Exercise 4.1

1. (i) All circles are .... (congruent, similar).
 (ii) All squares are .... (similar, congruent).
 (iii) All .... triangles are similar (isosceles, equilaterals):
 (iv) Two triangles are similar, if their corresponding angles are .... (proportional, equal)
 (v) Two triangles are similar, if their corresponding sides are .... (proportional, equal)
 (vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles are and (b) their corresponding sides are .... (equal, proportional).

 Sol:
 (i) All circles are similar
 (ii) All squares are similar
 (iii) All equilateral triangles are similar
 (iv) Two triangles are similar, if their corresponding angles are equal
 (v) Two triangles are similar, if their corresponding sides are proportional
 (vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.

2. Write the truth value (T/F) of each of the following statements:
 (i) Any two similar figures are congruent.
 (ii) Any two congruent figures are similar.
 (iii) Two polygons are similar, if their corresponding sides are proportional.
 (iv) Two polygons are similar if their corresponding angles are proportional.
 (v) Two triangles are similar if their corresponding sides are proportional.
 (vi) Two triangles are similar if their corresponding angles are proportional.

 Sol:
 (i) False
 (ii) True
 (iii) False
 (iv) False
 (v) True
 (vi) True

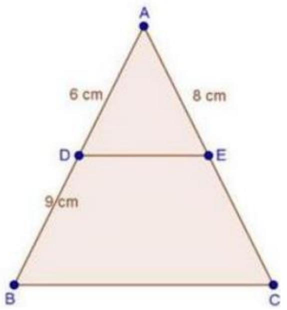
Exercise 4.2

1. In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC
 (i) If AD = 6 cm, DB = 9 cm and AE = 8 cm, find AC.
 (ii) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm, find AE
 (iii) If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, find AE

(iv) If AD = 4, AE = 8, DB = x – 4, and EC = 3x – 19, find x.
(v) If AD = 8cm, AB = 12 cm and AE = 12 cm, find CE.
(vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.
(vii) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.
(viii) If $\frac{AD}{BD} = \frac{4}{5}$ $and\ EC = 2.5\ cm, find\ AE$
(ix) If AD = x, DB = x − 2, AE = x + 2 and EC = x − 1, find the value of x.
(x) If AD = 8x − 7, DB = 5x − 3, AE = 4x − 3 and EC = (3x − 1), find the value of x.
(xi) If AD = 4x − 3, AE = 8x – 7, BD = 3x – 1 and CE = 5x − 3, find the volume of x.
(xii) If AD = 2.5 cm, BD = 3.0 cm and AE = 3.75 cm, find the length of AC.

Sol:

(i)



We have,

DE || BC

Therefore, by basic proportionally theorem,

We have $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{6}{9} = \frac{8}{EC}$

$\Rightarrow \frac{2}{3} = \frac{8}{EC}$

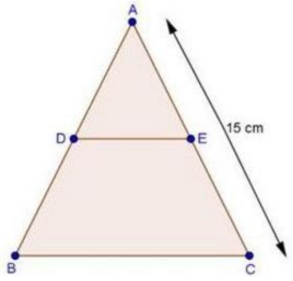
$\Rightarrow$ EC = $\frac{8\times3}{2}$

$\Rightarrow$ EC = 12 cm

$\Rightarrow$ Now, AC = AE + EC = 8 + 12 = 20 cm

∴ AC = 20 cm

(ii)



We have,

$\frac{AD}{DB} = \frac{3}{4}$ $and\ DE\ ||BC$

$Therefore, by\ basic\ proportionality\ theorem, we\ have$

$\frac{AD}{DB} = \frac{AE}{EC}$

Adding 1 on both sides, we get

$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\frac{3}{4} + 1 = \frac{AE+EC}{EC}$

$\Rightarrow \frac{3+4}{4} = \frac{AC}{EC}$ [∵ AE + EC = AC]

$\Rightarrow \frac{7}{4} = \frac{15}{EC}$

$\Rightarrow \text{EC} = \frac{15\times4}{7}$

$\Rightarrow \text{EC} = \frac{60}{7}$

Now, $\text{AE} + \text{EC} = \text{AC}$

$\Rightarrow \text{AE} + \frac{60}{7} = 15$

$\Rightarrow \text{AE} = 15 - \frac{60}{7}$

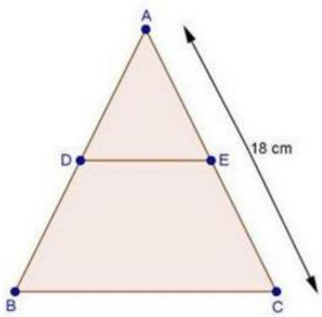
$= \frac{105-60}{7}$

$= \frac{45}{7}$

$= 6.43$ cm

∴ AE = 6.43 cm

(iii)



We have,

$\frac{AD}{DB} = \frac{2}{3}$ $and\ DE\ ||\ BC$

$Therefore, by\ basic\ proportionality\ theorem, we\ have,$

$\frac{AD}{DB} = \frac{EC}{AE}$

$\Rightarrow \frac{3}{2} = \frac{EC}{AE}$

Adding 1 on both sides, we get

$\frac{3}{2} + 1 = \frac{EC}{AE} + 1$

$\Rightarrow \frac{3+2}{2} = \frac{EC+AE}{AE}$

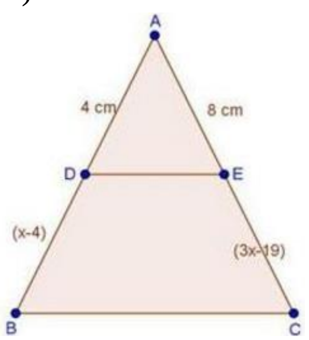
$\Rightarrow \frac{5}{2} = \frac{AC}{AE}$ [∵ AE + EC = AC]

$\Rightarrow \frac{5}{2} = \frac{18}{AE}$ [∵ AC = 18]

$\Rightarrow AE = \frac{18\times2}{5}$

$\Rightarrow AE = \frac{36}{5} = 7.2\ cm$

(iv)



We have,

DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{4}{x-4} = \frac{8}{3x-19}$

⇒ 4(3x – 19) = 8(x – 4)

⇒ 12x – 76 = 8x – 32

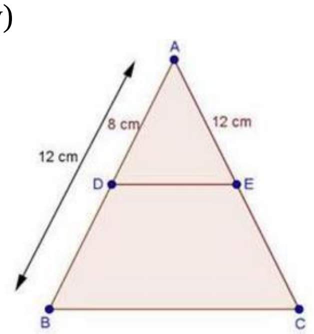
⇒ 12x – 8x = −32 + 76

⇒ 4x = 44

$\Rightarrow x = \frac{44}{4} = 11cm$

∴ x = 11 cm

(v)



We have,

AD = 8cm, AB = 12 cm

∴ BD = AB – AD

= 12 – 8

⇒ BD = 4 cm

And, DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{BD} = \frac{AE}{CE}$

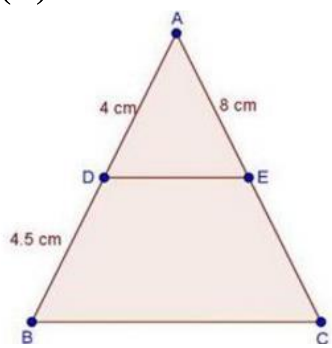
⇒ $\frac{8}{4} = \frac{12}{CE}$

⇒ CE = $\frac{12 \times 4}{8} = \frac{12}{2}$

⇒ CE = 6cm

∴ CE = 6cm

(vi)



We have,

DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

⇒ $\frac{4}{4.5} = \frac{8}{EC}$

⇒ $EC = \frac{8 \times 4.5}{4}$

⇒ EC = 9cm

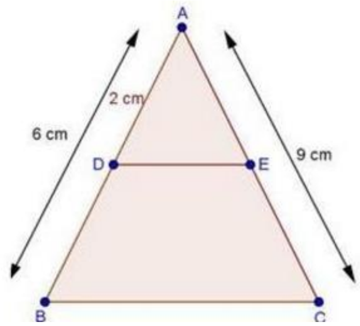
Now, AC = AE + EC

= 8 + 9

= 17 cm

∴ AC = 17 cm

(vii)



We have,

AD = 2 cm, AB = 6 cm

∴ DB = AB – AD

= 6 – 2

⇒ DB = 4 cm

And, DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

Taking reciprocal on both sides, we get,

$\frac{DB}{AD} = \frac{EC}{AE}$

$\frac{4}{2} = \frac{EC}{AE}$

Adding 1 on both sides, we get

$\frac{4}{2} + 1 = \frac{EC}{AE} + 1$

$\Rightarrow \frac{4+2}{2} = \frac{EC+AE}{AE}$

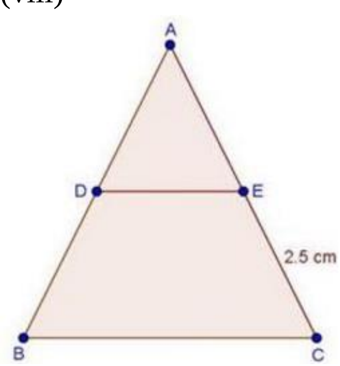
$\Rightarrow \frac{6}{2} = \frac{AC}{AE}$ [∵ EC + AE = AC]

$\Rightarrow \frac{6}{2} = \frac{9}{AE}$ [∵ AC = 9cm]

$AE = \frac{9\times 2}{6}$

$\Rightarrow AE = 3cm$

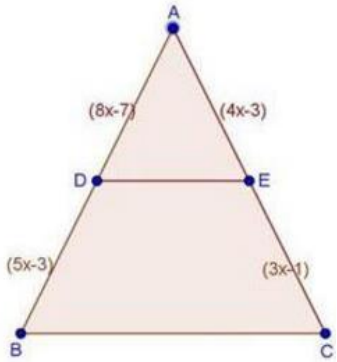
(viii)



We have, DE || BC

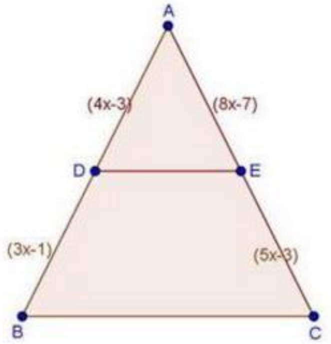
Therefore, by basic proportionality theorem,
We have,
$\frac{AD}{BD} = \frac{AE}{EC}$
$\Rightarrow \frac{4}{5} = \frac{AE}{2.5}$
$\Rightarrow$ AE = $\frac{4 \times 2.5}{5}$
$\Rightarrow$ AE = 2cm

(ix)



We have,
DE || BC
Therefore, by basic proportionality theorem,
We have,
$\frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$
$\Rightarrow x(x-1) = (x+2)(x-2)$
$\Rightarrow x^2 - x = x^2 - (2)^2$ [∵ (a – b) (a + b) = $a^2 - b^2$]
$\Rightarrow -x = -4$
$\Rightarrow$ x = 4 cm
∴ x = 4 cm

(x)



We have,

DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

$\Rightarrow (8x-7)(3x-1) = (4x-3)(5x-3)$

$\Rightarrow 24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$

$\Rightarrow 24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$

$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2[2x^2 - x - 1] = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow 2x^2 - 2x + 1x - 1 = 0$

$\Rightarrow 2x(x-1) + 1(x-1) = 0$

$\Rightarrow (2x + 1)(x - 1) = 0$

$\Rightarrow 2x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow x = -\frac{1}{2}$ *or* $x = 1$

$x = -\frac{1}{2}$ *is not possible*

$\therefore x = 1$

(xi)

We have, DE || BC

Therefore, by basic proportionality theorem,

We have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$\Rightarrow (4x-3)(5x-3) = (8x-7)(3x-1)$

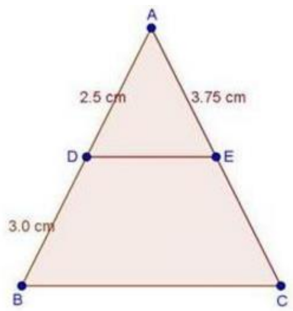
$\Rightarrow 4x(5x-3) - 3(5x-3) = 8x(3x-1) - 7(3x-1)$

$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$\Rightarrow 4x^2 - 2x - 2 = 0$

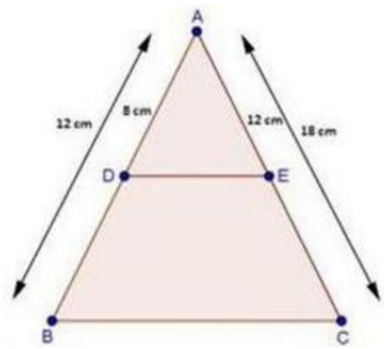
$\Rightarrow 2(2x^2 - x - 1) = 0$
$\Rightarrow 2x^2 - x - 1 = 0$
$\Rightarrow 2x^2 - 2x + 1x - 1 = 0$
$\Rightarrow 2x(x-1) + 1(x-1) = 0$
$\Rightarrow (2x + 1)(x - 1) = 0$
$\Rightarrow 2x + 1 = 0$ or $x - 1 = 0$
$\Rightarrow x = -\frac{1}{2}$ or $x = 1$
$x = -\frac{1}{2}$ is not possible
$\therefore x = 1$

(xii)



We have, DE || BC
Therefore, by basic proportionality theorem, we have,
$\frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{2.5}{3.0} = \frac{3.75}{EC}$
$\Rightarrow EC = \frac{3.75 \times 3}{2.5} = \frac{375 \times 3}{250}$
$\Rightarrow EC = \frac{15 \times 3}{10}$
$= \frac{45}{10} = 4.5\ cm$
Now, $AC = AE + EC = 3.75 + 4.5 = 8.25$
$\therefore AC = 8.25$ cm

2. In a ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE || BC:
(i) AB = 2cm, AD = 8cm, AE = 12 cm and AC = l8cm.
(ii) AB = 5.6cm, AD = 1.4cm, AC= 7.2 cm and AE = 1.8 cm.
(iii) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm and AE = 2.8 cm.
(iv) AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm.
Sol:



AB = 12 cm, AD = 12 cm and AC = 18 cm.

∴ DB = AB – AD

= 12 – 8

⇒ DB = 4 cm

And, EC = AC – AE

= 18 – 12

⇒ EC = 6 cm

Now, $\frac{AD}{DB} = \frac{8}{4} = \frac{2}{1}$ [∵ DB = 4 cm]

And, $\frac{AE}{EC} = \frac{12}{6} = \frac{2}{1}$ [∵ EC = 6 cm]

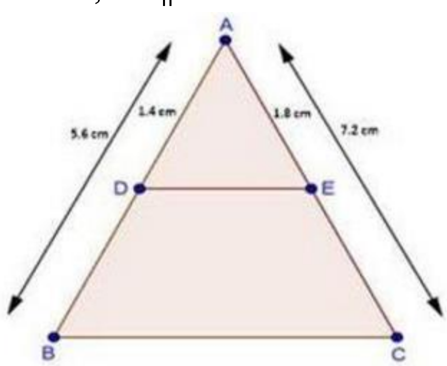
⇒ $\frac{AD}{DB} = \frac{AE}{EC}$

Thus, DE divides sides AB and AC of ∆ABC in the same ratio.

Therefore, by the converse of basic proportionality theorem,

(ii)

We have, DE || BC



We have,

AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

∴ DB = AB – AD

= 5.6 – 1.4

⇒ DB = 4.2 cm

And, EC = AC – AE

= 7.2 – 1.8

⇒ EC = 5.4 cm

Now, $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ [∵ DB = 4.2 cm]

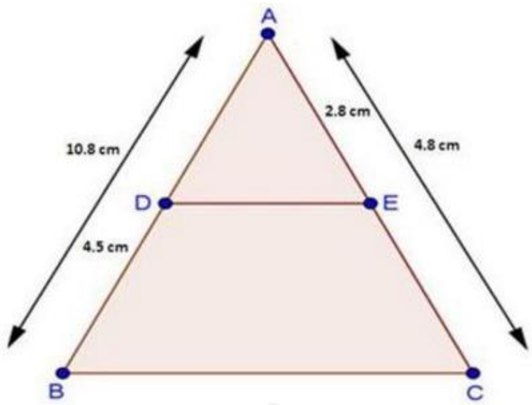
And, $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$ [∵ EC = 5.4 cm]

Thus, DE divides sides AB and AC of ΔABC in the same ratio.

Therefore, by the converse of basic proportionality theorem,

(iii)

We have,



We have,

AB = 10.8cm, BD = 4.5cm, AC = 4.8 cm and AE = 2.8cm

∴ AD = AB – DB = 10.8 – 4.5

⇒ AD = 6.3 cm

And, EC = AC – AE

= 4.8 – 2.8

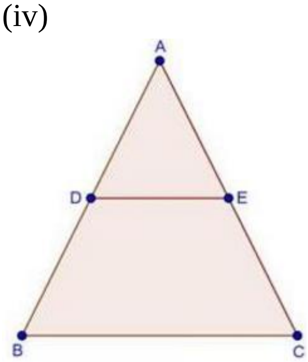
⇒ EC = 2 cm

Now, $\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$ [∵ AD = 6.3 cm]

And, $\frac{AE}{EC} = \frac{2.8}{2} = \frac{28}{20} = \frac{75}{5}$ [∵ EC = 2 cm]

Thus, DE divides sides AB and AC of ΔABC in the same ratio. Therefore, by the converse of basic proportionality theorem.

