JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

2 April 2025 Shift – 2

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. If the image of the point P(1, 0, 3) in the line joining the points A(4, 7, 1) and B(3, 5, 3) is Q(α, β, γ), then α + β + γ is equal to

(1) $\frac{47}{3}$ (2) $\frac{46}{3}$

(3) 18 (4) 13

Ans. **(2)**

Sol. P(1, 0, 3)

A(4, 7, 1), B(3, 5, 3)

Line AB $\Rightarrow \frac{x-3}{1}=\frac{y-5}{2}=\frac{z-3}{-2}=\lambda$

Let foot of perpendicular of P on AB be

$R \equiv (\lambda + 3, 2\lambda + 5, -2\lambda + 3)$

$\Rightarrow (\lambda + 3 - 1)(1) + (2\lambda + 5 - 0)(2) + (-2\lambda + 3 - 3)(-2) = 0$

$\Rightarrow \lambda + 2 + 4\lambda + 10 + 4\lambda = 0$

$\Rightarrow \lambda = -\frac{4}{3}$

$\Rightarrow R \equiv \left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

$Q \equiv \left(\frac{10}{3}-1,\frac{14}{3}-0,\frac{34}{3}-3\right) \equiv \left(\frac{7}{3},\frac{14}{3},\frac{25}{3}\right)$

$\Rightarrow \alpha+\beta+\gamma = \frac{7+14+25}{3}=\frac{46}{3}$

2. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function, If $10\int_1^x f(t)dt = 5xf(x) - x^5 - 9$ for all $x \ge 1$, then the value of $f(3)$ is :

(1) 18 (2) 32

(3) 22 (4) 26

Ans. **(2)**

Sol. $10\frac{d}{dx}\int_1^x f(t)dt = \frac{d}{dx}\left(5xf(x)-x^5-9\right)$

$\Rightarrow 10f(x) = 5f(x) + 5x f'(x) - 5x^4$

$\Rightarrow f(x) + x^4 = x f'(x)$

$\Rightarrow y + x^4 = x\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}+y\left(-\frac{1}{x}\right)=x^3$

$\Rightarrow ye^{\int -\frac{1}{x}dx} = \int x^3 e^{\int -\frac{1}{x}dx} + c$

$\Rightarrow ye^{-\ell n|x|} = \int x^3 e^{-\ell n|x|} + c$

$\Rightarrow \frac{y}{|x|} = \int \frac{x^3}{|x|} + c$

$\Rightarrow \frac{y}{x} = \int x^2 + c$

$\Rightarrow \frac{y}{x} = \frac{x^3}{3} + c$

Put x = 1 in given equation

$\Rightarrow 0 = 5f(1) - 1 - 9 \Rightarrow f(1) = 2$

$\Rightarrow \frac{2}{1}=\frac{1}{3}+c \Rightarrow c=\frac{5}{3}$

$\Rightarrow \frac{f(3)}{3}=\frac{27}{3}+\frac{5}{3}$

$\Rightarrow f(3) = 32$

3. The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is :

(1) 4 (2) 10

(3) 6 (4) 8

Ans. **(1)**

Sol. $a_2 + a_4 + ... + a_n = 30$ ....(1)

$a_1 + a_3 + ... + a_{n-1} = 24$ ....(2)

(1) – (2)

$(a_2 - a_1) + (a_4 - a_3) ... (a_n - a_{n-1}) = 6$

$\Rightarrow \frac{n}{2}d = 6 \Rightarrow nd = 12$

$a_n - a_1 = (n-1)d = \frac{21}{2}$

$\Rightarrow nd - d = \frac{21}{2} \Rightarrow 12 - \frac{21}{2} = d$

$\Rightarrow d = \frac{3}{2}$, $n = 8$

Sum of odd terms $= \frac{4}{2}\left[2a + (4-1)3\right] = 24$

$\Rightarrow a = \frac{3}{2}$

A.P. $\Rightarrow \frac{3}{2}, 3, \frac{9}{2}, 6, \frac{15}{2}, 9, \frac{21}{2}, 12$

no. of integer terms = 4

4. Let A = {1, 2, 3,...., 10} and R be a relation on A such that R = {(a, b) : a = 2b + 1}. Let (a_1, a_2), (a_2, a_3), (a_3, a_4), ..., (a_k, a_{k+1}) be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k, for which such a sequence exists, is equal to :

(1) 6 (2) 7

(3) 5 (4) 8

Ans. (3)

Sol. $a = 2b + 1$

$2b = a - 1$

$R = \{(3, 1), (5, 2), ..., (99, 49)\}$

Let $(2m + 1, m)$, $(2\lambda - 1, \lambda)$ are such ordered pairs.

According to the condition

$m = 2\lambda - 1 \Rightarrow m =$ odd number

$\Rightarrow 1^{st}$ element of ordered pair (a, b)

$a = 2(2\lambda - 1) + 1 = 4\lambda - 1$

Hence $a \in \{3, 7, ..., 99\}$

$\Rightarrow \lambda \in \{1, 2, ..., 25\}$

$\Rightarrow$ set of sequence

$$\left\{(4\lambda-1, 2\lambda-1), (2\lambda-1, \lambda-1), \left(\lambda-1, \frac{\lambda-2}{2}\right), ...\right\}$$

2^{nd} element of each ordered pair $= \frac{\lambda - 2^{r-2}}{2^{r-2}}$

For maximum number of ordered pairs in such sequence

$\frac{\lambda - 2^{r-2}}{2^{r-2}} = 1$ or 2 ; $1 \le \lambda \le 25$

$\lambda = 2^{r-1}$ or $\lambda = 3.2^{r-2}$

Case-I : $\lambda = 2r - 1$

$\lambda = 2, 2^2, 2^3, 2^4$

$r = 2, 3, 4, 5$

Hence maximum value of r is 5 when $\lambda = 16$

Case-II : $\lambda = 3.2^{r-2}$

$\lambda = 3, 6, 12, 24$

$r = 2, 3, 4, 5$

Hence maximum value of r is 5 when $\lambda = 24$

5. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{4}{\sqrt{17}}$ (2) $\frac{\sqrt{3}}{16}$

(3) $\frac{3}{\sqrt{19}}$ (4) $\frac{\sqrt{5}}{7}$

Ans. (1)

Sol. $2b = \frac{1}{4}(2ae)$

$\frac{b}{a} = \frac{e}{4}$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{e^2}{16}}$

$e^2\left(1 + \frac{1}{16}\right) = 1$

$e = \frac{4}{\sqrt{17}}$

6. The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point (7, 6, 2) and the line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point (5, 3, 4). The shortest distance between the lines L_1 and L_2 is :

(1) $\frac{23}{\sqrt{38}}$ (2) $\frac{21}{\sqrt{57}}$

(3) $\frac{23}{\sqrt{57}}$ (4) $\frac{21}{\sqrt{38}}$

Ans. (1)

Sol. $L_1 : \left(7\hat{i} + 6\hat{j} + 2k\right) + \lambda\left(-3\hat{i} + 2\hat{j} + 4k\right)$

$L_2 : \left(5\hat{i} + 3\hat{j} + 4k\right) + \lambda\left(2\hat{i} + \hat{j} + 3k\right)$

Distance between skew lines

$$= \frac{\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right).\left(2\hat{i} + 17\hat{j} - 7\hat{k}\right)}{\sqrt{342}}$$

$$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

7. Let (a, b) be the point of intersection of the curve $x^2 = 2y$ and the straight line $y - 2x - 6 = 0$ in the second quadrant. Then the integral $I = \int_a^b \frac{9x^2}{1+5^x}dx$ is equal to :

(1) 24 (2) 27

(3) 18 (4) 21

Ans. (1)

Sol. $x^2 = 2y$ & $y = 2x + 6$

$x^2 = 4x + 12$

$x^2 - 4x - 12 = 0 \Rightarrow \begin{array}{c|c} x = 6 & \text{if } x = -2 \\ y = 18 & y = 2 \end{array}$

$\therefore$ (6, 18) & (–2, 2)

Here (6, 18) Rejected because (a, b) lies in 2nd quadrant

$\therefore$ a = –2 & b = 2

$$\therefore I = \int_{-2}^{2} \frac{9x^2}{1+5^x}dx = \int_{-2}^{2} \frac{9.5^x.x^2}{1+5^x}dx$$

$$\therefore 2I = \int_{-2}^{2} 9x^2 dx = 18\int_0^2 x^2 dx = 18\left(\frac{x^3}{3}\right)_0^2$$

2I = 48

$\therefore \boxed{I = 24}$

8. If the system of equation

$2x + \lambda y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \mu z = 9$

has infinitely many solutions, then $(\lambda^2 + \mu^2)$ is equal to :

(1) 22 (2) 18

(3) 26 (4) 30

Ans. (3)

Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$

$\Rightarrow 2(2\mu + 5) + \lambda(-4 - 3\mu) + 3(7) = 0$

$\Rightarrow 4\mu - 3\lambda\mu - 4\lambda + 31 = 0$ ......(1)

$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0$

$\Rightarrow 2(-17) + \lambda(1) + 5(7) = 0$

$\Rightarrow \lambda = -1$

from equation (1)

$4\mu + 3\mu + 4 + 31 = 0 \Rightarrow \boxed{\mu = -5}$

$\therefore \boxed{\lambda^2 + \mu^2 = 26}$

9. If $\theta \in \left[-\frac{7\pi}{6}, \frac{4\pi}{3}\right]$, then the number of solutions of $\sqrt{3}\operatorname{cosec}^2\theta - 2\left(\sqrt{3} - 1\right)\operatorname{cosec}\theta - 4 = 0$, is equal to

