CLASS : XI CHAPTER : CONIC SECTIONS

EXERCISE 11.4.

In each of exercises 1 to 6, find the coordinates of foci, vertices, the eccentricity and length of Latus Rectum of the hyperbolas :

Q No 1 $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Sol. The given eqn of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$\therefore$ $a^2 = (4)^2$ and $b^2 = 9$

$\Rightarrow$ $a = 4$ and $b = 3$

$\therefore$ $c^2 = a^2 + b^2 \Rightarrow c = \sqrt{16+9} = \sqrt{25} = 5$.

$\therefore$ foci are $(\pm 5, 0)$ [$\because$ foci are $(\pm c, 0)$]

Vertices are $(\pm a, 0)$ ie $(\pm 4, 0)$

Eccentricity $e = \frac{c}{a} = \frac{5}{4}$.

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$.

Q No 2 $\frac{y^2}{9} - \frac{x^2}{27} = 1$

Sol Given eqn is $\frac{y^2}{9} - \frac{x^2}{27} = 1$.

$\Rightarrow$ $a^2 = 9$ and $b^2 = 27$

$\Rightarrow$ $a = 3$ and $b = 3\sqrt{3}$

$\therefore$ $c^2 = a^2 + b^2 \Rightarrow c^2 = 9 + 27 \Rightarrow c^2 = 36 \Rightarrow c = 6$.

$\therefore$ foci are $(0, \pm c)$ ie $(0, \pm 6)$

Vertices are $(0, \pm a)$ ie $(0, \pm 3)$

Eccentricity $e = \frac{c}{a} = \frac{6}{3} = 2$

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2 \times 27}{3} = 18$.

Q No 3 $9y^2 - 4x^2 = 36$.

Sol: The equation of hyperbola is $9y^2 - 4x^2 = 36$

or $\frac{y^2}{4} - \frac{x^2}{9} = 1$

$\Rightarrow a^2 = 4$ and $b^2 = 9$

$\Rightarrow a = 2$ and $b = 3$

$\therefore c = \sqrt{a^2+b^2} = \sqrt{4+9} = \sqrt{13}$.

$\therefore$ foci are $(0, \pm c)$ ie $(0, \pm\sqrt{13})$ and Vertices are $(0, \pm a)$ ie $(0, \pm 2)$

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times 9}{2} = 9$.

Q No 4 $16x^2 - 9y^2 = 576$

Sol. Given eqn. is $16x^2 - 9y^2 = 576$

or $\frac{x^2}{36} - \frac{y^2}{64} = 1$

$\therefore$ Here $a^2 = 36 \Rightarrow a = 6$

and $b^2 = 64 \Rightarrow b = 8$

$\therefore c = \sqrt{a^2+b^2} = \sqrt{36+64} = \sqrt{100} = 10$

$\therefore$ foci are $(\pm c, 0)$ ie $(\pm 10, 0)$

Vertices are $(\pm a, 0)$ ie $(\pm 6, 0)$

Eccentricity $e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times 64}{6} = \frac{64}{3}$

Q No 5 $5y^2 - 9x^2 = 36$

Sol. Given eqn is $5y^2 - 9x^2 = 36$ or $\frac{5y^2}{36} - \frac{9x^2}{36} = 1$

or $\frac{y^2}{36/5} - \frac{x^2}{4} = 1$

Here $a^2 = \frac{36}{5} \Rightarrow a = \sqrt{\frac{36}{5}} = \frac{6}{\sqrt{5}}$

and $b^2 = 4 \Rightarrow b = 2$

$c = \sqrt{a^2+b^2} \Rightarrow c = \sqrt{\frac{36}{5}+4} = \sqrt{\frac{36+20}{5}} = \sqrt{\frac{56}{5}} = \frac{2\sqrt{14}}{\sqrt{5}}$

$\therefore$ foci are $(0, \pm c)$ ie $\left(0, \pm \frac{2\sqrt{14}}{\sqrt{5}}\right)$

Vertices are $(0, \pm a)$ ie $\left(0, \pm \frac{6}{\sqrt{5}}\right)$

Eccentricity $e = \frac{c}{a} = \frac{2\sqrt{14}}{\sqrt{5}} \times \frac{\sqrt{5}}{6} = \frac{\sqrt{14}}{3}$

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2 \times 4 \times \sqrt{5}}{6} = \frac{4\sqrt{5}}{3}$

QNo. 6 $49y^2 - 16x^2 = 784$

Sol: Given eqn is $49y^2 - 16x^2 = 784$ or $\frac{y^2}{16} - \frac{x^2}{49} = 1$

$\therefore a^2 = 16$ and $b^2 = 49$

$\Rightarrow a = 4$ and $b = 7$.