(iv)



We have,

DE || BC

We have, AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm

Now $\frac{AD}{BD} = \frac{5.7}{9.5} = \frac{57}{95}$

$\Rightarrow \frac{AD}{BD} = \frac{3}{5}$

And, $\frac{AE}{EC} = \frac{3.3}{5.5} = \frac{33}{55}$

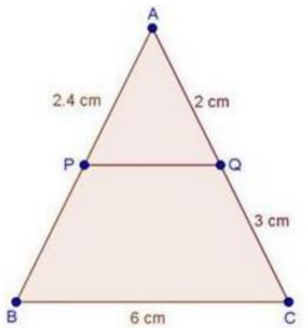
$\Rightarrow \frac{AE}{EC} = \frac{3}{5}$

Thus DE divides sides AB and AC of ΔABC in the same ratio.

Therefore, by the converse of basic proportionality theorem. We have DE || BC

3. In a ΔABC, P and Q are points on sides AB and AC respectively, such that PQ || BC. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm and BC = 6 cm, find AB and PQ.

Sol:



We have || BC

Therefore, by BPT

We have,

$\frac{AP}{PB} = \frac{AQ}{QC}$

$\frac{2.4}{PB} = \frac{2}{3}$

$\Rightarrow$ PB = $\frac{3\times2.4}{2} = \frac{3\times24}{2} = \frac{3\times6}{5} = \frac{18}{5}$

$\Rightarrow$ PB = 3.6 cm

Now, AB = AP + PB

= 2.4 + 3.6 = 6cm

Now, In ΔAPQ and ΔABC

∠A = ∠A [common]

∠APQ = ∠ABC [∵ PQ || BC ⇒ Corresponding angles are equal]

⇒ ΔAPQ ~ ΔABC [By AA criteria]

$\Rightarrow \frac{AB}{AP} = \frac{BC}{PQ}$ [corresponding sides of similar triangles are proportional]

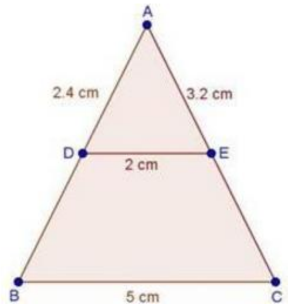
$\Rightarrow$ PQ = $\frac{6\times2.4}{6}$

$\Rightarrow$ PQ = 2.4 cm

Hence, AB = 6 cm and PO = 2.4 cm

4. In a ΔABC, D and E are points on AB and AC respectively such that DE || BC. If AD = 2.4cm, AE = 3.2 cm, DE = 2cm and BC = 5 cm, find BD and CE.

Sol:



We have,

DE || BC

Now, In ΔADE and ΔABC

∠A = ∠A [common]

∠ADE = ∠ABC [∵ DE || BC ⇒ Corresponding angles are equal]

⇒ ΔADE ~ ΔABC [By AA criteria]

⇒ $\frac{AB}{BC} = \frac{AD}{DE}$ [corresponding sides of similar triangles are proportional]

⇒ AB = $\frac{2.4 \times 5}{2}$

⇒ AB = 1.2 × 5 = 6.0 cm

⇒ AB = 6 cm

∴ BD = 6 cm

BD = AB – AD

= 6 – 2.4 = 3.6 cm

⇒ DB = 3.6 cm

Now,

$\frac{AC}{BC} = \frac{AE}{DE}$ [∵ Corresponding sides of similar triangles are equal]

⇒ $\frac{AC}{5} = \frac{3.2}{2}$

⇒ $AC = \frac{3.2 \times 5}{2} = 1.6 \times 5 = 8.0\ cm$

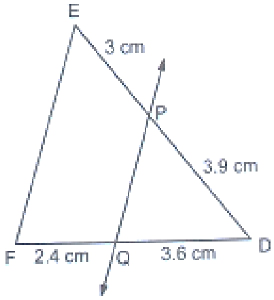
⇒ AC = 8 cm

∴ CE = AC – AE

= 8 – 3.2 = 4.8 cm

Hence, BD = 3.6 cm and CE = 4.8 cm

5. In below Fig., state if PQ || EF.



Sol:

We have,

DP = 3.9 cm, PE = 3cm, DQ = 3.6 cm and QF = 2.4 cm

Now, $\frac{DP}{PE} = \frac{3.9}{3} = \frac{1.3}{1} = \frac{13}{10}$

And, $\frac{DQ}{QF} = \frac{3.6}{2.4} = \frac{36}{24} = \frac{3}{2}$

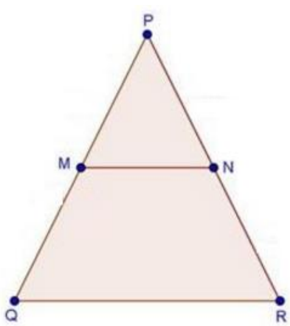
$\Rightarrow \frac{DP}{PE} \neq \frac{DQ}{QF}$

So, PQ is not parallel to EF

6. M and N are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether MN || QR

(i) PM = 4cm, QM = 4.5 cm, PN = 4 cm and NR = 4.5 cm

Sol:



(i) We have, PM = 4cm, QM = 4.5 cm, PN = 4 cm and NR = 4.5 cm

Hence, $\frac{PM}{QM} = \frac{4}{4.5} = \frac{8}{9}$

Also, $\frac{PN}{NR} = \frac{4}{4.5} = \frac{8}{9}$

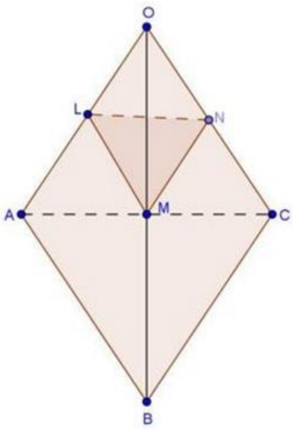
Hence, $\frac{PM}{QM} = \frac{PN}{NR}$

By converse of proportionality theorem

MN || QR

7. In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that LM || AB and MN || BC but neither of L, M, N nor of A, B, C are collinear. Show that LN ||AC.

Sol:



We have,

LM || AB and MN || BC

Therefore, by basic proportionality theorem,

We have,

$\frac{QL}{AL} = \frac{OM}{MB}$ ...(i)

$and, \frac{ON}{NC} = \frac{OM}{MB}$ ...(ii)

$Comparing\ equation\ (i) and\ equation\ (ii), we\ get,$

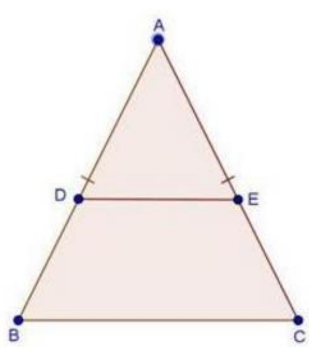
$\frac{ON}{AL} = \frac{ON}{NC}$

Thus, LN divides sides OA and OC of ΔOAC in the same ratio. Therefore, by the converse of basic proportionality theorem,

we have, LN || AC

8. If D and E are points on sides AB and AC respectively of a ΔABC such that DE || BC and BD = CE. Prove that ΔABC is isosceles.

Sol:



We have, DE || BC

Therefore, by BPT, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{DB}$ [∵BD = CE]

⇒ AD = AE

Adding DB on both sides

⇒ AD + DB = AE + DB

⇒ AD + DB = AE + EC [∴ BD = CE]

⇒ AB = AC

⇒ Δ ABC is isosceles

Exercise 4.3

1. In a ΔABC, AD is the bisector of ∠A, meeting side BC at D.

(i) If BD = 2.5cm, AB = 5cm and AC = 4.2cm, find DC.

(ii) If BD = 2cm, AB = 5cm and DC = 3cm, find AC.

(iii) If AB = 3.5 cm, AC = 4.2 cm and DC = 2.8 cm, find BD.

(iv) If AB = lo cm, AC =14 cm and BC =6 cm, find BD and DC.

(v) If AC = 4.2 cm, DC = 6 cm and 10 cm, find AB

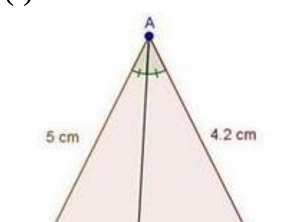
(vi) If AB = 5.6 cm, AC = 6cm and DC = 3cm, find BC.

(vii) If AD = 5.6 cm, BC = 6cm and BD = 3.2 cm, find AC.

(viii) If AB = 10cm, AC = 6 cm and BC = 12 cm, find BD and DC.

Sol:

(i)



We have,

∠BAD = ∠CAD

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

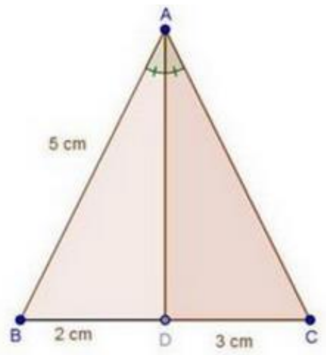
$\Rightarrow \frac{2.5}{DC} = \frac{5}{4.2}$

$\Rightarrow DC = \frac{2.5\times4.2}{5}$

$= \frac{25\times42}{5\times100} = \frac{5\times42}{100} = \frac{210}{100} = 2.1\ cm$

∴ DC = 2.1 cm

(ii)



We have,

AD is the bisector of ∠A

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

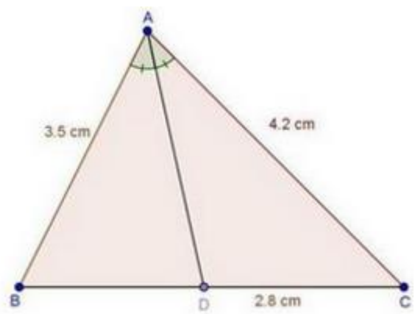
$\therefore\ \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{2}{3} = \frac{5}{AC}$

$\Rightarrow$ AC = $\frac{5\times3}{2} = \frac{15}{2}$

$\Rightarrow$ AC = 7.5 cm

(iii)



In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

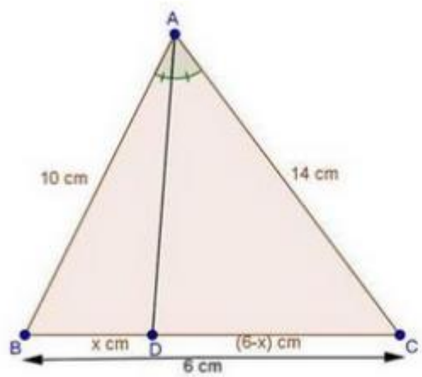
$\Rightarrow \frac{BD}{2.8} = \frac{3.5}{4.2}$

$= \frac{3.5\times2}{3}$

$= \frac{7}{3} = 2.33\ cm$

$\therefore BD = 2.3\ cm$

(iv)



In ∆ABC, AD is the bisector of ∠A

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$

⇒ 14x = 10(6 – x)

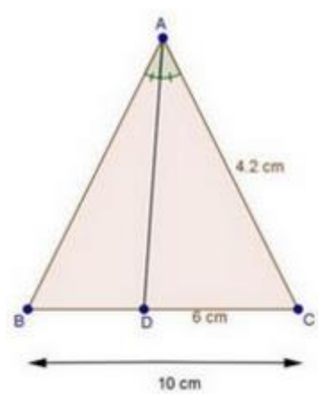
⇒ 24x = 60

$\Rightarrow x = \frac{60}{24} = \frac{5}{2} = 2.5cm$

Since, DC = 6 – x = 6 – 2.5 = 3.5 cm

Hence, BD = 2.5cm, and DC = 3.5 cm

(v)



We have,

BC = 10 cm, DC = 6 cm and AC = 4.2 cm

∴ BD = BC – DC = 10 – 6 = 4 cm

⇒ BD = 4 cm

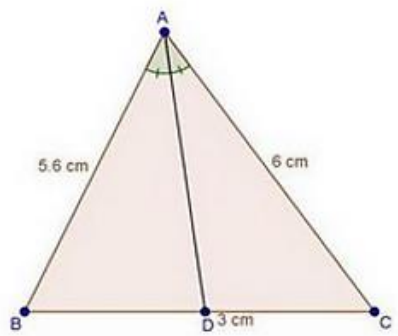
In ΔABC, AD is the bisector of ∠A.
We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{4}{6} = \frac{AB}{4.2}$ [∵ BD = 4 cm]

⇒ AB = 2.8 cm

(vi)



We have, In ΔABC, AD is the bisector of ∠A.
We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{BD}{3} = \frac{5.6}{6}$

⇒ BD = $\frac{5.6\times 3}{6} = \frac{5.6}{2} = 2.8cm$

⇒ BD = 2.8 cm

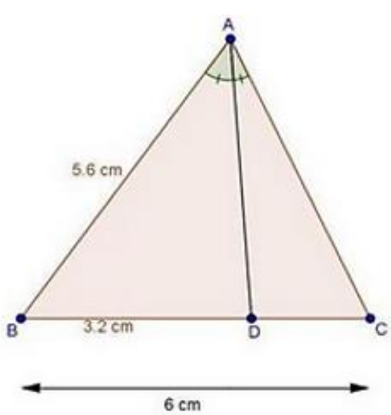
Since, BC = BD + DC

= 2.8 + 3

= 5.8 cm

∴ BC = 5.8 cm

(vii)



We have,

In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the containing the angle.

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$

$\frac{5.6}{AC} = \frac{3.2}{6-3.2}$ [∵ DC = BC – BD]

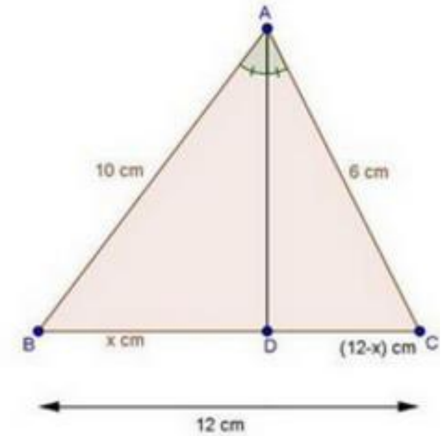
$\Rightarrow \frac{5.6}{AC} = \frac{3.2}{2.8}$

$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2}$

$= \frac{5.6 \times 7}{8} = 0.7 \times 7$

$= 4.9\ cm$

(viii)



In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{x}{12-z} = \frac{10}{6}$

⇒ 6x = 10(12 – x)

⇒ 6x = 120

$\Rightarrow x = \frac{120}{16} = 7.5\ cm$

∴ BD = 7.5 cm and DC = 12 – x = 12 – 7.5 = 4.5 cm

Hence, BD = 7.5 cm and DC = 4.5 cm

2. In Fig. 4.57, AE is the bisector of the exterior ∠CAD meeting BC produced in E. If AB = 10cm, AC = 6cm and BC = 12 cm, find CE.

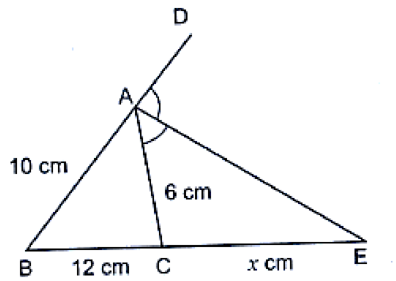


Fig. 4.57

Sol:

In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{x}{12-x} = \frac{10}{6}$

$\Rightarrow 6(12 + x) = 10x$

$\Rightarrow 72 + 6x = 10x$

$\Rightarrow 4x - 72$

$\Rightarrow x = \frac{72}{4} = 18\ cm$

$\therefore CE = 18$ cm

3. In Fig. 4.58, ΔABC is a triangle such that $\frac{AB}{AC} = \frac{BD}{DC}, \angle B = 70°, \angle C = 50°. Find\ \angle BAD.$

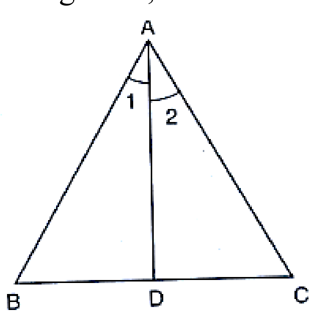


Fig. 4.58

Sol:

We have, if a line through one vertex of a triangle divides the opposite side in the ratio of the other two sides, then the line bisects the angle at the vertex.

