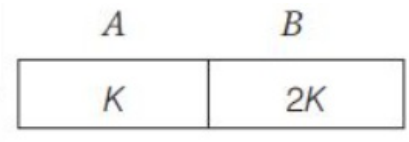
CGPET 2022

Solved Paper

Question 1

Two rods A and B of same dimension but different materials are welded together as shown in figure. Their thermal conductivities are K and $2K$. The thermal conductivity of the composite rod will be



Options:

A. $\frac{2}{3}K$

B. $\sqrt{2}K$

C. $3K$

D. $\frac{4}{3}K$

Answer: D

Solution:

Solution:
In series, equivalent thermal resistance of series combination is $R_S = R_1 + R_2$

$$\Rightarrow \frac{l_1 + l_2}{K_s A} = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A}$$

$\leftarrow l \rightarrow \leftarrow l \rightarrow$
K 2K

In given case, $l_1 = l_2 = l$ and $K_1 = K, K_2 = 2K$ So, we have

$$\frac{2l}{K_s A} = \frac{l}{KA} + \frac{l_2}{2KA}$$

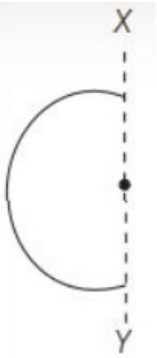
$$\Rightarrow \frac{2}{K_s} = \frac{1}{K} + \frac{1}{2K}$$

$$\Rightarrow \frac{2}{K_s} = \frac{3}{2K}$$

or $K_s = \frac{4K}{3}$

Question 2

A rod of length L and mass M is bent into form of a semi-circular ring (as shown in figure). The moment of inertia about XY-axis is



Options:

A. $\frac{ML^2}{2\pi^2}$

B. $\frac{ML^2}{\pi^2}$

C. $\frac{ML^2}{4\pi^2}$

D. $\frac{2ML^2}{\pi^2}$

Answer: A

Solution:

Solution:
Moment of inertia of a semi-circular ring (using standard results) is
$I = \frac{1}{2}MR^2 \ ...(i)$
As rod of length L is bent into semi-circular ring. Then, $\pi R = L$ or $R = \frac{L}{\pi}$
So, moment of inertia is given by
$I = \frac{1}{2}M \cdot (\frac{L}{\pi})^2$
or
$I = \frac{ML^2}{2\pi^2}$

Question 3

The average life T and the decay constant λ of a radioactive nucleus are related by the expression

Options:

A. $T\lambda = 1$

B. $T = \frac{0.693}{\lambda}$

C. $\frac{T}{\lambda} = 1$

D. $T = \frac{c}{\lambda}$

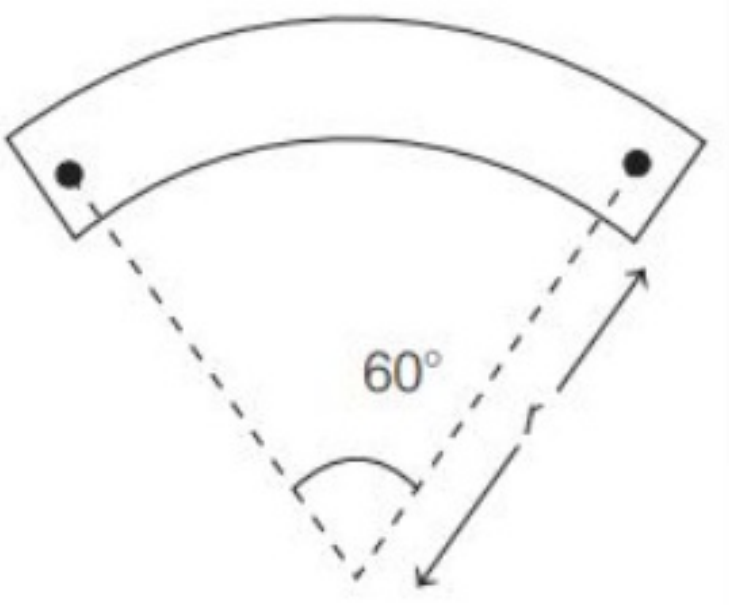
Answer: A

Solution:

Solution:
Decay constant λ and average life T are reciprocal of each other, i.e.
$\lambda = \frac{1}{T}$
$\Rightarrow \lambda T = 1$

Question 4

A bar magnet of length l and magnetic dipole moment M is bent in the form of an arc as shown in figure. The new magnetic dipole moment will be



Options:

A. $\frac{M}{2}$

B. M

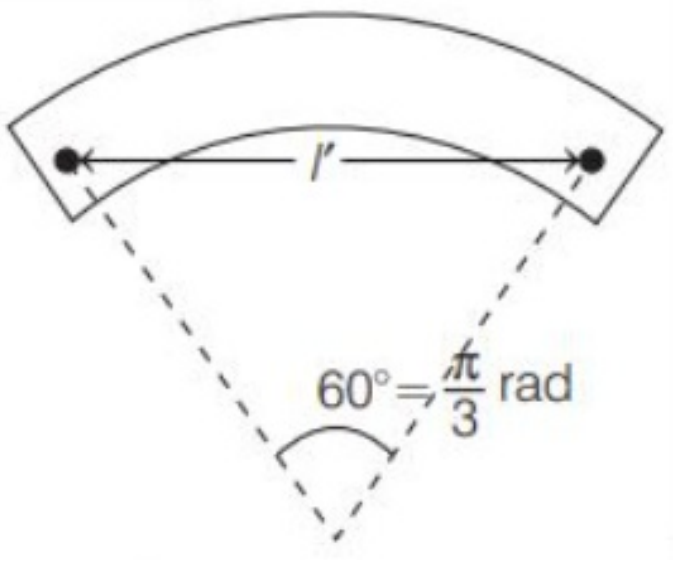
C. $\frac{3M}{\pi}$

D. $\frac{2M}{\pi}$

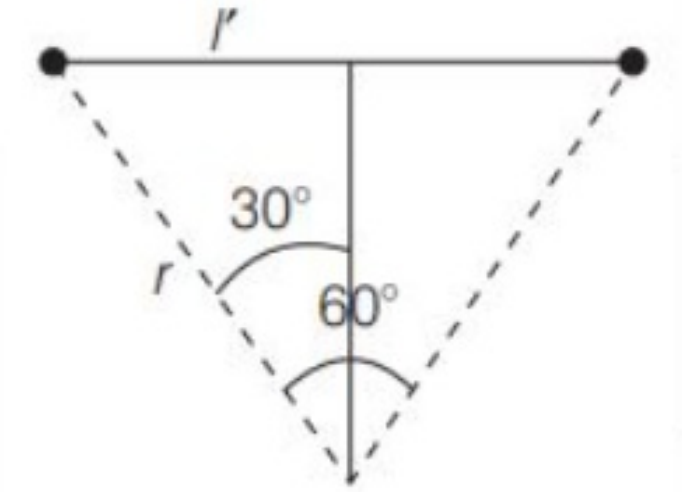
Answer: C

Solution:

Solution:
Initially, magnetic dipole moment of magnet is
$M = \mu \cdot l$
$\leftarrow$ l $\rightarrow$
After bending $M' = \mu . l'$ . . . (ii)

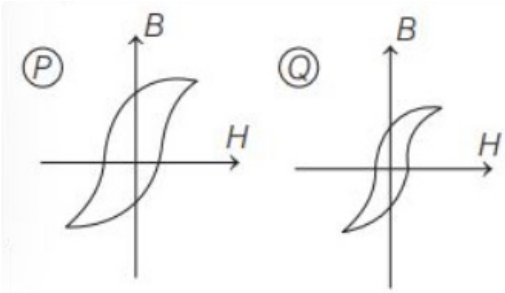


Here, $l = \frac{\pi}{3} . r$ or $\pi = \frac{3l}{\pi}$ . . . (iii)
$\therefore$ M $= \mu \times 2r \sin 30°$
$= \mu \times 2 \times \frac{3l}{\pi} \times \frac{1}{2}$ . . . [From Eqs (iii)]
$\frac{3\mu l}{\pi} = \frac{3M}{\pi}$ . . . [From Eqs (i)]



Question 5

$B-H$ curve of two different materials P and Q are shown in figure. These materials are used to make magnets for electric generator, transformer and electromagnetic core.
Then, it is proper to use



Options:

A. P for electric generators and transformer

B. P for electromagnets and Q for electric generators

C. P for transformers and Q for electric generators or electromagnets

D. Q for electromagnets and transformers

Answer: D

Solution:

Solution:
In $B-H$ curve of material P has large retentivity and coercivity, so it can be used for making permanent magnet. Material Q has less retentivity and coercivity, so it can be used for making core of electromagnetic and transformers.

Question 6

For the angle of minimum deviation of a prism to be equal to its refracting angle, the prism must be made of a material whose refractive index

Options:

A. lies between $\sqrt{2}$ and 1

B. lies between 2 and $\sqrt{2}$

C. is less than 1

D. is greater than 2

Answer: B

Solution:

Solution:
Given, $\delta_m = A$
Now, refractive index of material of prism is

$$n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{2A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

or $n = 2\cos\frac{A}{2}$

As, $0 \le \cos\frac{A}{2} \le 1$

$\Rightarrow 0 \le n \le 2$

Question 7

Three masses of $2\,\text{kg}, 4\,\text{kg}$ and $4\,\text{kg}$ are placed at the three points $(1,0,0), (1,1,0)$ and $(0,1,0)$, respectively. The position vector of its centre of mass is

Options:

A. $\frac{3}{5}\hat{\mathbf{i}} + \frac{4}{5}\hat{\mathbf{j}}$

B. $3\,i + \hat{\hat{\mathbf{j}}}$

C. $\frac{2}{5}\hat{\mathbf{i}} + \frac{4}{5}\hat{\mathbf{j}}$

D. $\frac{1}{5}\hat{\mathbf{i}} + \frac{4}{5}\hat{\mathbf{j}}$

Answer: A

Solution:

Solution:
7. (a) Coordinates by centre of mass are given by

$$\overline{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$
$$= \frac{2\times 1 + 4\times 0 + 4\times 1}{2+4+4} = \frac{6}{10} = \frac{3}{5}$$

and

$$\overline{y} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$
$$= \frac{2\times 0 + 4\times 1 + 4\times 1}{2+4+4} = \frac{8}{10} = \frac{4}{5}$$

Also, $\overline{z} = \frac{m_1z_1 + m_2z_2 + m_3z_3}{m_1 + m_2 + m_3}$

$$= \frac{2\times 0 + 4\times 0 + 4\times 0}{2+4+4} = 0$$

Position vector of Centre of mass is

$$r = x\hat{i} + y\hat{j} + z\hat{k} = 35\,\hat{i} + \frac{4}{5}\hat{j}$$

Question 8

The wavelength of X-rays is the order of

Options:

A. centimetre

B. micron (10^{-6}m)

C. angstrom (10^{-10}m)

D. metre

Answer: C

Solution:

Solution:
X-rays are electromagnetic waves with wavelength range, 0.1Å to 100Å.

Question 9

Photoelectric cell is a device which converts

Options:

A. light energy into electrical energy

B. chemical energy into electrical energy

C. electrical energy into light energy

D. magnetic energy into electrical energy

Answer: A

Solution:

Solution:
A photoelectric cell or a photocell converts light energy into electrical energy.

Question 10

The angular speed of a flywheel making 120 rev / min is

Options:

A. 2π rad / s

B. $4\pi^2$ rad / s

C. π rad / s

D. 4π rad / s

Answer: D

Solution:

Solution:
Angular speed
$= 120$ rpm
$= \frac{120 \times 2\pi}{60}$ rad / $s = 4\pi$ rad / s

Question 11

The capacity of a parallel plate capacitor depends on

Options:

A. nature of the plates

B. distance between the plates

C. thickness of the plates

D. potential difference between the plates

Answer: B

Solution:

Solution:
Capacity of a parallel plate capacitor is
$C = \frac{\varepsilon_0 A}{d}$
where,
A =
area of plates and
d =
distance between plates.
So,
$C \propto A$
and
$C \propto \frac{1}{d}$

Question 12

A point mass m is placed inside a spherical shell of radius R and mass M at a distance $\frac{R}{2}$ from the centre of the shell. The gravitational force exerted by the shell on the point mass is

Options:

A. $\frac{GMm}{R^2}$

B. $-\frac{GMm}{R^2}$

C. zero

D. $4\frac{GMm}{R^2}$

Answer: C

Solution:

Solution:
Inside a hollow sphere, gravitational field is zero. So, force exerted on a mass placed anywhere inside shell is always zero.

Question 13

A ray of light travelling in a direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$ is incident on a plane mirror.After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is

Options:

A. 30°

B. 45°

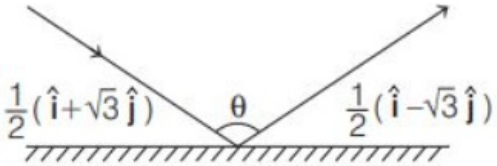
C. 60°

D. 120°

Answer: C

Solution:

Solution:



Let θ is angle between incident ray vector and reflected ray vector.
Then, using $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$
We have,
$\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j}) \cdot \frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$
$= \frac{1}{2}(\sqrt{1^2+3}) \cdot \frac{1}{2}(\sqrt{1^2+3}) \cdot \cos\theta \Rightarrow \frac{1}{4}(1-3) = \frac{1}{4} \times 4 \times \cos\theta$
$\Rightarrow \cos\theta = \frac{-1}{2}$
or $\theta = 120^\circ$
So, angle of incidence $= \frac{\theta}{2}$
60°

Question 14

The energy related per fission of a $_{92}U^{235}$ nucleus is nearly

Options:

A. 200 eV

B. 20 eV

C. 200 MeV

D. 2000 eV

Answer: C

Solution:

Solution:
Energy released per fission of $^{235}_{92}U$ is around $200\,MeV$.

Question 15

If the number of holes and electrons are n_h and n_e respectively, then the correct relation for pure semiconductor

Options:

A. $n_h > n_e$

B. $n_h < n_e$

C. $n_h = n_e$

D. $n_h^2 = n_e^2$

Answer: C

Solution:

Solution:
For a pure semiconductor number of holes and electrons are equal at every temperature.
$\therefore n_h = n_e$

Question 16

The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}\text{m} / s^2$. In the third second is

Options:

A. $\frac{10}{3}\text{m}$

B. $\frac{19}{3}m$

C. 6m

D. 4m

Answer: A

Solution:

Solution:
Given, $u = 0, a = 4/3 \text{ms}^{-2}$
Distance travelled by particle in first $3s$ is
$s_3 = ut + \frac{1}{2}at^2$
$= \frac{1}{2} \times \frac{4}{3} \times 3^2 = 6\text{m}$
and distance travelled in first $2s$ is
$s_2 = \frac{1}{2} \times \frac{4}{3} \times 2^2 = 8/3\text{m}$
So, distance travelled in 3rd second
$= s_3 - s_2 = 6 - \frac{8}{3} = \frac{10}{3}\text{m}$

Question 17

A spring of force constant 800N/m has an extension of 5cm. The work done in extending it from 5cm to 15cm is

Options:

A. 16J

B. 8J

C. 32J

D. 24J

Answer: B

Solution:

Solution:
For a spring work of extension is given by
$W = \frac{1}{2}k(x_f^2 - x_i^2)$
Here, $k =$ spring constant $= 800\text{N/m}$
$x_i =$ initial extension $= 5\text{cm}$
$= 5 \times 10^{-2}\text{m}$
$x_f =$ final extension $= 15\text{cm}$
$= 15 \times 10^{-2}\text{m}$
So, work done,
$W = \frac{1}{2} \times 800[(15 \times 10^{-2})^2 - (5 \times 10^{-2})^2]$
$= 8\text{J}$

Question 18

The dimensional formula of Boltzmann's constant is

Options:

A. $[\mathrm{ML^2T^{-2}\theta^{-1}}]$

B. $[\mathrm{ML^2T^{-2}}]$

C. $[\mathrm{ML^0T^{-2}\theta^{-1}}]$

D. $[ML^{-2}T^{-1}\theta^{-1}]$

Answer: A

Solution:

Solution:
Energy of a gas molecule per degree of freedom is $\mathrm{E} = \frac{1}{2}k_{\mathrm{B}}\mathrm{T}$
So, or
$k_{\mathrm{B}} = \frac{2\mathrm{E}}{\mathrm{T}}$
$[k_{\mathrm{B}}] = \frac{[\mathrm{E}]}{[\mathrm{T}]} = \frac{[\mathrm{ML^2T^{-2}}]}{[\theta]}$
$= [\mathrm{ML^2T^{-2}\theta^{-1}}]$

Question 19

The magnetic flux linked with a loop is $\varphi = 6t^2 + 7t + 1$, where φ is in milli weber and t is in second. What will be emf induced at $t = 2s$?

Options:

A. $-49\,\mathrm{mV}$

B. $-39\,\mathrm{mV}$

C. $-31\,\mathrm{mV}$

D. $-19\,\mathrm{mV}$

Answer: C

Solution:

Solution:
Induced emf is given by $\mathrm{E} = -\frac{d}{dt}\varphi_{\mathrm{B}}$
$\Rightarrow \mathrm{E} = -\frac{d}{dt}(6t^2 + 7t + 1) = -[12t + 7]$
So, $\mathrm{E}(\text{at } t = 2s)$
$= -(12 \times 2 + 7) = -31\,\mathrm{mV}$

Question 20

A body of mass $1000\,\mathrm{kg}$ is moving horizontally with a velocity $50\mathrm{m}/s$. A mass of $250\,\mathrm{kg}$ is added. Find the final velocity.

Options:

A. $40\text{m} / s$

B. $20\text{m} / s$

C. $30\sqrt{2}\text{m} / s$

D. $50\text{m} / s$

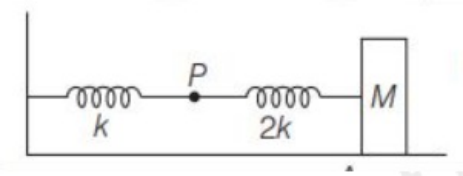
Answer: A

Solution:

Solution:
Momentum is conserved in the process of adding mass, so
$m_i v_i = m_f v_f$
$\Rightarrow 1000 \times 50 = (1000 + 250)\, v_f$
$\Rightarrow v_f = \frac{1000 \times 50}{1250}$
$= 40\text{ms}^{-1}$

Question 21

The mass M oscillates in simple harmonic motion with amplitude A as shown in figure. The amplitude of point P is



Options:

A. $\frac{A}{2}$

B. 2A

C. $\frac{A}{3}$

D. $\frac{2A}{3}$

Answer: D

Solution:

Solution:
Let x_1 and x_2 are displacements of springs, then
$A = x_1 + x_2 \quad ...(i)$
Also, force on both springs is same
$\Rightarrow k_1 x_1 = k_2 x_2$
$\Rightarrow kx_1 = 2kx_2$ or $x_2 = \frac{x_1}{2}$
Substituting in Eq. (i), we have

$x_1 = \frac{2}{3}A$

Question 22

Same force acts on two bodies of different masses 3 kg and 5 kg initially at rest. The ratio of times required to acquire same final velocity is

Options:

A. 5:3

B. 25:9

C. 9:25

D. 3:5

Answer: D

Solution:

Solution:
Let force on bodies is F, then accelerations of bodies are
$a_1 = \frac{F}{3}$ and $a_2 = \frac{F}{5}$
Final velocities are given by $v = u + at$ with $u = 0; v = at$
$\therefore v_1 = a_1 t_1$
and $v_2 = a_2 t_2$
As final velocities are given equal
$\Rightarrow v_1 = v_2$
or $a_1 t_1 = a_2 t_2$
$\Rightarrow \frac{t_1}{t_2} = \frac{a_2}{a_1} = \frac{F/5}{F/3} = \frac{3}{5}$

Question 23

The quantity which remains unchanged in a transformer is

Options:

A. voltage

B. current

C. frequency

D. None of these

Answer: C

Solution:

Solution:
In a transformer, voltage or current can be increased or decreased but input frequency is always equals to output frequency.