(1) 6 (2) 8

(3) 10 (4) 7

Ans. (1)

Sol. $$\operatorname{cosec}\theta = \frac{2\left(\sqrt{3}-1\right) \pm \sqrt{4\left(3+1-2\sqrt{3}\right)+16\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3}-1\right) \pm \sqrt{16+8\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3}-1\right) \pm \left(2+2\sqrt{3}\right)}{2\sqrt{3}}$$

$\text{cosec}\theta = 2$ or $\frac{-2}{\sqrt{3}}$

$\therefore \sin\theta = \frac{1}{2}$ or $\frac{-\sqrt{3}}{2}$

$\therefore \sin\theta = \frac{1}{2}$ has 3 solutions & also $\sin\theta = \frac{-\sqrt{3}}{2}$ has 3 solutions in $\left[\frac{-7\pi}{6}, \frac{4\pi}{3}\right]$

10. Given three indentical bags each containing 10 balls, whose colours are as follows :

	Red	Blue	Green
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

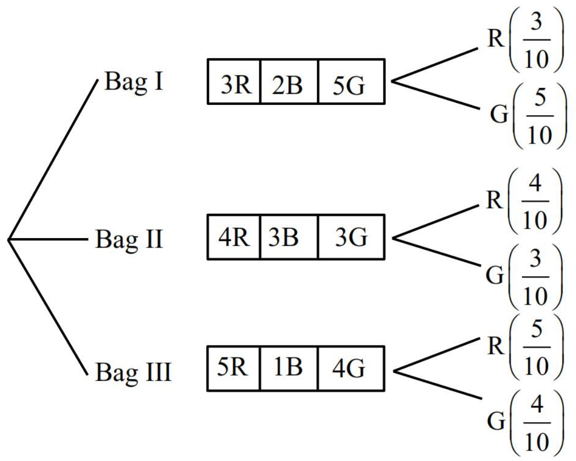
A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the balls is Green, the probability that it is from bag III is q, then the value of $\left(\frac{1}{p}+\frac{1}{q}\right)$ is :

(1) 6 (2) 9

(3) 7 (4) 8

Ans. (3)

Sol.



$$p = P\left(\frac{B_I}{R}\right) = \frac{\frac{1}{3}\left(\frac{3}{10}\right)}{\frac{1}{3}\left(\frac{3}{10}+\frac{4}{10}+\frac{5}{10}\right)} = \frac{1}{4}$$

$$q = \left(\frac{B_{III}}{G}\right) = \frac{\frac{1}{3}\left(\frac{4}{10}\right)}{\frac{1}{3}\left(\frac{5}{10}+\frac{3}{10}+\frac{4}{10}\right)} = \frac{1}{3}$$

$$\therefore \frac{1}{p}+\frac{1}{q} = 7$$

11. If the mean and the variance of 6, 4, a, 8, b, 12, 10, 13 are 9 and 9.25 respectively, then a + b + ab is equal to :

(1) 105 (2) 103

(3) 100 (4) 106

Ans. (2)

Sol. $\because$ mean = 9

$\therefore 53 + a + b = 72$

$\Rightarrow a + b = 19$

$\because \sigma^2 = \frac{37}{4}$ and $(\bar{X})^2 + \sigma^2 = \frac{\sum x_i^2}{N}$

$\Rightarrow 81 + \frac{37}{4} = \frac{529+a^2+b^2}{8}$

$\Rightarrow 648 + 74 = 529 + a^2 + b^2$

$\Rightarrow a^2 + b^2 = 193$

$\because a + b = 19 \Rightarrow a^2 + b^2 + 2ab = 361$

$\Rightarrow 2ab = 168$

$\Rightarrow ab = 84$

$\therefore a + b + ab = 103$

12. If the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$ is (a, b), then $(1 + a)^2 + b^2$ is equal to :

(1) 26 (2) 29

(3) 25 (4) 30

Ans. (1)

Sol. $x + |x| > 0 \quad \Rightarrow x \in (0, \infty)$ ....(1)

$\& \ 10 + 3x - x^2 > 0$

$\Rightarrow x^2 - 3x - 10 < 0$

$\Rightarrow x \in (-2, 5)$ ....(2)

from (1) & (2) $x \in (0, 5)$

$\therefore a = 0 \ \& \ b = 5$

$\therefore (1 + a^2) + b^2 = 1 + 25 = 26$

13. $4\int_0^1\left(\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}\right)dx - 3\log_e\left(\sqrt{3}\right)$ is equal to :

(1) $2+\sqrt{2}+\log_e\left(1+\sqrt{2}\right)$

(2) $2-\sqrt{2}-\log_e\left(1+\sqrt{2}\right)$

(3) $2+\sqrt{2}-\log_e\left(1+\sqrt{2}\right)$

(4) $2-\sqrt{2}+\log_e\left(1+\sqrt{2}\right)$

Ans. (2)

Sol. $4\int_0^1 \frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}dx - 3\ln\sqrt{3}$

$= 4\int_0^1 \frac{\sqrt{3+x^2}-\sqrt{1+x^2}}{(3+x^2)-(1-x^2)}dx - \frac{3}{2}\ln 3$

$= 2\left[\left\{\frac{x}{2}\sqrt{3+x^2}+\frac{3}{2}\ln\left(x+\sqrt{3+x^2}\right)\right\}_0^1\right.$

$\left. - \left\{\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\ln(x+\sqrt{1+x^2})\right\}_0^1\right] - \frac{3}{2}\ln 3$

$= 2\left[\left\{\frac{1}{2}\sqrt{4}+\frac{3}{2}\ln(1+\sqrt{4})\right\}-\left\{0+\frac{3}{2}\ln\sqrt{3}\right\}\right.$

$\left. -\left\{\frac{1}{2}\sqrt{2}+\frac{1}{2}\ln(1+\sqrt{2})\right\}+\left\{0+\frac{1}{2}(0)\right\}\right]-\frac{3}{2}\ln 3$

$=2\left[1+\frac{3}{2}\ln 3-\frac{3}{4}\ln 3-\frac{1}{\sqrt{2}}-\frac{1}{2}\ln(1+\sqrt{2})\right] - \frac{3}{2}\ln 3$

$= 2+3\ln 3 - \frac{3}{2}\ln 3 - \sqrt{2} - \ln(1+\sqrt{2}) - \frac{3}{2}\ln 3$

$= 2-\sqrt{2}-\ln(1+\sqrt{2})$

14. If $\lim_{x\to 0}\frac{\cos(2x)+a\cos(4x)-b}{x^4}$ is finite, then (a+b) is equal to :

(1) $\frac{1}{2}$

(2) 0

(3) $\frac{3}{4}$

(4) –1

Ans. (1)

Sol. $\lim_{x\to 0}\frac{\cos 2x + a\cos 4x - b}{x^4}$ = finite

$L = \frac{\left\{1-\frac{(2x)^2}{\lfloor 2}+\frac{(2x)^4}{\lfloor 4}....\right\}+a\left\{1-\frac{(4x)^2}{\lfloor 2}+\frac{(4x)^4}{\lfloor 4}....\right\}-b}{x^4}$

$L = \frac{(1+a-b)-x^2(2+8a)+x^4\left(\frac{2}{3}+\frac{32}{3}a\right)+x^6(\ )....}{x^4}$

$\therefore$ 1 + a – b = 0 and 2 + 8a = 0 $\Rightarrow a = -\frac{1}{4}$

b = a + 1

$= -\frac{1}{4}+1=\frac{3}{4}$

$\therefore a+b = -\frac{1}{4}+\frac{3}{4}=\frac{1}{2}$

15. If $\sum_{r=0}^{10}\left(\frac{10^{r+1}-1}{10^r}\right).{}^{11}C_{r+1} = \frac{\alpha^{11}-11^{11}}{10^{10}}$, then α is equal to :

(1) 15

(2) 11

(3) 24

(4) 20

Ans. (4)

Sol. $\sum_{r=0}^{10}\left(\frac{10^{r-1}-1}{10^r}\right){}^{11}C_{r+1}$

$=\sum_{r=0}^{10}\left(10-\frac{1}{10^r}\right){}^{11}C_{r+1}$

$=10\sum_{r=0}^{10}{}^{11}C_{r+1}-10\sum\left({}^{11}C_{r+1}\left(\frac{1}{10}\right)^{r+1}\right)$

$=10\left[{}^{11}C_1+{}^{11}C_2+.....+{}^{11}C_{11}\right]$

$-10\left[{}^{11}C_1\left(\frac{1}{10}\right)^1+{}^{11}C_2\left(\frac{1}{10}\right)^2+.....+{}^{11}C_{11}\left(\frac{1}{10}\right)^{11}\right]$

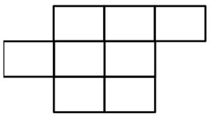
$=10\left[2^{11}-1\right]-10\left[\left(1+\frac{1}{10}\right)^{11}-1\right]$

$=10(2)^{11}-10-\frac{11^{11}}{10^{10}}+10$

$=\frac{(20)^{11}-11^{11}}{10^{10}}$

$\therefore \alpha = 20$

16. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is :

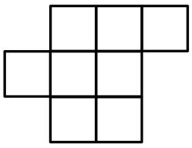


(1) 5880 (2) 960

(3) 840 (4) 5760

Ans. (4)

Sol.