$\therefore \angle 1 = \angle 2$

In ΔABC

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 70° + 50° = 180°$ [$\because \angle B = 70°$ and $\angle C = 50°$]

$\Rightarrow \angle A = 180° - 120° = 60°$

$\Rightarrow \angle 1 + \angle 2 = 60°$

$\Rightarrow \angle 1 + \angle 1 = 60°$ [$\because \angle 1 = \angle 2$]

$\Rightarrow 2\angle 1 = 60°$
$\Rightarrow \angle 1 = 30°$
$\therefore \angle BAD = 30°$

4. In ΔABC (Fig., 4.59), if ∠1 = ∠2, prove that $\frac{AB}{AC} = \frac{BD}{DC}$.

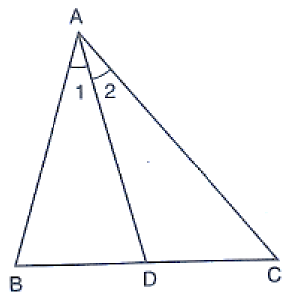
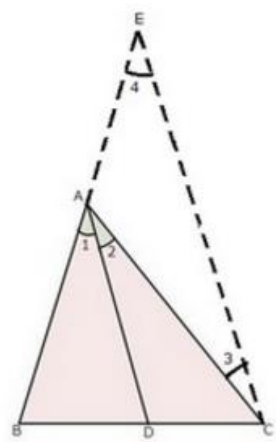


Fig. 4.59

Sol:



Given: A ΔABC in which ∠1 = ∠2
To prove: $\frac{AB}{AC} = \frac{BD}{DC}$
Construction: Draw CE || DA to meet BA produced in E.
Proof: since, CE || DA and AC cuts them.
$\therefore \angle 2 - \angle 3$ .... (i) [Alternate angles]
And, $\angle 1 - \angle 4$ ....(ii) [Corresponding angles]
But, $\angle 1 - \angle 2$ [Given]
From (i) and (ii), we get
$\angle 3 - \angle 4$
Thus, in ΔACE, we have
$\angle 3 = \angle 4$
$\Rightarrow AE = AC$ ... (iii) [Sides opposite to equal angles are equal]
Now, In ΔBCE, we have
DA || CE

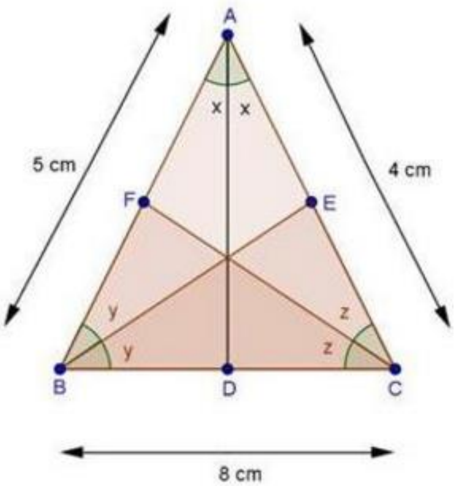
$\Rightarrow \frac{BD}{\boldsymbol{DC}} = \frac{BA}{\boldsymbol{AE}}$ [Using basic proportionality theorem]

$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$ [∵ BA – AB and AE – AC from (iii)]

Hence, $\frac{AB}{AC} = \frac{BD}{DC}$

5. D, E and F are the points on sides BC, CA and AB respectively of ΔABC such that AD bisects ∠A, BE bisects ∠B and CF bisects ∠C. If AB = 5 cm, BC = 8 cm and CA = 4 cm, determine AP, CE and BD.

Sol:



In ΔABC, CF bisects ∠C.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{AF}{FB} = \frac{AC}{BC}$

$\Rightarrow \frac{AF}{5-AF} = \frac{4}{8}$ [∵ FB = AB – AF = 5 – AF]

$\Rightarrow \frac{AF}{5-AF} = \frac{1}{2}$

⇒ 2AF = 5 – AF

⇒ 2AF + AF = 5

⇒ 3AF = 5

⇒ AF = $\frac{5}{3}$ cm

Again, In ΔABC, BE bisects ∠B.

$\therefore \frac{AE}{EC} = \frac{AB}{BC}$

$\Rightarrow \frac{4-CE}{CE} = \frac{5}{8}$ [∵ AE = AC – CE = 4 – CE]

$\Rightarrow 8(4 - CE) = 5 \times CE$

⇒ 32 – 8CE = 5CE

⇒ 32 = 13CE

⇒ CE = $\frac{32}{13}$ cm
Similarly,
$\frac{BD}{DC} = \frac{AD}{AC}$
⇒ $\frac{BD}{8-BD} = \frac{5}{4}$ [∵ DC = BC – BD = 8 – BD]
⇒ 4BD = 40 – 5BD
⇒ 9BD = 40
⇒ BD = $\frac{40}{9}$ cm
Hence, AF = $\frac{5}{3}$ cm, CE = $\frac{32}{13}$ cm and BD = $\frac{40}{9}$ cm.

6. In fig., 4.60, check whether AD is the bisector of ∠A of ΔABC in each of the following:

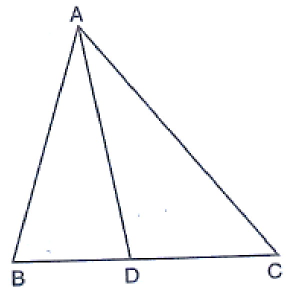


Fig. 4.60

(i) AB = 5cm, AC = 10cm, BD = 1.5 cm and CD = 3.5 cm
(ii) AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm
(iii) AB = 8 cm, AC = 24 cm, BD = 6 cm and BC = 24cm
(iv) AB = 6 cm, AC = 8cm, BD = l.5 cm and CD= 2 cm.
(v) AB = 5 cm, AC = l2 cm, BD = 2.5 cm and BC = 9cm

Sol:
Now,
$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7}$
And, $\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$
⇒ $\frac{BD}{CD} \neq \frac{AB}{AC}$
⇒ AD is not the bisector of ∠A.
Now,
$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$
And, $\frac{BD}{CD} = \frac{1.6}{2.4} = \frac{2}{3}$
⇒ $\frac{AB}{AC} = \frac{BD}{CD}$
⇒ AD is the bisector of ∠A.
Now, $\frac{AB}{AC} = \frac{8}{24} = \frac{1}{3}$

And, $\frac{BD}{CD} = \frac{BD}{BC-BD}$ [∵ CD = BC – BD]

$= \frac{BD}{24-6}$

$= \frac{6}{18}$

$= \frac{1}{3}$

$\therefore \frac{AB}{AC} = \frac{BD}{CD}$

$\therefore AD\ is\ the\ bisector\ of\ \angle A\ of\ \Delta ABC.$

$\frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$

And, $\frac{BD}{CD} = \frac{2.5}{BC-BD}$ [∵ CD = BC – BD]

$= \frac{2.5}{9-2.5}$

$= \frac{2.5}{6.5}$

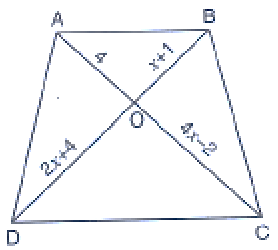
$= \frac{1}{3}$

$\therefore \frac{AB}{AC} \neq \frac{BD}{CD}$

∴ AD is not the bisector of ∠A of ΔABC.

Exercise 4.4

1. (i) In below fig., If AB || CD, find the value of x.



Sol:

Since diagonals of a trapezium divide each other proportionally.

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{4}{4x-2} = \frac{x+1}{2x+4}$

$\Rightarrow 4(2x+4) = (x+1)(4x-2)$

⇒ 8x + 16 = x(4x – 2) +1(4x – 2)

⇒ 8x + 16 = $4x^2$ + 2x – 2

$\Rightarrow 4x^2 + 2x - 8x - 2 - 16 = 0$

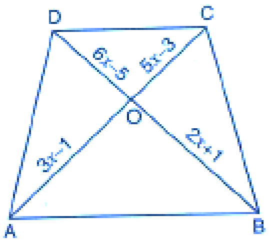
$\Rightarrow 4x^2 - 6x - 18 = 0$

$\Rightarrow 2[2x^2 - 3x - 9] = 0$

$\Rightarrow 2x^2 - 3x - 9 = 0$

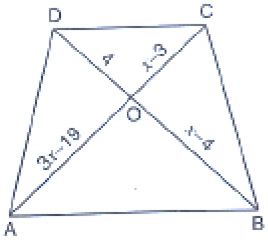
$\Rightarrow 2x(x-3)+3(x-3)=0$
$\Rightarrow (x-3)(2x+3)=0$
$\Rightarrow x-3=0$ or $2x+3=0$
$\Rightarrow x=3$ or $x=-\frac{3}{2}$
$\Rightarrow$ x = 3 or $x\ =-\frac{3}{2}$
$x\ =-\frac{3}{2}$ is not possible, because OB = x + 1 = $-\frac{3}{2}+1=-\frac{1}{2}$
Length cannot be negative
$\therefore \frac{AO}{OC}=\frac{BO}{OD}$

(ii) In the below fig., If AB || CD, find the value of x.



$\Rightarrow \frac{3x-1}{5x-3}=\frac{2x+1}{6x-5}$
$\Rightarrow$ (3x – 1) (6x – 5) = (2x + 1) (5x – 3)
$\Rightarrow$ 3x (6x – 5) – 1(6x – 5) = 2x (5x – 3) + 1 (5x – 3)
$\Rightarrow 18x^2-15x-6x+5=10x^2-6x+5x-3$
$\Rightarrow 8x^2-20x+8=0$
$\Rightarrow 4(2x^2-5x+2)=0$
$\Rightarrow 2x^2-4x-1x+2=0$
$\Rightarrow 2x(x-2)-1(x-2)=0$
$\Rightarrow (2x-1)(x-2)=0$
$\Rightarrow$ 2x – 1 = 0 or x – 2 = 0
$\Rightarrow x=\frac{1}{2}\ or\ x=2$
$x=\frac{1}{2}$ is not possible, because, OC = 5x – 3
$=5\left(\frac{1}{2}\right)-3$
$=\frac{5-6}{2}=-\frac{1}{2}$

(iii) In below fig., AB || CD. If OA = 3x – 19, OB = x – 4, OC = x – 3 and OD = 4, find x.



Since diagonals of a trapezium divide each other proportionally.

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{3x-19}{x-3} = \frac{x-4}{4}$

$\Rightarrow 4(3x - 19) = (x - 4)(x - 3)$

$\Rightarrow 12x - 76 = x(x - 3) - 4(x - 3)$

$\Rightarrow 12x - 76 = x^2 - 3x - 4x + 12$

$\Rightarrow x^2 - 7x - 12x + 12 + 76 = 0$

$\Rightarrow x^2 - 19x + 88 = 0$

$\Rightarrow x^2 - 11z - 8z + 88 = 0$

$\Rightarrow x(x - 11) - 8(x - 11) = 0$

$\Rightarrow (x - 11)(x - 8) = 0$

$\Rightarrow x - 11 = 0$ or $x - 8 = 0$

$\Rightarrow x = 11$ or $x = 8$

Exercise 4.5

1. In fig. 4.136, ΔACB ~ ΔAPQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.

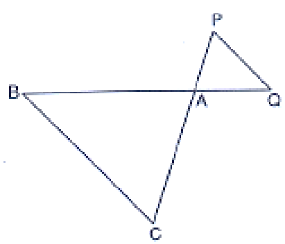


Fig. 4.136

Sol:

Given ΔACB ~ ΔAPQ

Then, $\frac{AC}{AP} = \frac{BC}{PQ} = \frac{AB}{AQ}$ [corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} = \frac{6.5}{AQ}$

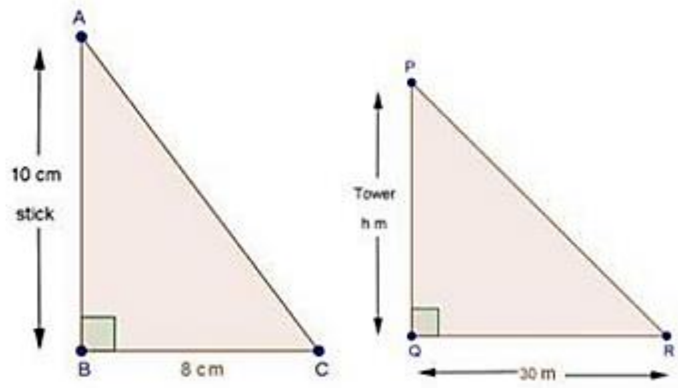
$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} \, and \, \frac{8}{4} - \frac{6.5}{AQ}$

$\Rightarrow AC = \frac{8}{4} \times 2.8 \, and \, AQ = 6.5 \times \frac{4}{8}$

⇒ AC = 5.6 cm and AQ = 3.25 cm

2. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time a shadow 30 m long. Determine the height of the tower.

Sol:



Length of stick = 10 cm
Length of shadow of stick = 8 cm
Length of shadow of tower = h cm
In ΔABC and ΔPQR
∠B = ∠Q = 90°
And, ∠C = ∠R [Angular elevation of sun]
Then, ΔABC ~ ΔPQR [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \frac{10cm}{8\ cm} = \frac{h\ cm}{3000}$

$\Rightarrow h = \frac{10}{8} \times 3000 = 3750\ cm = 37.5\ m$

3. In Fig. 4.137, AB || QR. Find the length of PB.

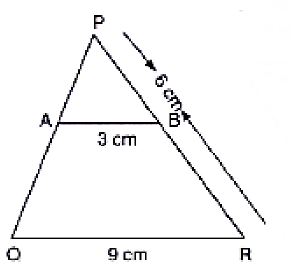


Fig. 4.137

Sol:

We have, ΔPAB and ΔPQR
∠P = ∠P [common]
∠PAB = ∠PQR [corresponding angles]

Then, ΔPAB ~ ΔPQR [By AA similarity]

$\therefore \frac{PB}{PR} = \frac{AB}{QR}$ [Corresponding parts of similar Δare proportional]

$\Rightarrow \frac{PB}{6} = \frac{3}{9}$

$\Rightarrow$ PB $= \frac{3}{9} \times 6 = 2\ cm$

4. In fig. 4.138, XY || BC. Find the length of XY

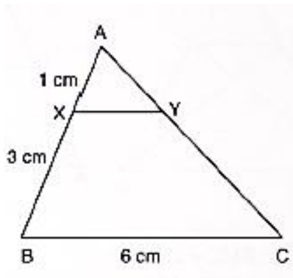


Fig. 4.138

Sol:

We have, XY || BC

In ΔAXY and ΔABC

∠A = ∠A [common]

∠AXY = ∠ABC [corresponding angles]

Then, ΔAXY ~ ΔABC [By AA similarity]

$\therefore \frac{AX}{AB} = \frac{XY}{BC}$ [Corresponding parts of similar Δ are proportional]

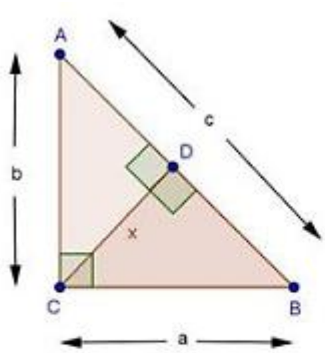
$\Rightarrow \frac{1}{4} = \frac{XY}{6}$

$\Rightarrow$ XY $= \frac{6}{4} = 1.5cm$

5. In a right angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx.

Sol:

We have: ∠C = 90° and CD ⊥ AB



In ΔACB and ΔCDB

∠B = ∠B [common]
∠ACB = ∠CDB [Each 90°]
Then, ΔACB ~ ΔCDB [By AA similarity]
$\therefore \frac{AC}{CD} = \frac{AB}{CB}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{b}{x} = \frac{c}{a}$
⇒ ab = cx

6. In Fig. 4.139, ∠ABC = 90° and BD ⊥ AC. If BD = 8 cm and AD = 4 cm, find CD.

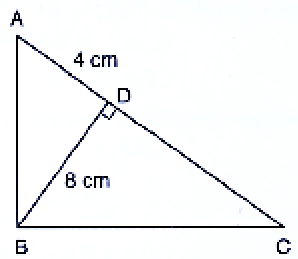


Fig. 4.139

Sol:
We have, ∠ABC = 90° and BD ⊥ AC
Now, ∠ABD + ∠DBC − 90° ...(i) [∵ ∠ABC − 90°]
And, ∠C + ∠DBC − 90° ...(ii) [By angle sum prop. in ΔBCD]
Compare equations (i) & (ii)
∠ABD = ∠C ...(iii)
In ΔABD and ΔBCD
∠ABD = ∠C [From (iii)]
∠ADB = ∠BDC [Each 90°]
Then, ΔABD ~ ΔBCD [By AA similarity]
$\therefore \frac{BD}{CD} = \frac{AD}{BD}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{8}{CD} = \frac{4}{8}$
$\Rightarrow \text{CD} = \frac{8\times8}{4} = 16\ cm$

7. In Fig. 4.14, ∠ABC = 90° and BD ⊥ AC. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.

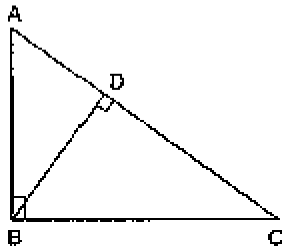


Fig. 4.140

Sol:

We have, ∠ABC = 90° and BD ⊥ AC

In ΔABC and ΔBDC

∠ABC = ∠BDC	[Each 90°]
∠C = ∠C	[Common]
Then, ΔABC ~ ΔBDC	[By AA similarity]
$\therefore \frac{AB}{BD} = \frac{BC}{DC}$	[Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4}$

$\Rightarrow$ BC = $\frac{5.7}{3.8} \times 8.1\ cm$

8. In Fig. 4.141, DE || BC such that AE = (1/4) AC. If AB = 6 cm, find AD.

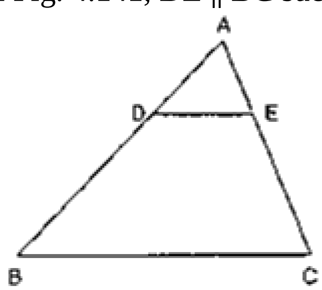


Fig. 4.141

Sol:

We have, DE || BC, AB = 6 cm and AE = $\frac{1}{4}\ AC$

In ΔADE and ΔABC

∠A = ∠A	[Common]
∠ADE = ∠ABC	[Corresponding angles]
Then, ΔADE ~ ΔABC	[By AA similarity]
$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$	[Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{AD}{6} = \frac{\frac{1}{4}AC}{AC}$	[∵ AE = $\frac{1}{4}\ AC\ given$]

$\Rightarrow \frac{AD}{6} = \frac{1}{4}$

$\Rightarrow$ AD = $\frac{6}{4}$ = 1.5 cm

9. In fig., 4.142, PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x}+\frac{1}{z}+\frac{1}{y}$

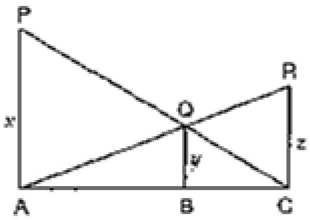


Fig. 4.142

Sol:

We have, PA ⊥ AC, QB ⊥ AC and RC ⊥ AC

Let, AB = a and BC = b

In ΔCQB and ΔCPA

∠QCB = ∠PCA [Common]

∠QBC = ∠PAC [Each 90°]

Then, ΔCQB ~ ΔCPA [By AA similarity]

$\therefore \frac{QB}{PA}=\frac{CB}{CA}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{y}{x}=\frac{b}{a+b}$ ….(i)

In Δ AQB and ΔARC

∠QAB = ∠RAC [common]

∠ABQ = ∠ ACR [Each 90°]

Then, ΔAQB ~ ΔARC [By AA similarity]

$\therefore \frac{QB}{RC}=\frac{AB}{AC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{y}{z}=\frac{a}{a+b}$ ….(ii)

Adding equations (i) & (ii)

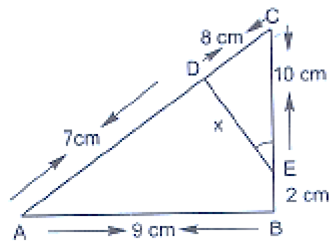
$\frac{y}{x}+\frac{y}{z}=\frac{b}{a+b}+\frac{a}{a+b}$

$\Rightarrow y\left(\frac{1}{x}+\frac{1}{z}\right)=\frac{b+a}{a+b}$

$\Rightarrow \text{y}\left(\frac{1}{x}+\frac{1}{z}\right)=1$

$\Rightarrow \frac{1}{x}+\frac{1}{z}=\frac{1}{y}$

10. In below fig., ∠A = ∠CED, Prove that ΔCAB ~ ΔCED. Also, find the value of x.



Sol:

We have, ∠A = ∠CED

In ΔCAB and ΔCED

∠C = ∠C [Common]

∠A = ∠CED [Given]

Then, ΔCAB ~ ΔCED [By AA similarity]

$\therefore \frac{CA}{CE} = \frac{AB}{ED}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{15}{10} = \frac{9}{x}$

$\Rightarrow x = \frac{10\times 9}{15} = 6\ cm$

11. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, what is the corresponding side of the other triangle?

Sol:

Assume ABC and PQR to be 2 triangles

We have,

ΔABC ~ ΔPQR

Perimeter of Δ ABC = 25 cm

Perimeter of Δ PQR = 15 cm

AB = 9 cm

PQ = ?

Since, ΔABC ~ ΔPQR

Then, ratio of perimeter of triangles = ratio of corresponding sides

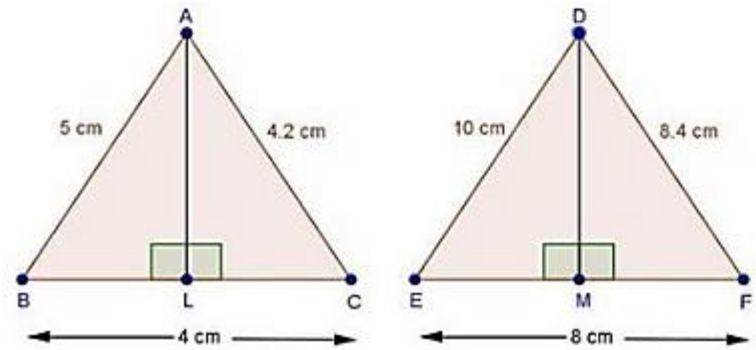
$\Rightarrow \frac{25}{12} = \frac{AB}{PQ}$

$\Rightarrow \frac{25}{15} = \frac{9}{PQ}$

$\Rightarrow$ PQ = $\frac{15\times 9}{25} = 5.4\ cm$

12. In ΔABC and ΔDEF, it is being given that: AB = 5 cm, BC = 4 cm and CA = 4.2 cm; DE=10cm, EF = 8 cm and FD = 8.4 cm. If AL ⊥ BC and DM ⊥ EF, find AL: DM.

Sol:



Since, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DE} = \frac{1}{2}$

Then, ΔABC ~ ΔDEF [By SSS similarity]

Now, In ΔABL ~ ΔDEM

∠B = ∠E [Δ ABC ~ ΔDEF]

∠ALB = ∠DME [Each 90°]

Then, ΔABL ~ ΔDEM [By AA similarity]

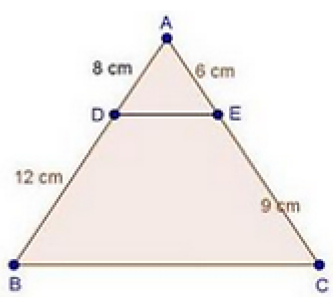
$\therefore \frac{AB}{DE} = \frac{AL}{DM}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{5}{10} = \frac{AL}{DM}$

$\Rightarrow \frac{1}{2} = \frac{AL}{DM}$

13. D and E are the points on the sides AB and AC respectively of a ΔABC such that: AD = 8 cm, DB = 12 cm, AE = 6 cm and CE = 9 cm. Prove that BC = 5/2 DE.

Sol:



We have,

$\frac{AD}{DB} = \frac{8}{12} = \frac{2}{3}$

And, $\frac{AE}{EC} = \frac{6}{9} = \frac{2}{3}$

Since, $\frac{AD}{DB} = \frac{AE}{EC}$

Then, by converse of basic proportionality theorem

DE || BC

In ΔADE and ΔABC

$\angle A = \angle A$ [Common]

$\angle ADE = \angle B$ [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ [Corresponding parts of similar Δ are proportional]

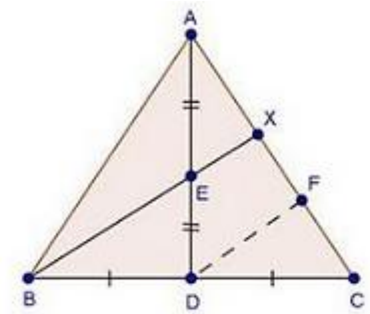
$\Rightarrow \frac{8}{20} = \frac{DE}{BC}$

$\Rightarrow \frac{2}{5} = \frac{DE}{BC}$

$\Rightarrow BC = \frac{5}{2}$ DE

14. D is the mid-point of side BC of a ΔABC. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that BE : EX = 3 : 1

Sol:



Given: In ΔABC, D is the mid-point of BC and E is the mid-point of AD.

To prove: BE : EX = 3 : 1

Const: Through D, draw DF || BX

Proof: In ΔEAX and ΔADF

$\angle EAX = \angle ADF$ [Common]

$\angle AXE = \angle DAF$ [Corresponding angles]

Then, ΔAEX ~ ΔADF [By AA similarity]

$\therefore \frac{EX}{DF} = \frac{AE}{AD}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{EX}{DF} = \frac{AE}{2AE}$ [AE = ED given]

$\Rightarrow DF = 2EX$ …. (i)

In ΔCDF and ΔCBX [By AA similarity]

$\therefore \frac{CD}{CB} = \frac{DF}{BX}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{1}{2} = \frac{DF}{BE+EX}$ [BD = DC given]

$\Rightarrow BE + EX = 2DF$

$\Rightarrow BE + EX = 4EX$

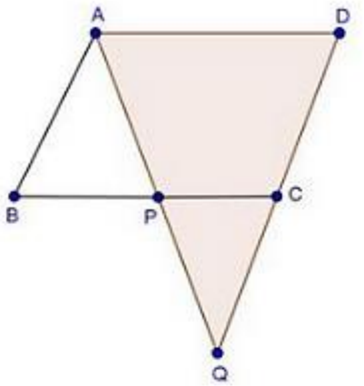
$\Rightarrow BE = 4EX – EX$ [By using (i)]

$\Rightarrow BE = 4EX – EX$

$\Rightarrow \frac{BE}{EX} = \frac{3}{1}$

15. ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q. Prove that the rectangle obtained by BP and DQ is equal to the AB and BC.

Sol:



Given: ABCD is a parallelogram

To prove: BP × DQ = AB × BC

Proof: In ΔABP and ΔQDA

∠B = ∠D [Opposite angles of parallelogram]

∠BAP = ∠AQD [Alternate interior angles]

Then, ΔABP ~ ΔQDA [By AA similarity]

$\therefore \frac{AB}{QD} = \frac{BP}{DA}$ [Corresponding parts of similar Δ are proportional]

But, DA = BC [Opposite sides of parallelogram]

Then, $\frac{AB}{QD} = \frac{BP}{BC}$

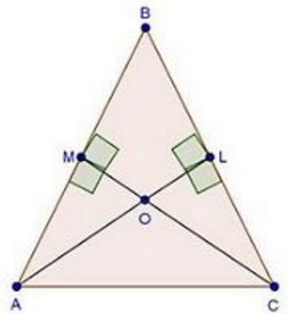
⇒ AB × BC = QD × BP

16. In ΔABC, AL and CM are the perpendiculars from the vertices A and C to BC and AB respectively. If AL and CM intersect at O, prove that:

(i) Δ OMA and ΔOLC

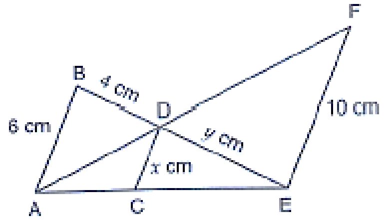
(ii) $\frac{OA}{OC} = \frac{OM}{OL}$

Sol:



We have,
AL ⊥ BC and CM ⊥ AB
In Δ OMA and ΔOLC
∠MOA = ∠LOC [Vertically opposite angles]
∠AMO = ∠CLO [Each 90°]
Then, ΔOMA ~ ΔOLC [By AA similarity]
$\therefore \frac{OA}{OC} = \frac{OM}{OL}$ [Corresponding parts of similar Δ are proportional]

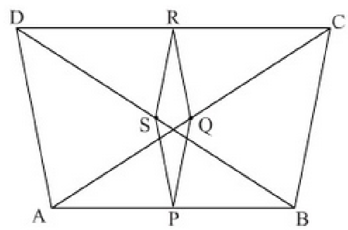
17. In Fig below we have AB || CD || EF. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm and DE = y cm, calculate the values of x and y.



Sol:
We have AB || CD || EF. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm and DE = y cm
In ΔECD and ΔEAB
∠CED = ∠AEB [common]
∠ECD = ∠EAB [corresponding angles]
Then, ΔECD ~ ΔEAB ….(i) [By AA similarity]
$\therefore \frac{EC}{EA} = \frac{CD}{AB}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{EC}{EA} = \frac{x}{6}$ ….(ii)
In ΔACD and ΔAEF
∠CAD = ∠EAF [common]
∠ACD = ∠AEF [corresponding angles]
Then, ΔACD ~ ΔAEF [By AA similarity]
$\therefore \frac{AC}{AE} = \frac{CD}{EF}$
$\Rightarrow \frac{AC}{AE} = \frac{x}{10}$ …(iii)
Add equations (iii) & (ii)
$\therefore \frac{EC}{EA} + \frac{AC}{AE} = \frac{x}{6} + \frac{x}{10}$
$\Rightarrow \frac{AE}{AE} = \frac{5x+3x}{30}$
$\Rightarrow 1 = \frac{8x}{30}$
$\Rightarrow x = \frac{30}{8} = 3.75$ cm
From (i) $\frac{DC}{AB} = \frac{ED}{BE}$

$\Rightarrow \frac{3.75}{6} = \frac{y}{y+4}$
⇒ 6y = 3.75y + 15
⇒ 2.25y = 15
$\Rightarrow y = \frac{15}{2.25} = 6.67\ cm$

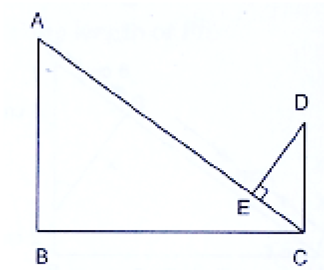
18. ABCD is a quadrilateral in which AD = BC. If P, Q, R, S be the mid-points of AB, AC, CD and BD respectively, show that PQRS is a rhombus.
Sol:



AD = BC and P, Q, R and S are the mid-points of sides AB, AC, CD and BD respectively, show that PQRS is a rhombus.
In ΔBAD, by mid-point theorem
PS ∥ AD and PS = $\frac{1}{2}$ AD …(i)
In ΔCAD, by mid-point theorem
QR ∥ AD and QR = $\frac{1}{2}\ AD$ …(ii)
Compare (i) and (ii)
PS ∥ QR and PS = QR
Since one pair of opposite sides is equal as well as parallel then
PQRS is a parallelogram ...(iii)
Now, In ΔABC, by mid-point theorem
PQ ∥ BC and PQ = $\frac{1}{2}$ BC …(iv)
And, AD = BC …(v) [given]
Compare equations (i) (iv) and (v)
PS = PQ …(vi)
From (iii) and (vi)
Since, PQRS is a parallelogram with PS = PQ then PQRS is a rhombus

19. In Fig. below, if AB ⊥ BC, DC ⊥ BC and DE ⊥ AC, Prove that Δ CED ∼ ABC.

[redacted]



Sol:

Given: AB ⊥ BC, DC ⊥ BC and DE ⊥ AC

To prove: ΔCED ~ ΔABC

Proof:

$\angle BAC + \angle BCA = 90°$ ...(i) [By angle sum property]

And, $\angle BCA + \angle ECD = 90°$ ...(ii) [DC ⊥ BC given]

Compare equation (i) and (ii)

$\angle BAC = \angle ECD$ ...(iii)

In ΔCED and ΔABC

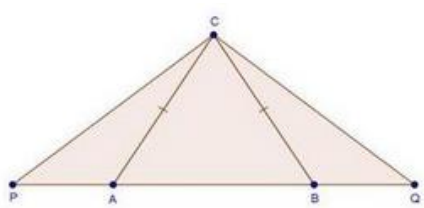
$\angle CED = \angle ABC$ [Each 90°]

$\angle ECD = \angle BAC$ [From (iii)]

Then, ΔCED ~ ΔABC [By AA similarity]

20. In an isosceles ΔABC, the base AB is produced both the ways to P and Q such that AP × BQ = AC^2. Prove that ΔAPC ~ ΔBCQ.

