

CHAPTER -02

INVERSE TRIGONOMETRIC FUNCTIONS

One Mark Questions

1. Write the domain of $f(x) = \sin^{-1}x$ [K]
2. Write the domain of $f(x) = \cos^{-1}x$ [K]
3. Write the range of $y = \cos^{-1}x$ [K]
4. Write the domain of $f(x) = \sec^{-1}x$ [K]
5. Write the principal value of branch of $f(x) = \sin^{-1}x$ [K]
6. Write the domain of $f(x) = \tan^{-1}x$ [K]
7. Write the set of all principal values of $\operatorname{cosec}^{-1}x$. [K]
8. If $x = \sin^{-1}y$, then find set values of y . [K]
9. Write a range of $f(x) = \sin^{-1}x$ other than $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. [U]
10. Write a range of $f(x) = \cos^{-1}x$ other than $[0, \pi]$. [U]
11. Find $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ [K]
12. If $y = \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$, then find value of y . [K]
13. Find the principal value $\sin^{-1}(-1)$. [K]
14. Find $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ [U]
15. Find $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ [U]
16. Find $\sin\left[\frac{1}{2}\sin^{-1}(-1)\right]$ [U]
17. Find the principal value of $\tan^{-1}(-\sqrt{3})$ [K]
18. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ [K]

19. Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ [K]
20. Find the principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ [K]
21. Find the principal value of $\cos^{-1}(-1)$ [K]
22. Find the principal value of $\sec^{-1}(-2)$ [K]
23. Write the set of value of x for which $2 \tan^{-1} x = \tan^{-1} \left[\frac{2x}{1-x^2} \right]$ holds. [K]
24. Write the set of value of x for which $2 \tan^{-1} x = \cos^{-1} \left[\frac{1-x^2}{1+x^2} \right]$ holds [K]
25. Find the value of $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x)$ $|x| \geq 1$ [U]
26. Find the value of $\sin(\tan^{-1} a + \cot^{-1} a)$ [U]
27. Find $\sin(\tan^{-1} x), |x| < 1$. [U]

Two Mark Questions

1. Prove that $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $x \in [-1, 1]$ [U]
2. Prove that $\sin^{-1}(-x) = -\sin^{-1} x$ where $x \in [-1, 1]$ [U]
3. Prove that $\tan^{-1}(-x) = -\tan^{-1} x$, $x \in \mathbb{R}$ [U]
4. Prove that $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$, $|x| \geq 1$ [U]
5. Prove that $\sec^{-1}(-x) = \pi - \sec^{-1} x$, $|x| \geq 1$ [U]
6. Prove that $\cot^{-1}(-x) = \pi - \cot^{-1} x$, $x \in \mathbb{R}$ [U]
7. Find the value of $\tan^{-1} 1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$ [U]
8. Find the value of $\cos^{-1} \frac{1}{2} + 2 \sin^{-1}\left(\frac{1}{2}\right)$ [K]
9. Find the values of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$ [K]
10. Find the value of $\tan^{-1} \sqrt{3} - \cot^{-1}(-\sqrt{3})$ [K]
11. Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ [U]

12. Find the value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ [U]
13. Find the value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ [U]
14. Evaluate $\sin^{-1}[\sin 110^\circ]$ [K]
15. Evaluate $\sin^{-1}\left[\sin \frac{2\pi}{3}\right]$ [K]
16. Evaluate $\cos^{-1}\left[\cos 13\frac{\pi}{6}\right]$ [K]
17. Evaluate $\tan^{-1}\left[\tan \frac{7\pi}{6}\right]$ [K]
18. Prove that $\sin^{-1} x = \operatorname{cosec}^{-1} \frac{1}{x}$ [U]
19. Prove that $\cos^{-1} x = \sec^{-1} \frac{1}{x}$ [U]
20. Prove that $\tan^{-1} x = \cot^{-1} \frac{1}{x}$ [U]
21. Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, $-1 \leq x \leq 1$ [U]
22. Prove that $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$, $|x| \geq 1$ [U]
23. Prove that $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, $-\infty < x < \infty$ [U]
24. Evaluate $\sin^{-1}\left[\sin \frac{3\pi}{5}\right]$ [U]
25. Evaluate $\cos^{-1}\left[\sin \frac{\pi}{9}\right]$ [U]
26. If $\sin\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right) = 1$, then find x. [U]
27. Express $\tan^{-1}\left[\frac{1}{\sqrt{x^2 - 1}}\right]$, $|x| > 1$, in the simplest form. [U]
28. Express $\tan^{-1}\left[\frac{x}{\sqrt{a^2 - x^2}}\right]$, $|x| < a$ in simplest form. [U]

