1

Multiple-Choice Questions: Type 1

*In this chapter, each question has **four** options (a, b, c and d), out of which **only one** option is correct.*

1.1 General Physics

1. A vernier callipers has 1-mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main-scale divisions. For this vernier callipers, the least count is

(a) 0.02 mm (b) 0.05 mm

(c) 0.1 mm (d) 0.2 mm

2. A quantity X is given by $\varepsilon_0 L \dfrac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of vacuum, L is the length, ΔV is the potential difference and Δt is the time interval. The dimensional formula for X is the same as that for

(a) resistance (b) charge

(c) voltage (d) current

3. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge whose pitch is 0.5 mm, with 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of this ball has a relative error of 2%, the relative percentage error in the density is

(a) 0.9 (b) 2.4

(c) 3.1 (d) 4.2

4. In the determination of Young modulus $\left(Y = \dfrac{4MgL}{\pi d^2 l}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer respectively. They have equal pitches of 0.5 mm. The number of divisions on each of their circular scales is 100. The maximum probable error in the measurement of Y arises due to the error in l and d both.

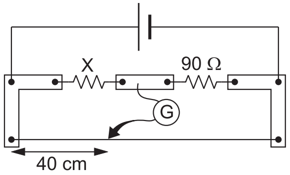
(a) The error in the measurement of d and l are equal.

(b) The error in the measurement of d is twice the error in the measurement of l.

(c) The error in the measurement of l is twice the error in the measurement of d.

(d) The error in the measurement of d is four times the error in the measurement of l.

5. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance (R) of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance (X) is



(a) (60 ± 0.15) Ω (b) (135 ± 0.56) Ω

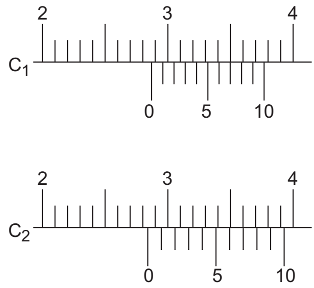
(c) (60 ± 0.25) Ω (d) (135 ± 0.23) Ω

6. The diameter of a cylinder is measured using a vernier callipers with no zero errors. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 divisions, equivalent to 2.45 cm. The 24th division of the vernier scale exactly coincides with one of the main-scale divisions. The diameter of the cylinder is

(a) 5.112 cm (b) 5.124 cm

(c) 5.136 cm (d) 5.148 cm

7. There are two vernier callipers, both of which have 1 cm divided into 10 equal divisions on the main scale. The vernier scale of one of the callipers (C_1) has 10 equal divisions, corresponding to 9 main-scale divisions. The vernier scale of the other calliper (C_2) has 10 equal divisions that correspond to 11 main-scale divisions. The readings of the two callipers are shown in the figure. The measured values by the callipers C_1 and C_2 are respectively



(a) 2.87 cm and 2.86 cm (b) 2.87 cm and 2.87 cm
(c) 2.87 cm and 2.83 cm (d) 2.85 cm and 2.82 cm

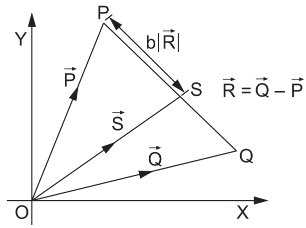
8. Consider an expanding sphere of instantaneous radius R whose mass (M) remains constant. The expansion is such that the instantaneous density (ρ) remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is also constant. The velocity (v) of any point on the surface of the expanding sphere is proportional to

(a) $R^{2/3}$ (b) R
(c) R^3 (d) $\frac{1}{R}$

9. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact of the stone with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ second, and he measures the depth of the well (L) to be 20 metres. Take the acceleration due to gravity (g) as 10 m s^{-2} and the velocity of sound as 300 m s^{-1}. Then the fractional error $\left(\frac{\delta L}{L}\right)$ in the measurement is close to

(a) 0.2% (b) 5%
(c) 1% (d) 3%

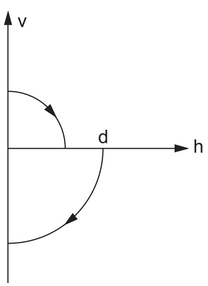
10. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be a point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among $\vec{P}, \vec{Q}$ and $\vec{S}$ is



(a) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$ (b) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

(c) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ (d) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

11. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting the subsequent motion and the air resistance, how its velocity (v) varies with the height (h) above the ground?

(a)


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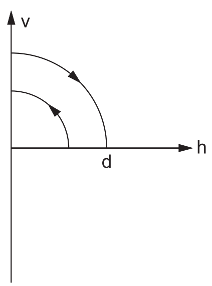

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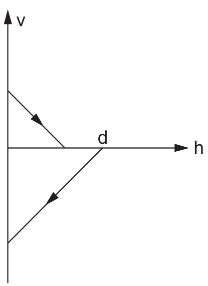
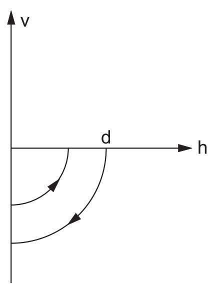
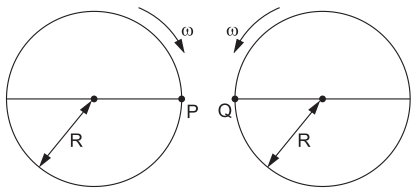
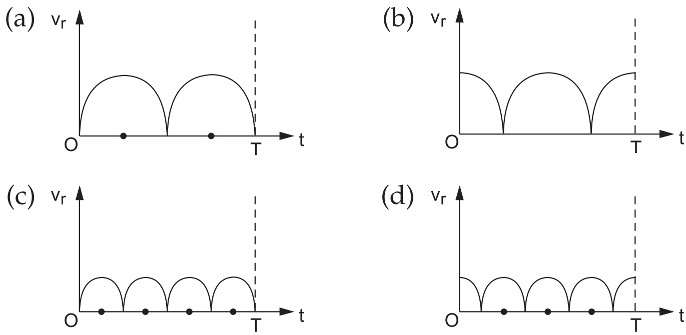


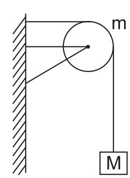
(d)


12. Two identical discs of the same radius R are rotating about their axes in opposite directions at the same angular speed ω. The discs are in the same horizontal plane. At the time $t = 0$, the points P and Q are facing each other, as shown in the figure. The relative speed between the two points P and Q is v_r. In each period of rotation (T) of the discs, v_r as a function of time is best represented by





13. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M, as shown in the figure. The force on the pulley by the clamp is given by

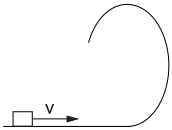


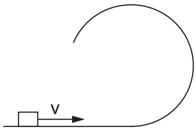
(a) $\sqrt{2}Mg$
(b) $\sqrt{2}mg$
(c) $\sqrt{(M+m)^2+m^2} \cdot g$
(d) $\sqrt{(M+m)^2+M^2} \cdot g$

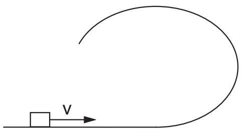
14. Two particles of masses m_1 and m_2 in projectile motion have the velocities $\vec{v_1}$ and $\vec{v_2}$ respectively at the time $t = 0$. They collide at $t = t_0$. Their velocities become $\vec{v_1'}$ and $\vec{v_2'}$ at $t = 2t_0$ while still moving in air. The value of $\left|\left(m_1\vec{v_1'} + m_2\vec{v_2'}\right) - \left(m_1\vec{v_1} + m_2\vec{v_2}\right)\right|$ is

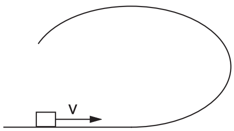
(a) zero
(b) $(m_1 + m_2)gt_0$
(c) $2(m_1 + m_2)gt_0$
(d) $\frac{1}{2}(m_1 + m_2)gt_0$

15. A small block is shot into each of the four tracks shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is the maximum in the case

(a) 

(b) 

(c) 

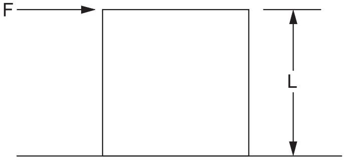
(d) 

16. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ and the gravity is neglected, the time after which the bead starts slipping is

(a) $\sqrt{\dfrac{\mu}{\alpha}}$ (b) $\dfrac{\mu}{\sqrt{\alpha}}$

(c) $\dfrac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

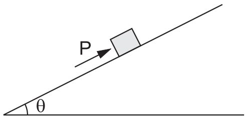
17. A cubical block of side length L rests on a rough horizontal surface having the coefficient of friction μ. A horizontal force F is applied on the block, as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



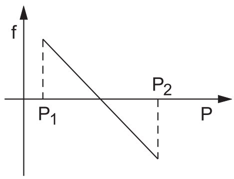
(a) infinitesimal (b) $\dfrac{mg}{4}$

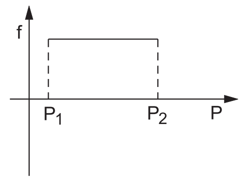
(c) $\dfrac{mg}{2}$ (d) $mg(1-\mu)$

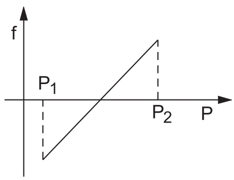
18. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ, where $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken

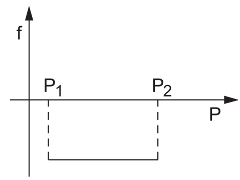


to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like

(a) 

(b) 

(c) 

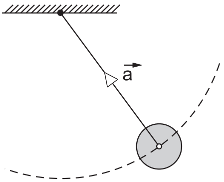
(d) 

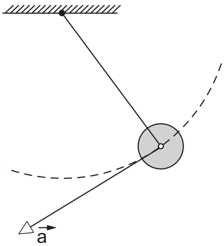
19. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth vertical wall of height h ($< l$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall, and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are respectively (when $g = 10\ \text{m s}^{-2}$)

(a) $\dfrac{\sqrt{3}}{16}$ and $\dfrac{16\sqrt{3}}{3}$ N (b) $\dfrac{3}{16}$ and $\dfrac{16\sqrt{3}}{3}$ N

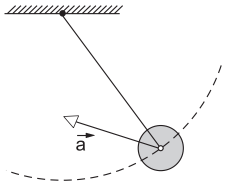
(c) $\dfrac{3\sqrt{3}}{16}$ and $\dfrac{8\sqrt{3}}{3}$ N (d) $\dfrac{3\sqrt{3}}{16}$ and $\dfrac{16\sqrt{3}}{3}$ N

20. A simple pendulum is oscillating without damping. When the displacement of the bob is less than the maximum, its acceleration vector $\vec{a}$ is correctly shown in the figure

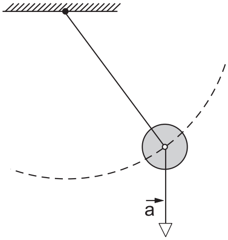
(a) 

(b) 

(c)



(d)

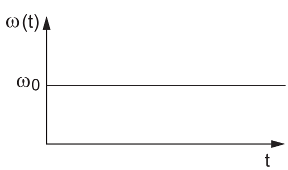


21. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

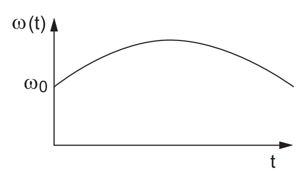
(a) up the incline while ascending and down the incline while descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending

22. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as

(a)



(b)



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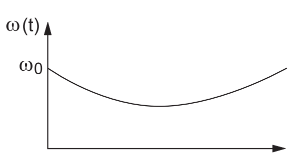
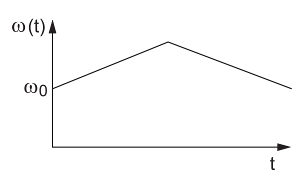


(d)



23. A ball of mass m = 0.5 kg is attached to the end of a string of length L = 0.5 m. The ball is rotated on a horizontal circular path about the vertical axis. The maximum tension that the string can bear is

324 N. The maximum possible value of the angular velocity of the ball is

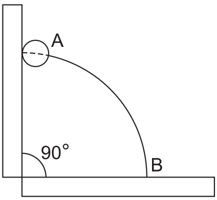
(a) 9 rad s^{-1} (b) 18 rad s^{-1}
(c) 27 rad s^{-1} (d) 36 rad s^{-1}

24. The work done on a particle of mass m by a force $K\left\{\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right\}$ when the particle is taken from the point $(a, 0)$ to another point $(0, a)$ along a circular path of radius a about the origin in the xy-plane is

(a) $\frac{2\pi K}{a}$ (b) $\frac{\pi K}{a}$

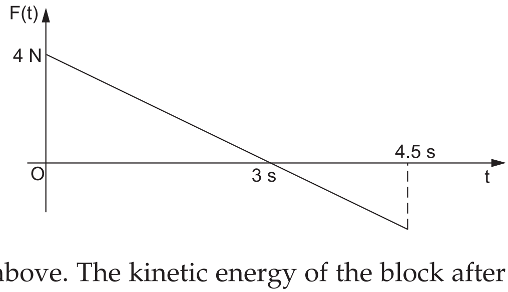
(c) $\frac{\pi K}{2a}$ (d) 0

25. A wire which passes through the hole in a small bead is bent in the form of a quarter of a circle. The wire is fixed vertically on the ground, as shown in the figure. The bead is released from near the top of this wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is



(a) always radially outwards
(b) always radially inwards
(c) initially radially outwards and later radially inwards
(d) initially radially inwards and later radially outwards

26. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in the figure above. The kinetic energy of the block after 4.5 seconds is

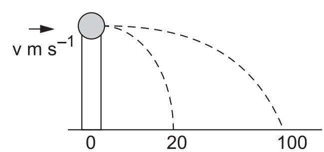


(a) 4.50 J (b) 7.50 J
(c) 5.06 J (d) 14.06 J

27. A moving point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1-kg mass reverses its direction and moves with a speed of 2 m s^{-1}. Which of the following statement is correct for the system of these two masses?

(a) The total momentum of the system is 4 kg m s^{-1}.
(b) The momentum of the 5-kg mass after collision is 4 kg m s^{-1}.
(c) The kinetic energy of the centre of mass is 0.75 J.
(d) The total kinetic energy of the system is 4 J.

28. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity v m s^{-1} in a horizontal direction, hits the centre of the ball. After the collision, the ball and the bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet, at a distance of 100 m from the foot of the post. The velocity v of this bullet is

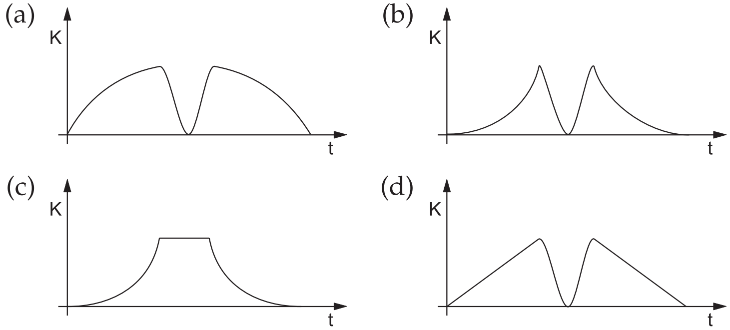


(a) 250 m s^{-1} (b) $250\sqrt{2}$ m s^{-1}
(c) 400 m s^{-1} (d) 500 m s^{-1}

29. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a perfectly inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed (u_0). The angle that the composite system makes with the horizontal immediately after the collision is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}+\alpha$
(c) $\frac{\pi}{4}-\alpha$ (d) $\frac{\pi}{2}$

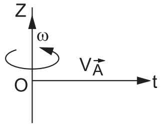
30. A tennis ball is dropped on a smooth horizontal surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with the time t most appreciably? (The figures are only illustrative and not drawn to the scale.)

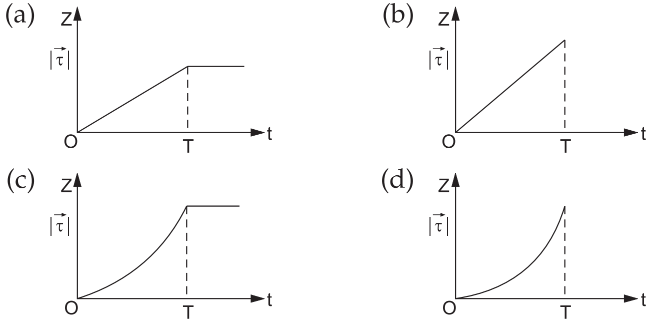


31. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m s^{-1} to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

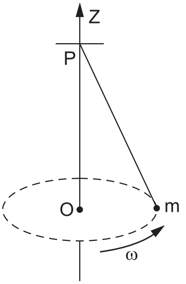
(a) 30 m s^{-1} (b) 20 m s^{-1}
(c) 10 m s^{-1} (d) 5 m s^{-1}

32. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with a constant angular speed ω, as shown in the figure. At $t = 0$, a small insect starts from O and moves with a constant speed v relative to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The magnitude of the torque ($|\vec{\tau}|$) on the system about O as a function of time (t) is best represented by





33. A small mass m is attached to a massless string whose other end is fixed at P, as shown in the figure. The mass is undergoing a circular motion in the xy-plane with the centre at O and a constant angular speed ω. If the angular momentum of the system calculated about O and P are respectively denoted by $\overrightarrow{L_O}$ and $\overrightarrow{L_P}$ then



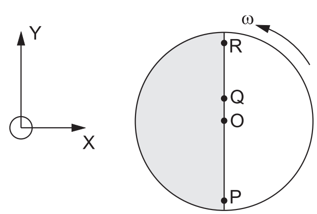
(a) $\overrightarrow{L_O}$ and $\overrightarrow{L_P}$ do not vary with time

(b) $\overrightarrow{L_O}$ varies with time, while $\overrightarrow{L_P}$ remains constant

(c) $\overrightarrow{L_O}$ remains constant, while $\overrightarrow{L_P}$ varies with time

(d) $\overrightarrow{L_O}$ and $\overrightarrow{L_P}$ both vary with time

34. Consider a disc rotating in the horizontal plane with a constant angular velocity ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side, as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the yz-plane and is the same for both the pebbles with respect to this disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius and (iii) ω remains constant throughout. Then,



(a) P lands in the shaded region and Q, in the unshaded region

(b) P lands in the unshaded region and Q, in the shaded region

(c) both P and Q land in the unshaded region

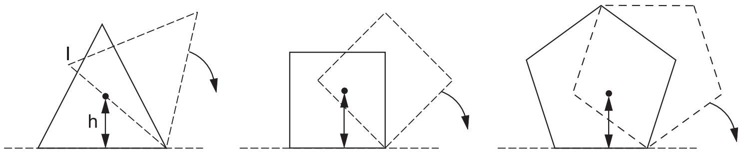
(d) both P and Q land in the shaded region

35. One end of a thick horizontal copper wire of length $2L$ and radius $2R$ is welded to an end of another thin horizontal copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(a) 0.25 (b) 0.50

(c) 2.00 (d) 4.00

36. Consider three regular polygons with the number of sides $n = 3$, 4 and 5 as shown in the figures.



The centre of mass of all the polygons is at the height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in the height of the locus of the centre of mass for each polygon is Δ. Then Δ depends on n and h as

(a) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (b) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

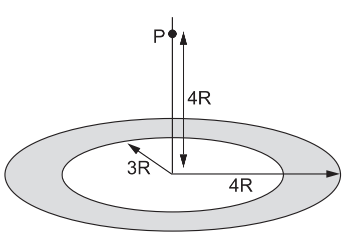
(c) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$ (d) $\Delta = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$

37. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbitting a few hundred kilometres above the earth's surface ($R_e = 6400$ km) will approximately be

(a) $\frac{1}{2}$ h (b) 1 h

(c) 2 h (d) 4 h

38. A thin uniform annular disc (see figure) of mass M has the outer radius $4R$ and the inner radius $3R$. The work required to take a unit mass from the point P on its axis to infinity is



(a) $\frac{2GM}{7R}(4\sqrt{2} - 5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2} - 5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2} - 1)$

39. A satellite is moving with a constant speed v in a circular orbit around the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of this object is

(a) $\frac{1}{2}mv^2$ (b) mv^2

(c) $\frac{3}{2}mv^2$ (d) $2mv^2$

40. A planet of radius $R = \frac{1}{10}$ (radius of the earth) has the same mass density as the earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kg m^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (taking the radius of the earth as 6×10^6 m and the acceleration due to gravity of the earth as 10 m s^{-2})

(a) 96 N (b) 108 N

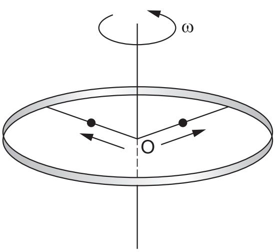
(c) 120 N (d) 150 N

41. A rocket is launched normal to the surface of the earth, away from the sun, along the line joining the sun and the earth. The sun is 3×10^5 times heavier than the earth and is at a distance 2.5×10^4 times larger than the radius of the earth. The escape velocity from the earth's gravitational field is v_e = 11.2 km s^{-1}. The minimum initial velocity required for the rocket to be able to leave the sun–earth system is close to

(a) 60 km s^{-1} (b) 40 km s^{-1}

(c) 70 km s^{-1} (d) 20 km s^{-1}

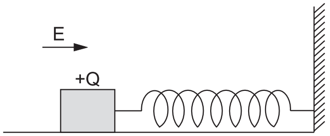
42. A ring of mass M and radius R is rotating with the angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $M/8$ at rest at O. These masses can move radially outwards along the two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $3R/5$ from O. At this instant, the distance of the other point mass from O is



(a) $\frac{2}{3}R$ (b) $\frac{3}{5}R$

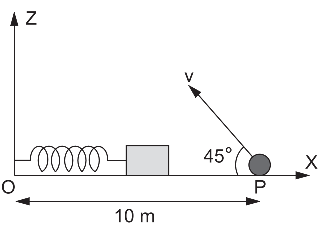
(c) $\frac{1}{3}R$ (d) $\frac{4}{5}R$

43. A wooden block performs SHM on a frictionless surface with a frequency of ν_0. The block carries a charge $+Q$ on its surface. If a uniform electric field $\vec{E}$ is switched on as shown then the SHM of the block will be of



(a) the same frequency and with a shifted mean position
(b) the same frequency and with the same mean position
(c) a changed frequency and with a shifted mean position
(d) a changed frequency and with the same mean position

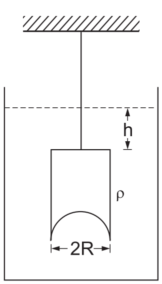
44. A small block is connected to one end of a massless spring of unstretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and then released. It then executes SHM with an angular frequency of $\omega = (\pi/3)$ rad s^{-1}. Simultaneously at $t = 0$, a small pebble is projected with a speed v from the point P at an angle of 45° as shown in the figure. The point P is at a horizontal distance of 10 cm from O. If this pebble hits the block at $t = 1$ s, the value of v is