Sol:



Given: In ΔABC, CA = CB and AP × BQ = AC2

To prove: ΔAPC ~ ΔBCQ

Proof:

AP × BQ = AC^2 [Given]

⇒ AP × BQ = AC × AC

⇒ AP × BQ = AC × BC [AC = BC given]

$\Rightarrow \frac{AP}{BC} = \frac{AC}{BQ}$ …(i)

Since, CA = CB [Given]

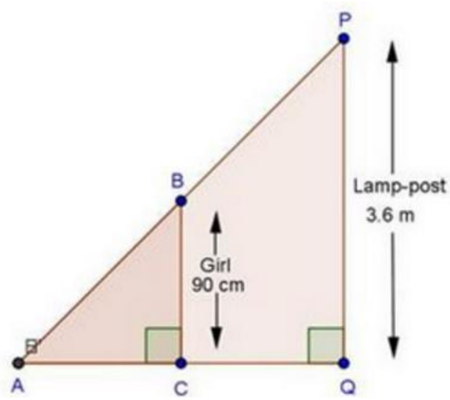
Then, ∠CAB = ∠CBA …(ii) [Opposite angles to equal sides]

Now, ∠CAB + ∠CAP = 180° …(iii) [Linear pair of angles]

And, ∠CBA + ∠CBQ = 180° …(iv) [Linear pair of angles]

Compare equation (ii) (iii) & (iv)
∠CAP = ∠CBQ …(v)
In ΔAPC and ΔBCQ
∠CAP = ∠CBQ [From (v)]
$\frac{AP}{BC} = \frac{AC}{BQ}$ [From (i)]
$Then, \Delta APC \sim \Delta BCQ$ [By SAS similarity]

21. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.
Sol:

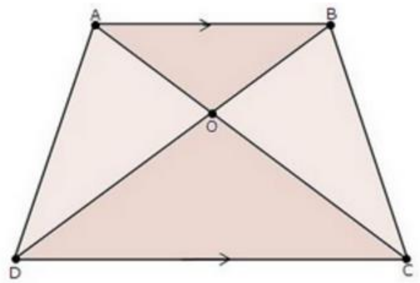


We have,
Height of girl = 90 cm = 0.9 m
Height of lamp-post = 3.6 m
Speed of girl = 1.2 m/sec
∴ Distance moved by girl (CQ) = Speed × Time
= 1.2 × 4 = 4.8m
Let length of shadow (AC) = x cm
In ΔABC and ΔAPQ
∠ACB = ∠AQP [Each 90°]
∠BAC = ∠PAQ [Common]
Then, ΔAB ~ ΔAPQ [By AA similarity]
$\therefore \frac{AC}{AQ} = \frac{BC}{PQ}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{x}{x+4.8} = \frac{0.9}{3.6}$
$\Rightarrow \frac{x}{x+4.8} = \frac{1}{4}$
⇒ 4x = x + 4.8
⇒ 4x – x = 4.8
⇒ 3x = 4.8
$\Rightarrow x = \frac{4.8}{3} = 1.6\ m$

∴ Length of shadow = 1.6m

22. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Sol:



We have,

ABCD is a trapezium with AB || DC

In ΔAOB and ΔCOD

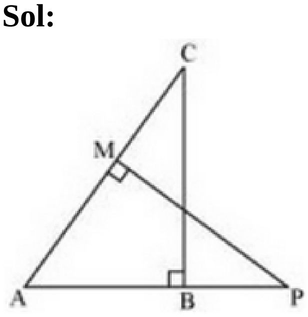
∠AOB = ∠COD	[Vertically opposite angles]
∠OAB = ∠OCD	[Alternate interior angles]
Then, ΔAOB ∼ ΔCOD	[By AA similarity]
∴ $\frac{OA}{OC} = \frac{OB}{OD}$	[Corresponding parts of similar Δ are proportional]

23. If ΔABC and ΔAMP are two right triangles, right angled at B and M respectively such that ∠MAP = ∠BAC. Prove that

(i) ΔABC ∼ ΔAMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Sol:



We have,

∠B = ∠M = 90°

And, ∠BAC = ∠MAP

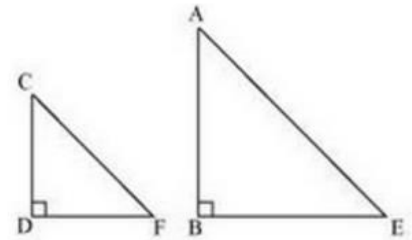
In ΔABC and ΔAMP

∠B = ∠M	[Each 90°]
∠BAC = ∠MAP	[Given]
Then, ΔABC ∼ ΔAMP	[By AA similarity]

$\therefore \frac{CA}{PA} = \frac{BC}{MP}$ [Corresponding parts of similar Δ are proportional]

24. A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol:



Let AB be a tower

CD be a stick, CD = 6m

Shadow of AB is BE = 28m

Shadow of CD is DF = 4m

At same time light rays from sun will fall on tower and stick at same angle.

So, ∠DCF = ∠BAE

And ∠DFC = ∠BEA

∠CDF = ∠ABE (tower and stick are vertical to ground)

Therefore Δ ABE ~ ΔCDF (By AA similarity)

So,

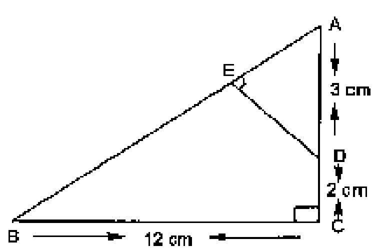
$\frac{AB}{CD} = \frac{BE}{DF}$

$\frac{AB}{6} = \frac{28}{4}$

AB = $28 \times \frac{6}{4} = 42m$

So, height of tower will be 42 metres.

25. In below Fig., ΔABC is right angled at C and DE ⊥ AB. Prove that ΔABC ~ ΔADE and Hence find the lengths of AE and DE.



Sol:

In ΔACB, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = (5)^2 + (12)^2$

$\Rightarrow AB^2 = 25 + 144 = 169$
$\Rightarrow$ AB $= \sqrt{169} = 13\ cm$
In ΔAED and ΔACB
∠A = ∠A [Common]
∠AED = ∠ACB [Each 90°]
Then, ΔAED ~ ΔACB [By AA similarity]
$\therefore \frac{AE}{AC} = \frac{DE}{CB} = \frac{AD}{AB}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{AE}{5} = \frac{DE}{12} = \frac{3}{13}$
$\Rightarrow \frac{AE}{5} = \frac{3}{13}$ and $\frac{DE}{12} = \frac{3}{13}$
$\Rightarrow$ AE $= \frac{15}{13}$ cm and DE $= \frac{36}{13}$ cm

Exercise 4.6

1. Triangles ABC and DEF are similar
 (i) If area $(\Delta ABC) = 16cm^2$, area $(\Delta DEF) = 25\ cm^2$ and BC = 2.3 cm, find EF.
 (ii) If area $(\Delta ABC) = 9cm^2$, area $(\Delta DEF) = 64\ cm^2$ and DE = 5.1 cm, find AB.
 (iii)If AC = 19cm and DF = 8 cm, find the ratio of the area of two triangles.
 (iv)If area $(\Delta ABC) = 36cm^2$, area $(\Delta DEF) = 64\ cm^2$ and DE = 6.2 cm, find AB.
 (v) If AB = 1.2 cm and DE = 1.4 cm, find the ratio of the areas of ΔABC and ΔDEF.

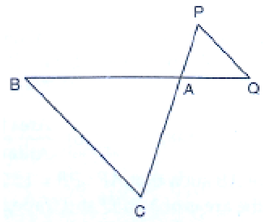
Sol:
(i)
We have,
ΔABC ~ΔDEF
Area $(\Delta ABC) = 16\ cm^2$,
Area $(\Delta DEF) = 25\ cm^2$
And BC = 2.3 cm
Since, ΔABC ~ ΔDEF
Then, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ DEF)} = \frac{BC^2}{EF^2}$ [By area of similar triangle theorem]
$\Rightarrow \frac{16}{25} = \frac{(2.3)^2}{EF^2}$
$\Rightarrow \frac{4}{5} = \frac{2.3}{EF}$ [By taking square root]
$\Rightarrow$ EF $= \frac{11.5}{4} = 2.875\ cm$
(ii)
We have,
ΔABC ~ ΔDEF
Area(ΔABC) = 9 cm^2
Area (ΔDEF) = 64 cm^2

And DE = 5.1 cm
Since, ΔABC ~ ΔDEF
Then, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{AB^2}{DE^2}$ [By area of similar triangle theorem]
$\Rightarrow \frac{9}{64} = \frac{AB^2}{(5.1)^2}$
$\Rightarrow \frac{3}{8} = \frac{AB}{5.1}$ [By taking square root]
$\Rightarrow$ AB $= \frac{3\times5.1}{8} = 1.9125\ cm$
(iii)
We have,
ΔABC ~ ΔDEF
AC = 19 cm and DF = 8 cm
By area of similar triangle theorem
$\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$
We have,
ΔABC ~ ΔDEF
AC = 19 cm and DF = 8 cm
By area of similar triangle theorem
$\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$
(iv)
We have, Area (ΔABC) = 36 cm^2
Area $(\Delta DEF) = 64\ cm^2$
DE = 6.2 cm
And, ΔABC ~ ΔDEF
By area of similar triangle theorem
$\frac{Area(\ \Delta ABC)}{Area\ (\Delta DEF)} = \frac{AB^2}{DE^2}$
$\Rightarrow \frac{36}{64} = \frac{AB^2}{(6.2)^2}$ [By taking square root]
$\Rightarrow$ AB = $\frac{6\times6.2}{8} = 4.65\ cm$
(v)
We have,
ΔABC ~ ΔDEF
AB = 1.2 cm and DF = 1.4 cm
By area of similar triangle theorem
$\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta\ DEF)} = \frac{AB^2}{DE^2}$
$= \frac{(1.2)^2}{(1.4)^2}$

$= \frac{1.44}{1.96}$

$= \frac{36}{49}$

2. In fig. below ΔACB ~ ΔAPQ. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ. Also, find the area (ΔACB): $area\ (\Delta APQ)$



Sol:

We have,

ΔACB ~ ΔAPQ

Then, $\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AC}{2.8} = \frac{10}{5} = \frac{6.5}{AQ}$

$\Rightarrow \frac{AC}{2.8} = \frac{10}{5}\ and\ \frac{10}{5} = \frac{6.5}{AQ}$

$\Rightarrow$ AC = $\frac{10}{5} \times 2.8\ and\ AQ = 6.5 \times \frac{5}{10}$

$\Rightarrow$ AC = 5.6 cm and AQ = 3.25 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta ACB)}{Area\ (\Delta APQ)} = \frac{BC^2}{PQ^2}$

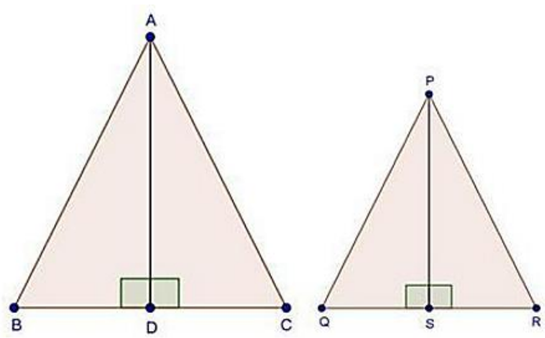
$= \frac{(10)^2}{(5)^2}$

$= \frac{100}{25}$

$= \frac{4}{1}$

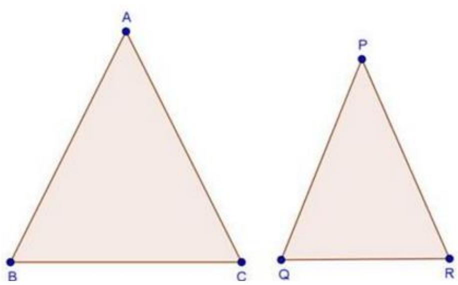
3. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. Find the ratio of their corresponding heights. What is the ratio of their corresponding medians?

Sol:



We have,
$\Delta ABC \sim \Delta PQR$
Area (ΔABC) = 81 cm^2,
Area (ΔPQR) = 49 cm^2
And AD and PS are the altitudes
By area of similar triangle theorem
$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$
$\Rightarrow \frac{81}{49} = \frac{AB^2}{PQ^2}$
$\Rightarrow \frac{9}{7} = \frac{AB}{PQ}$ ….(i) [Taking square root]
$In\ \Delta ABD\ and\ \Delta PQS$
$\angle B = \angle Q$ [$\Delta ABC \sim \Delta PQR$]
$\angle$ ADB = $\angle$PSQ [Each 90°]
Then, $\Delta ABD \sim \Delta PQS$ [By AA similarity]
$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ …(ii) [Corresponding parts of similar Δ are proportional]
Compare (1) and (2)
$\frac{AD}{PS} = \frac{9}{7}$
$\therefore$ Ratio of altitudes = $\frac{9}{7}$
Since, the ratio of the area of two similar triangles is equal to the ratio of the squares of the squares of their corresponding altitudes and is also equal to the squares of their corresponding medians.
Hence, ratio of altitudes = Ratio of medians = 9 : 7

4. The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.
Sol:



We have,
$\Delta ABC \sim \Delta PQR$
Area(ΔABC) = 169 cm^2
Area(ΔPQR) = 121 cm2
And AB = 26 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{AB^2}{PQ^2}$

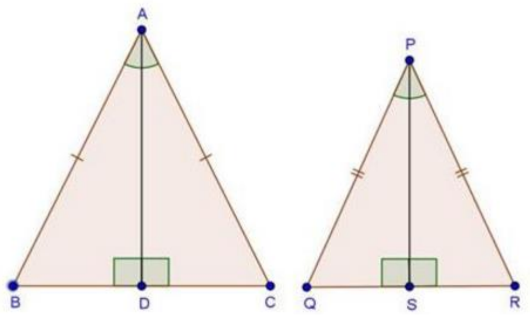
$\Rightarrow \frac{169}{121} = \frac{(26)^2}{PQ^2}$

$\Rightarrow \frac{13}{11} = \frac{26}{PQ}$ [Taking square root]

$\Rightarrow$ PQ = $\frac{11}{13} \times 26 = 22\ cm$

5. Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25. Find the ratio of their corresponding heights.

Sol:



Given: AB = AC, PQ = PQ and ∠A = ∠P

And, AD and PS are altitudes

And, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{36}{25}$ …(i)

To find: $\frac{AD}{PS}$

Proof: Since, AB = AC and PQ = PR

Then, $\frac{AB}{AC} = 1$ and $\frac{PQ}{PR} = 1$

$\therefore \frac{AB}{AC} = \frac{PQ}{PR}$

$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$ …(ii)

In ΔABC and ΔPQR

∠A = ∠P [Given]

$\frac{AB}{PQ} = \frac{AC}{PR}$ [From (2)]

$Then, \Delta ABC \sim \Delta PQR$ [By SAS similarity]

$\therefore \frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{AB^2}{PQ^2}$ ….(iii) [By area of similar triangle theorem]

$Compare\ equation\ (i) and\ (iii)$

$\frac{AB^2}{PQ^2} = \frac{36}{25}$

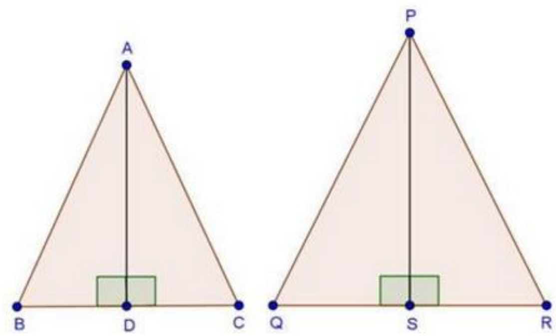
$\Rightarrow \frac{AB}{PQ} = \frac{6}{5}$ ….(iv)

In ΔABD and ΔPQS

∠B = ∠Q	[ΔABC ~ ΔPQR]
∠ADB = ∠PSQ	[Each 90°]
Then, ΔABD ~ ΔPQS	[By AA similarity]
$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$	
$\Rightarrow \frac{6}{5} = \frac{AD}{PS}$	[From (iv)]

6. The areas of two similar triangles are 25 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 2.4 cm, find the corresponding altitude of the other.