= Total – [(All in R_1 and R_3) + (All in R_2 and R_3) + (All in R_1 and R_2)]

$= {}^{8}C_5 \cdot \lfloor 5 - \left\{\lfloor 5 + \lfloor 5 + {}^{6}C_5 \cdot \lfloor 5\right\}$

$= \lfloor 5(56-1-1-6) = 120(48)$

= 5760

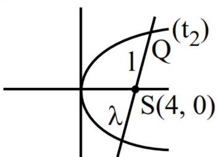
17. Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be (1, –4). If the focus of the parabola divides the chord PQ in the ratio m : n, gcd(m, n) = 1, then $m^2 + n^2$ is equal to :

(1) 17 (2) 10

(3) 37 (4) 26

Ans. (1)

Sol. $y^2 = 16x$; a = 4 focus S ≡ (4, 0)



$2at_1 = -4$

$\Rightarrow 2(4)t_1 = -4$

$\Rightarrow t_1 = -\frac{1}{2}$

$\because t_1 t_2 = -1$

$\Rightarrow t_2 = 2$

$\therefore Q\left(at_2^2, 2at_2\right) = (16, 16)$

Let, S divides PQ internally in λ : 1 ratio

$\therefore \frac{16\lambda - 4}{\lambda + 1} = 0$

$\lambda = \frac{1}{4} = \frac{m}{n}$

$\therefore m^2 + n^2 = 1 + 16 = 17$

18. Let $\vec{a} = 2\hat{i} - 3\hat{j} + k$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5k$ and a vector $\vec{c}$ be such that $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12k$ and $\vec{a}.\vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $|\vec{a}.\vec{d}|$ is equal to :

(1) 18 (2) 12

(3) 9 (4) 15

Ans. (4)

Sol. $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 3 & 2 & 5 \end{vmatrix}$

$= -17\hat{i} - 7\hat{j} + 13\hat{k}$

$(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3j + 12\hat{k}$

$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{c} \times \vec{b}) = -18\hat{i} - 3\hat{j} + 12\hat{k}$

$\Rightarrow \vec{b} \times \vec{c} = (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (\vec{a} \times \vec{b})$

$= (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (-17i - 7\hat{j} + 13\hat{k})$

$\vec{b} \times \vec{c} = -\hat{i} + 4\hat{j} - \hat{k}$

$\therefore \vec{a} \cdot \vec{d} = \vec{a} \cdot (\vec{b} \times \vec{c}) = (2i - 3j + k) \cdot (-\hat{i} + 4\hat{j} - \hat{k})$

= –2 – 12 – 1 = –15

$\therefore |\vec{a} \cdot \vec{d}| = 15$

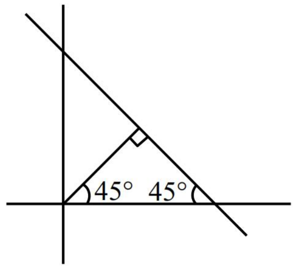
19. Let the area of the triangle formed by a straight Line L : x + by + c = 0 with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x-axis, then the value of $b^2 + c^2$ is:

(1) 90 (2) 93

(3) 97 (4) 83

Ans. (3)

Sol. $\frac{x}{-c}+\frac{y}{-c/b}=1$



$\therefore$ area of triangle $=\frac{1}{2}\left|\frac{c^2}{b}\right|=48$

$\left|\frac{c^2}{b}\right|=96$

$\because -c=-\frac{c}{b}$

$\Rightarrow b=1 \quad \therefore c^2=96$

$\therefore b^2+c^2=97$

20. Let A be a 3 × 3 real matrix such that $A^2(A-2I)-4(A-I)=O$, where I and O are the identity and null matrices, respectively. If $A^5=\alpha A^2+\beta A+\gamma I$, where α, β and γ are real constants, then $\alpha+\beta+\gamma$ is equal to:

(1) 12 (2) 20

(3) 76 (4) 4

Ans. **(1)**

Sol. $A^3-2A^2-4A+4I=0$

$A^3=2A^2+4A-4I$

$A^4=2A^3+4A^2-4A$

$=2(2A^2+4A-4I)+4A^2-4A$

$A^4=8A^2+4A-8I$

$A^5=8A^3+4A^2-8A$

$=8(2A^2+4A-4I)+4A^2-8A$

$A^5=20A^2+24A-32I$

$\therefore \alpha=20, \beta=24, \gamma=-32$

$\therefore \alpha+\beta+\gamma=12$

SECTION-B

21. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}+2y\sec^2 x=2\sec^2 x+3\tan x.\sec^2 x$ such that $y(0)=\frac{5}{4}$. Then $12\left(y\left(\frac{\pi}{4}\right)-e^{-2}\right)$ is equal to __________.

Ans. **(21)**

Sol. $\text{I.F.}=e^{\int 2\sec^2 x dx}$

$=e^{2\tan x}$

Solution of diff. eq.

$y.e^{2\tan x}=\int e^{2\tan x}(2\sec^2 x+3\tan x.\sec^2 x)dx$

$y.e^{2\tan x}=\int e^{2\tan x}.(2\sec^2 x)dx+\int e^{2\tan x}.(3\tan x.\sec^2 x)dx$

$y.e^{2\tan x}=e^{2\tan x}.2\tan x-\int e^{2\tan x}.2\sec^2 x\times 2\tan x dx+\int e^{2\tan x}.3\tan x.\sec^2 x dx$

$y.e^{2\tan x}=2\tan x.e^{2\tan x}-\int e^{2\tan x}.\tan x\sec^2 x dx$

$y.e^{2\tan x}=2\tan x.e^{2\tan x}-\frac{\tan x.e^{2\tan x}}{2}+\frac{e^{2\tan x}}{4}+C$

$y=2\tan x-\frac{\tan x}{2}+\frac{1}{4}+Ce^{-2\tan x}$

$x=0, y=\frac{5}{4}$

$c=1$

$y\left(\frac{\pi}{4}\right)=\frac{7}{4}+e^{-2}$

Then $12\left(y\left(\frac{\pi}{4}\right)-e^{-2}\right)=12\left(\frac{7}{4}\right)=21$

22. If the sum of the first 10 terms of the series $\frac{4.1}{1+4.1^4}+\frac{4.2}{1+4.2^4}+\frac{4.3}{1+4.3^4}+\ldots$ is $\frac{m}{n}$, where $\gcd(m,n)=1$, then $m+n$ is equal to ________.

Ans. **(441)**

Sol. $T_r=\frac{4.r}{1+4.r^4}$

$T_r=\frac{4.r}{(2r^2+2r+1)(2r^2-2r+1)}$

$T_r=\frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2+2r+1)(2r^2-2r+1)}$

$$T_r = \frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1}$$

$$T_1 = \frac{1}{1} - \frac{1}{5}$$

$$T_2 = \frac{1}{5} - \frac{1}{13}$$

$$\vdots$$

$$T_{10} = \frac{1}{181} - \frac{1}{221}$$

$$S_{10} = 1 - \frac{1}{221} = \frac{220}{221} = \frac{m}{n}$$

$m + n = 441$

23. If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x - y)^2 + 3y^2$ is equal to __________.

Ans. **(3)**

Sol. $y = \cos\left(\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{x}{2}\right)$

$$y = \frac{1}{2} \times \frac{x}{2} - \sqrt{1 - \frac{1}{4}}\sqrt{1 - \frac{x^2}{4}}$$

$4y = x - \sqrt{3}\sqrt{4 - x^2}$

$3(4 - x^2) = x^2 + 16y^2 - 8xy$

$12 - 3x^2 = x^2 + 16y^2 - 8xy$

$4x^2 + 16y^2 - 8xy = 12$

$x^2 + 4y^2 - 2xy = 3$

$x^2 + y^2 - 2xy - 3y^2 = 3$

$(x - y)^2 + 3y^2 = 3$

24. Let A(4, –2), B(1, 1) and C(9, –3) be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is __________.

Ans. **(3)**

Sol. Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix}$

= 6 square units

Maximum area of AFDE = $\frac{1}{2} \times 6$ = 3 sq. units

25. If the set of all $a \in R - \{1\}$, for which the roots of the equation $(1 - a)x^2 + 2(a - 3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to _________.

Ans. **(7)**

Sol. Both the roots are positive

$D \geq 0$

$4(a - 3)^2 - 4 \times 9(1 - a) \geq 0$

$a^2 - 6a + 9 - 9 + 9a \geq 0$

$a^2 + 3a \geq 0$

$a(a + 3) \geq 0$

$a \in (-\infty, -3] \cup [0, \infty)$ ……(i)

$$-\frac{b}{2a} > 0$$

$$\frac{2(a-3)}{2(a-1)} > 0$$

$a \in (-\infty, 1) \cup (3, \infty)$ ……..(ii)

$f(0) = 9 > 0$

Equation (i) $\cap$ (ii)

$a \in (-\infty, -3] \cup [0, 1)$

$2\alpha + \beta + \gamma - 6 + 0 + 1 = 7$

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

26. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Net dipole moment of a polar linear isotropic dielectric substance is not zero even in the absence of an external electric field.

Reason (R) : In absence of an external electric field, the different permanent dipoles of a polar dielectric substance are oriented in random directions.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **(A)** is correct but **(R)** is not correct

(2) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(4) **(A)** is not correct but **(R)** is correct

Ans. (4)

Sol. A : Since polar dielectrics are randomly oriental $\vec{P}_{net} = \vec{0}$.

R : If $\vec{E}$ is absent, polar dielectric remain polar & are randomly oriented.