(a) $\sqrt{50}$ m s^{-1} (b) $\sqrt{51}$ m s^{-1}

(c) $\sqrt{52}$ m s^{-1} (d) $\sqrt{53}$ m s^{-1}

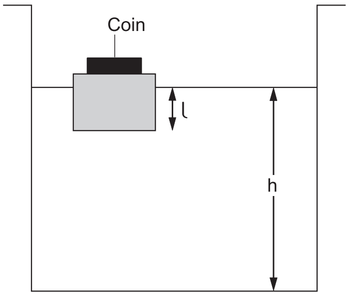
45. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ. It stays vertical inside the liquid. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is



(a) Mg (b) $Mg - V\rho g$

(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g(V + \pi R^2 h)$

46. A wooden block, with a coin placed on its top, floats in water, as shown in the figure. The distance l and h are shown. After some time the coin falls into the water. Then,

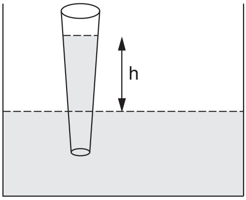


(a) l decreases and h increases
(b) l increases and h decreases
(c) both l and h increase
(d) both l and h decrease

47. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in the half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water then the correct statement is that the shell is

(a) more than half-filled if ρ_c is less than 0.5
(b) more than half-filled if ρ_c is more than 1
(c) half-filled if ρ_c is more than 0.5
(d) less than half-filled if ρ_c is less than 0.5

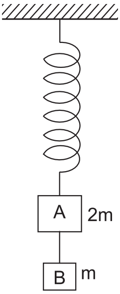
48. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is s, its density is ρ and its contact angle with glass is θ, the value of h will be



(a) $\dfrac{2s}{b\rho g}\cos(\theta - \alpha)$ (b) $\dfrac{2s}{b\rho g}\cos(\theta + \alpha)$

(c) $\dfrac{2s}{b\rho g}\cos\left(\theta - \dfrac{\alpha}{2}\right)$ (d) $\dfrac{2s}{b\rho g}\cos\left(\theta + \dfrac{\alpha}{2}\right)$

49. Two blocks A and B of masses $2m$ and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring, as shown in the figure. The magnitude of acceleration of A and B immediately after the string is cut are respectively



(a) g and $\frac{g}{2}$ (b) $\frac{g}{2}$ and g

(c) g and g (d) $\frac{g}{2}$ and $\frac{g}{2}$

50. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of the stars A and B respectively,

(a) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$ (b) $T_A > T_B$ if $r_A > r_B$

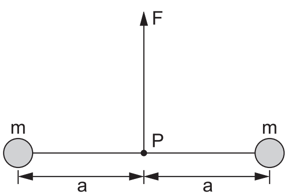
(c) $T_A > T_B$ if $m_A > m_B$ (d) $T_A = T_B$

51. A solid sphere of mass M, radius R and having moment of inertia as I is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then the radius of the disc will be

(a) $\frac{2R}{\sqrt{15}}$ (b) $R\sqrt{\frac{2}{15}}$

(c) $\frac{4R}{\sqrt{15}}$ (d) $\frac{R}{4}$

52. Two particles each of mass m are tied at the ends of a light string of length $2a$. The whole system is kept on a smooth horizontal surface with the string held tight so that each particle remains at a distance a from the centre P (as shown in the figure). Now the midpoint of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of the acceleration when the seperation between them becomes $2x$ is



(a) $\frac{F}{2m} \cdot \frac{a}{\sqrt{a^2 - x^2}}$ (b) $\frac{F}{2m} \cdot \frac{x}{\sqrt{a^2 - x^2}}$

(c) $\frac{F}{2m} \cdot \frac{x}{a}$ (d) $\frac{F}{2m} \cdot \frac{\sqrt{a^2 - x^2}}{x}$

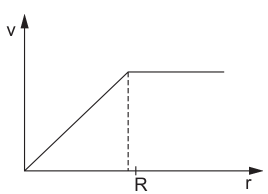
53. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $3v^2/4g$ with respect to the initial position. The object is a

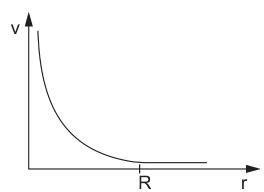
(a) ring (b) solid sphere
(c) hollow sphere (d) disc

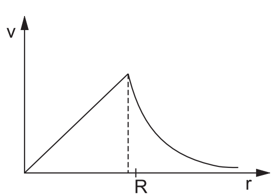
54. A spherically symmetric gravitational system of particles has a mass density

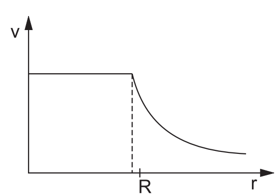
$$\rho = \begin{cases} \rho_0 \text{ for } r < R \\ 0 \text{ for } r > R \end{cases},$$

where ρ_0 is a constant. A test mass can undergo a circular motion under the influence of the gravitational field of particles. Its speed v as a function of the distance r (where $0 < r < \infty$) from the centre of the system is represented by

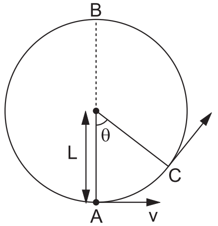
(a)


(b)


(c)


(d)


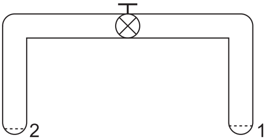
55. A bob of mass m is suspended by a massless string of length L. The horizontal velocity v at the position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is the half of that at A satisfies



(a) $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

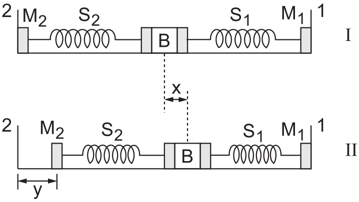
(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (d) $\frac{3\pi}{4} < \theta < \pi$

56. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. The end 1 has a hemispherical soap bubble of radius r. The end 2 has a subhemispherical soap bubble, as shown in the figure. Just after opening the valve,



(a) air from the end 1 flows towards the end 2: no change occurs in the volume of the soap bubbles
(b) air from the end 1 flows towards the end 2: the volume of the soap bubble at the end 1 decreases
(c) no change occurs
(d) air from the end 2 flows towards the end 1: the volume of the soap bubble at the end 1 increases

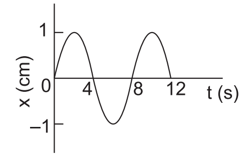
57. A block B is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$ respectively (see the figure I). The other ends are attached to the identical supports M_1 and M_2 which are not fixed to the walls. The springs and supports have negligible masses. There is no friction anywhere. The block B is displaced towards the wall 1 by a small distance x (see the figure II) and then released. The block returns and moves a maximum distance y towards the wall 2. The displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is



(a) 4 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

58. The x–t graph of a particle undergoing a simple harmonic motion is shown here. The acceleration of the particle at $t = \frac{4}{3}$ s is



(a) $\frac{\sqrt{3}\pi^2}{32}$ cm s^{-2} (b) $\frac{-\pi^2}{32}$ cm s^{-2}

(c) $\frac{\pi^2}{32}$ cm s^{-2} (d) $-\frac{\sqrt{3}\pi^2}{32}$ cm s^{-2}

59. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then,

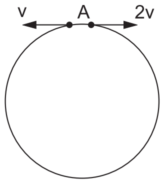
(a) the block will start sliding down the plane at $\theta = 30°$

(b) the block will remain at rest on the plane up to a certain θ and then it will topple

(c) the block will start sliding down the plane at $\theta = 60°$ and continue to do so at higher angles

(d) the block will start sliding down the plane at $\theta = 60°$ and, on further increasing θ, it will topple at a certain θ

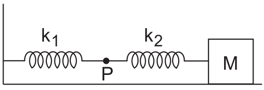
60. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$ respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, will these two particles again reach the point A?



(a) Four (b) Three

(c) Two (d) One

61. A block M shown in the figure oscillates in a simple harmonic motion with amplitude A. The amplitude of the point P is



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$

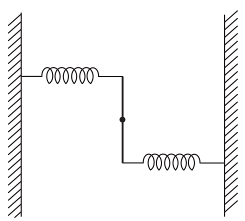
(c) $\dfrac{k_1 A}{k_1 + k_2}$ (d) $\dfrac{k_2 A}{k_1 + k_2}$

62. A piece of wire is bent in the shape of the parabola $y = kx^2$ (the y-axis being vertical), with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

(a) $\dfrac{a}{gk}$ (b) $\dfrac{a}{2gk}$

(c) $\dfrac{2a}{gk}$ (d) $\dfrac{a}{4gk}$

63. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants (k). The springs are fixed to rigid supports, as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and then released. The frequency of the oscillation is



(a) $\dfrac{1}{2\pi}\sqrt{\dfrac{2k}{M}}$ (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{M}}$

(c) $\dfrac{1}{2\pi}\sqrt{\dfrac{6k}{M}}$ (d) $\dfrac{1}{2\pi}\sqrt{\dfrac{24k}{M}}$

1.2 Heat and Thermodynamics

1. When a block of iron floats in mercury at 0 °C, a fraction k_1 of its volume is submerged; while at the temperature 60 °C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion for iron is γ_{Fe} and that for mercury is γ_{Hg}, the ratio k_1/k_2 can be expressed as

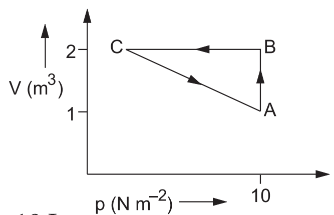
(a) $\dfrac{1 + (60\ °\text{C})\gamma_{\text{Fe}}}{1 + (60\ °\text{C})\gamma_{\text{Hg}}}$ (b) $\dfrac{1 - (60\ °\text{C})\gamma_{\text{Fe}}}{1 + (60\ °\text{C})\gamma_{\text{Hg}}}$

(c) $\dfrac{1+(60\,°\text{C})\gamma_{\text{Fe}}}{1-(60\,°\text{C})\gamma_{\text{Hg}}}$ (d) $\dfrac{1+(60\,°\text{C})\gamma_{\text{Hg}}}{1+(60\,°\text{C})\gamma_{\text{Fe}}}$

2. Starting with the same initial conditions, an ideal gas expands from the volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric, and W_3 if purely adiabatic. Then,

(a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$
(c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$

3. An ideal gas is taken through the cycle A → B → C → A, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process C → A is



(a) –5 J (b) –10 J
(c) –15 J (d) –20 J

4. 5.6 L of helium gas at STP is adiabatically compressed to 0.7 L. Taking the initial temperature to be T_1, the work done in the process is

(a) $\frac{9}{8}RT_1$ (b) $\frac{3}{2}RT_1$

(c) $\frac{15}{8}RT_1$ (d) $\frac{9}{2}RT_1$

5. A mixture of 2 mol of helium gas (atomic mass = 4 amu) and 1 mol of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\dfrac{v_{\text{rms(He)}}}{v_{\text{rms(Ar)}}}\right)$ is

(a) 0.32 (b) 0.45
(c) 2.24 (d) 3.16

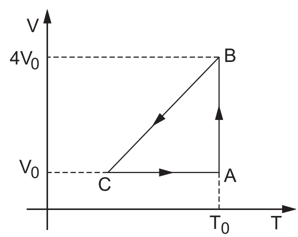
6. Two moles of ideal helium gas are in a rubber balloon at 30 °C. The balloon is fully expandable and can be assumed to require no energy for its expansion. The temperature of the gas in the balloon is slowly changed to 35 °C. The amount of heat required in raising the temperature is nearly (take $R = 8.31$ J mol^{-1} K^{-1})

(a) 62 J (b) 104 J (c) 124 J (d) 208 J

7. A real gas behaves like an ideal gas if its

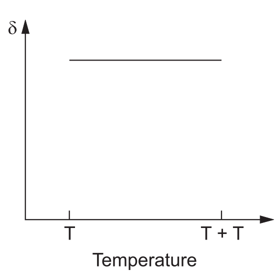
(a) pressure and temperature are both high
(b) pressure and temperature are both low
(c) pressure is high and temperature is low
(d) pressure is low and temperature is high

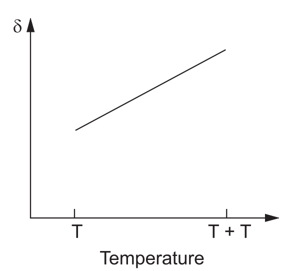
8. One mole of an ideal gas in the initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is p_0. Choose the correct statement from the following.

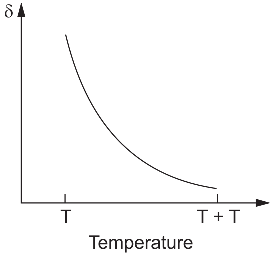


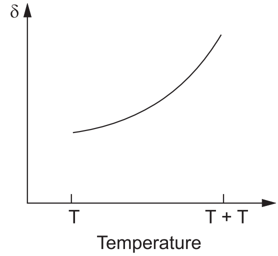
(a) The internal energies at A and B are different.
(b) The work done by the gas in process AB is $p_0V_0 \ln 4$.
(c) The pressure at C is $\frac{p_0}{4}$.
(d) The temperature at C is $\frac{T_0}{4}$.

9. An ideal gas is initially at a thermodynamic temperature T and has a volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, the pressure remaining constant. Which of the following graphs shows how the quantity $\delta = \Delta V/(V\Delta T)$ varies with temperature?

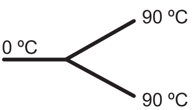
(a)


(b)


(c) 

(d) 

10. Three rods made of the same material and having the same cross section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0 °C and 90 °C respectively. The temperature of the junction of the three rods will be



(a) 45 °C (b) 60 °C

(c) 30 °C (d) 20 °C

11. An ideal black body at the room temperature is thrown into a furnace. It is observed that

(a) initially it is the darkest body and at later times the brightest

(b) it is the darkest body at all times

(c) it cannot be distinguished at all times

(d) initially it is the darkest body and at later times it cannot be distinguished

12. Three very large plates of same area are kept parallel and close to each other. They are considered ideal black surfaces and have very high thermal conductivity. The first and the third plates are maintained at the temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e., the second) plate under the steady-state condition is

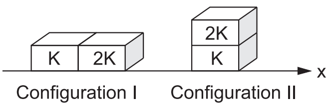
(a) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ (b) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$

(c) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ (d) $(97)^{\frac{1}{4}} T$

13. Two nonreactive monatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures when enclosed in a vessel kept at a constant temperature is 4 : 3. The ratio of their densities is

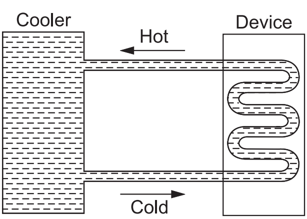
(a) 1 : 4 (b) 1 : 2
(c) 6 : 9 (d) 8 : 9

14. Two rectangular blocks having identical dimensions can be arranged either in Configuration I or in Configuration II, as shown in the figure. One of the blocks has the thermal conductivity *K* and the other, 2*K*. The temperature difference between the ends along the *x*-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in Configuration I. The time required in transporting the same amount of heat in Configuration II is



(a) 2.0 s (b) 3.0 s
(c) 4.5 s (d) 6.0 s

15. A water cooler of storage capacity 120 litres can cool water at a constant rate of *P*. In a closed circulation system (as shown schematically in this figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of power (thermal load). The temperature of the water fed into the device cannot exceed 30 °C and the entire 120 litres of water is initialy cooled to 10 °C. The entire system is thermally insulated. The minimum value of *P* for which the device can be operated for 3 hours (taking the specific heat capacity of water = 4.2 kJ kg^{-1} K^{-1} and the density of water = 1000 kg m^{-3}) is



(a) 1600 W (b) 2067 W
(c) 2533 W (d) 3933 W

16. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at the pressure $p_i = 10^5$ Pa and the volume $V_i = 10^{-3}$ m^3 changes to a final state at $p_f = \left(\frac{1}{32}\right) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that p^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at p_i followed by an isochoric process at V_f. The amount of the heat supplied to the system in the two-step process is approximately

(a) 112 J (b) 294 J
(c) 588 J (d) 813 J

17. Parallel rays of light of the intensity $I = 912$ W m^{-2} are incident on a spherical black body kept in the surroundings of temperature 300 K. Take the Stefan–Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ W m^{-2} K^{-4}, and assume that the energy exchange with the surroundings is only through radiations. The final steady-state temperature of the black body is close to

(a) 330 K (b) 660 K
(c) 990 K (d) 1550 K

18. The ends Q and R of two thin wires PQ and RS are soldered (joined) together. Initially each of the wires has a length of 1 m at 10 °C. Now the end P is maintained at 10 °C, while the end S is heated and maintained at 400 °C. The system is thermally insulated from its surroundings. If the thermal conductivity of the wire PQ is twice that of the wire RS, and the coefficient of linear thermal expansion for PQ is 1.2×10^{-5} K^{-1}, the change in the length of the wire PQ is

(a) 0.78 mm (b) 0.90 mm
(c) 1.56 mm (d) 2.34 mm

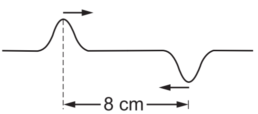
19. An ideal gas is expanding such that pT^2 = constant. The coefficient of volume expansion for the gas is

(a) $\frac{1}{T}$ (b) $\frac{2}{T}$

(c) $\frac{3}{T}$ (d) $\frac{4}{T}$

1.3 Sound Waves

1. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other, as shown in the figure. The speed of each pulse is 2 cm s^{-1}. After 2 s, the total energy of the pulses will be



(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential

2. A point mass is subjected to two simultaneous sinusoidal displacements in the x-directions: $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + 2\pi/3\right)$. Adding a third sinusoidal displacement $x(t) = B \sin (\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are respectively

(a) $\sqrt{2}A$ and $\frac{3\pi}{4}$ (b) A and $\frac{4\pi}{3}$
(c) $\sqrt{3}A$ and $\frac{5\pi}{6}$ (d) A and $\frac{\pi}{3}$

3. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin (\pi x/L) \sin \omega t$ and the energy is E_1, and in another experiment the displacement is $y_2 = A \sin (2\pi x/L) \sin 2\omega t$ and the energy is E_2. Then

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

4. Two vibrating strings made of the same material but of lengths L and $2L$ have the radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes—the one of length L with frequency ν_1 and the other with frequency ν_2. The ratio ν_1/ν_2 is equal to

(a) 2 (b) 4
(c) 8 (d) 1

5. A sonometer wire resonates with a given tuning fork, forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork, forming three antinodes for the same positions of the bridges. The value of M is

(a) 25 kg (b) 5 kg

(c) 12.5 kg (d) $\frac{1}{25}$ kg

6. A student is performing an experiment with a resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38 °C in which the speed of sound is 336 m s^{-1}. The zero of the metre scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is

(a) 14.0 cm (b) 15.2 cm

(c) 16.4 cm (d) 17.6 cm

7. A hollow pipe of length 0.8 m is closed at one end. At its open end, a 0.5-m-long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 m s^{-1}, the mass of the string is

(a) 5 grams (b) 10 grams

(c) 20 grams (d) 40 grams

8. A train moves towards a stationary observer with a speed of 34 ms^{-1}. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 ms^{-1}, the frequency registered is f_2. If the speed of sound is 340 ms^{-1} then the ratio f_1/f_2 is

(a) $\frac{18}{19}$ (b) $\frac{1}{2}$

(c) 2 (d) $\frac{19}{18}$

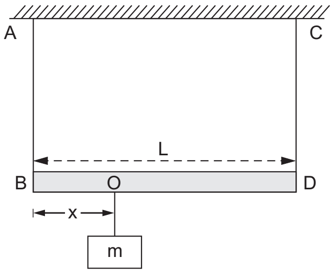
9. A siren placed at a railway platform is emitting sound waves of 5 kHz frequency. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey by a different train B, he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of the train B to that of the train A is

(a) 242 : 252 (b) 2 : 1
(c) 5 : 1 (d) 11 : 6

10. A police car with a siren of frequency of 8 kHz is moving with a uniform velocity of 36 km h^{-1} towards a tall building which reflects the sound waves. The speed of sound in air is 320 m s^{-1}. The frequency of the siren heard by the car driver is

(a) 8.50 kHz (b) 8.25 kHz
(c) 7.75 kHz (d) 7.50 kHz

11. A massless rod is suspended by two identical strings, AB and CD, of equal lengths. A block of mass m is suspended from the point O such that BO = x, as shown in the figure. Further, it is observed that the frequency of the first harmonic (fundamental frequency) in AB is equal to the 2nd harmonic in CD. Then the length of BO is



(a) $\frac{L}{5}$ (b) $\frac{4L}{5}$

(c) $\frac{3L}{4}$ (d) $\frac{L}{4}$

12. In the experiment to determine the speed of sound using a resonance column,

(a) the prongs of the tuning fork are kept in a vertical plane
(b) the prongs of the tuning fork are kept in a horizontal plane
(c) in one of the two resonances observed, the length of the

resonating air column is close to the wavelength of sound in air

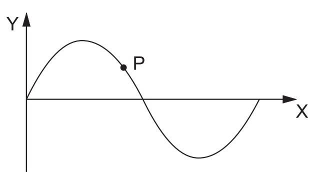
(d) in one of the two resonances observed, the length of the resonating air column is close to half the wavelength of sound in air

13. A vibrating string of length l under the tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited with a tuning fork of frequency n. When the tension of the string is slightly increased, the beat frequency reduces to 2 beats per second. Assuming the speed of sound in air to be 340 m s^{-1}, the frequency n of the tuning fork is

(a) 344 Hz (b) 336 Hz

(c) 117.3 Hz (d) 109.3 Hz

14. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm s^{-1}. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snapshot of the wave is shown in the figure. The velocity of the point P when its displacement is 5 cm is

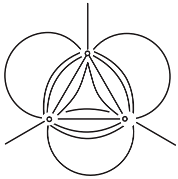


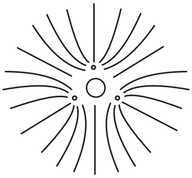
(a) $\frac{\sqrt{3}\pi}{50}\,\hat{j}$ m s^{-1} (b) $-\frac{\sqrt{3}\pi}{50}\,\hat{j}$ m s^{-1}

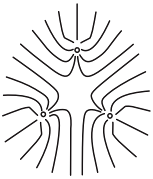
(c) $\frac{\sqrt{3}\pi}{50}\,\hat{i}$ m s^{-1} (d) $-\frac{\sqrt{3}\pi}{50}\,\hat{i}$ m s^{-1}

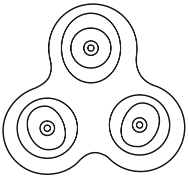
1.4 Electrostatics

1. Three positive charges each having the value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as

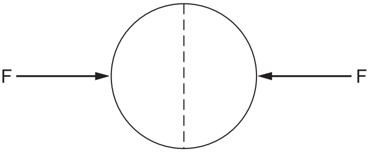
(a) 

(b) 

(c) 

(d) 

2. A uniformly charged thin spherical shell of radius R carries a uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with a force F (see the figure). F is proportional to



(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$

(b) $\frac{1}{\varepsilon_0}\sigma^2 R$

(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$

(d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

3. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5$ V m^{-1}. When the field is switched off, the drop is observed to fall with a terminal velocity of 2×10^{-3} m s^{-1}. Given g = 9.8 m s^{-2}, viscosity of the air = 1.8×10^{-5} N s m^{-2} and density of oil = 900 kg m^{-3}, the magnitude of q is

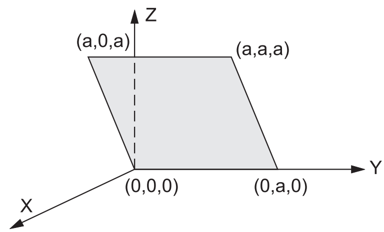
(a) 1.6×10^{-19} C

(b) 3.2×10^{-19} C

(c) 4.8×10^{-19} C

(d) 8.0×10^{-19} C

4. Consider the electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area shown in the figure due to this field is



(a) $2E_0a^2$ (b) $\sqrt{2}\,E_0a^2$

(c) E_0a^2 (d) $\dfrac{E_0a^2}{\sqrt{2}}$

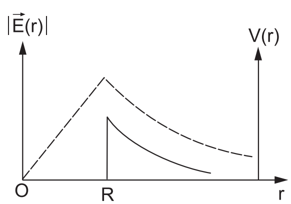
5. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a d.c. voltage source of potential difference X. A proton is released from the rest, midway between the two plates. It is found to move at 45° to the vertical just after the release. Then X is nearly

(a) 1×10^{-5} V (b) 1×10^{-7} V

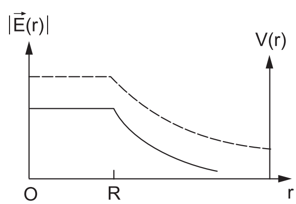
(c) 1×10^{-9} V (d) 1×10^{-10} V

6. Consider a thin spherical shell of radius R with its centre at the origin, carrying a uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre is best represented by which graph?

