

According to the question

$A = 140\ cm^2,\ b_1 = 25\ cm,\ h = 7\ cm$

Area of a trapezium $= \frac{1}{2} \times h \times (b_1 + b_2)$

So, $140 = \frac{1}{2} \times 7 \times (25 + b_2)$

$$\frac{2 \times 140}{7} = 25 + b_2$$

$40 = 25 + b_2$

$b_2 = 40 - 25$

The second side $b_2 = 15$ cm

Exercise 14.3

Q. 1. Find the area of a rhombus where diagonals are of 24 cm & 10 cm.

Q. 2. Find the area of a rhombus where one side is 7.5 cm and the perpendicular from the apex is 4 cm.

Q. 3. Find the area of a trapezium where parallel sides are of 20 m and 8 m and the distance between there two sides are 12 cm.

Q. 4 What will be the area of the trapezium where base is 30 cm and 24.4 cm, if its perpendicular from apex is 1.5 cm

Q. 5. The area of a trapezium is 105 cm^2 and its height is 7 cm, if base of the one of the sides out of the two parallel arms is 6 cm more than the other, find the length of the two parallel sides.

Area of a Rectangular path:

Usually we need to find the area of the verandah around a school building, the road around a farm, the surrounding space of a play-ground etc., what do we do in these conditions?

Fig.14.10

In fig.14.10. we see a rectangular piece of farm that is surrounded by a road on all sides. What will you do, if you need to find the area of this road?

It is clear that we are getting two rectangles, so we can subtract the area of the smaller rectangle from the area of the bigger rectangle.

Area of the rectangular path = Area of the bigger rectangle – Area of the smaller rectangle

Example 10 :

A rectangular farm is 90 m. in length and 65 m. in width. On all the four sides there is path of 5 m. wide find the area of the path.

Solution :

It is clear from the picture that the area of the path =

Area of the rectangle ABCD - Area of the rectangle EFGH

Therefore , Area of the path = (AB × BC) – (EF × FG)

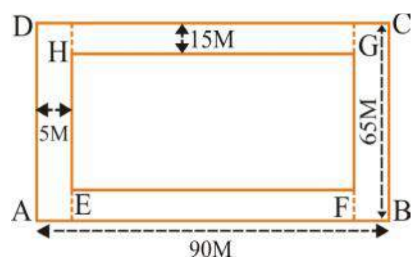


Fig. 14.11

Here, AB = AE + EF + FB

AB = 5 + 90 + 5

AB = 100 meter

Similarly, BC = 5 + 65 + 5

= 75 m.

Now the area of the path = (AB × BC) – (EF × FG)

= 100 × 75 – 90 × 65

= 7500 – 5850

= 1650 meter^2

Example 11 :

The length of a wall is 15.5 m and its width is 9m, There are two doors of 3m×1.5m size and two windows of 2m×1m size. What will be the cost of white washing this wall at the rate of Rs. 5 per square meter.

Solution :

First we need to find the area of the wall that is to be actually white washed.

Therefore, Area of the wall to be white washed

= Area of the complete wall – Area of (2 doors + 2 windows)

Area of the wall = Length × breadth

= 15.5 × 9

= 139.5 m^2

Area of one door = length × breadth

= 3 × 1.5

= 4.5 m^2

Area of two doors = 2 × 4.5 m^2

= 9.0 m^2

Area of one window = length × breadth

= 2 × 1

= 2 m^2

Area of two windows = 2 × 2 m^2

= 4 m^2

Area of the wall to be white washed

= 139.5 – (9.0 +4)

= 139.5 - 13.0

= 126.5 m^2

Since the cost of white washing is Rs. 5/- square meter

Therefore, total cost = 126.5 × 5

= Rs. 632.50

Example 12 :

The length and breadth of a rectangular area is 35m and 24m respectively. There is a 2m wide road along its length and 1m wide road along its breadth. Find the area of the road.

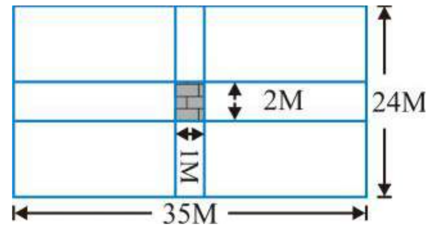


Fig. 14.12

Solution :

Area of the road along the length = $35 \times 2 = 70m^2$

Area of the road along the breadth = 24×1

= $24\ m^2$

Area of the shaded part = 2×1

= $2m^2$

(The shaded part comes in both the roads, So we will subtract that area once)

The area of the road = $70 + 24 - 2$

= $94 - 2$

= $92\ m^2$

Exercise 14.4

Q. 1. One 25 cm long and 10 cm wide picture is surrounded by a 2 cm wide frame on all the sides; find the area of the frame.

Q. 2. A rectangular playground measures 35m × 25m. A road 3m wide along the length and 2 m wide along the width goes through the middle of the ground. Find the area of the road.

Q.3 A basketball ground is 28 m long and 15 m wide. A 5 m wide gallery for spectators has to be built on all its sides of the ground. Find the area of the spectator gallery and the cost of making that gallery at Rs. 5.25 per square meter.

Area of a circular path:

In our previous classes, we have learnt about the circle. If the radius of the circle is 'r' then circumference $C = 2\pi r$ and the area= πr^2

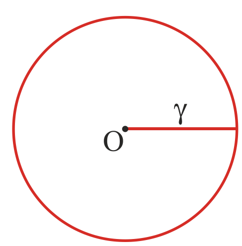


Fig. 14.13

Since, π is a constant and its value is 22/7 or 3.14.

Example 13 :

Find the circumference and area of the wheel of a bicycle where radius is 21cm.

Solution :

The wheel of a bicycle is circular, therefore the circumference of a wheel

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 21 \text{ cm} = 132 \text{ cm}$

Area of the wheel $= \pi r^2$

$$= \frac{22}{7} \times (21)^2$$

$$= \frac{22}{7} \times 21 \times 21$$

$$= \frac{22}{\cancel{7}} \times \overset{3}{\cancel{21}} \times 21$$

$$= 1386 \text{ cm}^2$$

Concentric Circle:

In fig. 14.14 are given two concentric circles. If we need to find the area of the shaded portion, what wall we do? Obviously we shall subtract the smaller area from the bigger area.

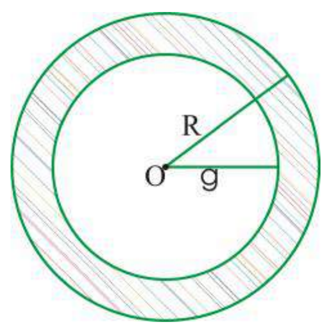


Fig. 14.14

Therefore:

Area of a circular path = Area of bigger circle – area of smaller circle

Example 14 :

A circular pond is 200 m in its radius. A circular path of 7 m width built along with the bank. Find the area of the path.

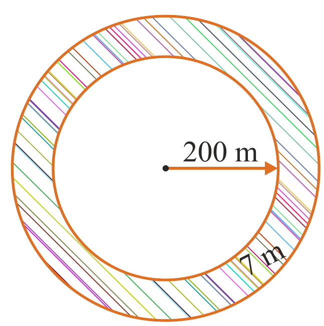


Fig. 14.15

Solution:

The area of a circular path =

Area of the bigger circle – area of the smaller circle

Radius of the smaller circle **r** = 200 m

Radius of the bigger circle **R** $= 200 + 7$ m

$= 207$ m

The area of the bank binding of the pond $= \text{®} R^2 - \text{®} r^2$

$= \text{®}[(207)^2 - (200)^2]$

$$= \frac{22}{7}(207 + 200)(207 - 200) \quad [\because (a^2 - b^2) = (a+b)(a - b)]$$

$$= \frac{22}{7}(407)(7) = 22\ (407)\ m^2 = 8954\ m^2$$

Exercise 14.5

Q. 1. The radius of two concentric circles are 9 cm and 12 cm respectively. Find the area of the circular path between the two circles.

Q.2. The area of a circle is 616 cm^2. There is a 2 m wide road on its edge. What will be area of that road?

Q.3 The radius of a circular cricket ground is 60 m. Around the ground a 7m circular gallery is to be built for spectators. What will be its area?

To find the Approximate Area of the Trapezium given on graph paper with the help of square grid method and verify the result with the formula method –

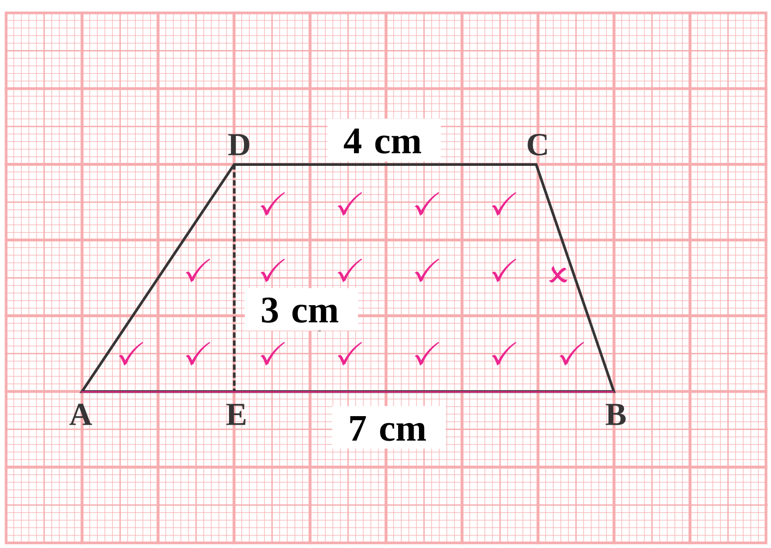


Fig. 14.16

For figure 14.16

Calculate the Approximate area of a Trapezium with the square grid.

In Trapezium ABCD

Number of complete squares = 14

Number of squares that are bigger than half in size = 2

Number of just half sized squares = 1

Area of the Trapezium = (No of Complete square + No of squares more than half in size) + $\frac{\text{No. of half sized squares}}{2}$

$= 14+2+\frac{1}{2}\times 1$

$= 16 + \frac{1}{2} \times 1$

$= 16 + \frac{1}{2}$

$= 16 + .5$

$= 16.5$ Sq. Cm.

Area of a Trapezium by formula method

In trapezium ABCD

Parallel Sides	AB	= 7cm.
and	CD	= 4cm.
Height of the Trapezium	DE	= 3cm.

Area of the Trapezium = ½ (Sum of the Parallel side) xHeight

$= \frac{1}{2}(AB + CD) \times DE$

$= \frac{1}{2}(7 + 4) \times 3$

$= \frac{1}{2} \times 11 \times 3$

$= \frac{33}{2}$

$= 16.5$ Sq. Cm.

It is clear that

Approximate Area of the Trapezium by square grid method = Area of the Trapezium by formula

To find Approximate Area of polygon given on graph paper by square grid –

In Polygon ABCDEFA

figure 14.17

The area of the figure 14.17 is done by this method.

- No. of complete squares = 21
- No. of squares that are bigger than half in size = 8
- No. of half sized square = 2

$$\text{No. of squares for} = (\text{No of Complete square} + \text{No of squares more than half in size}) + \frac{\text{No. of half sized squares}}{2}$$

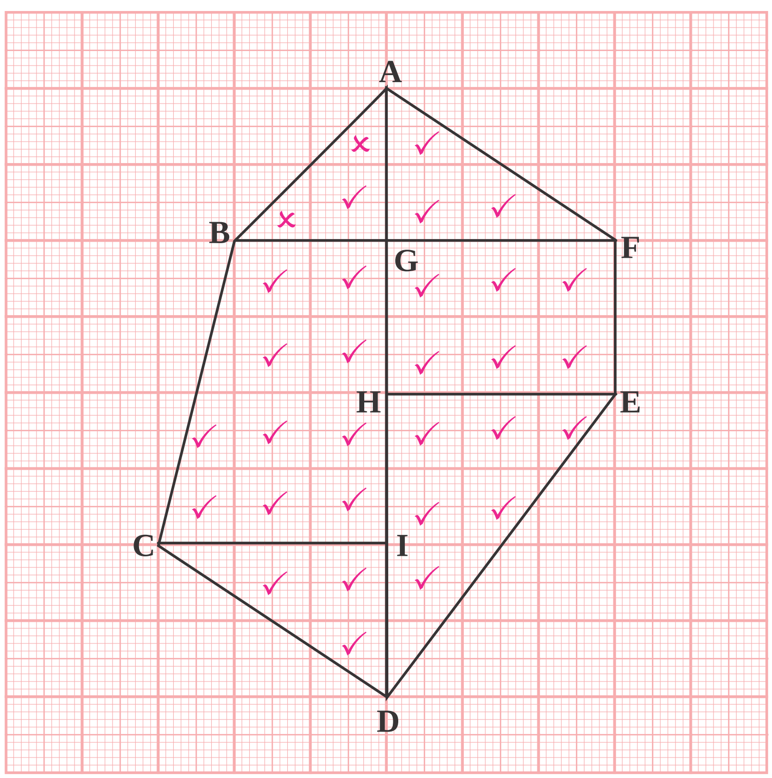


Fig. 14.17

$= 21+8+\frac{1}{2}\times 2$

$= 29 + \frac{1}{2}\times 2$

$= 29 + 1$

$= 30$

Therefore Area of the polygon ABCDEFA $= 30cm^2$

Calculation of Area of Trapezium by formula

Area of polygon ABCDEFA = Area of ΔAGB + Area of Trapezium BGIC + Area of ΔCID + Area of ΔDHE + Area of Rectangle HEFG + Area of ΔGFA

$$= \frac{1}{2} BG \times GA + \frac{1}{2} (BG + CI) \times GI + \frac{1}{2} CI \times ID + \frac{1}{2} HE \times HD + HE \times HG + \frac{1}{2} GF \times AG$$

$$= \frac{1}{2} \times 2 \times 2 + \frac{1}{2} (2 + 3) \times 4 + \frac{1}{2} \times 3 \times 2 + \frac{1}{2} \times 3 \times 4 + 3 \times 2 + \frac{1}{2} \times 3 \times 2$$

$$= 2 + (5 \times 2) + 3 + (3 \times 2) + 6 + 3$$

$$= 2 + 10 + 3 + 6 + 6 + 3$$

$$= 30cm^2$$

It is clear that

Approximate area of the polygon by square grid = Area of the polygon by formula method

We have learnt

1. Area of rhombus = base × height
2. The diagonal of a rhombus divides the rhombus into two equal triangles.
3. Area of triangle = $\frac{1}{2}$ × base × height

4. Area of equalatral triangle = $\frac{\sqrt{3}}{4}(side)^2$

5. Area of a rhombus = Base × Height

= ½ × 1st diagonal × 2nd diagonal

= ½ × d_1 × d_2

6. Area of a trapezium = ½ (sum of parallel sides) × height

= ½× (b_1+b_2)×h

7. Area of a circle = π × $(\text{radius})^2$

= πr^2 (here, π =constant = $\frac{22}{7}$ = 3.14 approx.)

8. Circumference of a circle = 2× π × radius

= $2\pi r$

Chapter—15

MENSURATION-II

Introduction

Some children were playing in the sand infront of the school.

Aashu and her friends also reached there. Aashu said "Let us take a mug full of sand each and make somethings", Rahim said "OK, but I don't know how to make a house or anything of that sort, let's make a rectangular block".

Everyone took a mug full of sand (equal amounts) and started making blocks. All these were completed in a short while but were of different measures. Anu asked "Why is this so?" All of us took equal amounts of sand, then why are the blocks of different sizes?" Rahim, looked closely at the blocks and said "I can see that the blocks that have a larger spread on the ground have lesser heights. Also there seems to be some relationship between the amount of sand and the length, width and height of the block". Aashu said, "We have learnt in earlier classes that **volume = Length × Breadth × Height**. So while the volume of the sand remained the same, the height, length and breadth changed. Let us measure the length, breadth and height of the blocks."

Figure 15.1

Everybody measured the length, breadth and height of their blocks. Using the formula Volume = Length × Breadth × Height, they found that the volume of sand used in each of the blocks was the same.

The three of them then gave different shapes to their sand blocks. Will the new shapes made by them have the same volume as that of the blocks? If so, then why? Write in your notebook about this.

Is Capacity also the Volume?

Children calculated the volume of the sand in the mug by finding the volume of sand in the block. The quantity of sand that can be put in a mug, is its capacity. Similarly, the amount of water in a bucket, or the amount of air in a room, are the capacities of the bucket and the room respectively. Therefore, the volume of the substance that can be contained in an empty space is the capacity of the empty space. Can you tell the capacity of the glass that you use for drinking water and the capacity of the bucket that you use while bathing? You cannot tell the capacity because you need to know the maximum amount of water that can be contained in the glass or in the bucket.

You would have understood what the capacity is. But how would you explain what the volume of an object is on the basis of your experience? Think and write in your notebook. Match your answer with those of your friends and find out the difference and similarity between your thinking and their thinking.

You have read about area in 6th class. The area of a figure represents the measure of the space covered by it on a plane. In a similar way, volume of an object represents the amount of space occupied by that object. Let us see how we measure the volume or the space occupied by an object.

Cuboid:

To find the volume we need to know the shape of the object. Let us first learn about the shapes of the objects whose volume is to be determined. Observe the things around you for example- copy, book, match box, chalk box and a brick. What is special about the shapes of all these objects?

Figure 15.2

The special features of these objects are that each of their surface is rectangular and the area of each surface is exactly the same as the area of the surface opposite it. Such type of shapes are called **cuboids**.

Activity 1

Make a list of any five cuboidal objects that are found in your surroundings. Check whether the areas of opposite surfaces are equal or not. Also check whether adjacent edges of cuboid make an angle of 90° with each other or not. Since adjacent edges of the cuboid make an angle of 90° with each other, therefore each of its surface is a rectangle. A cuboid is therefore also called a **rectangular solid**.

Figure 15.3

Take a notebook or book that looks like the following:

Figure 15.4

The object in your hand is a cuboid. At many points of this object, three edges are meeting. Count and write the number of such points.

As you are observing in the above figure A, B, C, D, E, F, G, H are such 8 points on the cuboid, these are called the vertices of a cuboid. At each such point three edges are meeting. Count and write the total number of surfaces present in the cuboid with you.

You would have noticed that the cuboid has 6 surfaces like - ABCD, its opposite surface EFGH, AEFB and its opposite surface DCGH and similarly AEHD and its opposite surface BFGC. In this way every cuboid has 6 surfaces and these are called faces of the Cuboid.

In this way AB, BC, CD, DA, EF, FG, GH, HE, DH, AE, BF, CG are the 12 edges (sides) of the cuboid.

Activity 2

Measure the edges of your mathematics book and write down the measurements. Answer the following questions:-

1. Are all edges of different measurement?
2. How many edges are of the same measurement?
3. How many different measures of edges do we have?

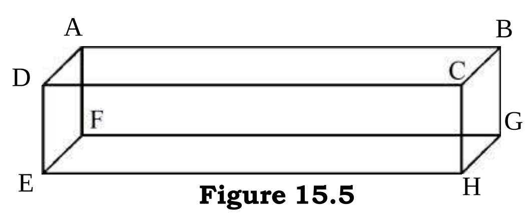


Figure 15.5

You can see in the figure 15.5 that DC=AB = FG=EH and AD=FE=GH=BC. Similarly AF=BG=ED=HC etc. In every cuboid there are three quarterplets (4 each) of equal edges and among these edges one edge may be considered as the length, the second edge as breadth and third as the height.

Let the length of the cuboid be AB, breadth be AD and height be AF. All these are of different measures but if the length breadth and height of a cuboid are equal then what will be the shape of the solid? Have you ever seen a solid of this shape?

Cube:

Concentrate on the figure 15.6. Measure its length, breadth and height.

What is the similarity between its length, breadth and height? What would you call this shape?

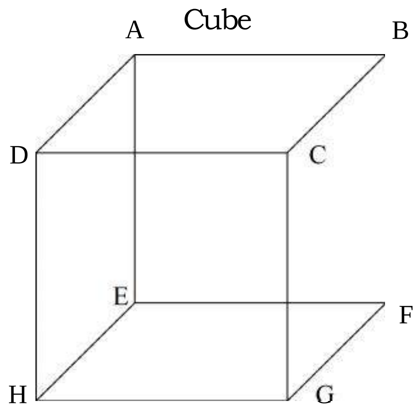


Figure 15.6

"A rectangular solid with equal length, breadth and height is called a Cube."

If the length, breadth and height of a cuboid are known then its volume can be calculated.

If the length, breadth and height of a cuboid are known then its volume = Length × breath × height.

Or $V = \ell \times b \times h$

V = Volume of cuboid

ℓ = length of cuboid

b = breadth of cuboid

h = height of cuboid

We have studied in the previous classes that the area of a rectangle = length × breadth and the volume of a cuboid = length ×breadth × height. Since the base is a rectangle we can also write volume of cuboid as = (Area of base) × height.

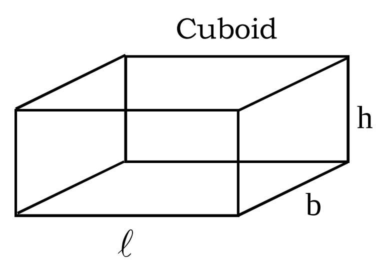


Figure 15.7

Length, breadth and height are equal in a cube therefore $\ell = b = h$.

Since, Volume of cube = side × side × side = $(\text{Side})^3 = S^3$ (S= side)

$$V = S^3 \ (S = \text{Side})$$

Unit of volume

As meter is the unit of length in the same way square meter or $(\text{meter})^2$ is the unit of area. Similarly, we would need a standard unit for volume also. If everyone uses different units for volume or capacity, then each will get a different value.

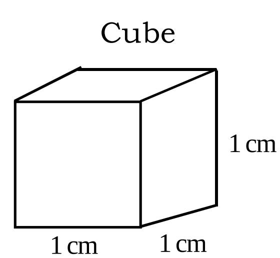


Figure 15.8

For example;

If a tank is filled by 50 small buckets then considering the small bucket as a unit, the volume of the tank will be 50 units or 50 small buckets. But if the same tank gets filled by 10 big buckets then taking the big bucket as the unit of volume, the volume of the tank will be 10 units or 10 big buckets. Therefore, there is a need of such a standard unit for volume, that has the same value at all places.

The unit of volume is a 1 cubic cm. This is equal to the volume of a cube of length1cm, breadth 1cm and height 1 cm. This can be written as 1 cm^3.

The unit of volume is also cubic meter. This is equal to the volume of a cube of length 1m, breadth1m and height1m. It can also be written as 1 $meter^3$. This is the standard unit of volume.

Relation between m^3 and cm^3

$1m^3 = 1m \times 1m \times 1m$

$= 100\ cm \times 100\ cm \times 100\ cm$

$= 100 \times 100 \times 100\ cm^3$

$= 1000000\ cm^3$

$= 10^6\ cm^3$

Activity 3

In the following figure, measure the edges of the cuboids and find their volumes.

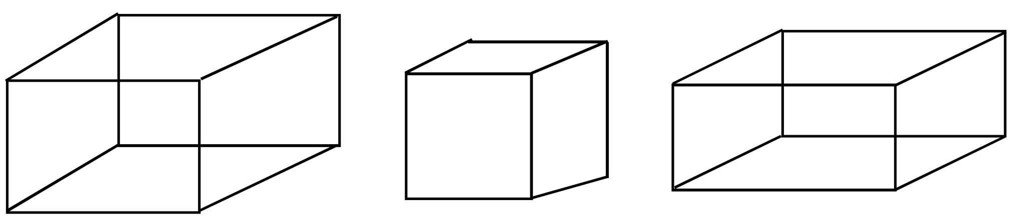


Figure 15.9

Example 1: The length breadth and height of a cuboid are 4cm, 3cm and 2cm respectively. Find its volume.

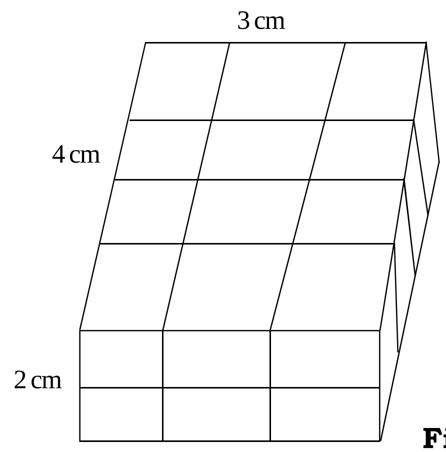


Figure 15.10

These are two layers of 1cm cubes. There are 12 cubes in each layer. Thus, there are 24 cubes. Hence, the volume of the cuboid is 24cm^3.

Solution: Given: length of Cuboid (ℓ) = 4 cm.

Breadth (b) = 3cm

Height (h) =2 cm

Volume of Cuboid V = $\ell \times b \times h$

$= 4 \times 3 \times 2$

$= 24\ cm^3$ or cubic centimeter

Example 2. If the side of a cube is 5 cm, find its volume.

Solution : Given : Length of one side of a Cube = 5 cm

$\therefore$ Volume of Cube = V = S^3 = S × S × S

= 5 × 5 × 5

= 125 cm^3

Example 3: Each side of a cube is 6 cm. How many cubes of side 2 cms can be cut out from this cube?

Solution : Given: The side of the cube = 6cm

The Volume of the cube = $(\text{Side})^3$

= $(6)^3$ = 6 × 6 × 6 = 216 cm^3

The side of cubes that have to be cut out = 2 cm

Volume of cube = $(\text{side})^3$

= $(2)^3$

= 2 × 2 × 2 = 8 cm^3

$\therefore$ Number of cubes that can be cut out.

$$= \text{Height} = \frac{\text{Volume of Bigger Cube}}{\text{Volume of the Cubes Cut Out}}$$

$$= \frac{216\text{cm}^3}{8\text{cm}^3}$$

Therefore, the number of cubes is 27.

Example 4: The volume of a wooden cuboid is 36 cm^3. If its length is 4 cm and breadth 3 cm then find its height.

Solution : Given: The volume of the wooden cuboid is = 36cm^3

Its Length= 4 cm

Its Breadth = 3 cm

Since volume of cuboid = Length × breadth × height

$$\therefore \text{Height} = \frac{\text{Vol.of cuboid}}{\text{Length} \times \text{breadth}}$$

$$= \frac{36}{4 \times 3} = 3\text{cm}$$ $\therefore$ It's height is 3cm.

The formula for calculating the length, breadth and height of a cuboid is as follows:-

1. $\text{Length} = \dfrac{\text{Volume}}{\text{Breadth} \times \text{Height}}$

2. $\text{Breadth} = \dfrac{\text{Volume}}{\text{Length} \times \text{Height}}$

3. $\text{Height} = \dfrac{\text{Volume}}{\text{Breadth} \times \text{Length}}$

Example 5: The length of a cuboid is 1 meter, breadth is 50 cm and height 20 cm, find its volume.

Solution: Here the units of length, breadth and height are different. In solving such questions their units have to be made same.

