EXERCISE 5.3

Find $\frac{dy}{dx}$ in the following :

Q No 1. $2x+3y = \sin x$.

Sol.

$2x+3y = \sin x$

Differentiating both sides w.r.t x, we get

$2+3\frac{dy}{dx} = \cos x \Rightarrow 3\frac{dy}{dx} = \cos x - 2$

$\Rightarrow \frac{dy}{dx} = \frac{1}{3}[\cos x - 2]$.

Q No. 2 $2x+3y = \sin y$

Sol. Differentiating both sides w.r.t x we get

$2+3\frac{dy}{dx} = \cos y \cdot \frac{dy}{dx}$

$\Rightarrow 2 = (\cos y - 3)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{2}{\cos y - 3}$

Q No. 3 $ax+by^2 = \cos y$

Sol Differentiating both sides w.r.t x we get

$a + b.2y\frac{dy}{dx} = -\sin y \cdot \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}[2by+\sin y] = -a. \Rightarrow \frac{dy}{dx} = \frac{-a}{2by+\sin y}$.

Q No. 4. $xy+y^2 = \tan x + y$.

Differentiating both sides w.r.t x, we get

$x\cdot\frac{dy}{dx} + y.1 + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$. [uv. form]

$\Rightarrow (x+2y-1)\frac{dy}{dx} = \sec^2 x - y$.

$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x - y}{x+2y-1}$.

Q No 5 $x^2+xy+y^2 = 100$.

Differentiating both sides w.r.t x, we get

$2x + x\cdot\frac{dy}{dx} + y.1 + 2y\frac{dy}{dx} = 0$

$(x+2y)\frac{dy}{dx} = -(2x+y)$

$\Rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$

QNo. 6: $x^3+x^2y+xy^2+y^3=81$

Differenting both sides w.r.t x, we get.

$$3x^2+\left[x^2\frac{dy}{dx}+y\cdot 2x\right]+\left[x\cdot 2y\frac{dy}{dx}+y^2\cdot 1\right]+3y^2\cdot\frac{dy}{dx}=0$$

$$\Rightarrow (x^2+2xy+3y^2)\frac{dy}{dx}=-(3x^2+2xy+y^2)$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(3x^2+2xy+y^2)}{(x^2+2xy+3y^2)}$$

QNo. 7: $\sin^2 y+\cos xy=k.$

Differentiating w.r.t x, we get

$$2\cdot\sin y\cdot\cos y\cdot\frac{dy}{dx}+(-\sin xy)\left[x\cdot\frac{dy}{dx}+y\cdot 1\right]=0$$

$$\Rightarrow \sin 2y\frac{dy}{dx}-x\cdot\sin(xy)\frac{dy}{dx}-y\sin(xy)=0$$

$$\Rightarrow \frac{dy}{dx}\left[\sin 2y-x\cdot\sin(xy)\right]=y\sin(xy)$$

$$\Rightarrow \frac{dy}{dx}=\frac{y\sin(xy)}{\sin 2y-x\cdot\sin(xy)}$$

QNo. 8: $\sin^2 x+\cos^2 y=1$

Differentiating both sides w.r.t x, we get.

$$2\cdot\sin x\cos x+2\cos y(-\sin y)\cdot\frac{dy}{dx}=0$$

$$\Rightarrow \sin 2x-\sin 2y\frac{dy}{dx}=0$$

$$\Rightarrow \sin 2x=\sin 2y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin 2x}{\sin 2y}.$$

QNo. 9 $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\cdot\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)\qquad\left[\because \frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}\right]$$

$$=\frac{1}{\sqrt{\frac{(1+x^2)^2-4x^2}{(1+x^2)^2}}}\times\frac{(1+x^2)(2)-(2x)(0+2x)}{(1+x^2)^2}$$

$$= \frac{1+x^2}{\sqrt{(1)^2+(x^2)^2+2x^2-4x^2}} \times \frac{2+2x^2-4x^2}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\cdot 2(1-x^2)}{\sqrt{(1-x^2)^2}\,(1+x^2)^2} = \frac{2\cdot(1+x^2)(1-x^2)}{(1-x^2)(1+x^2)^2} = \frac{2}{1+x^2}$$