29. Prove that $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \frac{1-x}{1+x}$, $x \in [0,1]$. [U]
30. Express $\tan^{-1} \left[\frac{3a^2x - x^3}{a^3 - 3ax^2} \right]$, $a > 0$, $\frac{-a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$ in simplest form [U]
31. Write $\tan^{-1} \left(\sqrt{\frac{1 - \cos x}{1 + \cos x}} \right)$, $0 < x < \pi$ in simplest form [U]
32. Write $\tan^{-1} \left(\sqrt{\frac{1 + \cos x}{1 - \cos x}} \right)$ $x \neq 2n\pi$ in simplest form [U]
33. Express $\cot^{-1} \left(\frac{1}{\sqrt{x^2 - 1}} \right)$, $x > 1$ in the simplest form [U]
34. Simplify $\tan^{-1} \left[\frac{3 \cos x - 4 \sin x}{4 \cos x + 3 \sin x} \right]$, if $\frac{3}{4} \tan x > -1$. [U]
35. Prove that $\tan^{-1} \left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x} \right) = \tan^{-1} \frac{a}{b} - x$ [U]
36. Prove that $2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$, $|x| \leq 1$ [U]
37. Prove that $2 \tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2}$, $x \geq 0$ [U]
38. Prove that $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$ $-1 < x < 1$ [U]
39. Prove that $\sin^{-1} \left(2x \sqrt{1-x^2} \right) = 2 \sin^{-1} x$, $\frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$ [U]
40. Prove that $\sin^{-1} \left(2x \sqrt{1-x^2} \right) = 2 \cos^{-1} x$, $\frac{1}{\sqrt{2}} \leq x \leq 1$ [U]
41. Prove that $3 \sin^{-1} x = \sin^{-1} (3x - 4x^3)$, $x \in \left[\frac{-1}{2}, \frac{1}{2} \right]$ [U]
42. Prove that $3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1 \right]$ [U]
43. Prove that $\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$, $|x| < 1$ [U]
44. Prove that $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$ [U]
45. Prove that $2 \tan^{-1} \left(\frac{1}{3} \right) = \tan^{-1} \frac{3}{4}$ [U]

46. Prove that $2 \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{24}{7}$. [U]
47. Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \frac{3}{4}$ [U]
48. Prove that $\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24} = \tan^{-1} \frac{1}{2}$ [U]
49. Simplify $\tan^{-1} \left[\frac{a \cos x - b \sin x}{b \cos x + a \sin x} \right]$, if $\frac{a}{b} \tan x > -1$. [U]

Three Mark Questions

1. Prove that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$, $xy < 1$ [U]
2. Prove that $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right)$, $xy > -1$ [U]
3. Prove that $2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$ [U]
4. Prove that $\tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$ [U]
5. Prove that $\tan^{-1} \left(\frac{x}{y} \right) - \tan^{-1} \left(\frac{x-y}{x+y} \right) = \frac{\pi}{4}$ [U]
6. Write $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$, $0 < x < \pi$ in simplest form [U]
7. Prove that $\tan^{-1} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right] = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$, $-\frac{1}{2} \leq x \leq 1$ [U]
8. Prove that $\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}$, $x \in \left(0, \frac{\pi}{4} \right)$ [U]
9. Express $\tan^{-1} \left(\frac{\cos x}{1-\sin x} \right)$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$ in the simplest form [U]
10. Prove that $\sin^{-1} \frac{8}{17} + \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{77}{36}$ [U]
11. Prove that $\sin^{-1} \frac{3}{5} - \sin^{-1} \frac{8}{17} = \cos^{-1} \frac{84}{85}$ [U]
12. Prove that $\tan^{-1} \frac{63}{16} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$ [U]

13. Prove that $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$ [U]

14. Prove that $\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$: [U]

15. Prove that $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$ [U]

16. Prove that $\tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2} = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right) \quad |x| < \frac{1}{\sqrt{3}}$ [U]

17. Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} + \tan^{-1} \left(\frac{4}{3} \right) = \frac{\pi}{2}$ [U]

18. Solve for x $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$: [U]

19. Solve for x : $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$ [U]

20. Solve : $2 \tan^{-1} (\cos x) = \tan^{-1} (2 \operatorname{cosec} x)$ [U]

21. Solve : $\tan^{-1} \left(\frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x \quad (x > 0)$ [U]

22. Solve : $\sin^{-1} (1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$ [U]

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