27. In a moving coil galvanometer, two moving coils M_1 and M_2 have the following particulars :

$R_1 = 5\ \Omega$, $N_1 = 15$, $A_1 = 3.6 \times 10^{-3}\ m^2$, $B_1 = 0.25$ T

$R_2 = 7\ \Omega$, $N_2 = 21$, $A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50$ T

Assuming that torsional constant of the springs are same for both coils, what will be the ratio of voltage sensitivity of M_1 and M_2 ?

(1) 1 : 1 (2) 1 : 4

(3) 1 : 3 (4) 1 : 2

Ans. (1)

Sol. Voltage sensitivity = $\frac{\theta}{V} = \frac{NAB}{cR}$

$$\text{Ratio} = \left(\frac{N_1A_1B_1}{N_2A_2B_2}\right)\frac{R_2}{R_1} = \frac{15\times3.6\times0.25}{21\times1.8\times0.5}\times\frac{7}{5} = \frac{1}{1}$$

28. The moment of inertia of a circular ring of mass M and diameter r about a tangential axis lying in the plane of the ring is :

(1) $\frac{1}{2}Mr^2$ (2) $\frac{3}{8}Mr^2$

(3) $\frac{3}{2}Mr^2$ (4) $2\ Mr^2$

Ans. (2)

Sol. Diameter is given as R.

$\therefore$ Radius = R/2

$$I_{tan\,gent} = \frac{3}{2}m\left(\frac{R}{2}\right)^2 = \frac{3}{8}mR^2$$

29. Two water drops each of radius 'r' coalesce to from a bigger drop. If 'T' is the surface tension, the surface energy released in this process is :

(1) $4\pi r^2T\left[2-2^{\frac{2}{3}}\right]$ (2) $4\pi r^2T\left[2-2^{\frac{1}{3}}\right]$

(3) $4\pi r^2T\left[1+\sqrt{2}\right]$ (4) $4\pi r^2T\left[\sqrt{2}-1\right]$

Ans. (1)

Sol. $2\times\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \Rightarrow r = 2^{1/3}R$

$U_i = 2 \times 4\pi R^2T$

$U_f = 4\pi r^2T = 4\pi R^2T2^{2/3}$

$\therefore$ Heat lost = $u_i - u_f = 4\pi R^2T[2-2^{2/3}]$

30. An electron with mass 'm' with an initial velocity $(t = 0)$ $\vec{v} = v_0\hat{i}$ $(v_0 > 0)$ enters a magnetic field $\vec{B} = B_0\hat{j}$. If the initial de-Broglie wavelength at $t = 0$ is λ_0 then its value after time 't' would be :

(1) $\frac{\lambda_0}{\sqrt{1-\frac{e^2B_0^2t^2}{m^2}}}$ (2) $\frac{\lambda_0}{\sqrt{1+\frac{e^2B_0^2t^2}{m^2}}}$

(3) $\lambda_0\sqrt{1+\frac{e^2B_0^2t^2}{m^2}}$ (4) λ_0

Ans. (4)

Sol. Magnetic field does not work

$\therefore$ Speed will not charge, so De-Broglie wavelength remains same.

31. A sinusoidal wave of wavelength 7.5 cm travels a distance of 1.2 cm along the x-direction in 0.3 sec. The crest P is at x = 0 at t = 0 sec and maximum displacement of the wave is 2 cm. Which equation correctly represents this wave ?

(1) $y = 2\cos(0.83x - 3.35t)$ cm
(2) $y = 2\sin(0.83x - 3.5t)$ cm
(3) $y = 2\cos(3.35x - 0.83t)$ cm
(4) $y = 2\cos(0.13x - 0.5t)$ cm

Ans. **(1)**

Sol. $v = \frac{\text{distance}}{\text{time}}$

$v = \frac{12}{0.3} = 4 \text{ cm/s}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.5} = \frac{4\pi}{15} = 0.83$

$v = \frac{\omega}{k} \Rightarrow \omega = vk = 4 \times \frac{4\pi}{15} = 3.35$

So $y = A\cos(kx - \omega t)$

32. Given a charge q, current I and permeability of vacuum μ_0. Which of the following quantity has the dimension of momentum ?

(1) qI/μ_0
(2) $q\mu_0 I$
(3) $q^2\mu_0 I$
(4) $q\mu_0/I$

Ans. **(2)**

Sol. $Q = AT$

$I = A$

$\mu_0 = MLT^{-2}A^{-2}$

$P = Q^x \mu_0^y I^z = [AT]^x [MLT^{-2}A^{-2}]^y [A]^z$

$MLT^{-1} = M^y L^y T^{x-2y} A^{-2y+z+x}$

Now; $y = 1$

$x - 2y = -1$

$-2y + z = 0$

$\therefore x = y = z = 1$

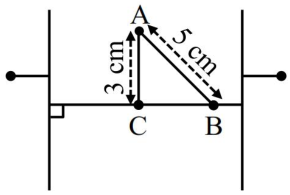
33. A solenoid having area A and length 'l' is filled with a material having relative permeability 2. The magnetic energy stored in the solenoid is :

(1) $\frac{B^2Al}{\mu_0}$
(2) $\frac{B^2Al}{2\mu_0}$
(3) B^2Al
(4) $\frac{B^2Al}{4\mu_0}$

Ans. **(4)**

Sol. $\frac{U}{V} = \frac{B^2}{2\mu_r u_0} \Rightarrow U = \frac{B^2}{4\mu_0}V = \frac{B^2}{4\mu_0}A\ell$

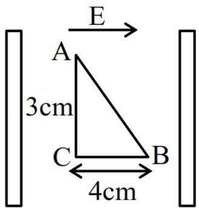
34. Two large plane parallel conducting plates are kept 10 cm apart as shown in figure. The potential difference between them is V. The potential difference between the points A and B (shown in the figure) is :



(1) $\frac{1}{4}V$
(2) $\frac{2}{5}V$
(3) $\frac{3}{4}V$
(4) 1 V

Ans. **(2)**

Sol.



Using $\Delta V = E(\Delta d)$

$V = E(10)$

$V_{AB} = E.4 = \frac{V}{10} \times 4 = \frac{2V}{5}$

35. Identify the characteristics of an adiabatic process in a monoatomic gas.

(A) Internal energy is constant.
(B) Work done in the process is equal to the charge in internal energy.
(C) The product of temperature and volume is a constant.
(D) The product of pressure and volume is a constant.
(E) The work done to change the temperature from T_1 to T_2 is proportional to $(T_2 - T_1)$

Choose the **correct** answer from the options given below :

(1) (A), (C), (D) only
(2) (A), (C), (E) only
(3) (B), (E) only
(4) (B), (D) only

Ans. **(3)**

Sol. $Q = \Delta U + W = 0 \Rightarrow -\Delta U = W$

$WD = -nC_v\Delta T \Rightarrow |WD| = nC_v\Delta T \propto T_2 - T_1$

$\therefore$ B & E [Only possibility]

36. Assuming the validity of Bohr's atomic model for hydrogen like ions the radius of Li^{++} ion in its ground state is given by $\frac{1}{X}a_0$, where X = ______.

(Where a_0 is the first Bohr's radius.)

(1) 2 (2) 1

(3) 3 (4) 9

Ans. **(3)**

Sol. $r = r_0 \frac{n^2}{z}$ & z = 3 for Li^{+2} and n = 1

$\therefore r = r_0 \frac{1^2}{3} = \frac{r_0}{3}$ $\therefore x = 3$

37. Energy released when two deuterons ($_1H^2$) fuse to form a helium nucleus ($_2He^4$) is :

(Given : Binding energy per nucleon of $_1H^2$ = 1.1 MeV and binding energy per nucleon of $_2He^4$ = 7.0 MeV)

(1) 8.1 MeV (2) 5.9 MeV

(3) 23.6 MeV (4) 26.8 MeV

Ans. **(3)**

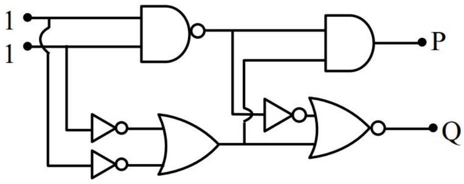
Sol. $\underset{1.1\text{MeV}}{_1H^2} + {_1H^2} \rightarrow \underset{7.0\text{MeV}}{_1He^4}$

$E_B = BE_{reactant} - BE_{product}$

$= 1.1 \times 2 + 1.1 \times 2 - 7 \times 4 = -23.6$ MeV

= Q = 23.6 MeV

38. In the digital circuit shown in the figure, for the given inputs the P and Q values are :

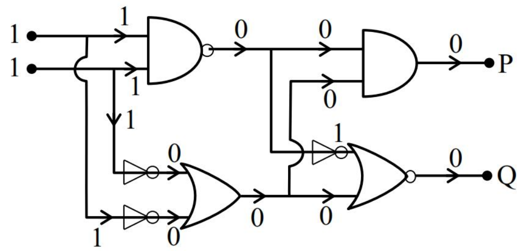


(1) P = 1, Q = 1 (2) P = 0, Q = 0

(3) P = 0, Q = 1 (4) P = 1, Q = 0

Ans. **(2)**

Sol.