Given length of cuboid = 1m = 100cm

Breadth = 50 cm

Height = 20 cm

Volume of cuboid = Length × breadth × height

= 100 × 50 × 20 = 100000 cm^3 = 10^5 cm^3

Example 6: If each edge of a cube is made 4 times, then how many times will the volume become?

Solution:

Let the edge of the first cube = S

Then volume of the first cube = S^3

After multiplying the edge by four, edge of the second cube = 4 × S = 4S

Then volume of second cube = $(\text{Side})^3 = (4S)^3$

= 4S × 4S × 4S

= 64 S^3

$$\therefore \frac{\text{Volume of second cube}}{\text{Volume of first cube}} = \frac{64S^3}{S^3}$$

$$= \frac{64}{1}$$

Therefore, the volume of the second cube is 64 times the volume of the first cube.

Exercise 15.1

1. Fill in the blanks:—

   (i) A cuboid has a total of __________ faces

   (ii) Length of a cuboid = $\dfrac{\text{Volume of the cuboid}}{\text{Breadth} \times \_\_\_\_\_\_}$

   (iii) 1 cubic m = _________ cubic cm.

2. A water tank is 3 m long, 2 m broad and 1 m deep. What is the capacity of the tank (in litre) if 1 cubic m = 1000 lt.

3. If the length, breadth and height of a tea box is 10 cm, 7 cm and 4 cm respectively then find its volume.

4. The length, breadth and height of a chalk box are 15 cm, 10 cm and 8 cm respectively. Find its volume.

5. Different measurements of cubical shapes are given below. Find their volumes-

S. No.	**Length**	**Breadth**	**Height**
(i)	10 cm	5 cm	3 cm
(ii)	4·5 cm	2·5 cm	2 cm
(iii)	8 m	4 m	2 m
(iv)	5 m	3 m	1.5 m
(v)	40 mm	35mm	25mm
(vi)	60mm	5cm	4cm
(vii)	12cm	70mm	20mm
(viii)	1m	25cm	150mm

6. The volume of a wooden cuboid is 480 cubic cm. Its length and breadth are 10 cm & 6 cm respectively. Find its height.

7. One side of a cube is 25 cm. How many cubes of side 5 cm can be cut out from the given cube?

8. The length breadth and height of a room are 5m, 4.5 m and 3 m respectively. Find the volume of air enclosed in the room.

9. A cuboidal diesel tank is of length 2 m, breadth 2m and depth 40 cm. Find its capacity in litres.

10. A swimming pool is 25 m long and 13 m broad. 325 cubic m water is poured into it. What will be the height of water in the pool?

11. The edge of a cubical dice is 1.2cm. Find its volume ?

12. A well is 8 m long 6 m broad and 9 m deep. It has water filled up to the height of 6 m. Find the capacity of the well and the present volume of water filled in the well?

13. A tank is 5 m. long, 4 m wide and 3m deep. Find the capacity of the tank. If the tank is filled with water upto 2 meters then find its volume?

14. A brick is 20 cm long, 10 cm broad and 6 cm high. How many bricks will be required to make a wall 60 m long, 0.25 m wide and 2 m high?

Surface Area of a Cuboid

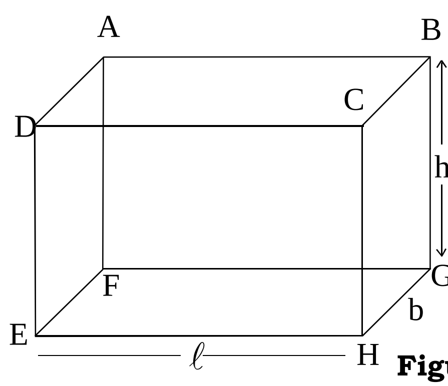


Figure 15.11

We have already seen that a cuboid has 6 rectangular faces. In these 6 faces we have 3 pairs of opposite faces. The opposite faces have equal area. This property is used to determine the surface area. If the length of a cuboid is ℓ, breadth is b and height is h, then the area of its upper and lower faces. (ABCD and EFGH) $= \ell \times b + \ell \times b$

$= \ell b + \ell b$

$= 2\ \ell b$

2. Area of the faces on the left and right side (BCHG and AFED)

$= b \times h + b \times h$

$= bh + bh$

$= 2\ bh$

3. Area of the front and back faces (CDEH and ABGF) $= h\ell + h\ell = 2\ h\ell$

Therefore, the total surface area of the cuboid = sum of the areas of all 6 faces of the cuboid.

$= 2\ell b + 2\ bh + 2h\ell$

$= 2\ (\ell b + bh + h\ell)$

Surface Area of a Cube

Surface area of a cuboid = 2 (b + bh + h)

We know that a cube is a special type of cuboid in which length, breadth and height are equal i.e. ℓ= b = h

Cube

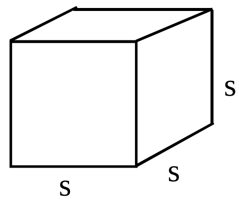


Figure 15.12

$\therefore$ Total surface area of a cube

= 2 (S.S + S.S + S.S)

= 2(3S$^{2)}$

= 6S^2

Example 7: Find the total surface area of a cuboid whose length, breadth and height are 9, 6, 2 cms respectively ?

Solution: Given: length of cube (ℓ) = 9 cm

Breadth (b) = 6 cm

Height (h) = 2 cm

Total surface area of cuboid

= 2 (ℓb + bh + hℓ)

= 2 (9 × 6+ 6 × 2 + 2×9) = 2 (54 + 12 + 18) = 2 (84) = 168 cm^2

Example 8: The edge of a cube is 5.5 cm long. Find its surface area ?

Solution: Given the Edge of the cube (S) = 5.5 cm

Surface area of the cube = 6.S^2 = 6 (5.5)^2

= 6 (5.5 × 5.5) = 6 (30.25) = 181.50 cm^2

Example 9: The Length, breadth and height of a chalk box are 10 cm, 7 cm and 6 cm respectively. Without considering the thickness of the cardboard, find the surface area of the cardboard used in making the chalk box ?

Solution: Since the chalk box is a cuboid therefore, the area of the cardboard used is equal to the surface area of the cuboid.

Given: Length of the cuboid (ℓ) = 10 cm

Breadth (b) = 7cm

Height (h) = 6 cm

Area of the cardboard used = surface area of the cuboid

= 2 (b+ bh + h) = 2 (10 × 7 + 7 × 6 + 6 × 10)

= 2 (70 + 42 + 60) = 344 cm^2

Exercise 15.2

1. Fill in the blanks:-
   (i) The total surface area of a cube of side 3 cms = ______ cm^2
   (ii) The area of the front faces of the cuboid is ______________
   (iii) A cuboid with equal length, breadth and height is called a _________
2. Find out the total surface area of the cuboid whose length, breadth and height are 6.5 cm, 4.5 cm and 2 cm respectively ?
3. A cuboid has length 15 feet, breadth 12 feet and height 9 feet. Find its total surface area ?
4. Find the surface area of a cuboid whose length is 0.5 m, breadth 25 cm and height 15 cm ?
5. Find the total surface area of cube having side 3.4 cm ?
6. Find the side of a cube, whose total surface area is 216cm^2 ?
7. The walls, floor and roof of a room are to be plastered. If the length, breadth and height of the room are 4.5 m, 3 m and 3.5 m respectively then calculate the area to be plastered ?
8. The measurements of an cuboidal oil box are 30 cm, 40 cm and 50 cm respectively. The cost of the tin sheet used in making the box is Rs. 10 per m^2. Find the cost of the tin required to make 20 such boxes ?
9. The edges of two cubes are 8 cm and 4 cm respectively. Calculate the ratio of their surface area ?

We have learnt

1. A rectangular solid that has length, breadth and height (3 dimensions), is called a cuboid.
2. A cuboid has 6 rectangular faces, 12 edges and 8 vertices.
3. A cuboid with equal length, breadth and height, is called a cube.
4. To determine the volume of a cuboid we multiply its length , breadth b and height h i.e. V= bh
5. Volume of a cube = S^3 (S is the edge or side of the cube)
6. The unit of volume is a cubic unit. It is equal to the volume of a cube having 1 unit length, 1 unit breadth and 1 unit height.
7. The sum of the areas of all the rectangular faces of a cuboid is called its totalsurface area and is given by = 2 (b + bh + h)
8. The total surface of a cube = $6.S^2$
9. $1m^3 = 10^6\ cm^3$

Chapter—16

FIGURES (TWO AND THREE DIMENSIONAL)

We have learnt about many types of figures and know their characteristic. Out of these figure, many of them are seen surrounding us directly or indirectly. We have learnt about, line segment. Line, triangle, quadrilateral and its kinds (Rhombus, rectangle, square, trapezium etc.) and about the figure with some more sides.

In previous classes, you have recognized about the figure found in your surroundings. Can tell where did you find a shape of rectangle?

And where did you find a triangle?

Triangle, all kinds of quadrilateral, polygons, circle etc. are made up of some kinds of faces, It mean they have length and breadth but no height. But in real, height is there. Then how will you exhibit these height in figure?

Come do and observe:

You are already familiar with following figure you also knew about their construction –

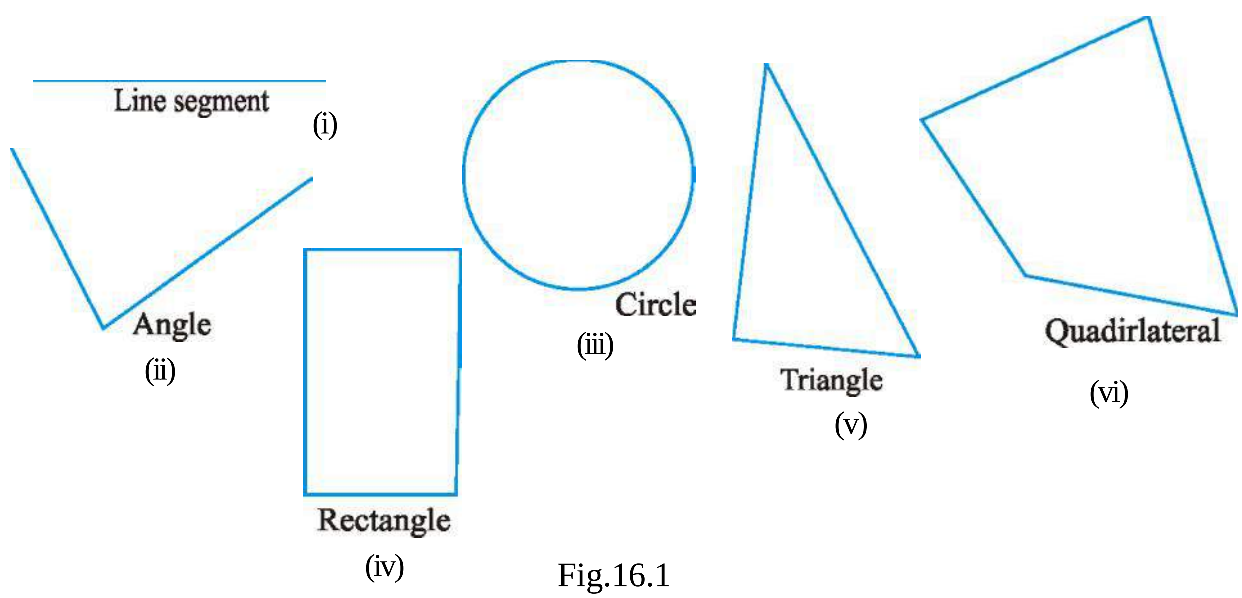


Fig.16.1

Can you draw brick, box like substances on paper?

Some students drawn figure of brick as follows-

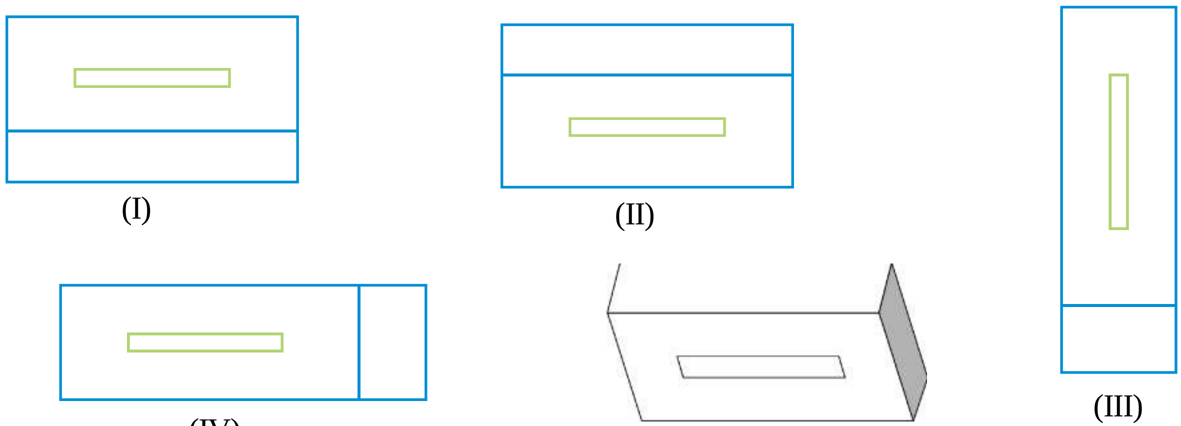


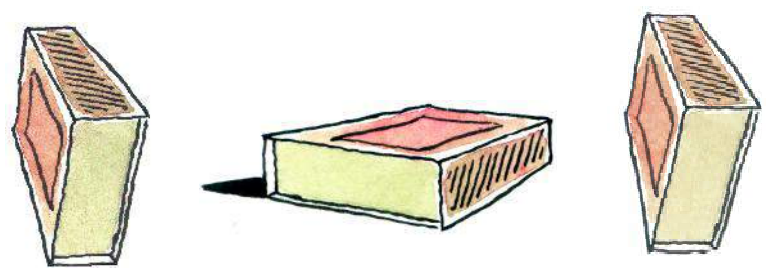
Fig.16.2 (V)

Are they all look like bricks? These all figures are different from each other. Can you tell why these are all different?

Activity 1

To understand this take one empty match box, make is stand on it combustible (gunpowder) face.

Now keep it on its larger face.



(i) (ii) (iii)

Fig.16.3

It is clear, that match box look differently now. Look at fig 16.3 (iii) in this figure, the match box is kept on smaller face. All the three figure are of match box but in different positions.

The figure of brick is also of different positions. Take bricks and keep them in different position according to the figure. Have you been able to keep those bricks according to the figure 16.2?

Activity -2

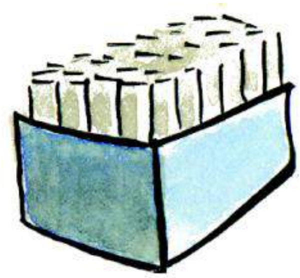


Fig. 16.4

You must have seen box of chalk. Chalks are kept straight in it. Take a full box of chalk and look from the above. You will be able to see the circular portions of the chalk but not its length.

If you draw its figure how it looks like? Anita has drawn its figure as shown in fig. 16.4.

If you draw the figure of an open box of chalk how will it look like? Now in it the upper portion of chalk will not seen.

Practice - 1

Take 5 items observe them from different positions and draw their figures.

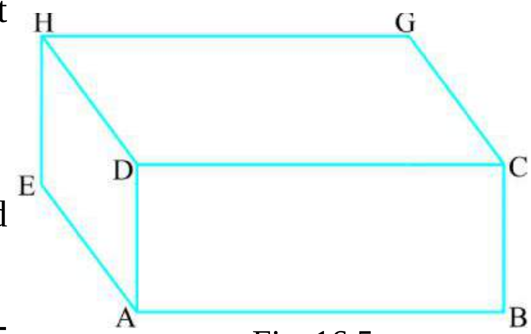


Fig. 16.5

Figure of an item with different position -

Come and observe the shape of the cuboid in fig. 16.5.

This cuboid figure if observed from different positions looks as follows:

If you observe it from the front

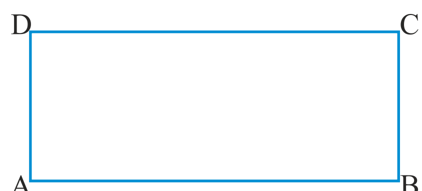


Fig. 16.5 (i)

If you observe it from the above fig. 16.5 (ii)

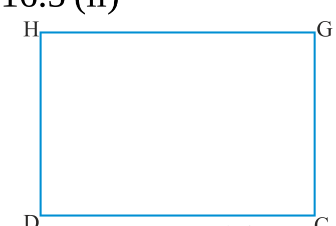


Fig. 16.5 (ii)

If you observe it from the left side

Fig.16.5 (iii)

If you prepare figure, after joining all the three true figures given above (16.5 i, ii, iii) then it should be like the previous one i.e. fig 16.5. As in the previous figure there is a particular kind of bending (angle) between the face of the figure. In the same way if all the three above figure are joined with same particular kind of bending (angle) again we get the same figure. Here, all the three face of the cuboid are joined with one another fig. 16.5 (iv).

Now if you join all the six face of the cuboid then will get the same figure as in fig. 16.5.

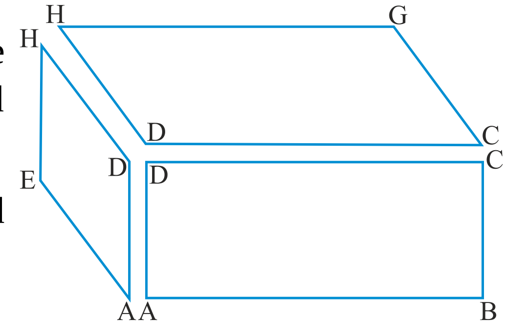
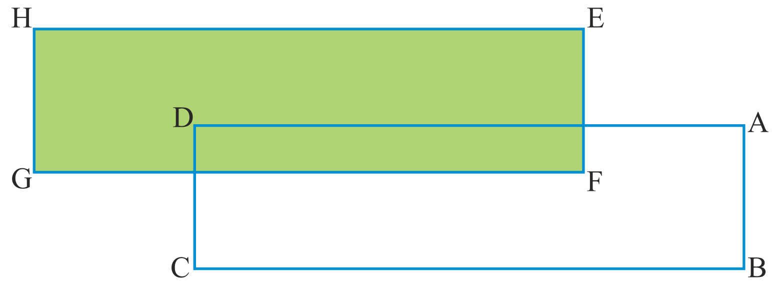


Fig.16.5 (iv)

Activity -3

Construct a figure of cuboid :-

Construction : Take piece of rectangular hard board. Keep the piece on a plane paper and draw an outline with a pencil. In this figure it is exhibited by ABCD. Now, according to the figure shift the hard board piece on the paper and again draw an outline with pencil. It is exhibited in the figure by EFGH. EFGH is shaded.



fp= 16.6 (i)

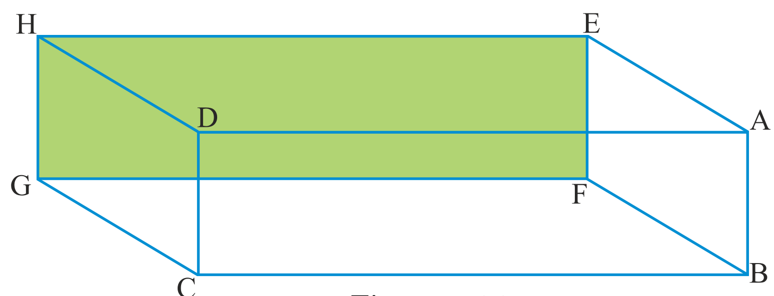


Fig. 16.6 (ii)

According to the figure 16.6 (ii) AE, BF, CG and DH are joined respectively which is the required figure of the cuboid.

In this, there are 6 rectangular face- ABCD, ABFE, BCGF, CDHG, DAEH and EFGH. Eight vertices A,B,C,D,E,F, G and H and 12, edges – AB, BC, CD, DA, AE, BF, CG, DH, EF, FG, GH and HE.

Activity -4

Make a figure of triangular prism:

Construction- Take are triangular hard board pieces and draw two different out lines with pencil, one is figure 'A' and another one is figure 'B' which is at a few distance from figure 'A'. Now label these according to figure 16.7 (iii)

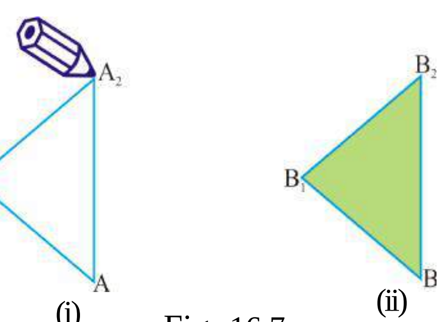


Fig. 16.7

Join AB, A_1, B_1, and A_2 B_2. Now the figure which obtained, are like figure 16.7(iv) (two and three dimensional) which is required triangular prism.

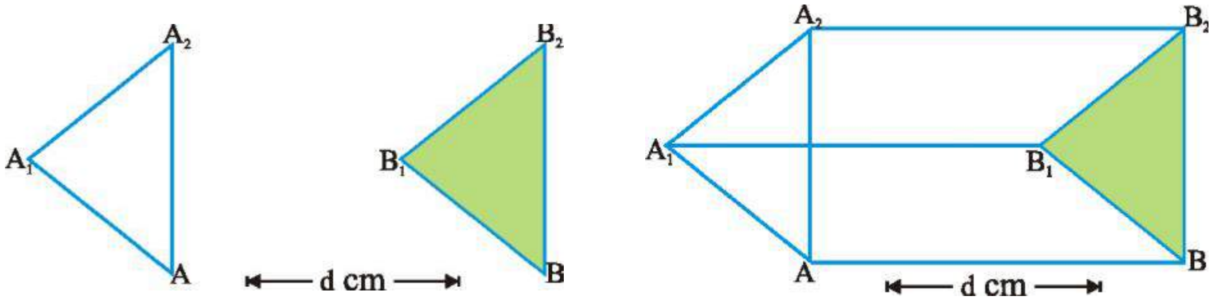


Fig. 16.7 (iii)

Fig. 16.7 (iv)

In this, there are 3 rectangular face ABB_2A_2, $A_1A_2B_2B_1$, and AA_1B_1B and two triangular face AA_1A_2 and BB_1B_2. There are 9 edges AB, A_1B_1, A_2B_2, AA, A_1A_2 A_2A, B, B_1 and B_2 and six verities A, A_1, A_2 and

Practice - 2

1. Construct a cube with the help of square hard board.
2. Construct a 4 cm long prism with the help of a triangular hard board.

Activity -5

Construct a cylindrical shape (cylinder)

Construction- Take one (circular disc) and draw circumference with pencil and determine the center point fig. 16.8 (i) and 9(ii). Now you have circular figure like figure 16.8 (ii) which is visible only when the disc is seen form the front.

Now you turn the disc a little and observe its diagonal position. Draw the same figure as it is visible. It will looks like figure 16.8 (iv)

After this, again you turn a little more and see it from the corners. Then you find this figure will look like figure 16.8 (iv).

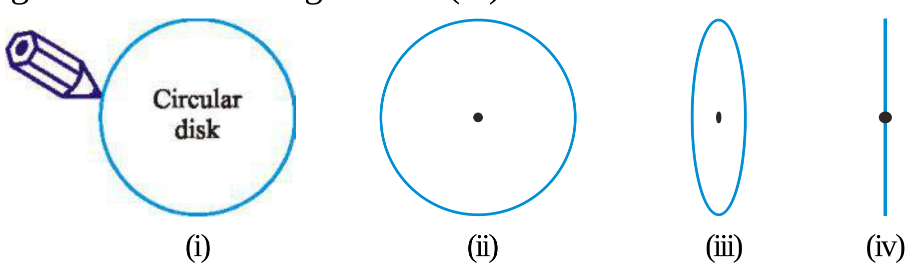


Fig. 16.8

Now draw two more figure as in fig 16.8 (iii) according to fig 16.8 (v) join the diameter A_1B_1 and diameter A_2B_2. After that join A_1B_2 and B_1B_2. In this way, you will obtain a cylindrical figure. In this figure both the ends have two spherical faces and the center part is curved.

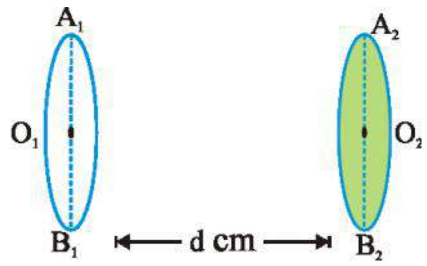


Fig. 16.8 (v)

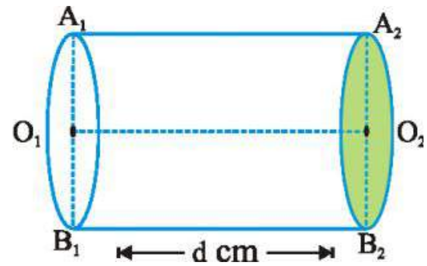
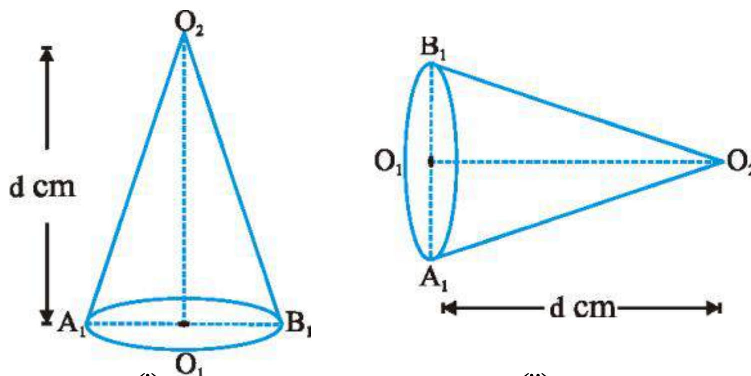


Fig. 16.8 (vi)

Activity -6

Construction of cone: construct a figure like fig. 16.8 (iii) and at some distance take a point ‘O_2’ in the canter. Join O_2A_1 and O_2. You will get a figure of cone fig. 16.9(i&ii) In this figure, one circular spherical face, one top part (vertex) and the bottom part is curved.



(i) Fig. 16.9 (ii)

Practice - 3

1. Construct a cylindrical structure of 5 cm length.
2. Construct a cone of conical structure of 3 cm height.
3. Prepare model of cylindrical structure or a cylinder and conical structure or ‘cone’ by folding papers.

Activity -7

Construction of quadrilateral:

Construction:

1. According to figure 16.10 (i) construct a triangle and shade it.
2. Take a point ‘P’ at some distance above the triangle according to fig. 16.10 (ii).

Now join the vertices A,B,C to the point ‘P’ respectively. Obtained figure will be like 16.10 (iii) It is the required figure quadrilateral.

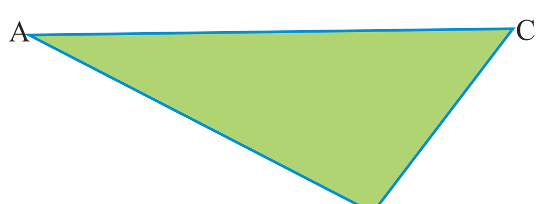


Fig. 16.10 (i)

In this figure there are four triangular face ABC, BCP, CAP and ABP, They are called triangular face also, In this figure there are 6 edges AB, BC, CA, AP, BP and CP and vertices A, B, C and P. There are 3 edges in each vertices.