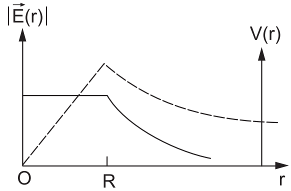
(a)



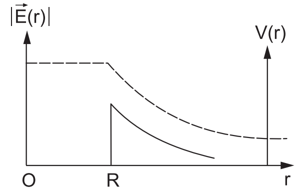
(b)



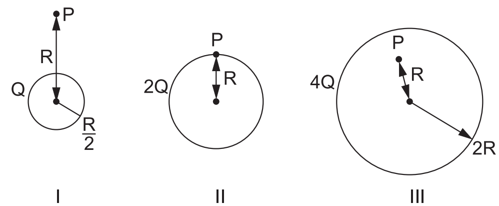
(c)



(d)



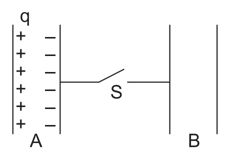
7. The charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres of radii $R/2$, R and $2R$ respectively as shown in the figure. If the magnitudes of electric field at the point P at a distance R from the centre of the spheres I, II and III are E_1, E_2 and E_3 respectively then



(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$

(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

8. Consider the situation shown in the figure. The capacitor A has a charge q on it, whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



(a) zero (b) $q/2$

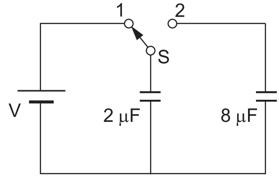
(c) q (d) $2q$

9. Two identical capacitors have the same capacitance C. One of them is charged to a potential V_1 and the other to V_2. The negative ends of the capacitors are connected. When the positive ends are also connected, the decrease in energy of the combined system is

(a) $\frac{1}{4}C(V_1^2 - V_2^2)$ (b) $\frac{1}{4}C(V_1^2 + V_2^2)$

(c) $\frac{1}{4}C(V_1 - V_2)^2$ (d) $\frac{1}{4}C(V_1 + V_2)^2$

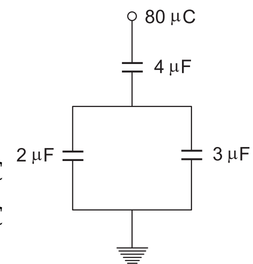
10. A 2-μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to the position 2 is



(a) 0% (b) 20%

(c) 75% (d) 80%

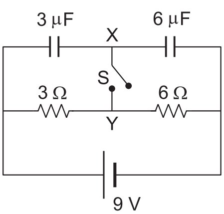
11. In the given circuit, a charge of 80 μC is given to the upper plate of the 4-μF capacitor. Then in the steady state, the charge on the upper plate of the 3-μF capacitor is



(a) +32 μC (b) +40 μC

(c) +48 μC (d) +80 μC

12. A circuit is connected as shown in the figure, with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is



(a) zero (b) 54 μC

(c) 27 μC (d) 81 μC

13. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. Choose the correct statement given below.

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
(d) No potential difference appears between the two cylinders when the same charge density is given to both the cylinders.

14. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then

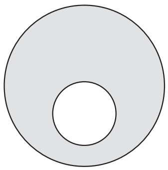
(a) negative and distributed uniformly over the surface of the sphere

(b) negative and appears only at the point on the sphere closest to the point charge
(c) negative and distributed nonuniformly over the entire surface of the sphere
(d) zero

15. A positive and a negative point charges of equal magnitudes are kept at $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, \frac{-a}{2}\right)$ respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is

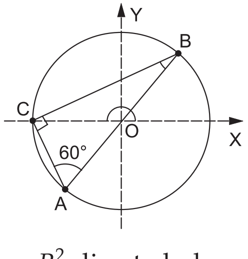
(a) positive
(b) negative
(c) zero
(d) dependent on the path connecting the initial and final positions

16. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume, as shown in the figure. The electric field inside the emptied space is



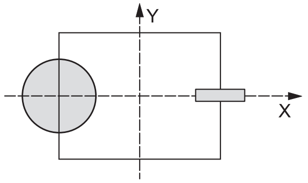
(a) zero everywhere
(b) nonzero and uniform
(c) nonuniform
(d) zero only at its centre

17. Consider a system of three charges $q/3$, $q/3$ and $-2q/3$ at the points A, B and C respectively, as shown in the figure. Take O as the centre of the circle of radius R, and $\angle CAB = 60°$. Which of the following option is correct?



(a) The electric field at the point O is $q/8\pi\varepsilon_0 R^2$ directed along the negative x-axis.
(b) The potential energy of the system is zero.
(c) The magnitude of the force between the charges at C and B is $q^2/54\pi\varepsilon_0 R^2$.
(d) The potential at the point O is $q/12\pi\varepsilon_0 R$.

18. A disc of radius $a/4$ having a uniformly distributed charge of 6 C is placed in the xy-plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge of 8 C is placed on the x-axis from $x = a/4$ to $5a/4$. Two point charges –7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$ respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is



(a) $\dfrac{-2\text{ C}}{\varepsilon_0}$ (b) $\dfrac{2\text{ C}}{\varepsilon_0}$

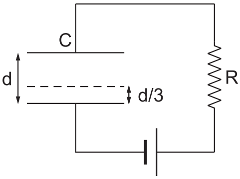
(c) $\dfrac{10\text{ C}}{\varepsilon_0}$ (d) $\dfrac{12\text{ C}}{\varepsilon_0}$

19. Three concentric metallic spherical shells of radii R, $2R$ and $3R$ are given the charges Q_1, Q_2 and Q_3 respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then the ratio of the charges given to the shells, i.e., $Q_1 : Q_2 : Q_3$, is equal to

(a) $1 : 2 : 3$ (b) $1 : 3 : 5$

(c) $1 : 4 : 9$ (d) $1 : 8 : 18$

20. A parallel-plate capacitor of capacitance C with plates of unit areas and separated by a distance of d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose that the liquid level decreases at a constant speed of V, the time constant as a function of time t is

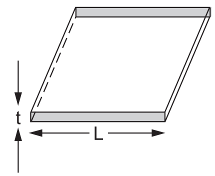


(a) $\dfrac{6\varepsilon_0 R}{5d + 3Vt}$ (b) $\dfrac{(15d + 9Vt)\,\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

(c) $\dfrac{6\varepsilon_0 R}{5d - 3Vt}$ (d) $\dfrac{(15d - 9Vt)\,\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

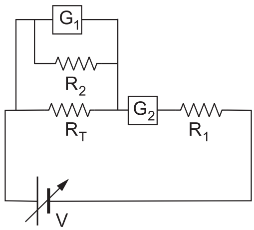
1.5 Current Electricity and Magnetism

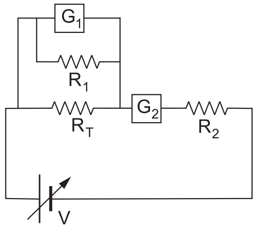
1. Consider a thin square sheet of side L and thickness t which is made of a material of resistivity ρ. The resistance between the two opposite faces shown by the shaded areas in the adjoining figure is

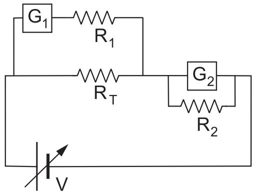


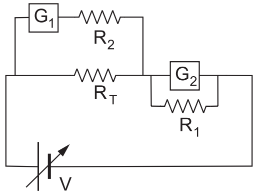
(a) directly proportional to L (b) directly proportional to t
(c) independent of L (d) independent of t

2. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

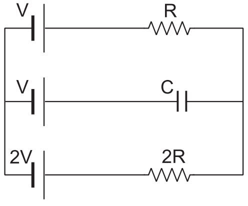
(a)


(b)


(c)


(d)


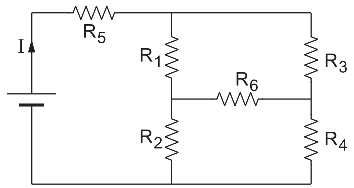
3. In the given circuit with a steady current, the potential drop across the capacitor must be



(a) V (b) $\dfrac{V}{2}$

(c) $\dfrac{V}{3}$ (d) $\dfrac{2V}{3}$

4. In the given circuit it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy



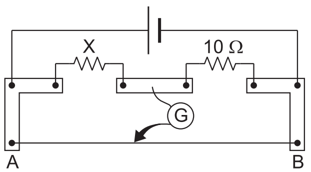
(a) $R_1R_2R_5 = R_3R_4R_6$

(b) $\dfrac{1}{R_5} + \dfrac{1}{R_6} = \dfrac{1}{R_1 + R_2} + \dfrac{1}{R_3 + R_4}$

(c) $R_1R_4 = R_2R_3$

(d) $R_1R_3 = R_2R_4 = R_5R_6$

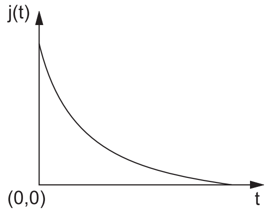
5. A meter bridge is set up, as shown, to determine an unknown resistance X using a standard 10-Ω resistor. The galvanometer G shows a null deflection when the tapping key is at the 52-cm mark. The end corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is

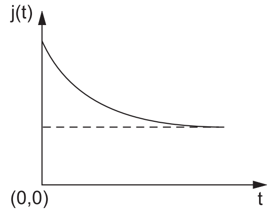


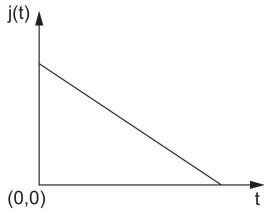
(a) 10.2 Ω (b) 10.6 Ω

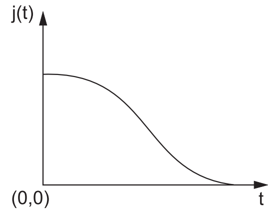
(c) 10.8 Ω (d) 11.1 Ω

6. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At the time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which of the following graphs best describes the subsequent variation of the magnitude of the current density $j(t)$ at any point in the material?

(a)


(b)


(c) 

(d) 

7. Incandescent bulbs are designed keeping in mind that the resistance of their filament increases with an increase in temperature. If at the room temperature, 100-W, 60-W and 40-W bulbs have filament resistances R_{100}, R_{60} and R_{40} respectively, the relation between these resistances is

(a) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ (b) $R_{100} = R_{40} + R_{60}$

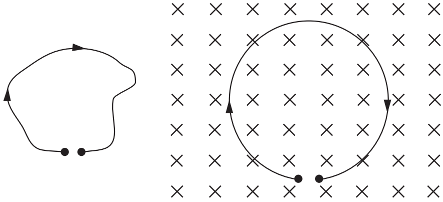
(c) $R_{100} > R_{60} > R_{40}$ (d) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

8. A particle of charge q and mass m moves in a circular orbit of radius r with the angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on

(a) ω and q (b) ω, q and m

(c) q and m (d) ω and m

9. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(a) IBL (b) $\frac{IBL}{\pi}$

(c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

10. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is

(a) $\dfrac{qbB}{m}$ (b) $\dfrac{q(b-a)B}{m}$

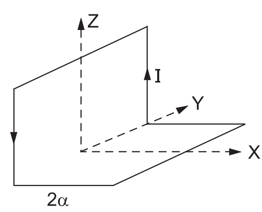
(c) $\dfrac{qaB}{m}$ (d) $\dfrac{q(b+a)B}{2m}$

11. A long straight wire along the z-axis carries a current I in the negative z-direction. The magnetic vector field $\vec{B}$ at a point having the coordinates (x, y) in the $z = 0$ plane is

(a) $\left(\dfrac{\mu_0 I}{2\pi}\right)\left(\dfrac{y\vec{i}-x\vec{j}}{x^2+y^2}\right)$ (b) $\left(\dfrac{\mu_0 I}{2\pi}\right)\left(\dfrac{x\vec{i}+y\vec{j}}{x^2+y^2}\right)$

(c) $\left(\dfrac{\mu_0 I}{2\pi}\right)\left(\dfrac{x\vec{j}-y\vec{i}}{x^2+y^2}\right)$ (d) $\left(\dfrac{\mu_0 I}{2\pi}\right)\left(\dfrac{x\vec{i}-y\vec{j}}{x^2+y^2}\right)$

12. A nonplanar loop of a conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point P$(a, 0, a)$ points in the direction

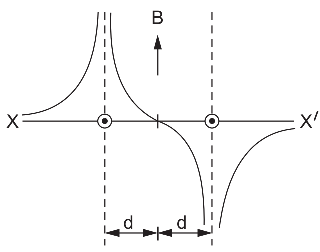


(a) $\dfrac{1}{\sqrt{2}}(-\vec{j}+\vec{k})$ (b) $\dfrac{1}{\sqrt{3}}(-\vec{j}+\vec{k}+\vec{i})$

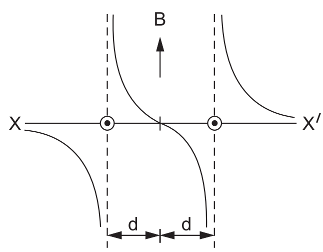
(c) $\dfrac{1}{\sqrt{3}}(\vec{i}+\vec{j}+\vec{k})$ (d) $\dfrac{1}{\sqrt{2}}(\vec{i}+\vec{k})$

13. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field B along the line XX' is given by

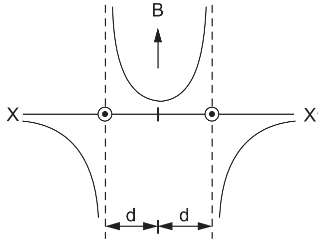
(a)



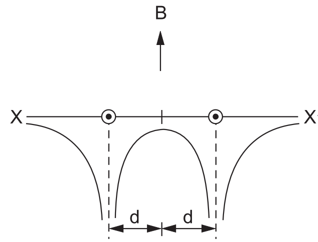
(b)



(c)



(d)



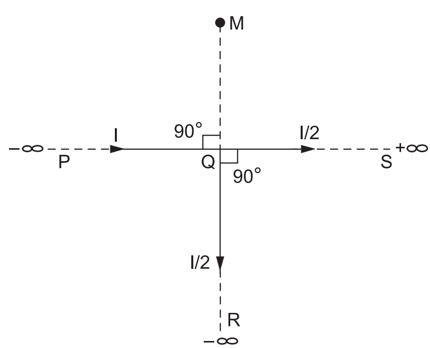
14. An a.c. voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased,

(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) the total impedance of the circuit is unchanged
(d) the total impedance of the circuit increases

15. An ionised gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+x$-direction and a magnetic field along the $+z$-direction then

(a) the positive ions deflect towards the $+y$-direction and negative ions towards the $-y$-direction
(b) all the ions deflect towards the $+y$-direction
(c) all the ions deflect towards the $-y$-direction
(d) the positive ions deflect towards the $-y$-direction and negative ions towards the $+y$-direction

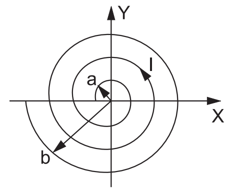
16. An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field strength due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current is $I/2$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field strength at M is now H_2. The ratio H_1/H_2 equals



(a) $\frac{1}{2}$ (b) 1

(c) $\frac{2}{3}$ (d) 2

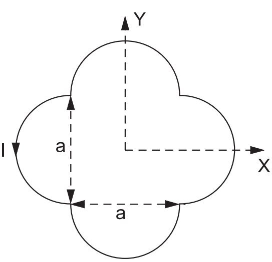
17. A long insulated copper wire is closely wound as a spiral of N turns. The spiral has the inner radius a and the outer radius b. The spiral lies in the xy-plane and a steady current I flows through the wire. The z-component of this magnetic field at the centre of the spiral is



(a) $\frac{\mu_0 NI}{2(b-a)} \ln \frac{b}{a}$ (b) $\frac{\mu_0 NI}{2(b-a)} \ln \frac{b+a}{b-a}$

(c) $\frac{\mu_0 NI}{2b} \ln \frac{b}{a}$ (d) $\frac{\mu_0 NI}{2b} \ln \frac{b+a}{b-a}$

18. A loop carrying a current I lies in the xy-plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is



(a) $a^2 I\hat{k}$

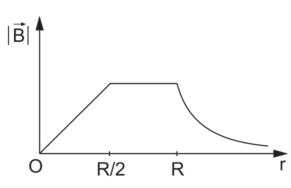
(b) $\left(\frac{\pi}{2}+1\right) a^2 I\hat{k}$

(c) $-\left(\frac{\pi}{2}+1\right) a^2 I\hat{k}$

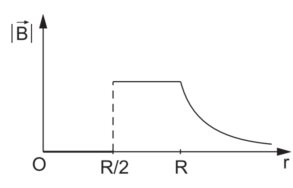
(d) $(2\pi+1)\, a^2 I\hat{k}$

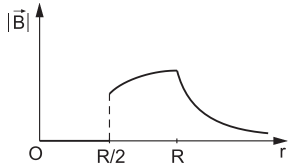
19. An infinitely long hollow conducting cylinder of inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$, as a function of the radial distance r from the axis is best represented by

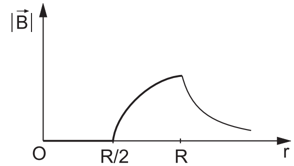
(a)



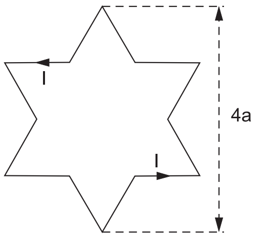
(b)



(c) 

(d) 

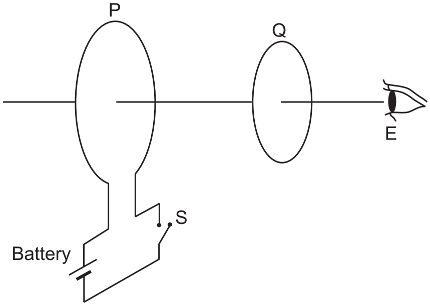
20. A symmetric star-shaped conducting wire loop is carrying a steady current I, as shown in the figure. The distance between any two diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the centre of the loop is



(a) $\dfrac{\mu_0 I}{4\pi a} \cdot 6(\sqrt{3}-1)$ (b) $\dfrac{\mu_0 I}{4\pi a} \cdot 3(\sqrt{3}-1)$

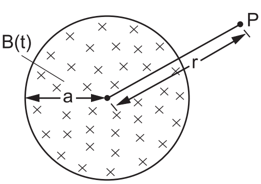
(c) $\dfrac{\mu_0 I}{4\pi a} \cdot 6(\sqrt{3}+1)$ (d) $\dfrac{\mu_0 I}{4\pi a} \cdot 3(2-\sqrt{3})$

21. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the directions of I_{Q1} and I_{Q2} (as seen by E) are



(a) respectively clockwise and anticlockwise
(b) both clockwise
(c) both anticlockwise
(d) respectively anticlockwise and clockwise

22. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at the point P at a distance r from the centre of the circular region



(a) is zero
(b) decreases as $\dfrac{1}{r}$

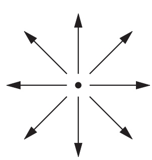
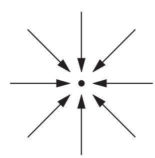
(c) increases as r
(d) decreases as $\dfrac{1}{r^2}$

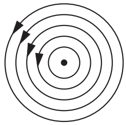
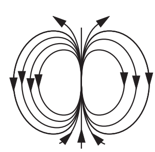
23. A coil having N turns is wound tightly in the form of a spiral with the inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic flux density at the centre is

(a) $\dfrac{\mu_0 NI}{b}$
(b) $\dfrac{2\mu_0 NI}{a}$

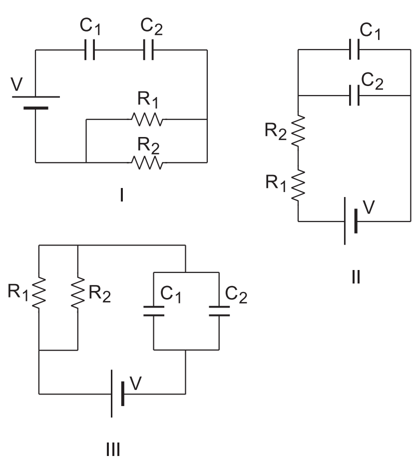
(c) $\dfrac{\mu_0 NI}{2(b-a)} \ln\dfrac{b}{a}$
(d) $\dfrac{\mu_0 I^N}{2(b-a)} \ln\dfrac{b}{a}$

24. Which of the patterns given below is valid for an electric field as well as a magnetic field?

(a) 
(b) 

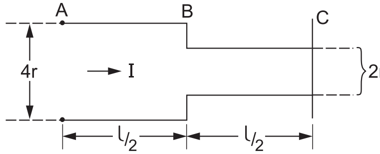
(c) 
(d) 

25. In the following circuits it is given that $R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$, $C_1 = 2\mu F$ and $C_2 = 4\mu F$. The time constants (in μs) for the circuits I, II and III are respectively



(a) 18, 8/9 and 4
(b) 18, 4 and 8/9
(c) 4, 8/9 and 18
(d) 8/9, 18 and 4

26. Consider a cylindrical element as shown in the figure. The current that flows through the element is I and the resistivity of the material of this cylinder is ρ. Choose the correct option.