Question 24

Electron-volt is the unit of

Options:

A. charge

B. potential

C. momentum

D. energy

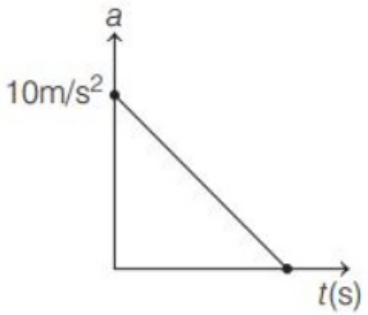
Answer: D

Solution:

Solution:
Electron-volt is unit of energy,
$1\,\text{eV} = 1.6 \times 10^{-19}\text{J}$

Question 25

A particles starts from rest. Its acceleration (a) versus time (t) graph is shown in the figure. The maximum speed of the particle will be



Options:

A. 110m / s

B. 55m / s

C. 550m / s

D. 660m / s

Answer: B

Solution:

Solution:
Final speed of particle
$= \frac{1}{2}at^2 =$ Area under $a - t$ graph
$= \frac{1}{2} \times 10 \times 11 = 55\text{m} / s$

Question 26

The number of electrons in 1C charge are

Options:

A. 5.46×10^{29}

B. 1.6×10^{-19}

C. 625×10^{18}

D. 9.5×10^{-16}

Answer: C

Solution:

Solution:
Using $\text{Q} = ne$,
$\text{Q} = 1\text{C}, e = 1.6 \times 10^{-19}\text{C}$
We have,
$n = \frac{\text{Q}}{e} = \frac{1}{1.6 \times 10^{-19}}$
$= 6.25 \times 10^{18}$ electrons

Question 27

A train moving at a speed of $220\text{m} / \text{s}$ towards a stationary object, emits a sound of frequency $1000\,\text{Hz}$. Some of the sound reaching the object gets reflected back to the train as echo. The frequency of the echo as detected by a person in the train is (velocity of sound in air $= 330\text{m} / \text{s}$)

Options:

A. $3500\,\text{Hz}$

B. $4000\,\text{Hz}$

C. $5000\,\text{Hz}$

D. $3000\,\text{Hz}$

Answer: C

Solution:

Frequency of reflected sound observed by an observer on moving train is

$f_2 = f_0(\frac{v + v_s}{v - v_s})$

Here,

f_0 = emitted sound′ s frequency

v_s = speed of scource

v = speed of sound

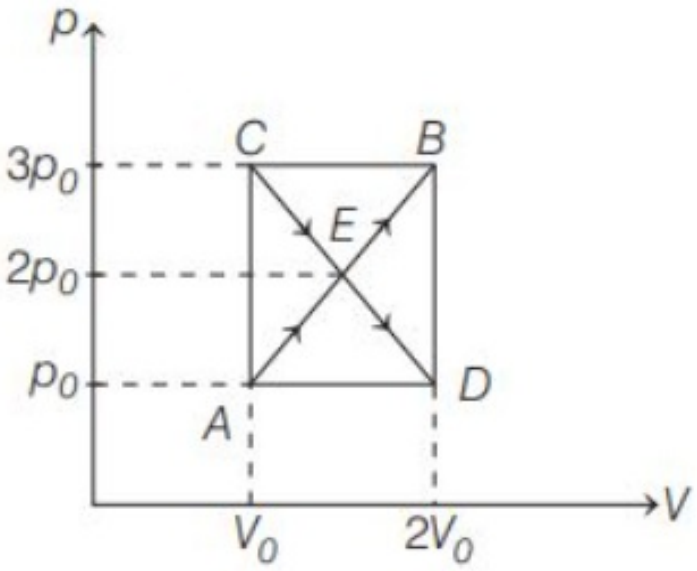
So, frequency of reflected sound in given case will be

$f_2 = 1000(\frac{330 + 220}{330 - 220}) = 1000 \times \frac{550}{110}$

$= 5000\,\text{Hz}$

Question 28

A thermodynamic system undergoes cyclic process $ABCDA$ as shown in figure. The work done by the system in the cycle is



Options:

A. $p_0 V_0$

B. $2p_0 V_0$

C. $\frac{p_0 V_0}{2}$

D. Zero

Answer: D

Solution:

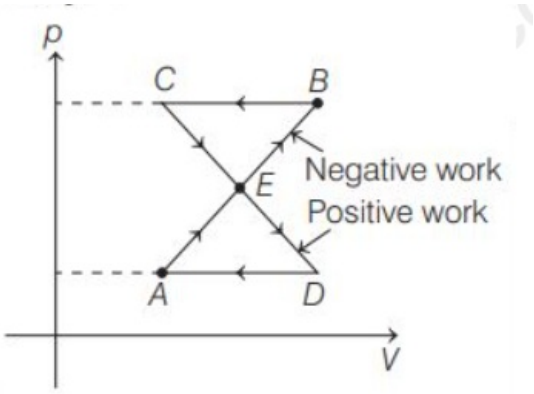
Solution:

Work done in a cyclic process

= Area under $p - V$ graph .

Also, clockwise cycle means work done is positive and anti-clockwise cycle means work done is negative.

In given cycle



Work done = W_{ABCDA}
$= W_{EBCE} + W_{AEDA}$
$= -(\frac{1}{2} \times V_0 \times p_0) + (\frac{1}{2} \times V_0 \times p_0) = 0$

Question 29

The lower end of a capillary tube of radius r is placed vertically in a water of density ρ and surface tension S. The rise of water in the capillary tube is upto height h, then heat evolved is

Options:

A. $+\frac{\pi^2 r^2 h^2 \rho g}{J}$

B. $+\frac{\pi r^2 h^2 \rho g}{2J}$

C. $-\frac{\pi^2 r^2 h^2 \rho g}{2J}$

D. $-\frac{\pi r^2 h^2 \rho g}{J}$

Answer: B

Solution:

Solution:
Heat evolved
= Work done by force of surface tension
Now, force due to surface tension,
F = surface tension × circumference of tub
$\Rightarrow F = 2\pi r \times T$ . . . [r = radius of tube]
where, surface tension,
$T = \frac{h\rho g r}{2}$
Also, displacement of Centre of mass of liquid
$= \frac{h}{2}$
So, work done by force of surface tension
W = force × displacement
$= 2\pi r \times T \times \frac{h}{2}$
$= 2\pi r \times \frac{h\rho g r}{2} \times \frac{h}{2}$

$\Rightarrow W = \frac{\pi r^2 h^2 \rho g}{2}$

Converting into units of heat,

$Q = \frac{\pi r^2 h^2 \rho g}{2J}$ . . . [$\because Q = \frac{W}{J}$]

Question 30

The current gain of transistor in CE mode is 49. What will be the current gain of the same transistor in CB mode?

Options:

A. 49

B. 0.98

C. 4.9

D. 98

Answer: B

Solution:

Solution:

Relation between current gain in CE mode (β) and current gain in CB mode (α) is

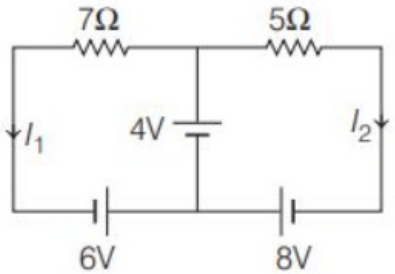
$\alpha = \frac{\beta}{1+\beta}$

Here, given $\beta = 49$

$\therefore \alpha = \frac{49}{1+49} = \frac{49}{50} = 0.98$

Question 31

In the given circuit, currents I_1 and I_2 are respectively



Options:

A. $\frac{8}{7}$A, $\frac{12}{5}$A

B. $\frac{8}{7}$A, $\frac{12}{7}$A

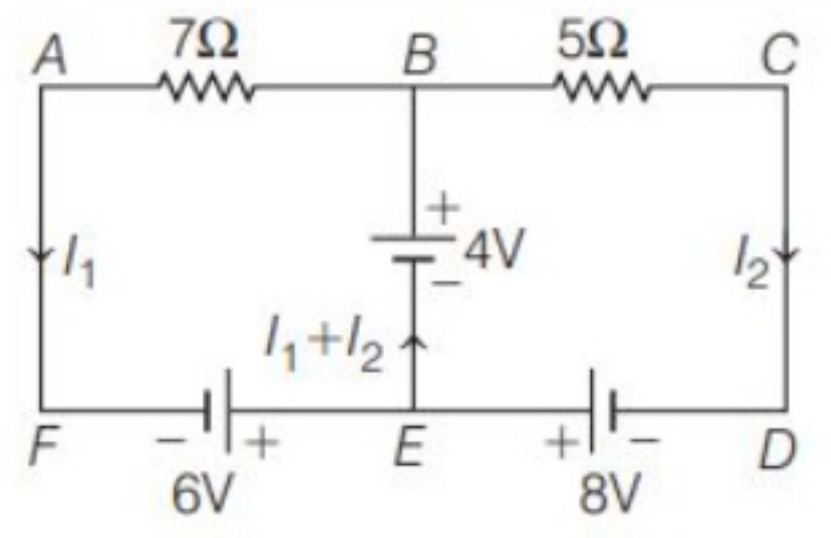
C. $\frac{10}{7}$A, $\frac{12}{5}$A

D. $\frac{10}{7}$A, $\frac{12}{7}$A

Answer: C

Solution:

Solution:



By KVL in loop ABEFA, we have
$\Sigma V = 0$
$\Rightarrow 6 + 4 - 7I_1 = 0$
$\Rightarrow I_1 = \frac{10}{7}$A
In loop BCDEB,
$\Rightarrow 8 + 4 - 5I_2 = 0$
$I_2 = \frac{12}{5}$A

Question 32

The root mean square velocity of hydrogen molecules at 300K is 1930m / s. The rms velocity of oxygen molecules at 1200K will be

Options:

A. 765m / s

B. 1065m / s

C. 965m / s

D. 865m / s

Answer: C

Solution:

Solution:

$v_{rms} \propto \sqrt{\frac{T}{M}}$

$\Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \times \sqrt{\frac{M_1}{M_2}}$

Here, $v_1 = 1930m / s T_1 = 300K$,

$M_1 = 2$ unit, $T_2 = 1200K, M_2 = 32$ unit

$\therefore v_2 = v_1 \sqrt{\left(\frac{T_2}{T_1} \times \frac{M_1}{M_2}\right)}$

$= 1930 \times \sqrt{(\frac{1200}{300} \times \frac{2}{32})}$

$= 1930 \times \sqrt{\frac{8}{32}}$

$= \frac{1930}{2} = 965m / s$

Question 33

A thin convex lens made from crown glass ($\mu = \frac{3}{2}$) has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2, respectively. The correct relation between the focal lengths, is

Options:

A. $f_2 > f_1 f_1$ become negative

B. f_1 and f_2 both become negative

C. $f_1 > f$ and f_2 become negative

D. $f_1 = f_2 < f$

Answer: C

Solution:

Solution:

If lens is dipped in a medium of higher refractive index, then its nature is changed. In given case,

$\frac{5}{3} > \frac{3}{2}$

So, convex lens behaves like a concave lens in second medium. Hence, f_2 is negative.

Also, when lens is in first medium ($n = \frac{4}{3}$), its power reduces or $f_1 > f$.

Question 34

The escape velocity of a body from the surface of the earth is v_e. The escape velocity of the same body from a height equal to $7R$ from earth's surface will be

Options:

A. $\frac{v_e}{\sqrt{2}}$

B. $\frac{v_e}{2}$

C. $\frac{v_e}{2\sqrt{2}}$

D. $\frac{v_e}{4}$

Answer: C

Solution:

Solution:
Let M = mass, R_e = radius of earth, then
$v_e = \sqrt{\frac{2GM}{R_e}}$
Now, when object is at a height of 7R from earth's surface, let escape speed needed is v_e', then by energy conservation,
PE + KE at separation ' $8R'$ from earth's Centre = Total energy at infinity
$\Rightarrow \frac{-GMm}{8R_e} + \frac{1}{2}m(v_e')^2 = 0$
$\Rightarrow \frac{1}{2}m(v_e')^2 = \frac{GMm}{8R_e}$
$\Rightarrow v_e' = \frac{1}{2\sqrt{2}} \cdot \left(\sqrt{\frac{2GM}{R_e}}\right)$
$= \frac{1}{2\sqrt{2}} \cdot v_e$

Question 35

The speed of light (c), gravitational constant (G) and Planck's constant (h) are taken as the fundamental units in a system. The dimension of time in this new system should be

Options:

A. $\left[G^{\frac{1}{2}} h^{\frac{1}{2}} c^{\frac{-5}{2}}\right]$

B. $\left[G^{\frac{-1}{2}} h^{\frac{1}{2}} c^{\frac{1}{2}}\right]$

C. $\left[G^{\frac{1}{2}}h^{\frac{1}{2}}c^{\frac{-3}{2}}\right]$

D. $\left[G^{\frac{1}{2}}h^{\frac{1}{2}}c^{\frac{1}{2}}\right]$

Answer: A

Solution:

Solution:
We have,
$\mathrm{T} = \mathrm{G}^{a} \cdot h^{b} \cdot c^{c}$
$\Rightarrow \mathrm{T} = [\mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}]^{a}[\mathrm{ML}^{2}\mathrm{T}^{-1}]^{b}[\mathrm{LT}^{-1}]^{c}$
Equating dimensions of similar physical quantities, we have
$-a + b = 0 \ \ldots(i)$
$3a + 2b + c = 0 \ \ldots(ii)$
$-2a - b - c = 1 \ \ldots(iii)$
Adding Eqs. (ii) and (iii), we have
$a + b = 1 \ \ldots(iv)$
From adding Eqs. (i) and (iv), we have
$b = \frac{1}{2}$
$\Rightarrow a = \frac{1}{2}$
Hence, $c = \frac{-5}{2}$
So, we can write
$[\mathrm{T}] = [\mathrm{G}^{1/2}h^{1/2}c^{-5/2}]$

Question 36

At two points P and Q on screen in Young's double slit experiment, wave from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$, respectively. The ratio of intensities at P and Q will be

Options:

A. 3:2

B. 2:1

C. $\sqrt{2}$:1

D. 4:1

Answer: B

Solution:

Solution:
Intensity in YDSE is given by
$\mathrm{I}_1 + \mathrm{I}_2 + 2\sqrt{\mathrm{I}_1}\sqrt{\mathrm{I}_2}\cos\varphi$
where, $\varphi =$ phase difference.
Here at point P,
$\mathrm{I}_1 = \mathrm{I}_2 = \mathrm{I}_0$ and $\varphi = 0$

$\Rightarrow I_P = 4I_0$
At point Q ,
$I_1 = I_2 = I_0$
$\Delta L = \frac{\lambda}{4}$ So $\varphi = \frac{2\pi}{4} = \frac{\pi}{2}$
$\Rightarrow I_Q = 2I_0$
$\therefore \frac{I_P}{I_Q} = \frac{4}{2} = \frac{2}{1}$

Question 37

3 moles of oxygen are mixed with 2 moles of helium. What will be the ratio of specific heats at constant pressure and constant volume for the mixture?

Options:

A. 6.7

B. 1.5

C. 4

D. None of these

Answer: B

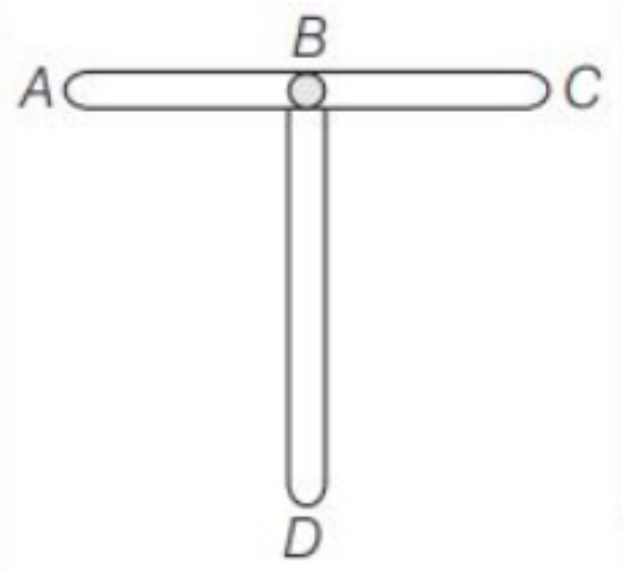
Solution:

Solution:
For a gas mixture,
$C_{p_{mix}} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$
and $C_{V_{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$
Here, $n_1 = 3, n_2 = 2$
$C_{V_1} = \frac{f}{2}R = \frac{5}{2}R; C_{V_2} = \frac{3}{2}R$
$C_{p_1} = (\frac{f}{2} + 1)R = \frac{7}{2}R;$
$C_{p_2} = \frac{5}{2}R$
Substituting values, we have
$C_{p_{mix}} = \frac{3 \times \frac{7}{2}R + 2 \times \frac{5}{2}R}{3 + 2} = \frac{31}{5}$
$C_{V\,mix} = \frac{3 \times \frac{5}{2}R + 2 \times \frac{3}{2}R}{3 + 2} = \frac{21}{5}$
Hence, ratio $\frac{C_{p_{mix}}}{C_{V_{mix}}} = \frac{31}{21} \approx \frac{3}{2}$ or 1.5

Question 38

Three conducting rods of same material and cross-section are shown in figure. Temperature of A, D and C are maintained at $20^\circ C$, $90^\circ C$ and

0 °C, respectively. The ratio of the lengths of BC and BD, if there is no flow in AB is



Options:

A. $\frac{2}{9}$

B. $\frac{9}{2}$

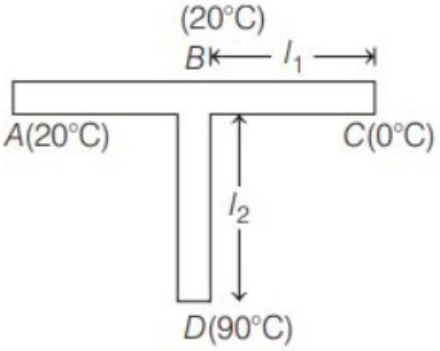
C. $\frac{2}{7}$

D. $\frac{7}{2}$

Answer: C

Solution:

Solution:



As there is no heat flow in AB this means junction B is at $20^\circ C$.
Hence, heat in flow from DB = heat out flow from BC.

$\Rightarrow H_{DB} = H_{BC}$

$\Rightarrow \frac{kA(\theta_D - \theta_B)}{l_2} = \frac{kA(\theta_B - \theta_C)}{l_1}$

As rods are of same material and area,

$\frac{\theta_D - \theta_B}{l_2} = \frac{\theta_B - \theta_C}{l_1}$

$\Rightarrow \frac{l_1}{l_2} = \frac{\theta_B - \theta_C}{\theta_D - \theta_B} = \frac{20 - 0}{90 - 20} = \frac{2}{7}$

Question 39

The force on a charge q placed in a uniform electric field E will be

Options:

A. $\frac{E}{q}$

B. qE

C. E

D. $\frac{q}{E}$

Answer: B

Solution:

Solution:

As, $E = \frac{F}{q}$

$\Rightarrow F = qE$

Question 40

Wattless current means

Options:

A. current is zero

B. the average power consumed in a cycle is zero

C. emf is zero

D. the phase difference between current and potential difference is zero

Answer: B

Solution:

Solution:
Wattless current means current of inductor or capacitor due to which there is no power loss. Average power consumed in an inductor or capacitor is zero.

Question 41

The hydrogen atom is in d-state. For this state, values of m are

Options:

A. 2, 1, 0

B. $-1, 0, 1$

C. $-2, -1, 0, 1, 2$

D. $-1, -2, -3, -4$

Answer: C

Solution:

Solution:
In d-state, $n = 3$
Hence, for a hydrogen atom is in d-state, l = 2 and values of magnetic quantum number m are
$m = (2l + 1) = 2(2) + 1 = 5$
$\therefore m = -2, 1, 0, 1, 2$

Question 42

A uniform cylindrical rod of length L, cross-sectional area A and Young's modulus Y is acted upon by the forces as shown in figure. The elongation of the rod is

Options:

A. $\frac{2FL}{3AY}$

B. $\frac{3FL}{5AY}$

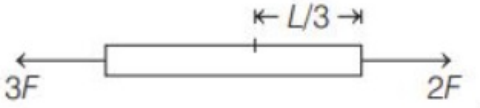
C. $\frac{3FL}{8AY}$

D. $\frac{8FL}{3AY}$

Answer: A

Solution:

Solution:
(Note Q does not match with diagram given)



Net force on portion of length $\frac{L}{3} = 2F$
So, Young's modulus is

$$Y = \frac{\frac{2F}{A}}{\Delta L/(L/3)} = \frac{2}{3} \cdot \frac{FL}{A\Delta L}$$
$$\Rightarrow \Delta L = \frac{2}{3}\frac{F \cdot L}{A \cdot Y}$$

Question 43

If the self-inductance of two coils are L_1 and L_2, respectively. The maximum possible mutual inductance is

Options:

A. L_1L_2

B. $\frac{L_1}{L_2}$

C. $\sqrt{\frac{L_1}{L_2}}$

D. $\sqrt{L_1L_2}$

Answer: D

Solution:

Solution:
For two coils, mutual inductance and self-inductances are related as
$M = k\sqrt{L_1 \cdot L_2}$
where, $k =$ coefficient of magnetic coupling.
Now, M is maximum when $k = 1$
$\Rightarrow M_{max} = \sqrt{L_1 \cdot L_2}$

Question 44

A particle of mass m is executing oscillations about the origin on the X-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a, then the time period T is

Options:

A. proportional to $\frac{1}{\sqrt{a}}$

B. independent of a

C. proportional to $\sqrt{a}$

D. proportional to $a^{3/2}$

Answer: A

Solution:

Solution:
Maximum potential energy,
$U_{max} = ka^3$ (..Given)
But $U_{max} = \frac{1}{2}m\omega^2a^2$
Equating, we have
$\frac{1}{2}m\omega^2a^2 = ka^3$
$\Rightarrow \omega^2 = \frac{2ka}{m} \Rightarrow \frac{4\pi^2}{T^2} = \frac{2ka}{m}$
$\Rightarrow T = \sqrt{\frac{4\pi^2m}{2ka}}$ or $T \propto \frac{1}{\sqrt{a}}$

Question 45

A heavy uniform chain lies on a horizontal table-top. If the coefficient of friction between the chain and table surface is 0.25, then the maximum fraction of length of the chain, that can hang over one edge of the table is

Options:

A. 20 %

B. 25 %

C. 35 %

D. 15 %

Answer: A

Solution:

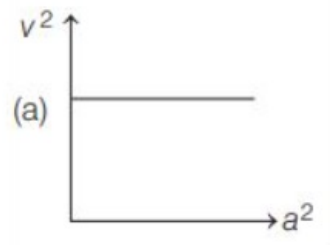
Solution:
Let $(\frac{1}{n})$ th part of chain hangs in equilibrium. Then, $(1-\frac{1}{n})$ th part lies on table.
In equilibrium, downward gravitational pull = Force of friction.
$\Rightarrow (\frac{1}{n}) \cdot \mathrm{L} \cdot x \cdot g = (1-\frac{1}{n}) \cdot L \cdot x \cdot g \cdot \mu$
Here, $x =$ mass per unit length, L = length of complete chain.
So, $n-1=\frac{1}{\mu}$
$\Rightarrow n=1+\frac{1}{\mu}=1+\frac{1}{0.25}=5$
$\therefore \frac{1}{5}$ th part of chain $= 20\%$ of total length can over hang.