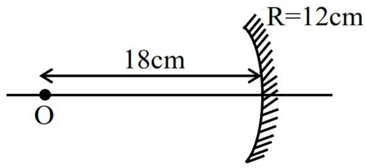
39. Two identical objects are placed in front of convex mirror and concave mirror having same radii of curvature of 12 cm, at same distance of 18 cm from the respective mirrors. The ratio of sizes of the images formed by convex mirror and by concave mirror is :

(1) 1/2 (2) 2

(3) 3 (4) 1/3

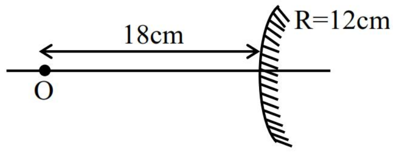
Ans. **(1)**

Sol.



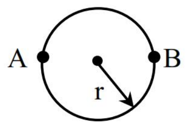
Using $m = \frac{f}{u - f}$

$m_1 = \frac{6}{18 - 6} = \frac{1}{2}$



$m_2 = \frac{6}{18 + 6} = \frac{1}{4}$ $\therefore \frac{m_2}{m_1} = \frac{1}{2}$

40. A sportsman runs around a circular track of radius r such that he traverses the path ABAB. The distance travelled and displacement, respectively, are



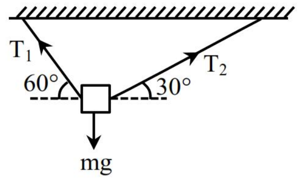
(1) 2r, 3πr (2) 3πr, πr

(3) πr, 3r (4) 3πr, 2r

Ans. **(4)**

Sol. Displacement = 2r

Distance = 2πr + πr = 3πr

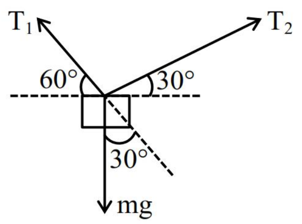
41. 

A body of mass 1kg is suspended with the help of two strings making angles as shown in figure. Magnitude of tensions T_1 and T_2, respectively, are (in N) :

(1) 5, $5\sqrt{3}$ (2) $5\sqrt{3}$, 5

(3) $5\sqrt{3}$, $5\sqrt{3}$ (4) 5, 5

Ans. (2)



Sol.

$T_1 = mg\cos 30°$

$T_2 = mg\sin 30°$

42. A bi–convex lens has radius of curvature of both the surfaces same as 1/6 cm. If this lens is required to be replaced by another convex lens having different radii of curvatures on both sides ($R_1 \neq R_2$), without any change in lens power then possible combination of R_1 and R_2 is :

(1) $\frac{1}{3}$cm and $\frac{1}{3}$cm (2) $\frac{1}{5}$cm and $\frac{1}{7}$cm

(3) $\frac{1}{3}$cm and $\frac{1}{7}$cm (4) $\frac{1}{6}$cm and $\frac{1}{9}$cm

Ans. (2)

Sol. This will happen when

$$\frac{1}{f_1} = \frac{1}{f_2}$$

$$(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-R_2}\right) = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}$$

43. If μ_0 and ε_0 are the permeability and permittivity of free space, respectively, then the dimension of $\left(\frac{1}{\mu_0\varepsilon_0}\right)$ is :

(1) L/T^2 (2) L^2/T^2

(3) T^2/L (4) T^2/L^2

Ans. (2)

Sol. $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \frac{1}{\mu_0\varepsilon_0} = C^2 = L^2T^{-2}$

44. Match **List-I with List-II**.

List-I	**List-II**
(A) Heat capacity of body	(I) $J\ kg^{-1}$
(B) Specific heat capacity of body	(II) JK^{-1}
(C) Latent heat	(III) $J\ kg^{-1}\ K^{-1}$
(D)Thermal conductivity	(IV) $Jm^{-1}K^{-1}s^{-1}$

Choose the **correct** answer from the options given below :

(1) (A)–(III), (B)–(I), (C)–(II), (D)–(IV)

(2) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)

(3) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)

(4) (A)–(II), (B)–(III), (C)–(I), (D)–(IV)

Ans. (4)

Sol. $C' = \frac{\Delta Q}{\Delta T} = JK^{-1}$

$$S = \frac{\Delta Q}{m\Delta T} = Jkg^{-1}K^{-1}$$

$$L = \frac{\Delta Q}{m} = Jkg^{-1}$$

$$\Delta Q = \frac{KA\Delta T}{L} \Rightarrow K = \frac{\Delta Q(L)}{A\Delta T} = Jm^{-1}K^{-1}s^{-1}$$

45. Consider a circular loop that is uniformly charged and has a radius $a\sqrt{2}$. Find the position along the positive z–axis of the cartesian coordinate system where the electric field is maximum if the ring was assumed to be placed in xy–plane at the origin :

(1) $\frac{a}{\sqrt{2}}$ (2) $\frac{a}{2}$

(3) a (4) 0

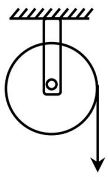
Ans. **(3)**

Sol. $E=\frac{KQr}{(x^2+R^2)^{3/2}}$

$\frac{dE}{dx}=0$

$\therefore x=\frac{R}{\sqrt{2}}=\frac{\sqrt{2}a}{\sqrt{2}}=a$

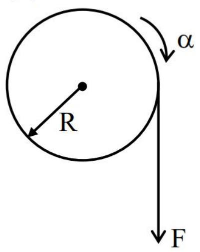
SECTION-B

46. 

A wheel of radius 0.2 m rotates freely about its center when a string that is wrapped over its rim is pulled by force of 10 N as shown in figure. The established torque produces an angular acceleration of 2 rad/s^2. Moment of inertia of the wheel is ________ kg m^2.

(Acceleration due to gravity = 10 m/s^2)

Ans. **(1)**



Sol.

$FR = I\alpha$

$\Rightarrow I=\frac{FR}{\alpha}=\frac{10\times0.2}{2}=1\ \text{kg–m}^2$

47. The internal energy of air in 4 m × 4 m × 3 m sized room at 1 atmospheric pressure will be ___ × 10^6 J. (Consider air as diatomic molecule)

Ans. **(12)**

Sol.

To find the internal energy of gas in the room.

$U=nC_vT=n\frac{5RT}{2}$

$=\frac{5}{2}PV=\frac{5}{2}\times10^5\times48=12\times10^6\,J$

48. A ray of light suffers minimum deviation when incident on a prism having angle of the prism equal to 60°. The refractive index of the prism material is $\sqrt{2}$. The angle of incidence (in degrees) is _____.

Ans. **(45)**

Sol. $\mu=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$, since A = 60° $\quad\therefore \delta m = 30°$

$\delta_m = 2i - A$ [as i = e]

$\Rightarrow i = 45°$

49. The length of a light string is 1.4 m when the tension on it is 5 N. If the tension increases to 7 N, the length of the string is 1.56 m. The original length of the string is _____ m.

Ans. **(1)**

Sol. $T=K(\ell-\ell_0)$

$\Rightarrow 5=K(1.4-\ell_0)$

$\Rightarrow 7=K(1.56-\ell_0)$

$\Rightarrow \frac{5}{1.4-\ell_0}=\frac{7}{1.56-\ell_0}$

$\therefore \ell_0 = 1\text{m}$

50. A satellite of mass 1000 kg is launched to revolve around the earth in an orbit at a height of 270 km from the earth's surface. Kinetic energy of the satellite in this orbit is ________ × 10^{10} J.

(Mass of earth = 6 × 10^{24} kg, Radius of earth = 6.4 × 10^6 m, Gravitational constant = 6.67 × 10^{-11} Nm^2 kg^{-2})

Ans. **(3)**

Sol. $KE=\frac{1}{2}mv^2=\frac{1}{2}m\frac{GM_e}{r}=\frac{GM_em}{2r}=\frac{GM_em}{2(R_E+h)}$

$=\frac{6.67\times10^{-11}\times6\times10^{24}\times6.4\times10^6}{2(6.4\times10^6+2.7\times10^5)}=3\times10^{10}\,J$

SECTION-A

51. When a concentrated solution of sulphanilic acid and 1-naphthylamine is treated with nitrous acid (273 K) and acidified with acetic acid, the mass (g) of 0.1 mole of product formed is :

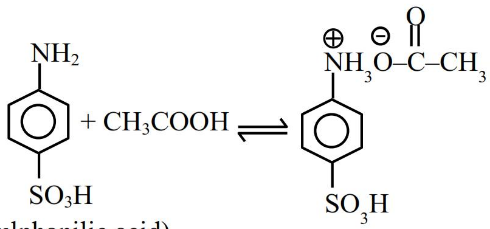
(Given molar mass in g mol^{-1} H : 1, C : 12, N : 14, O : 16, S : 32)

(1) 343 (2) 330

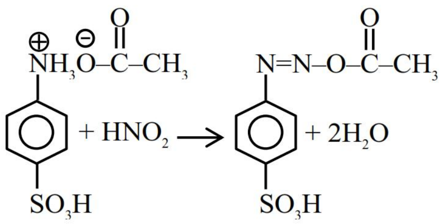
(3) 33 (4) 66

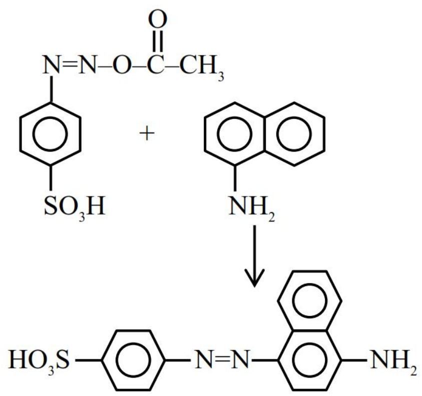
Ans. (3)

Sol.