(a) The power loss in the second half is four times the power loss in the first half.

(b) The voltage drop in the first half is twice the voltage drop in the second half.

(c) The current densities in the two halves are equal.

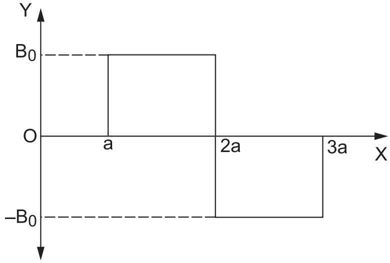
(d) The electric field in both the halves is the same.

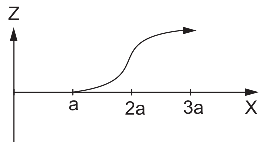
27. A resistance of 2 Ω is connected across one gap of a metre bridge (the length of the wire being 100 cm), and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is

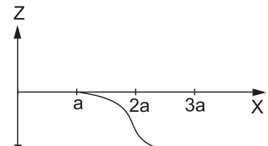
(a) 3 Ω (b) 4 Ω

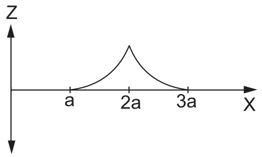
(c) 5 Ω (d) 6 Ω

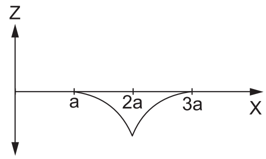
28. A magnetic field $\vec{B} = B_0\hat{j}$ exists in the region $a < x < 2a$ and $\vec{B} = -B_0\hat{j}$ exists in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at $x = a$. The trajectory of the charge in this region can be like

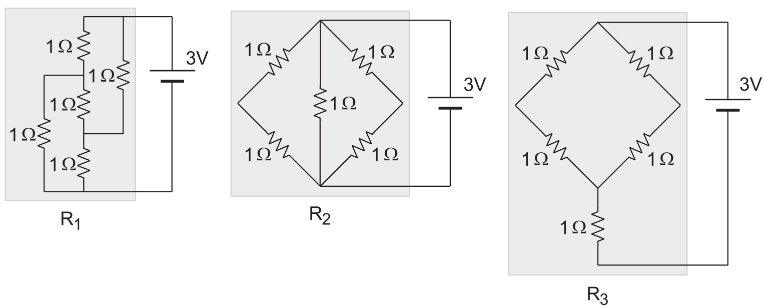


(a) 

(b) 

(c) 

(d) 

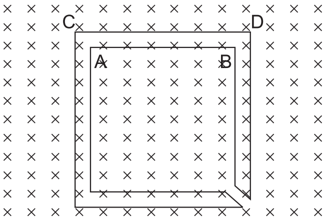
29. 

The figures above show three resistor configurations R_1, R_2 and R_3 connected to 3-V batteries. If the powers dissipated by the configurations R_1, R_2 and R_3 are respectively P_1, P_2 and P_3 then

(a) $P_1 > P_2 > P_3$ (b) $P_1 > P_3 > P_2$

(c) $P_2 > P_1 > P_3$ (d) $P_3 > P_2 > P_1$

30. The adjoining figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time, and I_1 and I_2 are the currents in the segments AB and CD. Then



(a) $I_1 > I_2$

(b) $I_1 < I_2$

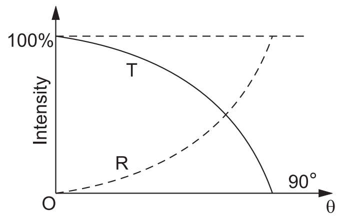
(c) I_1 is in the direction $\overrightarrow{BA}$ and I_2 is in the direction $\overrightarrow{CD}$

(d) I_1 is in the direction $\overrightarrow{AB}$ and I_2 is in the direction $\overrightarrow{DC}$

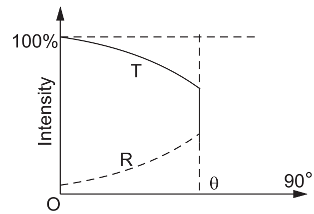
1.6 Ray Optics and Wave Optics

1. A light ray travelling in a glass medium is incident on a glass–air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, each as a function of θ, are plotted. The correct graph is

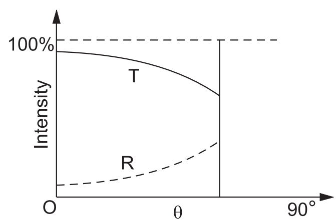
(a)



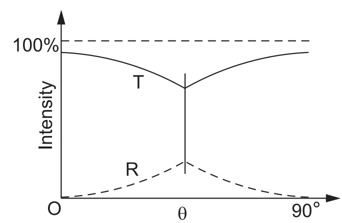
(b)



(c)



(d)



2. The image of an object formed by a plano-convex lens at a distance of 8 m behind the lens is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of curvature of the curved surface is

(a) 1 m (b) 2 m
(c) 3 m (d) 4 m

3. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is

(a) 30° (b) 60°
(c) 40° (d) 75°

4. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

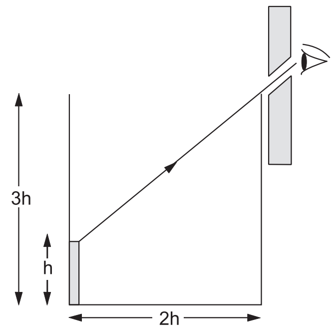
(a) virtual and at a distance of 16 cm from the mirror

(b) real and at a distance of 16 cm from the mirror

(c) virtual and at a distance of 20 cm from the mirror

(d) real and at a distance of 20 cm from the mirror

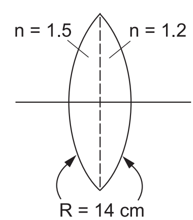
5. An observer can see through a pinhole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is $3h$ and its radius is h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is



(a) $\frac{5}{2}$ (b) $\sqrt{\frac{5}{2}}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{3}{2}$

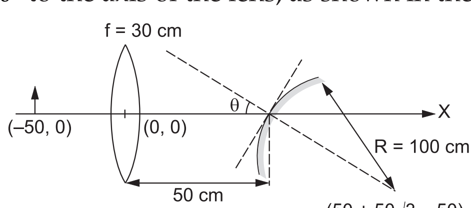
6. A biconvex lens is formed with two thin plano-convex lenses, as shown in the figure. The refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this biconvex lens, for an object distance of 40 cm the image distance will be



(a) –280.0 cm (b) 40.0 cm

(c) 21.5 cm (d) 13.3 cm

7. A small object is placed at 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror with radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle θ = 30° to the axis of the lens, as shown in the figure.

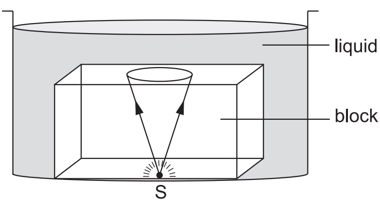


If the origin of the coordinate system is taken to be the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are

(a) $(25, 25\sqrt{3})$ (b) $\left(\frac{125}{3}, \frac{25}{3}\right)$

(c) $(0, 0)$ (d) $(50 - 25\sqrt{3}, 25)$

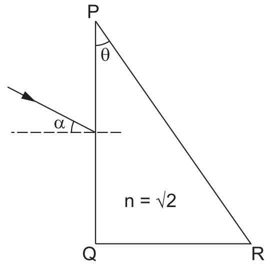
8. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive-index liquid, as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



(a) 1.21 (b) 1.30

(c) 1.36 (d) 1.42

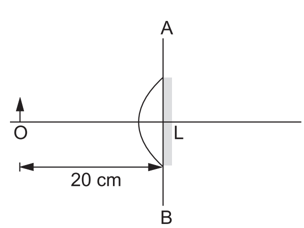
9. A parallel beam of light is incident from air at an angle α on the side PQ of a right-angled triangular prism of refractive index $n = \sqrt{2}$. The beam of light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is



(a) 15° (b) 22.5°

(c) 30° (d) 45°

10. A point object is placed at a distance of 20 cm from a plano-convex lens of focal length 15 cm. If the plane surface is silvered, the image will be formed at



(a) 60 cm left of AB

(b) 30 cm left of AB

(c) 12 cm left of AB

(d) 60 cm right of AB

11. 

A light beam is travelling from Region I to Region IV (refer to the figure). The refractive indices in Regions I, II, III and IV are n_0, $\frac{n_0}{2}$, $\frac{n_0}{6}$ and $\frac{n_0}{8}$ respectively. The angle of incidence (θ) for which the beam just misses entering Region IV is

(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{8}\right)$

(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

12. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake in the line of fall of the ball is looking at the ball. At an instant when the ball is 12.8 m above the water surface, the fish sees the speed of the ball as

(a) 9 m s^{-1} (b) 12 m s^{-1}

(c) 16 m s^{-1} (d) 21.33 m s^{-1}

13. A biconvex lens of focal length f forms a circular image of the sun of radius r in its focal plane. Therefore,

(a) $\pi r^2 \propto f$ (b) $\pi r^2 \propto f^2$

(c) if the lower half of the lens is covered with a black sheet, the area of the image is equal to $\frac{\pi r^2}{2}$

(d) if f is doubled, the intensity will increase

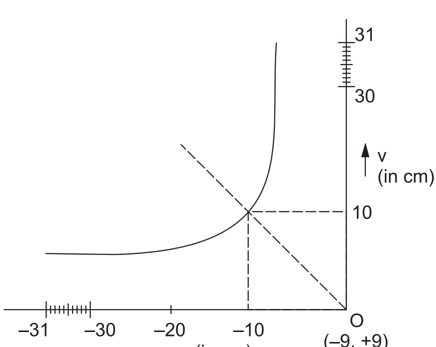
14. The graph of the image distance versus the object distance of a point from a convex lens is shown. Then the focal length of the lens is

(a) (0.50 ± 0.05) cm

(b) (0.50 ± 0.10) cm

(c) (5.00 ± 0.05) cm

(d) (5.00 ± 0.10) cm



15. In an experiment to determine the focal length (f) of a concave mirror by the u–v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards the left, the image appears to the right of the object pin. Then

(a) $x < f$
(b) $f < x < 2f$
(c) $x = 2f$
(d) $x > 2f$

16. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be

(a) only a reflected ray and no refracted rays
(b) only a refracted ray and no reflected rays
(c) a reflected ray and a refracted ray, and the angle between them would be less than $180° - 2\theta$
(d) a reflected ray and a refracted ray, and the angle between them would be greater than $180° - 2\theta$

17. Two beams of red and violet lights are made to pass separately through a prism (the angle of the prism being 60°). In the position of the minimum deviation, the angle of refraction will be

(a) 30° for each light
(b) greater for the violet light
(c) greater for the red light
(d) the same for both the colours of light but not equal to 30°

18. In a double-slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern,

(a) the intensities of both the maxima and minima increase
(b) the intensity of the maxima increases and the minima has the zero intensity
(c) the intensity of the maxima decreases but that of the minima increases
(d) the intensity of the maxima decreases and the minima has the zero intensity

19. Young's double-slit experiment is carried out by using green, red and blue lights, one at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

20. In Young's double-slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer) corresponding to any point having half the peak intensity is

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$
(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

1.7 Modern Physics

1. A pulse of light of 100 ns duration is absorbed completely by a small object initially at rest. The power of the pulse is 30 mW and the speed of light is 3×10^8 m s^{-1}. The final momentum of the object is

(a) 0.3×10^{-17} kg m s^{-1} (b) 1.0×10^{-17} kg m s^{-1}
(c) 3.0×10^{-17} kg m s^{-1} (d) 9.0×10^{-17} kg m s^{-1}

2. A metal surface is illuminated by light of two wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to those wavelengths are u_1 and u_2 respectively. If $u_1 : u_2 = 2 : 1$ and $hc = 1240$ eV nm, the work function of the metal is nearly

(a) 3.7 eV (b) 3.2 eV
(c) 2.8 eV (d) 2.5 eV

3. In a historical experiment to determine Planck constant, a metal surface was irradiated with lights of different wavelengths. The energies of the photoelectrons emitted were measured by applying a stopping potential. The relevant data for wavelengths (λ) of the incident light and the corresponding stopping potentials (V_0) are given below.

λ	V_0
0.3 μm	2.0 V
0.4 μm	1.0 V
0.5 μm	0.4 V

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 1.6 \times 10^{-19}$ C, the value of Planck constant found in such an experiment is

(a) 6.0×10^{-34} (b) 6.4×10^{-34}
(c) 6.6×10^{-34} (d) 6.8×10^{-34}

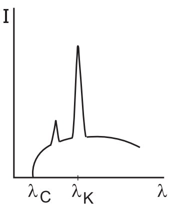
4. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true?

(a) Its kinetic energy increases, and the potential and total energies decrease.
(b) Its kinetic energy decreases but the potential energy increases, and thus the total energy remains the same.
(c) Its kinetic and total energies decrease, and the potential energy increases.
(d) Its kinetic, potential and total energies decrease.

5. Electrons each having the energy 80 keV are incident on the tungsten target of an X-ray tube. The K-shell electrons of tungsten have –72.5 keV energy. The X-rays emitted by the tube contain only

(a) a continuous X-ray spectrum (breamsstrahlung) with a minimum wavelength of ~0.0155 nm
(b) a continuous X-ray spectrum (breamsstrahlung) with all wavelengths
(c) the characteristic X-ray spectrum of tungsten
(d) a continuous X-ray spectrum (breamsstrahlung) with a minimum wavelength of ~0.0155 nm and the characteristic X-ray spectrum of tungsten

6. The intensity (I) of X-rays from a Coolidge tube is plotted against the wavelength (λ), as shown in the figure. The minimum wavelength found is λ_C and the wavelength of the K_α line is λ_K. As the accelerating voltage is increased,



(a) $\lambda_K - \lambda_C$ increases
(b) $\lambda_K - \lambda_C$ decreases
(c) λ_K increases
(d) λ_K decreases

7. The wavelength of the first spectral line in the Balmer series of a hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of a singly ionised helium atom is

(a) 1215 Å
(b) 1640 Å
(c) 2430 Å
(d) 4687 Å

8. If λ_{Cu} is the wavelength of the K_α *x*-ray line of copper (atomic number: 29) and λ_{Mo} is the wavelength of the K_α *x*-ray line of molybdenum (atomic number: 42) then the value of the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to

(a) 1.99
(b) 2.14
(c) 0.50
(d) 0.48

9. Which of the following processes represents a gamma decay?

(a) ${}^{A}_{Z}X + \gamma \rightarrow {}^{A}_{Z-1}X + a + b$
(b) ${}^{A}_{Z}X + {}^{1}_{0}n \rightarrow {}^{A-3}_{Z-2}X + c$
(c) ${}^{A}_{Z}X \rightarrow {}^{A}_{Z}X + f$
(d) ${}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}X + g$

10. An accident in a nuclear laboratory resulted in depositions of a certain amount of a radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times the permissible level allowed for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

(a) 64
(b) 90
(c) 108
(d) 120

11. A photoelectric material having the work function ϕ_0 is illuminated with light of wavelength $\lambda < \dfrac{hc}{\phi_0}$. The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in the de Broglie wavelength. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(a) Δ_d^3/λ^2
(b) Δ_d^2/λ^2
(c) Δ_d/λ
(d) Δ_d^3/λ

12. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}.$$

The measured masses of a neutron, ${}^1_1\text{H}$, ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u respectively. Given that the radii of both the ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ nuclei are the same; 1 u = 931.5 MeV/c^2, where c is the speed of light and $e^2/4\pi\varepsilon_0$ = 1.44 MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ is purely due to their electrostatic energy, the radius of either of the nuclei is

(a) 2.85 fm (b) 3.03 fm

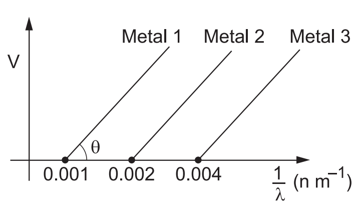
(c) 3.42 fm (d) 3.80 fm

13. Given a sample of radium–226 having a half-life of 4 days, find the probability that a nucleus disintegrates after two half-lives.

(a) 1 (b) $\frac{1}{2}$

(c) 1.5 (d) $\frac{3}{4}$

14. The graph between $1/\lambda$ and the stopping potentials (V) of three metals having the work functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photoelectric effect is plotted as shown in the figure. Which of the following statements is correct? (Here λ is the wavelength of the incident light.)



(a) The ratio of the work functions is $\phi_1 : \phi_2 : \phi_3 = 1:2:3$.

(b) The ratio of the work functions is $\phi_1 : \phi_2 : \phi_3 = 4:2:1$.

(c) tan θ is directly proportional to hc/e, where h is Planck constant and c is the speed of light.

(d) Violet light can eject photoelectrons from Metals 2 and 3.

15. In the options given below, let E denote the rest-mass energy of a nucleus and n a neutron. The correct option is

(a) $E\left({}^{236}_{92}\mathrm{U}\right) > E\left({}^{137}_{53}\mathrm{I}\right) + E\left({}^{97}_{39}\mathrm{Y}\right) + 2E(\mathrm{n})$

(b) $E\left({}^{236}_{92}\mathrm{U}\right) < E\left({}^{137}_{53}\mathrm{I}\right) + E\left({}^{97}_{39}\mathrm{Y}\right) + 2E(\mathrm{n})$

(c) $E\left({}^{236}_{92}\mathrm{U}\right) < E\left({}^{140}_{56}\mathrm{Ba}\right) + E\left({}^{94}_{36}\mathrm{Y}\right) + 2E(\mathrm{n})$

(d) $E\left({}^{236}_{92}\mathrm{U}\right) = E\left({}^{140}_{56}\mathrm{Ba}\right) + E\left({}^{94}_{36}\mathrm{Y}\right) + 2E(\mathrm{n})$

16. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

(a) 802 nm (b) 823 nm

(c) 1882 nm (d) 1648 nm

17. Electrons with de Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is

(a) $\lambda_0 = \dfrac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \dfrac{2h}{mc}$

(c) $\lambda_0 = \dfrac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

18. Which of the following statements is wrong in the context of X-rays generated from an X-ray tube?

(a) The wavelength of the characteristic X-rays decreases when the atomic number of the target increases.

(b) The cut-off wavelength of the continuous X-rays depends on the atomic number of the target.

(c) The intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube.

(d) The cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

19. A radioactive sample S_1, having an activity of 5 μC_i, has twice the number of nuclei as another sample S_2, which has an activity of 10 μC_i. The half-lives of S_1 and S_2 can be

(a) 20 years and 5 years respectively

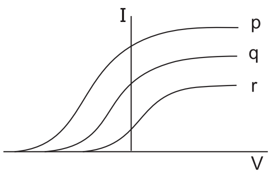
(b) 20 years and 10 years respectively

(c) 10 years each

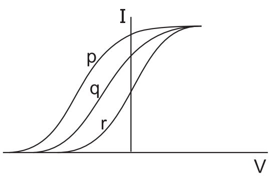
(d) 5 years each

20. Photoelectric-effect experiments are performed using three different metal plates p, q and r having the work functions ϕ_p = 2.0 eV, ϕ_q = 2.5 eV and ϕ_r = 3.0 eV respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct *I–V* graphs for the experiment is (taking *hc* = 1240 eV nm)

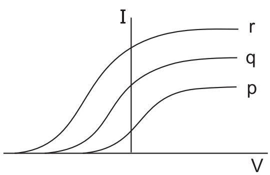
(a)



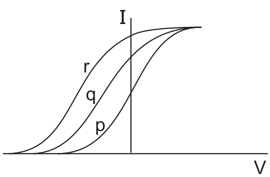
(b)



(c)



(d)



Answers

1.1 General Physics

1. d	**2.** d	**3.** c	**4.** a	**5.** c	**6.** b	**7.** c
8. b	**9.** c	**10.** c	**11.** a	**12.** a	**13.** d	**14.** c
15. a	**16.** a	**17.** c	**18.** a	**19.** d	**20.** c	**21.** b
22. b	**23.** d	**24.** d	**25.** d	**26.** c	**27.** c	**28.** d
29. a	**30.** b	**31.** c	**32.** b	**33.** c	**34.** c	**35.** c
36. d	**37.** c	**38.** a	**39.** b	**40.** b	**41.** b	**42.** d
43. a	**44.** a	**45.** d	**46.** d	**47.** d	**48.** d	**49.** b
50. d	**51.** a	**52.** b	**53.** d	**54.** c	**55.** d	**56.** b
57. c	**58.** d	**59.** b	**60.** c	**61.** d	**62.** b	**63.** c

1.2 Heat and Thermodynamics

1. a	**2.** a	**3.** a	**4.** a	**5.** d	**6.** d	**7.** d
8. b	**9.** c	**10.** b	**11.** d	**12.** c	**13.** d	**14.** a
15. b	**16.** c	**17.** a	**18.** a	**19.** c		

1.3 Sound Waves

1. b	**2.** b	**3.** c	**4.** d	**5.** a	**6.** b	**7.** b
8. d	**9.** b	**10.** a	**11.** a	**12.** a	**13.** a	**14.** a

1.4 Electrostatics

1. c	**2.** a	**3.** d	**4.** c	**5.** c	**6.** d	**7.** c
8. a	**9.** c	**10.** d	**11.** c	**12.** c	**13.** a	**14.** d
15. c	**16.** b	**17.** c	**18.** a	**19.** b	**20.** a	

1.5 Current Electricity and Magnetism

1. c	**2.** c	**3.** c	**4.** c	**5.** b	**6.** a	**7.** d
8. c	**9.** c	**10.** b	**11.** a	**12.** d	**13.** b	**14.** b
15. c	**16.** c	**17.** a	**18.** b	**19.** d	**20.** a	**21.** d
22. b	**23.** c	**24.** c	**25.** d	**26.** a	**27.** a	**28.** a
29. c	**30.** d					

1.6 Ray Optics and Wave Optics

1. c	**2.** c	**3.** a	**4.** b	**5.** b	**6.** b	**7.** a
8. c	**9.** a	**10.** c	**11.** b	**12.** c	**13.** b	**14.** d
15. b	**16.** c	**17.** a	**18.** a	**19.** d	**20.** b	

1.7 Modern Physics

1. b	**2.** a	**3.** b	**4.** a	**5.** d	**6.** a	**7.** a
8. b	**9.** c	**10.** c	**11.** a	**12.** c	**13.** d	**14.** c
15. a	**16.** b	**17.** a	**18.** b	**19.** a	**20.** a	

Hints and Solutions

1.1 General Physics

1. LC = 1 MSD – 1 VSD = 1 mm $- \dfrac{16}{20}$ mm = 0.2 mm.

2. $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{Q}{L}$.

$$\therefore \; [\varepsilon_0 LV] = [\varepsilon_0 L\Delta V] = [Q].$$

$$\therefore \quad [X] = \left[\frac{\varepsilon_0 L\Delta V}{\Delta t}\right] = \left[\frac{Q}{\Delta t}\right] = [\text{current}].$$

3. Diameter = 2.5 mm + $\dfrac{0.5\text{ mm}}{50} \times 20 = 2.7$ mm.

$$\therefore \text{percentage error in density} = \left(\frac{dm}{m} + \frac{3dr}{r}\right) \times 100$$

$$= 2 + \left(3 \times \frac{0.01}{2.70}\right) \times 100 = 3.1.$$

4.

$$\Delta d = \Delta l = \text{LC} = \frac{\text{pitch}}{\text{total cap division}} = \frac{0.5\text{ mm}}{100}.$$

$$Y = \frac{4MgL}{\pi d^2 l}.$$

$$\therefore \quad \frac{\Delta Y}{Y} = \frac{\Delta l}{l} + 2\left(\frac{\Delta d}{d}\right) = \frac{0.5\text{ mm}/100}{0.25\text{ mm}} + 2 \cdot \left(\frac{0.5\text{ mm}/100}{0.5\text{ mm}}\right)$$

$$= \frac{0.5 \times 10^{-2}}{0.25} + \frac{0.5 \times 10^{-2}}{0.25}.$$

Hence, the error in the measurement of d and l are equal.