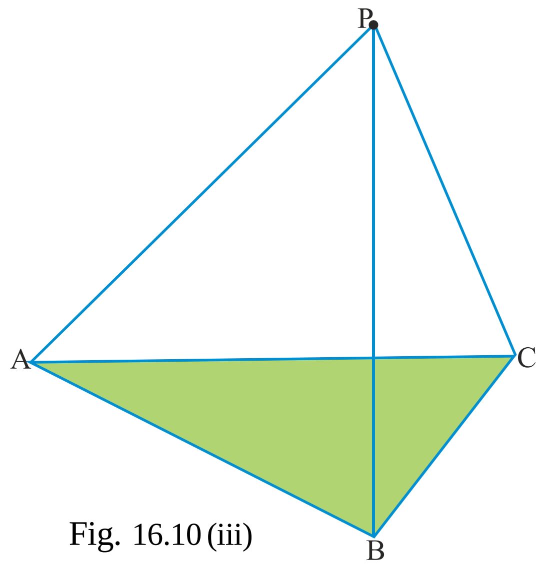


Fig. 16.10 (iii)

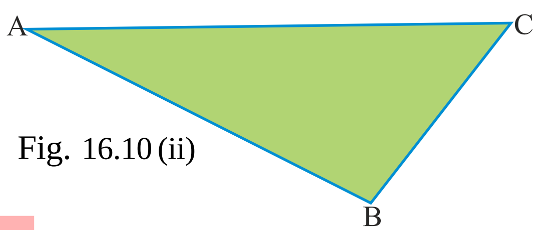


Fig. 16.10 (ii)

Activity -8

Construction of pyramid –

Construction

1. According to figure 16.11 (i),

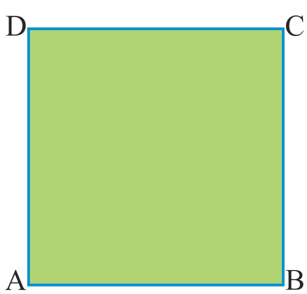


Fig. 16.11 (i)

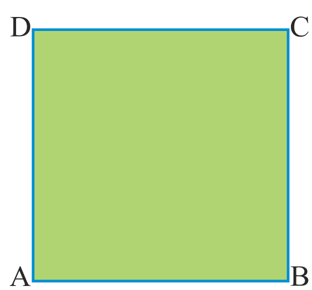


Fig. 16.11 (ii)

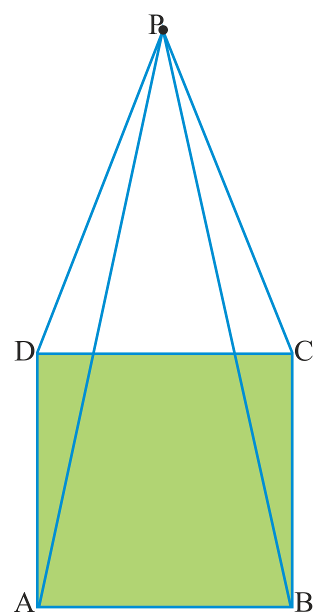


Fig. 16.11 (iii)

construct a figure of square and shade it.

2. Take a point p at some distance near about from the center, according to fig. 16.11 (ii).

3. Now join each vertices to the point ‘p’ obtained figure will be like figure 16.11 (iii). This figure is pyramid.

In this there are one square face ABCD and four triangular face ABP, BCP CDP and

DAP. There are 8 edges AB, BC, CD, DA, AP, BP, CP and DP and 5 vertices A, B, C, D and P also present in this figure.

Exhibit the hidden faces with the help of dotted lines:

There are some figures of cuboid given below. The main figure of cuboid is 16.12 (a) remaining other figures of cuboid are made by observing the coboid from different position. In such conditions, some parts of cuboid (vertex, edges and faces) are not seen these are exhibited by dotted lines.

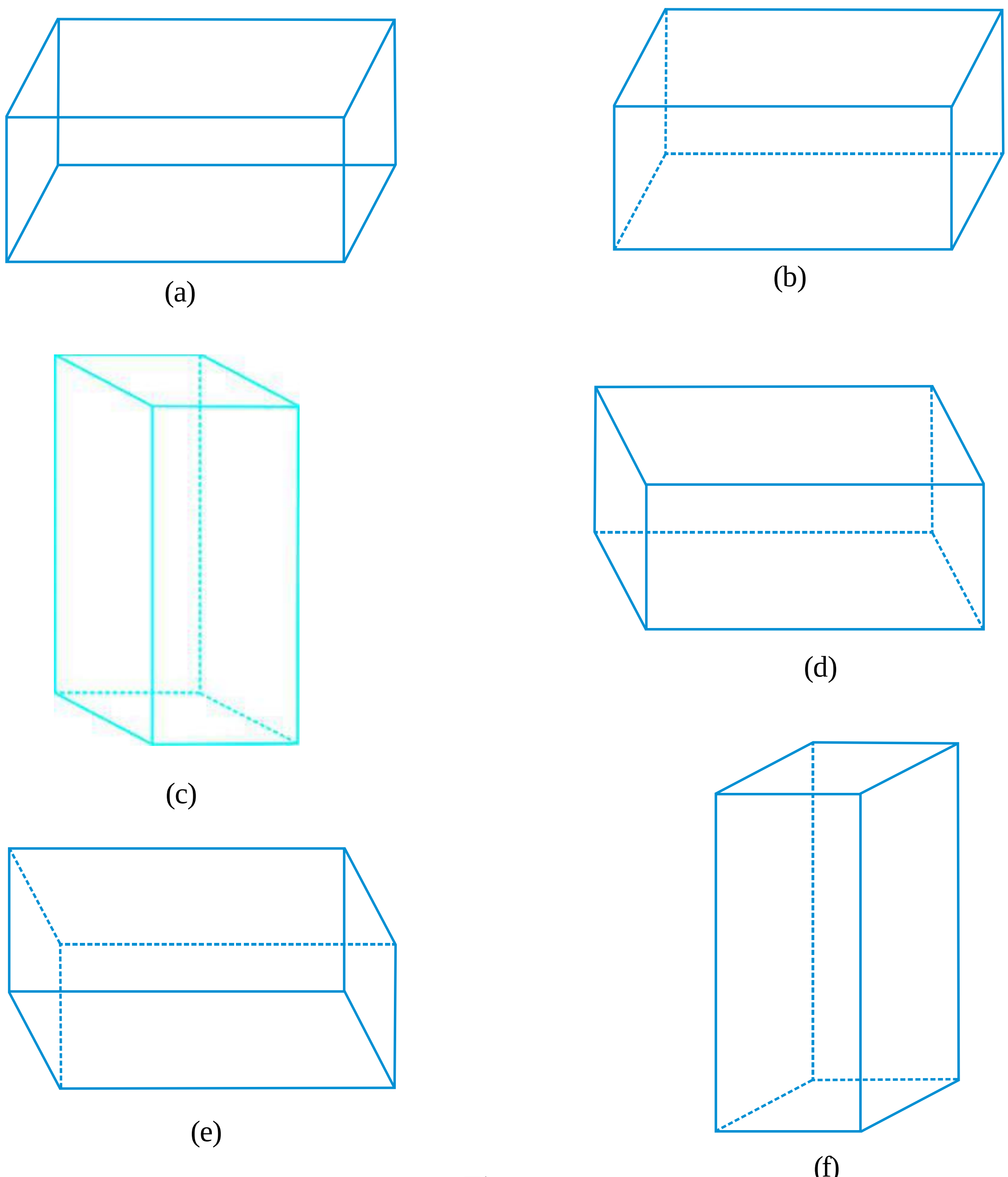
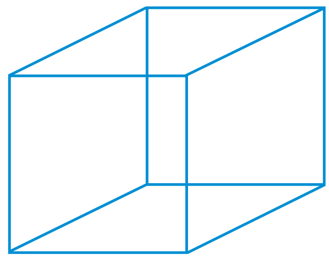


Fig. 16.12

Practice - 4

Now can you show the hidden faces by dotted lines in the following figure.

(A) Cube



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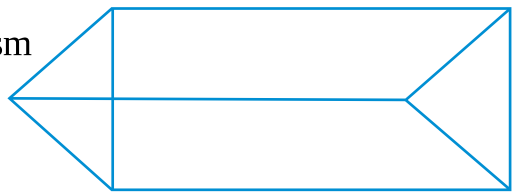
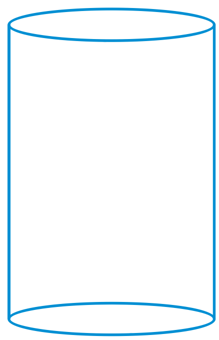
(B) Triangular Prism



____

(C) Cylinder



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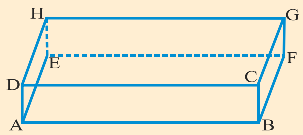
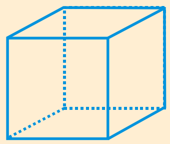
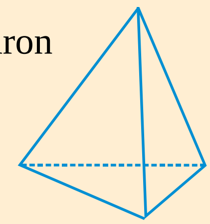
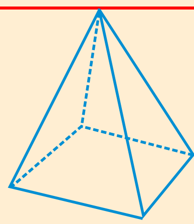
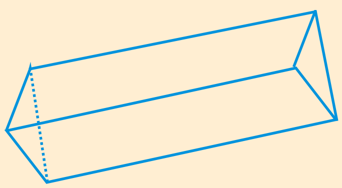
Recognition and calculation of vertices, faces and edges of the given figure.

Activity -9

In the given figure, name the vertices, face and edges, recognize them and write their numbers in the table.

Here a relation among the number of vertices, faces and edges of cuboid has been established. Now you establish the relation among the number of related parts in the remaining figure.

Table 16.1

S.No.	Figure and its name	Vertex(V)	Edge(E)	Face(F)	V-E+F
1.	Cuboid 	8	12	6	8-12+6=2
2.	Cube 				
3.	Tetra Hedron 				
4.	Pyramid 				
5.	Prism 				

After completing this table you will find there for every polygonal (figure made by 4 or more than 4 face) the value of V – E+F will always be 2. This relationship was established by Euler. So this relationship is named Euler relation after his name.

Activity 10

Construct a cuboid of any measurement and made one pyramid on the upper face, with a radius which is less than the breadth of cuboid. The figure, made by you, is like fig.16.13 in which there is a cuboid and a pyramid is seen.

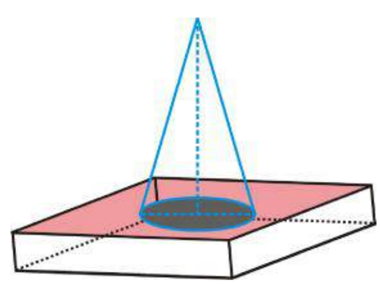
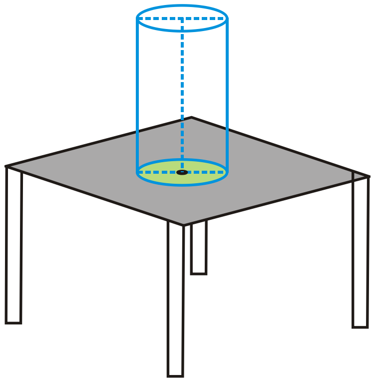


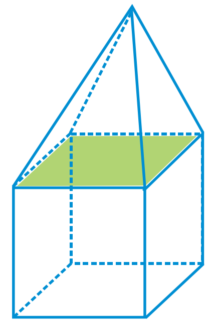
Fig. 16.13

Activity -11

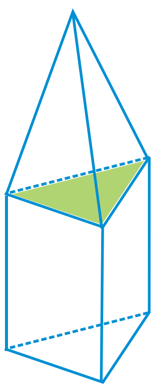
In each figure given below, there are more than one figure is joined. Recognize the figure and write their name.



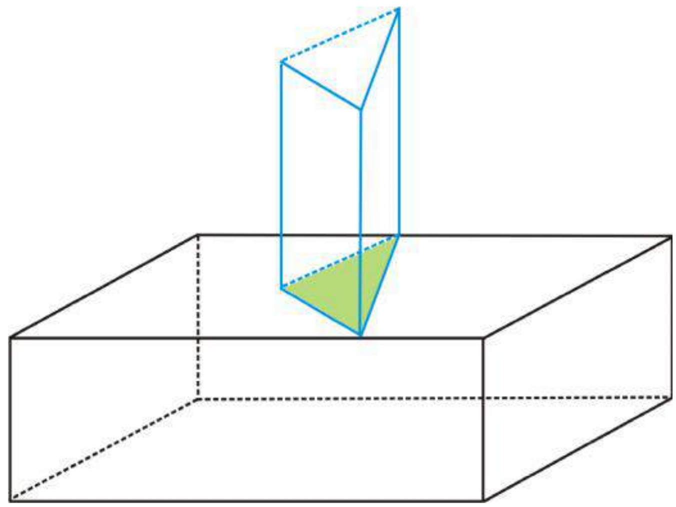
(a) Cylinder and table



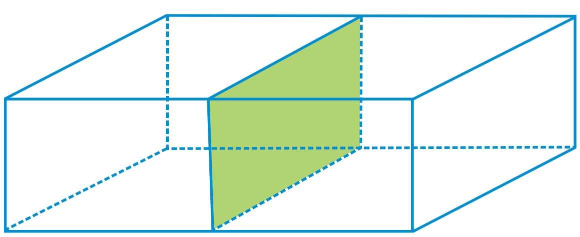
(b) ----------------



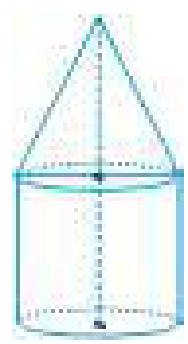
(c) ----------------



(d) ----------------



(e) ----------------



(f) ----------------

Fig. 16.14

Secondary figure to make a model:

The face of the following figure are shown separately. With the help of these you can make a model by cutting pieces of paper

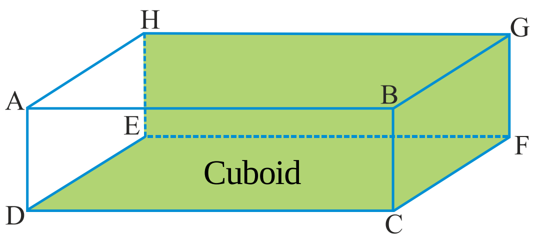
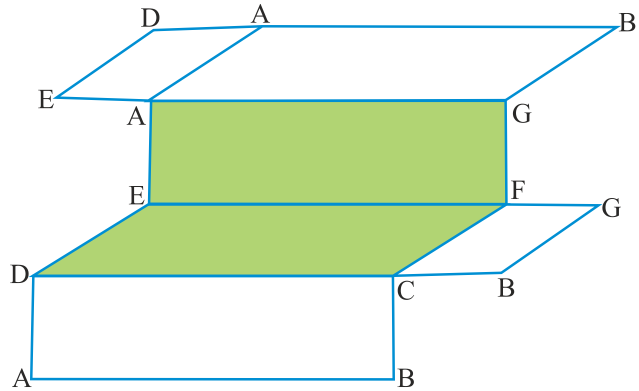


Fig. 16.15



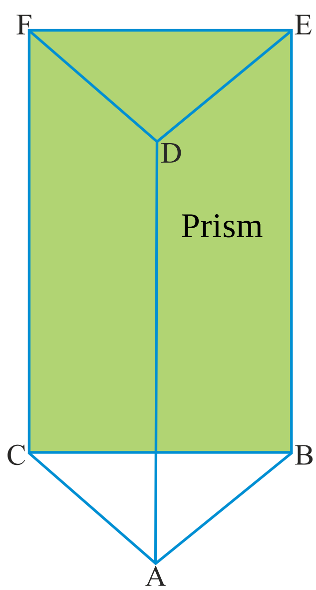
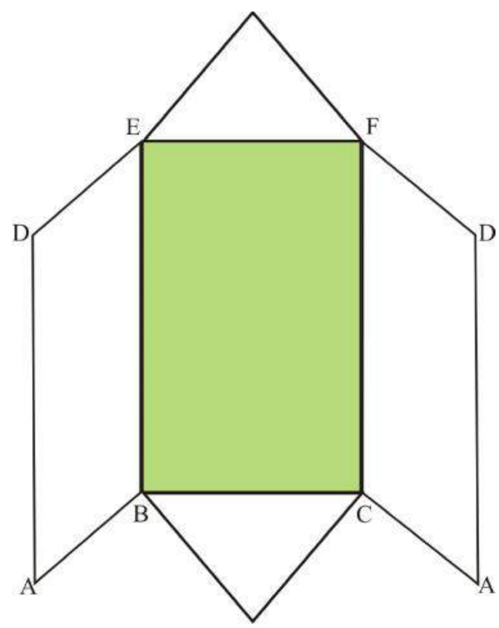
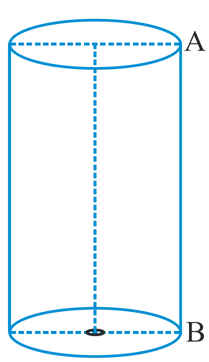


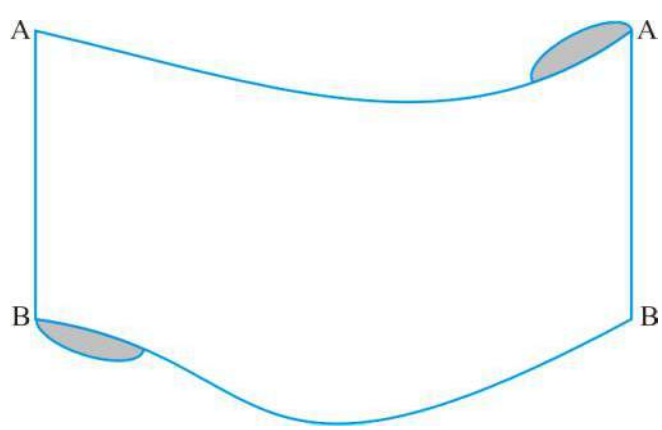
Fig. 16.16

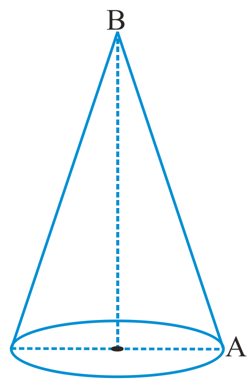




Cylinder

Fig. 16.17





Cone

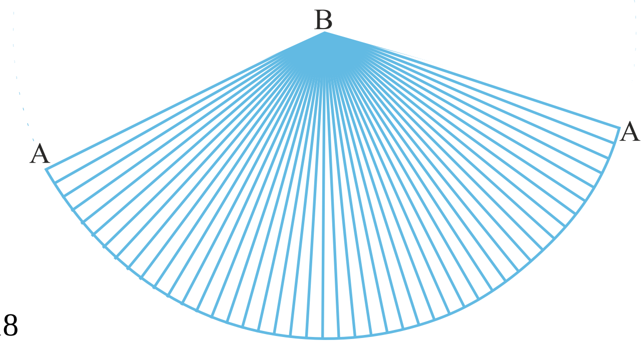
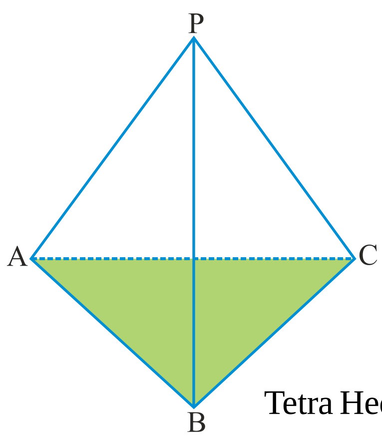


Fig. 16.18



Tetra Hedron

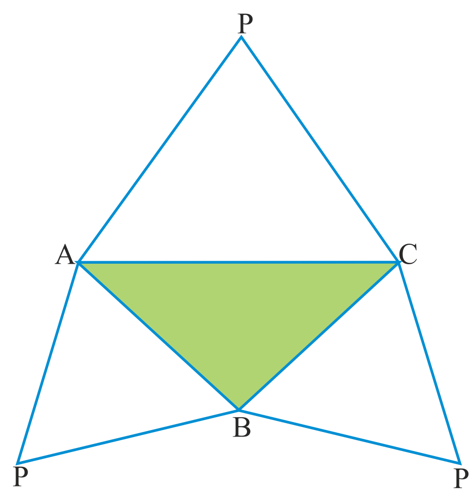
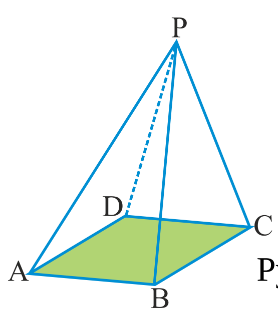


Fig. 16.19



Pyramid

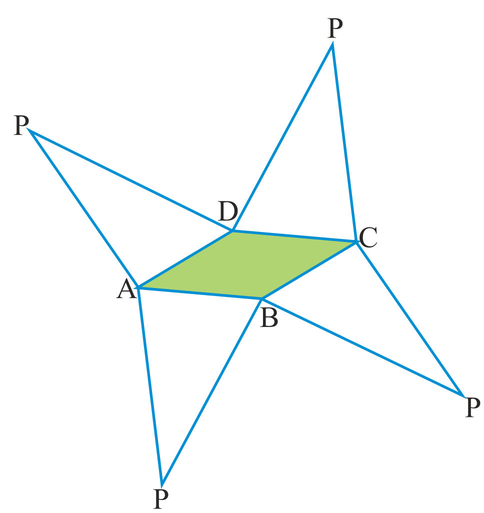
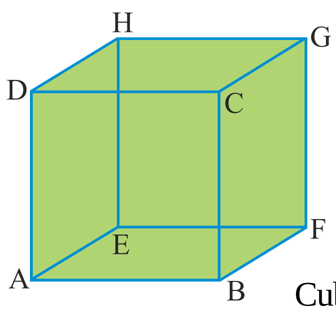
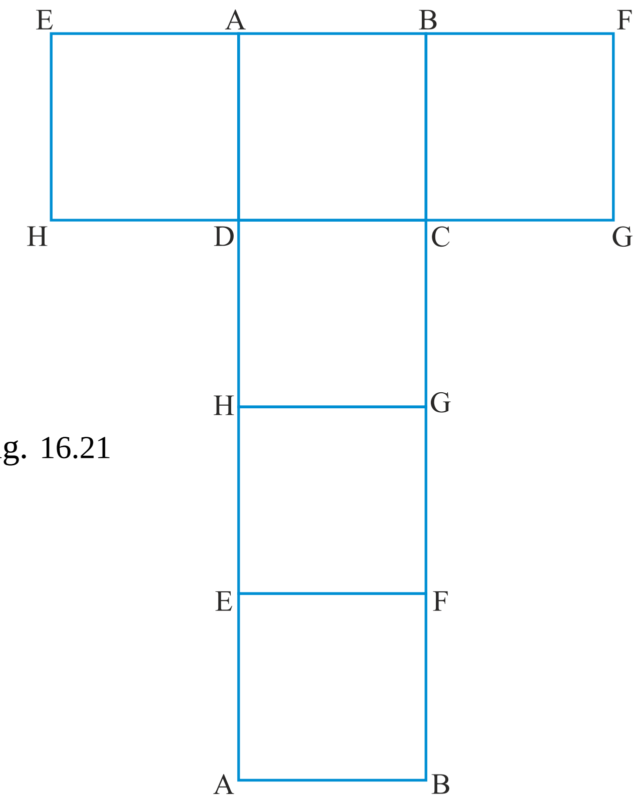


Fig. 16.20



Cube

Fig. 16.21



Tip :- The cube can be separated like face of cuboid and cuboid can be separated like the faces of the cube.

Exercise – 16

1. Construct (prepare) a cube with the help of square of 3 cm.
2. Construct a cylinder of 5 cm length.
3. Construct two triangles at a distance of 5 cm in your copy with the help of a triangular hard board and construct a triangular prism with the help of it.
4. Construct a quadrilateral in your copy.
5. If these are 4 faces and 4 vertices in a (polygonal) can you tell how many edges will be there in it.

Chapter—17

PLAYING WITH NUMBERS

We have learnt a lot about numbers. We have learnt to write the biggest numbers. We have also learnt that the numbers that are used counting called Nat weal numbers. Is we add zero in Nat weal number we get the group of whole numbers a 1, 2, 3 ————————. All the qualities of natural numbers exist in whole number. Add the group of whole number is indicated by "W". The whole number is indicated in groups as followers W = {0, 1, 2, 3 ————} on the number lines whole numbers are indicated as ————————————

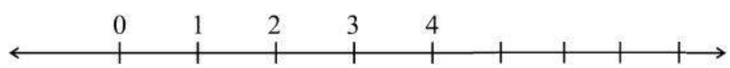


Fig. 17.1

We can solve some questions in whole number.

Q.1 Do you know which is the largest whole number?

Q.2 Which is the smallest whole square odd number?

Q.3 How many four digit number can be formed with 2, 5, 7 and 9.

....

Q.4 Formed the largest and smallest number by using the digits 2, 4, 6, 8?

Q.5 What is the difference between largest four digit numbers and five digit number.

Q.6 By using 3, 7, 9 as numbers which largest three digit number and smallest can be formed.

1. Ragani says that if we add the largest and the smallest number from the above method, it can be divided by 11.

2. Fatima says that this is only true for a two-digit- number. Verify the statement of Fatima.

3. Ramesh said that he does not know about the addition but if any three digit number is reversed, and if we subtract the smaller number from the larger

number, the remainder can be divided by 9 and 11. Is this correct?

4. Jyoti said that not only three digit number, but there are other number and number formed by them when we subtract the smaller number from the largest number the remainder will be divisible by 9.

Let us play a game -

You think of a number and subtract the sum of digits from the number. Is this number divisible by 9?

How does it happen? Let us find out the reason.

Suppose you have thought of number 7324 then according to statement.

$$= 7324 - (7 + 3 + 2 + 4)$$
$$= 7324 - 16$$
$$= 7308$$

Now the number is divisible by 9 [The sum of digits of this number is divisible by 9]. Similarly you can play the mathematical games with your friends.

Sum of whole number -

1. Come, lets see the sum of two whole numbers

18+12 = 30 (whole number)

22+19 = 41 (whole number)

24+68 = 92 (whole number)

Here 30, 41, 92 are also whole number.

Thus we see that sum of two.

Whole numbers is also a whole number.

Does it happen always?

You also find out sum of some more whole number.

Think, does it happens that sum of two whole numbers is not a whole number?

You will see that sum of two whole numbers is always a whole number.

If a and b are two whole number then their sum c will also be a whole number. Means a+b = c, this property is known as clouser property.

2. Come, lets add the two whole number

25+43 = 68 (Whole number)

Now change the order of the number and add.

43+25 = 68 (Whole number)

Are both their totals are same ?