(Sulphanilic acid)





(Red-azo dye)

0.1 mole of red-azo dye (Molar Mass = 327 gm/mol) will have 32.7 gm mass. Nearly 33 gm.

52. The d-orbital electronic configuration of the complex among $[Co(en)_3]^{3+}$, $[CoF_6]^{3-}$, $[Mn(H_2O)_6]^{2+}$ and $[Zn(H_2O)_6]^{2+}$ that has the highest CFSE is :

(1) $t_{2g}^6e_g^0$ (2) $t_{2g}^6e_g^4$

(3) $t_{2g}^3e_g^2$ (4) $t_{2g}^4e_g^2$

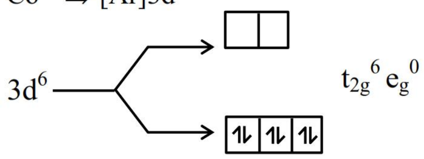
Ans. (1)

Sol. In $[Co(en)_3]^{+3}$ S.F.L. is present and hence highest value of CFSE

In rest all complexes WFL is present hence CFSE will be low.

$[Co(en)_3]^{+3} \Rightarrow Co^{+3}$ and en (SFL)

$Co^{+3} \Rightarrow [Ar]3d^6$



53. Given below are two statements :

Statement (I) : Neopentane forms only one monosubstituted derivative.

Statement (II) : Melting point of neopentane is higher than n-pentane

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is correct but **Statement II** is incorrect

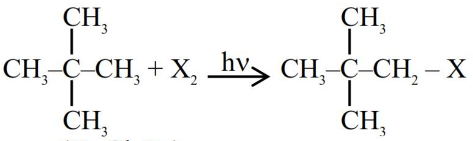
(2) Both **Statement I** and **Statement II** are correct

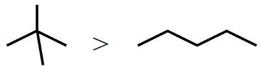
(3) Both **Statement I** and **Statement II** are incorrect

(4) **Statement I** is incorrect but **Statement II** is correct

Ans. (2)

Sol. Both Statement-I and Statement-II are correct.



melting point 

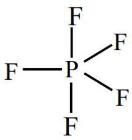
neo-pentane > n-pentane

54. Which among the following molecules is (a) involved in sp^3d hybridization, (b) has different bond lengths and (c) has lone pair of electrons on the central atom ?

(1) PF_5 (2) XeF_4

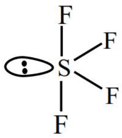
(3) SF_4 (4) XeF_2

Ans. (3)

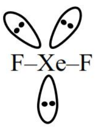
Sol. PF_5 :- 

(a) Hybridisation = sp^3d
(b) All bonds are not identical
(c) No lone pair on central atom

XeF_4 :- sp^3d^2 Hybridisation

SF_4 :- 

(a) Hybridisation = sp^3d
(b) All bonds are not identical
(c) 1 lone pair on central atom

XeF_2 :- 

(a) Hybridisation = sp^3d
(b) All bonds are identical
(c) 3 lone pair on central atom

55. Formation of $Na_4[Fe(CN)_5NOS]$, a purple coloured complex formed by addition of sodium nitroprusside in sodium carbonate extract of salt indicates the presence of :

(1) Sodium ion (2) Sulphate ion

(3) Sulphide ion (4) Sulphite ion

Ans. (3)

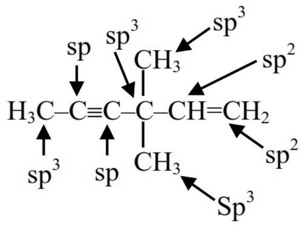
Sol. $\underset{\text{Sodium sulphide}}{Na_2S} + \underset{\text{Sodium nitro Prusside}}{Na_2[Fe(CN)_5NO]} \rightarrow \underset{\text{Violet Colour}}{Na_4[Fe(CN)_5NOS]}$

56. In 3, 3-dimethylhex-1-ene-4-yne, there are ____ sp^3, ______ sp^2 and ______ sp hybridised carbon atoms respectively :

(1) 4, 2, 2 (2) 3, 3, 2

(3) 2, 4, 2 (4) 2, 2, 4

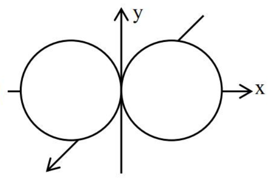
Ans. (1)

Sol. 

4-sp^3, 2-sp^2, 2-sp

57. Which of the following statements are **true** ?

(A) The subsidiary quantum number *l* describes the shape of the orbital occupied by the electron.

(B) 

is the boundary surface diagram of the $2p_x$ orbital.

(C) The + and – signs in the wave function of the $2p_x$ orbital refer to charge.

(D) The wave function of $2p_x$ orbital is zero everywhere in the xy plane.

(1) (B) and (D) only

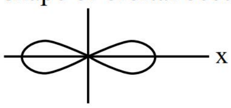
(2) (A), (B) and (C) only

(3) (C) and (D) only

(4) (A) and (B) only

Ans. (4)

Sol. (A) Azimuthal quantum number (ℓ) indicates the shape of orbital occupied by the electron

(B) 

p_x orbital

(C) The + and – sign in the wave function of $2p_x$ orbital refer to the sign (Phase) of the wave function , not the charge

(D) The wave function of $2p_x$ orbital will be zero in yz plane (Nodal plane).

58. The type of hybridization and the magnetic property of $[MnCl_6]^{3-}$ are :

(1) d^2sp^3, paramagnetic with four unpaired electrons

(2) sp^3d^2, paramagnetic with four unpaired electrons

(3) d^2sp^3, paramagnetic with two unpaired electrons

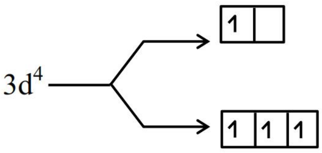
(4) sp^3d^2, paramagnetic with two unpaired electrons

Ans. **(2)**

Sol. $[MnCl_6]^{3-}$ contains Mn^{+3}

Mn^{+3} :- $[Ar]3d^4$

Ligand $\Rightarrow Cl^-$ (WFL)



Hybridisation = sp^3d^2

4 unpaired electrons

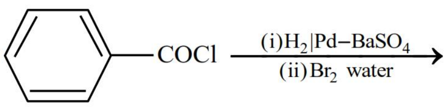
59. Consider the following reactions. From these reactions which reaction will give carboxylic acid as a major product ?

(A) $R-C\equiv N \xrightarrow[\text{mild condition}]{(i)H^+/H_2O}$

(B) $R-MgX \xrightarrow[(ii)H_3O^+]{(i)CO_2}$

(C) $R-C\equiv N \xrightarrow[(ii)H_3O^+]{(i)SnCl_2/HCl}$

(D) $R{\cdot}CH_2{\cdot}OH \xrightarrow{PCC}$

(E) 

Choose the **correct** answer from the options given below :

(1) A and D only

(2) A, B and E only

(3) B, C and E only

(4) B and E only

Ans. **(4)**

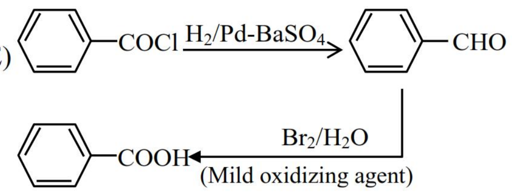
Sol. (A) $R-C\equiv N \xrightarrow[\text{mild condition}]{(i)H^+/H_2O} R-CONH_2$

Under mild condition amide is formed because this reaction is typically slow if further more heat will supplied then it gets convert in to –COOH.

(B) $R-MgX \xrightarrow[(ii)H_3O^+]{(i)CO_2} R-COOH$

(C) $R-C\equiv N \xrightarrow[(ii)H_3O^+]{(i)SnCl_2/HCl} R-CHO$

(D) $R{\cdot}CH_2{\cdot}OH \xrightarrow{PCC} R-CHO$

(E) 

60. Electronic configuration of four elements A, B, C and D are given below :

(A) $1s^22s^22p^3$ (B) $1s^22s^22p^4$

(C) $1s^22s^22p^5$ (D) $1s^22s^22p^2$

Which of the following is the correct order of increasing electronegativity (Pauling's scale) ?

(1) A < D < B < C

(2) A < C < B < D

(3) A < B < C < D

(4) D < A < B < C

Ans. **(4)**

Sol. N :- $1s^2\ 2s^2\ 2p^3$ (Electronegativity = 3)

O :- $1s^2\ 2s^2\ 2p^4$ (Electronegativity = 3.5)

F :- $1s^2\ 2s^2\ 2p^5$ (Electronegativity = 4)

C :- $1s^2\ 2s^2\ 2p^2$ (Electronegativity = 2.55)

Correct order = C > B > A > D

61. Match **List-I** with **List-II**

	List-I (Purification technique)		**List-II (Mixture of organic compounds)**
(A)	Distillation (simple)	(I)	Diesel + Petrol
(B)	Fractional distillation	(II)	Aniline + Water
(C)	Distillation under reduced pressure	(III)	Chloroform + Aniline
(D)	Steam distillation	(IV)	Glycerol + Spent-lye

Choose the **correct** answer from the options given below :

(1) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(2) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

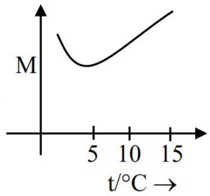
Ans. (4)

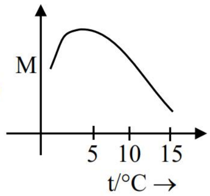
Sol.