$\therefore c = \sqrt{a^2 + b^2} = \sqrt{16 + 49} = \sqrt{65}$

$\therefore$ foci are $(0, \pm c) = (0, \pm\sqrt{65})$

Vertices are $(0, \pm a)$ ie $(0, \pm 4)$

$\therefore$ Eccentricity $e = \frac{c}{a} = \frac{\sqrt{65}}{4}$.

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2 \times 49}{4} = \frac{49}{2}$.

In each of exercise from 7 to 15 find the equation of parabola satisfying the given conditions.

QNo7. Vertices $(\pm 2, 0)$, foci $(\pm 3, 0)$

Sol. Since foci are on x-axis

$\therefore$ Eqn of hyperbola is of form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ --- (1)

Since Vertices are $(\pm 2, 0)$

$\therefore a = 2 \Rightarrow a^2 = 4$.

Also foci are $(\pm 3, 0)$

$\therefore c = 3 \Rightarrow c^2 = 9$

Now $c^2 = a^2 + b^2 \Rightarrow 9 = 4 + b^2 \Rightarrow b^2 = 5$.

$\therefore$ from (1) $\frac{x^2}{4} - \frac{y^2}{5} = 1$

which is required eqn.

Q No. 8: Vertices $(0, \pm 5)$, foci $(0, \pm 8)$

Sol. Since foci are on y-axis

$\therefore$ Eqn of reqd hyperbola is of form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ --- (1)

Since vertices are $(0, \pm 5)$

$\Rightarrow a = 5 \Rightarrow a^2 = 25$

Also foci are $(0, \pm 8)$

$\therefore c = 8 \Rightarrow c^2 = 64$

$\therefore c^2 = a^2 + b^2 \Rightarrow 64 = 25 + b^2 \Rightarrow b^2 = 39$

$\therefore$ from (1) $\frac{y^2}{25} - \frac{x^2}{39} = 1$ which is reqd eqn. of hyperbola.

Q No 10. Foci $(\pm 5, 0)$, the transverse axis is of length 8

Sol Since the foci are on x-axis

$\therefore$ Eqn of hyperbola is of form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ --- (1)

Foci are $(\pm 5, 0)$

$\therefore c = 5 \Rightarrow c^2 = 25$

Also length of transverse axis = 8

ie $2a = 8 \Rightarrow a = \frac{8}{2} = 4. \Rightarrow a^2 = 16$

$\therefore c^2 = a^2 + b^2 \Rightarrow 25 = 16 + b^2 \Rightarrow b^2 = 9$

$\therefore$ From (1) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

which is required eqn.

Q No 9. Vertices $(0, \pm 3)$, foci $(0, \pm 5)$

Sol. Since foci are on y-axis.

$\therefore$ Eqn of hyperbola is of form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ --- (1)

Since Vertices are $(0, \pm 3)$

$\therefore a = 3 \Rightarrow a^2 = 9$

Also foci are $(0, \pm 5)$ $\Rightarrow c = 5 \Rightarrow c^2 = 25$

Now $c^2 = a^2 + b^2 \Rightarrow 25 = 9 + b^2 \Rightarrow b^2 = 16$

$\therefore$ from (1) $\frac{y^2}{9} - \frac{x^2}{16} = 1$ which is required eqn.

Q No 11 Foci $(0, \pm 13)$, the conjugate axis is of length 24.

Sol. Since foci are on y-axis

$\therefore$ Eqn of hyperbola is of form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ ... (1)

Foci are $(0, \pm 13)$

$\therefore$ $c = 13 \Rightarrow c^2 = 169.$

Also length of conjugate axis $= 24.$

$\therefore$ $2b = 24 \Longrightarrow b = 12 \Rightarrow b^2 = 144$

Now $c^2 = a^2 + b^2 \Longrightarrow 169 = a^2 + 144 \Rightarrow a^2 = 25$

$\therefore$ From (1) $\frac{y^2}{25} - \frac{x^2}{144} = 1.$ which is required eqn.

