Product of Algebraic Expressions

Exercise 23:

Solution 1:

Monomials	Binomials	Trinomials
$7x$	$3a + 4b$	$x^2 - 5x + 6$
-4	$5m - 7$	$y^3 - 2y^2 + 3$
	$9 - 4x$	$p^2 - 2pq + 6q^2$

Solution 2:

1. $2(3 + 5) = 2 \times \underline{\mathbf{3}} + 2 \times \underline{\mathbf{5}}$
2. $6(\underline{\mathbf{7}} + \underline{\mathbf{4}}) = 6 \times 7 + 6 \times 4$
3. $10\ (\underline{\mathbf{9}} - \underline{\mathbf{7}}) = 10 \times 9 - 10 \times 7$

Solution 3:

The pairs of like terms are:
7y, 2y
5ab, 9ab
$11ax^2$, $10x^2a$

Solution 4:

1.

$$\begin{array}{rrrrrrr} & & p^2 & - & 2p & + & 7 \\ + & p^3 & + & 2p^2 & & + & 3 \\ \hline & p^3 & + & 3p^2 & - & 2p & + & 10 \end{array}$$

2.

$$\begin{array}{rrrrrrr} & & m^2 & - & 3m & - & 9 \\ + & m^3 & & + & 5m & - & 4 \\ \hline & m^3 & + & m^2 & + & 2m & - & 13 \end{array}$$

3.

$$\begin{array}{rrrrrrrr} & 2x^3 & - & 3x^2y & + & 5xy^2 & - & 4y^3 \\ (+) & -x^3 & + & 2x^2y & - & 4xy^2 & + & 3y^3 \\ \hline & x^3 & - & x^2y & + & xy^2 & - & y^3 \end{array}$$

Solution 5:

1. $4mn^2 - 2mn^2 + mn^2$
   $= (4 - 2 + 1)mn^2$
   $= 3mn^2$

2. $(2y^3 - 5y^2 + 9y) + (y^2 - 3y + 8)$
   $= 2y^3 - 5y^2 + 9y + y^2 - 3y + 8$
   $= 2y^3 - 5y^2 + y^2 + 9y - 3y + 8$
   $= 2y^3 - 4y^2 + 6y + 8$

3. $(6p^2 - pq + 5q^2) - (3p^2 + 2pq - q^2)$
   $= 6p^2 - pq + 5q^2 - 3p^2 - 2pq + q^2$
   $= 6p^2 - 3p^2 - pq - 2pq + 5q^2 + q^2$
   $= 3p^2 - 3pq + 6q^2$

Exercise 24:

Solution 1:

1. $4x \times 6y = (4 \times 6) \times (x \times y) = 24xy$
2. $3a \times 8b = (3 \times 8) \times (a \times b) = 24ab$
3. $2x^2 \times 7x = (2 \times 7) \times (x^2 \times x) = 14x^3$
4. $7x^3 \times 9 = (7 \times 9) \times x^3 = 63x^3$
5. $8 \times \frac{1}{4}x = \left(8 \times \frac{1}{4}\right) \times x = 2x$
6. $\frac{1}{2}x^2 \times \frac{1}{3}y^2 = \left(\frac{1}{2} \times \frac{1}{3}\right) \times (x^2 \times y^2) = \frac{1}{6}x^2y^2$
7. $3ax \times 7a^2x^2 = (3 \times 7) \times (a \times a^2) \times (x \times x^2) = 21a^3x^3$
8. $\frac{2}{3}p^2 \times \frac{9}{4}q^3 = \left(\frac{2}{3} \times \frac{9}{4}\right) \times (p^2 \times q^3) = \frac{3}{2}p^2q^3$
9. $7qy \times 4rx = (7 \times 4) \times (qy \times rx) = 28qyrx$
10. $3x^2y \times \frac{5}{6}x^2y = \left(3 \times \frac{5}{6}\right) \times (x^2y \times x^2y) = \frac{5}{2}x^4y^2$

Exercise 25:

Solution 1:

1. $4 \times (x + 2)$

$= (4 \times x) + (4 \times 2)$
$= 4x + 8$

2. $7 \times (p + q)$

$= (7 \times p) + (7 \times q)$
$= 7p + 7q$

3. $m(x + y)$

$= (m \times x) + (m \times y)$
$= mx + my$

4. $3p \times (p^2 - 4)$

$= 3p \times p^2 - (3p \times 4)$
$= 3p^3 - 12p$

5. $(2x^2 - 3x) \times 3xy$

$= (2x^2 \times 3xy) - (3x \times 3xy)$
$= 6x^3y - 9x^2y$

Exercise 26:

Solution 1(1):

Horizontal arrangement:

$6 \times (y - 3)$
$= 6 \times y - 6 \times 3$
$= 6y - 18$

Vertical arrangement:

$$\begin{array}{rr} & y - 3 \\ \times & 6 \\ \hline & 6y - 18 \\ \hline \end{array}$$

Solution 1(2):

Horizontal arrangement:

$1 \times (a^2 - a)$

$= 1 \times a^2 - 1 \times a$

$= a^2 - a$

Vertical arrangement:

$$\begin{array}{rrr} & a^2 & - & a \\ \times & & & 1 \\ \hline & a^2 & - & a \\ \hline \end{array}$$

Solution 1(3):

Horizontal arrangement:

$(3x^2 + 7x) \times 2x$

$= (3x^2 \times 2x) + (7x \times 2x)$

$= 6x^3 + 14x^2$

Vertical arrangement:

$$\begin{array}{rrrr} & 3x^2 & + & 7x \\ \times & & & 2x \\ \hline & 6x^3 & + & 14x^2 \\ \hline \end{array}$$

Solution 1(4):

Horizontal arrangement:

$(5a^2 - 3b^2) \times 4ab$

$= (5a^2 \times 4ab) - (3b^2 \times 4ab)$

$= 20a^3b - 12ab^3$

Vertical arrangement:

$$\begin{array}{rrrr} & 5a^2 & - & 3b^2 \\ \times & & & 4ab \\ \hline & 20a^3b & - & 12ab^3 \\ \hline \end{array}$$

Solution 1(5):

Horizontal arrangement:

$(5m^2n - n^2) \times 4mn$

$= (5m^2n \times 4mn) - (n^2 \times 4mn)$

$= 20m^3n^2 - 4mn^3$

Vertical arrangement:

$$\begin{array}{rcl} & 5m^2n & - \quad n^2 \\ \times & & 4mn \\ \hline & 20m^3n^2 & - \quad 4mn^3 \end{array}$$

Exercise 27:

Solution 1(1):

Horizontal arrangement :

$(x + 2) \times (x + 3)$

$= x \times (x + 3) + 2 \times (x + 3)$

$= x^2 + 3x + 2x + 6$

$= x^2 + 5x + 6$

Vertical arrangement:

$$\begin{array}{rcl} & x & + \quad 2 \\ x & x & + \quad 3 \\ \hline & 3x & + \quad 6 \end{array}$$

$$+$$

$$\begin{array}{rcl} x^2 & + & 2x \\ \hline x^2 & + & 5x + 6 \end{array}$$

Solution 1(2):

Horizontal arrangement :

$(y + 4) \times (y + 1)$

$= y \times (y + 1) + 4 \times (y + 1)$

$= y^2 + y + 4y + 4$

$= y^2 + 5y + 4$

Vertical arrangement:

$$\begin{array}{rrrr} & y & + & 4 \\ \times & y & + & 1 \\ \hline & y & + & 4 \\ + & & & \\ \hline y^2 & + & 4y & \\ \hline y^2 & + & 5y & + 4 \end{array}$$

Solution 1(3):

Horizontal arrangement:

$(p + 3) \times (p - 3)$

$= p \times (p - 3) + 3 \times (p - 3)$

$= p^2 - 3p + 3p - 9$

$= p^2 - 9$

Vertical arrangement:

$$\begin{array}{rrrr} & p & + & 3 \\ \times & p & - & 3 \\ \hline & -3p & - & 9 \\ + & & & \\ \hline p^2 & + & 3p & \\ \hline p^2 & & & - 9 \end{array}$$