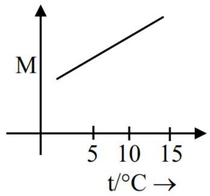
	List-I (Purification technique)		**List-II (Mixture of organic compounds)**
(A)	Distillation (simple)	(III)	Chloroform + Aniline
(B)	Fractional distillation	(I)	Diesel + Petrol
(C)	Distillation under reduced pressure	(IV)	Glycerol + Spent-lye
(D)	Steam distillation	(II)	Aniline + Water

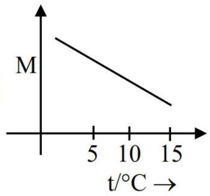
62. 'x' g of NaCl is added to water in a beaker with a lid. The temperature of the system is raised from 1°C to 25°C. Which out of the following plots, is best suited for the change in the molarity (M) of the solution with respect to temperature ?

[Consider the solubility of NaCl remains unchanged over the temperature range]

(1)


(2)


(3)


(4)


Ans. (2)

Sol. $\underset{1°C}{\text{Water}} \xrightarrow[\text{decreases}]{\text{Volume}} \underset{4°C}{\text{Water}} \xrightarrow[\text{thermal expansion}]{\text{Volume increases}} \underset{25°C}{\text{Water}}$

$$\text{Molarity} = \frac{n_{solute}}{(\text{Volume of solution})_{\ell}}$$

Hence

$$1°C \xrightarrow[\text{increases}]{\text{molarity}} 4°C \xrightarrow[\text{decreases}]{\text{molarity}} 25°C$$

63. Arrange the following in order of magnitude of work done by the system / on the system at constant temperature :

(a) $|w_{reversible}|$ for expansion in infinite stage.

(b) $|w_{irreversible}|$ for expansion in single stage.

(c) $|w_{reversible}|$ for compression in infinite stage.

(d) $|w_{irreversible}|$ for compression in single stage.

Choose the **correct** answer from the options given below :

(1) a > b > c > d

(2) d > c = a > b

(3) c = a > d > b

(4) a > c > b > d

Ans. (2)

Sol. For isothermal process

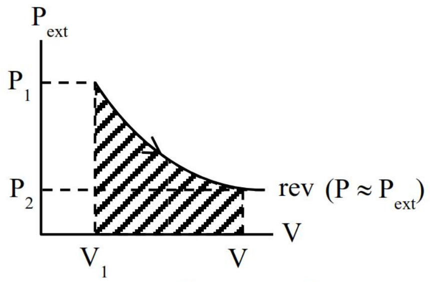
$|W_{reversible}|_{expansion} = |W_{reversible}|_{compression}$

$= -\,nRT\ln\dfrac{V_f}{V_i}$

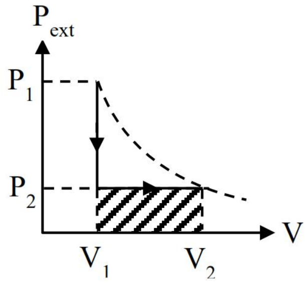
$|W_{irreversible}|_{expansion} < |W_{irreversible}|_{compression}$

d > c = a > b

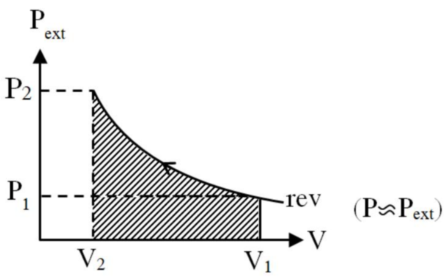
$|W_{irreversible}|_{expansion} = -\,P_{ext}\,(V_f - V_i)$



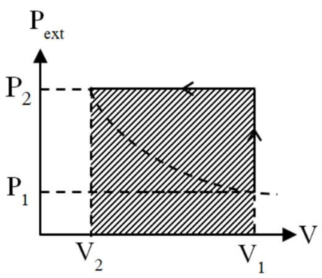
(Reversible expansion)



(Single step irreversible)



(Reversible)



(Single step irreversible)

Graphical representation

We can compare work by area of PV graph.

64. Reactant A converts to product D through the given mechanism (with the net evolution of heat) :

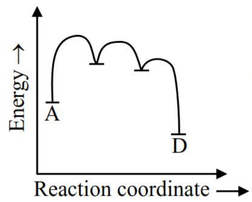
$A \rightarrow B$ slow ; $\Delta H = +ve$

$B \rightarrow C$ fast ; $\Delta H = -ve$

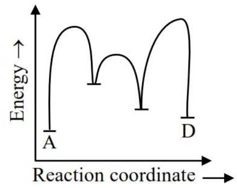
$C \rightarrow D$ fast ; $\Delta H = -ve$

Which of the following represents the above reaction mechanism ?

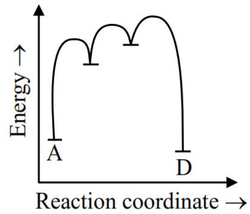
(1)



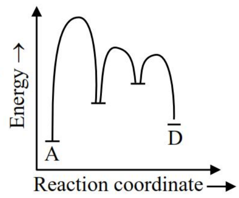
(2)



(3)

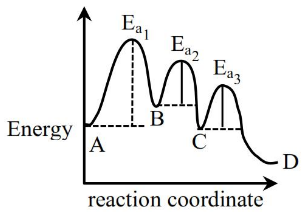


(4)



Ans. **(1)**

Sol.



$A \rightarrow B$ Slow

$\Delta H = +ve \quad E_{a_1} \rightarrow High$

$B \rightarrow C$ Fast

$\Delta H = -ve \quad E_{a_2} \rightarrow Low$

$C \rightarrow D$

$\Delta H = -ve \quad E_{a_3} \rightarrow Low$

65. The nature of oxide (TeO_2) and hydride (TeH_2) formed by Te, respectively are :

(1) Oxidising and acidic

(2) Reducing and basic

(3) Reducing and acidic

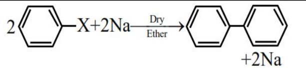
(4) Oxidising and basic

Ans. **(1)**

Sol. TeO_2 is oxidizing in nature because it can be reduced from +4 oxidation state to lower oxidation state.

TeH_2 due to less bond dissociation energy easily breaks and hence acidic in nature.

66. Match **List-I** with **List-II**

List-I (Reaction)		List-II (Name of reaction)	
(A)		(I)	Lucas reaction
(B)	$ArN_2^+X^- \xrightarrow[HCl]{Cu} ArCl + N_2 \uparrow + CuX$	(II)	Finkelstein reaction
(C)	$C_2H_5Br + NaI \xrightarrow[Acetone]{Dry} C_2H_5I + NaBr$	(III)	Fittig reaction
(D)	$CH_3C(OH)(CH_3)CH_3 \xrightarrow[ZnCl_2]{HCl} CH_3C(Cl)(CH_3)CH_3$	(IV)	Gatterman reaction

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

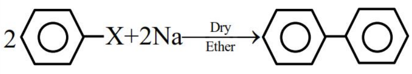
(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Ans. (2)

Sol.

List-I (Reaction)		List-II (Name of reaction)	
(A)		(III)	Fittig reaction
(B)	$ArN_2^+X^- \xrightarrow[HCl]{Cu} ArCl + N_2 \uparrow + CuX$	(IV)	Gatterman reaction
(C)	$C_2H_5Br + NaI \xrightarrow[Acetone]{Dry} C_2H_5I + NaBr$	(II)	Finkelstein reaction
(D)	$CH_3C(OH)(CH_3)CH_3 \xrightarrow[ZnCl_2]{HCl} CH_3C(Cl)(CH_3)CH_3$	(I)	Lucas reaction

67. Consider the following chemical equilibrium of the gas phase reaction at a constant temperature : $A(g) \rightleftharpoons B(g) + C(g)$

If p being the total pressure, K_p is the pressure equilibrium constant and α is the degree of dissociation, then which of the following is true at equilibrium ?

(1) If p value is extremely high compared to K_p, $\alpha \approx 1$

(2) When p increases α decreases

(3) If k_p value is extremely high compared to p, α becomes much less than unity

(4) When p increases α increases

Ans. (2)

Sol. $A(g) \rightleftharpoons B(g) + C(g)$

	A(g)	B(g)	C(g)
t = 0	a	0	0
t = t	a (1 – α)	aα	aα

a moles of A(g) taken initially and at time

Now moles fraction of A(g), B(g) and C(g) are

$$X_A = \frac{a - a\alpha}{a + a\alpha} = \frac{1-\alpha}{1+\alpha}$$

$$X_B = \frac{a\alpha}{a + a\alpha} = \frac{\alpha}{1+\alpha}$$

$$X_C = \frac{a\alpha}{a + a\alpha} = \frac{\alpha}{1+\alpha}$$

Now if P is total pressure then partial pressure of A(g), B(g) and C(g) are

$$P_A = \left(\frac{1-\alpha}{1+\alpha}\right)P$$

$$P_B = \left(\frac{\alpha}{1+\alpha}\right)P$$

$$P_C = \left(\frac{\alpha}{1+\alpha}\right)P$$

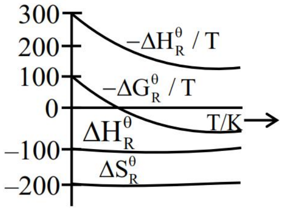
$$K_P = \frac{\left(\frac{\alpha}{1+\alpha}\right)P\left(\frac{\alpha}{1+\alpha}\right)P}{\left(\frac{1-\alpha}{1+\alpha}\right)P}$$

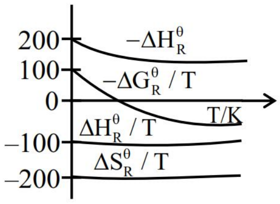
$$K_P = \frac{\alpha^2 P}{1-\alpha^2}$$

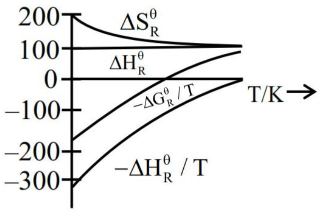
As K_P is only function of temperature.