Q No 12. Foci $(\pm 3\sqrt{5}, 0)$, the latus rectum is of length 8

Sol Since foci are on x-axis

$\therefore$ Eqn of hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ --(1)

Foci are $(\pm 3\sqrt{5}, 0)$

$\therefore$ $c = 3\sqrt{5} \Rightarrow c^2 = 45.$

Also Length of latus rectum $= 8$

ie $\frac{2b^2}{a} = 8 \Longrightarrow b^2 = 4a.$

Now $c^2 = a^2 + b^2$

$\Rightarrow 45 = a^2 + 4a \Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow (a+9)(a-5) = 0 \Rightarrow a = -9, 5$

Rejecting $a = -9$ as a cannot be negative

$a = 5 \Rightarrow a^2 = 25$ and $b = 4a = 4 \times 5 = 20$

$\therefore$ From (1) $\frac{x^2}{25} - \frac{y^2}{20} = 1.$

which is required eqn.

Q No. 13. Foci $(\pm 4, 0)$, the latus rectum is of length 12.

Since foci are on x-axis

$\therefore$ Eqn of hyper-bola is of form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

Foci are $(\pm 4, 0)$ $\therefore$ $c = 4 \Rightarrow c^2 = 16$

Also length of Latus Rectum = 12

i.e $\frac{2b^2}{a} = 12 \Rightarrow b^2 = 6a$

Now $c^2 = a^2 + b^2 \Rightarrow 16 = a^2 + 6a$

$\Rightarrow a^2 + 6a - 16 = 0$

$\Rightarrow (a+8)(a-2) = 0 \Rightarrow a = -8, 2$

Rejecting $a = -8$ as a can not be negative.

$\therefore a = 2 \Rightarrow a^2 = 4$

Also $b^2 = 6a = 6 \times 2 = 12$.

$\therefore$ from (1) $\frac{x^2}{4} - \frac{y^2}{6} = 1$, reqd eqn.

Q No. 14 Vertices are $(\pm 7, 0)$, $e = \frac{4}{3}$

Sol. Since the vertices are on x-axis

$\therefore$ Eqn of hyperbola is of form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ---(1)

Vertices are $(\pm 7, 0)$

$\therefore a = 7 \Rightarrow a^2 = 49$

Now $e = \frac{4}{3} \Rightarrow \frac{c}{a} = \frac{4}{3} \Rightarrow c = \frac{4}{3} \times a = \frac{4}{3} \times 7 = \frac{28}{3}$.

Now $c^2 = a^2 + b^2$

$\Rightarrow \frac{784}{9} = 49 + b^2 \Rightarrow b^2 = \frac{784}{9} - 49 = \frac{784 - 441}{9} = \frac{343}{9}$

$\therefore$ from (1) $\frac{x^2}{49} - \frac{y^2}{\frac{343}{9}} = 1$

or $\frac{x^2}{49} - \frac{9y^2}{343} = 1$

Which is required equation.

Q No 15. Foci $(0, \pm\sqrt{10})$, passing through $(2, 3)$

Sol. Since foci are on y-axis

$\therefore$ Equation of hyperbola is of form

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \text{---(1)}$$

foci are $(0, \pm\sqrt{10})$

$\therefore$ $c = \sqrt{10} \Rightarrow c^2 = 10$

$\therefore$ $c^2 = a^2 + b^2 \Rightarrow a^2 + b^2 = 10$ --- (2)

Also since hyperbola passes through (2,3)

$$\frac{9}{a^2} - \frac{4}{b^2} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{4}{10-a^2} = 1 \quad \text{(using (2))}$$

$$\Rightarrow 90 - 9a^2 - 4a^2 = a^2(10 - a^2)$$

$$\Rightarrow a^4 - 23a^2 + 90 = 0 \Rightarrow (a^2 - 18)(a^2 - 5) = 0$$

$$\Rightarrow a^2 = 18, 5$$

Now $b^2 = 10 - a^2$

$\therefore$ When $a^2 = 18$, $b^2 = 10 - 18 = -8$ which is not possible

$\therefore$ $a^2 = 5$ and $b^2 = 10 - 5 = 5$

$\therefore$ From (1) $\frac{y^2}{5} - \frac{x^2}{5} = 1$

which is required eqn.

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