Question 46

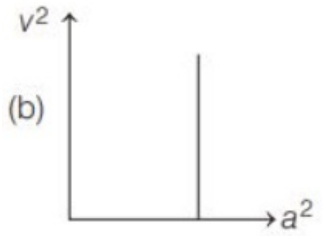
A mass m is performing linear simple harmonic motion, then which of the following graph represents correctly the variation of acceleration a corresponding to linear velocity v ?

Options:

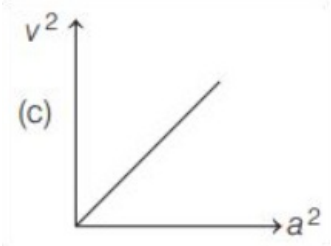
A.



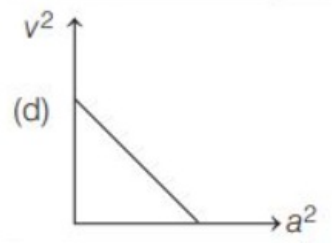
B.



C.



D.



Answer: D

Solution:

Solution:
In SHM , velocity, $v = \omega\sqrt{A^2 - y^2}$
$v^2 = \omega^2(A^2 - y^2)$ . . . (i)
and acceleration, $a = -\omega^2 y$
$\Rightarrow a^2 = \omega^4 y^2$ . . . (ii)
From Eqs . (i) and (ii) , we have
$\Rightarrow v^2 = \omega^2 A^2 - \omega^2 y^2$
$\Rightarrow v^2 = \omega^2 A^2 - \omega^2 a^2$
$\Rightarrow v^2 = -\omega^2 a^2 + \omega^2 A^2$
Comparing with $y = mx + c$
Above equation is a straight line with negative slope and positive intercept.

Question 47

If mass of a body, M on the surface of the earth, then the mass of the same body on the surface of the moon is

Options:

A. $\frac{M}{6}$

B. Zero

C. M

D. None of these

Answer: C

Solution:

Solution:
Mass is quantity of matter and its value doesnot change from place-to-place.

Question 48

A magnet is cut in three equal parts by cutting it perpendicular to its length. The time period of original magnet is T_0 in a uniform magnetic field B.
Then, the time period of each part in the same magnetic field is

Options:

A. $\frac{T_0}{2}$

B. $\frac{T_0}{3}$

C. $\frac{T_0}{4}$

D. T_0

Answer: D

Solution:

Solution:

Let magnetic moment of magnet is M and its moment of inertia is I. Then, magnetic moment of each part will be $\frac{M}{3}$, and moment of inertia of each part will be $\frac{I}{3}$.

As, $T \propto \sqrt{\frac{I}{M}}$

$\Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{I_2}{I_1}} \times \sqrt{\frac{M_1}{M_2}}$

$= \sqrt{\frac{1}{3}} \times \sqrt{\frac{3}{1}} = 1$

Hence, $T_2 = T_1$

Question 49

For a satellite, if the time of revolution is T, then its kinetic energy is proportional to

Options:

A. $\frac{1}{=}$

B. $\frac{1}{0}$

C. $\frac{1}{0}$

D. $T^{-\frac{2}{3}}$

Answer: D

Solution:

Solution:

We have,

$E_k = \frac{1}{2}mv^2$

where, $v = \frac{2\pi r}{T}$

$\therefore E_k = \frac{1}{2}m(\frac{2\pi r}{T})^2$

$\Rightarrow E_k \propto \frac{r^2}{T^2}$

As for a satellite, $T^2 \propto r^3$

$\Rightarrow r^2 \propto T^{\frac{4}{3}}$

$\Rightarrow E_k \propto \frac{T^{\frac{4}{3}}}{T^2} = T^{\frac{4}{3}-2}$

$\Rightarrow E_k \propto T^{-\frac{2}{3}}$

Question 50

If g is acceleration due to gravity on the surface of the earth, its value at the height equal to double the radius of the earth is

Options:

A. g

B. $\frac{g}{2}$

C. $\frac{g}{3}$

D. $\frac{g}{9}$

Answer: D

Solution:

Solution:

Acceleration of gravity at height h above earth's surface is

$g_h = g(\frac{R}{R+h})^2$

At $h = 2R$,

$g_h = g(\frac{R}{R+2R})^2$

$\Rightarrow g_h = \frac{g}{9}$

Question 51

Correct order of electronegativity is

Options:

A. F > O > Cl = N

B. F > Cl > O > N

C. Cl > F > O < N

D. None of the above

Answer: A

Solution:

Solution:
Left to right group 1 to 17 electronegativity
increase while top to bottom decreases.
Fluorine in group 17 at positioned in IInd period.
Hence, correct order of electronegativity is
F > O > Cl = N

Question 52

Correct order of electronegativity is

Options:

A. F > O > Cl = N

B. F > Cl > O > N

C. Cl > F > O < N

D. None of the above

Answer: A

Solution:

Solution:
Copper (Cu) and mercury (Hg) have positiveelectrode potential, $E° = \text{positive}$. Hence, showsproperty of auto-reduction. Auto-reductionprocess in which sulphone ores of lesselectropositive metals like Hg, Pb, Cu etc. areheated in the air so as to convert part of the oreinto oxide or sulphate.

Question 53

A de-polariser used in dry cell is

Options:

A. manganese dioxide

B. potassium hydroxide

C. ammonium chloride

D. zinc chloride

Answer: A

Solution:

Solution:
The term ‘de-polariser’ has been used todenote a substance used in a primary cell toprevent a buid up of hydrogen gas bubbles. Abattery de-polariser takes up electrons duringdischarge of the cell, therefore it is always anoxidizing agent (Manganese dioxide) Manganesedioxide is used as de-polarizes in dry cell.
$2NH_4^+ + 2Mn_2O_2 \rightarrow Mn_2O_3 + H_2O + 2NH_3$

Question 54

Which of following is an anionic detergent?

Options:

A. Sodium stearate

B. Sodium lauryl sulphate

C. Cetyltrimethyl ammonium bromide

D. Glycerol oleate

Answer: B

Solution:

Solution:
Sodium lauryl sulphate is an anionicdetergent. It has anion at the soluble end of thechain, it is a sodium salt of the sulphonated longchain of lauryl alcohol.

Question 55

Which statement is correct for dinitrogen pentoxide?

Options:

A. In the gas phase it decomposes into NO_2, NO and O_2.

B. Solid N_2O_5 is ionic.

C. Covalent in solution and gas phase.

D. Coloured deliquescent solid.

Answer: B

Solution:

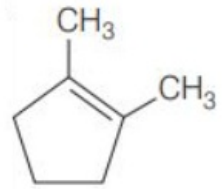
Solution:
In gaseous form N_2O_5 dissociate as:
$2N_2O_5 \rightarrow 4NO_2 \rightarrow O_2$ hence (a) is incorrect.In solid form N_2O_5 is ionic, hence (b) is correct.There are both covalent and coordinate covalentbonds, hence (c) is incorrect.
It is a colourless compound, hence (d) isincorrect.
So, statement (b) is correct for dinitrogen petoxide.

Question 56

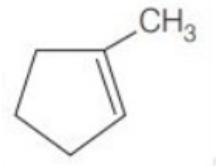
Which compound would give 5-keto, 2-methyl hexanal upon ozonolysis?

Options:

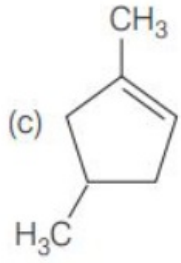
A.



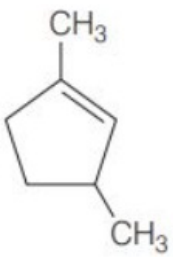
B.



C.



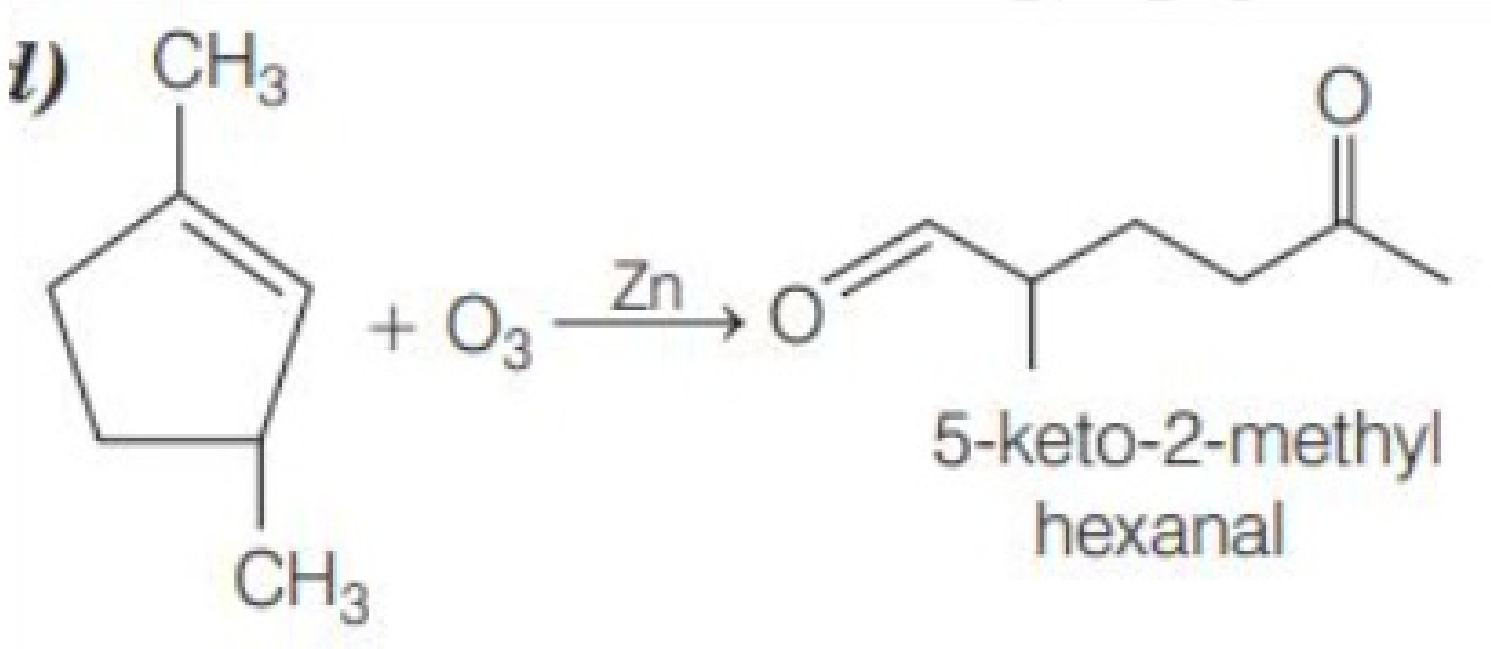
D.



Answer: D

Solution:

Solution:



Question 57

Anhydrone is

Options:

A. $KClO_4$

B. $NaClO_4$

C. $Mg(ClO_4)_2$

D. $HClO_4 + H_2SO_4$

Answer: C

Solution:

Solution:
Magnesium perchlorate $Mg(ClO_4)_2$ is dehydrating agent and known as anhydrase.

Question 58

In case of following Bronsted base, the correct order of basic strength is

Options:

A. $ClO_4^- > ClO_3^- > ClO_2^- > ClO^-$

B. $ClO^- > ClO_2^- > ClO_3^- > ClO_4^-$

C. $ClO_3^- > ClO_2^- > ClO_4^- < ClO^-$

D. $ClO_2^- > ClO_3^- > ClO^- > ClO_4^-$

Answer: B

Solution:

Solution:
According Bronsted concept of acid and base, strongest acid form weakest conjugate base. $HClO_4$ is strongest and HOCl is weakest acid.
$ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$ is the correct increasing order of Bronsted base strength with an increase in the number of oxygen atoms, the extent of delocalization of negative charge by $p\pi - d\pi$ bonding increases.

Question 59

When 10 mL of 10M solution of H_2SO_4 and 100 mL of 1M solution of NaOH are mixed, the resulting solution will be

Options:

A. acidic

B. neutral

C. alkaline

D. Cannot be predicted

Answer: A

Solution:

Solution:
One mole of H_2SO_4 will give $= 2$ moles of H^+
10 mole of H_2SO_4 will give $= 2 \times 10$ moles of H^+
10 mole H_2SO_4 will give $= 2 \times 10 \times 10$ moles of $H^+ = 200$ moles of H^+
One mole of NaOH will give $= 1$ mole of OH^-
100 mL of NaOH will give $= 1 \times 100$ moles of OH^-
In neutralisation H^+ will remains in solution, hence solution will be acidic.

Question 60

HIO_3 on heating at 170 °C give

Options:

A. I_2

B. I_2O_5

C. I_2O_4

D. HIO_4

Answer: B

Solution:

Solution:
On heating HIO_3 at $170^\circ C$
$2HIO_3 \rightarrow I_2O_5 + H_2O$

Question 61

Among the following compounds that is both paramagnetic and coloured

Options:

A. $K_2Cr_2O_7$

B. $(NH_4)_2[TiCl_6]$

C. $VOSO_4$

D. $K_3[Cu(CN)_4]$

Answer: C

Solution:

Solution:
In $VOSO_4$ oxidation state of vanadium is $+4$.
$= [Ar]3d^3 4s^2$
$V^{4+} = [Ar]3d^1 4s^0$
Therefore, it is paramagnetic.
In $K_2Cr_2O_7$ oxidation state of Cr = +6, unpaired electron = 0
In $(NH_4)_2[TiCl_6]$ oxidation state of Ti = +4, unpaired electron = 0
In $K_3[Cu(CN)_4]$ oxidation state of Cu = +1, unpaired electron = 0
In $K_2Cr_2O_7, (NH_4)_2[TiCl_6]$ and $K_3[Cu(CN)_4]$ in
coloured but diamagnetic but $VOSO_4$ paramagnetic and coloured.

Question 62

An element A has face centered cubic structure with edge length of 361 pm. The radius of atom A is

Options:

A. 127.6 pm

B. 180.5 pm

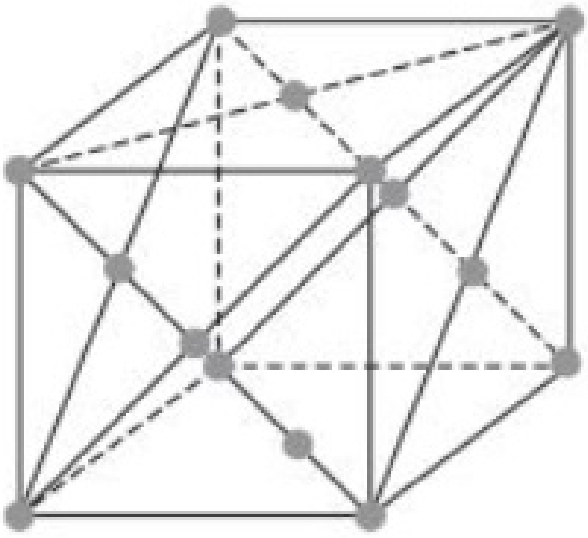
C. 160.5 pm

D. 64 pm

Answer: A

Solution:

Solution:
For face centered cubic structure
$a\sqrt{2} = 4r$
a = edge length



$$r = \frac{a \times \sqrt{2}}{4}$$
$$= \frac{361 \times 1.41}{4}$$
$$= 127.6\,\text{pm}$$

Question 64

IUPAC name of the complex $K_3[Al(C_2O_4)_3]$ is

Options:

A. potassium alumino oxalate

B. potassium trioxalatoaluminate (III)

C. potassium aluminium (III) oxalate

D. potassium trioxalato aluminate (VI)

Answer: B

Solution:

Solution:
IUPAC name of $K_3[Al(C_2O_4)_3]$ is potassiumtrioxalatoaluminate (III).

Question 65

Number of electrons transferred in each case when $KMnO_4$ acts as an oxidising agent to give $MnO_2, Mn^{2+}, Mn(OH)_3, MnO_4^{2-}$ are respectively

Options:

A. $3, 5, 4, 1$

B. 4, 3, 1, 5

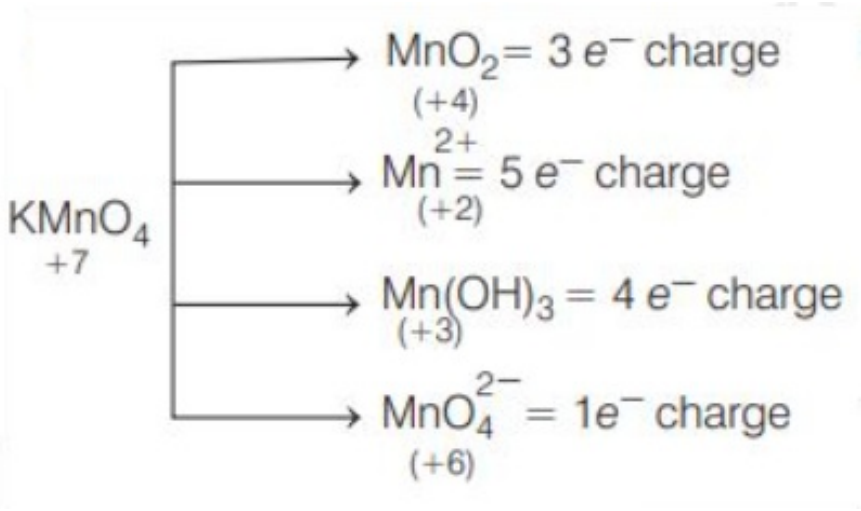
C. $1, 3, 4, 5$

D. $5, 4, 3, 1$

Answer: A

Solution:

Solution:



Hence, 3541 is correct.

Question 66

In following compounds, which one contains least oxidation state of iron?

Options:

A. $K_3[Fe(OH)_6]$

B. $K_2[FeO_4]$

C. $FeSO_4(NH_4)_2SO_4 \cdot 6H_2O$

D. $K_3[Fe(CN)_6]$

Answer: C

Solution:

Solution:
$K_3[Fe(OH)_6]$ oxidation state of Fe = +3
$K_2[FeO_4]$ oxidation state of Fe = +6
$FeSO_4(NH_4)_2SO_4 \cdot 6H_2O$ oxidation state of Fe = +2
$K_3[Fe(CN)_6]$ oxidation state Fe = +3
Hence, $FeSO_4(NH_4)_2SO_4 \cdot 6H_2O$ contains least oxidation state of iron.

Question 67

The set of quantum number not possible for an electron in an atom is

Options:

A. $n = 1, l = 1, m = 1, s = +\frac{1}{2}$

B. $n = 1, l = 0, m = 0, s = +\frac{1}{2}$

C. $n = 1, l = 0, m = 0, s = -\frac{1}{2}$

D. $n = 2, l = 0, m = 0, s = +\frac{1}{2}$

Answer: A

Solution:

Solution:
Azimuthal quantum number (l) have 0 to $(n-1)$ values.
where, n is principal quantum number.
When $n = 1, l$ will always be 0
Hence, set a is not possible.