Alternatively

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Put $x = \tan\theta \Rightarrow \tan^{-1}x = \theta$.

$$\therefore \; y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta)$$

$$= 2\theta = 2\tan^{-1}x$$

$$\therefore \; \frac{dy}{dx} = \frac{2}{1+x^2} \quad \left[\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right]$$

QNo. 10:

$$y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \; -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Put $x = \tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

As $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$$\Rightarrow -\frac{1}{\sqrt{3}} < \tan\theta < \frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{6} < \theta < \frac{\pi}{6} \Rightarrow -\frac{\pi}{2} < 3\theta < \frac{\pi}{2}$$

$$\therefore \; y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta)$$

$$= 3\theta = 3\tan^{-1}x.$$

$$\therefore \; \frac{dy}{dx} = 3\cdot\frac{d}{dx}(\tan^{-1}x) = 3\times\frac{1}{1+x^2} = \frac{3}{1+x^2}.$$

QNo 11

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right); \; 0 < x < 1$$

Put $x = \tan\theta$.

As $0 < x < 1 \Rightarrow 0 < \tan\theta < 1 \Rightarrow 0 < \theta < \frac{\pi}{4} \Rightarrow 0 < 2\theta < \frac{\pi}{2}$

$$\therefore \; y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$$

$$\therefore \; \frac{dy}{dx} = \frac{2}{1+x^2}.$$

Q No 12. $y = \sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$; $0 < x < 1$

$$\Rightarrow \quad y = \frac{\pi}{2} - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \qquad \left[\because \sin^{-1}x + \cos^{-1}x = \pi/2\right]$$

$$\Rightarrow \quad y = \frac{\pi}{2} - 2\tan^{-1}(x) \quad [\text{As in Q.No 11}]$$

$$\Rightarrow \quad \frac{dy}{dx} = 0 - 2\cdot\frac{1}{1+x^2} = -\frac{2}{1+x^2}$$

Q No. 13 $y = \cos^{-1}\left(\dfrac{2x}{1+x^2}\right)$; $-1 < x < 1$

$$\Rightarrow y = \frac{\pi}{2} - \sin^{-1}\left(\frac{2x}{1+x^2}\right) \qquad \left[\because \sin^{-1}x + \cos^{-1}x = \pi/2\right]$$

$$\Rightarrow y = \frac{\pi}{2} - 2\tan^{-1}(x) \quad [\text{As in Q No. 9}]$$

$\because \cancel{-1} < x < 1$

$$\therefore \frac{dy}{dx} = 0 - 2\cdot\frac{1}{1+x^2} = -\frac{2}{1+x^2}$$

Q No. 14. $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Sol: Let $x = \sin\theta$, $-\pi/2 < \theta < \pi/2$

As $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}} \Rightarrow -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}} \Rightarrow -\frac{\pi}{4} < \theta < \pi/4$

$$\Rightarrow \quad -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}$$

$$\therefore y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right) = \sin^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$$

$$= \sin^{-1}\left(2\sin\theta\cdot\cos\theta\right) = \sin^{-1}(\sin 2\theta) = 2\theta.$$

$$\Rightarrow y = 2\cdot\sin^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

Q No 15.

$y = \sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$; $0 < x < \frac{1}{\sqrt{2}}$

Sol

Let $\cos^{-1}x = \theta$ ie $x = \cos\theta$, $0 \leq \theta \leq \pi$

As $0 < x < \frac{1}{\sqrt{2}}$

$\therefore \quad 0 < \cos\theta < \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{\pi}{2} > \theta > \frac{\pi}{4} \Rightarrow \pi > 2\theta > \pi/2$$

$$\therefore y = \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$$

$$= \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta), \ \pi/2 < \theta < \pi$$

$$\Rightarrow y = 2\theta = 2\cos^{-1}x$$

$$\therefore \frac{dy}{dx} = 2 \cdot \frac{d}{dx}(\cos^{-1}x) = 2\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$= -\frac{2}{\sqrt{1-x^2}}$$

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