One more time add two whole number

10487+368 = 10588 (whole number)

Now change the order of numbers and add.

368+10487 = 10855 (whole number)

Is there any difference in their totals?

It means

25+43+= 43+25 = 68

10487 + 368 = 368 +10487 = 10855

Therefore, sum of two whole numbers remains same even if there order is changed.

If a and b are two whole numbers then their sum (a+b) and after change in order then sum of (b+a) will give the same result.

It means, a+b= b+a we call this as ***commutative property of addition*** .

3. Let us add '0' to any whole number.

Is there any change in value of the number? In any whole number a it we are adding a whole number to 0, in both case the value is same.

It means a+0= 0+a = a. Due to this special property of 0 it is called ***identity element of sum***.

Now we do the sum of three whole numbers, like 17+19+44 we can find out their total in two ways.

We sum up first two numbers then we add the third number to the sum of first two numbers.

(17+29) + 44 = 46+44 = 90

4. Either we can add the last two whole numbers and then add the first number to their sum.

17 + (29+44) = 17 + 73 + 90

In both condition the total is same ?

Therefore (17+29) + 44 = 17 + (29+44)

Therefore, if a, b, c are three whole numbers, then

$$a+b+c = (a+b) + c = a + (b+c)$$

*This is called **associative law of addition**.*

This activity can be done with four or more than four number see whether their sum is same or not?

Rules for subtracting of whole numbers

1. The rules for subtracting of two numbers-

   15-8 = 7 (Whole number)

   25-14 = 11 (Whole number)

   18-18 = 0 (Whole number)

   16-23 = ? (Is this will be a whole number?)

Do you always get a whole number by subtracting two whole numbers if not why?

Yes, if we subtract a small whole number from a larger whole number or if we subtract two similar number than we get the whole number. But when we subtract a larger whole number from a small whole number than we do not get a whole number.

If a and b are two whole numbers and a>b or a=b than a-b = c will be a whole number and if a<b than a-b will not be a whole number.

2. Let us subtract three whole numbers 25, 8, 6. It can be subtracted in two ways. Lets do it-

(25 – 8)– 6	25 – (8 – 6)
= 17 – 6	= 25 – 2
= 11	= 23

Is the value same in both conditions?

Take some more numbers and solve it your self what result (conclusion) do you get from it? This only, that *while subtracting the order of numbers cannot be changed*.

3. Let us subtract 0 from a whole number

   5-0 = 5,   18-0 = 18

Try to subtract zero from some more whole numbers. Do you get the same numbers?

So if a is the whole numbers than

a – 0 = a

So it we subtract 0 from any whole numbers we get the same whole number.

4. Now if 15 – 15 = 0, 23 – 23 = 0

Do the same activity with other whole numbers. Have you ever get any other number than “0”? If we subtract any whole number from the same whole number the result is zero.

If a is any whole number than a J a = 0.

Multiplication of whole numbers

1. Let us multiply two whole numbers

18 × 8 = 144, 29 × 12 = 348

41 × 7 = 287, 86 × 4 = 344

We see that 144, 348, 287, 344 are all whole numbers. Multiplication of two whole numbers result into a whole number.

Is it always happens so.

Try to multiply two whole numbers. On multiplying two whole number have you not received any whole number.

So we come to the conclusion that multiplication of two whole numbers result into a whole number.

If a and b are two numbers the multiplicand of this whole numbers is always be whole numbers. So **a × b = c** is called ***obscure law of multiplication***.

2. Let us take two whole number 5 and 8. What multiplcant do you get by multiplying this numbers?

5 × 8 = 40

Change in their order and multiply

8 × 5 = 40

Is there difference in their product?

Take some more whole numbers and multiply them.

Change their order and multiply.

Does it happen, that there is change in their multiplicand ?

The multiplicand remains same if we are multiplying two whole numbers with and without changing their orders.

If a and b are two whole numbers then the multiplicand $a \times b$ and after change of order $b \times a$ will remain same. This is ***commutative property of multiplication***.

3. Now take three whole numbers and multiply.

The multiplication can be done in following two ways-

$4 \times 5 \times 6 = (4 \times 5) \times 6 = 4 \times (5 \times 6)$

$= 20 \times 6 \qquad = 4 \times 30$

$= 120 \qquad = 120$

Is the value of multiplication coming same in both conditions? If yes, than take few more whole numbers and multiply them in same ways.

Do you get the same value for all multiplication?

Similarly take 4 numbers and multiply. If three or more numbers are multiply in any order the value of multiplication remains same.

If a, b and c are three whole numbers then **$(a \times b) \times c = a \times (b \times c)$**. This is ***associative property of multiplication***.

4. Now multiply any whole number with 0 and see the result.

$8 \times 0 = 0$, $19 \times 0 = 0$, $0 \times 15 = 0$

$29 \times 0 = 0$, $45 \times 0 = 0$, $48 \times 0 = 0$

Now if you multiply any whole number with 0, will you always get value as 0?

*It means **multiplication of any whole number with 0 will give the value as 0. If a is any whole numbers then $a \times 0 = 0$**.*

5. Similarly, multiply any whole number with 1. What is the value of that whole number.

If any whole number is multiplied with 1 we get the same number.

*If 'a' is any whole number then **$a \times 1 = a$** because this special property of 1 it is called **multiplication identity**.*

6. Multiply the following numbers-

The multiplication can be done in two ways.

$5(8 + 4)$

$= 5(8 + 4)$	$= 5(8 + 4)$
$= 5(12)$	$= 5 \times 8 + 5 \times 4$

= 60		= 40 + 20
		= 60

Do you get the same value in both conditions?

Similarly,

5(8 – 4)

= 5 × (8 – 4)		= 5 (8 – 4)
= 5 × 4		= 5 × 8 – 5 × 4
= 20		= 40 – 20
		= 20

If a, b, c are whole numbers then $a(b \pm c) = a \times b \pm a \times c$.This is called ***distributive property of multiplication in addion/substraction***.

Similarly take any three numbers and solve it by both method and see whether you get same value not?

Division of whole numbers-

1. We know that division is opposite of multiplication. Let us see, how?

40 ÷ 4 = 10 ⇒ 10 × 4 = 40

21 ÷ 3 = 7 ⇒ 7 × 3 = 21

Come let's divide few more whole numbers and see.

20 ÷ 5 = 4 and remainder 0

25 ÷ 4 = 6 and remainder 1

The value gained by division of whole number will not be always a whole number, means we will not get 0 as remainder always. So we can say that **dividing a whole number with another whole number will not always give value as whole number**.

2. We know that

15 ÷ 15 = 1 28 ÷ 28 = 1 49 ÷ 49 = 1

Therefore dividing the whole number with same number (except zero) will always give the value of division as 1.

If a is any whole number (except 0) then **a ÷ a = 1**

Now 15 ÷ 1 = 15 28 ÷ 1 = 28

40 ÷ 1 = 40

If we are dividing any whole number by 1 we will always get value of

division as the same number.

If a is a whole number then a ÷ 1 = a

Fractional number

Come let us divide 21 by 4 and see.

The division of 21 by 4 is written 21/4 and such number are called fractional.

Fractional number - Fractional numbers are those numbers whose numerator and denominator are natural numbers

The diagrams given below indicate that one unit is divided in many parts and out of that some parts are taken.

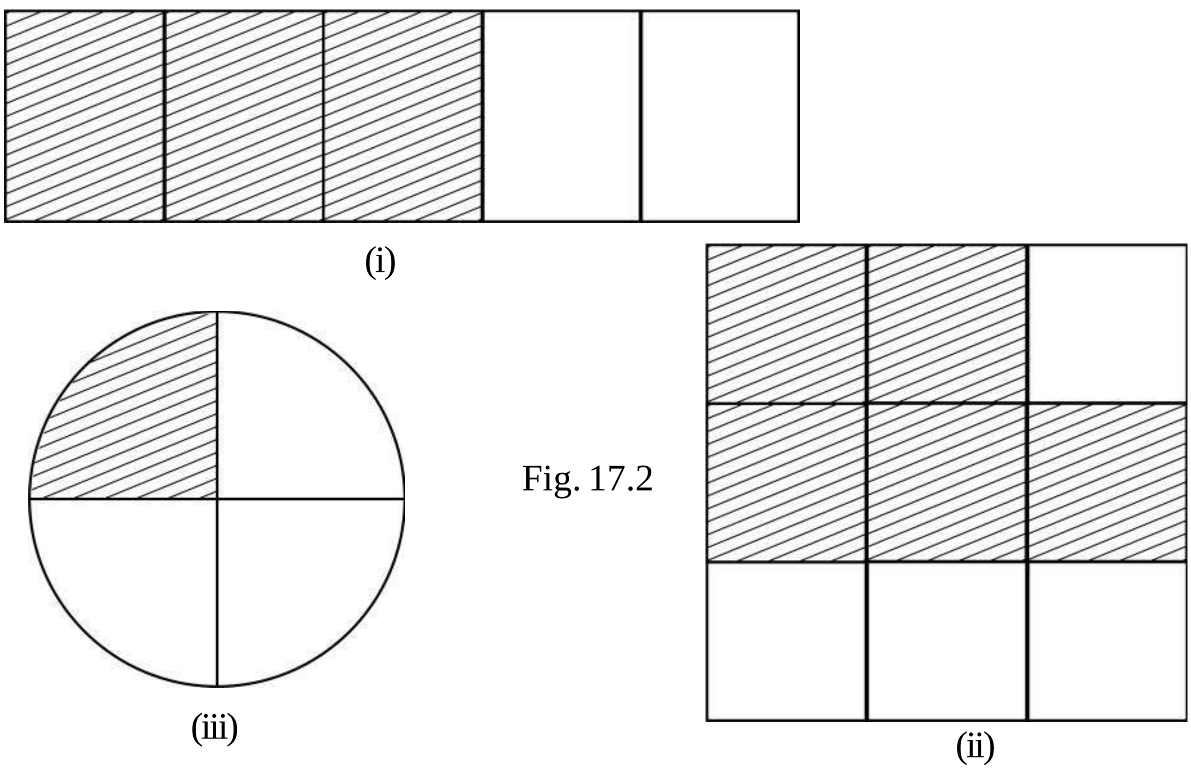


Fig. 17.2

Write the shaded and blank portions of able diagram in form of fractions.

	Shaded part	Total parts	Fraction
(i)	3	5	$\frac{3}{5}$
(ii)	----------------	----------------	----------------
(iii)	----------------	----------------	----------------

These are all perfect fractions. Perfect fractions are those fractions in which the value of numerator is less than denominator. Those fractions whose numerator greater than denominator are called imperfect fractions. In this fraction these can be many complete units. In a fraction how many complete units and incomplete units are there to show them mixed fractions are used.

Like $\frac{8}{3}$ can be shown by the help of diagram in following way.

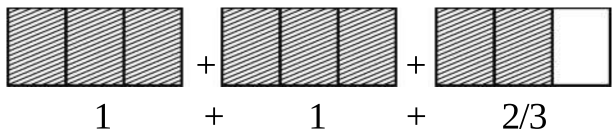


Fig. 17.3

In this three units are divided there equal parts and out of that 2 complete units and 2 of three parts of third unit is taken.

We can also write the mixed fraction as per rule of division like 78/37 in which 78 is dividend and 37 is divisor.

```
37)  78  (2
     74
   ------
      4      remainder
```

Then we can write in form of mixed fraction as $2\frac{4}{37}$.

When the numerator of fractions are equal then with increase in value of denominator the value of fraction decreases like $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \frac{1}{6}$.... ,and when the denominator of fraction are equal then with increase in value of numerator the value of fraction increase like $\frac{1}{8} < \frac{2}{8} < \frac{3}{8} < \frac{4}{8} < \frac{5}{8} < \frac{6}{8} < \frac{7}{8}$

Exercise 17.1

Q.1 Find out perfect fractions.

(i) $\frac{16}{5}$ (ii) $\frac{12}{13}$ (iii) $\frac{78}{41}$ (iv) $\frac{6}{7}$

Q.2 Show the fractions in form of diagrams.

(i) $\frac{6}{5}$ (ii) $\frac{3}{8}$ (iii) $\frac{7}{11}$ (iv) $\frac{4}{15}$

Q.3 Find complete units in the following fractions and also write in form of mixed fraction?

(i) $\frac{14}{9}$ (ii) $\frac{89}{12}$ (iii) $\frac{119}{18}$ (iv) $\frac{267}{61}$

Q.4 Write the fraction in ascending order.

(i) $\frac{8}{9}, \frac{6}{9}, \frac{4}{3}, \frac{2}{5}$ (ii) $\frac{1}{9}, \frac{4}{9}, \frac{2}{9}, \frac{8}{9}, \frac{7}{9}$

Fraction on number line -

Like whole number we can also show fraction on number line. Fractional numbers $\frac{1}{3}, \frac{4}{3}, \frac{8}{9}$ and $\frac{13}{9}$ are shown on number line in the following way.

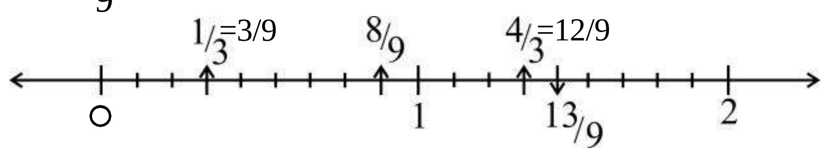


Fig. 17.3

In the above number line shown fraction $\frac{4}{3}$ and $\frac{13}{9}$ are near but there are many different fractional numbers in between them like :- $\frac{25}{18}, \frac{37}{27}, \frac{51}{36}$ etc.

Integer-

Negative numbers are required while subtracting whole number and we get the group of integers and when we add whole number and the group of negative number. We get a group of whole numbers. In integer, positive and negative number have zero also. The group of integers is shown by I as -

I = {...., –4, –3, –2, –1, 0, 1, 2, 3, 4, ...}

Showing integer on numbers line-

Take a number line and mark its centre as zero. Mark the positive numbers on the right side of zero and negative numbers on the left side of zero. There is no bigger or smaller number in this.

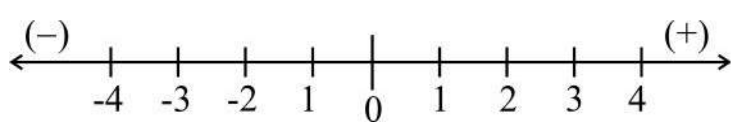


Fig. 17.4

Rational number -

Rational number are those which can be written $\frac{p}{q}(q \neq 0)$ This number can be either positive or negative.

All number $\frac{4}{-5}, \frac{6}{4}, \frac{-13}{4}, \frac{8}{1}, \frac{-9}{-1}, \frac{0}{-7}, ---$ are rational numbers.

All fractional numbers are rational numbers and all integers are rational number. We can compare the rational numbers and can also be shown on the number line. Rational number can be changed into standard form for example the standard form of $\frac{16}{20}$ is $\frac{4}{5}$.

Exercise 17.2

Q.1 Show the following fractional number on number line.

(i) $\frac{3}{5}$ (ii) $\frac{6}{5}$ (iii) $\frac{7}{8}$

Q.2 Write this rational number in the form of integers.

(i) $\frac{8}{1}$ (ii) $\frac{-12}{1}$ (iii) $\frac{20}{1}$

(iv) $\frac{-39}{1}$ (v) $\frac{59}{1}$

Q.3 Show the largest number in the following pairs of number.

(i) $\frac{5}{9}$ and 0 (ii) $\frac{-6}{7}$ and 0

(iii) $\frac{-5}{3}$ and $\frac{17}{-10}$ (iv) $\frac{6}{-5}$ and $\frac{-13}{-8}$

Q.4 What should be added to the numerator of $\frac{4}{9}$ to get $\frac{2}{3}$.

Q.5 What should be subtracted from the denominator of $\frac{5}{6}$ to get 1.

Q.6 The numerator of any fraction is 2 more then the denominator. If the numerator is 5 what will be the fraction?

Activity 1

How many three digit number can be formed by using the number 0, 4, 7 ? Write in the following table-

Table 1

S.No.	Number	Expended from in the place value	Does the number have three digit
1.	047	000 + 40 + 7	No
2.	407	400 + 00 + 7	Yes
3.	------		----------------
4.	------	----------------	----------------
5.	------	----------------	----------------
6.	------	----------------	----------------

Observation - 1. Which are the numbers formed with three digit numbers.

2. Why is 047, not a three digit numbers?

Since '0' does not have any value if it is placed before any integer's number. Therefore 047=47 which is two digit number.

Activity 2

Complete the table.

S.No.	Value of a, b, c	100 ò a + 10 ò b + c	Number
1.	a = 9, b = 2, c = 8	$100 \times 9 + 10 \times 2 + 8$	900 + 20 + 8 = 928
2.	a = 3, b = 0, c = 4	....	......................= 304
3.	a = 0, b = 7, c = 5	....	....
4.	a = .., b = .., c = ..	....	....
5.	a = .., b = .., c = ..	....	....

Practice

1. How many three digit numbers can be formed by using only three digits? Write in ascending order.

2. Write the following number in the form of 100 a + 10b + c.

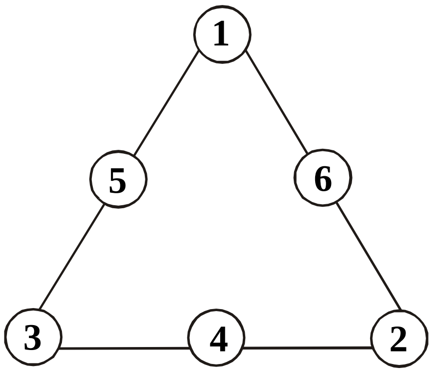
(i) 376 (ii) 850 (iii) 69 (iv) 207

Some Mathematical game

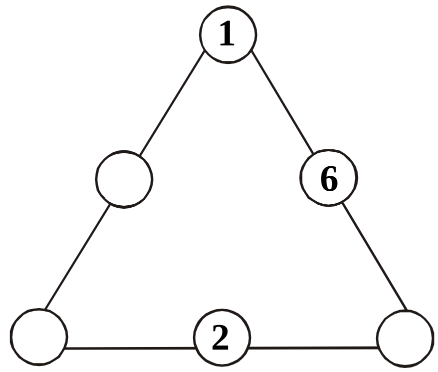
Activity 3

(Magic Triangle)

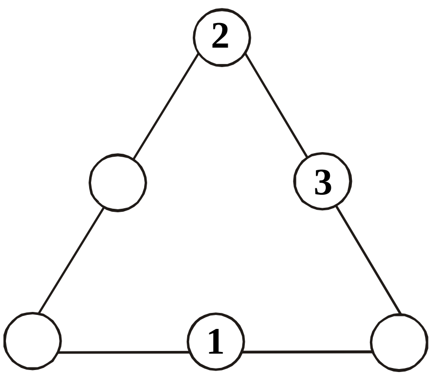
In the given triangle fill 1, 2, 3, 4, 5, 6 in such a way that the addition of the numbers on each side is the same –



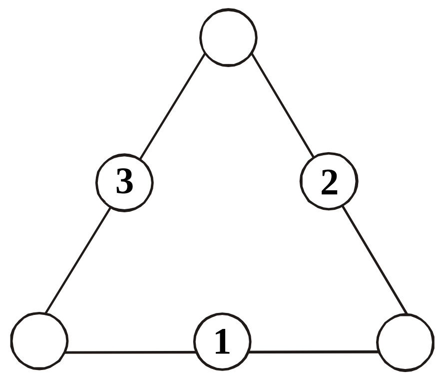
Addition - 9



Addition - 10



Addition - 11



Addition - 12

In the above triangle we can see that sum of numbers 1, 2, 3, 4, 5, 6 it arranged in a various ways, will be the same

You can also make various groups of these digits and get the same addition.

Activity -4

Complete the given sequence

(i) 1, 2, 3, __, __, __, __, 8

(ii) 3, 5, 7, __, __, 13, __

(iii) 26, 23, 20, __, __, __, 8, __

(iv) 7, 12, 18, __, __, 42, __, __, 75

(v) 1, 4, 9, __, 25, __, __, 64, __

Activity 5

Continuing the following series fill in the blanks.

$1\frac{1}{2} \times 3 = 1\frac{1}{2} + 3$

$1\frac{1}{3} \times 4 = 1\frac{1}{3} + \square$

$1\frac{1}{\square} \times 5 = \square + 5$

$\square \times \square = 1\frac{1}{5} + \square$

After completing the above series make any two series of own.

Queries

Guessing the number with asking anything. Bharti ask her friend Jayanti to think of a three digit number whose first and last digits are not same and after that interchange the digit and make the new numbers and find the difference of these two number. Now interchange the digits of answer received and add it to the answer. After telling this Bharti tells to Jayanti that after doing the above exercise you are having with you final sum as 1089 Jayanti was astonished.

How Bharti came to know that the remainder with her was 1089.

Come, lets solve the problem & see

Let Jayanti have thought of number as 102

After interchanging the digits, number is 201

The difference of number =

$$\begin{array}{r} 201 \\ -\ 102 \\ \hline 099 \end{array}$$

Now interchanging the digits of answer = 990

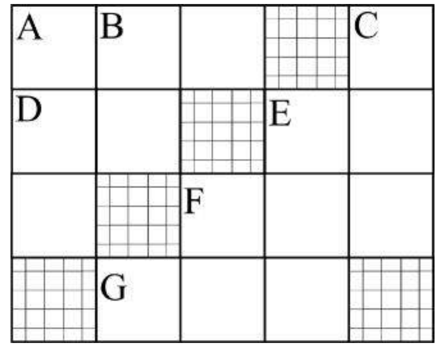
And total of both =

$$\begin{array}{r} 099 \\ +\ 990 \\ \hline 1089 \end{array}$$

In this way you can also play games with you friends.

Activity 6

Fill the logic as for information



Left to right.

A- five times of square of six

D- 1 less than square of ten

E- 10 more than square of nine

F- 4 less than eight hundred.

G- Cube of nine

Top to bottom

A- Square of fourteen

B- Two numbers respectively

C- Cube of six

E- The largest number of three digit

F- Twice of square of multiplicand of two smallest prime numbers respectively.

Logic

All the students of calls 8th were telling their age out of them Anju and Raju refused to tell their age. Then Rahul and Vivek said, Alright. You don't tell us your age but answer few questions and we will tell your age. Anju and Raju agreed for same.

Now Vivek and Rahul asked Anju to do the following.

Anju, you double your age and add five to it and whatever answer your get multiply it with so. After this add Raju's age to that number and add number of days (365) of a year to that number. Whatever is the answer substract 615 out of that? Now tell us what number is remaining with you.

After doing all the above mentioned things Anju and Raju gave the answer. With this answer Rahul and Vivek told about the age of Anju and Raju.

Now Anju and Raju were wondering that they have done the calculation of their age on their own and haven't told anything about it, but how Rahul and Vivek came to know about their age.

Now Anju and Raju were ages to know how Rahul and Vivek know of about their age. They asked them.

Rahul and Vivek explained them the method.

Anju's Let us take your age to be 14 years.

Now multiply your age with 2 $= 14 \times 2 = 28$

Add five to the number $= 28 + 5 = 33$

Now multiply the number with 50 $= 33 \times 50 = 1650$

Now add Raju's age (assume 13 years) 1650 in above numbers

$= 1650 + 13 = 1663$

Add number of days of year it $= 1663 + 365 = 2028$

Now subtract 615 from it $= -\underline{615}$

1413

In the answer received first two digit shows age of Anju. And remaining two digits show age of Raju.

Play similar games with your friends.

Activity 7

Magic square

In the given magic square use numbers from 1 to 16 and fill in the blank boxes such that the sum of numbers should be 34 however you add it. (That is from left to night, top to bottom, inclined etc.)

16			13
	10		
9		7	12
	15		1

Sum = 34

Similarly take any other 16 numbers in series and try to make Magic square.

Logic

Write any three digit number and again after previous numbers write the same member and make it a six digit number. Now divide it with 7, 11 and 13. Do you get the answer as same number which you have taken earlier? How is it possible? Think about in and find reason.

Exercise 17.3

1. By using digits 0, 3, 5 how many numbers can be formed. Select the number formed by two digits and three digits out of these numbers.

2. Solve the questions given below.

(A) $37 \times 3 =$ ---------------
$37 \times 6 =$ ---------------
$37 \times 9 =$ ---------------
$37 \times 12 =$ --------------

(B) $1\,2\,3\,4\,5\,6\,7\,9 \times 9 =$ ---------------
$1\,2\,3\,4\,5\,6\,7\,9 \times 18 =$ ---------------
$1\,2\,3\,4\,5\,6\,7\,9 \times 27 =$ ---------------
$1\,2\,3\,4\,5\,6\,7\,9 \times 36 =$ ---------------

----

3 Complete the given series.

(a) 2, 5, 10, 17, __, __, 50, __, 82, __
(b) 0, 1, 1, 2, 3, 5, 8, __, 21, __, 55, __,
(c) 125, 120, 114, 107, __, __, __, 69, __
(d) 20, 15, 11, __, __, 5

4. See the following relations carefully.

$043 = 0^1 + 4^2 + 3^3 = 0 + 16 + 27 = 043$
$135 = 1^1 + 3^2 + 5^3 = 1 + 9 + 125 = 135$
$2427 = 2^1 + 4^2 + 2^3 + 7^4 = 2 + 16 + 8 + 2401 = 2427$

Solve the following as above -
063, 175, 518, 1306

5. In how many ways do you get 100 by taking number 1 to 9 in series and using the signs? Like

6. In the given square find out the value of A, B, C, D ……… J by adding.

					Sum
	57	49	30	A	B
	32	C	30	27	114
	28	D	26	29	E
	13	15	17	F	71
	37	35	G	H	120
Sum	I	136	128	130	J

7. Vijay has multiplied a number with 5 and then subtracted 5 from it and then divided the answer with 5. Now tell what is the number Vijay got? Is the numbers got by Vijay less then 1 then the previously taken number? How is it possible?

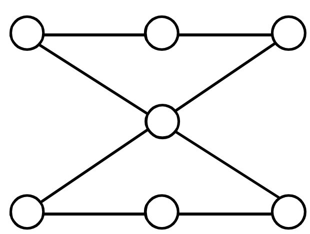
8. Sunita has taken a three digit number as 258 and made it a six digit number 258258. Then she divided this number with 7, 11 and 13 respectively. What is the quotient got by Sunita? Is it the initial number taken by her? How is it possible?

9. House number of Simon's is 57. After doubling the number he added 5 and multiplied the new number with 50 and then he added his friend Kailash's age 15 and number of days (365) to that answer and then subtracted 615 out of new answer received. After doing so is the answer received is 5715? Is this Simon's house number and Kailash's age in the answer? How it is possible?

10. Take the number 2 five times and use signs +, –, × and ÷ and get the answer as 3 and 7. Similarly, try to get other numbers also.

11. 7 non divisible numbers are given. Use these numbers in the given figure in such a way that the sum of its any side should be 41.

    5, 7, 11, 13, 17, 19, 23



Verification rule of Divisibility

A number is completely divisible by another number or not, to know this we use division method. But, do you know that there are some simple rules also. With the help of these rules we can find divisibility by any particular number.