Solution 1(4):

Horizontal arrangement:

$(a+b)\times(b+d)$

$= a\times(b+d)+b\times(b+d)$

$= ab+ad+b^2+bd$

Vertical arrangement:

$$\begin{array}{rrrrr} & & a & + & b \\ & & b & + & d \\ \hline & ad & & + & bd \\ + & & & & \\ ab & + & b^2 & & \\ \hline ab & + & ad & + & b^2 + bd \end{array}$$

Solution 1(5):

Horizontal arrangement:

$(m+n)(m-n)$

$= m\times(m-n)+n\times(m-n)$

$= m^2-mn+mn-n^2$

$= m^2-n^2$

Vertical arrangement:

$$\begin{array}{rrrrr} & & m & + & n \\ & \times & m & - & n \\ \hline & - & mn & - & n^2 \\ + & & & & \\ m^2 & + & mn & & \\ \hline m^2 & & & - & n^2 \end{array}$$

Solution 1(6):

Horizontal arrangement:

$(4-p)(5+p^2)$

$= 4 \times (5+p^2) - p \times (5+p^2)$

$= 20 + 4p^2 - 5p - p^3$

Vertical arrangement:

$$\begin{array}{rrrrr} & & 4 & - & p \\ & & 5 & + & p^2 \\ \hline & 4p^2 & & - & p^3 \\ + & & & & \\ 20 & + & & - & 5p \\ \hline 20 & + & 4p^2 & - & 5p - p^3 \end{array}$$

Solution 1(7):

Horizontal arrangement:

$(2x+y)(3x+2y)$

$= 2x \times (3x+2y) + y \times (3x+2y)$

$= 6x^2 + 4xy + 3xy + 2y^2$

$= 6x^2 + 7xy + 2y^2$

Vertical arrangement:

$$\begin{array}{rrrrr} & & 2x & + & y \\ & x & 3x & + & 2y \\ \hline & & 4xy & + & 2y^2 \\ + & & & & \\ & 6x^2 & + & 3xy & \\ \hline & 6x^2 & + & 7xy & + 2y^2 \end{array}$$

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Solution 1(8):

Horizontal arrangement:

$(3p^2 + q)(4p - 2q)$

$= 3p^2 \times (4p - 2q) + q \times (4p - 2q)$

$= 12p^3 - 6p^2q + 4pq - 2q^2$

Vertical arrangement:

$$\begin{array}{rrrrr} & & 3p^2 & + & q \\ & & 4p & - & 2q \\ \hline & - & 6p^2q & & - \ 2q^2 \\ + & & & & \\ 12p^3 & & + \ 4pq & & \\ \hline 12p^3 & - & 6p^2q & + & 4pq \ - \ 2q^2 \end{array}$$

Solution 1(9):

Horizontal arrangement:

$(2 + y)(3 + y)$

$= 2 \times (3 + y) + y \times (3 + y)$

$= 6 + 2y + 3y + y^2$

$= 6 + 5y + y^2$

Vertical arrangement:

$$\begin{array}{rrrr} & 2 & + & y \\ \times & 3 & + & y \\ \hline & 2y & + & y^2 \\ + & & & \\ & 6 & + & 3y \\ \hline & 6 & + & 5y \ + \ y^2 \end{array}$$

Solution 1(10):

Horizontal arrangement:

$(a+4b)(a-4b)$

$= a \times (a-4b) + 4b \times (a-4b)$

$= a^2 - 4ab + 4ab - 16b^2$

$= a^2 - 16b^2$

Vertical arrangement:

$$\begin{array}{rrrrr} & & a & + & 4b \\ & \times & a & - & 4b \\ \hline & - & 4ab & - & 16b^2 \\ + & & & & \\ & a^2 & + & 4ab & \\ \hline & a^2 & & - & 16b^2 \end{array}$$