5. For balance with unknown resistance X,

$$\frac{X}{R} = \frac{l}{100\text{ cm} - l} = \frac{40\text{ cm}}{60\text{ cm}}$$

$$\Rightarrow \quad \frac{X}{90\ \Omega} = \frac{40}{60}$$

$$\Rightarrow \quad X = \frac{40}{60} \times 90\ \Omega = 60\ \Omega.$$

Now, $X = R\left(\dfrac{l}{100\text{ cm} - l}\right)$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{\Delta l}{l} + \frac{\Delta l}{100\text{ cm} - l} = \frac{0.1}{40} + \frac{0.1}{60} = \frac{1}{240}.$$

$$\therefore \quad \Delta X = \frac{1}{240} \times 60\ \Omega = 0.25\ \Omega$$

$$\Rightarrow \quad X = (60 \pm 0.25)\,\Omega.$$

6. Main scale division (MSD) = 0.05 cm

and vernier scale division (VSD) = $\dfrac{4.9\text{ cm}}{100}$ = 0.049 cm.

∴ least count (LC) = 1 MSD – 1 VSD = 0.05 cm – 0.049 cm = 0.001cm.

$$\begin{aligned} \Rightarrow \quad \text{Diameter} &= \text{MS} + \text{VC} \times \text{LC} \\ &= (5.10 + 24 \times 0.001)\text{ cm} \\ &= 5.124\text{ cm.} \end{aligned}$$

7. With the first callipers,

MSD = 2.8 cm and VSD × LC = 7 × $\dfrac{1}{10}$ mm = 0.07 cm.

∴ the reading is 2.87 cm.

With the second callipers,

MSD = 2.8 cm and vernier scale reading = 7 × $\dfrac{-0.1}{10}$ cm = –0.07 cm.

∴ the reading is 2.80 + 0.10 – 0.07 = 2.83 cm.

8. $M = \dfrac{4}{3}\pi R^3 \rho = \text{constant}$

$$\Rightarrow \quad \frac{4}{3}\pi 3R^2\left(\frac{dR}{dt}\right)\rho + \frac{4}{3}\pi R^3 \frac{d\rho}{dt} = 0$$

$$\Rightarrow \quad \frac{dR}{dt} = -\frac{R}{3\rho}\frac{d\rho}{dt}$$

$$\Rightarrow \quad v \propto R.$$

9. $$T = \sqrt{\frac{2L}{g}} + \frac{L}{v}$$

$$\Rightarrow \quad \frac{\delta T}{\delta L} = \frac{1}{\sqrt{2gL}} + \frac{1}{v}.$$

$$\therefore \quad \delta L = \frac{\delta T}{\dfrac{1}{\sqrt{2gL}} + \dfrac{1}{v}} = \frac{\delta T}{\dfrac{1}{20} + \dfrac{1}{300}} = \frac{150}{8}\delta T.$$

$$\therefore \quad \frac{\delta L}{L}\times 100\% = \frac{150}{8}\frac{\delta T}{L}\times 100\%$$

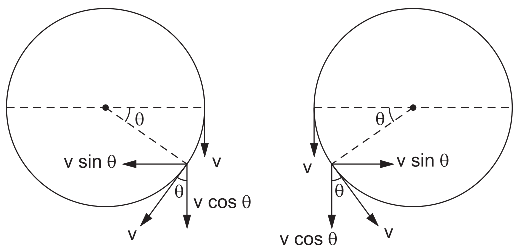
$$= \frac{150}{8}\times\frac{0.01}{20}\times 100\% = \frac{15}{16}\% \approx 1\%.$$

10. $\vec{S} = \vec{P} + b|\vec{R}| = \vec{P} + b|\vec{Q} - \vec{P}| = \vec{P}(1-b) + b\vec{Q}$.

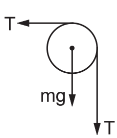
11. v is negative when the ball is falling and positive when it bounces up. Also, $v = -\sqrt{2g(d-h)}$ for downward motion and $v = \sqrt{g(d-2h)}$ for upward motion.

12. Relative velocity $= |\vec{v}_r| = 2v\sin\theta = 2v\sin\omega t$.

In each rotation, the relative speed becomes zero twice and reaches the maximum value twice.



13. For the block of mass M to be in equilibrium, the tension in the string is $T = Mg$. The force on the pulley by the clamp must balance all the other forces acting on it, i.e., the resultant of the forces shown in the figure. This has a magnitude of $\sqrt{(Mg+mg)^2 + (Mg)^2} = g\sqrt{(M+m)^2 + M^2}$.



14. The momentum of the two-particle system at $t = 0$ is given by

$$\vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2.$$

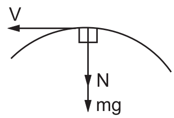
A collision between the two does not affect the total momentum of the system.

A constant external force $(m_1 + m_2)g$ acts on the system. The impulse given by this force in the time interval from $t = 0$ to $t = 2t_0$ is $(m_1 + m_2)g \times 2t_0$.

$\therefore$ the absolute change in momentum in this interval is

$$|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)| = 2(m_1 + m_2)gt_0.$$

15. The blocks will have the same speed, say, equal to v, at the highest point of each track, as they all rise to the same height. Let R be the radius of curvature of a track and N be the normal reaction of the track at the highest point of the track.



Centripetal force $= N + mg = \dfrac{mv^2}{R}$.

N will be the maximum when R is the minimum. This occurs when the track is most sharply curved.

16. The linear acceleration of the bead is $a = L\,\alpha$.

$\therefore$ the reaction force on the bead due to the rod is $N = ma = mL\alpha$.

After the time t, the angular velocity of the bead is $\omega = \alpha t$.

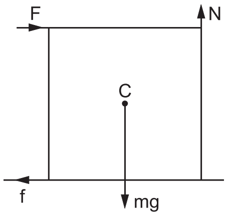
$\therefore$ the centripetal acceleration of the bead is $\omega^2 L = \alpha^2 t^2 L$.

$\therefore$ the force of friction at the limiting position is $\mu N = \mu mL\alpha$.

$\therefore$ for slipping,

$$\mu mL\alpha = m\alpha^2 t^2 L \qquad \text{or} \qquad t = \sqrt{\mu/\alpha}.$$

17. When F is applied, the normal reaction (N) of the floor moves to the right. The cube topples when N reaches its edge.

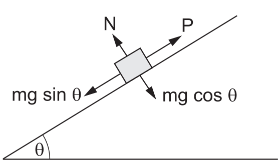


Here, $N = mg$ and the force of friction is $f = F$.

Taking the torque about the centre C,

$$F \times \frac{L}{2} + f \times \frac{L}{2} = N \times \frac{L}{2} \qquad \text{or} \qquad 2F = mg.$$

18. The force of friction, f, becomes zero when $P = mg \sin\theta$. For $P < mg \sin\theta$, f acts upwards along the incline.



19. Taking the torque about A,

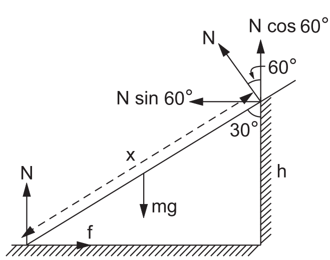
$$N\left(\frac{h}{\sin 60^\circ}\right) = mg\,\frac{l}{2}\cos 60^\circ$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}}Nh = \frac{1}{4}mgl \qquad \text{...(i)}$$

Again, $N + N\cos 60^\circ = mg$

$$\Rightarrow \quad \frac{3}{2}N = mg$$

$$\Rightarrow \quad N = \frac{2}{3}mg.$$



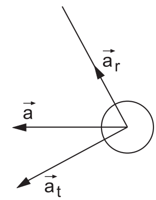
Substituting N in (i),

$$\frac{2}{\sqrt{3}}\cdot\frac{2}{3}mgh = \frac{1}{4}mgl$$

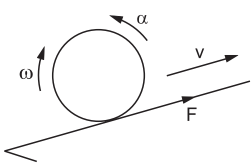
$$\Rightarrow \qquad \frac{h}{l} = \frac{3\sqrt{3}}{16}.$$

Now, friction $= f = N \sin 60° = \frac{2}{3}mg\cdot\frac{\sqrt{3}}{2} = \frac{mg}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$ N.

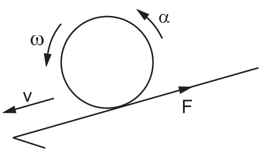
20. In the given position, the bob is moving along a circular path with some speed and hence has some radial acceleration $\vec{a}_r$. It also has some tangential acceleration $\vec{a}_t$. As $\vec{a}$ is the resultant of these two, it will be directed somewhere between them.



21. When the cylinder rolls up the incline, its angular velocity ω is clockwise and decreasing. This requires an anticlockwise angular acceleration α, which is provided by the force of friction (F) acting up the incline.



When the cylinder rolls down the incline, its angular velocity ω is anticlockwise and increasing. This requires an anticlockwise angular acceleration α, which is provided by the force of friction (F) acting up the incline.



22. The angular momentum (L) of the system is conserved; i.e., $L = I\omega$ = constant.

When the tortoise walks along a chord, it first moves closer to the centre and then away from the centre. Hence, I first decreases and then increases. As a result, ω will first increase and then decrease. Also, the change in ω will be a nonlinear function of time.

23. A centripetal force is provided by the component of the tension along the horizontal. So,

$$T\sin\theta = m\omega^2 r = \frac{Tr}{L} \;\Rightarrow\; \omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324\ \text{N}}{(0.5\ \text{kg})\,(0.5\ \text{m})}} = 36\ \text{rad s}^{-1}.$$

24. Work done $= \int \vec{F}\,.\,\vec{dr} = \int \vec{F}\,.\,(dx\hat{i} + dy\hat{j})$

$$= \frac{K}{a^3}\int_a^0 x\,dx + \frac{K}{a^3}\int_0^a y\,dy \qquad [\because x^2 + y^2 = a^2]$$

$$= \frac{K}{a^3}\left(-\frac{a^2}{2} + \frac{a^2}{2}\right) = 0.$$

[Note: $\vec{F} = \frac{K}{a^3}(x\hat{i} + y\hat{j}) = \frac{K}{a^3}\vec{r}$ (radial).
Any radial force does zero work in circular motion.]

25. For energy conservation, $\frac{1}{2}mv^2 = mgR(1 - \cos\theta)$, and for circular motion, $mg\cos\theta - N = \frac{mv^2}{R}$.

Simplifying, $N = mg(3\cos\theta - 2)$.

Initially, $\theta = 0$, i.e., $N = mg$ (radially inward).

As θ increases, N decreases. At $\theta = \cos^{-1}\left(\frac{2}{3}\right) \approx 48°$, N reduces to zero, and afterwards N acts radially outward.

26. The area under the F–t curve is 4.5 kg m s^{-1} = p.

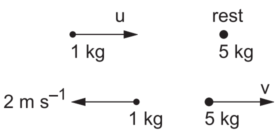
$\therefore \text{KE} = \frac{p^2}{2m} = 5.06$ J.

27. $u = -2$ m s^{-1} + $5v$.

$v + 2$ m s^{-1} = $-1(0 - u) = u$.

Solving, $u = 3$ m s^{-1} and $v = 1$ m s^{-1}.

KE of CM = $\frac{1}{2}$ (6 kg) (0.5 m s^{-1})^2 = 0.75 J.



28. $h = \frac{1}{2}gt^2 \Rightarrow 5 = \frac{1}{2}(10)t^2 \Rightarrow t = 1$ s.

From $x = v_x t$ we have v_{ball} = 20 m s^{-1} and v_{bullet} = 100 m s^{-1}.

Conserving linear momentum, 0.01 v = 0.01 × 100 m s^{-1} + 0.2 × 20 m s^{-1}
$\Rightarrow v$ = 500 m s^{-1}.

29. At the highest point, the velocity of the 1st particle is $\vec{v}_1 = (u_0\cos\alpha)\hat{i}$.

At the position of collision $\left(H = \frac{u_0^2\sin^2\alpha}{2g}\right)$, the velocity of the 2nd particle is

$$\vec{v}_2 = \sqrt{u_0^2 - 2gH}\ \hat{j} = \sqrt{u_0^2 - 2g\left(\frac{u_0^2\sin^2\alpha}{2g}\right)}\ \hat{j} = (u_0\cos\alpha)\ \hat{j}.$$

Conserving linear momentum,

$$2m\vec{v} = m\vec{v}_1 + m\vec{v}_2 = m(u_0\cos\alpha)(\hat{i} + \hat{j})$$

$$\Rightarrow \quad \vec{v} = \frac{u_0\cos\alpha}{2}(\hat{i} + \hat{j}). \quad \therefore\ \theta = 45°.$$

30. Kinetic energy $= K = \frac{1}{2}mv^2$.

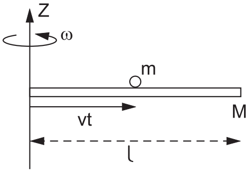
$$\therefore \frac{dK}{dt} = \frac{1}{2}m\left(2v \cdot \frac{dv}{dt}\right).$$

31. $$v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(10\text{ kg})(14\text{ m s}^{-1}) + (4\text{ kg})(0)}{10\text{ kg} + 4\text{ kg}} = 10\text{ m s}^{-1}.$$

32. Angular momentum $= L = I\omega = m(vt)^2\omega = mv^2\omega t^2$.

$$\therefore \text{torque} = \tau = \frac{dL}{dt} = 2mv^2\omega t.$$

So, $\tau \propto t$, which represents a straight line passing through the origin.



33. $\overrightarrow{L_O}$ has a constant magnitude and a constant direction (perpendicular to the plane of rotation); $\overrightarrow{L_P}$ has a constant magnitude, but its direction continuously changes with time.

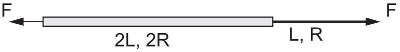
34. At $t = \frac{T}{8} = \frac{1}{8}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{4\omega}$,

the x-coordinate of P is $(\omega R)\left(\frac{\pi}{4\omega}\right) = \frac{\pi R}{4} > R\cos 45°$.

$\therefore$ both P and Q land in the unshaded region.

35. If x_1 and x_2 be the elongations in the thick and thin wires respectively, the Young modulus

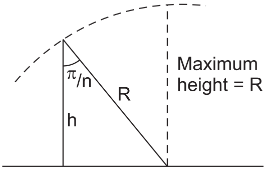
$$Y = \frac{F/A_1}{x_1/L_1} = \frac{F/A_2}{x_2/L_2}.$$



$$\therefore \quad \frac{x_2}{x_1} = \frac{A_1}{A_2}\cdot\frac{L_2}{L_1} = \frac{2}{1}.$$

36. $\cos\left(\frac{\pi}{n}\right) = \frac{h}{R}$

$$\Rightarrow \Delta = R - h = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right].$$



37. For a satellite in a circular orbit around the earth, the time period T depends on the radius r as $T^2 \propto r^3$. Here, for the geostationary satellite, $T_1 = 24$ h and $r_1 = 36000$ km. For the spy satellite, $r_2 = 6400$ km and if the time period be T_2,

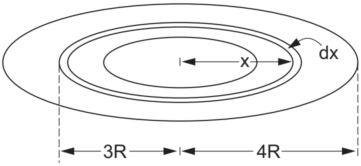
$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} \text{ or } T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = (24\text{ h})\left(\frac{6400\text{ km}}{36000\text{ km}}\right)^{3/2} \approxeq 2\text{ h}.$$

38. Mass per unit area $= \sigma = \dfrac{M}{\pi(4R)^2 - (3R)^2}$,

area of ring $= 2\pi x dx$,

and mass of ring $= dm = 2\pi\sigma x dx$.

$$\therefore\ V_p = -G\int_{3R}^{4R} \frac{dm}{[(4R)^2 + x^2]^{1/2}}.$$



39. For the object to escape, $\text{KE} + \left(-\dfrac{GMm}{r}\right) = 0.$

For circular motion, $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$.

So, $\dfrac{GMm}{r} = mv^2 =$ required KE.

40. The value of g at a distance r (where $r < R$) from the centre of a planet is

$$g = g_0\frac{r}{R} = \frac{4}{3}G\pi r\rho.$$

The force required to keep the wire at rest is

$$F = \text{weight of the wire} = \int dm{\cdot}g$$

$$= \int_{\frac{4R}{5}}^{R} (\lambda dr)\left(\frac{4}{3}\pi G\rho r\right) = \frac{4}{3}G\pi\rho\lambda\left.\frac{r^2}{2}\right|_{4R/5}^{R}$$

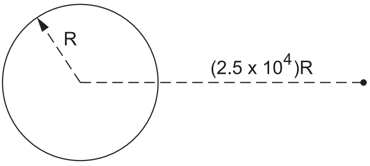
$$= \frac{4}{3}G\pi\rho\lambda\frac{9R^2}{50}.$$

Taking $\rho =$ density of the earth $= \dfrac{M_e}{\frac{4}{3}\pi R_e^3}$ and $R = \dfrac{R_e}{10}$,

we obtain $\quad F = \dfrac{9}{5000}gR_e\lambda = 108\ \text{N}.$

41. The escape speed from the earth is

$$v_e = \sqrt{2gR_e} = \sqrt{\frac{2GM_e}{R}}$$
$$= 11.2\ \text{km s}^{-1}.$$



If v_0 be the escape speed from the earth–sun system then

$$\frac{1}{2}mv_0^2 - \frac{GmM_e}{R} - \frac{GM_S m}{(2.5\times10^4)R} \geqslant 0.$$

For the rocket to just escape,

$$\frac{1}{2}mv_0^2 = \frac{GmM_e}{R} + \frac{GM_S m}{(2.5 \times 10^4)R}$$

$$\Rightarrow \quad \frac{v_0^2}{2} = \frac{GM_e}{R} + \frac{G \times 3 \times 10^5 M_e}{(2.5 \times 10^4)R} = \frac{13GM_e}{R}$$

$$\Rightarrow \quad v_0 = \sqrt{13\left(\frac{2GM_e}{R}\right)} = \sqrt{13}\, v_e = \sqrt{13} \times 11.2 \text{ km s}^{-1}$$

$$= 40.4 \text{ km s}^{-1} \approx 40 \text{ km s}^{-1}.$$

42. Conserving the angular momentum of the system,

$$MR^2\,\omega = MR^2 . \frac{8}{9}\omega + \frac{M}{8}\left(\frac{3}{5}R\right)^2 \times \frac{8\omega}{9} + \frac{M}{8}x^2\left(\frac{8\omega}{9}\right) \Rightarrow x = \frac{4}{5}R.$$

43. The frequency $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$ of a spring–block system is independent of the external fields (e.g., gravity).

44. Time of flight $= T = \frac{2u\sin\theta}{g}$, i.e., $1 \text{ s} = \frac{2u \cdot \sin 45°}{10}$.

$\therefore u = 5\sqrt{2} \text{ m s}^{-1} = \sqrt{50} \text{ m s}^{-1}$.

45. Net upward buoyancy force on the cylinder

= weight of the liquid displaced by it

= $\rho g V$

= (upward force on the bottom) – (downward force on the top).

$\therefore$ the force on the bottom is $\rho g V + (h\rho g)\pi R^2 = \rho g(V + \pi R^2 h)$.

46. When the coin slips into the water, the wooden block moves up and l decreases. When the coin was floating, it displaced water equal to its own weight. When inside the water, it displaces water equal to its own volume. As its density is greater than that of water, it displaced more water in the first case. Hence, h decreases when it falls into the water.

47. $mg + (V_l\rho)g = \frac{V_0}{2} \times \rho \times g \quad \Rightarrow V_l = \frac{V_0}{2} - \frac{m}{\rho}$.

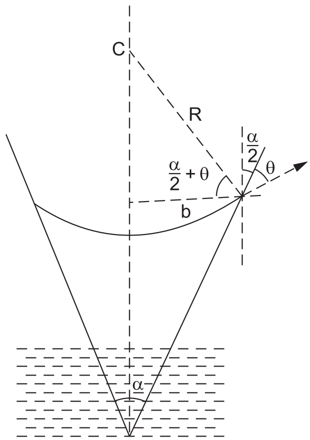
Hence, $V_l < \frac{V_0}{2}$.

48. From the geometry of the figure,

$$\frac{b}{R} = \cos\left(\theta + \frac{\alpha}{2}\right).$$

If p_0 = atmospheric pressure then

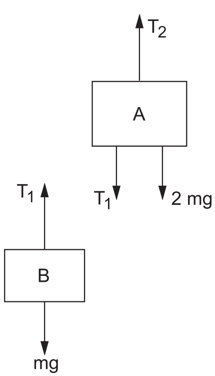
$$p_0 - \frac{2s}{R} + h\rho g = p_0$$



$$\Rightarrow \qquad \frac{2s}{R} = h\rho g$$

$$\Rightarrow \qquad h = \frac{2s}{R\rho g} = \frac{2s}{b\rho g}\cos\left(\theta+\frac{\alpha}{2}\right).$$

49. From the free-body diagram (FBD) of block A,



$$T_2 = T_1 + 2mg.$$

Similarly, for the block B,

$$T_1 = mg \text{ and } T_2 = 3mg.$$

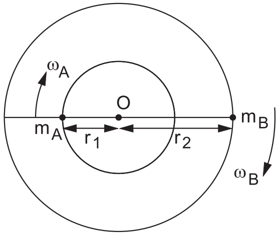
When the string is cut, $T_1 = 0$.