Question 68

In the dichromate dianion $(Cr_2O_7)^{2-}$

Options:

A. $4\,Cr-O$ bonds are equivalent

B. $6\,Cr$ - O bonds are equivalent

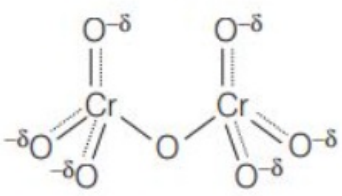
C. All $Cr-O$ bonds are equivalent

D. All $Cr-O$ bonds are non-equivalent

Answer: B

Solution:

Solution:
In $(Cr_2O_7)^{2-}$ dichromate dianion.



In Cr_2O_7, 6 $Cr-O$ bonds are equivalent and exhibit double bond character $Cr-O-Cr$ and bond is different.

Question 69

$\alpha-D(+)$ glucose and β-D $(+)$ glucose are

Options:

A. conformers

B. epimers

C. anomers

D. enantiomers

Answer: C

Solution:

Solution:
$\alpha-D(+)$ glucose and $\beta-D(+)$ glucose are anomers. An anomer of a saccharide only differs in its structure at the anomeric carbon. Anomeric carbon being the functional group of the carbohydrate, which is usually the carboxyl group attached to it.

Question 70

When $CH_2=CH-COOH$ is reduced with, $LiAlH_4$ the compound obtained will be

Options:

A. CH_3CH_2COOH

B. $CH_2=CH-CH_2OH$

C. $CH_3-CH_2-CH_2OH$

D. CH_3-CH_2-CHO

Answer: B

Solution:

Solution:
Due to reduction by $LiAlH_4$ $-COOH$ group will reduce into $-CH_2OH$ without affect carbon-carbon double bond.
$CH_2=CH-COOH+4H \xrightarrow{LiAlH_4} CH_2=CH-CH_2OH+H_2O$

Question 71

A substance forms Zwitter ion. It can have functional groups

Options:

A. $-NH_2, -COOH$

B. $-NH_2, -SO_3H$

C. Both (a) and (b)

D. None of the above

Answer: C

Solution:

Solution:
Those compound which have one acidic and other basic group in their structure can form Zwitter ion.
When one hydrogen change their position from acidic group to basic, Zwitter ion form.
Hence, $-NH_2, -COOH$ and $-NH_2, -SO_3H$ functional groups can form Zwitter ion.

Question 72

One mole of $Co(NH_3)_5Cl_3$, gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of $AgCl$. The structure of the complex is

Options:

A. $[Co(NH_3)_5Cl]Cl_2$

B. $[Co(NH_3)_3Cl_3] \cdot 2NH_3$

C. $[Co(NH_3)_4Cl_2]Cl \cdot NH_3$

D. $[Co(NH_3)_4Cl]Cl_2 \cdot NH_3$

Answer: A

Solution:

Solution:
$Co(NH_3)Cl_3$ complex compound gives 2 moles of $AgCl$ by adding excess of $AgNO_3$
It exhibit only $2Cl^-$ in present at primary valency other Cl^- is present as ligand, hence molecular formula will be $[Co(NH_3)_5Cl]Cl_2$

Question 73

$N_2(g) + 2H_2(g) \rightleftharpoons 2NH_3(g)$;
$[\Delta H = -93.6\,kJ]$
The yield of ammonia does not Increase when

Options:

A. pressure is increased

B. pressure is decreased

C. temperature is lowered

D. volume of reaction vessel is decreased

Answer: B

Solution:

Solution:
$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g); \Delta H = -93.6\,kJ$ In above reaction, number of molecules are reducing from left to right. According to Le-Ch atelier when we create any boundation. Then reaction will shift in a direction to neutralize the effect of boundation. When we reduce the pressure in above reaction then reaction will shift into backward direction where number of molecules are increasing. Hence, the yield of ammonia does not increase when pressure is decreased.

Question 74

Which pair of the following will not show common ion effect?

Options:

A. $H_2S + HNO_3$

B. $H_2S + HCl$

C. $NH_4Cl + NH_4OH$

D. $KCN + HCN$

Answer: A

Solution:

Solution:
HNO_3 is oxidizing agent, hence if will not suitable to create common ion effect. Therefore, $H_2S + HNO_3$ combination will not exhibit common ion effect.
Common Ion effect When a weak electrolyte is ionise with any strong electrolyte having one ion common than ionisation of weak electrolyte in further depressed due to presence of common ion of strong electrolyte.

Question 75

The binding energy of the hydrogen nucleus is

Options:

A. $13.6\,\text{MeV}$

B. $200\,\text{MeV}$

C. $24\,\text{MeV}$

D. Zero MeV

Answer: A

Solution:

Solution:
Binding energy $= mc^2 = \Delta mc^2$
Δm = mass defect
c = velocity of light
For hydrogen nucleus, it $13.6\,\text{MeV}$.
Binding energy $= \Delta m \times 931.48\,\text{MeV}$.
Mass defect in hydrogen nucleus in $0.0146 \times 931.48\,\text{MeV} = 13.6\,\text{MeV}$

Question 76

The question contains statement I and statement II of the four choices given after the statements, choose the one that best describe the two statements.
Statement I In strongly acidic solution, aniline becomes more reactive towards electrophile.
Statement II The amino group being completely protonated in strongly acidic solution, the lone pair of electrons on the nitrogen is no longer available for resonance.

Options:

A. Statement I is false and statement II is true.

B. Statement I is true and statement II is true; statement II is correct explanation of statement I.

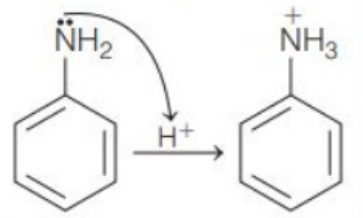
C. Statement I is true and statement II is true; statement | is not a correct explanation of Statement I.

D. Statement I is true; statement II is false.

Answer: A

Solution:

Solution:



In strongly acidic solution aniline is not morereactive for electrophile.Due to gaining H + from acidic solution, NH_2 isprotonated, hence not available for resonance.Hence statement 1 is wrong and 2 is right.

Question 77

Pair of species having identical shape for molecule is

Options:

A. CF_4, SF_4

B. BF_3, PCl_3

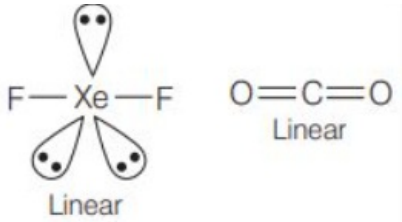
C. XeF_2, CO_2

D. PF_5, IF_5

Answer: C

Solution:

Solution:



Question 78

When mercuric iodide is added to aqueous solution of potassium iodide

Options:

A. freezing point is raised

B. freezing point is depressed

C. freezing point remains constant

D. freezing point may raised or depressed

Answer: A

Solution:

Solution:
By HgI_2 is added in KI solution.
$HgI_2 + 2KI \rightarrow K_2[HgI_4]$
the number of ions decreases, thus decreasing the van′t Hoff ← factor
$\Delta T_f = i \times K_f \times m$
Since, i has decreased, it results in less depression in the freezing point causing the freezing point to rise.

Question 79

When the electrons of hydrogen atoms return to L-shell from shell of higher energy, the series of lines in the spectrum are of

Options:

A. Pfund series

B. Lyman series

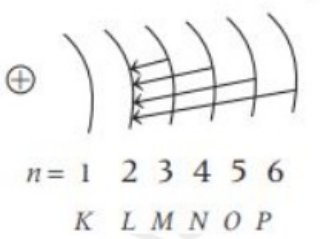
C. Brackett series

D. Balmer series

Answer: D

Solution:

Solution:



When electron returns to $n = 2$ i.e.
L shell than Balmer series of spectrum exhibit.

Question 80

Benzene reacts with H_2 at 150 °C at 30 atm pressure in the presence of Ni catalyst to give

Options:

A. cyclohexene

B. cyclohexane

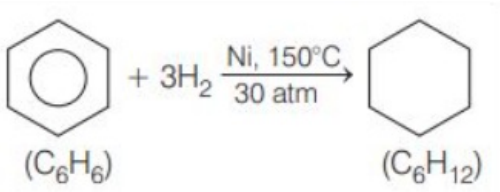
C. n-hexane

D. No reaction occur

Answer: B

Solution:

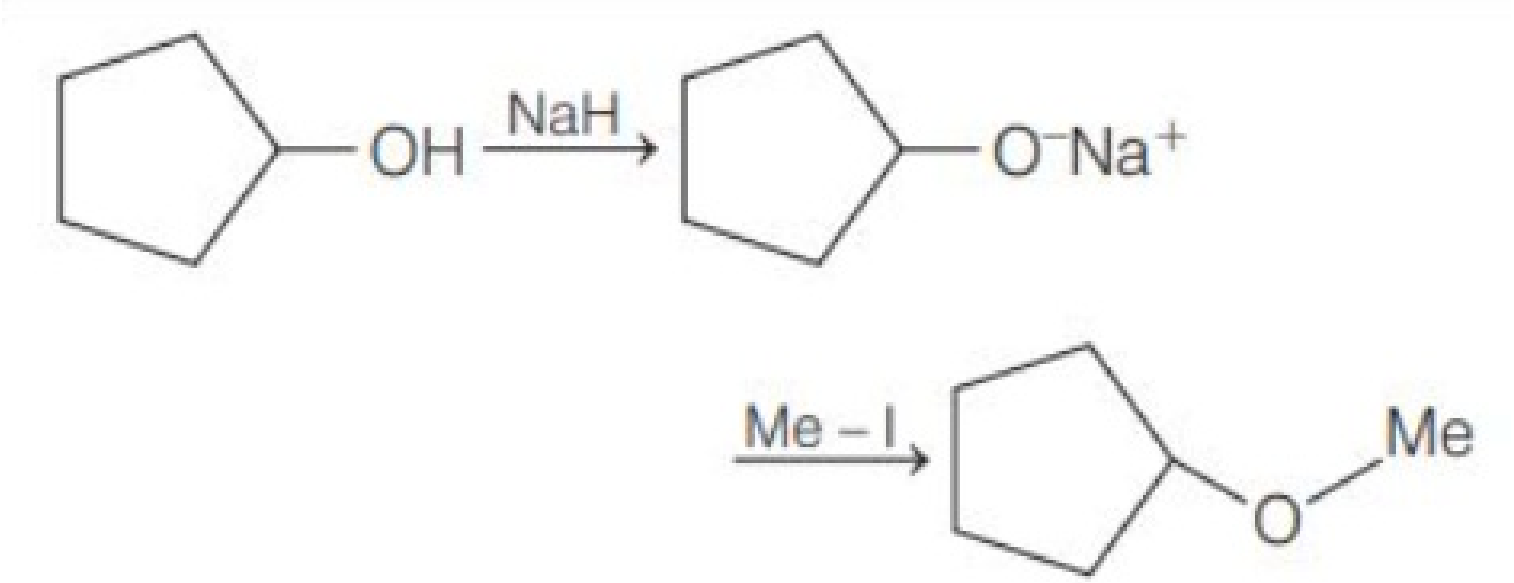
Solution:



In pressure of Ni due to complete hydrogenation of benzene gives cyclohexane.

Question 81

The reaction



can be classified as

Options:

A. Williamson's ether synthesis reaction

B. alcohol formation reaction

C. dehydration reaction

D. Williamson alcohol synthesis reaction

Answer: A

Solution:

Solution:
Answer is Williamson's a ether synthesis reaction, where is first step $-O-H$ group in change into $\bar{O}-Na^{+}$ group changes into $-O-CH_3^{-}$.

Question 82

An organic compound, containing $C = 39.13\%, H = 2.69\%$ and remaining % were oxygen. Its vapour density was found to be 46 . What will be its molecular formula?

Options:

A. $C_3H_8O_3$

B. C_3H_6O

C. C_6H_6O

D. $C_7H_6O_2$

Answer: A

Solution:

Solution:
C = 39.13
H = 269
$O = 100 - (39.13 + 269)$
$= 100 - 41.82 = 58.18\%$
Divided in atomic mass :
$C = \frac{39.13}{12}$ $H = \frac{2.69}{1}$ $O = \frac{58.18}{16}$
= 3, 3, 3

Vapour density
= 46
Molecular mass
$= 2 \times 46 = 92$
Molecular formula
$= C_3H_8O_3$
$= 36 + 8 + 48 = 92$
Compound have 3 carbons and 3 oxygens.

Question 83

$^{235}_{92}U$ belongs to IIIrd B group of periodic table. It loses one α-particle and two β-particles, the new element belongs to the group.

Options:

A. I B

B. I A

C. III B

D. III A

Answer: C

Solution:

Solution:
When radioactive elements emits α-particle then it shifts 2 groups in left hand side and by emitting are β-particle it shift one group right. IB – IIB – IIIB Continuous release of one α and two β-particles. It will returns to its original group, therefore new elements also belongs to III B group

---

Question 84

Identify the gas which is adsorbed by activated charcoal.

Options:

A. H_2

B. N_2

C. O_2^2

D. SO_2

Answer: D

Solution:

Solution:
Hydrogen, oxygen and nitrogen are not adsorb at activated charcoal. SO_2 will be adsorbed at activated charcoal.

Question 85

If a is the initial concentration of the reactant and n be the order of the reaction, then half-life of the reaction will be

Options:

A. a^{1-n}

B. a^{n-1}

C. a^n

D. a^{n+1}

Answer: A

Solution:

Solution:

$t_{1/2} = \frac{1}{a^{(n-1)}} = a^{(1-n)}$

Question 86

Consider following statements:
A. The absolute value of internal energy cannot be evaluated.
B. Entropy is a state function and extensive property.
C. Temperature is an extensive property.
D. The standard enthalpy of formation of an element (ΔH_f) is always considered to zero.
E. For an adiabatic process, $\Delta S > 0$. Wrong statements are

Options:

A. A and B

B. A and D

C. C and E

D. B and D

Answer: C

Solution:

Solution:
Temperature is an intensive property and for adiabatic process $\Delta S = 0$.
Hence, wrong statements are C and E.

Question 87

2 moles of PCl_5 is decomposed in 2L closed vessel. At equilibrium the percentage decomposition of PCl_5 is 40%. The value of equilibrium constant is

Options:

A. 0.267

B. 0.0267

C. 2.67

D. 26.7

Answer: A

Solution:

Solution:
$PCl_5 \rightleftharpoons PCl_3 + Cl_2$
$\frac{(a-x)}{V} \quad \frac{x}{V} \frac{x}{V}$
$K_C = \frac{x^2}{(a-x)V}$
$\because x = 40\%$
$\therefore 2\,\text{mole} = 80\%$
$V = 2L$
$= \frac{0.8 \times 0.8}{(2 - 0 \cdot 8) \times 2}$
$= \frac{0.64}{1 \cdot 2 \times 2} = \frac{0.64}{2.4}$
$\Rightarrow K_C = 0 \cdot 267$

Question 88

The dissociation energy of CH_4 is 360 kcal mol^{-1} and that of ethane is 620 kcal mol^{-1}. The bond energy of C − C bond is

Options:

A. 260 kcal

B. 980 kcal

C. 90 kcal

D. 80 kcal

Answer: D

Solution:

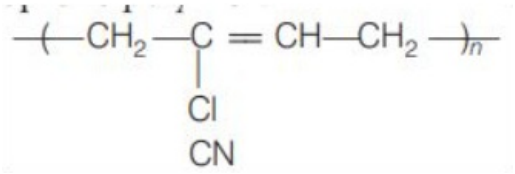
Solution:
In CH_4 there are $4C - H$ bond.
Bond energy $= 360$
Hence, bond energy of one $C - H$ bond
$= \frac{360}{4} = 90$
In C_2H_6 there are $6C - H$ bond and one $C - C$
$620 = 6 \times 90 + C - C$
$620 - 540 = C - C$
Hence, $C - C = 80$ kcal

Question 89

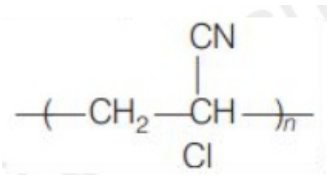
Which of the following structure represents neoprene polymer?

Options:

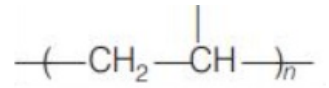
A.



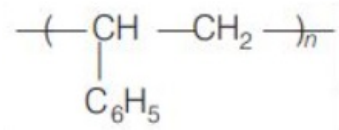
B.



C.



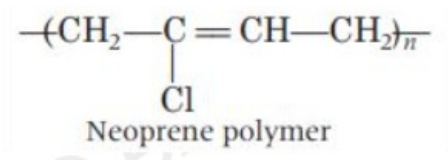
D.



Answer: A

Solution:

Solution:



Question 90

This question contains statement I and statement II, of the four choices given after the statements. Choose the one that best describes the two statements.
Statement I Aromatic acids do not undergo Friedel-Crafts reaction.
Statement II COOH is *m*-directing group.

Options:

A. Statement I is false, statement II is true.

B. Statement I is true, statement | is true, statement II is correct explanation of statement I.

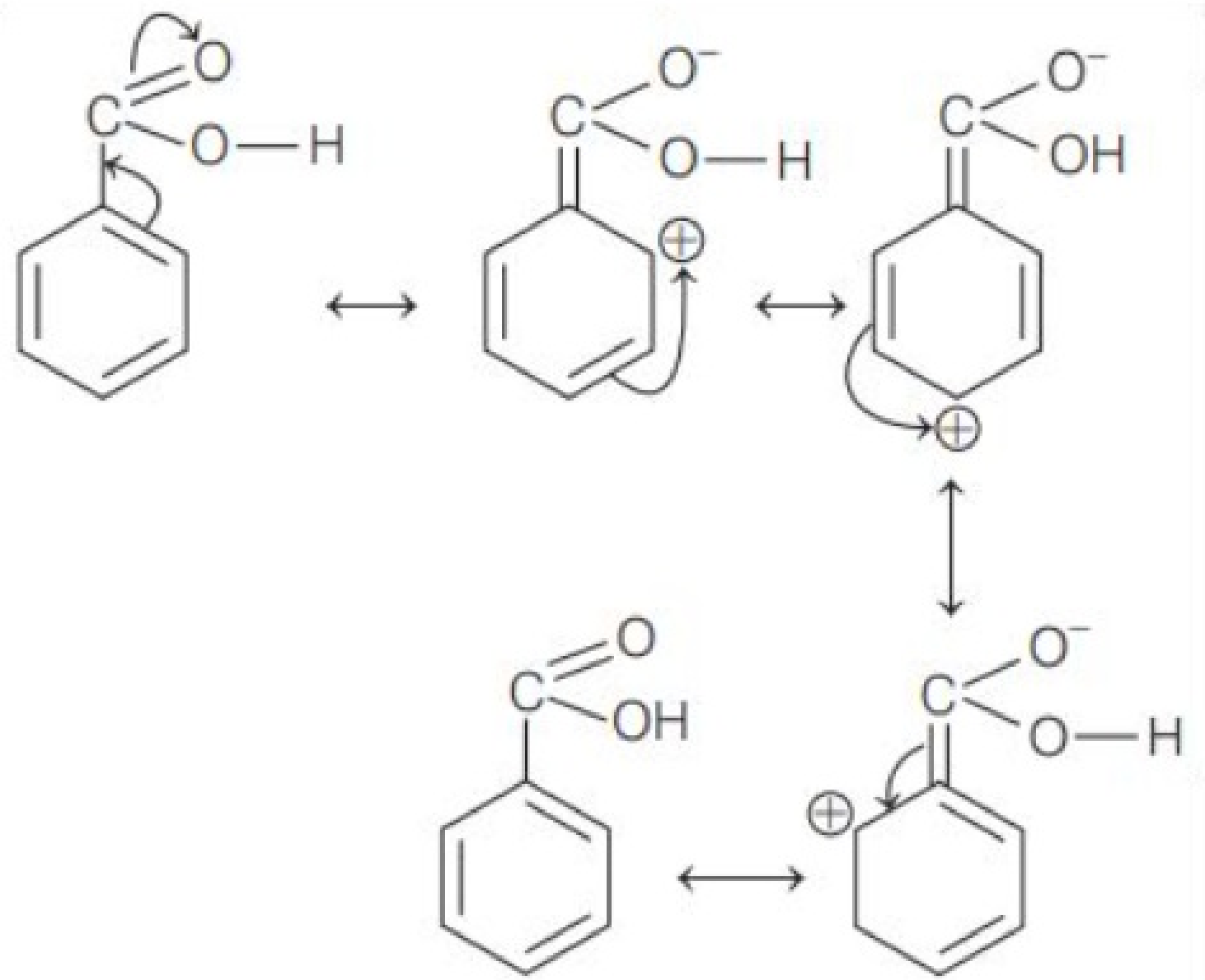
C. Statement I is true, statement II is true, statement II is not a correct explanation of statement I.