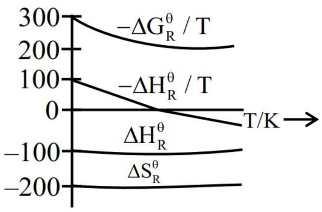
So as $P \uparrow$ $\alpha \downarrow$

68. Which of the following graphs correctly represents the variation of thermodynamic properties of Haber's process ?

(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. $N_2(g) + 3H_2(g) \rightarrow 2NH_3$

$\Delta H° = –ve$

$\Delta S° = –ve$

(As gaseous moles decreases).

(1) As temperature increases $\frac{-\Delta H_R^o}{T}$, decreases

(2) $\Delta G° = –RT \ln K_{eq}$

$R \ln K_{eq} = -\frac{\Delta G^o}{T}$

(on increasing temperature in exothermic reaction K_{eq} decreases)

$\Delta H°$ and $\Delta S°$ are almost constant with temperature.

69. A tetrapeptide "*x*" on complete hydrolysis produced glycine (Gly), alanine (Ala), valine (Val), leucine (Leu) in equimolar proportion each. The number of tetrapeptides (sequences) possible involving each of these amino acids is

(1) 16 (2) 32

(3) 8 (4) 24

Ans. **(24)**

Sol. The number of tetrapeptides (sequences) possible involving each of these amino acids (glycine, alanine, valine, leucine) ; It has three (3) peptides linkage the number of permutations in which they can be arranged

$= 4 \times 3 \times 2 \times 1$

$= 24$

70. In Dumas' method for estimation of nitrogen, 0.5 gram of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound (Aqueous tension at 300 K = 15 mm Hg) is

(1) 1.257 (2) 20.87

(3) 18.67 (4) 12.57

Ans. **(4)**

Sol. Pressure of N_2 gas = (715 – 15)

= 700 mmHg

$$n_{N_2} = \frac{PV}{RT}$$

$$n_{N_2} = \frac{700 \times 60 \times 10^{-3}}{760 \times 0.0821 \times 300}$$

$= 2.24 \times 10^{-3}$ mol

Mass of $N_2 = 2.24 \times 10^{-3} \times 28g$

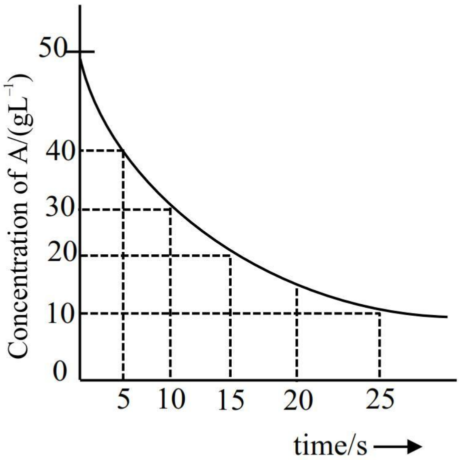
$= 0.06272$ g

$$\%N_2 = \frac{0.06272}{0.5} \times 100 \simeq 12.57$$

SECTION-B

71. For the reaction A → B the following graph was obtained. The time required (in seconds) for the concentration of A to reduce to 2.5 g L^{-1} (if the initial concentration of A was 50 g L^{-1}) is _______. (Nearest integer)

Given : log 2 = 0.3010



Ans. (43)

Sol. As it is difficult to predict order using data provided in graph.

For specific time interval 0 - 5 sec, 5 – 10 sec and 10 – 15 sec. order comes to be zero, but graph is not a straight line.

Assuming 1st order kinetics

$$K = \frac{1}{t}\ln\frac{A_0}{A_t}$$

$$K = \frac{1}{10}\ln\frac{40}{20}$$

Time required to reduce to 2.5 g/L

$$K = \frac{1}{t}\ln\frac{50}{2.5}$$

$$\frac{1}{10}\ln 2 = \frac{1}{t}\ln 20$$

$$t = \frac{1.3010\times 10}{0.3010} = 43.3 \text{ sec.}$$

72. 0.2 % (w/v) solution of NaOH is measured to have resistivity 870.0 mΩ m. The molar conductivity of the solution will be _______$\times 10^2$ mS dm^2 mol^{-1}. (Nearest integer)

Ans. (23)

Sol. Given : Concentration of NaOH = 0.2% (w/v)

∴ 0.2 g of NaOH in 100 ml of solution.

Molarity of NaOH solution

$$= \frac{\text{moles of solute}}{V_{ml}} \times 1000$$

$$= \frac{0.2/40}{100}\times 1000 = = \frac{0.2}{40\times 100}\times 1000 = \frac{2}{40}\text{ M}$$

Given resistivity of solution = 870 m ohm m

$= 870 \times 10^{-3}$ ohm m

$= 870 \times 10^{-3} \times 10$ ohm dm

$= 870 \times 10^{-2}$ ohm dm

= 8.7 ohm dm

Now conductivity

$$K = \frac{1}{\rho} = \frac{1}{8.7}\text{ ohm}^{-1}\text{ dm}^{-1}$$

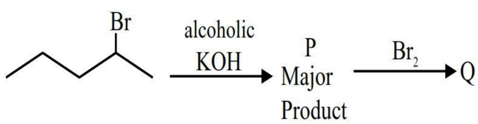
Now molar conductivity of solution is

$$\lambda_m = \frac{K}{M} = \frac{\frac{1}{8.7}}{\frac{2}{40}} = \frac{40}{2\times 8.7} = 2.29\text{ S dm}^2\text{ mol}^{-1}$$

2.29×10^3 m S dm^2 mol^{-1}

$= 22.9 \times 10^2$ m S dm^2 mol^{-1}

$= 23 \times 10^2$ m S dm^2 mol^{-1}

73. 

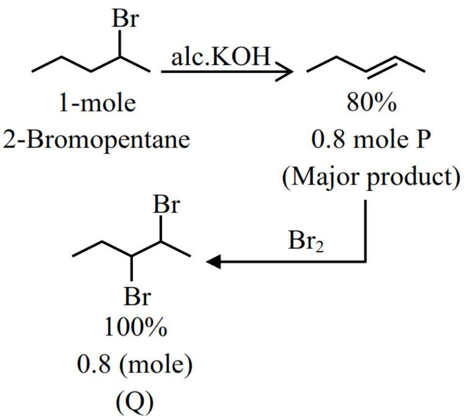
Consider the above sequence of reactions. 151 g of 2-bromopentane is made to react. Yield of major product P is 80% whereas Q is 100%.

Mass of product Q obtained is _________g.

(Given molar mass in g mol^{-1} H: 1, C : 12, O : 16, Br : 80)

Ans. (184)

Sol.



Molecular mass of Q = 230g mol^{-1}

Mass of Q = 0.8 × 230

= 184 g

74. When 1 g each of compounds AB and AB_2 are dissolved in 15 g of water separately, they increased the boiling point of water by 2.7 K and 1.5 K respectively. The atomic mass of A (in amu) is_______$\times 10^{-1}$(Nearest integer)

(Given : Molal boiling point elevation constant is 0.5 K kg mol^{-1})

Ans. (25)

Sol. For AB

$\Delta T_b = 2.7$ K

$2.7 = 1 \times 0.5 \times m$

$m = \frac{27}{5}$

Let molar mass of AB = x.

So $\frac{1/x}{15} \times 1000 = \frac{27}{5}$

x = 12.34

For AB_2

$\Delta T_b = 1.5$ K

$1.5 = 1 \times 0.5 \times m$

m = 3

Let molar mass of AB_2 = y

So $\frac{1/y}{15} \times 1000 = 3$

$y = \frac{1000}{45}$

y = 22.22

Now let a and b be atomic masses of A and B respectively, then

A + b = 12.34 …. (i)

A + 2b = 22.22 …. (ii)

B = 22.22 – 12.34 = 9.88

Now a = 12.34 – 9.88 = 2.46

$= 24.6 \times 10^{-1} = 25 \times 10^{-1}$

75. The spin-only magnetic moment value of M^{n+-} ion formed among Ni, Zn Mn and Cu that has the least enthalpy of atomisation is_____. (in nearest integer)

Here n is equal to the number of diamagnetic complexes among $K_2[NiCl_4]$, $[Zn(H_2O)_6]Cl_2$, $K_3[Mn(CN)_6]$ and $[Cu(PPh_3)_3I]$

Ans. (0)

Sol. $K_2[NiCl_4] \Rightarrow sp^3$, Paramagnetic

$[Zn(H_2O)_6]Cl_2 \Rightarrow sp^3d^2$, Diamagnetic

$K_3[Mn(CN)_6] \Rightarrow d^2sp^3$, Paramagnetic

$[Cu(PPh_3)_3I] \Rightarrow sp^3$, Diamagnetic

Hence the value of n is 2

Least value of enthalpy of atomisation among Ni, Zn, Mn and Cu is of Zn

Zn^{+2} :- $[Ar]3d^{10}$

$\mu = 0$