$\therefore$ for B, $mg = ma_B \Rightarrow a_B = g$.

For A, $3mg - 2mg = 2ma_A \Rightarrow a_A = \frac{g}{2}$.

50. The gravitational attraction provides the required centripetal force.

Thus, $\dfrac{Gm_1m_2}{(r_1+r_2)^2} = m_A\omega_A^2r_1 = m_B\omega_B^2r_2$.



Further, the system rotates about the centre of mass O. So,

$$m_Ar_1 = m_Br_2$$

$$\Rightarrow \qquad \omega_A = \omega_B.$$

$$\therefore \qquad T_A = T_B.$$

51. Since the mass of the sphere is unchanged, we have

$$\left(\frac{2}{5}MR^2\right)_{\text{sphere}} = \left(\frac{1}{2}MR_0^2 + MR_0^2\right)_{\text{disc}}$$

or $$\frac{2}{5}R^2 = \frac{3}{2}R_0^2.$$

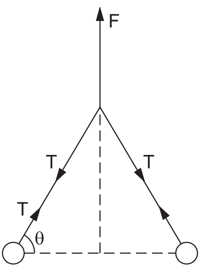
$\therefore$ the radius of the disc is $R_0 = \dfrac{2R}{\sqrt{15}}\cdot$

52. $$2T\sin\theta = F$$

and $$T\cos\theta = m\cdot a_x.$$

$\therefore$ $$2\tan\theta = \frac{F}{ma_x}$$

$\Rightarrow$ $$a_x = \frac{F}{2m\tan\theta} = \frac{F}{2m}\left(\frac{x}{\sqrt{a^2 - x^2}}\cdot\right)$$



53. By the law of conservation of mechanical energy,

$$\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = mgh = mg\left(\frac{3v^2}{4g}\right)$$

$\Rightarrow$ $$1 + \frac{K^2}{R^2} = \frac{3}{2}\cdot$$

$\therefore$ $\dfrac{K^2}{R^2} = \dfrac{1}{2}$, which is true for a disc.

54. The gravitational system represents a solid sphere of radius R having a uniform density of $\rho = \rho_0$.

For $r < R$, force $= \dfrac{G\frac{4}{3}\pi r^3 \rho m}{r^2} = \dfrac{mv^2}{r}\cdot$

$\therefore$ $v \propto r$ (a straight line from $r = 0$ to $r = R$).

For $r > R$, $F = \dfrac{G\frac{4}{3}\pi R^3 \rho m}{r^2} = \dfrac{mv^2}{r}$, i.e., $v \propto \dfrac{1}{\sqrt{r}}\cdot$

$\therefore$ the correct plot is the option (c).

55. The critical speed at A is $\sqrt{5gL}$.

Conserving energy from A to C,

$$\frac{1}{2}m\left(\sqrt{5gL}\right)^2 = \frac{1}{2}mv_c^2 + mgL(1 - \cos\theta)$$

or $\cos\theta = -\dfrac{7}{8}$ $\left[\because v_C = \dfrac{v_A}{2} = \dfrac{1}{2}\sqrt{5gL}\right]$

$= -0.87.$

since $\cos\dfrac{3\pi}{4} = -\dfrac{1}{\sqrt{2}} = -0.71,$

we get $\dfrac{3\pi}{4} < \theta < \pi.$

56. The pressure inside a spherical soap bubble exceeds the outer (atmospheric) pressure p_0 by $4S/R$, where S is the surface tension of the soap solution, and R is the radius.

Thus, the pressure inside the bubble is $p = p_0 + \dfrac{4S}{R}.$

$\because\ R_2 > R_1, \quad \therefore\ p_2 < p_1.$

Hence, air flows from 1 to 2.

57. By the law of conservation of mechanical energy,

$$\frac{1}{2}kx^2 = \frac{1}{2}(4k)\,y^2$$

$$\Rightarrow \quad \frac{y}{x} = \frac{1}{2}.$$

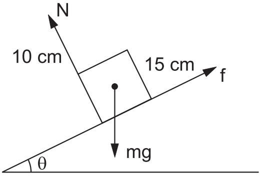
58. From the given x–t graph of SHM, displacement $= x = A \sin \omega t$.

$\therefore\ v = A\omega\cos\omega t$ and $a = -A\omega^2 \sin\omega t$.

Given that $A = 1$ cm, $\omega = \dfrac{2\pi}{8}$ rad s^{-1} and $t = \dfrac{4}{3}$ s

$$\Rightarrow\ a = -(1\text{ cm})\left(\frac{2\pi}{8}\text{ s}^{-1}\right)^2 \sin\left(\frac{2\pi}{8}\cdot\frac{4}{3}\right) = -\frac{\sqrt{3}\,\pi^2}{32}\text{ cm s}^{-2}.$$

59. The block slides down the plane when



$$mg\sin\theta \ge f = \mu N = \mu mg\cos\theta$$

or $\tan\theta \ge \mu = \sqrt{3} = 1.732.$

The block topples without sliding when

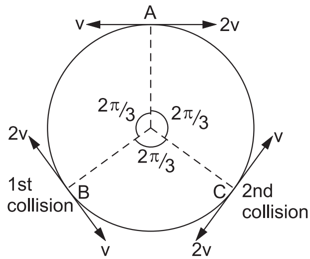
$$f\left(\frac{15}{2}\right) \ge N(5) \qquad \text{(taking torque about CM)}$$

or $\mu N\left(\dfrac{15}{2}\right) \ge N(5)$

or $\mu \ge \dfrac{2}{3} = 0.67$ or $\tan\theta \ge 0.67.$

Thus, the block will neither slip nor topple till $\theta \le \tan^{-1}(0.67)$, and when θ exceeds this, it topples before sliding occurs.

60. The angular displacement of the faster particle is 2θ and that of the slower particle is θ. So,



$$2\theta + \theta = 2\pi$$

$$\Rightarrow \quad \theta = \frac{2\pi}{3}.$$

Hence, the first collision will occur at B, where the velocities are interchanged due to equal masses and their elastic collision. The second collision will occur at C and the third at A. Thus, two collisions will occur before meeting at A.

61. The springs are in series; so restoring force is the same in both the springs. Hence,

$$k_1x_1 = k_2x_2.$$

Given that $\quad x_1 + x_2 = A.$

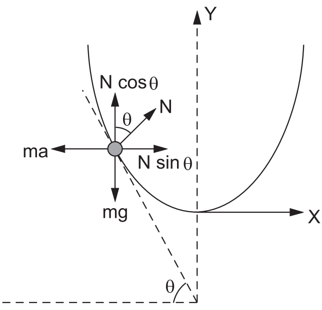
$$\therefore \quad x_1 = \frac{Ak_2}{k_1 + k_2}.$$

62. Forces acting on the bead as shown in the free-body diagram are

(i) weight (mg)

(ii) normal reaction (N)

(iii) pseudo force (ma).



Resolving N, we get for balance

$$N \sin\theta = ma \text{ and } N\cos\theta = mg.$$

$$\therefore \quad \tan\theta = \frac{a}{g}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{a}{g}.$$

But $\quad y = kx^2.$

So, $$\frac{dy}{dx} = 2kx$$

$$\Rightarrow \quad x = \frac{a}{2gk}.$$

63. When the rod is turned through an angle θ, the restoring torque (opposing θ) is

$$\tau = -2k\left(\frac{L}{2}\theta\right)\frac{L}{2} = I\frac{d^2\theta}{dt^2}.$$

$$\therefore \qquad \frac{d^2\theta}{dt^2} = \frac{\frac{kL^2}{2}(-\theta)}{\frac{ML^2}{12}} = -\left(\frac{6k}{M}\right)\theta.$$

This represents an angular SHM for which

$$\omega = \sqrt{\frac{6k}{M}} = \frac{2\pi}{T} \text{ or } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}.$$

1.2 Heat and Thermodynamics

1. Let ρ_{Fe} and ρ_{Hg} denote respectively the densities of iron and mercury at 0 °C, and m be the mass of the block.

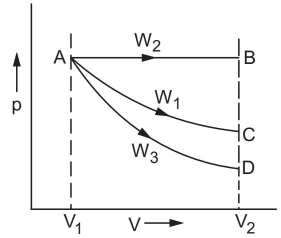
   $\therefore$ the volume of the block is m/ρ_{Fe} and that of the displaced mercury is $k_1 m/\rho_{Fe}$.

   $$\therefore \left(\frac{k_1 m}{\rho_{Fe}}\right)\rho_{Hg} = m \quad \text{or} \quad k_1 = \frac{\rho_{Fe}}{\rho_{Hg}}. \quad \text{Also, } \rho_t = \frac{\rho_0}{1+\gamma t}.$$

   $$\therefore \; k_2 = \frac{\rho_{Fe}}{1+(\gamma_{Fe}\times 60\,°\text{C})} \times \frac{1+(\gamma_{Hg}\times 60\,°\text{C})}{\rho_{Hg}} = k_1 \frac{1+(60\,°\text{C})\gamma_{Hg}}{1+(60\,°\text{C})\gamma_{Fe}}$$

   $$\text{or} \quad \frac{k_1}{k_2} = \frac{1+(60\,°\text{C})\,\gamma_{Fe}}{1+(60\,°\text{C})\,\gamma_{Hg}}.$$

2. The three processes are plotted on a p–V diagram. AB is isobaric, AC is isothermal, and AD is adiabatic. In each case, the work done is equal to the area under the curve.



3. From the first law of thermodynamics, $Q = \Delta U + W$. For a cyclic process, $\Delta U = 0$. Also, $Q = 5$ J.

   $W = W_{AB} + W_{BC} + W_{CA}$.

   $W_{AB} = p\Delta V = (10 \text{ N m}^{-2})(1 \text{ m}^3) = 10$ J and $W_{BC} = 0$.

   $\therefore \quad 5 \text{ J} = 10 \text{ J} + W_{CA} \qquad \text{or} \qquad W_{CA} = -5$ J.

4. $TV^{\gamma-1}$ = constant and γ for helium (a monatomic gas) is $\frac{5}{3}$.

   $\therefore \quad T_1(5.6)^{2/3} = T_2(0.7)^{2/3}$, hence $T_2 = 4T_1$.

   $$\therefore \quad W = \frac{nR}{\gamma-1}(T_2 - T_1) = \frac{1}{4}\frac{R}{2/3}(4T_1 - T_1) = \frac{9}{8}RT_1.$$

5. $\dfrac{V_{\text{rms(He)}}}{V_{\text{rms(A}_r)}} = \dfrac{\sqrt{3RT/m_{\text{He}}}}{\sqrt{3RT/m_{\text{Ar}}}} = \sqrt{\dfrac{m_{\text{Ar}}}{m_{\text{He}}}} = \sqrt{\dfrac{40}{4}} = \sqrt{10} = 3.16.$

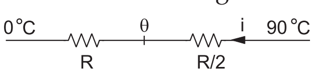
6. $\Delta Q = nC_{\text{P}}\Delta T = n\left(\dfrac{\gamma}{\gamma-1}R\right)\Delta T = 2\left(\dfrac{5}{2}R\right)\Delta T = 2\left(\dfrac{5}{2}\times\dfrac{25}{3}\right)\times 5\ \text{J} = 208\ \text{J}.$

7. At low pressures, the number of molecules per unit volume decreases, so that the 'point mass' approximation holds. Also, at high temperatures, the average speeds increase.

8. (Internal energy) $\propto$ (absolute temperature).
$\therefore\ U_{\text{A}} = U_{\text{B}}$.
In the isothermal process AB, $W = nRT \ln(V_{\text{f}}/V_{\text{i}}) = p_0V_0 \ln(V_{\text{B}}/V_{\text{A}})$.

9. $pV = nRT$. At constant pressure, $p\Delta V = nR(\Delta T)$.
Dividing, $\dfrac{\Delta V}{V} = \dfrac{\Delta T}{T}$ or $\delta = \dfrac{\Delta V}{V(\Delta T)} = \dfrac{1}{T}$ or $\delta \propto \dfrac{1}{T}$.

10. As the three rods are made of the same material and have the same dimensions, they have the same thermal resistance, say R. Let θ be the temperature of the junction.
The circuit becomes as shown in the figure below.

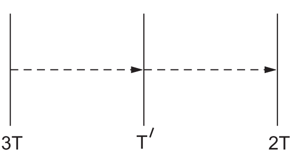


$$\therefore \qquad i = \frac{90\ °\text{C} - \theta}{R/2} = \frac{\theta - 0\ °\text{C}}{R}$$

or $\quad 180\ °\text{C} - 2\theta = \theta \quad$ or $\quad \theta = 60\ °\text{C}$.

11. A furnace behaves as a black body.

12. In the steady state, absorption and emission by the middle plate will be the same. Thus,



$$\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$$

$$\Rightarrow \qquad T' = \left(\frac{97}{2}\right)^{1/4} T.$$

13. By the gas laws: $\quad pV = nRT = \dfrac{m}{M}RT$

$$\Rightarrow \qquad pM = \frac{m}{V}RT = \rho RT$$

$$\Rightarrow \qquad \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}\times\frac{M_1}{M_2} = \frac{4}{3}\times\frac{2}{3} = \frac{8}{9}.$$

14. Let the thermal resistances in the two configurations be R_1 and R_2 respectively.

In Configuration I,

$$R_1 = \frac{L}{KA} + \frac{L}{2KA} = \frac{3L}{2KA}.$$

In Configuration II,

$$\frac{1}{R_2} = \frac{1}{\frac{L}{KA}} + \frac{1}{\frac{L}{2KA}} = \frac{3KA}{L}$$

$$\Rightarrow \quad R_2 = \frac{L}{3KA}.$$

$$\because \quad \Delta Q_1 = \Delta Q_2,$$

$$\therefore \quad \left(\frac{\Delta\theta}{R_1}\right)t_1 = \left(\frac{\Delta\theta}{R_2}\right)t_2$$

$$\Rightarrow \quad t_2 = \left(\frac{R_2}{R_1}\right)t_1 = 2 \text{ s}.$$

15. Heat generated = (3 kW) (3 h) = $3 \times 10^3 \times 3 \times 3600$ J = 324×10^5 J.

Heat absorbed by water = $mc_w\Delta\theta$ = (120)(4200)(20) J = 100.8×10^5 J.

Heat absorbed by the coolant,

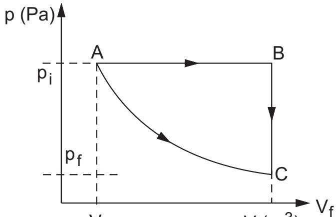
$$Pt = (324 \times 10^5 - 100.8 \times 10^5) \text{ J}$$

$$\Rightarrow \quad P = \frac{223.2 \times 10^5 \text{ J}}{3 \times 3600 \text{ s}} = 2067 \text{ W}.$$

16. Given that p^3V^5 = constant $\Rightarrow pV^{5/3}$ = constant.

$\therefore$ $\gamma = 5/3$ (for a monatomic gas).

For the isobaric process AB,



$$\Delta Q_1 = nc_p\,\Delta T = n\left(\frac{5}{2}R\right)\Delta T$$

$$= \frac{5}{2}p\,\Delta V$$

$$= \frac{5}{2}(10^5 \text{Pa})\,(7 \times 10^{-3} \text{m}^3)$$

$$= 1750 \text{ J (heat absorbed)}.$$

For the isochoric process BC,

$$\Delta Q_2 = nc_V\Delta T = n\left(\frac{3}{2}\right)R\,\Delta T$$

$$= \frac{3}{2}(Vdp) = \frac{3}{2}(8 \times 10^{-3} \text{ m}^3)\left(\frac{1}{32} - 1\right) \times 10^5 \text{ Pa}$$

$$= -1162.5 \text{ J (heat expelled)}.$$

So, the net heat supplied to the system is 1750 J + (–1162.5 J)

$$= 587.5 \text{ J} \approx 588 \text{ J}.$$

17. In the steady state,

the rate of energy loss = rate of energy incidence

$$\Rightarrow \quad \pi R^2 \cdot I = \sigma \cdot 4\pi R^2 \cdot (T^4 - T_0^4)$$

$$\Rightarrow \quad I = 4\sigma(T^4 - T_0^4).$$

$$\therefore \quad T^4 - T_0^4 = 40 \times 10^8 \text{ K}^4$$

$$\Rightarrow \quad T^4 - 81 \times 10^8 \text{ K}^4 = 40 \times 10^8 \text{ K}^4$$

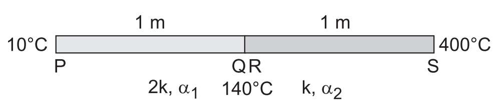
$$\Rightarrow \quad T^4 = 121 \times 10^8 \text{ K}^4$$

$$\Rightarrow \quad T \approx 330 \text{ K}.$$

18. For an element dx at a distance x from the end P, the change in length

$$dl = \alpha_1\, dx(\theta - 10) \text{ or } \Delta l = \int dl,$$

where θ is the temperature at the distance x from the end P.



Temperature gradient in PQ $= \dfrac{\theta - 10}{x} = \dfrac{140 - 10}{1}$

$$\Rightarrow \quad \theta = 130x + 10$$

$$\Rightarrow \quad \Delta l = \int_0^1 (130x)\alpha_1\, dx = 130\,\alpha_1 \left.\frac{x^2}{2}\right|_0^1 = 65 \times 1.2 \times 10^{-5} \text{ m}$$

$$= 78.0 \times 10^{-5} \text{ m} = 0.78 \text{ mm}.$$

19. Given that pT^2 = constant. But $pV = nRT$.

$$\therefore \quad \left(\frac{nRT}{V}\right)T^2 = \text{constant}.$$

$$\Rightarrow \quad T^3 = KV. \qquad \text{...(1)}$$

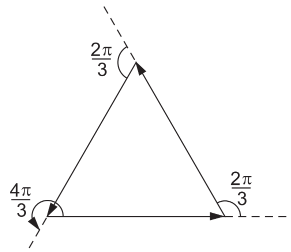
Differentiating, $\quad 3T^2 dT = KdV. \qquad \text{...(2)}$

Dividing (2) by (1), $\quad \dfrac{dV}{V} = \dfrac{3dT}{T}.$

$$\therefore \quad \gamma = \frac{1}{V}\frac{dV}{dT} = \frac{3}{T}.$$

1.3 Sound Waves

1. After 2 s, the pulses will overlap completely. The string becomes straight and therefore does not have any potential energy. Its entire energy must be kinetic.
2. Conclude from the vector triangle (equilateral).



3. A stationary wave has the equation of the form

$$y = A \sin kx \sin \omega t.$$

Here, for y_1, we have $k_1 = \frac{\pi}{L}$ and $\omega_1 = \omega$.

$$\therefore \quad v_1 = \frac{\omega_1}{k_1} = \frac{\omega L}{\pi}.$$

For y_2, we have $k_2 = \frac{2\pi}{L}$ and $\omega_2 = 2\omega$.

$$\therefore \quad v_2 = \frac{\omega_2}{k_2} = \frac{\omega L}{\pi} = v_1.$$

Thus, the wave velocities are the same in both the cases. Also, they have the same amplitude. The frequency for y_2 is twice the frequency for y_1.

Now, since energy $\propto$ (frequency)^2,

$$\therefore \quad E_2 = 4E_1.$$

4. $\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{T}{A\rho}} = \frac{1}{2l}\sqrt{\frac{T}{\pi R^2 \rho}} = \frac{1}{2lR}\sqrt{\frac{T}{\pi\rho}}.$

As T and ρ are the same for both the wires, $\nu \propto \frac{1}{lR}$.

Here, $LR = L \times 2r = 2L \times r$. $\quad \therefore \nu_1 = \nu_2$.

5. The frequency of vibration of a string is given by

$$n = \frac{N}{2l}\sqrt{\frac{T}{m}},$$

where N = number of loops (or segments) = number of antinodes. The other symbols have their usual meanings.

Here, $\quad n = \frac{5}{2l}\sqrt{\frac{9g}{m}} = \frac{3}{2l}\sqrt{\frac{Mg}{m}} \quad$ or $\quad M = 25$ kg.

6. $\frac{\lambda}{4} = L + e$

$\Rightarrow L = \frac{\lambda}{4} - e = \frac{v}{4f} - 0.6R = 16.4 \text{ cm} - 1.2 \text{ cm} = 15.2 \text{ cm}.$

7. $\dfrac{V_{\text{sound}}}{2l_{\text{pipe}}} = 2\left(\dfrac{1}{2l_{\text{string}}}\right)\sqrt{\dfrac{T}{m}},$

mass of the string = ml_s = 10 g.

8. $n_o = \dfrac{n_s V}{V - v_s}.$

Here, $f_1 = \dfrac{n_s V}{340 \text{ m s}^{-1} - 34 \text{ m s}^{-1}}$ and $f_2 = \dfrac{n_s V}{340 \text{ m s}^{-1} - 17 \text{ m s}^{-1}}.$

$\therefore \quad \dfrac{f_1}{f_2} = \dfrac{340 \text{ m s}^{-1} - 17 \text{ m s}^{-1}}{340 \text{ m s}^{-1} - 34 \text{ m s}^{-1}} = \dfrac{19}{18}.$

9. When an observer approaches a stationary source, which is emitting sound waves of frequency n_s, he hears a frequency n, given by

$$n = n_s\left(1 + \frac{v}{V}\right),$$

where v = velocity of the observer and V = velocity of sound.

Here, let v_A and v_B be the velocities of the trains A and B.

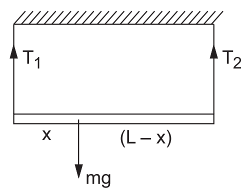
For A, we have n = 5.5 kHz = (5 kHz) $\left(1 + \dfrac{v_A}{V}\right)$ or $\dfrac{v_A}{V} = 0.1.$

For B, we have n = 6 kHz = (5 kHz) $\left(1 + \dfrac{v_B}{V}\right)$ or $\dfrac{v_B}{V} = 0.2.$

$\therefore \quad \dfrac{v_B}{v_A} = 2$ or $v_B : v_A = 2 : 1.$

10. $f' = \left(\dfrac{v + v_o}{v - v_s}\right) f = \left(\dfrac{320 + 10}{320 - 10}\right)(8 \times 10^3 \text{ Hz}) = 8.5 \text{ kHz}.$

11. $T_1 + T_2 = mg,\ T_1 x = T_2 (L - x),$



$f_1 = \dfrac{1}{2l}\sqrt{\dfrac{T_1}{\mu}}$ and $f_2 = \dfrac{2}{2l}\sqrt{\dfrac{T_2}{\mu}},$

where μ = linear mass density and $f_1 = f_2$.

$\therefore \quad T_1 = 4T_2$ or $T_1 x = \dfrac{T_1}{4}(L - x)$ or $x = \dfrac{L}{5}.$

12. The prongs of the tuning fork are kept in a vertical plane in order to set up longitudinal standing waves in the air column.