D. Statement I is true, statement II is false.

Answer: C

Solution:

Solution:
$-COOH$ group is *m*-directing it is explained on the basis of resonance there is positive charge at *o* and *p*. Hence, electrophilic attack is not possible at *o* and *p* only *m* position remains for electrophilic substitution.



Aromatic acid do not undergo Friedel-Crafts reaction.

Question 91

How many structural isomers are possible for C_4H_9Br ?

Options:

A. 2

B. 3

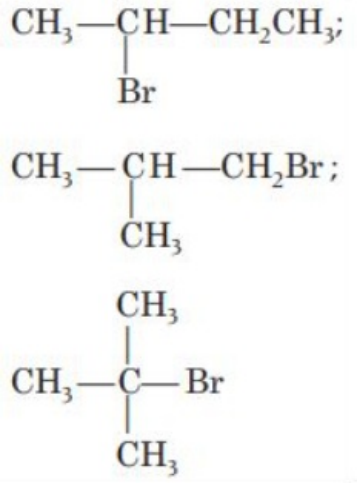
C. 4

D. 5

Answer: C

Solution:

Solution:
4 isomers are possible for C_4H_9Br
1) $CH_3CH_2CH_2CH_2Br$;



Question 92

The ionisation potential of which element is highest?

Options:

A. H

B. He

C. Ar

D. F

Answer: B

Solution:

Solution:
He has highest ionization potentialIP increases left to rightIP decreases top to bottom.He is present extremely right and upper positionin periodic table.

Question 93

pH of a solution is 10 . The number of hydrogen ions present per millilitre of solution is

Options:

A. 6.023×10^{23}

B. 6.023×10^{13}

C. 6.023×10^{10}

D. 6.023×10^{7}

Answer: B

Solution:

Solution:
pH = 10 of a solution means $[H^+] = 1 \times 10^{-10}$ mole per litre
1 mole number of $H^+ = 6.023 \times 10^{23}$
10^{-10} mole = $H^+ = 6.023 \times 10^{23} \times 10^{-10}$
$= 6.023 \times 10^{13}$

Question 94

Among the following which are mismatched?

	Order	Unit of k
(A)	Zero order	$\text{mol L}^{-1}\ \text{time}^{-1}$
(B)	First order	mol L^{-1}
(C)	Second order	$\text{Lmol}^{-1}\text{time}^{-1}$
(D)	Third order	$\text{L}^2\text{mol}^{-1}\text{time}^{-1}$
(E)	nth order	$\text{mol}^{1-n}\text{L}^{n-1}\text{time}^{-1}$

Options:

A. A and C

B. *B* and *D*

C. A and E

D. C and E

Answer: B

Solution:

Solution:
Unit of $k = \text{mol}^{(1-h)}\text{L}^{(n-1)}$ time $^{-1}$
For Ist order reaction $= \text{mol}^{(1-h)}\text{L}^{1-1}$ time $^{-1}$
$k = \text{time}^{-1}$
Hence, B is mismatched.
For IIIrd order reaction,
$= \text{mol}^{(1-3)}\text{L}^{3-1}\text{time}^{-1} = \text{mol}^{-2}\text{L}^{2}\text{time}^{-1}$
Hence, D is mismatched.

Question 95

Which one of the following will act as the best protective colloid?

Options:

A. Gelatin

B. Starch

C. gum arabic

D. Egg albumin

Answer: A

Solution:

Solution:
Lower the gold number better is protecting power of hydrophilic colloid.
Gelatin has minimum value of gold number. So, gelatin act as the best protective colloid.

Question 96

$\text{Na}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)$ then ΔH is equal to

Options:

A. $\Delta U + 2RT$

B. $\Delta U - 2RT$

C. $\Delta U + RT$

D. $\Delta U - RT$

Answer: B

Solution:

Solution:
$N_2 + 3H_2 \rightleftharpoons 2NH_3$
$\therefore \Delta H = \Delta E + nRT$
where, ΔH = Change in enthalpy
ΔE = Change in external energy
$= \Delta U + (-2)RT = \Delta U - 2RT$

Question 97

Match List-I with List-II and select the correct answer using following codes: Substance
A. Crystals of tartaric acid
B. Fe_2O_3
C. Rochelle salt
D. KH_2PO_4
E. Fe_3O_4
Properties
1. Ferrimagnetism
2. Ferroelectricity
3. Piezoelectricity
4. Anti-ferromagnetism
5. Pyroelectricity
Codes

Options:

A. 1 2 3 4 5

B. 5 4 3 2 1

C. 4 5 1 2 3

D. 3 2 5 4 1

Answer: B

Solution:

Solution:
A. Crystals of tartaric acid $\rightarrow$ 5. Pyroelectricity
B. $Fe_2O_3 \rightarrow$ 4. Anti-ferro magnetism
C. Rochelle salt $\rightarrow$ 3. Piezoelectricity
D. $KH_2PO_4 \rightarrow$ 2. Ferroelectricity
E. $Fe_3O_4 \rightarrow$ 1. Ferrimagnetism

Question 98

The factor $\frac{\Delta T_f}{K_f}$ represents

Options:

A. molarity

B. molality

C. normality

D. formality

Answer: B

Solution:

Solution:

$\therefore \Delta T_f = \frac{K_f \times W_B \times 1000}{M_B \times W_A}$

where,

W_B = amount of solute in gram

W_A = amount of solvent in gram

M_B = molecular mass of solute

ΔT_f = depression in freezing point

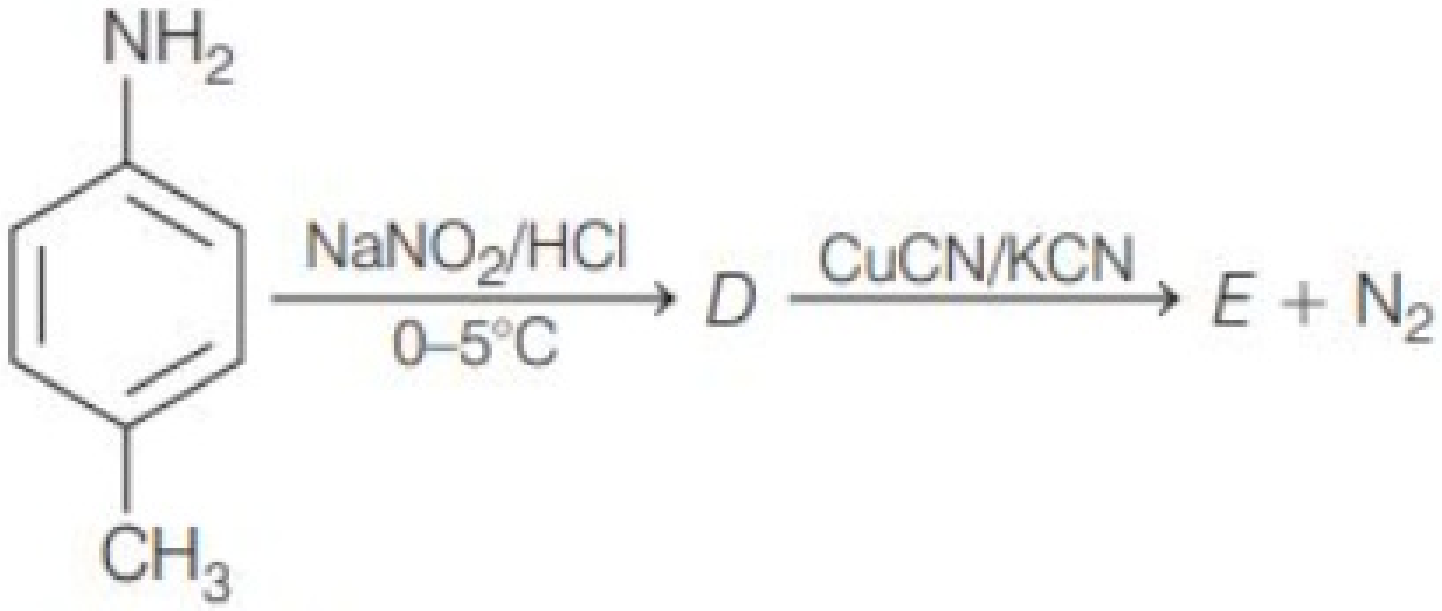
K_f = molar depression constant

$\frac{\Delta T_f}{K_f} = \frac{W_B \times 1000}{M_B \times W_A}$

$\frac{\Delta T_f}{K_f}$ = molality

Question 99

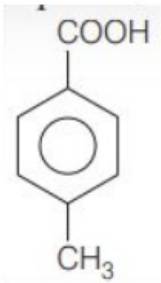
In the reaction,



the product E is

Options:

A.



B.

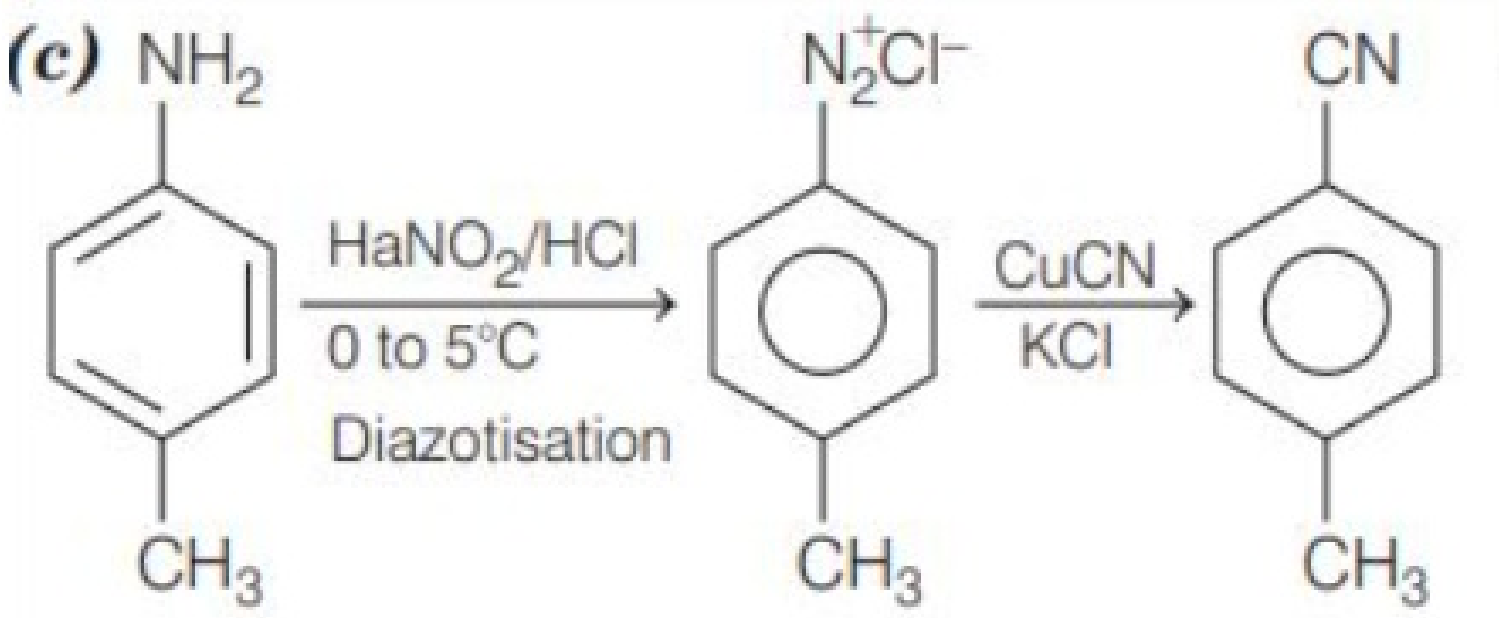
C.

D.

Answer: C

Solution:

Solution:



Question 100

The process of converting hydrated alumina $Al(OH)_3$ into anhydrous alumina is called

Options:

A. roasting

B. smelting

C. dressing

D. calcination

Answer: D

Solution:

Solution:
In the process of converting hydrated alumina $Al(OH)_3$ into anhydrous alumina Al_2O_3 is called calcination.
$2Al(OH)_3 \xrightarrow{\Delta} Al_2O_3 + 3H_2O$

Question 101

The angle of elevation of the top of a tower at any point on the ground is 30° and that at a point $10m$ nearer to tower from earlier is 60°. Then, the height of the tower is

Options:

A. 15m

B. 10m

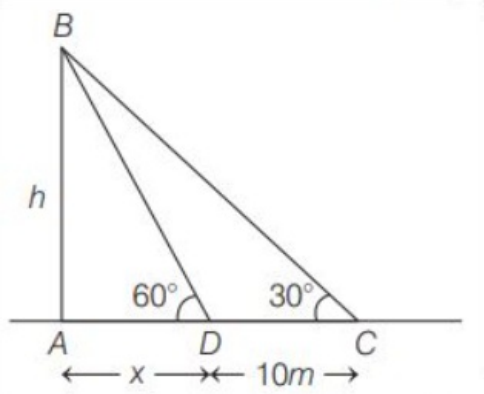
C. $5\sqrt{3}$m

D. None of these

Answer: C

Solution:

Solution:
Let AB be the tower of height hm and C and D are the points where the angle of elevation of the top of the tower be 30° and 60°. Also, CD = 10m and let AD be xm



In $\triangle ABD$

$\tan 60^\circ = \frac{AB}{AD}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow x = \frac{h}{\sqrt{3}} \ldots . (i)$

Now, in $\triangle ABC$,

$\tan 30^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+10}$

$\Rightarrow x + 10 = \sqrt{3}h$

$\Rightarrow \frac{h}{\sqrt{3}} + 10 = \sqrt{3}h \ldots [\text{From Eq}.(i)]$

$\Rightarrow h + 10\sqrt{3} = 3h$

$\Rightarrow 2h = 10\sqrt{3}$

$\Rightarrow h = 5\sqrt{3}$

So, height of tower is $5\sqrt{3}$m.

Question 102

If [.] denotes the greatest integer function, then $\int_0^{1.5} [x^2]\,dx =$

Options:

A. $2 + \sqrt{2}$

B. $2-\sqrt{2}$

C. $1+\sqrt{2}$

D. None of these

Answer: B

Solution:

Solution:

Let $I = \int_0^{1.5} [x^2]\,dx$

$= \int_0^1 [x^2]\,dx + \int_1^{\sqrt{2}} [x^2]\,dx + \int_{\sqrt{2}}^{1.5} [x^2]\,dx$

$= \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{1.5} 2\,dx$

$= 0 + [x]_1^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{1.3}$

$= (\sqrt{2}-1) + 2(1.5-\sqrt{2})$

$= \sqrt{2}-1+3-2\sqrt{2} = 2-\sqrt{2}$

Question 103

Argument of the difference of two complex numbers $3+4i$ and $-3-4i$ is

Options:

A. $\frac{\pi}{2}$

B. $\tan^{-1}\frac{4}{3}$

C. $\tan^{-1}\frac{3}{4}$

D. None of the above

Answer: B

Solution:

Solution:

Let $z_1 = 3+4i$ and $z_2 = -3-4i$

$\therefore z = z_1 - z_2$

$= (3+4i) - (-3-4i)$

$= 3+4i+3+4i = 6+8i$

Let θ be the argument of z, then

$\tan\theta = \frac{8}{6}$

$\therefore \theta = \tan^{-1}\frac{4}{3}$

Question 104

Length of perpendicular from the point $(1, 2, 3)$ to the line $\frac{x-2}{4} = \frac{y-3}{5} = \frac{z-4}{6}$

Options:

A. $\sqrt{\frac{6}{77}}$

B. $\frac{6}{\sqrt{77}}$

C. $\frac{6}{77}$

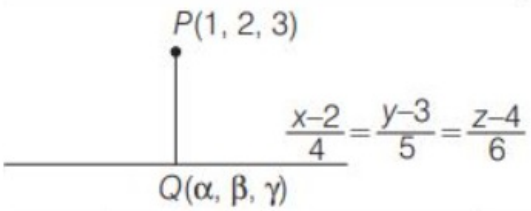
D. None of these

Answer: A

Solution:

Solution:

Let $P(1, 2, 3)$ and $Q(\alpha, \beta, \gamma)$, where Q is foot of perpendicular to the line $\frac{x-2}{4} = \frac{y-3}{5} = \frac{z-4}{6} = \lambda$ from P



Now, coordinates of Q are $(4\lambda + 2, 5\lambda + 3, 6\lambda + 4)$

$\therefore$ DR´s of line joining PQ are

$< 4\lambda + 2 - 1, 5\lambda + 3 - 2, 6\lambda + 4 - 3 >$

i.e., $< 4\lambda + 1, 5\lambda + 1, 6\lambda + 1 >$

Again PQ is perpendicular to the given line, so

$4(4\lambda + 1) + 5(5\lambda + 1) + 6(6\lambda + 1) = 0$

$\Rightarrow 16\lambda + 4 + 25\lambda + 5 + 36\lambda + 6 = 0$

$\Rightarrow 77\lambda = -15 \Rightarrow \lambda = \frac{-15}{77}$

$\therefore$ Coordinates of Q are

$(4 \times \frac{-15}{77} + 2, 5 \times \frac{-15}{77} + 3, 6 \times \frac{-15}{77} + 4)$

i.e., $(\frac{94}{77}, \frac{156}{77}, \frac{218}{77})$

Now, PQ $= \sqrt{(\frac{94}{77} - 1)^2 + (\frac{156}{77} - 2)^2 + (\frac{218}{77} - 3)^2}$

$= \sqrt{(\frac{17}{77})^2 + (\frac{2}{77})^2 + (\frac{-13}{77})^2}$

$= \sqrt{\frac{289 + 4 + 169}{77^2}}$

$= \sqrt{\frac{462}{77 \times 77}} = \sqrt{\frac{6}{77}}$

Question 105

Barcode reader is a/an device.

Options:

A. Output

B. Input

C. Display

D. Storage

Answer: B

Solution:

Solution:
Barcode reader is an input device.