Let us see, some rules. (Further in this chapter we write "divisible" only in place of completely divisible)

1. Verification of Divisibility by 2

If the unit place of any number has 0, 2, 4, 6 and 8, then this number is completely divisible by 2.

20, 62, 34, 26, 18, ... are divisible by 2.

21, 63, 33, 35, 17, ... are not divisible by 2.

here, 18 is divisible by 2. Let us verify this by division.

2)18(9
−18
0

Therefore, it is completely divisible by 2.

2) 21(10
− 2
01
00
1

Whether the 21 is also divisible by 2, Remainder is 1
Therefore, it is not completely divisible by 2.

2. Verification of divisibility by 3

If the sum of all the digits of a number is divisible by 3, then the number is divisible by 3. For example: In 111111,the sum of all the digits would be 1 + 1 + 1 + 1 + 1 + 1 = 6, therefore the number is divisible by 3.

Similarly in 5112 the sum of all the digits 5 + 1 + 1 + 2 = 9 so this number is divisible by 3.

The digits in 412 will give the sum 7, therefore this number will not be divisible by 3.

3. Verification of divisibility by 6

If any number is divisible by 2 and 3 separately, then the number will be divisible by 6.

216, it is divisible by 2 (The digit in unit's place is 2)
216 is divisible by 3 .(The sum of digits is 9)
So, it will be divisible by 6.
643212, is divisible by 2 (because the digit in unit's a place is 2)
is divisible by 3 (because the sum of digits is 18).
So, the number is divisible by 6.

4. Verification of divisibility by 9

If the sum of digits of a number is divisible by 9, then the number will also be divisible by 9.

The number 3663, is divisible by 9 (because the sum is 3 + 6 + 6 + 3 = 18, divisible by 9).

1827, is divisible by 9 (the sum of digits is 18, which is divisible by 9).
1227, is divisible by 9 (the sum of digits is 12, which is not divisible by 9).

5. Verification of divisibility by 5

If the digit in the unit's place is 0 or 5, the number will be divisble by 5.

e.g. 1045, is divisible by 5 (because the digit in the unit's place is 5).

940, is divisible by 5 (because the digit in the unit's place is 0).

6. Verification of divisibility by 10

If any number has 0 in its unit's place, then the number is divisible by 10.

Example:

1000, is divisible by 10 (digit in the unit's place is 0).

2130, is divisible by 10 (digit in the unit's place is 0).

5003, is not divisible by 10 (digit in the unit's place is 3).

7. Verification of divisibility by 4

If the number made by the ten's and unit's place digits of any number is divisible by 4 or the ten's and unit's place has zero, then the number is divisible by 4.

For example:

In 79412, the digits in ten's and unit's place are 1 and 2, so the number made by these two digits is 12. Since 12 is divisible by 4, therefore, 79412 will be divisible by 4.

1300 will be divisible by 4 (because the digits in ten's and unit's place are 0).

413 will not be divisible by 4 (because the digits 13 is not completely divisible by 4).

8. Verification of divisibility by 8

If the number made by unit's, ten's and hundredth places is divisible by 8, or the number contains 0 in all these three places, then the number would be divisible by 8.

31000	(divisible by 8)
1816	(divisible by 8, because 816 is divisible by 8)
12317	(not divisible by 8, because 317 is not divisible by 8)

9. Verification of divisibility by 7

Take a number and double its last digit. Now subtract this doubled number from the rest of the digits of the original number.

Repeat the process till the result is a one digit or two digit number. If the obtained number is divisible by 7, then the original number also be divisible by 7.

In 1729, the last digit is 9, twice of 9 is 18

172 - 18 = 154

In 154, the last digit is 4, twice of 4 is 8

15 - 8 = 7, 7 is the last digit.

Therefore the number will be divisible by 7.

10. Verification of divisibility by 11

For any number find out the sum of the digits in the odd places and the sum of the digits in the even places. If the difference between the sum of digits at odd places and the sum of digits in even places are 0, 11 or multiple of 11, then the number would be divisible by 11.

Example: In 856592,

the sum of digits in odd places

$= 8 + 6 + 9 = 23$

the sum of digits in even places

$= 5 + 5 + 2 = 12$

The difference between the two sums :

$23 - 12 = 11$

Therefore, the number is divisible by 11.

Exercise 17.4

1. Verify the given numbers are divisible by 2 or not.

   (i) 252 (ii) 457 (iii) 436 (iv) 2509 (v) 94241

2. Verify the given numbers are divisible by 3 or not.

   (i) 324 (ii) 2500 (iii) 20325 (iv) 83812 (v) 94241

3. Which the given number are divisible by 5

   (i) 932 (ii) 815 (iii) 6570 (iv) 45864 (v) 77129

4. Which the given number are divisible by 7

   (i) 560 (ii) 791 (iii) 5623 (iv) 7007

We have learnt

1. Sum of two whole numbers is always a whole number.
2. Sum of two whole numbers and sum of after interchanging the order of numbers are always same.
3. If zero is added to any whole number and any whole number is added to zero, the value of number will be same.
4. If a, b and c are three whole numbers then $(a+b) + c = a + (b+c)$.
5. If a and b are two whole numbers and if $a>b$ or $a = b$ then is case of $a - b$, we will get a whole number.
6. If a is any whole number then $a - 0 = a$
7. If a is any whole number then $a - a = 0$
8. Multiplicand of two whole number is also a whole number. If

   $a \times b = c$

   If a and b are two whole numbers then their multiplicand c is also a whole number.
9. Multiplicand of two whole numbers and multiplicand after change in their order will remain same

   $a \times b = b \times a$
10. Multiplicand of three whole numbers are same in different conditions $(a\times b) \times c = a \times (b \times c)$.
11. If a is any whole number then multplication of a with 0 will give product as 0 i.e. $a\times 0 = 0$.
12. If a is any whole number and if it is multiplied by 1 then product will be whole number $a \times 1 = a$.

Chapter—18

OPERATIONS ON RATIONAL NUMBER

You have learnt that the numbers which can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ o, are called rational numbers.

In class 6, we learnt addition, subtraction, multiplication and division of positive fractions. Let us learn all these operations in detail.

Addition of Rational Numbers

Figure 18.1

A watermelon seller divided one watermelon into 10 equal parts. From these, Sujeet bought 2 parts, Uma bought 3 parts and Akanksha bought 3 parts, how many parts have been sold?

From a total of 10 parts, Sujeet has taken 2 parts = 2/10

From a total of 10 parts, Uma has taken 3 parts = 3/10

Akanksha has taken 3 parts =3/10

Therefore, total parts that Sujeet, Uma and Akanksha have taken = 2/10 +3/10 +3/10 = (2+3+3)/10 =8/10

The seller sold 8 parts out of 10 or 8/10 parts of the watermelon.

Let us learn to add two rational numbers with the help of a figure.

X 0	X	X
X 0	X	X
X 0	X	X
0		
0		

Figure 18.2

Example 1: Add $\frac{3}{5}$ and $\frac{1}{3}$

Take a rectangle, to represent 3/5 draw 4 horizontal lines spaced such that the rectangle is divided into 5 equal parts. From these, label three parts by the symbol 'x'. Now to represent 1/3, draw 2 vertical lines spaced such that the rectangle is divided into 3 equal parts. From these 3 parts, label one by '0'. Now the rectangle has been divided into 15 parts. Add the parts labeled by 'x' and the parts labeled by '0'.

Number of 'x' blocks + Number of '0' blocks = 9+5 = 14

14 parts out of 15 are marked or the marked blocks are $=\frac{14}{15}$

And 3/5+1/3 = (9 +5)/15 = 14/15

In the same way to find $\frac{3}{5}-\frac{1}{3}$, subtract the number of '0' blocks from the number of 'x' blocks or 9-5 = 4 parts and the total no of blocks is 15.

Therefore $\frac{3}{5}-\frac{1}{3}=\frac{9-5}{15}=\frac{4}{15}$

Similarly, add and subtract the following rational numbers using figures. Write the answer in lowest form.

(i) $\frac{3}{7}+\frac{1}{4}$ (ii) $\frac{2}{5}+\frac{1}{3}$ (iii) $\frac{3}{7}-\frac{1}{4}$ (iv) $\frac{2}{5}-\frac{1}{3}$

(v) $\frac{5}{6}+\frac{2}{3}$ (vi) $\frac{1}{4}-\frac{2}{3}$

Let us discuss the answers of the questions solved by you.

Ans (i) In solving this question, you drew horizontal lines in a rectangle to divide it into 7 equal parts. Of these 7 parts, you label 3 by 'X' .Now, by drawing 3 vertical lines you divide the rectangle into 4 equal parts and label one of them by '✓'. In this way the rectangle has been divided into 28 equal parts of which 12 blocks are marked by 'X' and 7 blocks are marked by '✓'.

Therefore, the sum of 3/7 and 1/4 will contain 12+7 = 19 blocks out of 28 blocks or $\frac{3}{7}+\frac{1}{4}=\frac{12}{28}+\frac{7}{28}=\frac{12+7}{28}=\frac{19}{28}$

Similarly, for $\frac{3}{7}-\frac{1}{4}=\frac{12}{28}-\frac{7}{28}=\frac{12-7}{28}=\frac{5}{28}$.

Ans (V): To solve this question, you divided the rectangle into 6 equal parts by drawing horizontal or vertical lines. You marked 5 parts with 'X' signs. Then you divided the rectangle into 3 equal parts as in previous questions and marked 2 of these with '✓' sign. Now the rectangle has been divided into 18 parts. It has 15 parts marked with 'X' and 12 parts marked with '✓'. Total number blocks marked by 'X' and '✓' =15 + 12 = 27

Therefore $\frac{5}{6}+\frac{2}{3}=\frac{15}{18}+\frac{12}{18}=\frac{27}{18}$

The lowest form of this will be $\frac{3}{2}$.

While solving this questions, Fatima told Raju, that last year, while adding and subtracting fractions, we converted the given fractions changed into common denominator forms.The denominator of the sum was always the product of the denominators of the two fractions.

In this method also the denominator of the sum is the product of the denominators of the two rational numbers. Raju said, that in the previous chapter we learnt that rational numbers can be expressed as $\frac{p}{q}$ or $\frac{r}{s}$, where p, q, r, s are integer and q ≠ o, s ≠ 0.

fractions with small denominators can be added by making equivalent fractions for eg $\frac{5}{6}+\frac{3}{8}$

$$\frac{5}{6}=\frac{5}{6},\frac{10}{12},\frac{15}{18},\frac{20}{24},\frac{25}{30}$$
$$\frac{3}{8}=\frac{3}{8},\frac{6}{16},\frac{9}{24},\frac{12}{32},\frac{15}{40}$$

Here equivalent fractions with same denominator are

$$\frac{5}{6} = \frac{20}{24} \ ; \ \frac{3}{8} = \frac{9}{24}$$

$$\frac{5}{6}+\frac{3}{8}=\frac{9}{24}+\frac{20}{24}=\frac{29}{24}$$

"Can we add or subtract these numbers by using the "common denominator method" Fatima said, "Let us try and find out".

For getting the common denominator of $\frac{p}{q}+\frac{r}{s}$, we multiplying the numerator and denominator of p/q by s and multiply the numerator and denominator of r/s by q.
(p/q) x (s/s) + (r/s) x (q/q)= (ps/qs)+ (rq/sq) = (ps+rq)/sq
In the same way find the sum of the following rational numbers.

(i) $\frac{m}{n}+\frac{r}{\ell}$ (ii) $\frac{a}{b}+\frac{q}{n}$ (iii) $\frac{s}{t}+\frac{c}{d}$

Add $\frac{3}{5}+\frac{-4}{7}$ by using common denominator method.

Here, the denominators are 5 and 7.Therefore, to reduce them into common denominator form, multiply the numerator and denominator of the first rational number by 7 and multiply the numerator and denominator of the second rational number by 5.

Therefore, $\frac{3}{5}=\frac{3}{5}\times\frac{7}{7}=\frac{21}{35}$ and $\frac{-4}{7}=\frac{-4\times5}{7\times5}=\frac{-20}{35}$

Thus, $\frac{3}{5}+\frac{-4}{7}=\frac{21}{35}+\frac{-20}{35}=\frac{21-20}{35}=\frac{1}{35}$

Some-times while solving by reducing into the common denominator form, we may get a common factor in the denominator.

Can you find the value of $\frac{5}{6}+\frac{3}{8}$

Radha started solving the problem. $\frac{5}{6}\times\frac{3}{8}+\frac{3}{8}\times\frac{6}{6}$

But Fatima did not like this method. She said, since 2 is a common factor between the denominators, there is no need to multiply the numerator and denominator by 2, it means we will multiply the numerator and denominator of $\frac{5}{2\times3}$ by $\frac{4}{4}$ and the numerator and denominator of $\frac{3}{2\times4}$ is multiplied by $\frac{3}{3}$ $\frac{5}{6}\times\frac{4}{4}+\frac{3}{8}\times\frac{3}{3}=\frac{20}{24}+\frac{9}{24}=\frac{29}{24}$

We can get the common denominator form of two fractions this way also.
Radha said that on reducing $\frac{3}{2\times5}+\frac{5}{2\times7}$ to the common denominator form the denominator will be 2 × 5 ×7. This is also the L.C.M. of the denominators.

Activity 1

Find the LCM of the denominators and perform the addition and subtraction of the rational numbers according to the operations mentioned in the activity.

Table 18.1

S, No	Rational number I	Rational number II	LCM of denomi -nators	$\dfrac{\frac{\text{Numerator of I}\times\text{LCM}}{\text{Denominator of I}}+\frac{\text{Numerator of II}\times\text{LCm}}{\text{Denominator of II}}}{\text{L C M of denominator}}$	Answer
1.	$\frac{4}{15}$	$\frac{7}{12}$	60	$\dfrac{\frac{4\times60}{15}+\frac{7\times60}{12}}{60}=\frac{4\times4+7\times5}{60}=\frac{16+35}{60}$	$\frac{51}{60}$ or $\frac{17}{20}$
2.	$\frac{7}{20}$	$\frac{3}{10}$	....	....	....
3.	$\frac{-7}{3}$	$\frac{11}{12}$	....	....	....
4.	$\frac{15}{8}$	$\frac{13}{12}$	....	....	....
5.	$\frac{6}{7}$	$\frac{5}{21}$	....	....	....

The sum obtained by the addition of two rational numbers follows certain rules. Let us learn them through the following examples. Fill the blanks in the following examples and examine table.

Activity

Closure Property

Table 18.2

S.No.	Rational Numbers	Add	Steps for Adding	Sum	Is this a rational Number?
1	$\frac{5}{7}$ and $\frac{4}{7}$	$\frac{5}{7}+\frac{4}{7}$	$\frac{5+4}{7}$	$\frac{9}{7}$	Yes
2	3 and $-\frac{6}{5}$	$\frac{3}{1}+\frac{-6}{5}$	$\frac{3\times5+(-6)\times1}{5}$	$\frac{9}{5}$	Yes
3	$-\frac{5}{13}$ and $\frac{5}{13}$	–	–	–	–
4	$\frac{1}{8}$ and $\frac{7}{8}$	–	–	–	–

It is clear from the above table that the **sum of two rational numbers is always a rational number. This is called Closure Property of addition.**

Take any two rational numbers and check whether their sum is a rational number or not?

Commutative Property

Suppose two rational numbers are

$$\frac{-5}{6} \text{ and } \frac{3}{4}, \text{ then } \frac{-5}{6}+\frac{3}{4}=\frac{-5\times2+3\times3}{12}=\frac{-10+9}{12}=\frac{1}{12}$$

$$\text{and } \frac{3}{4}+\left(\frac{-5}{6}\right)=\frac{3\times3+(-5)\times2}{12}=\frac{9+(-10)}{12}=\frac{1}{12}$$

Therefore, -5/6+3/4 = 3/4 + (-5/6)

Fill up the blanks of the following table

Table 18.3

S.No.	Rational Numbers	Sum of rational numbers	Sum of rational numbers on changing their order	Are their sums equal in both the situations
1.	$\frac{1}{8}$ and $\frac{7}{8}$	$\frac{1}{8}+\frac{7}{8}=\frac{1+7}{8}=\frac{8}{8}$	$\frac{7}{8}+\frac{1}{8}=\frac{7+1}{8}=\frac{8}{8}$	Yes
2.	$\frac{-3}{8}$ and $\frac{5}{16}$	$-\frac{3}{8}+\frac{5}{16}=$------	$\frac{5}{16}+\left(\frac{-3}{8}\right)=$---------	------
3.	$-\frac{7}{15}$ and $-\frac{8}{25}$	$-\frac{7}{15}+\frac{-8}{25}=$------	$\frac{-8}{25}+\frac{-7}{15}=$---------	------
4.	$\frac{p}{q}$ and $\frac{r}{s}$	$\frac{p}{q}+\frac{r}{S}=$------	$\frac{r}{s}+\frac{p}{q}=$---------	------

From the above table we get that sum of two rational numbers is equal to the sum of the rational numbers obtained by changing the order of addition of the rational number.

The sum of two rational numbers remains same if the order of their addition is changed. This property is called Commutative Property of addition in rational numbers.

So, if $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers then $\frac{p}{q}+\frac{r}{s} = \frac{r}{s}+\frac{p}{q}$

If $\frac{3}{4}+\frac{-5}{8} = x+\frac{3}{4}$ then, what is the value of x ?

Associative Property

Let the three rational numbers be $\frac{4}{5},\frac{2}{7}$ and $\frac{-3}{8}$. We can add these numbers in two ways.

First Method

$$\frac{4}{5}+\left(\frac{2}{7}+\frac{-3}{8}\right)=\frac{4}{5}+\left(\frac{2\times8-3\times7}{56}\right)$$

$$= \frac{4}{5} + \left(\frac{16-21}{56}\right) = \frac{4}{5} - \frac{5}{56}$$

$$= \frac{4\times56-5\times5}{280} = \frac{224-25}{280} = \frac{199}{280}$$

Second Method

$$\left(\frac{4}{5}+\frac{2}{7}\right)+\left(\frac{-3}{8}\right)=\left(\frac{4\times7+2\times5}{35}\right)+\left(\frac{-3}{8}\right)$$

$$=\left(\frac{28+10}{35}\right)-\left(\frac{3}{8}\right)$$

$$= \frac{38}{35}-\frac{3}{8} = \frac{38\times8-3\times35}{280}$$

$$= \frac{304-105}{280} = \frac{199}{280}$$

(Addition of rational number follows Associative Property)

Here $\frac{4}{5}+\left(\frac{2}{7}+\frac{-3}{8}\right)=\left(\frac{4}{5}+\frac{2}{7}\right)+\left(\frac{-3}{8}\right)$

In adding three rational numbers, if we add third number to the sum of first two numbers, we get the same value as we obtain when we add first to sum of second and third number. This rule is called Associative Property of addition of rational numbers.

Activity 3

Find the value of the following:-

(1) $\frac{1}{11}+\left(\frac{5}{6}+\frac{7}{12}\right)$ and $\left(\frac{1}{11}+\frac{5}{6}\right)+\frac{7}{12}$

(2) $\frac{3}{4}+\left(\frac{-5}{3}+\frac{4}{5}\right)$ and $\left(\frac{3}{4}+\frac{-5}{3}\right)+\frac{4}{5}$

(3) $\frac{-2}{3}+\left(\frac{1}{5}+\frac{3}{4}\right)$ and $\left(\frac{-2}{3}+\frac{1}{5}\right)+\frac{3}{4}$

Do we get the same value in both the situations?

From the above activity, we find that solution obtained in both the situations is the same. Therefore, we can say that addition of rational numbers follows Associative Property.

Sum of Zero with other rational numbers

We know that on adding 0 to another integer, there is no change in the value of the integer. Let us add 0 to rational numbers-

$$\frac{3}{5}+0=\frac{3}{5}+\frac{0}{5}=\frac{3+0}{5}=\frac{3}{5}$$

Similarly, $0+\frac{-4}{9}=\frac{-4}{9}$

Does there exist any number other than '0' which when added to another rational number does not change its value?

Thus we know that there is no number except 0 which when added to a rational number leaves the value of the rational number unchanged.

Because of this property '0' is called the **Additive Identity** for rational numbers. If $\frac{p}{q}$ is any rational number, $\frac{p}{q}+0=\frac{p}{q}$.

Additive inverse

$\frac{11}{15}$ and $\frac{-11}{15}$ are two rational numbers. Their sum $\frac{11}{15}+\left(\frac{-11}{15}\right)=\frac{11-11}{15}=0$

Given below are two rational numbers, one is positive and the other one is negative. Find the sum of these rational numbers.

(i) $\frac{-13}{36}+\frac{13}{36}=$ ______

(ii) $\frac{289}{295}+\frac{-289}{295}=$ ______

For each rational number, there always exists a rational number such that the sum of the two numbers is zero. That number is called the **Additive inverse** of the given number.

For example, the additive inverse of $\frac{3}{5}$ is $\frac{-3}{5}$

additive inverse of $\frac{17}{19}$ is $\frac{-17}{19}$

To obtain the additive inverse of any number we add such a number to the given number that the sum is zero. For example

$\frac{-5}{7} + x = 0$ or $x = \frac{5}{7}$.

Therefore, the additive inverse of $\frac{-5}{7}$ is $\frac{5}{7}$

Exercise 18.1

1. Add the following rational numbers –

   (i) $\frac{3}{2}, \frac{13}{17}$ (ii) $\frac{-7}{9}, \frac{-3}{4}$ (iii) $\frac{3}{4}, \frac{-2}{5}$

2. Use commutative property to fill up the following blanks:-

   (i) $\frac{-5}{9} + \frac{4}{7} = \frac{4}{7} + \_\_\_\_$ (iii) $\frac{-11}{29} + \frac{6}{31} = \_\_\_ + \_\_\_$

   (iii) $\frac{-15}{7} + \_\_\_ = \frac{13}{19}\_\_\_$ (iv) $\frac{5}{6} + \left(-\frac{7}{9}\right) = -\frac{7}{9} + \_\_\_\_$

3. Show that $\left(\frac{-2}{5} + \frac{4}{9}\right) + \frac{-3}{4} = \frac{-2}{5} + \left(\frac{4}{9} + \frac{-3}{4}\right)$. Which property is used in this?

4. Simplify -

   (i) $\frac{3}{7} + \frac{4}{9} + \frac{-6}{11}$ (ii) $\frac{-1}{6} + \frac{-2}{3} + \frac{-1}{3}$

   (iii) $\frac{5}{14} + \frac{2}{-7} + \frac{-3}{2}$

5. What should be added with $\frac{-7}{12}$ so that the sum is 0 ?

6. Fill up the blanks:–

   (i) Additive inverse of $\frac{-5}{7} = \_\_\_\_\_$

   (ii) $\frac{4}{17} + \frac{-4}{17} = \_\_\_\_$

(iii) $0+\frac{39}{51}=$ ______

(iv) Additive inverse of $\frac{42}{17}=$ ______

7. The following problems are related to some rule. Write the appropriate property that is used in the each blank-

(i) $\frac{13}{15}+\frac{4}{8}=\frac{4}{8}+\frac{13}{15}$ (..........................)

(ii) $\frac{2}{19}+\left(\frac{-3}{17}+\frac{4}{13}\right)=\left(\frac{2}{19}+\frac{-3}{17}\right)+\frac{4}{13}$ (..........................)

(iii) $\frac{p}{q}+0=\frac{p}{q}$ (..........................)

(iv) $\frac{-r}{s}+\frac{r}{s}=0$ (..........................)

8. Think of some rational numbers, verify the use of commutative and associative laws of addition for these rational numbers.

Subtraction of Rational Numbers

In class 6, while subtracting one fraction from the other, we found the answer by making the denominators equal. The operation of subtraction is opposite of the operation of addition. Subtraction of one number from the other number is the sum of the number and the additive inverse of the second number. Let us understand this through the following examples-

Examples 2: Subtract $\frac{1}{4}$ from $\frac{3}{8}$

$$\frac{3}{8}-\frac{1}{4}=\frac{3\times1-1\times2}{8}=\frac{3-2}{8}=\frac{1}{8} \qquad \text{(LCM of 4, 8 is 8)}$$

Addition of the given number with the additive inverse of $\frac{1}{4}$ is

$$\frac{3}{8}+\left(\frac{-1}{4}\right)=\frac{3}{8}+\frac{(-1)}{4}=\frac{3\times1+(-1)\times2}{8}=\frac{3-2}{8}=\frac{1}{8}$$

Thus, both the solutions are equal.

Now, subtract $\frac{11}{13}$ from $\frac{7}{19}$ and add $\frac{7}{19}$ to the additive inverse of $\frac{11}{13}$, then examine your answers.

We can even subtract rational numbers by using a number line. Let us see-

Example 3: Subtract $\frac{7}{12}$ from $\frac{5}{6}$

Solution: Here, the denominators are not equal. Therefore, before solving we will make the denominators equal.

$$\frac{5}{6}=\frac{5\times 2}{6\times 2}=\frac{10}{12} \quad (\therefore \text{LCM of 6 and 12 is 12})$$

$$\frac{7}{12}=\frac{7\times 1}{12\times 1}=\frac{7}{12}$$

We divide 1 unit on the number line into 12 equal parts. Firstly, to show $\frac{10}{12}$ we move 10 parts to the right of 0. Since we have to now subtract $\frac{7}{12}$ from, we return 7 parts to the left of the10th part.

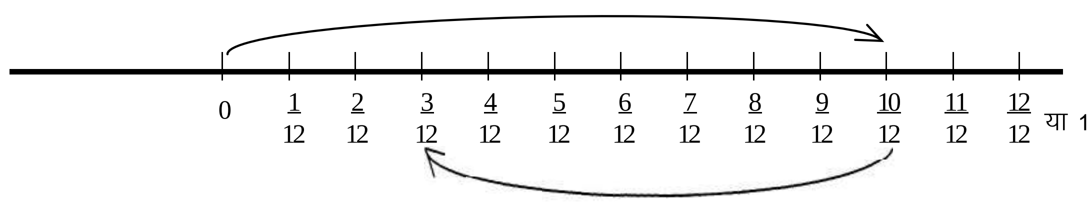


Figure 18.3

We reach $\frac{3}{12}$. Similarly, we get $\frac{3}{12}$ after subtracting $\frac{7}{12}$ from $\frac{5}{6}$

$$\frac{5}{6}-\frac{7}{12}=\frac{10}{12}-\frac{7}{12}$$

$$=\frac{10-7}{12}=\frac{3}{12}=\frac{1}{4}$$

Example 4: Subtract $\frac{-3}{9}$ from $\frac{4}{9}$

Solution: Since the subtraction of a rational number means the addition of the additive inverse of that rational number.