13. The frequency of the third harmonic of the closed pipe is

$$f = 3\left(\frac{v}{4l}\right) = \frac{3 \times 340 \text{ m s}^{-1}}{4 \times 0.75 \text{ m}} = 340 \text{ Hz}.$$

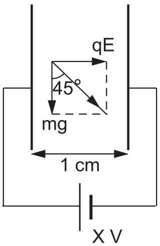
Since the increase in tension reduces the beat frequency from 4 s^{-1} to 2 s^{-1},

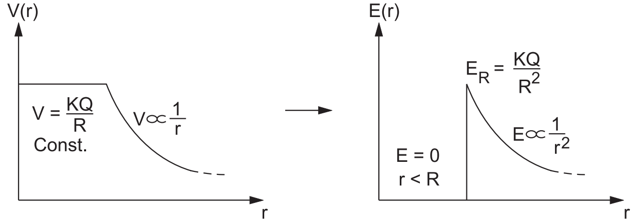
$\therefore \quad n - 340 \text{ Hz} = 4 \text{ Hz} \quad \Rightarrow \quad n = 344 \text{ Hz}.$

14.
$$y = A\sin(\omega t \pm \phi).$$
$$\because \quad A = 10 \text{ cm and } y = 5 \text{ cm},$$
$$\therefore \quad \sin(\omega t \pm \phi) = \frac{1}{2} = \sin 30°$$
$$\Rightarrow \quad \omega t \pm \phi = 30°.$$
Here,
$$\omega = \frac{2\pi}{T} = 2\pi\left(\frac{V}{\lambda}\right) = 2\pi\left(\frac{10 \text{ cm s}^{-1}}{50 \text{ cm}}\right) = \frac{2\pi}{5} \text{ rad s}^{-1}.$$
$\therefore$ the velocity of the point P is
$$v_P = \frac{dy}{dt} = A\omega\cos(\omega t \pm \phi) = (10 \text{ cm})\left(\frac{2\pi}{5}\text{ s}^{-1}\right)(\cos 30°)$$
$$= 2\pi\sqrt{3} \text{ cm s}^{-1} = \frac{\pi\sqrt{3}}{50} \text{ m s}^{-1}.$$
$$\therefore \quad \vec{v}_P = \frac{\pi\sqrt{3}}{50}\hat{j} \text{ m s}^{-1}.$$

1.4 Electrostatics

1. The tangent drawn to a line of force at any point on it must give the direction of the electric intensity at that point. Consider a point midway between any two of the charges. Here, the resultant intensity is only the intensity due to the third charge and must point away from the third charge. This is satisfied only in the sketch (c).
2. Only the option (a) has the dimensions of force.
3. $qE = \frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v$.
4. Flux = (field) (projected area) = $E_0 a^2$.
5.
$$qE = mg$$
$$\Rightarrow \quad eE = m_P g$$
$$\Rightarrow \quad (1.602 \times 10^{-19} \text{ C})\frac{x}{10^{-2}\text{m}} = (1.672 \times 10^{-27} \text{ kg})(10 \text{ m s}^{-2})$$
$$\Rightarrow \quad x \approx 1 \times 10^{-9} \text{ V}.$$



6.


7. The electric field at a point outside the dielectric sphere is $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ and that at a point inside is $E = E_{\text{surface}} \times \dfrac{r}{R}$, where r is the distance from the centre.

$\therefore\ E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2},\ E_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{R^2}$ and $E_3 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{4Q}{4R^2}\dfrac{R}{2R}$.

$\therefore\ E_1 : E_2 : E_3 = 2 : 4 : 1.$

8. The $\pm q$ charges appearing on the inner surfaces of A are bound charges. As B is without charge initially and is isolated, the charges on A will not be affected on closing the switch S. No charge will flow into B.

9. The initial energy of the system is given by $E_\text{i} = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2$.

When the capacitors are joined, they reach a common potential equal to $\dfrac{\text{total charge}}{\text{total capacity}} = \dfrac{CV_1 + CV_2}{2C} = \dfrac{V_1 + V_2}{2} = V.$

The final energy of the system is $E_\text{f} = \frac{1}{2}(2C)V^2$.

$\therefore$ the decrease in energy is $E_\text{i} - E_\text{f} = \frac{1}{4}C(V_1 - V_2)^2$.

10. Fractional loss in energy $= \dfrac{\Delta U}{U}$,

where $\Delta U = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2 = \dfrac{1}{2}\dfrac{2\times 8}{10}(V - 0)^2 = \dfrac{8}{10}V^2$.

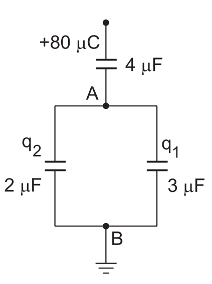
Initial energy $= U = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(2)V^2 = V^2$.

$\therefore$ per cent dissipation $= \dfrac{\Delta U}{U} \times 100\%$.

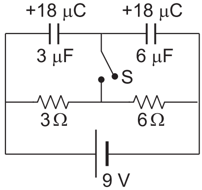
11. The potential difference across AB is

$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{80\ \mu\text{C}}{5\ \mu\text{F}} = 16\ V.$$

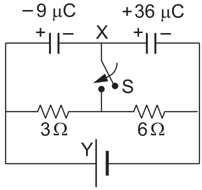
$$\therefore \quad q_1 = (16\ \text{V})\,(3\ \mu\text{F}) = 48\ \mu\text{C}.$$



12. Consider the charge distribution when (i) S is open, (ii) S is closed.



(i)



(ii)

Applying the junction rule at X, the charge that flows from Y to X is (36 – 9) μC = 27μC.

13. If λ is the linear charge density given to the inner cylinder then according to Gauss's law,

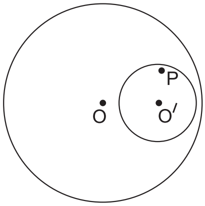
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ for } a < r < b.$$

$$\therefore \text{ potential difference} = \Delta V = -\int \vec{E}\cdot\vec{dr} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}.$$

14. Solution not required.

15. By symmetry, the xy-plane through the origin is equipotential; so at the points A(–a, 0, 0) and B(0, a, 0), $V_A = V_B = 0$. Hence, $W = 0$.

16. Let P be a point in the cavity.



Let $\overrightarrow{\mathrm{OP}} = \vec{r}_1$, $\overrightarrow{\mathrm{O'P}} = \vec{r}_2$ and $\overrightarrow{\mathrm{OO'}} = \vec{a}$.

The field at P due to the entire sphere is $\vec{E}_1 = \frac{\rho}{3\varepsilon_0}(\overrightarrow{\mathrm{OP}})$, and that due to the cavity is $\vec{E}_2 = -\frac{\rho}{3\varepsilon_0}(\overrightarrow{\mathrm{O'P}})$.

$$\therefore \text{ the net field at P is } \vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}(\overrightarrow{\mathrm{OP}} - \overrightarrow{\mathrm{O'P}}) = \frac{\rho}{3\varepsilon_0}(\overrightarrow{\mathrm{OP}} + \overrightarrow{\mathrm{PO'}})$$

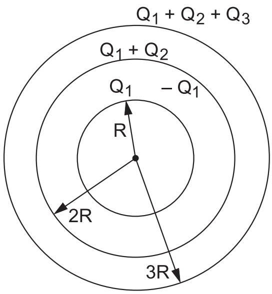
$$= \frac{\rho}{3\varepsilon_0}(\overrightarrow{\mathrm{OO'}}) = \text{constant.}$$

Thus, the field in the cavity is nonzero and uniform.

17. $$F_{BC} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{(2R\sin 60°)^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}.$$

18. The charges enclosed in the cubical surface are (i) half the charge on the disc, (ii) the point charge at $\left(\frac{a}{4}, -\frac{a}{4}, 0\right)$, and (iii) the charge on the rod from $\frac{a}{4}$ to $\frac{a}{2}$, which is $\frac{8\,\mathrm{C}}{4}$. Thus, the electric flux is $\phi = \frac{1}{\varepsilon_0}\Sigma q = \frac{1}{\varepsilon_0}\left[3\,\mathrm{C} - 7\,\mathrm{C} + \frac{8\,\mathrm{C}}{4}\right] = -\frac{2\,\mathrm{C}}{\varepsilon_0}$.

19. The charge distribution on the outer surfaces of the three concentric conducting shells are shown in the figure. For the charge densities to be equal,



$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2} = \text{constant}.$$

$\therefore\ Q_1 = K, Q_1 + Q_2 = 4K$ and $Q_1 + Q_2 + Q_3 = 9K$.

$\therefore\ Q_2 = 3K$ and $Q_3 = 5K$.

$\therefore\ Q_1 : Q_2 : Q_3 = 1 : 3 : 5$.

20. Effective capacitance $= C = \dfrac{\varepsilon_0 A}{d - y + \dfrac{y}{K}}$.

$\because\ A = 1$ and $K = 2$,

$$\therefore\ C = \frac{\varepsilon_0}{d - \dfrac{y}{2}}.$$

Now, $\dfrac{dy}{dt} = -V$

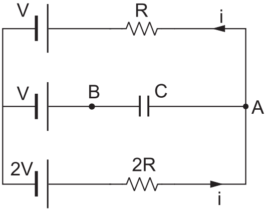
$$\Rightarrow \int_{d/3}^{y} dy = -\int_0^t V\,dt \quad \Rightarrow\ y = \frac{d}{3} - Vt.$$

$$\therefore\quad C = \frac{\varepsilon_0}{d - \dfrac{1}{2}\left(\dfrac{d}{3} - Vt\right)} = \frac{\varepsilon_0}{\dfrac{5}{6}d + \dfrac{Vt}{2}} = \frac{6\varepsilon_0}{5d + 3Vt}.$$

$$\therefore\quad \text{time constant} = \tau = RC = \frac{6\varepsilon_0 R}{5d + 3Vt}.$$

1.5 Current Electricity and Magnetism

1. As the sheet is square, $R = \rho\dfrac{l}{A} = \rho\dfrac{L}{Lt} = \dfrac{\rho}{t}$.
2. G_1 acts as a voltmeter due to the high resistance R_1 in series. The small resistance R_2 in parallel with G_2 acts as a shunt.



3. Moving anticlockwise from A,

$$iR + V - 2V + 2iR = 0$$

or $\quad 3iR = V \quad$ or $\quad i = \dfrac{V}{3R}.$

Then, $V_A - V_B = iR + V - V = iR$

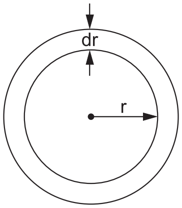
$$= \frac{V}{3} = \text{potential drop across C.}$$

4. As I is independent of R_6, no current flows through R_6. This requires that the junction of R_1 and R_2 is at the same potential as the junction of R_3 and R_4. This must satisfy the condition $R_1/R_2 = R_3/R_4$, as in the Wheatstone bridge.

5. $X(48\text{ cm} + 2\text{ cm}) = (10\ \Omega)\ (52\text{ cm} + 1\text{ cm})$

$$\Rightarrow \quad X = \frac{(10\ \Omega)\ (53\text{ cm})}{50\text{ cm}} = 10.6\ \Omega.$$

6. For an infinite line charge,



$$E = \frac{\lambda}{2\pi\varepsilon r} = -\frac{dV}{dr}$$

or $$dV = -\frac{\lambda}{2\pi\varepsilon}\frac{dr}{r}.$$

The current through the elemental cylindrical shell of radius r and thickness dr, is

$$I = \frac{|dV|}{dR} = \frac{\dfrac{\lambda}{2\pi\varepsilon}\dfrac{dr}{r}}{\dfrac{dr}{\sigma\cdot 2\pi rl}} = \frac{\sigma\lambda l}{\varepsilon}.$$

Since the current is radially outward,

$$I = \frac{d}{dt}(\lambda l) = -\frac{\lambda\sigma l}{\varepsilon} \Rightarrow \frac{d\lambda}{\lambda} = -\frac{\sigma}{\varepsilon}\,dt$$

$$\Rightarrow \quad \lambda = \lambda_0 e^{-\left(\frac{\sigma}{\varepsilon}\right)t}.$$

$$\therefore \quad j(t) = \frac{I}{A} = \frac{I}{2\pi rl} = \frac{\sigma\lambda}{2\pi\varepsilon r} = \frac{\sigma\lambda_0}{2\pi\varepsilon r}\,e^{-\left(\frac{\sigma}{\varepsilon}\right)t}.$$

This corresponds to the graph (a).

7. Power $= P = \dfrac{V^2}{R}.$

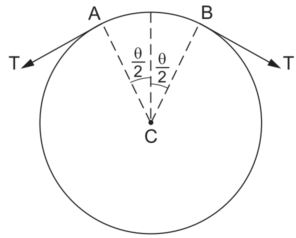
As resistance increases with temperature, the equality $\dfrac{1}{R_{100}} = \left(\dfrac{1}{R_{40}}\right) + \left(\dfrac{1}{R_{60}}\right)$ does not hold exactly. Hence, $\dfrac{1}{R_{100}} > \dfrac{1}{R_{60}} > \dfrac{1}{R_{40}}$ is the best option.

8. The effective current is $i = q \cdot \dfrac{\omega}{2\pi}$ and the area is $A = \pi r^2$.

The magnetic moment is $\mu = Ai = \dfrac{1}{2} q\omega r^2$.

The angular momentum is $L = I\omega = mr^2\omega$. $\therefore \dfrac{\mu}{L} = \dfrac{q}{2\,m}$.

9. $L = 2\pi r \quad \Rightarrow \quad r = \dfrac{L}{2\pi}$.



(Length of the element AB) $= dl = r\theta = \left(\dfrac{L}{2\pi}\right)\theta$.

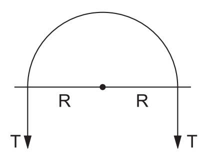
(Outward ampere force on AB) $\Rightarrow BI\,dl$.

(Inward force on AB due to tension T)

$= 2T \sin(\theta/2) = T\theta$.

Equating each of the two forces for equilibrium $T = \dfrac{BIL}{2\pi}$.

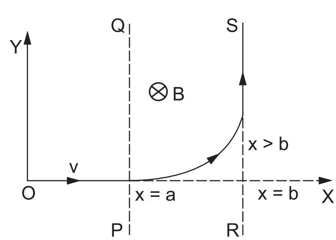
Alternative method



$2T = IB(2R) = IB\left(\dfrac{L}{\pi}\right)$

$\Rightarrow \quad T = \dfrac{IBL}{2\pi}$.

10. In the figure, the z-axis points out of the paper, and the magnetic field is directed into the paper, existing in the region between PQ and RS. The particle moves in a circular path of radius r in the magnetic field. It can just enter the region $x > b$ for $r \geq b - a$.



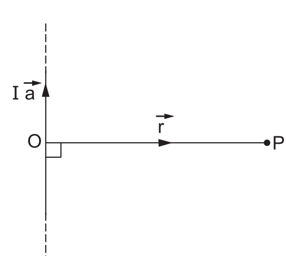
Now, $mv = Bqr$

or $\quad r = \dfrac{mv}{Bq} \geq b - a$

or $\quad v \geq \dfrac{q(b-a)B}{m}$

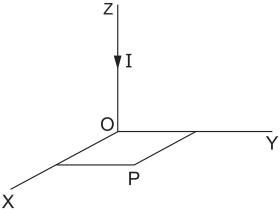
or $\quad v_{\text{min}} = \dfrac{q(b-a)B}{m}$.

11. To find the magnetic field at a point P due to a long straight wire carrying the current I, let $\vec{a}$ be a unit vector in the direction of current flow, and $\vec{r}$ be the vector, normal to $\vec{a}$, from the wire to P. Then the magnetic field at P is



$$\vec{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{\vec{a} \times \vec{r}}{r^2}.$$

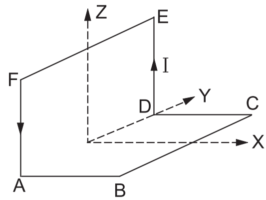
Here, $\vec{a} = -\vec{k}$ and $\vec{r} = x\vec{i} + y\vec{j}$.



$$\therefore \quad \vec{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{(-\vec{k}) \times (x\vec{i} + y\vec{j})}{r^2}$$

$$= \left(\frac{\mu_0 I}{2\pi}\right)\left(\frac{y\vec{i} - x\vec{j}}{x^2 + y^2}\right).$$

12. The conductors BC and EF will together produce a magnetic field at P, which lies in the xz-plane and is equally inclined to the x- and z-axes. The conductors AB and CD will together produce a magnetic field at P directed along the z-axis and of the magnitude B_0 (say). The conductors DE and FA will produce a field at P with the same magnitude B_0, and directed along the x-axis. Thus, the field at P will have equal components in the x- and z-directions, and no component in the y-direction. It must therefore point in the direction $(1/\sqrt{2})\,(\vec{i} + \vec{k})$.



13. The magnetic field lines due to a straight current constitute concentric circles. Hence, the magnetic field between the two wires act in opposite directions. Similarly, the magnetic field to the left of the first wire and that to the right of the second wire are oppositely directed. Hence the correct graph is (b).

14. Impedance = $Z = \left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]^{1/2}$ and current = $I = \frac{V}{Z}$.

When ω increases, Z decreases and I increases.

As power $\propto I^2$, the bulb glows brighter.

15. In the electric field $\vec{E}$, force $= q\vec{E} \Rightarrow \vec{V} \propto q\vec{E} \Rightarrow \vec{V} \propto qE\hat{i}$.

In the magnetic field $\vec{B}$, force $\vec{F} = q(\vec{V} \times \vec{B}) \Rightarrow \vec{F} \propto q(qE\hat{i}) \times B\hat{k}$.

$$\Rightarrow \quad F \propto q^2 EB(-\hat{j}) \qquad [\because\ \hat{i} \times \hat{k} = -\hat{j}].$$

As $F \propto q^2$, so both the positive and the negative ions get deflected along the $-y$-direction.

16. The magnetic field strength at M due to QR is zero in all cases. When QS carries $I/2$ current, the field strength at M due to QS is $H_1/2$. Then

$$H_2 = H_1 + \frac{H_1}{2} = \frac{3H_1}{2} \quad \text{or} \quad \frac{H_1}{H_2} = \frac{2}{3}.$$

17. $B_0 = \int_a^b \frac{\mu_0 IN}{2r(b-a)}\,dr = \frac{\mu_0 IN}{2(b-a)}\int_a^b \frac{dr}{r} = \frac{\mu_0 IN}{2(b-a)}\ln\frac{b}{a}\cdot$

18. Area of the current loop, $A = a^2 + 4 \times \frac{\pi(a/2)^2}{2} = a^2 + \frac{\pi a^2}{2}\cdot$

$\therefore$ magnetic moment $= \vec{m} = I\vec{A} = I\left(\frac{\pi}{2}+1\right)a^2\hat{k}.$

19. In the cavity $\left(\text{where } r < \frac{R}{2}\right)$, $B = 0$.

Outside the cylinder (where $r > R$), $B = \frac{\mu_0 I}{2\pi r}\cdot$

In the material of the cylinder $\left(\text{where } \frac{R}{2} < r < R\right)$,

$$B\cdot 2\pi r = \mu_0\left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right]J$$

$$\Rightarrow \quad B = \frac{\mu_0 J}{2r}\left(r^2 - \frac{R^2}{4}\right)\cdot$$

For $\quad r = \frac{R}{2}$, $B = 0$; and for $r = R$,

$$B = \frac{\mu_0 J}{2R}\left(R^2 - \frac{R^2}{4}\right) = \frac{3}{8}\mu_0 JR.$$

Thus, $\quad B = 0$ for $r < \frac{R}{2}$;

$B = \frac{\mu_0 J}{2}\left(r - \frac{R^2}{4r}\right)$ ... for $\frac{R}{2} \le r < R$ (nonlinear increase);

and B decreases inversely with r for $r > R$.

20. The total magnetic field at the centre is

$$B = 12 \times \text{field due to each wire}$$

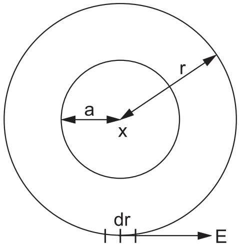
$$= 12\left(\frac{\mu_0 I}{4\pi a}\right)(\sin 60^\circ - \sin 30^\circ).$$

21. When S is closed, the clockwise current I_P flowing through P produces a magnetic field B directed from E to Q and then to P. This field B, by Lenz's law, causes an anticlockwise current I_{Q1} in Q. When S is opened, B decreases. This induces a clockwise current I_{Q2} in Q.

22. Construct a concentric circle of radius r. The induced electric field (E) at any point on this circle is equal to that at P. For this circle,

$$\oint \vec{E} \cdot d\vec{\ell} = 2\pi r E = \text{induced emf} = \frac{d\Phi}{dt} = \pi a^2 \dot{B}(t).$$

As the RHS is independent of r, we have $E \propto \frac{1}{r}$.

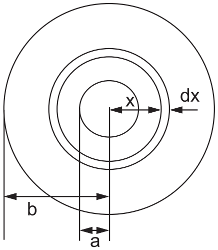


23. The number of turns per unit length is $\frac{N}{b-a}$.

Consider an elemental ring of radius x and width dx.

The number of turns in the ring is $dN = \frac{N\,dx}{b-a}$.

The magnetic field at the centre due to the ring is



$$dB = \frac{\mu_0 (dN) I}{2x} = \frac{\mu_0 I}{2} \cdot \frac{N\,dx}{b-a} \cdot \frac{1}{x}.$$

$\therefore$ the field at the centre is $\int dB = \frac{\mu_0 NI}{2(b-a)} \int_a^b \frac{dx}{x} = \frac{\mu_0 NI}{2(b-a)} \cdot \ln\frac{b}{a}$.

24. The pattern in the option (c) corresponds to a magnetic field (due to a straight steady current) as well as an electric field (inside a solenoid carrying a time-varying electric current).

25. For the circuit I, $\tau_1 = C_{eq} \times R_{eq} = \left(\frac{C_1 C_2}{C_1 + C_2}\right)\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \frac{8}{6} \times \frac{2}{3} = \frac{8}{9}\,\mu\text{s}.$

For the circuit II, $\tau_2 = (C_1 + C_2)(R_1 + R_2) = 6 \times 3 = 18\,\mu\text{s}.$

For the circuit III, $\tau_3 = (C_1 + C_2)\left(\frac{R_1 R_2}{R_1 + R_2}\right) = 6 \times \frac{2}{3} = 4\,\mu\text{s}.$

26. $R = \rho\left(\frac{l}{A}\right)$; so $\frac{R_1}{R_2} = \frac{A_2}{A_1} = \frac{\pi r^2}{\pi 4 r^2} = \frac{1}{4}$.