Question 106

The function $f(x) = x^2 - x + 1$ in the interval $(0, 1)$ is

Options:

A. monotonic decreasing

B. monotonic increasing

C. neither decreasing nor increasing

D. None of the above

Answer: C

Solution:

Solution:
We have,
$f(x) = x^2 - x + 1$
$\therefore f'(x) = 2x - 1$
For $f(x)$ to be increasing
$f'(x) > 0$
$2x - 1 > 0$
$x > \frac{1}{2} \Rightarrow x \in (\frac{1}{2}, \infty)$
and $f(x)$ to be decreasing
$f'(x) < 0$
$2x - 1 < 0$
$x < \frac{1}{2} \Rightarrow x \in (-\infty, \frac{1}{2})$
So, $f(x)$ is neither increasing nor decreasing in the interval $(0, 1)$

Question 107

If $\gamma_1 = 3\,\hat{i} + \hat{j} - 4\,\hat{k}$ and $\gamma_2 = 2\,\hat{i} + 5\,\hat{j} + 8\,\hat{k}$ are any two vectors then the unit vector parallel to $\gamma_2 - \gamma_1$ is

Options:

A. $\frac{-\hat{i}+2\hat{j}+13\hat{k}}{\sqrt{174}}$

B. $\frac{\hat{i}+4\hat{j}-12\hat{k}}{\sqrt{161}}$

C. $\frac{\hat{i}-2\hat{j}+13\hat{k}}{\sqrt{174}}$

D. $\frac{-\hat{i}+4\hat{j}+12\hat{k}}{\sqrt{161}}$

Answer: D

Solution:

Solution:
We have,
$\gamma_1 = 3\hat{i}+\hat{j}-4\hat{k}$ and $\gamma_2 = 2\hat{i}+5\hat{j}+8\hat{k}$
Now, $\gamma_2-\gamma_1 = (2\hat{i}+5\hat{j}+8\hat{k})-(3\hat{i}+\hat{j}-4\hat{k})$
$= -\hat{i}+4\hat{j}+12\hat{k}$
Again, $|\gamma_2-\gamma_1| = \sqrt{(-1)^2+(4)^2+(12)^2}$
$= \sqrt{1+16+144} = \sqrt{161}$
Unit vector parallel to $\gamma_2-\gamma_1 = \frac{\gamma_2-\gamma_1}{|\gamma_2-\gamma_1|}$
$= \frac{-\hat{i}+4\hat{j}+12\hat{k}}{\sqrt{161}}$

Question 108

For the following linear differential equation, the integrating factor will be $x(x^2+1)\frac{dy}{dx} = y(1-x^2)+x^3\log x$

Options:

A. $\left(\frac{x^2+1}{x}\right)$

B. $\frac{x^2-1}{x^2}$

C. $\frac{x^2+1}{x^2-1}$

D. None of these

Answer: A

Solution:

Solution:
We have,
$x(x^2+1)\frac{dy}{dx} = y(1-x^2) + x^3 \log x$
$\Rightarrow x(x^2+1)\frac{dy}{dx} - (1-x^2)y = x^3 \log x$
$\Rightarrow \frac{dy}{dx} - \frac{(1-x^2)}{x(x^2+1)}y = \frac{x^2 \log x}{x^2+1}$
$\therefore$ Integrating factor $= e^{\int \frac{-(1-x^2)}{x(x^2+1)}dx}$
$= e^{\int\left(\frac{2x}{x^2+1} - \frac{1}{x}\right)dx}$
$= e^{(\log(x^2+1) - \log x)}$
$= e^{\log\left(\frac{x^2+1}{x}\right)}$
$= \frac{x^2+1}{x}$

Question 109

The parametric representation $y = 4t + 1,\ x = 4 + t^2$ represents

Options:

A. a parabola with focus at $(1, 4)$

B. an ellipse with centre at $(4, 1)$

C. a parabola with vertex $(4, 1)$

D. None of the above

Answer: C

Solution:

Solution:
We have,
$y = 4t + 1, x = 4 + t^2$
$\Rightarrow t = \frac{y-1}{4}, t^2 = x - 4$
$\therefore \left(\frac{y-1}{4}\right)^2 = x - 4$
$\Rightarrow (y-1)^2 = 16(x-4)$
which is a equation of parabola with vertex $(4, 1)$

Question 110

If $sin^{-1}\frac{2x}{1+x^2} - 2\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{2}$, then x is equal to

Options:

A. 0

B. 1

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: B

Solution:

Solution:
We have,
$\sin^{-1}(\frac{2x}{1+x^2}) - 2\cos^{-1}(\frac{1-x^2}{1+x^2}) + 2\tan^{-1}(\frac{2x}{1-x^2}) = \frac{\pi}{2}$
$\Rightarrow 2\tan^{-1}x - 2(2\tan^{-1}x) + 2(2\tan^{-1}x) = \frac{\pi}{2}$
$\Rightarrow 2\tan^{-1}x - 4\tan^{-1}x + 4\tan^{-1}x = \frac{\pi}{2}$
$\Rightarrow 2\tan^{-1}x = \frac{\pi}{2}$
$\Rightarrow \tan^{-1}x = \frac{\pi}{4}$
$\Rightarrow x = \tan\frac{\pi}{4}$
$\therefore x = 1$

Question 111

A line with direction ratios $2, 7, -5$ is drawn to intersect the lines $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$ and $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$. Then, the length intercepted on it is

Options:

A. 78

B. $\sqrt{78}$

C. 38

D. None of these

Answer: B

Solution:

Solution:
Let
$L_1: x+3/-3 = y-3/2 = z-6/4 = r_1$
and
$L_2: x-5/3 = y-7/-1 = z+2/1 = r_2$

Again, let
A and B are the points of intersection of the line with L_1 and L_2. Then,
$A(-3r_1-3, 2r_1+3, 4r_1+6)$ and
$B(3r_2+5, -r_2+7, r_2-2)$
Now, DR´s of AB are
$< 3r_2+5+3r_1+3, -r_2+7-2r_1-3, r_2-2-4r_1-6 >$

i.e., $< 3r_2 + 3r_1 + 8, -r_2 - 2r_1 + 4, r_2 - 4r_1 - 8 >$
But DR′s o f AB are $< 2, 7, -5 >$.
$\therefore \frac{3r_2 + 3r_1 + 8}{2} = \frac{-r_2 - 2r_1 + 4}{7} = \frac{r_2 - 4r_1 - 8}{-5}$
$\Rightarrow 7(3r_2 + 3r_1 + 8) = 2(-r_2 - 2r_1 + 4)$
and $-5(-r_2 - 2r_1 + 4) = 7(r_2 - 4r_1 - 8)$

$\Rightarrow 23r_2 + 25r_1 = -48$
and $-2r_2 + 38r_1 = -36$
$\therefore r_1 = -1$ and $r_2 = -1$

So, coordinates of A are
$(0, 1, 2)$
and B are
$(2, 8, -3)$
$\therefore$ Length of intercept,
AB $= \sqrt{(2-0)^2 + (8-1)^2 + (-3-2)^2}$
$= \sqrt{4 + 49 + 25} = \sqrt{78}$

Question 112

If $a = 5, b = 3, c = 2$ in $\triangle ABC$, then $\triangle ABC$ is a

Options:

A. right angled triangle

B. equilateral triangle

C. acute angled triangle

D. None of the above

Answer: D

Solution:

Solution:
We have,
$a = 5, b = 3, c = 2$
Now, $b + c = 3 + 2 = 5 = a$
$\therefore$ABC , are collinear.

Question 113

If the forces $\mathbf{F_1 = 2\,\hat{i} - 4\,\hat{j} + 4\,\hat{k}}$ and $\mathbf{F_2 = 2\,\hat{i} - 3\,\hat{j} - 6\,\hat{k}}$ acting together displace a particle from $\mathbf{P(4, -2, -6)}$ to the point $\mathbf{Q(10, -2, 2)}$, then the work done by the forces is

Options:

A. 8 units

B. 0 unit

C. 4 units

D. None of these

Answer: A

Solution:

Solution:
We have,
$\mathbf{F}_1 = 2\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}, \mathbf{F}_2 = 2\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} - 6\,\hat{\mathbf{k}}$
$\therefore$ Net force, $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$
$= (2\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}) + (2\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} - 6\,\hat{\mathbf{k}})$
$= 4\,\hat{\mathbf{i}} - 7\,\hat{\mathbf{j}} - 2\,\hat{\mathbf{k}}$
Again, $\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP}$
$= (10\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (4\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$
$= 6\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$
Now, work done $= \mathbf{F} \,.\, \mathbf{PQ}$
$(4\,\hat{\mathbf{i}} - 7\,\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \,.\, (6\,\hat{\mathbf{i}} - 0\,\hat{\mathbf{j}} + 8\hat{\mathbf{k}})$
$24 + 0 - 16$
8 Units

Question 114

Which of the following device is used for gaming?

Options:

A. Mic

B. Mouse

C. Joystick

D. Light Pen

Answer: C

Solution:

Solution:
Joystick is used for gaming purpose.

Question 115

Solution of differential equation

$$\frac{dy}{dx} = e^{x-y}(e^x - e^y)$$

Options:

A. $e^{y+x} = e^{e^x} - xe^{e^x} + c$

B. $e^y e^{e^x} = e^x e^{e^x} - e^{e^x} + c$

C. $e^{x-y} = e^{e^x} + x^2 e^{e^x} + c$

D. None of the above

Answer: B

Solution:

Solution:
We have,
$\frac{dy}{dx} = e^{x-y}(e^x - e^y)$
$\Rightarrow \frac{dy}{dx} = e^{2x-y} - e^x$
$\Rightarrow e^y \frac{dy}{dx} = e^{2x} - e^x \cdot e^y$
$\Rightarrow e^y \frac{dy}{dx} + e^x e^y = e^{2x}$
Put $e^y = v$
$\Rightarrow e^y \frac{dy}{dx} = \frac{dv}{dx}$
$\therefore \frac{dv}{dx} + e^x v = e^{2x}$
Now, IF $= e^{\int e^x dx} = e^{e^x}$
Hence, solution is given as
$v \cdot e^{e^x} = \int e^{2x} \cdot e^{e^x} dx + C$
$= \int e^x e^{e^x} e^x dx + C$
Put $e^x = t$
$\Rightarrow e^x dx = dt$
$\therefore ve^{e^x} = \int te^t dt + c$
$ve^{e^x} = te^t - \int e^t \cdot 1 dt + c$
$ve^{e^x} = te^t - e^t + c$
$e^y e^{e^x} = e^x e^{e^x} - e^{e^x} + c$
which is required solution.

Question 116

Out of $(2n+1)$ tickets numbered from 1 to $2n+1$, three are drawn at random. The probability that the numbers on them are in arithmetic progression will be

Options:

A. $\frac{3n}{5n^2-1}$

B. $\frac{3n}{4n^2-1}$

C. $\frac{n}{4n^2+1}$

D. None of the above

Answer: B

Solution:

Solution:
Out of $(2n+1)$ tickets, we have $(n+1)$ tickets having odd number written on them and n tickets have even number. For three numbers to be in AP either two number from odd or two numbers from even numbers must be selected.

$\therefore$ Required probability $= \frac{{}^{n+1}C_2 + {}^{n}C_2}{{}^{2n+1}C_3}$

$= \frac{\frac{(n+1)n}{2\times 1} + \frac{n(n-1)}{2\times 1}}{\frac{(2n+1)(2n)(2n-1)}{3\times 2\times 1}}$

$= \frac{3[n(n+1)+n(n-1)]}{2n(2n+1)(2n-1)}$

$= \frac{3n(n+1+n-1)}{2n(2n+1)(2n-1)}$

$= \frac{3n}{4n^2-1}$

Question 117

A 28m long wire is broken into two pieces. One piece is folded as square and another as circle. The lengths of two pieces will be, when the combined areas made by both figures will be minimum

Options:

A. $\frac{112}{4+\pi}, \frac{28\pi}{4+\pi}$

B. $\frac{124}{8+\pi}, \frac{26\pi}{8+\pi}$

C. $\frac{224}{6+\pi}, \frac{14\pi}{6+\pi}$

D. None of these

Answer: A

Solution:

Solution:
Let the length of the side of the square be $x\ m$ and the radius of the circle be y metres. It is given that the length of the wire is 28m

$\therefore 4x + 2\pi y = 28$

$\Rightarrow 2x + \pi y = 14 \ \ldots(i)$

Let A be the combined area of the square and circle. Then,

$A = x^2 + \pi y^2 \ \ldots(ii)$

$\Rightarrow A = x^2 + \pi(\frac{14-2x}{\pi})^2 \ \ldots[\text{Using Eq}(i)]$

$= x^2 + \frac{1}{\pi}(14-2x)^2$

$\Rightarrow \frac{dA}{dx} = 2x + \frac{2}{\pi}(14-2x)(-2)$

$= 2x - \frac{4}{\pi}(14-2x)$

and $\frac{d^2A}{dx^2} = 2 - \frac{4}{\pi}(-2) = 2 + \frac{8}{\pi}$

The critical numbers of A are given by

$\frac{dA}{dx} = 0$
$\therefore 2x - \frac{4}{\pi}(14 - 2x) = 0$
$\Rightarrow x = \frac{28}{\pi + 4}$
Clearly,
$(\frac{d^2A}{dx^2})_{x = \frac{28}{\pi + 4}} = 2 + \frac{8}{\pi} > 0$
Thus, A is minimum when
$x = \frac{28}{\pi + 4}$
. Putting $x = \frac{28}{\pi + 4}$ in Eq . (i) we get
$y = \frac{14}{\pi + 4}$
So, length of the two piece of wire are
$4x = 4 \times \frac{28}{4 + \pi} = \frac{112}{4 + \pi}$m
and
$2\pi y = 2\pi \times \frac{14}{\pi + 4} = \frac{28\pi}{\pi + 4}$m

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Question 118

If the ellipse $x^2 + 9y^2 = 9$ meets the ellipse $b^2x^2 + y^2 = b^2, b > 1$ in n points, then n is equal to

Options:

A. 0

B. 2

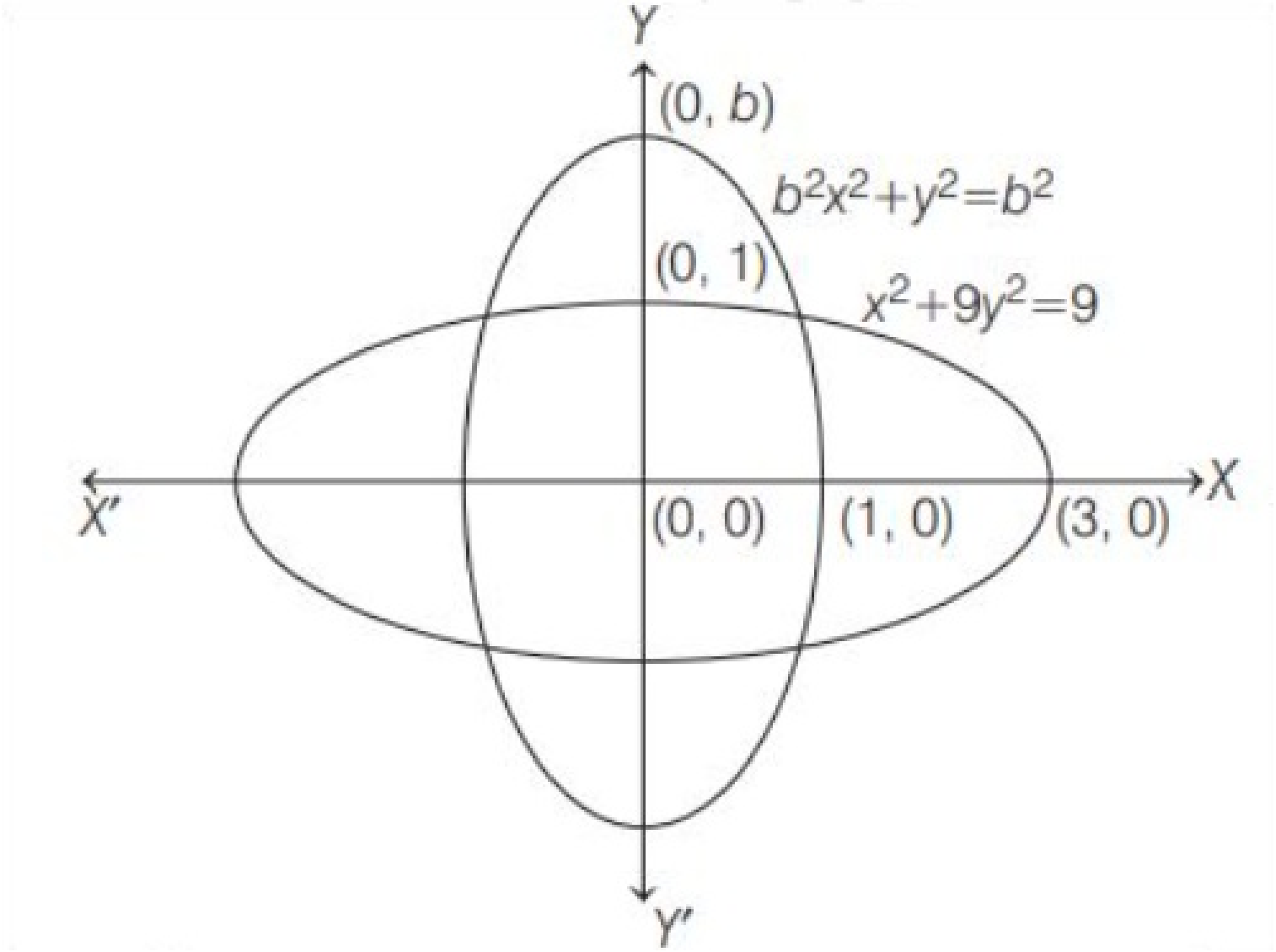
C. 4

D. 9

Answer: C

Solution:

Solution:
We have,
$x^2 + 9y^2 = 9$
$\Rightarrow \frac{x^2}{9} + \frac{y^2}{1} = 1$
and $b^2x^2 + y^2 = b^2$
$\Rightarrow \frac{x^2}{1} + \frac{y^2}{b^2} = 1$ and $b > 1$



So, from graph it is clear both curves meet at 4points. Hence, $x = 4$

Question 119

Which of the following natural element is the primary element in computer chips?

Options:

A. Silicon

B. Carbon

C. Iron

D. Uranium

Answer: A

Solution:

Solution:
Silicon is the natural elements which is usedas primary element in computer chips.

Question 120

What is the name of programs that control the computer system?

Options:

A. Hardware

B. Keyboard

C. Software

D. Mouse

Answer: C

Solution:

Solution:
Software control the computer system

Question 121

When $x^{300}+1$ is divided by x^3+1, then remainder is

Options:

A. 2

B. -2

C. 3

D. None of these

Answer: A

Solution:

Solution:
Let $P(x) = x^{300}+1$ and $q(x) = x^3+1$
Now, $q(x) = 0$
$\Rightarrow x^3+1=0$
$\Rightarrow x^3=-1$
Again, $P(x) = x^{300}+1$
$=(x^3)^{100}+1$
$=(-1)^{100}+1$
$=1+1=2$
So, when $x^{300}+1$ is divided by x^3+1
Remainder $=2$

Question 122

The maximum value of the function $f(x) = (\frac{1}{x})^x$ is

Options:

A. e^e

B. $(\frac{1}{e})^e$

C. $e^{\frac{1}{e}}$

D. None of the above

Answer: C

Solution:

Solution:

(c) Let $y = (\frac{1}{x})^x = x^{-x}$

$\Rightarrow \log y = -x \log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -(1 + \log x)$

$\Rightarrow \frac{dy}{dx} = -y(1 + \log x)$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-dy}{dx}(1 + \log x) - \frac{y}{x} = y(1 + \log x)^2 - \frac{y}{x}$

$= x^{-x}(1 + \log x)^2 - \frac{x^{-x}}{x}$

$= x^{-x}(1 + \log x)^2 - x^{-x-1}$

At points of local maximum and local minimum, we must have

$\frac{dy}{dx} = 0$

$-y(1 + \log x) = 0$

$\Rightarrow 1 + \log x = 0$

$\Rightarrow \log x = -1$

$\Rightarrow x = e^{-1}$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x = e^{-1}} = (\frac{1}{e})^{-\frac{1}{e}}(1 + \log \frac{1}{e})^2 - (\frac{1}{e})^{-\frac{1}{e}-1}$

$= (e^{-1})^{-\frac{1}{e}}(1 - \log e)^2 - (e^{-1})^{-\frac{1}{e}-1}$

$= -e^{\frac{1}{e}+1} < 0$

So, $x = \frac{1}{e}$ is a point of local maximum. The local maximum value of y is obtained by putting $x = \frac{1}{e}$ in y and is equal to $e^{\frac{1}{e}}$.