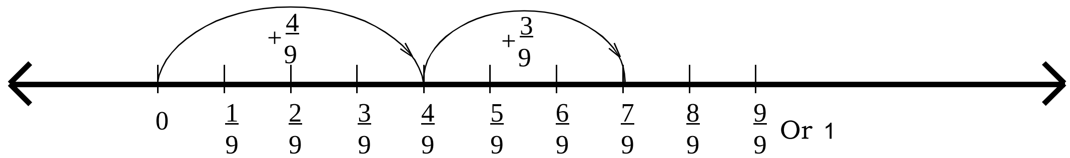


Figure 18.4

Therefore, subtraction of $\frac{-3}{9}$ means the addition of the additive inverse of $\frac{-3}{9}$,that is 3/9

$$\frac{4}{9}-\left(\frac{-3}{9}\right)=\frac{4}{9}+\frac{3}{9}=\frac{4+3}{9}=\frac{7}{9}$$

We divide the number line between 0 and 1 into 9 equal parts. We move 4 parts to the right of 0, and then again move 3 parts in the same direction. Thus we reach the 7^{th} part, equal to $\frac{7}{9}$.

Therefore, $\frac{4}{9}-\left(\frac{-3}{9}\right)=\frac{7}{9}$

Example 5: What should be added to $\frac{5}{9}$ so that the sum is $\frac{2}{3}$?

Solution: Let the sum of $\frac{5}{9}$ and $\frac{p}{q}$ be $\frac{2}{3}$

$$\frac{5}{9}+\frac{p}{q}=\frac{2}{3}$$

Add the additive inverse of $\frac{5}{9}$ to the both sides.

$$\frac{5}{9}+\frac{p}{q}+\left(\frac{-5}{9}\right)=\frac{2}{3}+\left(\frac{-5}{9}\right)$$

Or $\frac{p}{q}=\frac{2}{3}+\left(\frac{-5}{9}\right)$

Or $\frac{p}{q}=\frac{2\times 3}{3\times 3}+\frac{-5\times 1}{9\times 1}$ (LCM of 3, 9 is 9)

Or $\frac{p}{q}=\frac{6}{9}+\frac{-5}{9}$

Or $\frac{p}{q}=\frac{6-5}{9}$

Or $\frac{p}{q}=\frac{1}{9}$

Thus on adding 1/9 to 5/9 the sum is 2/3.

Example 6: What should be subtracted from $\frac{11}{13}$ to get $\frac{5}{26}$?

Solution: Subtracting p/q from 11/13 yields 5/26.

$$\frac{11}{13}-\frac{p}{q}=\frac{5}{26}$$

Add the additive inverse of $\frac{11}{13}$ to the both sides

Or $\frac{11}{13}-\frac{p}{q}+\left(-\frac{11}{13}\right)=\frac{5}{26}+\left(-\frac{11}{13}\right)$

Or $-\frac{p}{q}=\frac{5}{26}+\left(\frac{-11}{13}\right)$

Or $-\frac{p}{q}=\frac{5\times1}{26\times1}+\left(\frac{-11\times2}{13\times2}\right)$ (L.C.M. of13, 26 is 26)

Or $-\frac{p}{q}=\frac{5}{26}+\left(\frac{-22}{26}\right)$

Or $-\frac{p}{q}=\frac{5-22}{26}$

Or $-\frac{p}{q}=\frac{-17}{26}$

Or $\frac{p}{q}=\frac{17}{26}$ (Multiplying both sides by -1)

Thus we get $\frac{5}{26}$ after subtracting $\frac{17}{26}$ from $\frac{11}{13}$

Example 7: Simplify $\frac{1}{4}+\left(\frac{-5}{9}\right)-\left(\frac{-7}{12}\right)$.

Solution: Here, we are given three rational numbers, where the operations of addition and subtraction are to be done simultaneously.

For solving such questions, we make the denominators of all the rational numbers in the question equal.

$\frac{1}{4}=\frac{1\times 9}{4\times 9}=\frac{9}{36}$ Here the L.C.M. of 4, 9 and 12 is 36.

$$\frac{-5}{9}=\frac{-5\times 4}{9\times 4}=\frac{-20}{36}$$

$$\frac{-7}{12}=\frac{-7\times 3}{12\times 3}=\frac{-21}{36}$$

$$\frac{1}{4}+\left(\frac{-5}{9}\right)-\left(\frac{-7}{12}\right)=\frac{9}{36}+\frac{-20}{36}-\left(\frac{-21}{36}\right)$$

$$=\frac{9-20+21}{36}=\frac{30-20}{36}$$

$$=\frac{10}{36}=\frac{5}{18}$$

Properties of subtraction in Rational Number

1. Closure property- We have seen the properties of addition of rational numbers. In the subtraction of rational numbers also some properties apply. Let us see the following example:-

Subtract $\frac{25}{36}$ from $\frac{11}{21}$

Here $\frac{11}{21}-\frac{25}{36}=\frac{11\times 12-25\times 7}{252}=\frac{132-175}{252}$ (Here, LCM of 21 and 36 is 252)

$=\frac{-43}{252}$,which is a rational number.

Here, $\frac{11}{21}, \frac{25}{36}$ and $\frac{-43}{252}$ all three are rational numbers. The operation of subtraction between any two rational numbers gives a rational number. Check this property by taking some rational numbers.

2. Subtraction of zero from rational number- If zero is subtracted from a rational number, the value of the rational number does not change.

For example $\frac{-21}{45} - 0 = \frac{-21}{45}$ and $\frac{5}{17} - 0 = \frac{5}{17}$

$$\boxed{\frac{P}{Q} - 0 = \frac{P}{Q}}$$

3. Commutative property :-

Find the value of the following:-

(i) $\frac{5}{12} - \frac{6}{13}$ and (ii) $\frac{6}{13} - \frac{5}{12}$

Here $\frac{5}{12} - \frac{6}{13} = \frac{5\times13}{12\times13} - \frac{6\times12}{13\times12}$

$= \frac{65}{156} - \frac{72}{156}$ (L.C.M. of 12 and 13 is 156)

$= \frac{65-72}{156} = \frac{-7}{156}$

And $\frac{6}{13} - \frac{5}{12} = \frac{6\times12}{13\times12} - \frac{5\times13}{12\times13} = (72/156) - (65/156)$

$= \frac{72-65}{156} = \frac{7}{156}$

Is $\frac{-7}{156}$ equal to $\frac{7}{156}$?

Therefore, $(5/12) - (6/13) \neq (6/13) - (5/12)$.

Hence, subtraction of rational numbers does not follow the commutative property.

Exercise 18.2

Q1. Subtract the first rational number from the second rational number.

(i) $\frac{4}{5}$ from $\frac{3}{4}$ (ii) $\frac{-1}{8}$ from $\frac{1}{4}$

(iii) $\frac{13}{24}$ from $\frac{-5}{12}$ (iv) $\frac{-7}{13}$ from $\frac{-8}{13}$

Q2. Solve

(i) $\frac{2}{9}+\frac{1}{3}-\frac{5}{9}$ (ii) $\frac{1}{5}-\frac{3}{7}+\frac{1}{2}$ (iii) $\frac{-1}{12}+\frac{3}{5}-6$

Q3. What should be added to $\frac{3}{8}$ so that the sum is $\frac{11}{12}$?

Q4. What should be subtracted from $\frac{13}{25}$ so that the difference is $\frac{19}{25}$?

Q5. Write true or false and give right statements for the false statements

(i) Additive inverse of $\frac{-3}{5}$ is $\frac{5}{3}$

(ii) $\frac{4}{5}-\frac{7}{9}=\frac{7}{9}-\frac{4}{5}$

(iii) The value of a number does not change after subtracting 0 from it.

(iv) Subtraction of a rational number means the addition of the additive inverse of that number.

Multiplication of Rational Number

While multiplying two fractions, you have seen that we multiply the denominator with the denominator and the numerator with the numerator. Since rational numbers are also composed of numerator and denominator, we multiply rational numbers in a similar way. Let us discuss the multiplication of two rational numbers through examples.

Example 8: Multiply $\frac{3}{4}$ and $\frac{7}{16}$ and write the value.

Solution: $\frac{3}{4}\times\frac{7}{16}=\frac{3\times7}{4\times16}=\frac{21}{64}$

Example 9: Multiply $\frac{-5}{7}$ and $\frac{13}{17}$ and write the value.

Solution: $\frac{-5}{7}\times\frac{13}{17}=\frac{-5\times13}{7\times17}=\frac{-65}{119}$

Example 10: Multiply $\frac{-9}{11}$ and $\frac{22}{27}$ and write the value.

Solution: $\frac{-9}{11} \times \frac{22}{27} = \frac{-9 \times 22}{11 \times 27} = \frac{-1 \times 2}{1 \times 3} = \frac{-2}{3}$

It is clear from the above examples that to find the product of two rational numbers, we multiply the numerator with the numerator and the denominator with the denominator.

If (p/q) and (r/s) are two rational numbers then (p/q) x (r/s) = (p x r)/ (q x s)

Example 11: Multiply $\frac{2}{3}, \frac{-6}{7}$ and $\frac{8}{15}$

Solution: $\frac{2}{3} \times \frac{-6}{7} \times \frac{8}{15} = \frac{2 \times -6 \times 8}{3 \times 7 \times 15} = \frac{-32}{105}$

In finding the product of more than two rational numbers, we multiply all numerators with each other and all denominators with the other denominators. If the rational numbers $\frac{p}{q}, \frac{r}{s}, \frac{u}{v}$ and $\frac{w}{z}$ etc are multiplied, then $\frac{p}{q} \times \frac{r}{s} \times \frac{u}{v} \times \frac{w}{z} = \frac{p \times r \times u \times w}{q \times s \times v \times z}$ and the product is written in its simplest form

Activity 4

Fill up the blanks in the table given below as directed:-

Table 18.4

S.No.	Rational Numbers	Multiplication of rational numbers	Product	On Changing the order of multiplication	Product	The obtained number is rational or not?
1.	$\frac{11}{15}, \frac{1}{4}$	$\frac{11}{15} \times \frac{1}{4}$	$\frac{11}{60}$	$\frac{1}{4} \times \frac{11}{15}$	$\frac{11}{60}$	Yes
2.	$\frac{-5}{8}, \frac{-7}{4}$	$\frac{-5}{8} \times \frac{-7}{4}$	----------	-------------		
3.	$\frac{-19}{12}, \frac{5}{13}$	$\frac{-19}{12} \times \frac{5}{13}$	---------	-------------		
4.	$\frac{4}{9}, \frac{-18}{5}$	------------	---------	------------		
5.	$\frac{31}{-6}, \frac{24}{7}$	-----------	---------	----------		

From the above, you find that on multiplying rational numbers, we get another rational number. Thus **the set of rational number is closed under multiplication.**

The change in the order of numbers being multiplied does not affect the product. This property is the commutative property of multiplication. Hence, if we have two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$, $\frac{p}{q}\times\frac{r}{s}=\frac{r}{s}\times\frac{p}{q}$

Think of two other rational numbers and verify whether their multiplication follows Commutative Property of multiplication or not.

Distributive property

Integers follow distributive property. Is this also applicable to rational numbers? Let us see through some examples.

Example 12: Simplify $\frac{2}{5}\times\left(\frac{3}{4}+\frac{1}{7}\right)$

Solution: First Method;

$$\frac{2}{5}\times\left(\frac{3}{4}+\frac{1}{7}\right)=\frac{2}{5}\left(\frac{3\times7+1\times4}{28}\right)$$

$$=\frac{2}{5}\left(\frac{21+4}{28}\right)$$

$$=\frac{2}{5}\left(\frac{25}{28}\right)=\frac{5}{14}$$

We can also solve the above problems in the following way.

Second Method:

$$\frac{2}{5}\times\left(\frac{3}{4}+\frac{1}{7}\right)=\frac{2}{5}\times\frac{3}{4}+\frac{2}{5}\times\frac{1}{7}=\frac{2\times3}{5\times4}+\frac{2\times1}{5\times7}$$

$$=\frac{6}{20}+\frac{2}{35}=\frac{6\times7+2\times4}{140}=\frac{42+8}{140}=\frac{50}{140}=\frac{5}{14}$$

The solutions of the first and the second method are the same, this means that

$$\frac{2}{5}\left(\frac{3}{4}+\frac{1}{7}\right)=\frac{2}{5}\times\frac{3}{4}+\frac{2}{5}\times\frac{1}{7}$$

This is distributive law for rational numbers

Think of any other three similar rational numbers and verify whether they follow the distributive property.

Therefore, if p/q, r/s, and u/v are three rational numbers then (p/q) (r/s + u/v) = (p/q) × (r/s) + (p/q) × (u/v)

Example 13: If x, y and z are three rational numbers then

$$x \times (y+z) = x \times y + x \times z$$

Where $x = \frac{-5}{8}, y = \frac{7}{9}, z = \frac{11}{12}$

L.H.S. = x × (y +z)

$= \frac{-5}{8} \times \left(\frac{7}{9} + \frac{11}{12}\right)$ (substituting the values of x, y and z)

$$= \frac{-5}{8} \times \left(\frac{7 \times 4 + 11 \times 3}{36}\right)$$

$$= \frac{-5}{8} \times \left(\frac{28 + 33}{36}\right)$$

$$= \frac{-5}{8} \times \frac{61}{36} = \frac{-305}{288}$$

R.H.S. $= x \times y + x \times z$

$$= \frac{-5}{8} \times \frac{7}{9} + \left(\frac{-5}{8}\right) \times \frac{11}{12}$$

$$= \frac{-35}{72} + \frac{-55}{96}$$

R.H.S. $= \frac{-35 \times 4 - 55 \times 3}{288}$ (L.C.M. of 72 and 96 is 288)

$$= \frac{(-140 - 165)}{288}$$

$$= \frac{-305}{288}$$

L.H.S. = R.H.S.

Multiplication of rational number with zero

Zero is a rational number, You can write it in many ways like 0/1, 0/-27, 0/q where q is an integer and $q \neq 0$

Multiplying 0 with any rational number like:-

$$\frac{-27}{84} \times \frac{0}{q} = \frac{0}{84q} = 0$$

Similarly, the multiplication of 0 with any other rational number yields 0.

Multiplicative identity

Can you think of a rational number which when multiplied with a rational number $\frac{p}{q}$ yields the product $\frac{p}{q}$

Radha said to Fatima, "We know that when we multiply a number with 1, the value of the number does not change and since one is also a rational number, which can be written as 1/1, -2/-2, 57/57 etc. Therefore, 1 is the rational number whose multiplication with $^{p}/_{q}$ (where $q \neq 0$) gives the product $^{p}/_{q}$."

Here '1' is called the **Multiplicative identity**.

Multiplicative inverse

$\frac{1}{3} \times \square = 1$, which number should be put in the blank box so that it's product is 1.

Your answer will be $\frac{3}{1}$.

Activity 5

Some problems are given below. Fill up the blank boxes with appropriate numbers.

(i) $\frac{1}{7} \times \square = 1$ (ii) $\square \times \frac{1}{7} = 1$

(iii) $\frac{1}{13} \times \square = 1$ (iv) $\square \times \frac{1}{13} = 1$

(v) $\frac{7}{13} \times \square = 1$ (vi) $\frac{13}{7} \times \square = 1$

You are observing that two rational numbers whose product is 1 (a multiplicative identity) are being multiplied. You also write below pairs of rational numbers whose product is 1 (the multiplicative identity.)

(1) $\frac{\quad}{\quad} \times \frac{\quad}{\quad} = 1$

(2) $\frac{\quad}{\quad} \times \frac{\quad}{\quad} = 1$

(3) $\frac{\quad}{\quad} \times \frac{\quad}{\quad} = 1$

(4) $\frac{\quad}{\quad} \times \frac{\quad}{\quad} = 1$

While filling up the above, Raju was thinking that to obtain the additive identity, we added the number to its additive inverse. Then similarly to get the multiplicative identity do we multiply the number with its multiplicative inverse? If this is so, then the numbers given above would be the multiplicative inverses of each other.

Thus, **if the product of two numbers is one then both numbers are the multiplicative inverses of each other.**

Let us see, how to find multiplicative inverse.

Example 14: What is the multiplicative inverse of $\frac{p}{q}$?

Solution. Let x be the multiplicative inverse of $\frac{p}{q}$.

$$\frac{p}{q} \times x = 1$$

Or $p\,x = q$

Or $x = \frac{q}{p}$.

Thus, the multiplicative inverse of $\frac{p}{q}$ is $\frac{q}{p}$.

That is, **we can obtain the multiplicative inverse of a number by changing the numerator into denominator and denominator into numerator.**

Let us see some examples:-

(1) $\frac{4}{3} \times \frac{3}{4} = 1$

(2) $\frac{-27}{53} \times \frac{53}{-27} = 1$

(3) $\frac{a}{b} \times \frac{b}{a} = 1$ or $\frac{b}{a} \times \frac{a}{b} = 1$

Thus, $\frac{b}{a}$ is the multiplicative inverse of $\frac{a}{b}$ and $\frac{a}{b}$ is also known as the reciprocal of reciprocal of $\frac{b}{a}$.

Write the multiplicative inverse or the reciprocal of the following:-

$$\frac{-4}{9}, \frac{2}{-7}, \frac{8}{15}, \frac{c}{d}, 4, -5$$

Does a multiplicative inverse exist for each rational number?

What will be the multiplicative inverse of 0? Think about it?

The multiplicative inverse of 0 does not exist, since we do not get 1 on multiplying 0 with any rational number.

Thus, 0 does not have any multiplicative inverse.

Exercise 18.3

Q1. Substitute the values given below and check if $\frac{p}{q} \times \frac{r}{s} = \frac{r}{s} \times \frac{p}{q}$.

(i) $\frac{p}{q} = \frac{-3}{7}, \frac{r}{s} = \frac{11}{15}$ (ii) $\frac{p}{q} = 2, \frac{r}{s} = \frac{13}{17}$

(iii) $\frac{p}{q} = \frac{-105}{13}, \frac{r}{s} = \frac{-5}{8}$ (iv) $\frac{p}{q} = \frac{-16}{3}, \frac{r}{s} = 0$

Q2. Substitute x, y and z and check whether $x \times (y + z) = x \times y + x \times z$

(i) $x = \frac{-1}{2}, y = \frac{5}{7}, z = \frac{-7}{4}$ (ii) $x = \frac{3}{2}, y = \frac{-8}{5}, z = \frac{17}{6}$

(iii) $x = 1, y = \frac{9}{5}, z = 0$

Q3. Fill in the blanks using commutative property :-

(i) $\frac{2}{3} \times 4 = 4 \times$ ___ (ii) $\frac{11}{19} \times$ ___ $= \frac{1}{2} \times$ ___

(iii) ___ $\times \frac{7}{9} =$ ___ $\times \frac{-3}{17}$

Q4. Use associative property and fill the blanks.

(i) $\frac{1}{2} \times \left(\frac{17}{6} \times \frac{2}{9}\right) = \left(\frac{1}{2} \times \frac{17}{6}\right) \times$ ___

(ii) $\frac{-1}{8}\times\left(\frac{-2}{5}\times\frac{1}{4}\right)=(\_\_\times\_\_)\times\frac{1}{4}$ (iii) $\frac{4}{7}\times\left(\frac{-25}{3}\times\frac{1}{5}\right)=(\_\_\times\_\_)\times\_\_$

Q5. Each of the questions given below relate to some property .Write the property in the blank against each.

Property

(i) $\frac{7}{12}\times\left(\frac{1}{9}+\frac{5}{3}\right)=\frac{7}{12}\times\frac{1}{9}+\frac{7}{12}\times\frac{5}{3}$ (____________________)

(ii) $\frac{5}{7}\times\left(\frac{25}{3}+\frac{4}{3}\right)=\frac{5}{7}\times\frac{25}{3}+\frac{5}{7}\times\frac{4}{3}$ (____________________)

(iii) $\frac{8}{11}\times\frac{3}{7}=\frac{3}{7}\times\frac{8}{11}$ (____________________)

(iv) $\frac{5}{3}\times\frac{3}{5}=1$ (____________________)

(v) $\frac{-3}{12}\times 1=1\times\left(\frac{-3}{12}\right)=\frac{-3}{12}$ (____________________)

Q6. Write the reciprocal of the following:-

(i) 4 (ii) $\frac{17}{5}$ (iii) $\frac{-6}{29}$ (iv) $\frac{p}{q}$

Q7. Write true or false:

(i) The product of a rational number and its reciprocal is one.

(ii) If the reciprocal of x is y, the reciprocal of y is 1/x.

(iii) The multiplicative inverse of a positive rational number is a negative rational number.

(iv) 0 is not the multiplicative inverse of any number.

Division of Rational Number

Radha and Fatima were playing the game find the multiplicative inverse. They were giving numbers to each other to find its multiplicative inverse. Radha noticed something new. She told Fatima “See in these examples multiplication of a number with its multiplicative inverse is in fact equivalent to dividing the number by itself.

Like $4 \times \frac{1}{4} = \frac{4}{4} = 4 \div 4$

$2 \times \frac{1}{2} = \frac{2}{2} = 2 \div 2$

Fatima said that this means that dividing by a number is the same as multiplying with the multiplicative inverse of the number.

$3 \div 4 = 3 \times$ (multiplicative inverse of 4)

$= 3 \times \frac{1}{4}$

Activity 6

Convert division into multiplication using multiplicative inverse.

e.g $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1}$

(i) $\frac{4}{3} \div \frac{3}{4} =$ (ii) $\frac{7}{9} \div \frac{8}{7} =$ (iii) $\frac{a}{x} \div \frac{b}{y} =$ (iv) $\frac{p}{q} \div \frac{r}{s} =$

We can conclude from the above that if we have to divide $\frac{x}{y}$ by $\frac{a}{b}$. Then by writing it as $\frac{x}{y} \times$ (Multiplicative inverse of $\frac{a}{b}$), we can get the answer.

$\frac{x}{y} \div \frac{a}{b} = \frac{x}{y} \times$ (Multiplicative inverse of $\frac{a}{b}$) $= \frac{x}{y} \times \frac{b}{a}$

Example 15: Solve the following

(i) $2 \div \frac{-2}{3}$ (ii) $\frac{-5}{4} \div \frac{15}{14}$ (iii) $\frac{23}{12} \div \frac{46}{36}$

Solution: (i) $2 \div \frac{-2}{3} = 2 \times \frac{3}{-2} = -3$ (ii) $\frac{-5}{4} \div \frac{15}{14} = \frac{-5}{4} \times \frac{14}{15} = \frac{-7}{6}$

(iii) $\frac{23}{12} \div \frac{46}{36} = \frac{23}{12} \times \frac{36}{46} = \frac{3}{2}$

Example 16: The product of two rational numbers is -21. If one of the number is $\frac{3}{10}$, find the second number ?

Solution: Let the second rational number be $\frac{p}{q}$

According to the question $\frac{3}{10} \times \frac{p}{q} = -21$

Multiplying both sides by $\frac{10}{3}$, (the multiplicative inverse of 3/10)

$$\frac{3}{10} \times \frac{p}{q} \times \frac{10}{3} = -21 \times \frac{10}{3}$$

$$\text{Or } \frac{p}{q} = \frac{-210}{3} = \frac{-70}{1}$$

Thus, the second number is $-\frac{210}{3}$ or $\frac{-70}{1}$

Exercise 18.4

Q.1. Divide -

(i) $\frac{1}{6}$ by $\frac{3}{4}$ (ii) $\frac{-8}{11}$ by $\frac{5}{9}$ (iii) -9 by $\frac{4}{7}$

(iv) $\frac{-102}{38}$ by $\frac{-17}{19}$ (v) $\frac{6}{15}$ by $\frac{8}{-35}$ (vi) $\frac{-60}{9}$ by -10

Q2. Simplify

(i) $\frac{4}{5} \div (-1)$ (ii) $\frac{95}{16} \div \frac{8}{19}$ (iii) $\left(\frac{-7}{8}\right) \div \left(\frac{-2}{15}\right)$

(iv) $\frac{21}{5} \div \frac{7}{-5}$ (v) $\frac{-6}{7} \div (-15)$ (vi) $-7 \div (-5)$

Q3. The product of two numbers is 12. If one number is $\frac{3}{5}$, find the second number?

Q4. Which number multiplied by $\frac{-9}{5}$ gives the product -11?

Q5. What should be multiplied by $\frac{-28}{39}$ so that the product is the multiplicative inverse of $\frac{3}{7}$?

Q6. There are 540 students in a school out of them $\frac{5}{9}$ are boys. How many girls are in the school

How many numbers are there between two rational numbers

Fatima and Kartik were solving questions related to ordering of rational numbers. Fatima said, "Kartik, we can write integers between – 15 and -8. These are -14, -13, -12, -11, -10, -9. Similarly, can we write rational numbers between two rational numbers?"

Kartik said, "Definitely, we can write many rational numbers between two rational numbers".

Fatima Said, "Yes! between $\frac{-15}{1}$ and $\frac{-8}{1}$ we have all the integers but besides that between $\frac{-15}{1}$ and $\frac{-14}{1}$ we also have $\frac{-29}{2}$. Kartik said, "There are many more rational numbers." Fatima, "Oh! Yes. There would be many, that it would be difficult to count them".

Do you agree with Kartik and Fatima? Is the statement of Fatima that there are countless rational numbers between $\frac{-15}{1}$ and $\frac{-8}{1}$ correct? Radha said, "How can this be? There is no fraction between $\frac{2}{5}$ and $\frac{3}{5}$."

Can you think which fractions lie between $\frac{2}{5}$ and $\frac{3}{5}$?

Kartik said, "Let us find out-

$$\frac{2}{5}=\frac{2\times2}{5\times2}=\frac{4}{10}$$

$$\frac{3}{5}=\frac{3\times2}{5\times2}=\frac{6}{10}$$

$\frac{5}{10}$ lies between the two rational numbers.

Immediately, Ramesh and Meena together said.

$$\frac{2}{5}=\frac{2\times3}{5\times3}=\frac{6}{15}$$

$$\frac{3}{5}=\frac{3\times3}{5\times3}=\frac{9}{15}$$

Now $\frac{7}{15},\frac{8}{15}$ are between these numbers.

Can you find at least 20 rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$?

So, how many rational numbers do we have between $\frac{5}{7}$ and $\frac{6}{7}$?

Anu saw a special thing that if we have two rational numbers with equal denominators and consecutive numerators like $\frac{5}{7}$ and $\frac{6}{7}$, then if these are multiplied by $\frac{2}{2}$, we get $\frac{10}{14}$ & $\frac{12}{14}$, $\frac{11}{14}$ is between $\frac{10}{14}$ and $\frac{12}{14}$. If we multiply them by $\frac{3}{3}$, we get $\frac{15}{21}$ and $\frac{18}{21}$ thus $\frac{16}{21}$, $\frac{17}{21}$, are two other rational numbers between them.

If we multiply the chosen numbers by $\frac{5}{5}$, We get 4 new rational numbers between $\frac{25}{35}$ and $\frac{30}{35}$.

Anu said - if I multiply them by $\frac{17}{17}$, we will get 16 new rational numbers between $\frac{5}{7}$ and $\frac{6}{7}$.

Do you agree with Anu's statement?

If we consider the above examples we can see that we have found many rational numbers between $\frac{5}{7}$ and $\frac{6}{7}$. Can you imagine how many rational numbers exist between $\frac{5}{7}$ and $\frac{6}{7}$?