Sol:



We have,

ΔABC ~ ΔPQR

Area (ΔABC) = 25 cm^2

Area (ΔPQR) = 36 cm^2

AD = 2.4 cm

And AD and PS are the altitudes

To find: PS

Proof: Since, ΔABC ~ ΔPQR

Then, by area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{25}{36} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{5}{6} = \frac{AB}{PQ}$ ….(i)

$In\ \Delta ABD\ and\ \Delta PQS$

∠B = ∠Q	[Δ ABC ~ ΔPQR]
∠ADB ~ ∠PSQ	[Each 90°]
Then, ΔABD ~ ΔPQS	[By AA similarity]
$\therefore \frac{AB}{PS} = \frac{AD}{PS}$	….(ii) [Corresponding parts of similar Δ are proportional]

Compare (i) and (ii)

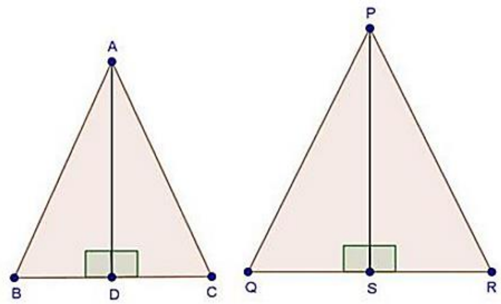
$\frac{AD}{PS} = \frac{5}{6}$

$\Rightarrow \frac{2.4}{PS} = \frac{5}{6}$

$\Rightarrow PS = \frac{2.4 \times 6}{5} = 2.88\ cm$

7. The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Sol:



We have,

$\Delta ABC \sim \Delta PQR$

AD = 6 cm

And, PS = 9 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$ …(i)

$In\ \Delta ABD\ and\ \Delta PQS$

$\angle B = \angle Q$ [$\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each 90°]

Then, $\Delta ABD \sim \Delta PQS$ [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AB}{PQ} = \frac{6}{9}$

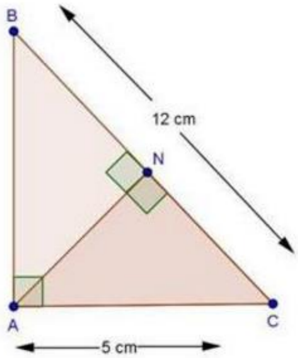
$\Rightarrow \frac{AB}{PQ} = \frac{2}{3}$ …(ii)

Compare equations (i) and (ii)

$\frac{Area(\Delta ABC)}{Area\ (\Delta PQR)} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

8. ABC is a triangle in which ∠A =90°, AN⊥ BC, BC = 12 cm and AC = 5cm. Find the ratio of the areas of ΔANC and ΔABC.

Sol:



In ΔANC and ΔABC

∠C = ∠C [Common]

∠ANC = ∠BAC [Each 90°]

Then, ΔANC ~ ΔBAC [By AA similarity]

By area of similarity triangle theorem

$\frac{Area\ (\Delta\ ANC)}{Area\ (\Delta BAC)} = \frac{AC^2}{BC^2}$

$= \frac{5^2}{12^2}$

$= \frac{25}{144}$

9. In Fig. 4.178, DE || BC

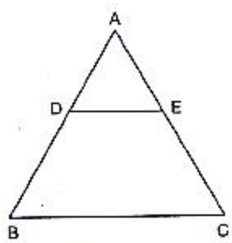


Fig. 4.178

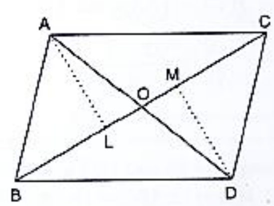


Fig. 4.179

(i) If DE = 4 cm, BC = 6 cm and Area (ΔADE) = 16 cm^2, find the area of ΔABC.

(ii) If DE = 4cm, BC = 8 cm and Area (ΔADE) = 25 cm^2, find the area of ΔABC.

(iii)If DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.

Sol:

We have, DE || BC, DE = 4 cm, BC = 6 cm and area $(\Delta ADE) = 16cm^2$

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

∴ By area of similar triangle theorem

$\frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{16}{Area\ (\Delta ABC)} = \frac{4^2}{6^2}$

$\Rightarrow Area\ (\Delta ABC) = \frac{16\times36}{16} = 36cm^2$

$we\ have, DE\ ||BC, DE = 4\ cm, BC = 8\ cm\ and\ area\ (\Delta ADE) = 25\ cm^2$

$In\ \Delta ADE\ and\ \Delta ABC$

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, $\Delta ADE \sim \Delta ABC$ [By AA similarity]

By area of similar triangle theorem

$\frac{Area\ (\Delta ADE)}{Area\ (\Delta\ ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{16}{Area\ (\Delta ABC)} = \frac{4^2}{6^2}$

⇒ Area (ΔABC) = $\frac{16\times36}{16} = 36\ cm^2$

We have, DE || BC, DE = 4 cm, BC = 8 cm and area (ΔADE) = $25cm^2$

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\Rightarrow \frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\frac{25}{Area\ (\Delta ABC)} = \frac{4^2}{8^2}$

$\Rightarrow Area\ (\Delta ABC) = \frac{25\times64}{16} = 100\ \ cm^2$

We have, DE || BC, and $\frac{DE}{BC} = \frac{3}{5}$ ….(i)

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠B [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\Rightarrow \frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{ar\ (\Delta ADE)}{ar(\Delta ADE)+ar(trap.DECB)} = \frac{3^2}{5^2}$ [From (i)]

⇒ 25ar (ΔADE) = $9ar\ (\Delta ADE) + 9ar\ (trap.DECB)$

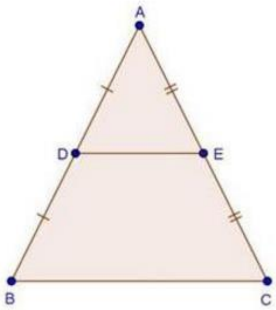
⇒ 25 ar (ΔADE – 9ar) (ΔADE) = 9ar (trap.DECB)

⇒ 16 ar(ΔADE) = 9 ar (trap. DECB)

$\Rightarrow \frac{ar\ (\Delta ADE)}{ar\ (trap.DECB)} = \frac{9}{16}$

10. In ΔABC, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC

Sol:



We have, D and E as the mid-points of AB and AC

So, according to the mid-point theorem

DE || BC and DE = $\frac{1}{2}BC$ …(i)

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠B [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$= \frac{\left(\frac{1}{2}BC\right)^2}{BC^2}$ [From (i)]

$= \frac{\frac{1}{4}BC^2}{BC^2}$

$= \frac{1}{4}$

11. In Fig., 4.179, ΔABC and ΔDBC are on the same base BC. If AD and BC intersect at O, prove that $\frac{area\ (\Delta ABC)}{area\ (\Delta DBC)} = \frac{AO}{DO}$

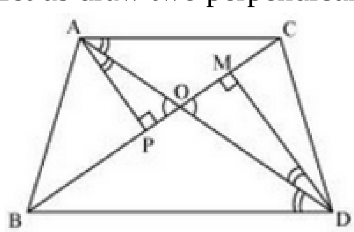
Sol:

We know that area of a triangle = $\frac{1}{2} \times Base \times height$

Since ΔABC and ΔDBC are one same base,

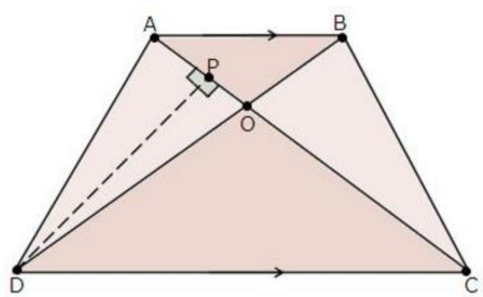
Therefore ratio between their areas will be as ratio of their heights.

Let us draw two perpendiculars AP and DM on line BC.



In ΔAPO and ΔDMO,
∠APO = ∠DMO (Each is 90°)
∠AOP = ∠DOM (vertically opposite angles)
∠OAP = ∠ODM (remaining angle)
Therefore ΔAPO ~ ΔDMO (By AAA rule)
Therefore $\frac{AP}{DM} = \frac{AO}{DO}$
Therefore $\frac{area\ (\Delta ABC)}{area\ (\Delta DBC)} = \frac{AO}{DO}$

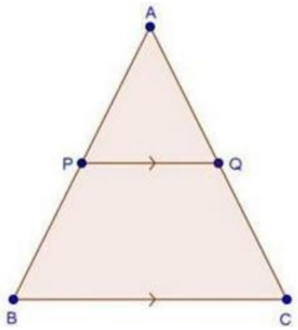
12. ABCD is a trapezium in which AB || CD. The diagonals AC and BD intersect at O. Prove that: (i) ΔAOB and ΔCOD (ii) If OA = 6 cm, OC = 8 cm,
Find:
(a) $\frac{area\ (\Delta AOB)}{area\ (\Delta COD)}$
(b) $\frac{area\ (\Delta AOD)}{area\ (\Delta COD)}$
Sol:



We have,
AB || DC
In ΔAOB and ΔCOD
∠AOB = ∠COD [Vertically opposite angles]
∠OAB = ∠OCD [Alternate interior angles]
Then, ΔAOB ~ ΔCOD [By AA similarity]
(a) By area of similar triangle theorem
$\frac{ar\ (\Delta AOB)}{ar\ (\Delta COD)} = \frac{OA^2}{OC^2} = \frac{6^2}{8^2} = \frac{36}{64} = \frac{9}{16}$
(b) Draw DP ⊥ AC
$\therefore \frac{area\ (\Delta AOD)}{area\ (\Delta COD)} = \frac{\frac{1}{2} \times AO \times DP}{\frac{1}{2} \times CO \times DP}$
$= \frac{AO}{CO}$
$= \frac{6}{8}$
$= \frac{3}{4}$

13. In ABC, P divides the side AB such that AP : PB = 1 : 2. Q is a point in AC such that PQ || BC. Find the ratio of the areas of ΔAPQ and trapezium BPQC.

Sol:



We have,

PQ || BC

And $\frac{AP}{PB} = \frac{1}{2}$

In ΔAPQ and ΔABC

$\angle A = \angle A$ [Common]

$\angle APQ = \angle B$ [Corresponding angles]

Then, $\Delta APQ \sim \Delta ABC$ [By AA similarity]

By area of similar triangle theorem

$\frac{ar\ (\Delta APQ)}{ar\ (\Delta ABC)} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{ar\ (\Delta APQ)}{ar(\Delta APQ)+ar(trap.BPQC)} = \frac{1^2}{3^2}\ \left[\frac{AP}{PB} = \frac{1}{2}\right]$

$\Rightarrow 9\text{ar}\ (APQ) = ar(\Delta APQ) + ar(trap.BPQC)$

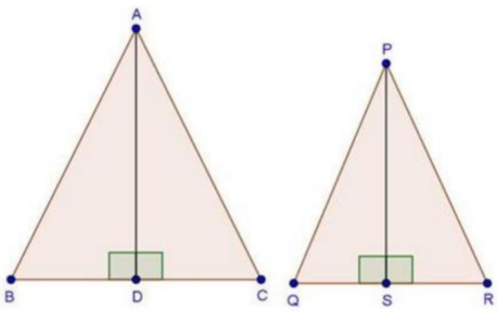
$\Rightarrow 9\text{ar}\ (APQ) - ar(\Delta APQ) + ar(trap.BPQC)$

⇒ 8ar(APQ) = ar(trap. BPQC)

$\Rightarrow \frac{ar\ (\Delta APQ)}{ar(trap.BPQC)} = \frac{1}{8}$

14. The areas of two similar triangles are 100 cm^2 and 49 cm^2 respectively. If the altitude the bigger triangle is 5 cm, find the corresponding altitude of the other.

Sol:



We have, $\Delta ABC \sim \Delta PQR$

Area(ΔABC) = 100 cm^2,

Area (ΔPQR) = 49 cm^2

AD = 5 cm

And AD and PS are the altitudes

By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{100}{49} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{10}{7} = \frac{AB}{PQ}$ ...(i)

$In\ \Delta ABD\ and\ \Delta PQS$

∠B = ∠Q [ΔABC ~ ΔPQR]

∠ADB = ∠PSQ [Each 90°]

Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ ...(ii) [Corresponding parts of similar Δ are proportional]

Compare (i) and (ii)

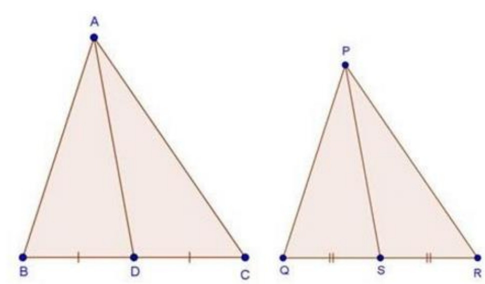
$\frac{AD}{PS} = \frac{10}{7}$

$\Rightarrow \frac{5}{PS} = \frac{10}{7}$

$\Rightarrow \text{PS} = \frac{5\times 7}{10} = 3.5\ cm$

15. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm, find the corresponding median of the other.

Sol:



We have,

ΔABC ~ ΔPQR

Area (ΔABC) = 121 cm^2,

Area (ΔPQR) = 64 cm^2

AD = 12.1 cm

And AD and PS are the medians

By area of similar triangle theorem

$\frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{121}{64} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{11}{8} = \frac{AB}{PQ}$ …(i)

Since, ΔABC ~ ΔPQR

Then, $\frac{AB}{PQ} = \frac{BC}{QR}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QS}$ [AD and PS are medians]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QS}$ …(ii)

In ΔABD and ΔPQS

∠B = ∠Q [ΔABC ~ ΔPQS]

$\frac{AB}{PQ} = \frac{BD}{QS}$ [From (ii)]

$Then, \Delta ABD \sim \Delta PQS$ [By SAS similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ …(iii) [Corresponding parts of similar Δ are proportional]

$Compare\ (i)\ and\ (iii)$

$\frac{11}{8} = \frac{AD}{PS}$

$\Rightarrow \frac{11}{8} = \frac{12.1}{PS}$

$\Rightarrow$ PS = $\frac{8 \times 12.1}{PS}$

$\Rightarrow$ PS = $\frac{8 \times 12.1}{PS} = 8.8\ cm$

16. If ΔABC ~ ΔDEF such that AB = 5 cm, area (ΔABC) = 20 cm^2 and area (ΔDEF) = 45 cm^2, determine DE.

Sol:

We have,

ΔABC ~ ΔDEF such that AB = 5 cm,

Area (ΔABC) = 20 cm^2 and area(ΔDEF) = 45 cm^2

By area of similar triangle theorem

$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{AB^2}{DE^2}$

$\Rightarrow \frac{20}{45} = \frac{5^2}{DE^2}$

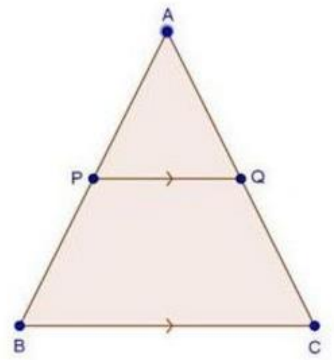
$\Rightarrow \frac{4}{9} = \frac{5^2}{DE^2}$

$\Rightarrow \frac{2}{3} = \frac{5}{DE}$ [Taking square root]

$\Rightarrow DE = \frac{3 \times 5}{2} = 7.5\ cm$

17. In ΔABC, PQ is a line segment intersecting AB at P and AC at Q such that PQ || BC and PQ divides ΔABC into two parts equal in area. Find $\frac{BP}{AB}$

Sol:



We have,

PQ || BC

And ar(ΔAPQ) = ar(trap. PQCB)

⇒ ar(ΔAPQ) = ar(ΔABC) – ar(ΔAPQ)

⇒ 2ar(ΔAPQ) = ar(ΔABC) …(i)

In ΔAPQ and ΔABC

∠A = ∠A [common]

∠APQ = ∠B [corresponding angles]

Then, ΔAPQ ~ ΔABC [By AA similarity]

∴ By area of similar triangle theorem

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta APQ)} = \frac{AP^2}{AB^2}$ [By using (i)]

$\Rightarrow \frac{1}{2} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AP}{AB^2}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AP}{AB}$ [Taking square root]

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB-BP}{AB}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB}{AB} - \frac{BP}{AB}$

$\Rightarrow \frac{1}{\sqrt{2}} = 1 - \frac{BP}{AB}$

$= \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{BP}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

18. The areas of two similar triangles ABC and PQR are in the ratio 9:16. If BC = 4.5 cm, find the length of QR.

Sol:

We have,

$\Delta ABC \sim \Delta PQR$

$\frac{area\ (\Delta ABC)}{area\ (\Delta PQR)} = \frac{BC^2}{QR^2}$

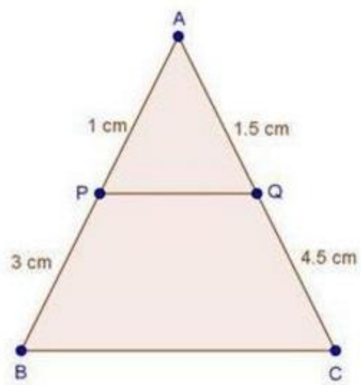
$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{QR^2}$

$\Rightarrow \frac{3}{4} = \frac{4.5}{QR}$ [Taking square root]

$\Rightarrow$ QR = $\frac{4 \times 4.5}{3} = 6cm$

19. ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 m, prove that area of ΔAPQ is one- sixteenth of the area of ABC.

Sol:



We have,

AP = 1 cm, PB = 3 cm, AQ = 1.5 cm and QC = 4.5 m

In ΔAPQ and ΔABC

∠A = ∠A [Common]

$\frac{AP}{AB} = \frac{AQ}{AC}$ [Each equal to $\frac{1}{4}$]

$Then, \Delta APQ \sim \Delta ABC$ [By SAS similarity]

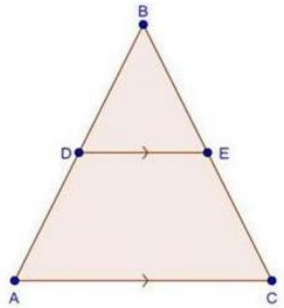
$By\ area\ of\ similar\ triangle\ theorem$

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1^2}{4^2}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16} \times ar(\Delta ABC)$

20. If D is a point on the side AB of ΔABC such that AD : DB = 3.2 and E is a Point on BC such that DE || AC. Find the ratio of areas of ΔABC and ΔBDE.