Question 123

The order and degree of following differential equation is

$$\frac{d^2y}{dx^2} = [y + (\frac{dy}{dx})^6]^{1/4}$$

Options:

A. order 2 and degree 6

B. order 1 and degree 6

C. order 2 and degree 4

D. None of the above

Answer: C

Solution:

Solution:
We have,
$\frac{d^2y}{dx^2} = [y + (\frac{dy}{dx})^6]^{\frac{1}{4}}$
$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^4 = y + (\frac{dy}{dx})^6$
$\therefore$ Order $= 2$, Degree $= 4$

Question 124

The sum of all 2-digit numbers which leave remainder 3 when divided by 5 is

Options:

A. 171

B. 99

C. 999

D. 111

Answer: C

Solution:

Solution:
Required numbers are $13, 18, 23, \ldots, 98$
These numbers from AP with $a = 13, d = 5, a_n = 98$
Now, $a_n = a + (n-1)d$
$\Rightarrow 98 = 13 + (n-1)(5)$
$\Rightarrow 85 = 5(n-1)$
$\Rightarrow 17 = n - 1$
$\Rightarrow n = 18$
Now, required sum $= S_n = \frac{n}{2}[a + a_n]$
$= \frac{18}{2}[13 + 98]$
$= 9 \times 111 = 999$

Question 125

The area bounded by curves $y = |\log_e|x||$ and $y = 0$ is

Options:

A. 2 sq units

B. 4 sq units

C. 8 sq units

D. None of these

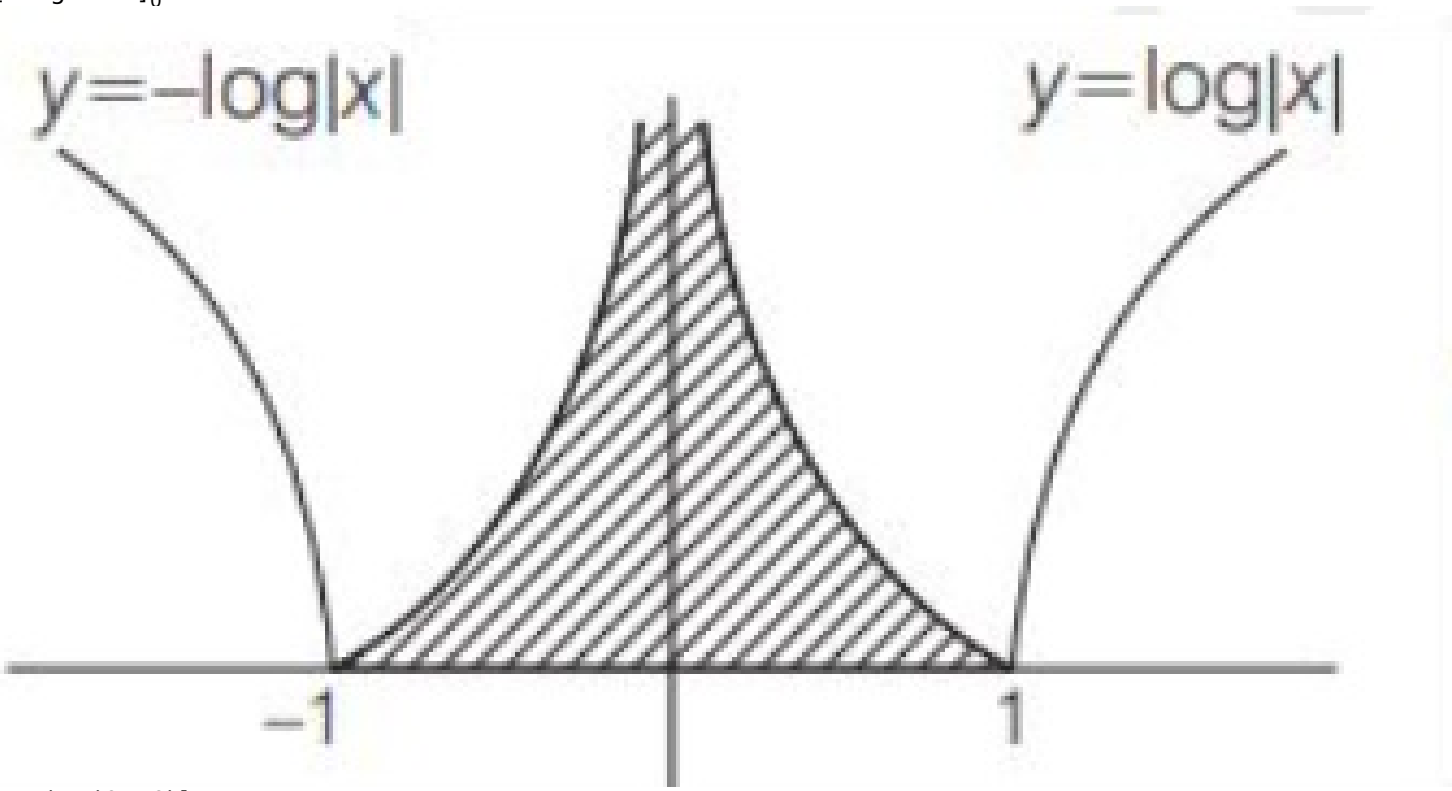
Answer: A

Solution:

Solution:

$\therefore$ Required area $= 2\int_0^1 \log x \, dx$

$= 2[x \log x - x]_0^1$



$2[(0-1)-(0-0)]$

2 sq units

Question 126

If $BC = a, CA = b$ and $AB = c$ are three sides of a triangle with $|a| = a$, $|b| = b$ and $|c| = c$, then $\frac{a^2 + b^2 - c^2}{2ab}$ is

Options:

A. cos A

B. cos B

C. sin C

D. cos C

Answer: D

Solution:

Solution:
In a $\triangle ABC$, with usual notations, we know that
$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Question 127

If position vectors of A and B are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$, then the position vector of point P dividing AB in the ratio 3:4 is

Options:

A. $\frac{\hat{i} - 9\hat{j} + \hat{k}}{7}$

B. $\frac{\hat{i} + \hat{j} - \hat{k}}{7}$

C. $\frac{9i - \hat{j} + \hat{k}}{7}$

D. None of these

Answer: D

Solution:

Solution:
$\mathbf{OP} = \frac{3\mathbf{OB} + 4\mathbf{OA}}{3+4}$
$= \frac{3[\hat{i} + \hat{j} - \hat{k}] + 4[2\hat{i} - \hat{j} + \hat{k}]}{7}$
$\frac{3\hat{i} + 3\hat{j} - 3\hat{k} + 8\hat{i} - 4\hat{j} + 4\hat{k}}{7}$
$\frac{11\hat{i} - \hat{j} + \hat{k}}{7}$

Question 128

The slope of the tangent at (x, y) to a curve passing through $(1, \frac{\pi}{4})$ is given by $\frac{y}{x} - \cos^2(\frac{y}{x})$, then the equation of the curve is

Options:

A. $y = x\tan^{-1}(\log(\frac{x}{e}))$

B. $y = x\tan^{-1}(\log(\frac{e}{x}))$

C. $y = x^2\tan^{-1}(\log(\frac{e}{x}))$

D. None of these

Answer: B

Solution:

Solution:

We have, $\frac{dy}{dx} = \frac{y}{x} - \cos^2(\frac{y}{x})$ . . . (i)

On putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in Eq . (i) ,

We get,

$v + x\frac{dv}{dx} = v - \cos^2 v$

$x\frac{dv}{dx} = -\cos^2 v$

$\int \sec^2 v dv = \int -\frac{1}{x} dx$

$\tan v = -\log x + c$

$\tan \frac{y}{x} = -\log x + c$

Since, curve passes through $(1, \frac{\pi}{4})$, So

$\tan \frac{\pi}{4} = -\log 1 + c$

$\Rightarrow c = 1$

$\therefore \tan \frac{y}{x} = -\log x + 1$

$\Rightarrow \tan \frac{y}{x} = \log(\frac{e}{x})$

$\Rightarrow y = x\tan^{-1}(\log(\frac{e}{x}))$

Question 129

The area of the region in lst quadrant bounded by the Y-axis $y = \frac{x}{4}, y = 1 + \sqrt{x}$ and $y = \frac{2}{\sqrt{x}}$ is

Options:

A. $\frac{8}{3}$ sq units

B. $\frac{11}{3}$ sq units

C. $\frac{2}{3}$ sq unit

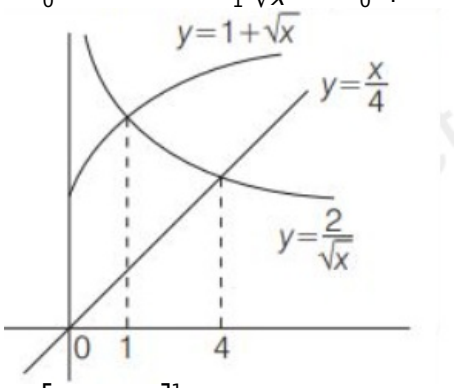
D. None of these

Answer: B

Solution:

Solution:
Required area

$= \int_0^1 (1+\sqrt{x})\,dx + \int_1^4 \frac{2}{\sqrt{x}}\,dx - \int_0^4 \frac{x}{4}\,dx$



$= \left[x + \frac{x^{3/2}}{(\frac{3}{2})}\right]_0^1 + ([2 \times 2\sqrt{x}]_1^4 - \left[\frac{x^2}{8}\right]_0^4.$

$= (1 + \frac{2}{3} - 0) + (8 - 4) - (2 - 0)$

$= \frac{5}{3} + 4 - 2 = \frac{11}{3}$ sq units

Question 130

$\lim_{x \to 0} \frac{x}{|x| + x^2}$ is equal to

Options:

A. 1

B. -1

C. does not exists

D. None of the above

Answer: C

Solution:

Solution:
We have,

LHL $= \lim_{x \to 0^-} \frac{x}{|x| + x^2}$

$= \lim_{h \to 0} \frac{(0-h)}{|0-h| + (0-h)^2}$

$= \lim_{h \to 0} \frac{-h}{h + h^2}$
$= \lim_{h \to 0} \frac{-1}{1 + h} = -1$
RHL $= \lim_{x \to 0^+} \frac{x}{|x| + x^2}$
$= \lim_{h \to 0} \frac{0 + h}{|0 + h| + |0 + h|^2}$
$= \lim_{h \to 0} \frac{h}{h + h^2} = \lim_{h \to 0} \frac{1}{1 + h} = 1$
$\therefore$ LHL $\neq$ RHL

Question 131

If $A + B = \frac{2\pi}{3}$, where $A, B \in R^+$, then the maximum value of $sin\ A + sin\ B$ is

Options:

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\frac{\sqrt{3}}{2}$

D. 2

Answer: B

Solution:

Solution:
Let $y = \sin A + \sin B$
$= \sin A + \sin(\frac{2\pi}{3} - A)$
$= \sin A + \sin\frac{2\pi}{3}\cos A - \cos\frac{2\pi}{3}\sin A$
$= \sin A + (\frac{\sqrt{3}}{2})\cos A - (\frac{-1}{2})\sin A$
$= \frac{3}{2}\sin A + \frac{\sqrt{3}}{2}\cos A$
$\therefore\ -\sqrt{(\frac{3}{2})^2 + (\frac{\sqrt{3}}{2})^2} \leq \frac{3}{2}\sin A + \frac{\sqrt{3}}{2}\cos A \leq \sqrt{(\frac{3}{2})^2 + (\frac{\sqrt{3}}{2})^2}$
$\Rightarrow -\sqrt{3} \leq y \leq \sqrt{3}$
So, maximum value of y is $\sqrt{3}$

Question 132

..... erases letters to the left to the cursor.

Options:

A. Backspace

B. Delete

C. Both

D. None of these

Answer: A

Solution:

Solution:
Backspace is used to erase lettes to the left ofthe cursor.

Question 133

If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$

Options:

A. $\frac{4}{3}$

B. $\frac{8}{3}$

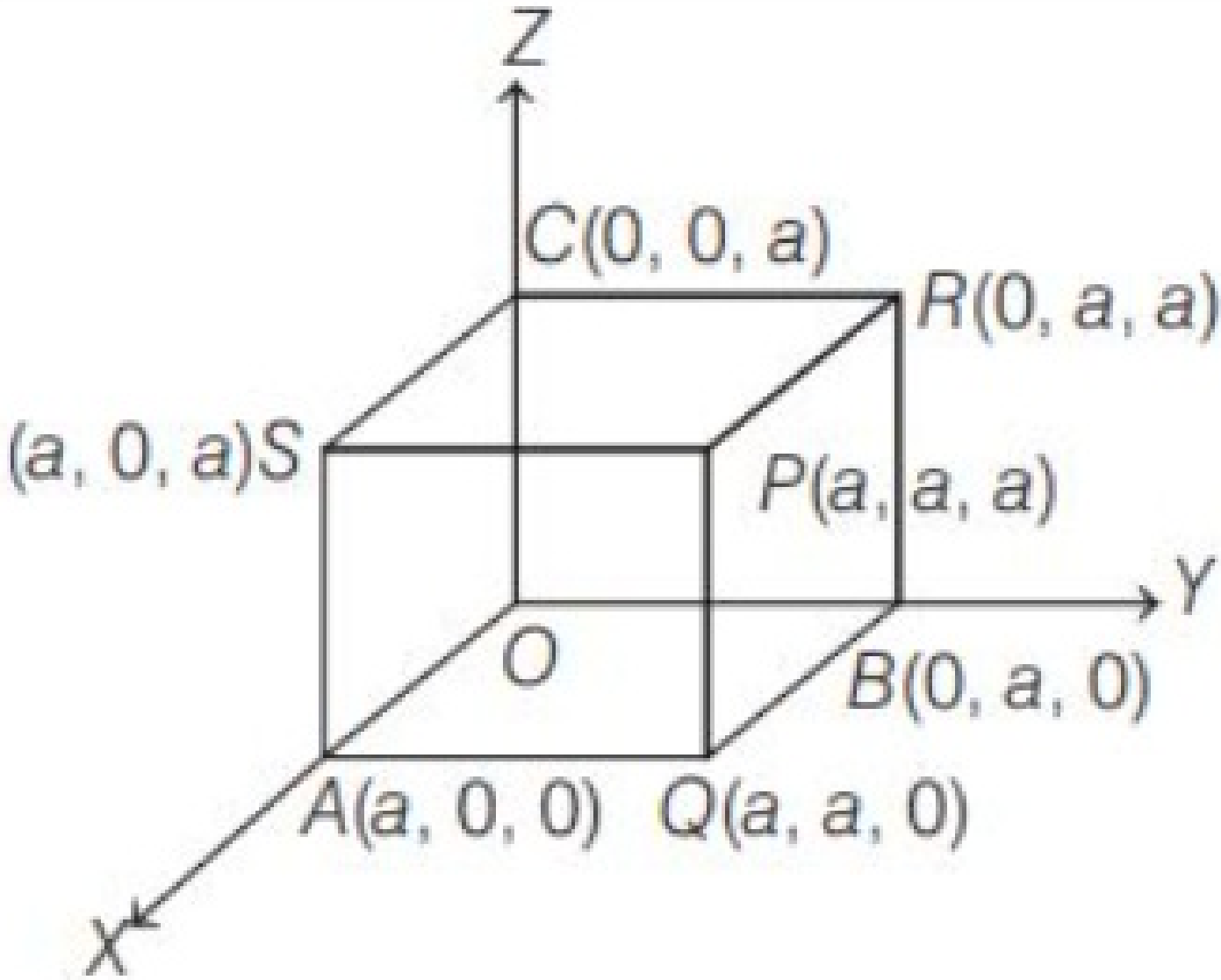
C. $\frac{3}{4}$

D. $\frac{3}{8}$

Answer: B

Solution:

Solution:
Let *a* be the length of an edge of the cube andlet one corner be at the origin.



Clearly, OP, AR, BS andCQ are the diagonals of the cube. The DR′s of OP, AR, BS and CQ are
$a-0, a-0, a-0$ i.e. a, a, a
$0-a, a-0, a-0$ i.e. $-a, a, a$
$a-0, 0-a, a-0$ i.e. $a, -a, a$
and
$a-0, a-0, 0-a$ i.e. $a, a, -a$ respectively.
Let $\alpha, \beta, \gamma, \delta$ be the angle made by a line whose DR′s are l, m, n with OP, AR, BS and CQ. Then,

$$\cos\alpha = \frac{a\cdot l + a\cdot m + a\cdot n}{\sqrt{a^2+a^2+a^2}\sqrt{l^2+m^2+n^2}}$$
$$= \frac{l+m+n}{\sqrt{3}\sqrt{l^2+m^2+n^2}}$$
$$\cos\beta = \frac{-a\cdot l + a\cdot m + a\cdot n}{\sqrt{a^2+a^2+a^2}\sqrt{l^2+m^2+n^2}}$$
$$= \frac{-l+m+n}{\sqrt{3}\sqrt{l^2+m^2+n^2}}$$
$$\cos\gamma = \frac{al - am + an}{\sqrt{a^2+a^2+a^2}\sqrt{l^2+m^2+n^2}}$$
$$= \frac{l-m+n}{\sqrt{3}\sqrt{l^2+m^2+n^2}}$$
$$\text{and } \cos\delta = \frac{al + am - an}{\sqrt{a^2+a^2+a^2}\sqrt{l^2+m^2+n^2}}$$
$$= \frac{l+m-n}{\sqrt{3}\sqrt{l^2+m^2+n^2}}$$
$$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$$
$$= \frac{(l+m+n)^2}{3(l^2+m^2+n^2)} + \frac{(-l+m+n)^2}{3(l^2+m^2+n^2)} + \frac{(l-m+n)^2}{3(l^2+m^2+n^2)} + \frac{(l+m-n)^2}{3(l^2+m^2+n^2)}$$
$$= \frac{4}{3}$$

Again,

$$1-\sin^2\alpha + 1 - \sin^2\beta + 1 - \text{si}^2\gamma + 1 - \sin^2\delta = \frac{4}{3}$$
$$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = 4 - \frac{4}{3} = \frac{8}{3}$$

Question 134

If $A + B + C = \frac{\pi}{6}$, then $\tan 6A + \tan 6B + \tan 6C =$

Options:

A. $\cot 6A \cdot \cot 6B \cdot \cot 6C$

B. 0

C. 1

D. $\tan 6A \cdot \tan 6B \cdot \tan 6C$

Answer: D

Solution:

Solution:
We have,
$A + B + C = \frac{\pi}{6}$
$\Rightarrow 6A + 6B + 6C = \pi$
$\Rightarrow 6A + 6B = \pi - 6C$
$\Rightarrow \tan(6A + 6B) = \tan(\pi - 6C)$
$\Rightarrow \frac{\tan 6A + \tan 6B}{1 - \tan 6A \cdot \tan 6B} = -\tan 6C$
$\Rightarrow \tan 6A + \tan 6B = -\tan 6C + \tan 6A \tan 6B \tan 6C$
$\Rightarrow \tan 6A + \tan 6B + \tan 6C = \tan 6A \tan 6B \tan 6C$

Question 135

The function $f(x) = \begin{cases} \frac{|x-3|}{x-3} & x \neq 3 \\ 0 & x = 3 \end{cases}$. is

Options:

A. discontinuous at $x = -2$

B. continuous at $x = -2$

C. continuous at $x = 3$

D. None of the above

Answer: B

Solution:

Solution:
We have

$$f(x) = \begin{cases} \frac{|x-3|}{x-3} & x \neq 3 \\ 0 & x = 3 \end{cases}.$$
$$= \begin{cases} \frac{-(x-3)}{x-3} & x < 3 \\ 0 & x = 3 \\ \frac{x-3}{x-3} & x > 3 \end{cases}$$
$$= \begin{cases} -1 & x < 3 \\ 0 & x = 3 \\ 1 & x > 3 \end{cases}$$
Now, at $x = 3$
$LHL = -1, RHL = 1, f(3) = 0$
So, $f(x)$ is not continuous at $x = 3$
Again, at $c = -2$
$f(x) = -1[\because$ when $x < 3, f(x) = -1]$
which is a polynomial function. So, $f(x)$ is continuous at $x = -2$

Question 136

$$\int x^{-11}(1+x^4)^{-1/2}\,dx =$$

Options:

A. $\frac{-1}{2}\left[\frac{(1+x^{-4})^{\frac{5}{2}}}{5} - \frac{2}{3}(1+x^{-4})^{\frac{3}{2}} + (1+x^{-4})\right] + c$

B. $\frac{-1}{2}\left[\frac{(1+x^{4})^{\frac{5}{2}}}{5} - \frac{2}{3}(1+x^{4})^{\frac{3}{2}} + (1+x^{4})\right] + c$

C. $\frac{1}{2}\left[\frac{(1+x^{4})^{\frac{5}{2}}}{5} + \frac{2}{3}(1+x^{4})^{\frac{3}{2}} + (1+x^{4})\right] + c$

D. None of the above

Answer: D

Solution:

Solution:
Let
$$I = \int x^{-11}(1+x^4)^{-\frac{1}{2}}\,dx$$
$$= \int \frac{1}{x^{11}(1+x^4)^{\frac{1}{2}}}\,dx$$
Put $x^2 = t$
$\Rightarrow 2x\,dx = dt$
$$\therefore I = \frac{1}{2}\int \frac{dt}{t^6(1+t^2)^{\frac{1}{2}}}$$
Put $t = \tan u$
$\Rightarrow dt = \sec^2 u\,du$

$$\therefore I = \frac{1}{2}\int \frac{\sec^2 u\, du}{\tan^6 u \cdot \sec u}$$
$$= \frac{1}{2}\int \frac{\sec u}{\tan^6 u} du$$
$$= \frac{1}{2}\int \frac{\cos^5 u}{\sin^6 u} du$$
$$= \frac{1}{2}\int \cot^5 u \operatorname{cosec} u\, d\, u$$
$$= \frac{1}{2}\int \cot^4 u \operatorname{cosec} u \cdot \cot u\, d\, u$$
$$= \frac{1}{2}\int (\operatorname{cosec}^2 u - 1)^2 \operatorname{cosec} u \cot u\, d\, u$$

Put $\operatorname{cosec} u = v$

$$\Rightarrow -\operatorname{cosec} u \cot u\, d\, u = dv$$

$$\therefore \mathrm{I} = \frac{-1}{2}\int (v^2 - 1)^2 dv$$
$$= \frac{-1}{2}\int (v^4 - 2v^2 + 1)\, dv$$
$$= \frac{-1}{2}\left[\frac{v^5}{5} - \frac{2v^3}{3} + v\right] + C$$
$$= \frac{-1}{2}\left[\frac{\operatorname{cosec}^5 u}{5} - \frac{2}{3}\operatorname{cosec}^3 u + \operatorname{cosec} u\right] + c$$
$$= \frac{-1}{2}\left[\frac{(t^2+1)^{5/2}}{5t^5} - \frac{2}{3}\frac{(t^2+1)^{3/2}}{t^3} + \frac{(t^2+1)^{1/2}}{t}\right] + c$$
$$= \frac{-1}{2}\left[\frac{(x^4+1)^{5/2}}{5x^{10}} - \frac{2}{3}\frac{(x^4+1)^{3/2}}{x^6} + \frac{(x^4+1)^{1/2}}{x^2}\right] + c$$

Question 137

If $D = \begin{vmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{vmatrix}$, then $\dfrac{D}{2(5!)^3} - 2$ is

Options:

A. a multiple of 5

B. a multiple of 6

C. 0

D. -5

Answer: A

Solution:

Solution:

We have,

$$\mathrm{D} = \begin{vmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{vmatrix}$$
$$= \begin{vmatrix} 5! & 6\times 5! & 7\times 6\times 5! \\ 6\times 5! & 7\times 6\times 5! & 8\times 7\times 6\times 5! \\ 7\times 6\times 5! & 8\times 7\times 6\times 5! & 9\times 8\times 7\times 6\times 5! \end{vmatrix}$$
$$= (5!)^3 \begin{vmatrix} 1 & 6 & 42 \\ 6 & 42 & 336 \\ 42 & 336 & 3024 \end{vmatrix}$$

Common 6 from R_2 and 42 from R_3,

$$(5!)^3 \times 6 \times 42 \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

On applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_2$, we get

$$(5!)^3 \times 6 \times 42 \begin{vmatrix} 1 & 6 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix}$$

$= (5!)^3 \times 6 \times 42 \times 1[16 - 14]$

$D = (5!)^3 \times 6 \times 42 \times 2$

$\Rightarrow \frac{D}{2(5!)^3} = 6 \times 42 = 252$

$\therefore \frac{D}{2(5!)^3} - 2 = 250$

which is a multiple of 5 .