Activity 7

1. Write 25 rational numbers between $\frac{1}{3}$ and $\frac{2}{3}$.

2. Can you write any two fractions such that there is no fraction between them?

Example 4: Write 10 rational numbers between $\frac{-4}{3}$ and $\frac{3}{4}$

Solution: Given rational numbers do not have equal denominator.

So, we make all the denominators equal than, $\frac{-4}{3} = \frac{-4 \times 4}{3 \times 4} = \frac{-16}{12}$

and $\frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$. Now $\frac{-16}{12}$ and $\frac{9}{12}$ are rational numbers with equal denominators. The difference between their numerators is 25.

So, there will be 24 rational numbers between $\frac{-4}{3}$ and $\frac{3}{4}$

$$\frac{-15}{12}, \frac{-14}{12}, \frac{-13}{12} \ldots\ldots\ldots \frac{-2}{12}, \frac{-1}{12}, \frac{0}{12}, \frac{1}{12}, \frac{2}{12} \ldots \frac{7}{12}, \frac{8}{12}$$

We can choose any 10 rational numbers from the above numbers.

If you have to find 25 rational numbers between $\frac{-4}{3}$ and $\frac{4}{3}$ what will you do?

Another Method

Example 5: Write 5 rational numbers between $\frac{1}{8}$ and $\frac{1}{2}$

Solution: $\frac{1}{2}$ is bigger and $\frac{1}{8}$ is smaller among them.

If we add both the two numbers and divide them by 2. Then, the resulting number is the central number (midpoint) between them.

First mid point between them is:-

$$= \frac{\frac{1}{8} + \frac{1}{2}}{2} = \frac{1}{2}\left(\frac{1}{8} + \frac{1}{2}\right) = \frac{1}{2}\left(\frac{1\times1}{8\times1} + \frac{1x4}{2x4}\right)$$

$$= \frac{1}{2}\left(\frac{1+4}{8}\right) = \frac{5}{16}$$

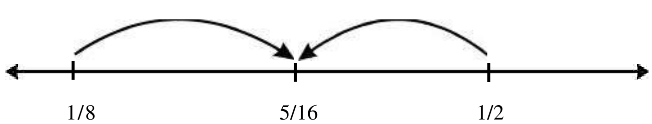


Figure 1.10

As shown in figure 1.10. This is exactly in the middle of $\frac{1}{8}$ and $\frac{1}{2}$.

To find more rational numbers we can find two more middle numbers between $\frac{1}{8}$ & $\frac{5}{16}$ and $\frac{5}{16}$ & $\frac{1}{2}$

The middle number between $\frac{1}{8}$ and $\frac{5}{16}$ is –

$$\frac{1}{2}\left(\frac{1}{8} + \frac{5}{16}\right) = \frac{1}{2}\left(\frac{1\times2}{2\times8} + \frac{5\times1}{16\times1}\right)$$

$$= \frac{1}{2}\left(\frac{2+5}{16}\right) = \frac{1}{2}\left(\frac{7}{16}\right) = \frac{7}{32}$$

The middle number between $\frac{5}{16}$ and $\frac{1}{2}$ is $\frac{1}{2}\left(\frac{5}{16}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{5\times1}{16\times1}+\frac{1\times8}{2\times8}\right) = \frac{1}{2}\left(\frac{5+8}{16}\right) = \frac{13}{32}$

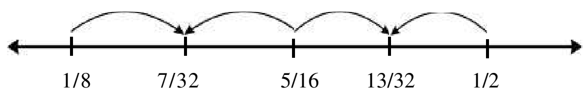


Figure 18.6

Now, We find the middle number between $\frac{1}{8}$ and $\frac{7}{32}$

The middle number between $\frac{1}{8}$ and $\frac{7}{32}$

$$\frac{1}{2}\left(\frac{1}{8}+\frac{7}{32}\right) = \frac{1}{2}\left(\frac{1\times4}{8\times4}+\frac{7\times1}{32\times1}\right) = \frac{1}{2}\left(\frac{4}{32}+\frac{7}{32}\right) = \frac{1}{2}\left(\frac{11}{32}\right) = \frac{11}{64}$$

The middle number between $\frac{13}{32}$ and $\frac{1}{2}$

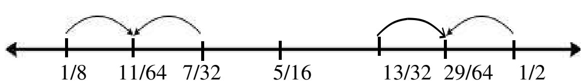


Figure 18.7

$$\frac{1}{2}\left(\frac{13}{32}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{13\times1}{32\times1}+\frac{1\times16}{2\times16}\right) = \frac{1}{2}\left(\frac{13}{32}+\frac{16}{32}\right) = \frac{29}{64}$$

Thus, five rational numbers between $\frac{1}{8}$ and $\frac{1}{2}$ are:- $\frac{11}{64}, \frac{7}{32}, \frac{5}{16}, \frac{13}{32}, \frac{29}{64}$

Rajni said, "This means that we can find at least one rational number between any two rational numbers." Rahul said, "That is not all, if we continue we can find as many rational numbers as we wish".

What do you think about this? Discuss among yourselves.

Example 6: Write three rational numbers between $\frac{-7}{3}$ and $\frac{5}{8}$

Solution : The middle rational number between $\frac{-7}{3}$ and $\frac{5}{8}$

$$= \frac{1}{2}\left(\frac{-7}{3}+\frac{5}{8}\right)$$

$$= \frac{1}{2}\left(\frac{-7\times8}{3\times8} = \frac{5\times3}{8\times3}\right) = \frac{1}{2}\left(\frac{-56+15}{24}\right) = \frac{-41}{48}$$

The middle rational number between $\frac{-7}{3}$ and $\frac{-41}{48}$

$$=\frac{1}{2}\left(\frac{-7}{3}+\frac{-41}{48}\right)=\frac{1}{2}\left(\frac{-7\times16}{3\times16}+\frac{-41\times1}{48\times1}\right)=\frac{1}{2}\left(\frac{-112+(-41)}{48}\right)=\frac{1}{2}\left(\frac{-153}{48}\right)=\frac{-153}{96}$$

The middle rational number between $\frac{-41}{48}$ and $\frac{5}{8}$

$$=\frac{1}{2}\left(\frac{-41}{48}+\frac{5}{8}\right)=\frac{1}{2}\left(\frac{-41\times1}{48\times1}+\frac{5\times6}{8\times6}\right)$$

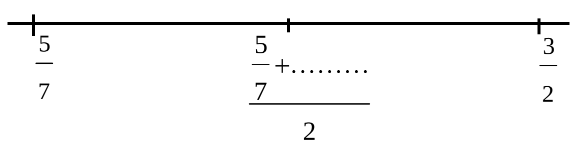
$$=\frac{1}{2}\left(\frac{-41+30}{48}\right)$$

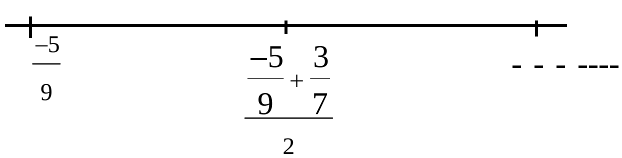
$$=\frac{1}{2}\left(-\frac{11}{48}\right)=\frac{-11}{96}$$

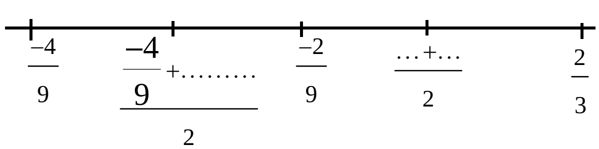
Thus three numbers between $\frac{-7}{3}$ and $\frac{5}{8}$ are $\frac{-153}{96}, \frac{-41}{48}$ and $\frac{-11}{96}$

EXERCISE 18.5

1. Fill up the blanks in following figure.

i) 

ii) 

iii) 

2. How many rational numbers can be written between two rational numbers? Explain.

3. Write 5 rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

4. Write 4 rational numbers between $\frac{1}{3}$ and $\frac{-2}{7}$.

5. Write 6 rational numbers between $\frac{-1}{6}$ and $\frac{3}{4}$

6. True or false -

   (i) $\frac{1}{10}$ is the middle number of $\frac{-1}{2}$ and $\frac{3}{5}$

   (ii) There is no rational number between $\frac{4}{5}$ and $\frac{6}{5}$

   (iii) There are only 3 rational numbers between 3 and 7

7. Write more questions in which you have to find rational numbers between two rational numbers. Give these questions to your friends ?

8. Write in your words what you have learnt in this chapter about rational numbers ?

We have learnt

1. If x and y are two rational numbers,

   (i) x + y is also a rational number

   (ii) x - y is also a rational number

   (iii) x × y is also a rational number

   (iv) x ÷ y is also a rational number (If y is non – zero)

2. If x and y are two rational numbers then

   (i) $x + y = y + x$

   (ii) $x \times y = y \times x$

   (iii) $x - y \neq y - x$ (except when x = y)

   (iv) $x \div y \neq y \div x$ (except when x = y and $x \neq 0$, $y \neq 0$)

3. If x, y and z are three rational numbers then

   (i) $(x+y)+z = x+(y+z)$

   (ii) $(x\times y)\times z = x\times(y\times z)$

4. If x, y and z are three rational numbers then

   (i) $x \times (y+z) = x \times y + x \times z$

(ii) $x \times (y-z) = x \times y - x \times z$

5. If x is any rational number, the following statements are true.
   (i) $x + 0 = 0 + x = x$ (ii) $x - 0 = x$
   (iii) $x \times 0 = 0 \times x = 0$ (iv) $x \times 1 = 1 \times x = x$ (v) $x \div 1 = x$

6. If $x = \frac{p}{q}$ is a non – zero rational number, the multiplicative inverse of x is $\frac{1}{x} = \frac{q}{p}$ is also a rational number ?

7. Dividing a rational number x by another rational number y is the same as multiplying x with the multiplicative inverse of y ?

   (x/y)/(a/b) = (x/y) × (multiplicative inverse of a/b) = (x/y) × (b/a)

   Or divisor ÷ dividend = Divisor × (reciprocal of dividend).

8. In adding the two rational numbers, we convert them into common denominator fractions and then add them.

9. There are infinite rational numbers between two rational numbers.

10. If we have two rational numbers with equal denominators, then we can easily find rational numbers whose number is one less than the difference of their numerators.

Chapter—19

MENSURATION-III

You see pipe of tap, roller of wood, refill of pen; tube light, battery of torch, like things daily. Can you tell the name of these fegures? What are the similarities in them?

Look at the following figure carefully and make their groups.

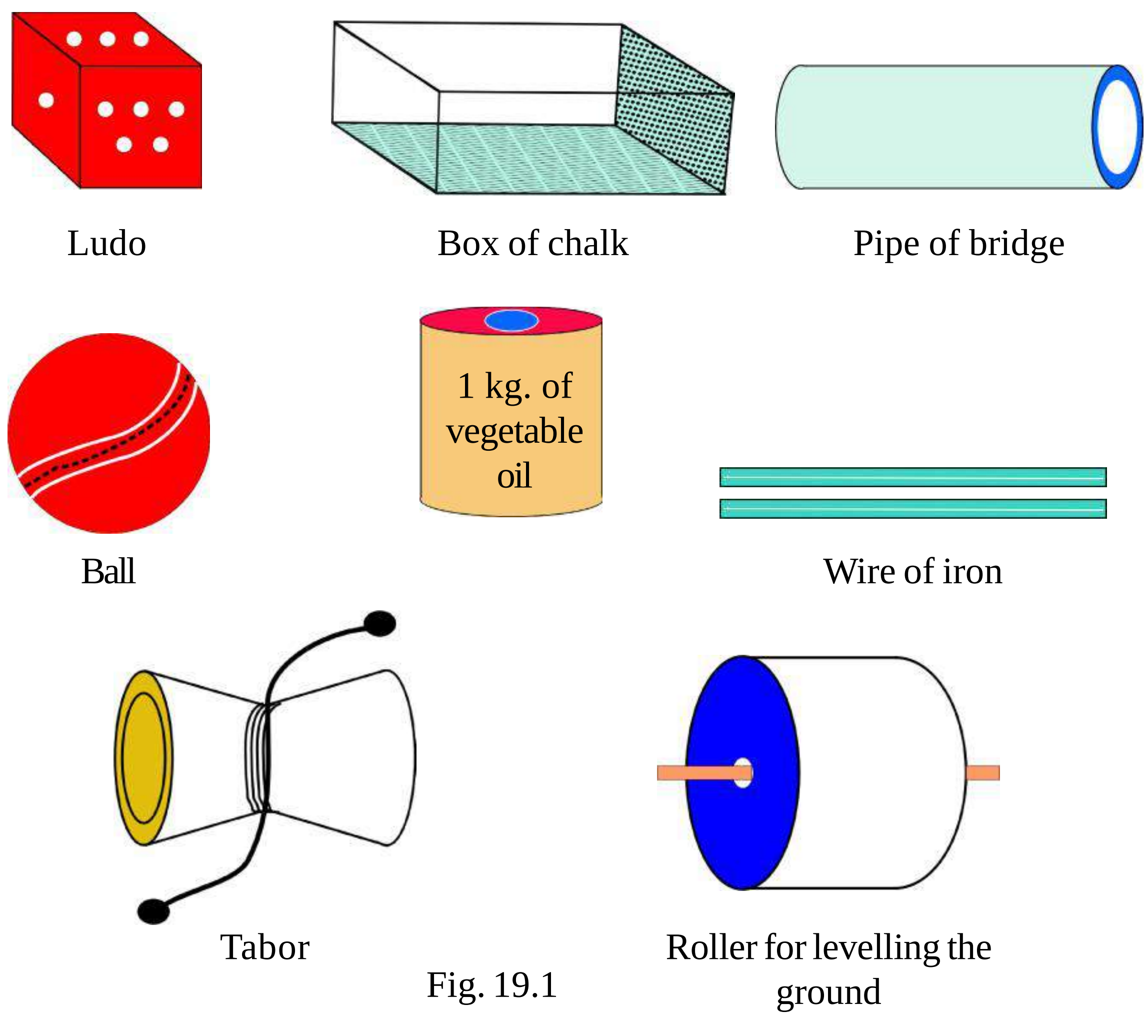


Fig. 19.1

On what basis we can identity the above figure? Discuss with your friends.

You will find two circular phases lying on the figure like pipe, roller etc, which are parallel and equal and the third surface is curved. These type of figures are called cylinder.

Let's discuss about a cylinder-

A cylinder is shown in the given figure. There are two circular ends in a cylinder which are parallel and equilibrium. These circular ends are called vertex and base. Left portion of cylinder i.e. both circular ends adjoin the cylindrical surface is called curved portion or curved surface of cylinder.

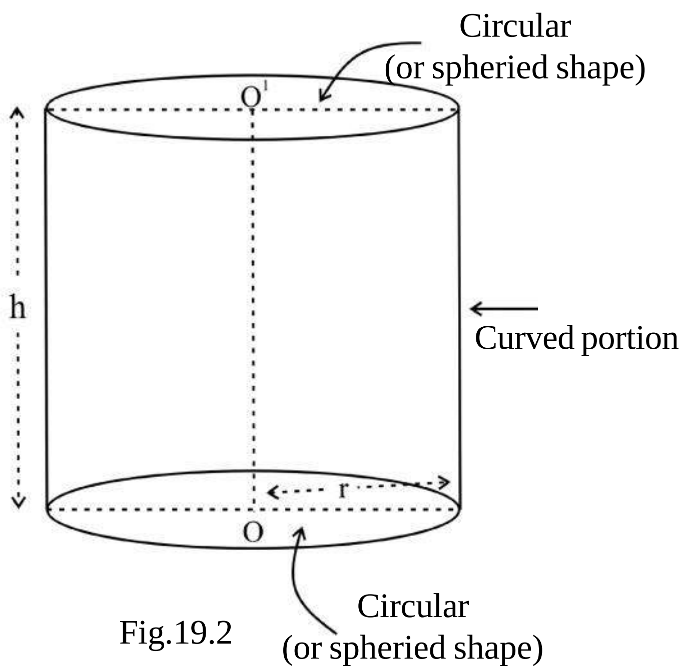


Fig.19.2

Base of cylinder or radius of upper base are radius of cylinder represented by 'r'.

The line segment which joins centre of base and vertex of cylinder, is altitude. This altitude is the height of cylinder which is represented by 'h'.

Volume of cylinder

In class 7th you have learnt to measur the volume of cuboid. Can you tell that how can we measure the volume of cuboid?

Monika: To find the volume of cuboids we have to multiply its length, breath and its height. i.e. ***volume of cuboid = length*** × ***breath*** × ***height***.

Sunil: But length × breath is equal to area of the base of the cuboids.

So, can we say that ***volume of cuboid = Area of base*** × ***height***

Can we find the volume of cylinder similarly? Discuss with your friends and teacher. You will find that this formula is also true for volume of cylinder. i.e., ***volume of cylinder = Area of base of cylinder*** × ***height***, let r is radius of base of cylinder. Since base of cylinder is circular so that the Area of base $= \pi r^2$

Now of the height of cylinder is h then volume of cylinder = Area of base × height = $\pi r^2 \times h$ = $\pi r^2 h$

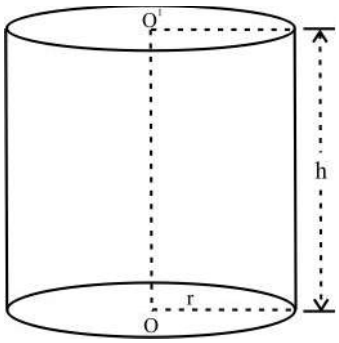
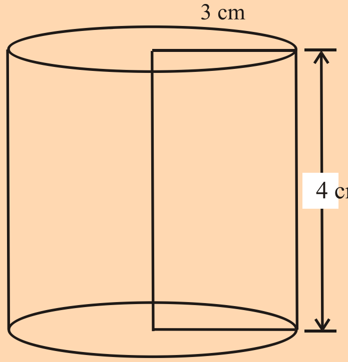
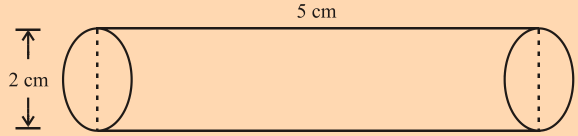


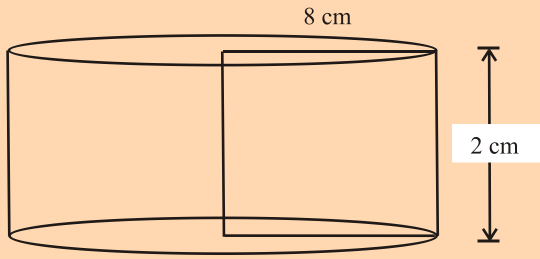
Fig. 19.3

Volume of cylinder (V)	= ®**r**^{2}×**h** cubic unit
V	= ®**r**^{2}**h** cubic unit

Activity 1.

Complete the following figure on the basis of given measures-

S.No.	Figure of cylinder	height or Length (h)	radius (r)	Volume (V)
1		----------	----------	----------
2		----------	----------	----------

3		----------	----------	----------

Example 1.

The diameter of base of cylinder is 14 cm and height in 15 cm. Find its volume?

Solution: According to question, diameter of base of cylinder = 14 cm.

$\Rightarrow$ Radius of base of cylinder (r) $= \frac{14}{2} = 7$ cm.

And height of cylinder (h) = 15 cm

$\therefore$ Volume of cylinder (V) $= \pi r^2 h$

$$= \frac{22}{7} \times (7)^2 \times 15$$

$$= \frac{22}{\cancel{7}_1} \times \overset{1}{\cancel{7}} \times 7 \times 15$$

$= 22 \times 7 \times 15$

$= 2310 \text{ cm}^3$ or cubic cm.

So, volume of cylinder is 2310 cm^3.

Example 2.

A circular well of radius 3.5 m is digged up to 20 m deep. Find out the volume of mud digged out from the well?

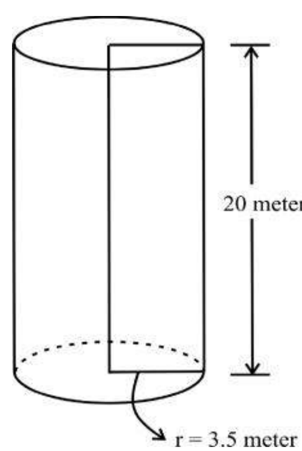


Fig. 19.4

Solution- According to question

Cylindrical well's radius (r) = 3.5 m

Well's height (h) = 20 m

Volume of mud digged out = volume of the well

$= \pi r^2 h$

$= \frac{22}{7} \times (3.5)^2 \times 20$

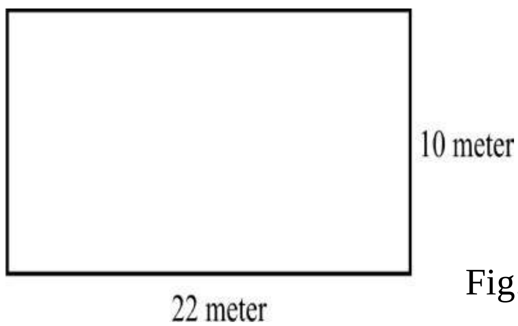
$= \frac{22}{\cancel{7}_1} \times \overset{0.5}{\cancel{3.5}} \times 3.5 \times 20$

$= 22 \times 0.5 \times 3.5 \times 20$

$= 770 \text{ cm}^3$

Example-3

22 m × 10 m rectangular sheet of iron rolled towards its length and formed a cylindrical pipe. Find the volume of pipe?



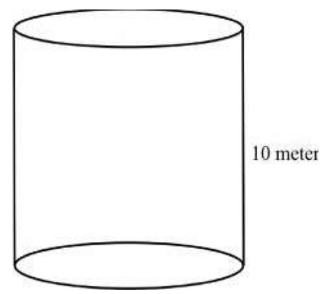


Fig. 19.5

Solution:

Since sheet of iron is rolled towards its length, so the height of pipe will be 10 m.

$\therefore$ height of cylindrical pipe h $= 10$ m

By folding the sheet the radius of the pipe formed be r metre then.

Length of rectangular sheet = circumference of the base of pipe.

$\Rightarrow \quad 22 = 2\pi r$

$\Rightarrow \quad 22 = 2 \times \frac{22}{7} \times r$

$\Rightarrow \quad \frac{22 \times 7}{2 \times 22} = r$

$\Rightarrow \quad r = 7/2$ meter

So the volume of pipe $= \pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 10$$

$$= \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} \times \cancel{10}^{5}$$

$$= 385 \text{ cubic meter}$$

Example 4.

Area of the base of a cylinder is 154 square cm. and height in 8 cm then find its volume?

Solution: According to the question,

Area of the base of cylinder $= 154 \text{ cm}^2$

height of cylinder $= 8$ cm

Volume of cylinder = Area of base × height

$= 154 \text{ cm}^2 \times 8 \text{ cm}$

$= 1232 \text{ cm}^3$

Exercise 19.1

Q-1 Fill in the blanks-

(i) The shape of the base of cylinder is ___________

(ii) Formula of the volume of cylinder is _______________

(iii) Radius and height of cylinder is 7 cm each. Volume of cylinder is _________

Q-2 Area of base of cylinder is 1386 cm^2. If its height in 13 cm then what will be the volume?

Q-3 With the given measurements find the volume of cylinder.

(i) Radius = 12 cm. height = 14 cm.

(ii) Radius = 2.8 cm, height = 5 cm.

(iii) Diameter = 20m, height = 21 m.

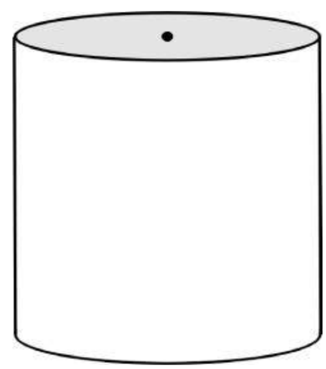
Q-4 If the diameter of a cylinder is half then find the ratio between the volume of new and the old one?

Q-5 A cylindrical tank is of radius 2.8 m and height in 3.5 m. Find the capacity of tank.

Q-6 How much volume of iron is required for making a solid iron rod with diameter 1.4 cm and length 90 cm.

Surface Area of Cylinder-

Take a closed cylindrical box of tin. Now tell, to find out the whole surface area of the box. Which of the areas we have to find?



Box of tin (Fig. 19.6)

There are three phase in a cylindrical box. Two of them are circular (upper base and lower base) and the third part in curved. Area of will be equal. If the radius of circular phase in r then are of each circular phase will be πr^2.

Now the question arises that how can we get the area of third phase or circular part.

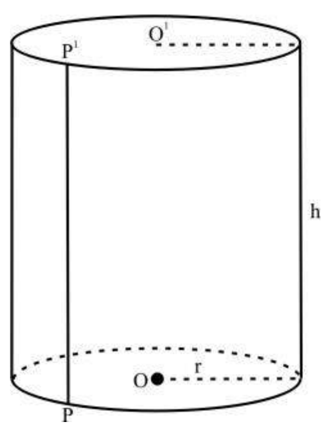


Fig. 19.7

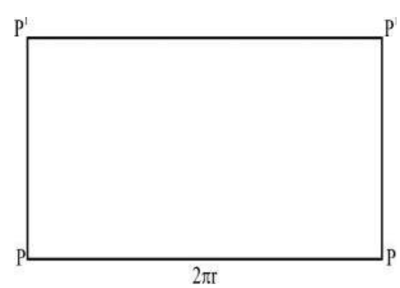


Fig. 19.8

To find the area of curved part take a line segment pp' in it (fig 15.7). Now cut and spread the curved part of box along with its length pp'. So we get a rectangular strip like fig 19.8. The length of resultant rectangular fig. will be equal to circumference of curved part and height will be equal to breath of curved part. Simultaneously area of rectangular part and area of curved part both will be equal.

Since the radius of curved part in r then fore its circumference in $= 2\pi r$

Now if the height of curved part (of box) in h then.

Area of curved part = Area of rectangular part
= (length × breath)
= Circumference of curved part × height
$= 2\pi r \times h = 2\pi rh$

Area of curved surface of cylinder = 2®rh

So whole surface area of cylindrical box
= Area of curved part + Area of base + Area of top
$= 2\pi rh + \pi r^2 + \pi r^2$
$= 2\pi rh + 2\pi r^2$
$= 2\pi r(r+h)$

Whole surface area of cylinder = 2®r (r+h)

Example 5.

Radius in 7cm. and height in 15 cm of a closed cylindrical tin box. Find the area of the in sheet used for making that tin box.

Solution- According to question

Radius of cylindrical box = 7cm.

and height h = 15 cm.

\ Area of used sheet = whole surface area of box
= whole surface area of cylinder.
$= 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 7 \times (15 + 7)$$

$$= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{7}^1 \times 22$$

$= 968 \text{ cm}^2$

Example-6

A solid cylindrical base whose radius is 5 cm and height is 21 cm. Find the curved surface and whole surface area.

Solution- According to question

Radius of cylindrical box = 5cm.

and height h = 21 cm.