Sol:



We have,

$\frac{AD}{DB} = \frac{3}{2}$

$\Rightarrow \frac{DB}{AD} = \frac{2}{3}$

In ΔBDE and ΔBAC

∠B = ∠B [common]

∠ BDE = ∠A [corresponding angles]

Then, ΔBDE ~ ΔBAC [By AA similarity]

By area of similar triangle theorem

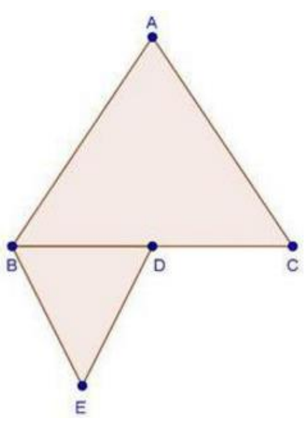
$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{AB^2}{BD^2}$

$= \frac{5^2}{2^2}$ $\left[\frac{AD}{DB} = \frac{3}{2}\right]$

$= \frac{25}{4}$

21. If ΔABC and ΔBDE are equilateral triangles, where D is the mid-point of BC, find the ratio of areas of ΔABC and ΔBDE.

Sol:



We have,

ΔABC and ΔBDE are equilateral triangles then both triangles are equiangular

∴ ΔABC ~ ΔBDE [By AAA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ABC)}{ar\ (\Delta BDE)} = \frac{BC^2}{BD^2}$

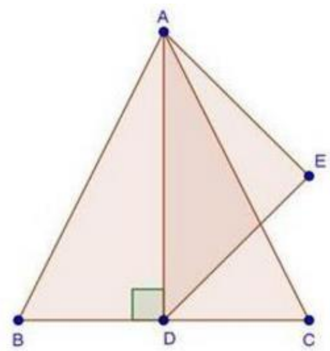
$= \frac{2(BD)^2}{BD^2}$ [D is the mid-point of BC]

$= \frac{4BD^2}{BD^2}$

$= \frac{4}{1}$

22. AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that Area (ΔADE): Area (ΔABC) = 3: 4

Sol:



We have,

ΔABC is an equilateral triangle

Then, AB = BC = AC

Let, AB = BC = AC = 2x

Since, AD ⊥ BC then BD = DC = x

In ΔADB, by Pythagoras theorem

$AB^2 = (2x)^2 - (x)^2$

$\Rightarrow AD^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow AD = \sqrt{3}x\ cm$

$Since, \Delta ABC\ and\ \Delta ADE\ both\ are\ equilateral\ triangles\ then\ they\ are\ equiangular$

∴ ΔABC ~ ΔADE [By AA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2}$

$= \frac{(\sqrt{3}x)^2}{(2x)^2}$

$= \frac{3x^2}{4x^2}$

$= \frac{3}{4}$

Exercise 4.7

1. If the sides of a triangle are 3 cm, 4 cm, and 6 cm long, determine whether the triangle is a right-angled triangle.

Sol:
We have,
Sides of triangle
AB = 3 cm
BC = 4 cm
AC = 6 cm
$\therefore AB^2 = 3^2 = 9$
$BC^2 = 4^2 = 16$
$AC^2 = 6^2 = 36$
Since, $AB^2 + BC^2 \neq AC^2$
Then, by converse of Pythagoras theorem, triangle is not a right triangle.

2. The sides of certain triangles are given below. Determine which of them right triangles are.
(i) a = 7 cm, b = 24 cm and c = 25 cm
(ii) a = 9 cm, b = l6 cm and c = 18 cm
(iii) a = 1.6 cm, b = 3.8 cm and c = 4 cm
(iv) a = 8 cm, b = 10 cm and c = 6 cm

Sol:
We have,
a = 7 cm, b = 24 cm and c = 25 cm
$\therefore a^2 = 49, b^2 = 576 \ and \ c^2 = 625$
Since, $a^2 + b^2 = 49 + 576$
= 625
= c^2
Then, by converse of Pythagoras theorem, given triangle is a right triangle.

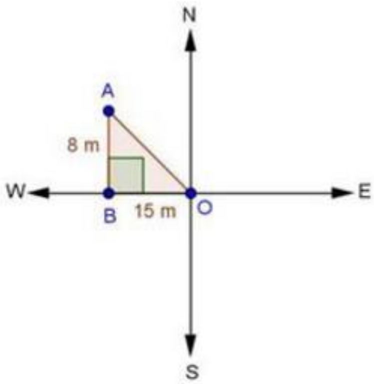
We have,
a = 9 cm, b = 16 cm and c = 18 cm
$\therefore a^2 = 81, b^2 = 256 \ and \ c^2 = 324$
Since, $a^2 + b^2 = 81 + 256 = 337$
$\neq c^2$
Then, by converse of Pythagoras theorem, given triangle is not a right triangle.

We have,
a = 1.6 cm, b = 3.8 cm and C = 4 cm
$\therefore a^2 = 64, b^2 = 100 \ and \ c^2 = 36$
Since, $a^2 + c^2 = 64 + 36 = 100 = b^2$

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

3. A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

Sol:



Let the starting point of the man be O and final point be A.

∴ In ΔABO, by Pythagoras theorem $AO^2 = AB^2 + BO^2$

⇒ $AO^2 = 8^2 + 15^2$

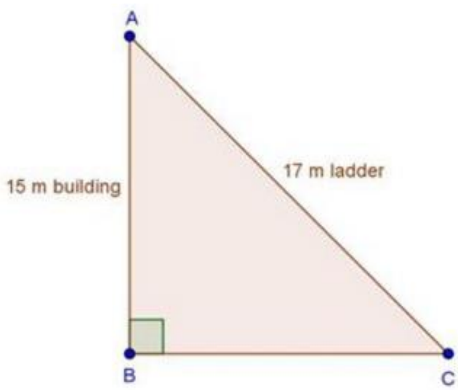
⇒ $AO^2 = 64 + 225 = 289$

⇒ AO = $\sqrt{289} = 17m$

∴ He is 17m far from the starting point.

4. A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Sol:



In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

⇒ $15^2 + BC^2 = 17^2$

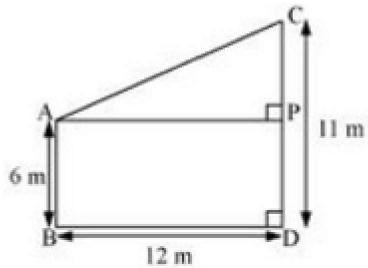
⇒ $225 + BC^2 = 17^2$

⇒ $BC^3 = 289 - 225$

⇒ $BC^2 = 64$

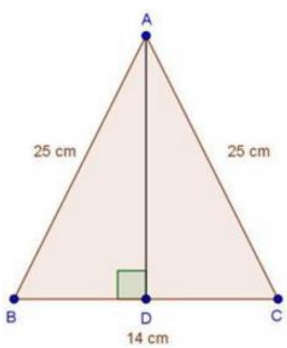
$\Rightarrow BC = 8\, m$
$\therefore Distance\ of\ the\ foot\ of\ the\ ladder\ from\ building = 8\, m$

5. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
Sol:



Let CD and AB be the poles of height 11 and 6 m.
Therefore CP = 11 – 6 = 5 m
From the figure we may observe that AP = 12m
In triangle APC, by applying Pythagoras theorem
$AP^2 + PC^2 = AC^2$
$12^2 + 5^2 = AC^2$
$AC^2 = 144 + 25 = 169$
AC = 13
Therefore distance between their tops = 13m.

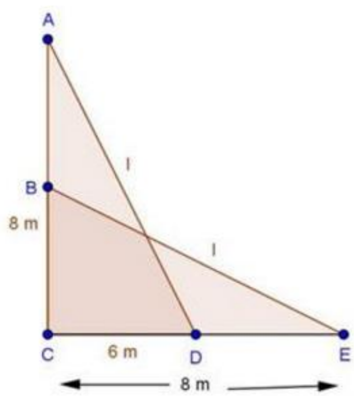
6. In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC.
Sol:



We have
AB = AC = 25 cm and BC = 14 cm
In ΔABD and ΔACD
∠ADB = ∠ADC [Each 90°]
AB = AC [Each 25 cm]

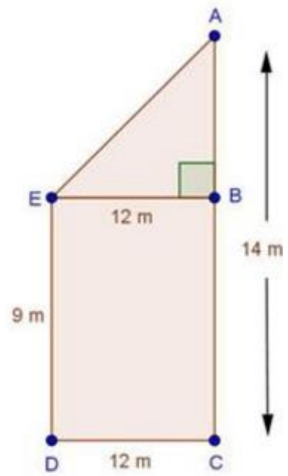
AD = AD [Common]
Then, ∆ABD ≅ ∆ACD [By RHS condition]
∴ BD = CD = 7 cm [By c.p.c.t]
In ∆ADB, by Pythagoras theorem
$AD^2 + BD^2 = AB^2$
$\Rightarrow AD^2 + 7^2 = 25^2$
$\Rightarrow AD^2 = 625 - 49 = 576$
$\Rightarrow AD = \sqrt{576} = 24\ cm$

7. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?
Sol:



Let, length of ladder be AD = BE = *l* m
In ∆ACD, by Pythagoras theorem
$AD^2 = AC^2 + CD^2$
$\Rightarrow\ l^2 = 8^2 + 6^2$ ….(i)
$In\ \Delta BCE, by\ pythagoras\ theorem$
$BE^2 = BC^2 + CE^2$
$\Rightarrow\ l^2 = BC^2 + 8^2$ ….(ii)
$Compare\ (i) and\ (ii)$
$BC^2 + 8^2 = 8^2 + 6^2$
$\Rightarrow BC^2 = 6^2$
$\Rightarrow BC = 6m$

8. Two poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
Sol:



We have,

AC = 14 m, DC = 12m and ED = BC = 9m

Construction: Draw EB ⊥ AC

∴ AB = AC – BC = 14 – 9 = 5m

And, EB = DC = 12 m

In ΔABE, by Pythagoras theorem,

$AE^2 = AB^2 + BE^2$

$\Rightarrow AE^2 = 5^2 + 12^2$

$\Rightarrow AE^2 = 25 + 144 = 169$

$\Rightarrow AE = \sqrt{169} = 13\ m$

$\therefore Distance\ between\ their\ tops = 13\ m$

9. Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig. 4.219

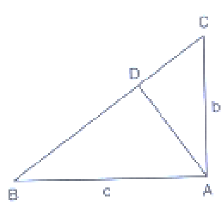


Fig. 4.219

Sol:

We have,

In ΔBAC, by Pythagoras theorem

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = c^2 + b^2$

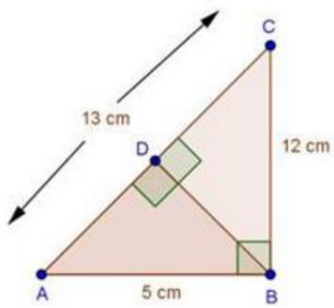
$\Rightarrow BC = \sqrt{c^2 + b^2}$ …(i)

$In\ \Delta ABD\ and\ \Delta CBA$

∠B = ∠B [Common]

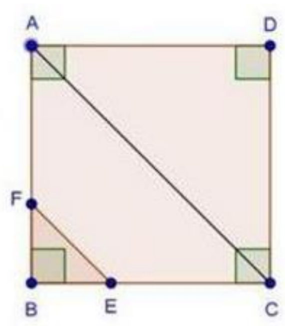
$\angle ADB = \angle BAC$ [Each 90°]
Then, $\Delta ABD \sim \Delta CBA$ [By AA similarity]
$\therefore \frac{AB}{CB} = \frac{AD}{CA}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{c}{\sqrt{c^2+b^2}} = \frac{AD}{b}$
$\Rightarrow AD = \frac{bc}{\sqrt{c^2+b^2}}$

10. A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.
Sol:



Let, AB = 5cm, BC = 12 cm and AC = 13 cm. Then, $AC^2 = AB^2 + BC^2$. This proves that ΔABC is a right triangle, right angles at B. Let BD be the length of perpendicular from B on AC.
Now, Area ΔABC = $\frac{1}{2}(BC \times BA)$
= $\frac{1}{2}(12 \times 5)$
= 30 cm^2
Also, Area of ΔABC = $\frac{1}{2} AC \times BD = \frac{1}{2}(13 \times BD)$
$\Rightarrow (13 \times BD) = 30 \times 2$
$\Rightarrow$ BD = $\frac{60}{13}$ cm

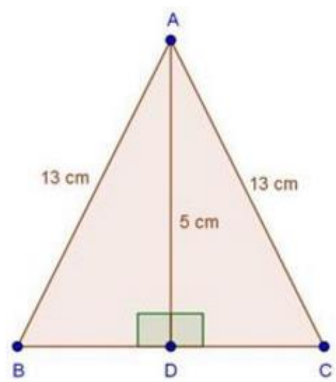
11. ABCD is a square. F is the mid-point of AB. BE is one third of BC. If the area of ΔFBE = 108 cm^2, find the length of AC.
Sol:



Since, ABCD is a square
Then, AB = BC = CD = DA = x cm
Since, F is the mid-point of AB
Then, AF = $FB = \frac{x}{2}\ cm$
Since, BE is one third of BC
Then, BE = $\frac{x}{3}$ cm
We have, area of ΔFBE = 108 cm^2
$\Rightarrow \frac{1}{2} \times BE \times FB = 108$
$\Rightarrow \frac{1}{2} \times \frac{x}{3} \times \frac{x}{2} = 108$
$\Rightarrow x^2 = 108 \times 2 \times 3 \times 2$
$\Rightarrow x^2 = 1296$
$\Rightarrow x = \sqrt{1296} = 36cm$
$In\ \Delta ABC, by\ pythagoras\ theorem\ AC^2 = AB^2 + BC^2$
$\Rightarrow AC^2 = x^2 + x^2$
$\Rightarrow AC^2 = 2x^2$
$\Rightarrow AC^2 = 2 \times (36)^2$
$\Rightarrow AC = 36\sqrt{2} = 36 \times 1.414 = 50.904\ cm$

12. In an isosceles triangle ABC, if AB = AC = 13 cm and the altitude from A on BC is 5 cm, find BC.

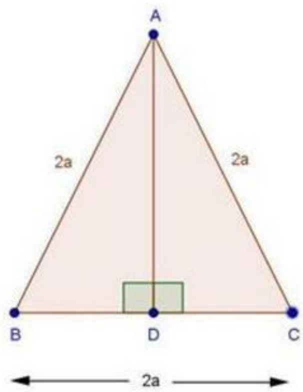
Sol:



In ΔADB, by Pythagoras theorem
$AD^2 + BD^2 = 13^2$
$\Rightarrow 25 + BD^2 = 169$
$\Rightarrow BD^2 = 169 - 2 = 144$
$\Rightarrow BD = \sqrt{144} = 12\ cm$
$In\ \Delta ADB\ and\ \Delta ADC$
∠ADB = ∠ADC [Each 90°]
AB = AC [Each 13 cm]

AD = AD [Common]
Then, ΔADB ≅ ΔADC [By RHS condition]
∴ BD = CD = 12 cm [By c.p.c.t]
Hence, BC = 12 + 12 = 24 cm

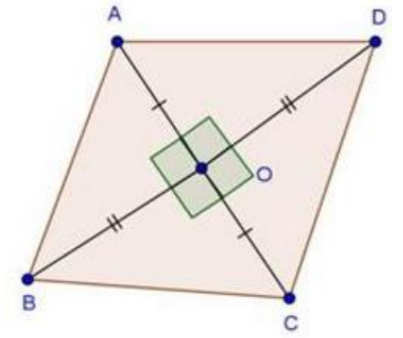
13. In a ΔABC, AB = BC = CA = 2a and AD ⊥ BC. Prove that
(i) AD = $a\sqrt{3}$ (ii) Area (ΔABC) = $\sqrt{3}\,a^2$
Sol:



(i) In ΔABD and ΔACD
∠ADB = ∠ADC [Each 90°]
AB = AC [Given]
AD = AD [Common]
Then, ΔABD ≅ ΔACD [By RHS condition]
∴ BD = CD = a [By c.p.c.t]
In ΔADB, by Pythagoras theorem
$AD^2 + BD^2 = AB^2$
$\Rightarrow AD^2 + (a)^2 = (2a)^2$
$\Rightarrow AD^2 + a^2 = 4a^2$
$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$
$\Rightarrow AD = a\sqrt{3}$

(ii) Area of ΔABC = $\frac{1}{2} \times BC \times AD$
$= \frac{1}{2} \times 2a \times a\sqrt{3}$
$= \sqrt{3}a^2$

14. The lengths of the diagonals of a rhombus are 24 cm and 10 cm. Find each side of the rhombus.
Sol:



We have,
ABCD is a rhombus with diagonals AC = 10 cm and BD = 24 cm
We know that diagonal of a rhombus bisect each other at 90°
∴ AO = OC = 5 cm and BO = OD = 12 cm
In ΔAOB, by Pythagoras theorem

$AB^2 = AO^2 + BO^2$

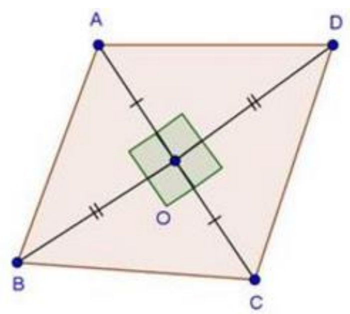
$\Rightarrow AB^2 = 5^2 + 12^2$

$\Rightarrow AB^2 = 25 + 144 = 169$

$\Rightarrow AB = \sqrt{169} = 13\ cm$

15. Each side of a rhombus is 10 cm. If one of its diagonals is 16 cm find the length of the other diagonal.

Sol:



We have,
ABCD is a rhombus with side 10 cm and diagonal BD = 16 cm
We know that diagonals of a rhombus bisect each other at 90°
∴ BO = OD = 8 cm
In ΔAOB, by pythagoras theorem

$AO^2 + BO^2 = AB^2$

$\Rightarrow AO^2 + 8^2 = 10^2$

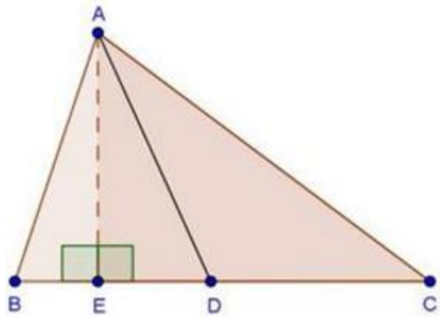
$\Rightarrow AO^2 = 100 - 64 = 36$

$\Rightarrow AO = \sqrt{36} = 6\ cm$ [By above property]

$hence, AC = 6 + 6 = 12\ cm$

16. In an acute-angled triangle, express a median in terms of its sides.

Sol:



We have,

In ΔABC, AD is a median.