Question 138

Differential coefficient of $\tan^{-1}(\frac{2x}{1-x^2})$ with respect to $\cos^{-1}(\frac{1-x^2}{1+x^2})$ is

Options:

A. 0

B. -1

C. 1

D. None of these

Answer: C

Solution:

Solution:

Let $u = \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x$

and $v = \cos^{-1}(\frac{1-x^2}{1+x^2}) = 2\tan^{-1}x$

$\therefore u = v \Rightarrow \frac{du}{dv} = 1$

Question 139

If a line is drawn through a fixed point $P(3,4)$ to cut the circle $2x^2 + 2y^2 - 32 = 0$ at A and B, then $PA \cdot PB =$

Options:

A. 16

B. 9

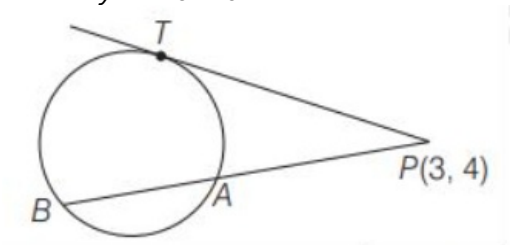
C. 41

D. 25

Answer: B

Solution:

Solution:
$2x^2 + 2y^2 - 32 = 0$
$\Rightarrow x^2 + y^2 - 16 = 0$



Now, length of tangent from $P(3,4)$ = PT
$= \sqrt{(3)^2 + (4)^2 - 16} = 3$
We know that
$PA \cdot PB = PT^2 = (3)^2 = 9$

Question 140

If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of two concurrent lines, then the direction ratios of the lines bisecting the angle between them are

Options:

A. $l_1 \pm m_1, l_2 \pm m_2, l_3 \pm m_3$

B. $m_1 \pm n_1, m_2 \pm n_2, m_3 \pm n_3$

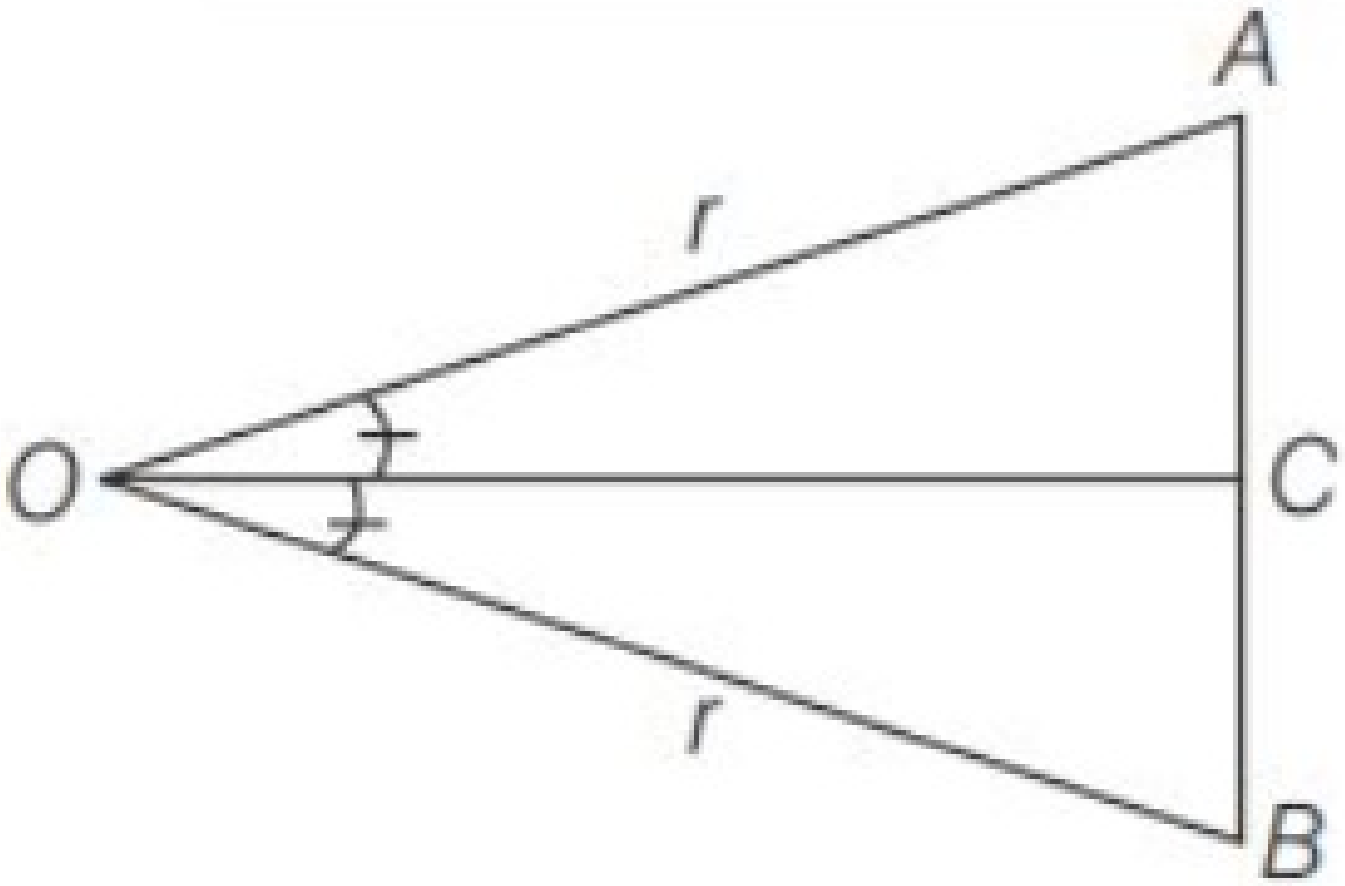
C. $l_1 \pm l_2, m_1 \pm m_2, n_1 \pm n_2$

D. $l_1 l_2, m_1 m_2, n_1 n_2$

Answer: C

Solution:

Solution:
Let O be the origin



Then, OA and OB are lines where DR′s are $< l_1, m_1, n_1 >$ and $< l_2, m_2, n_2 >$ respectively. Let OA = OB = r. Again OC denotes the angle bisector of $\angle AOB$. Then, C will be the mid-point of AB.

$\therefore$ Coordinates of C

$= \left(\frac{l_1 r + l_2 r}{2}, \frac{m_1 r + m_2 r}{2}, \frac{n_1 r + n_2 r}{2} \right)$

Now, DR′s of OC $= \frac{l_1 r + l_2 r}{2} - 0, \frac{m_1 r + m_2 r}{2} - 0, \frac{n_1 r + n_2 r}{2} - 0$

$= \frac{r(l_1 + l_2)}{2}, \frac{r(m_1 + m_2)}{2}, \frac{r(n_1 + n_2)}{2}$

$\therefore$ DR′s of OC $= l_1 + l_2 : m_1 + m_2 : n_1 + n_2$

Similarly, DR's of obtuse $\angle AOB$ will be $l_1 - l_2,\ m_1 - m_2, n_1 - n_2$

So, DR′s of angle bisector will be $l_1 \pm l_2, m_1 \pm m_2,\ n_1 \pm n_2$

Question 141

If $\int \frac{1 - x^7}{x(1 + x^7)} \mathrm{dx} = \mathrm{a} \ln \left| x \right| + b \ln x^7 + 1 \left| + c \right.$ **then**

Options:

A. $a = 1$ and $b = \frac{-2}{7}$

B. $a = 1$ and $b = \frac{2}{7}$

C. $a = -1$ and $b = \frac{-2}{7}$

D. None of the above

Answer: A

Solution:

Solution:

Let $I = \int \frac{1-x^7}{x(1+x^7)}\,dx$

$= \int \frac{2-(1+x^7)}{x(1+x^7)}\,dx$

$= 2\int \frac{1}{x(1+x^7)}\,dx - \int \frac{1}{x}\,dx$

$= 2\int \frac{x^6}{x^7(1+x^7)}\,dx - \ln x + c_1$

Again, let $I_1 = \int \frac{x^6}{x^7(1+x^7)}\,dx$

Put $x^7 = t$ and $7x^6\,dx = dt$

$\therefore I_1 = \frac{1}{7}\int \frac{dt}{t(1+t)}$

$= \frac{1}{7}\int (\frac{1}{t} - \frac{1}{1+t})\,dt$

$= \frac{1}{7}\ln t - \frac{1}{7}\ln(1+t) + c_2$

$= \frac{1}{7}\ln x^7 - \frac{1}{7}\ln(1+x^7) + c_2$

$= \ln x - \frac{1}{7}\ln(1+x^7) + c_2$

Now, $I = 2I_1 - \ln x + c_1$

$= 2\ln x - \frac{2}{7}\ln(1+x^7) - \ln x + c_1 + 2c_2$

$= \ln x - \frac{2}{7}\ln(1+x^7) + c,$

where, $c = c_1 + 2c_2$

$\therefore a = 1, b = -\frac{2}{7}$

Question 142

The curve $y = \sqrt{x^2-4}$ is defined in the interval $[2, 4]$. Find the coordinates of point P, such that the tangent at point P is parallel to the chord joining two points of the curve

Options:

A. $(\sqrt{2}, \sqrt{4})$

B. $(4, 2\sqrt{3})$

C. $(\sqrt{6}, \sqrt{2})$

D. None of these

Answer: C

Solution:

Solution:

Let $y = f(x) = \sqrt{x^2-4}$ on $[2, 4]$

According to LMVT ,

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$= \frac{f(4)-f(2)}{4-2}$

$= \frac{\sqrt{16-4}-\sqrt{4-4}}{2}$

$= \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\therefore \frac{2c}{2\sqrt{c^2-4}} = \sqrt{3} \ \ldots \left[\because f'(x) = \frac{2x}{2\sqrt{x^2-4}}\right]$

$\Rightarrow c = \sqrt{3} \times \sqrt{c^2-4}$

$\Rightarrow c^2 = 3(c^2-4)$

$\Rightarrow c^2 = 3c^2 - 12$

$\Rightarrow 2c^2 = 12$

$\Rightarrow c^2 = 6$

$\Rightarrow c = \sqrt{6} \ \ldots [\because x \in [2,4]]$

Again $f(c) = \sqrt{6-4} = \sqrt{2}$

So, required point is $(\sqrt{6}, \sqrt{2})$.

Question 143

If $\hat{i} - \hat{j}$, $\hat{j} - \hat{k}$, $\hat{k} - \hat{i}$ and $\hat{i} + \hat{j} + \hat{k}$ are the vertices of a tetrahedron, then its volume is

Options:

A. $\frac{1}{6}$

B. 6

C. 1

D. None of these

Answer: D

Solution:

Solution:

Let A, B, C, D are vertices of tetrahedron and $\mathbf{OA} = (\hat{i} - \hat{j}), \mathbf{OB} = \hat{j} - \hat{k}, \mathbf{OC} = \hat{k} - \hat{i}$ and $\mathbf{OD} = \hat{i} + \hat{j} + \hat{k}$. Then,

$\mathbf{AB} = -\hat{i} + 2\hat{j} - \hat{k}, \mathbf{AC} = -2\hat{i} + \hat{j} + \hat{k}, \mathbf{AD} = 2\hat{j} + \hat{k}$

$\therefore$ Volume of tetrahedron $= \frac{1}{6}[\mathrm{ABC}]$

$= \frac{1}{6}\begin{vmatrix} -1 & 2 & -1 \\ -2 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$= \frac{1}{6}[-1(1-2) - 2(-2-0) - 1(-4-0)]$

$\frac{1}{6}[1+4+4]$

$= \frac{9}{6} = \frac{3}{2}$ cubic unit

Question 144

The area of triangle is $10\mathrm{m}^2$. If its two vertices are $(0,0), (5,0)$ and the third vertex lie on the line $x = 5$, then the third vertex is

Options:

A. $(4,5)$

B. $(5,4)$

C. $(0,4)$

D. $(5,5)$

Answer: B

Solution:

Solution:
Let the vertices of the triangle are $A(0,0)$, $B(5,0)$ and $C(5,y)$.
Now, ar$(\triangle ABC) = 10$ sq units

$$\therefore \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & y & 1 \end{vmatrix} = \pm 10$$

$$\Rightarrow 1[5y - 0] = \pm 20$$
$$\Rightarrow 5y = \pm 20$$
$$\Rightarrow y = \pm 4$$

So, third vertex is $(5, \pm 4)$.

Question 145

The slope of a curve, passing through $(3,4)$ at any point is reciprocal of twice the ordinate of that point. Then, the curve is

Options:

A. parabola

B. hyperbola

C. ellipse

D. None of the above

Answer: A

Solution:

Solution:
We have,

$$\frac{dy}{dx} - \frac{1}{2y}$$

$$2ydy = dx$$
$$\Rightarrow \int 2ydy - \int dx$$
$$\Rightarrow y^2 = x + c$$

Since, curve passes through $(3,4)$

$$\therefore (4)^2 = 3 + c$$
$$\Rightarrow c = 13$$

$\therefore$ Equation of curve is :

$$y^2 = x + 13$$

Which is a equation of parabola.

Question 146

The three points $(1, 2, 3), (-2, 3, 5)$ and $(7, 0, -1)$

Options:

A. are collinear

B. form a right angled triangle

C. form an equilateral triangle

D. are equidistant from each other

Answer: A

Solution:

Solution:
Let $A(1, 2, 3), B(-2, 3, 5)$ and $C(7, 0, -1)$
Then, $\mathbf{AB} = (-2 - 1)\hat{\mathbf{i}} + (3 - 2)\hat{\mathbf{j}} + (5 - 3)\hat{\mathbf{k}}$
$-3\hat{\mathbf{i}} + \hat{\mathbf{j}} = 2\hat{\mathbf{k}}$
and $\mathbf{BC} = (7 + 2)\hat{\mathbf{i}} + (0 - 3)\hat{\mathbf{j}} + (-1 - 5)\hat{\mathbf{k}}$
$-9\hat{\mathbf{i}} + 3\mathbf{j} - 6\hat{\mathbf{k}}$
$3(-3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
$\mathbf{BC} = -3\mathbf{AB}$

Question 147

The product of perpendiculars drawn from any point on $4x^2 - 9y^2 = 144$ upon its asymptotes is

Options:

A. 144/3

B. 36

C. 16

D. 52

Answer: A

Solution:

Solution:
We have,

$4x^2 - 9y^2 = 144$
$\Rightarrow \frac{x^2}{36} - \frac{y^2}{16} = 1$
which is equation of hyperbola.
Now, equation of asymptotes of hyperbola
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $y = \pm \frac{b}{a}x$
$\therefore$ Equation of asymptotes of given hyperbola are
$y = \pm\frac{4}{6}x$
$\Rightarrow y = \pm\frac{2}{3}x$
So, equation of asymptotes are $3y + 2x = 0$ and $3y - 2x = 0$
Let (h, k) be any point of hyperbola. Then,
Distance of (h, k) from $3y + 2x = 0$ is given by
$d_1 = \left|\frac{2h + 3k}{\sqrt{4 + 9}}\right| = \left|\frac{2h + 3k}{\sqrt{13}}\right|$
and distance of (h, k) from $3y - 2x = 0$ is given by
$d_1 = \left|\frac{2h + 3k}{\sqrt{4 + 9}}\right| = \left|\frac{2h + 3k}{\sqrt{13}}\right|$ and distance of (h, k) from $3y - 2x = 0$ is given by
$d_2 = \left|\frac{-2h + 3k}{\sqrt{4 + 9}}\right|$
$= \left|\frac{-2h + 3k}{\sqrt{13}}\right|$
Now, $d_1 d_2 = \left|\frac{2h + 3k}{\sqrt{13}}\right| \times \left|\frac{-2h + 3k}{\sqrt{13}}\right|$
$= \left|\frac{-4h^2 + 9k^2}{13}\right|$
$= \left|\frac{-(4h^2 - 9k^2)}{13}\right|$ . . . [$\because (h, k)$ lies on hyperbola]
$= \left|\frac{-144}{13}\right| = \frac{144}{13}$

Question 148

If $f(x) = sin\ |x| + \cos|x|$, then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx =$

Options:

A. 2

B. 4

C. 8

D. None of the above

Answer: B

Solution:

Solution:

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx$
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)\,dx$
$= 2\int_{0}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)\,dx$

$$= 2\int_0^{\frac{\pi}{2}} (\sin x + \cos x)\, dx$$
$$= 2[-\cos x + \sin x]_0^{\pi/2}$$
$$= 2[(0+1) - (-1+0)]$$
$$= 4$$

Question 149

In the expansion of $(2 + 3x - 4x^3)^{2034}$ the sum of all coefficients is

Options:

A. 2034

B. -2034

C. 1

D. -1

Answer: C

Solution:

Solution:
Sum of all coefficient
$= (2 + 3 - 4)^{2034}$
$= (1)^{2034} = 1$

Question 150

A bag contains 5 balls and of these it is equally likely that $0, 1, 2, 3, 4, 5$ are white. A ball drawn and if found to be white. The chance that it is the only white ball, will be

Options:

A. $\frac{1}{12}$

B. $\frac{1}{6}$

C. $\frac{1}{15}$

D. None of these

Answer: C

Solution:

Solution:
150. (c) Let E_1 : Event that bag contains 0 white ball.
E_2 : Event that bag contains 1 white ball.
E_3 : Event that bag contains 2 white ball.
E_4 : Event that bag contains 3 white ball.
E_5 : Event that bag contains 4 white ball.
E_6 : Event that bag contains 5 white ball.
A : Event that drawn ball is white ball.
Now, $P(E_1)P(E_2) = P(E_3) = P(E_4) = P(E_5) = P(E_6) = \frac{1}{6}$

$$P\left(\frac{A}{E_1}\right) = 0$$

$$P\left(\frac{A}{E_2}\right) = \frac{^1C_1}{^5C_1} = \frac{1}{5}$$

$$P\left(\frac{A}{E_3}\right) = \frac{^2C_1}{^5C_1} = \frac{2}{5}$$

$$P\left(\frac{A}{E_4}\right) = \frac{^3C_5}{^5C_1} = \frac{3}{5}$$

$$P\left(\frac{A}{E_5}\right) = \frac{^4C_1}{^5C_1} = \frac{4}{5}$$

$$P\left(\frac{A}{E_5}\right) = \frac{^5C_1}{^5C_1} = \frac{5}{5} = 1$$

Required probability

$$= \frac{P(E_2/A)P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + P(E_4)P(A/E_4) + P(E_5)P(A/E_5) + P(E_6)P(A/E_6)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{5}}{\frac{1}{6} \times 0 + \frac{1}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{2}{5} + \frac{1}{6} \times \frac{3}{5} + \frac{1}{6} \times \frac{4}{5} + \frac{1}{6} \times 1}$$

$$= \frac{1}{1+2+3+4+5} = \frac{1}{15}$$

Question 63

Among the following which are mismatched?

	Molecule	Hybridisation
A.	ClO_2^-	sp^3
B.	NH_3	sp^2
C.	SF_6	sp^3d^2
D.	SF_4	sp^3d
E.	XeF_4	sp

Options:

A. A and C

B. *B* and *E*

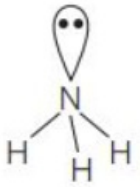
C. C and E

D. *A* and *D*

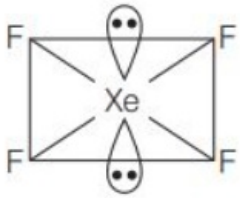
Answer: B

Solution:

Solution:
NH_3 is sp^3-hybrid having a lone pair.



XeF_4 is sp^3d^2-hybrid having 2 lone pairs.



Hence, B and E are mismatched.