Solution 1(11):

Horizontal arrangement :

$(10m-7n)(10m+7n)$

$= 10m \times (10m+7n) - 7n \times (10m+7n)$

$= 100m^2 + 70mn - 70mn - 49n^2$

$= 100m^2 - 49n^2$

Vertical arrangement:

$$\begin{array}{rrrrr} & & 10m & - & 7n \\ & \times & 10m & + & 7n \\ \hline & & 70mn & - & 49n^2 \\ + & & & & \\ & 100m^2 & - & 70mn & \\ \hline & 100m^2 & & - & 49n^2 \end{array}$$

Solution 1(12):

Horizontal arrangement:

$(3p - 5q)(3p + 5q)$

$= 3p \times (3p + 5q) - 5q \times (3p + 5q)$

$= 9p^2 + 15pq - 15pq - 25q^2$

$= 9p^2 - 25q^2$

Vertical arrangement:

$$\begin{array}{rrrr} & 3p & - & 5q \\ \times & 3p & + & 5q \\ \hline & 15pq & - & 25q^2 \\ + & & & \\ & 9p^2 & - & 15pq \\ \hline & 9p^2 & & - \ 25q^2 \end{array}$$

Exercise 28:

Solution 1(1):

$4(y^2 - 2y + 7)$

$= (4 \times y^2) - (4 \times 2y) + (4 \times 7)$

$= 4y^2 - 8y + 28$

Solution 1(2):

$6(2a^2 - 3a - 5)$

$= (6 \times 2a^2) - (6 \times 3a) - (6 \times 5)$

$= 12a^2 - 18a - 30$

Solution 1(3):

$7x(3x^2 - 5x + 8)$

$= (7x \times 3x^2) - (7x \times 5x) + (7x \times 8)$

$= 21x^3 - 35x^2 + 56x$

Solution 1(4):

$(m^2 – 4m + 6)2m$
$= (m^2 \times 2m) – (4m \times 2m) + (6 \times 2m)$
$= 2m^3 – 8m^2 + 12m$

Solution 1(5):

$(4p^2 + p + 3)9p$
$= (4p^2 \times 9p) + (p \times 9p) + (3 \times 9p)$
$= 36p^3 + 9p^2 + 27p$

Solution 1(6):

$3a(a^2 + 4a – 6)$
$= (3a \times a^2) + (3a \times 4a) – (3a \times 6)$
$= 3a^3 + 12a^2 – 18a$

Solution 1(7):

$5q(2a + 3b – 4c)$
$= (5q \times 2a) + (5q \times 3b) – (5q \times 4c)$
$= 10aq + 15bq – 20cq$

Solution 1(8):

$4m(3mn – 2n^2 + 5)$
$= (4m \times 3mn) – (4m \times 2n^2) + (4m \times 5)$
$= 12m^2n – 8mn^2 + 20m$

Solution 1(9):

$(x^2y^2 + xy + 1)xy$
$= (x^2y^2 \times xy) + (xy \times xy) + (1\times xy)$
$= x^3y^3 + x^2y^2 + xy$

Solution 1(10):

$11p^2q(p^2 – 3pq + 8q^2)$
$= (11p^2q \times p^2) – (11p^2q \times 3pq) + (11p^2q \times 8q^2)$
$= 11p^4q – 33p^3q^2 + 88p^2q^3$

Exercise 29:

Solution 1(1):

$(3y - 2)(y^2 - 5y + 7)$
$= 3y \times (y^2 - 5y + 7) - 2 \times (y^2 - 5y + 7)$
$= (3y \times y^2 - 3y \times 5y + 3y \times 7) - (2 \times y^2 - 2 \times 5y + 2 \times 7)$
$= 3y^3 - 15y^2 + 21y - 2y^2 + 10y - 14$
$= 3y^3 - 17y^2 + 31y - 14$

Solution 1(2):

$(a^2 - ab + b^2)(a + b)$
$= (a^2 - ab + b^2) \times a + (a^2 - ab + b^2) \times b$
$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$
$= a^3 + b^3$