Draw AE ⊥ BC

In ΔAEB, by pythagoras theorem

$AB^2 = AE^2 + BE^2$

$\Rightarrow AB^2 = AD^2 - DE^2 + (BD - DE)^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$

$\Rightarrow AB^2 = AD^2 + BD^2 - 2BD \times DE$

$\Rightarrow AB^2 = AD^2 + \frac{BC^2}{4} - BC \times DE$ ….(i) [BC = 2BD given]

Again, In ΔAEC, by pythagoras theorem

$AC^2 = AE^2 + EC^2$

$\Rightarrow AC^2 = AD^2 - DE^2 + (DE + CD)^2$ [By Pythagoras theorem]

$\Rightarrow AC^2 = AD^2 + CD^2 + 2CD \times DE$

$\Rightarrow AC^2 = AD^2 + \frac{BC^2}{4} + BC \times DE$ ….(ii) [BC = 2CD given]

$Add\ equations\ (i)\ and\ (ii)$

$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2}$

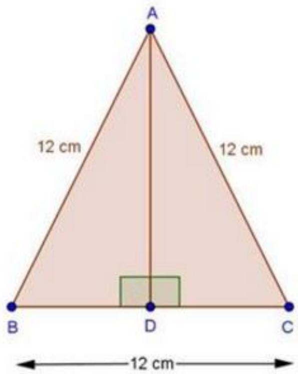
$\Rightarrow 2AB^2 + 2AC^2 = 4AD^2 + BC^2$ [Multiply by 2]

$\Rightarrow 4AD^2 = 2AB^2 + 2AC^2 - BC^2$

$\Rightarrow AD^2 = \frac{2AB^2 + 2AC^2 - BC^2}{4}$

17. Calculate the height of an equilateral triangle each of whose sides measures 12 cm.

Sol:



We have,

ΔABC is an equilateral Δ with side 12 cm.

Draw AE ⊥ BC

In ΔABD and ΔACD

∠ADB = ∠ADC [Each 90°]

AB = AC [Each 12 cm]

AD = AD [Common]

Then, ΔABD ≅ ΔACD [By RHS condition]

$\therefore AD^2 + BD^2 = AB^2$

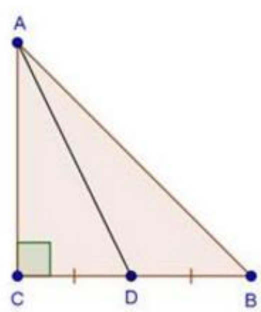
$\Rightarrow AD^2 + 6^2 = 12^2$

$\Rightarrow AD^2 = 144 - 36 = 108$

⇒ AD = $\sqrt{108} = 10.39$ cm

18. In right-angled triangle ABC in which ∠C = 90°, if D is the mid-point of BC, prove that $AB^2 = 4\,AD^2 - 3\,AC^2$.

Sol:



We have,

∠C = 90° and D is the mid-point of BC

In ΔACB, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = AC^2 + (2CD)^2$ [D is the mid-point of BC]

$AB^2 = AC^2 + 4CD^2$
$\Rightarrow AB^2 = AC^2 + 4(AD^2 - AC^2)$ [In ΔACD, by Pythagoras theorem]
$\Rightarrow AB^2 = AC^2 + 4AD^2 - 4AC^2$
$\Rightarrow AB^2 = 4AD^2 - 3AC^2$

19. In Fig. 4.220, D is the mid-point of side BC and AE ⊥ BC. If BC = a, AC = b, AB = c, ED = x, AD = p and AE = h, prove that:

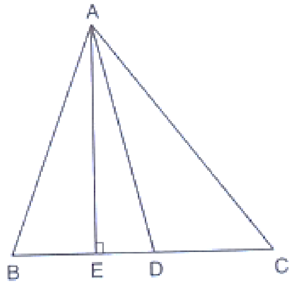


Fig. 4.220

(i) $b^2 = p^2 + ax + \frac{a^2}{4}$
(ii) $c^2 = p^2 - ax + \frac{a^2}{4}$
(iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

Sol:
We have, D as the mid-point of BC
(i) $AC^2 = AE^2 + EC^2$
$b^2 = AE^2 + (ED + DC)^2$ [By pythagoras theorem]
$b^2 = AD^2 + DC^2 + 2DC \times ED$
$b^2 = p^2 + \left(\frac{a}{2}\right)^2 + 2\left(\frac{a}{2}\right) \times x$ [BC = 2CD given]
$\Rightarrow b^2 = p^2 + \frac{a^2}{4} + ax$ ...(i)
(ii) In ΔAEB, by pythagoras theorem
$AB^2 = AE^2 + BE^2$
$\Rightarrow c^2 = AD^2 - ED^2 + (BD - ED)^2$ [By pythagoras theorem]
$\Rightarrow c^2 = p^2 - ED^2 + BD^2 + ED^2 - 2BD \times ED$
$\Rightarrow c^2 = p^2 + \left(\frac{a}{2}\right)^2 - 2\left(\frac{a}{2}\right) \times x$ ...(ii)
(iii) Add equations (i) and (ii)
$b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

20. In Fig., 4.221, ∠B < 90° and segment AD ⊥ BC, show that

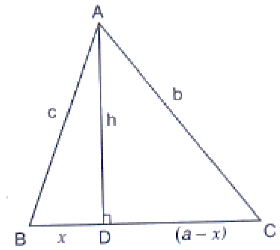


Fig. 4.221

(i) $b^2 = h^2 + a^2 + x^2 - 2ax$

(ii) $b^2 = a^2 + c^2 - 2ax$

Sol:

In ΔADC, by pythagoras theorem

$AC^2 = AD^2 + DC^2$

$\Rightarrow b^2 = h^2 + (a - x)^2$

$\Rightarrow b^2 = h^2 + a^2 + x^2 - 2ax$

$\Rightarrow b^2 = a^2 + (h^2 + x^2) - 2ax$

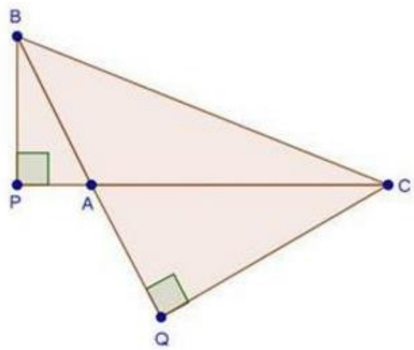
$\Rightarrow b^2 = a^2 + c^2 - 2ax$ by Pythagoras theorem

21. In ΔABC, ∠A is obtuse, PB ⊥AC and QC ⊥ AB. Prove that:

(i) AB × AQ = AC × AP

(ii) $BC^2 = (AC \times CP + AB \times BQ)$

Sol:



Then, ΔAPB ~ ΔAQC [By AA similarity]

$\therefore \frac{AP}{AQ} = \frac{AB}{AC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow AP \times AC = AQ \times AB$ ...(i)

(ii) In ΔBPC, by pythagoras theorem

$BC^2 = BP^2 + PC^2$

$\Rightarrow BC^2 = AB^2 - AP^2 + (AP + AC)^2$ [By pythagoras theorem]

$\Rightarrow BC^2 = AB^2 + AC^2 + 2AP \times AC$ …(ii)

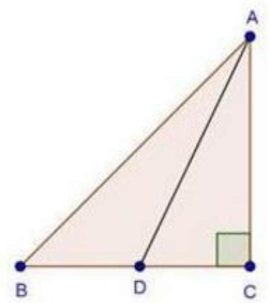
$In\ \Delta BQC, by\ pythagoras\ theorem,$

$BC^2 = CQ^2 + BQ^2$

$\Rightarrow BC^2 = AC^2 - AQ^2 + (AB + AQ)^2$ [By pythagoras theorem]

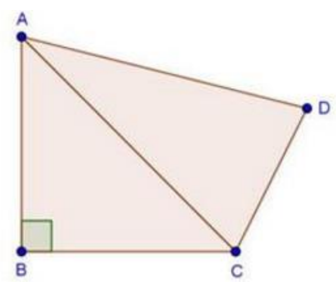
$\Rightarrow BC^2 = AC^2 - AQ^2 + AB^2 + AQ^2 + 2AB \times AQ$
$\Rightarrow BC^2 = AC^2 + AB^2 + 2AB \times AQ$ …(iii)
$Add\ equations\ (ii) \&\ (iii)$
$2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$
$\Rightarrow 2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$
$\Rightarrow 2BC^2 = 2AC[AC + AP] + AB[AB + AQ]$
$\Rightarrow 2BC^2 = 2AC \times PC + 2AB \times BQ$
$\Rightarrow BC^2 = AC \times PC + AB \times BQ$ [Divide by 2]

22. In a right ΔABC right-angled at C, if D is the mid-point of BC, prove that $BC^2 = 4(AD^2 - AC^2)$
Sol:



To prove: $BC^2 = 4[AD^2 - AC^2]$
We have, ∠C = 90° and D is the mid-point of BC.
LHS = BC^2
= $(2CD)^2$ [D is the mid-point of BC]
= $4CD^2$
= $4[AD^2 - AC^2]$ [In ΔACD, by pythagoras theorem]
= RHS

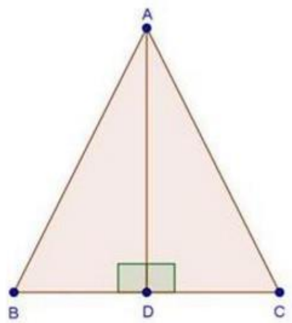
23. In a quadrilateral ABCD, ∠B = 90°, $AD^2 = AB^2 + BC^2 + CD^2$, prove that ∠ACD = 90°.
Sol:



We have, ∠B = 90° and $AD^2 = AB^2 + BC^2 + CD^2$
$\therefore AD^2 = AB^2 + BC^2 + CD^2$ [Given]
But $AB^2 + BC^2 = AC^2$ [By pythagoras theorem]

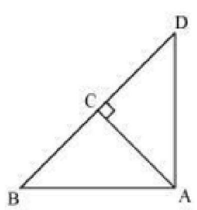
Then, $AD^2 = AC^2 + CD^2$
By converse of by pythagoras theorem
∠ACD = 90°

24. In an equilateral ΔABC, AD ⊥ BC, prove that $AD^2 = 3BD^2$.
Sol:



We have, ΔABC is an equilateral Δ and AD ⊥ BC
In ΔADB and ΔADC
∠ADB = ∠ADC [Each 90°]
AB = AC [Given]
AD = AD [Common]
Then, ΔADB ≅ ΔADC [By RHS condition]
∴ BD = CD = $\frac{BC}{2}$ …(i) [corresponding parts of similar Δ are proportional]
In, ΔABD, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$
$\Rightarrow BC^2 = AD^2 + BD^2$ [AB = BC given]
$\Rightarrow [2BD]^2 = AD^2 + BD^2$ [From (i)]
$\Rightarrow 4BD^2 - BD^2 = AD^2$
$\Rightarrow 3BD^2 = AD^2$

25. ΔABD is a right triangle right angled at A and AC ⊥ BD. Show that:
(i) $AB^2 = CB \times BD$
(ii) $AC^2 = DC \times BC$
(iii) $AD^2 = BD \times CD$
(iv) $\frac{AB^2}{AC^2} = \frac{BD}{DC}$
Sol:



(i) In ΔADB and ΔCAB

∠DAB = ∠ACB = 90°

∠ABD = ∠CBA (common angle)

∠ADB = ∠CAB (remaining angle)

So, ΔADB ~ ΔCAB (by AAA similarity)

Therefore $\frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$

(ii) Let ∠CAB = x

In ΔCBA

∠CBA = 180° $-$ 90° $- x$

∠CBA = 90° $- x$

Similarly in ΔCAD

∠CAD = 90° $- \angle CAD = 90° - x$

∠CDA = 90° $-$ ∠CAB

$= 90° - x$

∠CDA = 180° −90° $- (90° - x)$

∠CDA = x

Now in ΔCBA and ΔCAD we may observe that

∠CBA = ∠CAD

∠CAB = ∠CDA

∠ACB = ∠DCA = 90°

Therefore ΔCBA ~ ΔCAD (by AAA rule)

Therefore $\frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = DC \times BC$

(iii) In ΔDCA & ΔDAB

∠DCA = ∠DAB (both are equal to 90°)

∠CDA = ∠ADB (common angle)

∠DAC = ∠DBA (remaining angle)

ΔDCA ~ ΔDAB (AAA property)

Therefore $\frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

(iv) From part (i) $AB^2 = CB \times BD$

From part (ii) $AC^2 = DC \times BC$

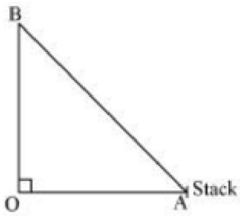
Hence $\frac{AB^2}{AC^2} = \frac{CB \times BD}{DC \times BC}$

$\frac{AB^2}{AC^2} = \frac{BD}{DC}$

Hence proved

26. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol:



Let OB be the pole and AB be the wire. Therefore by pythagoras theorem,

$AB^2 = OB^2 + OA^2$

$24^2 = 18^2 + OA^2$

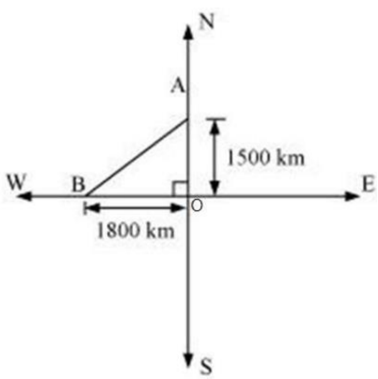
$OA^2 = 576 - 324$

OA = $\sqrt{252} = \sqrt{6 \times 6 \times 7} = 6\sqrt{7}$

Therefore distance from base = $6\sqrt{7}\ m$

27. An aeroplane leaves an airport and flies due north at a speed of 1000km/hr. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after 1 hours?

Sol:



Distance traveled by the plane flying towards north in $1\frac{1}{2}$ hrs

= $1000 \times 1\frac{1}{2} = 1500\ km$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}\ hrs$

= $1200 \times 1\frac{1}{2} = 1800\ km$

Let these distances are represented by OA and OB respectively.

Now applying Pythagoras theorem

Distance between these planes after $1\frac{1}{2}\ hrs$ AB = $\sqrt{OA^2 + OB^2}$

= $\sqrt{(1500)^2 + (1800)^2} = \sqrt{2250000 + 3240000}$

$= \sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$

So, distance between these planes will be $300\sqrt{61}$ km, after $1\frac{1}{2}\ hrs$

28. Determine whether the triangle having sides (a $-$ 1) cm, $2\sqrt{a}$ cm and (a + 1) cm is a right-angled triangle.

Sol:

Let ABC be the Δ with

AB = (a – 1) cm BC = $2\sqrt{a}$ cm, CA = (a + 1) cm

Hence, $AB^2 = (a-1)^2 = a^2 + 1 - 2a$

$BC^2 = \left(2\sqrt{a}\right)^2 = 4a$

$CA^2 = (a+1)^2 = a^2 + 1 + 2a$

Hence $AB^2 + BC^2 = AC^2$

$So\ \Delta\ ABC\ is\ right\ angled\ \Delta\ at\ B.$