Surface area of cylinder = $2\pi rh$

$$= 2 \times \frac{22}{\cancel{7}_1} \times 5 \times \cancel{21}^3$$

$= 2 \times 22 \times 5 \times 3$

$= 660 \text{ cm}^2$

And whole surface area of cylinder = $2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 5 \times (21+5)$$

$$= 2 \times \frac{22}{7} \times 5 \times 26$$

$= 817.14 \text{ cm}^2$

Example 7.

Volume of a cylinder is 36π cm^3 and area of the base of cube in 9π cm^2 find the whole surface area of cylinder.

Solution- Let the radius of base of a cylinder be **r** cm and height is h cm.

So the area of base of cylinder = πr^2

$9\pi = \pi r^2$

$9 = r^2$

$r = \sqrt{9}$

$r = 3$ cm

and volume of cylinder = $\pi r^2 h$

$36\pi = \pi(3)^2 \times h$

$36 = 9h$

$36/9 = h$

$h = 4$ cm

Therefore whole surface area of cylinder $= 2\pi r\,(r + h)$

$= 2 \times \pi \times 3(3 + 4)$

$= 6\pi \times 7$

$= 42\pi \text{ cm}^2$

Example 8.

A cylindrical pipe whose diameter is 14 cm. and height is 20 cm. Find the cost to paint curved surface @ Rs. 2 per 100 cm^2.

Solution-

According to question

Diameter of pipe $= 14$ Cm.

$\Rightarrow$ Radius r $= \frac{14}{2} = 7$ cm.

and height of pipe (h) $= 20$ cm.

$\therefore$ Curved surface area of pipe $= 2\pi rh$

$$= 2 \times \frac{22}{\not{7}} \times \not{7} \times 20 = 880 \text{ cm}^2$$

It is given that the cost of painting 100 cm^2 = Rs. 2

$\therefore$ Total cost of painting the pipe $= \frac{88\not{0} \times 2}{10\not{0}}$ = Rs. 17.60

Example-9

Circumference of base of a cylinder is 132 cm and its height in 2 m. Find the area of curved surface.

Solution: According to question

Circumference of base = 132 cm

Height of cylinder (h) = 2 m = 200 cm.

Area of curved surface = ?

Curved surface area of cylinder =

= Circumference of base × height

$= 132 \times 200 = 26400 \text{ cm}^2$

$\therefore$ Curved surface Area of cylinder = 26400 cm^2.

Exercise 19.2

Q-1 Find the curved surface and whole surface area of a cylinder whose measures are given below-

(i) Radius = 7 cm, height = 24 cm

(ii) Diameter = 20 m, height = 21 m.

(iii) Radius = 10.5 cm, height = 35 cm

(iv) Radius = 14 cm, height = 1 m

Q-2 Circumference of base of a cylindrical tank in 176 cm and height in 30 cm. Then find its curved surface area.

Q-3 Volume of a cylinder is 44 Cubic cm and radius is 2 cm. Find its whole surface area.

Q-4 A well whose diameter is 14m and 25m deep. Find the volume of mud taken out of the well. Also find the cost of plastering the well from inside at the rate of Rs. 3 per square metre.

Q-5 Circumference of the base of a cylinder and height is 44m. Find its curved surface.

Q-6 Area of curved part of a cylinder is 1000 π square cm and its diameter is 20 cm find the height of the given cylinder?

We have learnt

1. Volume of cylinder = Area of base × height

   $= \pi r^2 h$

2. Curved surface of a cylinder = Circumference of base × height

   $= 2\pi rh$

3. Whole surface of a cylinder = 2 × Area of base + curved surface

   $= 2\pi r^2 + 2\pi rh$

   $= 2\pi r(r + h)$

4. Area of a unit is always in square unit such as square centimeter, square meter etc and the volume of a unit is always in cubic unit such as cubic centimeter, cubic meter etc.

5. There are three phases in a solid cylinder. Two of them are circular (upper base and lower base) and the third one in curved phase.

ANSWERSHEET

Answersheet 1.1

1. (i) Not a perfect square (ii) perfect square (iii) perfect square (iv) perfect square

   (v) Not a perfect square

Answersheet 1.2

1. (i) Perfect cube (ii) not a perfect cube (iii) Perfect cube (iv) Perfect cube

   (v) not a perfect cube (vi) not a perfect cube

2. 2 3. 13 4. 11 5.(ii) 10 (v) 484 (vi) 169

Answersheet 1.3

1. (i) 19 (ii) 20 (iii) 28 (iv) 32 (v) 48 (vi) 84, 2. 16

Answersheet 1.4

1. (i) 23 (ii) 37 (iii) 32 (vi) 76 (v) 30 (vi) 89 (vii) 225 (viii) 603

2. 43 row or column 3. 38m lenght or width

Answersheet 1.5

1. (i) 2.7 (ii) 4.1 (iii) 3.05

2. (i) 95 (ii) 2.24 (iii) 2.64

Answersheet 1.6

(i) 5 (ii) 7 (iii) 11 (vi) 13 (v) 21 (vi) 55 (vii) 17 (viii) 35

Answersheet 2.1

1. (a) -125 (b) -1024 (c) 64 (d) 729

2. (a) 5^6 (b) $-(15)^{26}$ (c) $-(12)^2$ (d) $-(p)^7$

Answersheet 2.2

1. (a) $\frac{1}{343}$ (b) $\frac{16}{25}$ (c) -3125 (d) $\frac{16}{9}$

2. (a) $-\left(\frac{5}{7}\right)^2$ (b) $\left(\frac{3}{5}\right)^3$ (c) $\left(\frac{3}{2}\right)^6$

4. (a) True (b) True (c) False (d) False

Answersheet 3.2

1. AE = 2 cm. 2. 2.5 cm. 3. 1.2 cm.

Answersheet 4.1

1. (i) $6x^2 + 25x + 14$ (ii) $6x^2 + 17x - 45$ (iii) $105x^2 - 104x + 12$

   (iv) $\frac{3}{2}x^2 + \frac{72}{5}xy - 6y^2$ (v) $7x^2 + 34xy - 5y^2$

2. (i) $9x^2 + 10xy + 3y^2$ (ii) $3x^2 - \frac{5}{4}x + \frac{1}{8}$

   (iii) $3x^3 - 5x^2y + 3xy^2 - 5y^3$ (iv) $a^2 + 2ab + b^2$

Answersheet 4.2

1. (i) $-3xy$ (ii) $3xy$ (iii) $+xy^2$ (iv) $-4a^3b$ (v) $-4a^2b^2c^2$

2. (i) $x^2 - 3x + 2$ (ii) $-a^2b+2a+3$

   (iii) $-3a^3+4a$ (iv) $\frac{1}{4}x^2 + \frac{1}{2}x - \frac{1}{2}$

   (v) $a+b+c$

Answersheet 4.3

1. (i) $8x^4-4x^3+15x^2-3x-15$ (ii) $12m^5-9m^3-6m^2+8m+16$

   (iii) $9m^4-m^3-16m^2-4m+16$ (iv) $12y^4-8y^3-6y^2+4$

2. (i) to (v) the remainder is zero. So, the divisor is a factor of dividend.

3. (i) Quotient = $x^2 + 3x + 6$, remainder = 10

   (ii) Quotient = $x^2 - 2x + 6$, remainder = -42

   (iii) Quotient = $2x^3 - 4x^2 + x + 5$, remainder = -21

   (iv) Quotient = $2x^2 - 2x + 3$, remainder = 12

4. (i) Quotient = $m - 1$, remainder = 5

   (ii) Quotient = $a^2 - 4a + 9$, remainder = -16

   (iii) Quotient = $3x^2 + 4x - 3$, remainder = +6

   (iv) Quotient = $2x + 3$, remainder = $-x + 3$

Answersheet 5.1

1. (i) major arc (ii) minor arc (iii) $\angle ADB, \angle ACB$ (iv) minor arc AB

   (v) Arc CDA (vi) minor CD (vii) $\angle ADB, \angle ADC, \angle BDC$

Answersheet 5.2

(i) $\angle A = 45^\circ$ (ii) $x = y = 33^\circ$ (iii) $p = 40^\circ$

(iv) $m = 72^\circ, n = 57^\circ$ (v) $u = 93^\circ, v = 87^\circ$

Answersheet 5.3

1. Acute angle 2. obtuse angle 3. 180°

Answersheet 5.4

1. $\angle PRQ = 90^\circ, \angle QPR = 50^\circ$
2. $\angle PYQ = \angle PXQ = 80^\circ$
3. $\angle P = 90^\circ, \angle Q = \angle R = 45^\circ$
4. $\angle BDC = 130^\circ$
5. $\angle PTR = \angle PSR = 50^\circ$

Answersheet 5.5

1. (a) $x = 130^\circ$ (b) $x = 124^\circ$ (c) $x = y = 37^\circ$
2. (i) $x = y = 52^\circ$ (ii) $x = 150^\circ, y = 75^\circ$
3. $\angle COD = 70^\circ$
4. PQ = 3.2 cm
5. (i) $x = 90^\circ, y = 115^\circ$ (ii) $x = 115^\circ, y = 65^\circ$

Answersheet 5.6

1. BM = 3.5 cm., AB = 7 cm.
2. x = 90°, y = 50°
3. PM = MQ = 4 cm.
4. (1) Bisects (2) perpendicular

Answersheet 6.1

1. 72
2. 60
3. 8
4. 30 kg.
5. 67
6. 5
7. 3
8. 5
9. $x = 8$

Answersheet 6.3

1. $\frac{1}{4}$
2. $\frac{1}{4}$
3. $\frac{1}{5}$
4. $\frac{1}{3}$
5. $\frac{1}{2}$
6. $\frac{1}{2}, \frac{1}{2}$

Answersheet 7.1

1. (i) Direct variation $\frac{1}{3}$ (ii) Direct variation $\frac{1}{1}$

(iii) It's not a direct variation. (iv) It's not a direct variation.

2. 100, 3, 250, 6 (3) $\frac{1}{7}$ (4) 1000 Km

5. (i), (ii), (iii), (iv), (v)

6. 60 ticket (7) 180 Km (8) Rs. 216

(9) 50 minute (10) 5 hours (11) Rs. 2000

(12) 29 days (13) 252 metre (14) Rs. 7.30

(15) 1.925 cm (16) $4\frac{4}{15}$ minute or 256 seconds (17) 18 words per minute

(18) 34 labours (19) 15.75 quintal

(20) (i) (b), (ii) (a), (iii) (d), (iv) (c)

Answersheet 7.2

1.

x	8	6	4	72	36
y	9	12	18	10	2

2.

Speed (in Km/hr)	4	8	16	32	64
Time taken(in minute)	80	40	20	10	5

3. 10 day 4. 81 day 5. 45 Km/hr

6. 10 horses 7. 45 day 8. 175 soldier

9. 10 day 10. 15 day

11. (iii), (iv), (v) 12. 80 gm 13. 800 litre

Answersheet 8.1

1. (i) $1, 5, t, 5t, t^2, 5t^2$ (ii) $1, 7, x, y, 7x, 7y, 7xy$

(iii) $1, 2, 7, 14, \ell, \ell^2, m, 2\ell, 2\ell^2, 2m, 7\ell, 7\ell^2, 7m, 14\ell, 14\ell^2, 14m, \ell m,$
$2\ell m, 7\ell m, 14\ell m, \ell^2 m, 2\ell^2 m, 7\ell^2 m, 14\ell^2 m$

(iv) $1, 3, 13, 39, \ell, 3\ell, 13\ell, 39\ell, m, 3m, 13m, 39m, n, 3n, 13n, 39n, \ell m,$
$3\ell m, 13\ell m, 39\ell m, \ell n, 3\ell n, 13\ell n, 39\ell n, mn, 3mn, 13mn, 39mn, \ell mn,$
$3\ell mn, 13\ell mn, 39\ell mn$

2. (i) H.C.F. = 5 (ii) H.C.F. =3 (iii) H.C.F. = 2a (iv) H.C.F. = m

3. (i) $6m\ell$ (ii) $4bc$ (iii) xy (iv) $7x$

(v) $11pq^2r$ (vi) x (vii) 1

Answersheet 8.2

1. (i) x^2 (ii) $5a^2$ (iii) $2c$ (iv) $16x, 9z$

(v) $6b^2, 4bc, 5a.$

2. (i) $2a(2x + 3ay)$ (ii) $a(a^4y + b^3)$ (iii) $q^2(pr - 2t)$

(iv) $-5\ell m(m + 2\ell n)$ (v) $5(m + n)(m - n)$

3. (i) $2(x + 3y)(xy+2)$ (ii) $(m - 2n)(5mn + 12)$

(iii) $(3x + 4)(2x^2 + 3y)$ (iv) $(5x^2 + 4y)(3x^2 + 2y^2)$

(v) $(x + 3)(x + 8)$ (vi) $(x - 4)(3x - 5)$

(vii) $(\ell - m)(2m + 3)$

4. (i) $x - 3y^2x$ (ii) $- 51x^3 + 153x^2$ (iii) $6a^3 - 8a^4$

(iv) $9m^2 - 9mn$ (v) $9t^3 - 63t^5$

Answersheet 9.1

(1) (i) $4a^2 + 12a +9$ (ii) $\frac{4}{25}m^2 + \frac{3}{5}m + \frac{9}{16}$

(2) (i) $x^2 - 10x + 25$ (ii) $\frac{9}{4}x^2 - \frac{12}{5}xy + \frac{16}{25}y^2$

(iii) $4a^2$ J $2a + \frac{1}{4}$ (iv) x^4 J $2x^2y^2 + y^4$

(3) (i) $16x^2$ J 25 (ii) $\frac{x^2}{4} - \frac{y^2}{9}$

(iii) b^4 J a^4 (iv) x^6 J y^6

(4) (i) $4a^2 + 20a + 25$ (ii) $\frac{4}{9}m^4 + \frac{10}{9}m^2n^2 + \frac{25}{36}n^4$

(iii) $64x^6$ J $80x^3y^3 + 25y^6$

(5) (i) 1681 (ii) 4761 (iii) 9409 (iv) 7056

(6) (i) 9975 (ii) 8096 (iii) 249991

(7) (i) 50 (ii) 31 (iii) –13 (iv) 13

(8) $9x^2$ J $30xy + 25y^2$

(9) $\frac{x^2}{9} + \frac{xy}{6} + \frac{y^2}{16}$

Answersheet 9.2

1. (i) $(2x + 5)(2x + 5)$ (ii) $(5a + 7b)(5a + 7b)$

(iii) $(3x + 1)(3x + 1)$ (iv) $(1 + 9a)(1 + 9a)$

(v) $(p + \frac{1}{2})(p + \frac{1}{2})$

(vi) $(6a + 11b)(6a + 11b)$

2. (i) $(a - 5b)(a - 5b)$ (ii) $(4x - 13)(4x - 13)$

(iii) $(11x - 4y)(11x - 4y)$ (iv) $(x - 15)(x - 15)$

(v) $(6a - 1)(6a - 1)$

3. (i) $(5a + 7b)(5a - 7b)$ (ii) $(3x + 11y)(3x - 11y)$

(iii) $(8a + 1)(8a - 1)$ (iv) $(1 + 4b)(1 - 4b)$

(v) $(\frac{4}{5}m + \frac{2}{3}n)(\frac{4}{5}m - \frac{2}{3}n)$

4. (i) $(x + 4y + 7)(x + 4y - 7)$ (ii) $(10 + 2a + 3b)(10 - 2a - 3b)$

(iii) $(2x + 5y + 6)(2x + 5y - 6)$ (iv) $(7x - 5y)(5y - x)$

(v) $(xy + 4)(xy - 4)$

5. (i) $(x - 5y)$ (ii) $3, x$ (iii) $7, 7$ (iv) $(a + b - 2)$

Answersheet 10.1

1. (a)- (iii) Rectangle (b)- (iii) Rectangle (c)- (ii) 120° (d)- (ii) 28 cm

2. (i) Square (ii) trapezium (iii) 6 (iv) (n–2)

4. 1080° 5. 120°, 60°, 60° 6. 80°, 80°, 100°, 100°

7. 18 cm, 24 cm

Answersheet 10.2

1. 50º 2. 95º 3. 72º, 108º, 72º, 108º

4. 60º, 90º, 120º 5. 36º, 72º, 108º, 144º

6. 80º, 100º, 80º, 100º

Answersheet 12.1

(1)	$x = 3$	(2)	$x = 1$	(3)	m = 2
(4)	$x = \frac{3}{2}$	(5)	y = 4	(6)	y = 2
(7)	k = –2	(8)	$x = 3$	(9)	$x = -0.5$

Answersheet 12.2

(1)	14, 28	(2)	10, 30	(3)	5 cm, 2 cm
(4)	62	(5)	34	(6)	$\frac{5}{7}$
(7)	14 years, 10 years	(8)	36 years, 12 years		

Answersheet 13.1

1.	(i)	Rs. 410	(ii)	Rs. 1986	(iii)	Rs. 1040
2.	(i)	Rs. 8427	(ii)	Rs. 2916	(iii)	Rs. 6480
3.	Rs. 5948.80		4.	Rs. 9261		

Answersheet 13.2

1. Rs. 81   2. Rs. 27783   3. Rs. 12   4. Rs. 405

Answersheet 13.3

1. (i) Rs. 7986, Rs. 1986   (ii) Rs. 1764, Rs. 164

   (iii) Rs. 11241.25, Rs. 2741.25   (iv) Rs. 23152.50, Rs. 3152.50

2. Rs. 729   3. Rs. 5000   4. 15%   5. 10%

6. Rs. 46.08

Answersheet 13.4

1.	Rs. 1820	2.	Rs. 1200	3.	Rs. 1444	4.	Rs. 340
5.	Rs. 12000	6.	Rs. 937.50	7.	Rs. 5400	8.	Rs. 13200

Answersheet 14.1

1. (i) 150 cm^2 (ii) 720 cm^2 (iii) 1800 cm^2

2. 185.5 cm^2 3. 15 cm^2 4. 20 m

Answersheet 14.2

(1) 42 cm^2 (2) 18.75 cm^2 (3) 8 cm (4) 4500 dm^2

Answersheet 14.3

(1) 120 cm^2 (2) 30 cm^2 (3) 168 cm^2 (4) 40.8 cm^2 (5) 12 cm, 18 cm

Answersheet 14.4

(1) 156 cm^2 (2) 149 cm^2 (3) 530 cm^2, Rs. 2782.50

Answersheet 14.5

(1) 198 cm^2 (2) 188.57 cm^2 (3) 2794 cm^2

Answersheet 15.1

1. (i) 6 (ii) height (iii) 10^6
2. 6000 litre 3. 280 cm^3 4. 1200 cm^3
5. (i) 150 cm^3 (ii) 22.5 cm^3 (iii) 64 m^3
   (iv) 22.5 m^3 (v) 35 cm^3 (vi) 120 cm^3
   (vii) 168 cm^3 (viii) 37500 cm^3
6. 8 cm 7. 125 pic 8. 67.5 m^3 9. 1600 litre
10. 1 m 11. 1.728 cm^3
12. 432 m^3 (capacity) 288 m^3
13. 60 m^3 , 40000 litre 14. 25000 bricks

Answersheet 15.2

1. (i) 54 (ii) equal (iii) cube 2. 102.5 cm^2 3. 846 $foot^2$ 4. 4750 cm^2 5. 69.36 cm^2

6. 6 cm 7. 79.50 cm^2 8. Rs. 188 9. 4:1

Answersheet 16

(5) 6

Answersheet 17.1

1. (ii) $\frac{12}{13}$, (iv) $\frac{6}{7}$

3. (i) 1, $1\frac{5}{9}$ (ii) 7, $7\frac{5}{12}$ (iii) 6, $6\frac{11}{18}$ (iv) 4, $4\frac{23}{61}$

4. (i) $\frac{2}{5}, \frac{6}{9}, \frac{8}{9}, \frac{4}{3}$ (ii) $\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{7}{9}, \frac{8}{9}$

Answersheet 17.2

2. (i) 8 (ii) - 12 (iii) 20 (iv) - 39 (v) 59

3. (i) $\frac{5}{9}$ (ii) 0 (iii) $-\frac{5}{3}$ (iv) $\frac{-13}{-8}$

4. 2 5. 1 6. $\frac{5}{3}$

Answersheet 17.3

1. 10, two digit numbers 30, 50, 35, 53, three digit numbers 305, 350, 503, 530

2. (A) 111, 222, 333, ..., ... etc.

   (B) 111111111, 222222222, ... etc.

3. (a) 26, 37, 65, 101

   (b) 13, 34, 89

   (c) 99, 90, 80, 57 (d) 8, 6

4. $0^1 + 6^2 + 3^3 = 0 + 36 + 27 = 63$ etc.

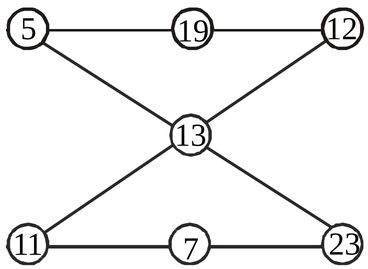
5. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \times 9 = 100$

6. A = 25, B = 161, C = 25, D = 12, E = 95, F = 26,

G = 25, H = 23, I = 167, J = 561

10. (i) 2 + 2 – 2 + 2/2 = 3 (ii) 22 ÷ 2 – 2 – 2 – 2 = 7

11.



Answersheet 17.4

1. (i) yes (ii) no (iii) yes (iv) no (v) no
2. (i) divisible (ii) not divisible (iii) divisible
   (iv) not divisible (v) divisible
3. 1–(ii) 815, (iii) 6570
4. (i) 560 (ii) 791 (iv) 7007

Answersheet 18.1

1. (i) $2\frac{9}{34}$ (ii) $-1\frac{19}{36}$ (iii) $\frac{7}{20}$
2. (i) $\frac{-5}{9}$ (ii) $\frac{6}{31}+\frac{-11}{29}$ (iii) $\frac{13}{9}, \frac{-15}{7}$ (iv) $\frac{5}{6}$
3. Asociative law
4. (i) $\frac{227}{693}$ (ii) $-1\frac{1}{6}$ (iii) $-1\frac{6}{14}$
5. $\frac{7}{12}$
6. (i) $\frac{5}{7}$ (ii) 0 (iii) $\frac{39}{51}$ (iv) $\frac{-42}{17}$

8. (1) Communtative law (ii) Asociative law
   (iii) Additive identity (iv) Additive inverse

Answersheet 18.2

1. (i) $-\frac{1}{20}$ (ii) $\frac{3}{8}$ (iii) $-\frac{23}{24}$ (4) $-\frac{1}{13}$

2. (i) 0 (ii) $\frac{19}{70}$ (iii) $-5\frac{29}{60}$

3. $\frac{13}{24}$ 4. $\frac{-6}{25}$ 5. (i) false (ii) false (iii) ture (iv) ture

Answersheet 18.3

3. (i) $\frac{2}{3}$ (ii) $\frac{1}{2}, \frac{11}{19}$ (iii) $\frac{-3}{17}, \frac{7}{9}$

4. (i) $\frac{2}{9}$ (ii) $\frac{-1}{8} \times \frac{-2}{5}$ (iii) $\left(\frac{4}{7} \times \frac{-25}{3}\right) \times \frac{1}{5}$

5. (i) Distributive law (ii) Distributive law
(iii) Communtative (iv) Multiplicative inverse
(v) Multiplicative indentity

6. (i) $\frac{1}{4}$ (ii) $-\frac{5}{17}$ (iii) $-\frac{29}{6}$ (4) $\frac{q}{p}$

7. (i) ture (ii) false (iii) false (iv) ture

Answersheet 18.4

1. (i) $\frac{2}{9}$ (ii) $\frac{-72}{55}$ (iii) $\frac{-63}{4}$ (iv) 3 (v) $-\frac{7}{4}$ (vi) $+\frac{2}{3}$

2. (i) $-\frac{4}{5}$ (ii) $\frac{1805}{128}$ (iii) $\frac{105}{16}$ (iv) -3 (v) $\frac{2}{35}$ (vi) $\frac{7}{5}$

3. 20 4. $\frac{55}{9}$ 5. $-\frac{13}{4}$

Answersheet 18.5

1. (i) $\frac{3}{2}$ (ii) $\frac{3}{7}$ (iii) $\frac{-2}{9}, \frac{-2}{9}, \frac{2}{3}$

3. $\frac{17}{48}, \frac{3}{8}, \frac{5}{12}, \frac{11}{24}, \frac{23}{48}$

4. $\frac{-35}{168}, \frac{-11}{84}, \frac{1}{42}, \frac{5}{28}, \frac{19}{56}$

5. $-\frac{5}{96}, \frac{1}{16}, \frac{7}{24}, \frac{25}{48}, \frac{61}{96}$

6. (i) ture (ii) false (iii) false

Answersheet 19.1

1. (i) Circular (ii) $\pi r^2 h$ (iii) 1078 cm^3

2. 20790 cm^3

3. (i) 6336 cm^3 (ii) 123.20 cm^3 (iii) 6600 m^3

4. 4 : 1

5. 86.24 m^3 6. 138.6 cm^3

Answersheet 19.2

1. (i) Curved surface = 1056 lseh^2, whole surface = 1364 cm^2

   (ii) Curved surface = 1320 m^2, whole surface = 1948.57 m^2

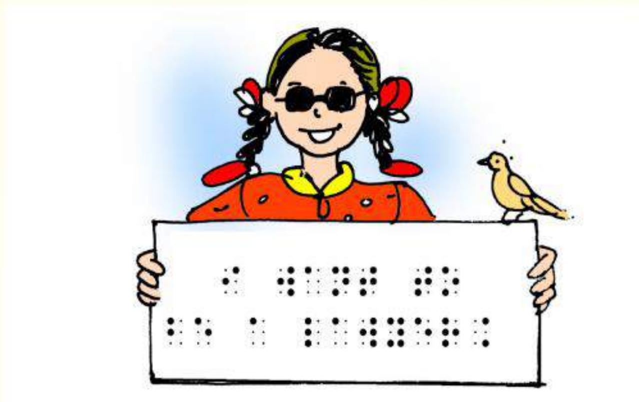
   (iii) Curved surface = 2310 cm^2, whole surface = 3003 cm^2

   (iv) Curved surface = 8800 cm^2, whole surface = 10032 cm^2

2. 5280 cm^2 3. 69.14 cm^2 4. 3850 m^3, Rs. 3300

5. 264 m^2 6. 50 cm

Braille An Introduction



Do you know what is written here?

It is: I want to be a lawyer.

Like devnaagri and Gurumukhi etc. Braille is also a script. Braille script is used by Blind persons to read and write. Braille was invented by Louis Braille in 1829. Braille script is based on six dots. These six dots are referred as the Braille cell. Each cell comprises of one Braille character. To write Braille script Blind person uses Stylus and Braille slate. Braille slate consist essentially of two metal or plastic plates hinged together to permit a sheet of paper to be inserted between the two plates. While writing on a Braille sheet (drawing sheet) it is to be written from right to left and then reverse the normal numbering of the Braille cell. Blind person reads these raised (embossed) dots with the help of their finger tip.

Braille cell

Total 63 combinations are possible using these 6 dots.
Some combinatios given below:

Braille Chart