Solution 1(3):

$(2p - 5)(3p^2 + p - 2)$
$= 2p \times (3p^2 + p - 2) - 5 \times (3p^2 + p - 2)$
$= 2p \times 3p^2 + 2p \times p - 2p \times 2 - 5 \times 3p^2 - 5 \times p + 5 \times 2$
$= 6p^3 + 2p^2 - 4p - 15p^2 - 5p + 10$
$= 6p^3 - 13p^2 - 9p + 10$

Solution 1(4):

$(m^2 - 3mn + n^2)(5mn + 4)$
$= (m^2 - 3mn + n^2) \times 5mn + (m^2 - 3mn + n^2) \times 4$
$= m^2 \times 5mn - 3mn \times 5mn + n^2 \times 5mn + m^2 \times 4 - 3mn \times 4 + n^2 \times 4$
$= 5m^3n - 15m^2n^2 + 5mn^3 + 4m^2 - 12mn + 4n^2$

Solution 1(5):

$(9y - 4x)(8y^2 - 5xy + 3x^2)$
$= 9y \times (8y^2 - 5xy + 3x^2) - 4x \times (8y^2 - 5xy + 3x^2)$
$= 9y \times 8y^2 - 9y \times 5xy + 9y \times 3x^2 - 4x \times 8y^2 + 4x \times 5xy - 4x \times 3x^2$
$= 72y^3 - 45xy^2 + 27x^2y - 32xy^2 + 20x^2y - 12x^3$
$= 72y^3 - 77xy^2 + 47x^2y - 12x^3$

Solution 1(6):

$(4m^2 + 6mn + 9n^2)(2m – 3n)$
$= (4m^2 \times 2m + 6mn \times 2m + 9n^2 \times 2m) – (4m^2 \times 3n + 6mn \times 3n + 9n^2 \times 3n)$
$= 8m^3 + 12m^2n + 18mn^2 – 12m^2n – 18mn^2 – 27n^3$
$= 8m^3 – 27n^3$

Solution 1(7):

$(p^2 + 2q)(7p^2 – pq – 11)$
$= p^2 \times (7p^2 – pq – 11) + 2q \times (7p^2 – pq – 11)$
$= (p^2 \times 7p^2 – p^2 \times pq – p^2 \times 11) + (2q \times 7p^2 – 2q \times pq – 2q \times 11)$
$= 7p^4 – p^3q – 11p^2 + 14p^2q – 2pq^2 – 22q$

Solution 1(8):

$(ab + a^2b + ab^2)(a + b)$
$= (ab + a^2b + ab^2) \times a + (ab + a^2b + ab^2) \times b$
$= (ab \times a + a^2b \times a + ab^2 \times a) + (ab \times b + a^2b \times b + ab^2 \times b)$
$= a^2b + a^3b + a^2b^2 + ab^2 + a^2b^2 + ab^3$
$= a^2b + a^3b + 2a^2b^2 + ab^2 + ab^3$

Solution 1(9):

$(x + 2y)(x^2 – 2xy + 4y^2)$
$= x \times (x^2 – 2xy + 4y^2) + 2y \times (x^2 – 2xy + 4y^2)$
$= x \times x^2 – x \times 2xy + x \times 4y^2 + 2y \times x^2 – 2y \times 2xy + 2y \times 4y^2$
$= x^3 – 2x^2y + 4xy^2 + 2x^2y – 4xy^2 + 8y^3$
$= x^3 + 8y^3$

Solution 1(10):

$(3a – 2b)(9a^2 + 6ab + 4b^2)$
$= 3a \times (9a^2 + 6ab + 4b^2) – 2b \times (9a^2 + 6ab + 4b^2)$
$= 3a \times 9a^2 + 3a \times 6ab + 3a \times 4b^2 – 2b \times 9a^2 – 2b \times 6ab – 2b \times 4b^2)$
$= 27a^3 + 18a^2b + 12ab^2 – 18a^2b – 12ab^2 – 8b^3$
$= 27a^3 – 8b^3$