Fourieor Series

* If $f(x)$ is piece wise continuous function in $[c, c+2l]$ then the Fouries series for the function, $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where

$$a_0 = \frac{1}{l} \int_{c}^{c+2l} f(x)\,dx$$

$$a_n = \frac{1}{l} \int_{c}^{c+2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{1}{l} \int_{0}^{c+2l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

* If $f(x)$ is even function in $(-l, l)$ then the Fourier series for the f^n is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

where $a_0 = \frac{2}{l} \int_{0}^{l} f(x)\,dx$, $a_n = \frac{2}{l} \int_{0}^{l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$

$$b_n = 0$$

* If $f(x)$ is odd function in $(-l, l)$ then the fourier series for the function $f(x)$ is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$

* The half-range fourier cosine series for the function $f(x)$ in $(0, l)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

where

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

* The half-range fourier sine series for the function $f(x)$ in $(0, l)$ is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

Question The fourier series for $f(x) = x$ in $(-\pi, \pi)$ is.

$f(x) = x$ is off function

$a_0 = 0, \quad a_n = 0$

$$\therefore \quad b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{\pi}\int_0^\pi x\sin\left(\frac{n\pi x}{\pi}\right)dx$$

$$= \frac{2}{\pi}\int_0^\pi x\sin(nx)\,dx$$

$D \to \oplus x \quad \ominus 1 \quad \oplus 0$

$I \to \sin nx \quad \frac{-\cos nx}{n} \quad -\frac{\sin nx}{n^2}$

$$= \frac{2}{\pi}\left[-\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^\pi$$

$$= \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n} + \frac{\sin n\pi}{n^2} + 0 - \frac{\sin n(0)}{n^2}\right]$$

$$b_n = \frac{-2\cos n\pi}{n} = \frac{(-1)^n\cdot(-2)}{n} = \frac{2(-1)^n(-1)}{n}$$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

the Fourier series for the Function is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$x = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n} \sin\left(\frac{n\pi x}{\pi}\right)$$

$$x = \sum_{n=1}^{\infty} \frac{2}{n} (-1)^{n+1} \sin(nx)$$

$$x = 2\left\{\frac{1}{1}\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x - - -\right\}$$

Question The Fourier series for for $f(x) = x^2$ in $(-\pi, \pi)$ is

$f(x) = x^2$ is even fn $b_n = 0$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$a_0 = \frac{2}{\pi}\int_0^\pi x^2\,dx \Rightarrow a_0 = \frac{2\pi^3}{3}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{2}{\pi}\int_0^\pi x^2 \cos\left(\frac{n\pi x}{\pi}\right)dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos(nx)\, dx$$

$$D \to \oplus x^2 \quad \ominus 2x \quad \oplus 2 \quad \ominus 0$$

$$I \to \cos(nx) \quad \frac{\sin nx}{n} \quad \frac{-\cos nx}{n^2} \quad \frac{-\sin nx}{n^3}$$

$$a_n = \frac{2}{\pi}\left\{ x^2 \frac{\sin nx}{n} + 2x \frac{\cos nx}{n^2} - \frac{2\sin nx}{n^3} \right\}\Bigg|_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left(\pi \frac{\sin n\pi}{n} + 2\pi \frac{\cos n\pi}{n^2} - \frac{2\sin n\pi}{n^3}\right) - (0+0+0)\}$$

$$a_n = \frac{2}{\pi}\left(0 + \frac{2\pi}{n^2}(-1)^n + 0\right)$$

$$a_n = \frac{4}{n^2}(-1)^n$$

the fourier series for $f(x) = x^2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$x^2 = \frac{2\pi^2/3}{2} + \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n \cos(nx)$$

$$x^2 = \frac{\pi^3}{3} + 4\left\{ -\frac{1}{1^2}\cos x + \frac{1}{2^2}\cos 2x - \frac{1}{3^2}\cos 3x + \cdots \right\}$$