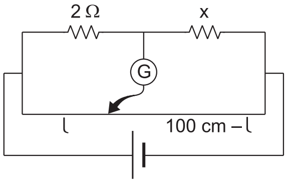
Ratio of power loss $= \frac{P_1}{P_2} = \frac{I^2 R_1}{I^2 R_2} = \frac{1}{4}$.

Ratio of voltage drop $= \frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{1}{4}$.

Ratio of current density $= \frac{J_1}{J_2} = \frac{I/A_1}{I/A_2} = \frac{A_2}{A_1} = \frac{1}{4}$.

Ratio of electric fields $= \frac{E_1}{E_2} = \frac{V_1/l_1}{V_2/l_2} = \frac{V_1}{V_2} = \frac{1}{4}$.

27. For balance,



$$\frac{2\,\Omega}{x} = \frac{l}{100\text{ cm} - l} \qquad ...(1)$$

and $$\frac{x}{2\,\Omega} = \frac{l + 20\text{ cm}}{80\text{ cm} - l}. \qquad ...(2)$$

Solving (1) and (2),

$$x = 3\,\Omega.$$

28. The magnetic force on a moving charge is given by $\vec{F} = q(\vec{v} \times \vec{B})$.

For $a < x < 2a$, $\vec{B} = B_0\hat{j}$. So, $\vec{F}_1 = qvB(\hat{k})$

$\Rightarrow$ the path is circular.

Similarly, for $2a < x < 3a$, $\vec{B} = -B\hat{j}$.

$\therefore$ $\vec{F}_2 = -qvB(\hat{k})$, and the correct graph is (a).

29. Power dissipation $= \frac{V^2}{R}$.

The equivalent resistance in the three configurations are

$$R_1 = 1\,\Omega,\ R_2 = \frac{1}{2}\,\Omega \text{ and } R_3 = 2\,\Omega.$$

$\therefore$ $P_1 = \frac{V^2}{1}$, $P_2 = \frac{2V^2}{1}$ and $P_3 = \frac{V^2}{2}$.

$\therefore$ $P_2 > P_1 > P_3$.

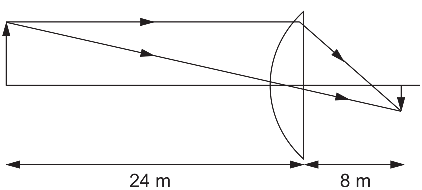
30. According to Lenz's law, a current will be induced in the anticlockwise direction so as to oppose the increasing magnetic flux into the paper. Thus, I_1 flows from A to B, and I_2 flows from D to C.

1.6 Ray Optics and Wave Optics

1. After the total internal reflection, there is no refracted ray.

2. The refractive index of the given lens material is

$$\mu = \frac{\lambda_a}{\lambda_m} = \frac{3}{2}.$$



Now, $\frac{1}{f} = (\mu - 1)\frac{1}{R} = \frac{1}{2R}$.

From the lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{2R} = \frac{1}{8\text{ m}} - \frac{1}{-24\text{ m}} = \frac{1}{6\text{ m}}$$

$$\Rightarrow \quad R = 3\text{ m}.$$

3. If θ be the angle between the directions of the incident ray and the reflected ray then

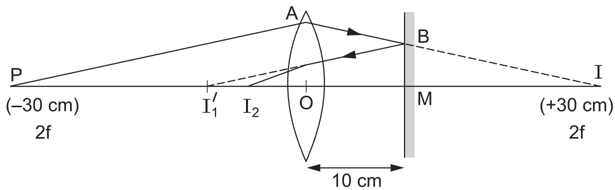
$$\cos\theta = \frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})\cdot\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$$

$$= \frac{1}{4}(1-3) = -\frac{1}{2} = \cos 120^\circ$$

$\Rightarrow \qquad \theta = 120^\circ$

$\therefore \quad$ angle of incidence $= i = \dfrac{180^\circ - \theta}{2} = 90^\circ - 60^\circ = 30^\circ.$

4.



(i) The real image of P is at I, where OI = 30 cm and MI = 30 – 10 = 20 cm.

(ii) The incident ray AB gets reflected by the mirror and would meet the axis at I_1', where $I_1'M$ = MI = 20 cm.

(iii) $I_1'O = I_1'M - OM = 20 - 10 = 10$ cm.

Finally, I_1' will act as a virtual object with $u = OI_1' = (+10\text{ cm})$ for the convex lens.

For the lens formula, $u = OI_1' = +10$ cm,

$v = ?, f = +15$ cm.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{v} - \frac{1}{10} = \frac{1}{15}$$

$\Rightarrow \qquad v = 6\text{ cm} = OI_2.$

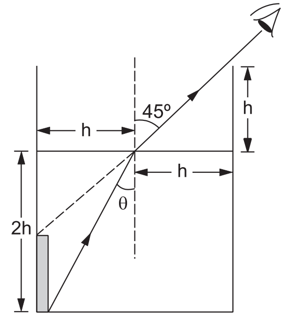
$\therefore$ distance from the mirror,

$$= I_2M = I_2O + OM = 6\text{ cm} + 10\text{ cm} = 16\text{ cm}.$$

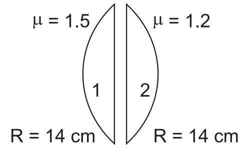
5. The line of sight of the observer remains constant, making an angle of 45° with the normal.

$$\sin\theta = \frac{h}{\sqrt{h^2+(2h)^2}} = \frac{1}{\sqrt{5}};$$

$$\mu = \frac{\sin 45^\circ}{\sin\theta} = \frac{1/\sqrt{2}}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}.$$



6. From the lens-maker's formula,



$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{14} - \frac{1}{\infty}\right) = \frac{1}{28}.$$

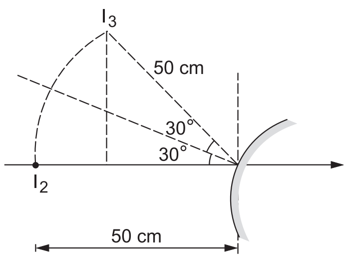
Similarly, $\frac{1}{f_2} = (1.2 - 1)\left(\frac{1}{\infty} - \frac{1}{-14}\right) = \frac{1}{70}.$

$\Rightarrow$ $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{28} + \frac{1}{70} = \frac{1}{20}.$$

Now, from the lens formula,

$$\frac{1}{v} = \frac{1}{20\text{ cm}} - \frac{1}{40\text{ cm}} = \frac{1}{40\text{ cm}} \quad \Rightarrow v = 40\text{ cm}.$$

7. For the first image formed by the lens, u = –50 cm, f = +30 cm, and so v = 75 cm. This is at 25 cm to the right of the mirror.



For final image by the mirror,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{25} + \frac{1}{v} = \frac{1}{50}$$

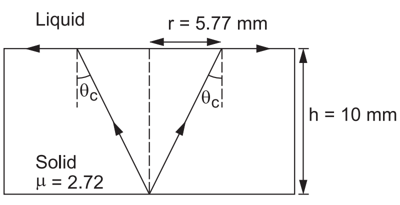
$\Rightarrow$ v = –50 cm, which is at I_2 when the mirror is straight. When tilted by 30°, the final image is at I_3, for which

$x = 50 - 50\cos 60° = 25$ cm

$y = 50\sin 60° = 25\sqrt{3}$ cm.

8. $$\tan\theta_c = \frac{r}{h} = \frac{5.77}{10} \approx \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $\theta_c = 30°.$



Now, $\frac{\mu_l}{\mu_s} = \sin\theta_c$

$\Rightarrow$ $\mu_l = \sin 30° \times \mu_s = \frac{1}{2} \times 2.72$

$= 1.36.$

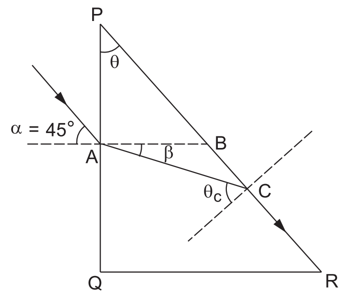
9. From Snell's law, $1\cdot(\sin 45°) = \sqrt{2}\sin\beta$

$\Rightarrow \quad \beta = 30°$. For critical angle θ_c,

$$\sin\theta_c = \frac{1}{\sqrt{2}} \text{ or } \theta_c = 45°.$$

Now, $\theta + (90° + \beta) + (90° - 45°) = 180°$,

$\Rightarrow \quad \theta = 45° - \beta = 15°$.



10. For a silvered lens, equivalent focal length

$$\frac{1}{f} = \frac{2}{f_l} + \frac{1}{f_m} = \frac{2}{15\text{ cm}} + \frac{1}{\infty} \quad \Rightarrow f = \frac{15}{2}\text{ cm}.$$

The silvered lens behaves as a concave mirror, so applying mirror formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{-20\text{ cm}} + \frac{1}{v} = -\frac{2}{15\text{ cm}}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{20\text{ cm}} - \frac{2}{15\text{ cm}} = \frac{-5}{60\text{ cm}}$$

$$\Rightarrow \quad v = -12\text{ cm (left of AB)}.$$

11. Let i be the angle of incidence at the interface of Regions III and IV. From Snell's law,

$$n_0 \sin\theta = \frac{n_0}{6}\sin i.$$

For TIR, $\quad i \geq i_c$ (critical angle)

$$\Rightarrow \quad \sin i \geq \frac{n_0/8}{n_0/6}$$

or $\quad \sin i \geq \dfrac{3}{4}.$

$$\therefore \quad n_0 \sin\theta = \frac{n_0}{6}\cdot\frac{3}{4} \quad \Rightarrow \sin\theta = \frac{1}{8}.$$

12. $$v_{\text{ball}}^2 = 2 \times (10\text{ m s}^{-2})\,(7.2\text{ m})$$

$$\Rightarrow \quad v_{\text{ball}} = 12\text{ m s}^{-1}.$$

To the fish, the image of the ball will appear as elevated, so

$$X_{\text{image of ball}} = \mu_w \times X_{\text{ball}}.$$

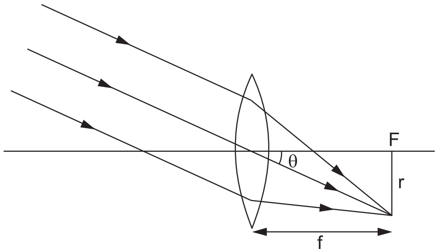
Differentiating, $\quad v_{\text{image}} = \dfrac{4}{3}(v_{\text{ball}}) = \dfrac{4}{3}(12\text{ m s}^{-1}) = 16\text{ m s}^{-1}.$

13. From the ray diagram,

$$\tan\theta = \frac{r}{f}$$

$$\Rightarrow \quad \pi r^2 = (\pi \tan^2\theta)\, f^2.$$

$$\therefore \quad \pi r^2 \propto f^2.$$



14. From the $u \sim v$ graph, $|u| = |v| = 10$ cm.

From lens formula, $f = 5.0$ cm.

The least count of the scale on the x- and y-axes is 0.1 cm.

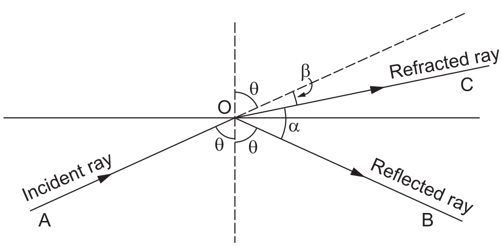
$\Rightarrow$ The option is (5.00 ± 0.10) cm.

15. Due to parallax, shift of image is more than that of the object as observed. So the image is nearer to the eye and $v > u$. Hence, the object lies between centre and focus $\Rightarrow f < x < 2f$.

16. From the figure showing the incident ray (AO), reflected ray (OB) and refracted ray (OC), we have

$$\alpha + \beta = 180° - 2\theta$$

$$\Rightarrow \quad \alpha < (180° - 2\theta).$$



17. For minimum deviation $r_1 = r_2 = r = \frac{A}{2} = 30°$; which is same for all wavelengths.

18. In interference between waves of equal amplitudes a, the minimum intensity is zero and the maximum intensity is proportional to $4a^2$. For waves of unequal amplitudes a and A $(A > a)$, the minimum intensity is nonzero and the maximum intensity is proportional to $(a + A)^2$, which is greater than $4a^2$.

19. Fringe width $\beta = \frac{D\lambda}{d}$, so $B \propto \lambda \Rightarrow \beta_R > \beta_G > V_B$.

20. If I_0 = intensity of each coherent source then

$$I = 2I_0\,(1 + \cos\phi) \text{ and } I_{max} = 4I_0.$$

$$\text{Now, } I = \frac{I_{max}}{2} = 2I_0 = 2I_0(1 + \cos\phi), \quad \text{so } \cos\phi = 0$$

$$\Rightarrow \quad \phi = \frac{2\pi}{\lambda}\Delta = (2n+1)\frac{\pi}{2}, \text{ hence } \Delta = (2n+1)\frac{\lambda}{4}.$$

1.7 Modern Physics

1. $t = 100$ ns $= 100 \times 10^{-9}$ s, $P = 30$ mW $= 30 \times 10^{-3}$ W, $c = 3 \times 10^{8}$ m s^{-1}.

$$\text{Momentum} = \frac{P \times t}{c} = \frac{(30 \times 10^{-3}\,W)\,(100 \times 10^{-9}\,\text{s})}{(3 \times 10^{8}\ \text{m s}^{-1})} = 1.0 \times 10^{-17}\ \text{m s}^{-1}.$$

2. For $\lambda_1 = 248$ nm, $E_1 = \dfrac{hc}{\lambda} = \dfrac{1240\ \text{eV nm}}{248\ \text{nm}} = 5$ eV.

For $\lambda_2 = 310$ nm, $E_2 = \dfrac{1240\ \text{eV nm}}{310\ \text{nm}} = 4$ eV.

$$\frac{(\text{KE})_1}{(\text{KE})_2} = \left(\frac{u_1}{u_2}\right)^2 = 4 = \frac{5\ \text{eV} - \phi_0}{4\ \text{eV} - \phi_0}$$

$\Rightarrow \quad \phi_0 = 3.7$ eV.

3. $$\frac{hc}{\lambda} = \phi + eV_0 \qquad \text{or} \qquad hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e\,(V_1 - V_2)$$

$$\Rightarrow \quad h = \frac{e\,(V_1 - V_2)\,\lambda_1\lambda_2}{c\,(\lambda_2 - \lambda_1)} = \frac{(1.6 \times 10^{-19}\,\text{C})\,(2\text{V} - 1\text{V})\,(0.3 \times 0.4) \times 10^{-12}\,\text{m}^2}{(3 \times 10^{8}\ \text{m s}^{-1})\,(0.4 - 0.3) \times 10^{-6}\,\text{m}}$$

$$= 6.4 \times 10^{-34}\ \text{J s}.$$

Alternatively: $V_0 = \dfrac{hc}{e\lambda} - \dfrac{\phi}{e}$, so the slope of $V_0 \sim \dfrac{1}{\lambda}$ graph is a constant $= \dfrac{hc}{e}\cdot$

Thus, $$\frac{hc}{e} = \frac{(2\text{V} - 1\text{V})}{\left(\dfrac{1}{0.3} - \dfrac{1}{0.4}\right)} \times 10^{-6}\,\text{m} = 1.2 \times 10^{-6}\,\text{Vm}$$

$$\Rightarrow \quad h = \frac{e}{c}\,(1.2 \times 10^{-6}\ \text{Vm}) = 6.4 \times 10^{-34}\ \text{J s}.$$

4. In H-atom, $E_{\text{total}} = -\dfrac{K}{n^2}$, KE $= \dfrac{K}{n^2}$ and potential energy $= -\dfrac{2K}{n^2}\cdot$

For transition from excited state ($n > 1$) to ground state ($n = 1$), kinetic energy of electron increases $\left(\dfrac{K}{n^2} \to K\right)$ whereas potential energy and total energy decrease (become more negative).

5. As the energy of the incident electrons is greater than the magnitude of the energy of the K-shell electrons, the target atoms will have vacancies in the K shell (K-shell electrons will be knocked out). This will cause emission of the entire characteristic spectrum of tungsten. Along with this, continuous X-rays will always be present, with

$$\lambda_c = \frac{1242\ \text{nm eV}}{80 \times 10^{3}\ \text{eV}} \cong 0.0155\ \text{nm}.$$

6. λ_K does not depend on the accelerating voltage. λ_c decreases with increase in accelerating voltage. Thus, as the accelerating voltage is increased, the difference $\lambda_K - \lambda_c$ will increase.

7. $$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{6561} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}, \; \frac{1}{\lambda} = 4R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{4}.$$

8. $$\frac{\lambda_{\text{Cu}}}{\lambda_{\text{Mo}}} = \left(\frac{Z_{\text{Mo}} - 1}{Z_{\text{Cu}} - 1}\right)^2.$$

9. In a gamma decay process, there is no change in either A or Z.

10. Activity $$A = A_0\left(\frac{1}{2}\right)^{t/T}$$

or $$\frac{A_0}{64} = A_0\left(\frac{1}{2}\right)^{t/T}$$

or $$\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{t/18\text{ days}} \quad \Rightarrow t = 108\text{ days.}$$

11. $$\frac{hc}{\lambda} = \phi_0 + \text{KE}_{\text{max}} = \phi_0 + \frac{p^2}{2m} = \phi_0 + \frac{h^2}{2m\lambda_{\text{d}}^2}$$

$$\Rightarrow \quad \frac{h^2}{2m\lambda_{\text{d}}^2} = \frac{hc}{\lambda} - \phi_0.$$

Differentiating, $$-\frac{3h^2\,d\lambda_{\text{D}}}{2m\lambda_{\text{D}}^3} = -\frac{hc}{\lambda^2}d\lambda$$

$$\Rightarrow \quad \frac{d\lambda_{\text{D}}}{d\lambda} \propto \frac{\lambda_{\text{D}}^3}{\lambda^2}.$$

12. Consider the reaction ${}^1_0n + {}^{15}_8\text{O} \rightarrow {}^{15}_7\text{N} + {}^1_1\text{H} + \text{Q}$,

so $$Q = (\Delta M)c^2 = (M_n + M_{\text{O}} - M_{\text{N}} - M_{\text{p}})\,c^2$$
$$= 0.003796 \times 931.5 = 3.5359\text{ MeV.} \qquad \text{...(1)}$$

From the given relation

$$\Delta Q = \Delta E = E_{{}^{15}_8\text{O}} - E_{{}^{15}_7\text{N}}$$

$$= \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}(8 \times 7 - 7 \times 6)$$

$$= \frac{3}{5}(1.44\text{ MeV}\cdot\text{fm}) \times \frac{14}{R} \qquad \text{...(2)}$$

Solving equations (1) and (2), $R = 3.42$ fm.

13. Probability P for the decay is

$$P = 1 - e^{-\lambda t}$$

For $$t = 2T_{1/2} = 2\left(\frac{\ln 2}{\lambda}\right)$$

$$P = 1 - e^{-\lambda \cdot 2\left(\frac{\ln 2}{\lambda}\right)}$$

$$= 1 - e^{-\ln 4} = 1 - \frac{1}{4} = \frac{3}{4}.$$

14. From Einstein's photoelectric equation

$$\frac{hc}{\lambda} = \phi + eV_s$$

$$\Rightarrow \quad V_s = \left(\frac{hc}{e}\right)\frac{1}{\lambda} - \frac{\phi}{e}.$$

This represents a straight line with slope $\tan\theta = \frac{hc}{e}$.

Work function $$\phi = h\nu_0 = \frac{hc}{\lambda_0}.$$

For metal 1, $$\frac{1}{\lambda_{01}} = \frac{\phi_1}{hc} = 0.001 \text{ nm}^{-1}.$$

For metal 2, $$\frac{1}{\lambda_{02}} = \frac{\phi_2}{hc} = 0.002 \text{ nm}^{-1}.$$

For metal 3, $$\frac{1}{\lambda_{03}} = \frac{\phi_3}{hc} = 0.004 \text{ nm}^{-1}.$$

$$\therefore \quad \phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4.$$

$$\lambda_{01} = 1000 \text{ nm}.$$

$$\lambda_{02} = \frac{1000}{2} \text{ nm} = 500 \text{ nm}.$$

$$\lambda_{03} = \frac{1000}{4} \text{ nm} = 250 \text{ nm}.$$

$\because \lambda_v \leq 400$ nm, photoelectrons can be ejected by metal 1 and metal 2 only and not by metal 3.

15. During a nuclear reaction, the rest mass energy of the main nucleus (U) must be greater than the rest mass energy of daughter nuclei in which it breaks up (as the conservation of momentum is always followed).

16. The largest wavelength (λ_1) in UV region of H-spectrum is the first line in Lyman series for the transition $n = 2$ to $n = 1$:

$$\frac{1}{122} = R\left(1 - \frac{1}{4}\right) = \frac{3}{4}R. \qquad \text{...(1)}$$

The smallest wavelength (λ_2) in IR region is the last line in Paschen series for the transition $n = \infty$ to $n = 3$:

$$\frac{1}{\lambda_2} = R\left(\frac{1}{9}-\frac{1}{\infty}\right) = \frac{R}{9}. \qquad ...(2)$$

$\therefore$ from (1) and (2), $\lambda_2 = 823.5$ nm.

17. For X-ray emission, cut-off wavelength

$$\lambda_0 = \frac{hc}{eV} \qquad ...(1)$$

But $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\,(eV)}} \Rightarrow eV = \frac{h^2}{2m\lambda^2}.$$

Equating eV, $$\frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2}$$

$\Rightarrow$ $$\lambda_0 = \frac{2m\,c\lambda^2}{h}.$$

18. $\lambda_{\text{cut off}} = \frac{hc}{eV}$, which is independent of the atomic number (Z)

$\Rightarrow$ option (b) is wrong.

19. $$A = \left|\frac{dN}{dt}\right| = \lambda N = \frac{\ln 2}{T} N$$

$\Rightarrow$ $$5\,\mu\,C_i = \frac{\ln 2}{T_1}\cdot 2N_0$$

$$10\,\mu\,C_i = \frac{\ln 2}{T_2}\cdot N_0$$

$\Rightarrow$ $$T_1 = 4T_2.$$

20. Energy of photon corresponding to wavelength λ is

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{\lambda}.$$

$\therefore$ $$\lambda_1 = 550 \text{ nm}, E_1 = \frac{1240}{550}\text{ eV} = 2.25 \text{ eV}$$

$$\lambda_2 = 450 \text{ nm}, E_2 = \frac{1240}{450}\text{ eV} = 2.75 \text{ eV}$$

$$\lambda_3 = 350 \text{ nm}, E_3 = \frac{1240}{350}\text{ eV} = 3.5 \text{ eV}$$

Since emission occurs when $E > \phi$,

$\phi_p = 2$ eV, all the three components cause photoemission

$\phi_q = 2.5$ eV, last two cause emission

$\phi_r = 3$ eV, only the last causes emission

$\therefore$ $I_p > I_q > I_r$.

❑