CBSE Test Paper 03

Chapter 13 Probability

1. A die is thrown 6 times. If ‘getting an odd number’ is a success, what is the probability of at most 5 successes?
   a. $\frac{63}{64}$
   b. $\frac{21}{64}$
   c. $\frac{49}{64}$
   d. $\frac{37}{64}$

2. Given that the events A and B are such that P(A) = $\frac{1}{2}$, P (A ∪ B) = $\frac{3}{5}$ and P(B) = p. Find p if they independent.
   a. $\frac{1}{4}$
   b. $\frac{1}{2}$
   c. $\frac{1}{5}$
   d. $\frac{1}{3}$

3. The conditional probability of an event E, given the occurrence of the event F is given by
   a. $\mathrm{P(E|F)} = \frac{P(E \cap F)}{P(E)},\ P(F) \neq 0$
   b. $\mathrm{P(E|F)} = \frac{P(E \cup^{F})}{P(F)},\ P(F) \neq 0$
   c. $\mathrm{P(E \mid F)}\ = \frac{P(E \cap F)}{P(F)},\ P(F) \neq 0$
   d. $\mathrm{P(E|F)} = \frac{P(E \cap^{F})}{P(F)},\ P(F) < 0$

4. Compute P(A|B), if P(B) = 0.5 and P (A ∩ B) = 0.32
   a. P(A|B) = $\frac{16}{33}$
   b. P(A|B) = $\frac{15}{27}$
   c. P(A|B) = $\frac{16}{25}$
   d. P(A|B) = $\frac{16}{29}$

5. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

    a. 4
    b. 1
    c. 2
    d. 5
6. The possibility of drawing a jack or a spade from a well shuffled standard deck of 52 playing cards is ________ .
7. If three unbiased coins are tossed, the probability of getting at least 2 tails is ________.
8. A subset of the sample space associated with a random experiment is called an ________ or a case.

9. A Box of oranges is inspected by examining three randomly selected oranges drown without replacement. If all the three oranges are good, the box is approved for sale, other wise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approve for sale.

10. Let X denote the no of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form where K is some unknown constant

$$P(\chi = x) = \begin{cases} 0.1\ if\ x = 0 \\ kx\ if\ x = 1,\ or\ 2 \\ K(5 - x)\ if\ x = 3\ or\ 4 \\ 0,\ otherwise \end{cases}.$$

    a. Find the value of K.
    b. What is the probability that you study at least two hours. Exactly two hours.At most two hours.

11. If A and B are events such that P(A|B)=P(B|A),then P(A)=P(B).

12. In a meeting 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take x = 0 if he opposed and x = 1 if he is in favour. Find E (x) and var (x).

13. If A and B are two events such that P (A) = $\frac{1}{4}$ P (B) = $\frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$ find P(not A and not B).

14. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

15. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark noted down and it is replaced. If 6 balls are drawn in this way, find the probability that:

    i. all will bear 'X' mark.
    ii. not more than 2 will bear 'Y' mark.
    iii. at least one ball will bear 'Y' mark.
    iv. the number of balls with 'X' mark and 'Y' mark will be equal.

16. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII, what is the probability that a student chosen randomly studies in class XII, given that the chosen student is a girl?

17. A black and a red die are rolled. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

18. Five bad oranges are accidentally mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence, find the mean and variance of the distribution.

Solution

1. a. $\frac{63}{64}$

   Explanation: p = probability of success = $\frac{3}{6}$ = $\frac{1}{2}$. q = probability of failure = 1 – p = 1 – $\frac{1}{2}$ = $\frac{1}{2}$. let x be the number of successes , then x has the binomial distribution with : n = 6 , p = $\frac{1}{2}$, q = $\frac{1}{2}$ $\therefore p(x=r)={}^{n}C_{r}q^{n-r}p^{r}$ P(atmost 5 successes) = P (x ≤ 5) = 1 - P(x > 5) = 1 – P(x = 6) =
   $1-{}^{6}C_{6}\left(\frac{1}{2}\right)^{6-6}\left(\frac{1}{2}\right)^{6}=1-\frac{1}{64}=\frac{63}{64}$.
2. c. $\frac{1}{5}$

   Explanation: Since A and B are independent events. Therefore,

   P(A ∩ B) = P(A).P(B)

   P(A ∪ B)=P(A) + P(B) - P(A ∩ B)

   $\Rightarrow\frac{3}{5}=\frac{1}{2}+$p - P(A).P(B) $\Rightarrow\frac{1}{10}=p-\frac{1}{2}p\Rightarrow p=\frac{1}{5}$
3. c. $\mathrm{P(E\,|\,F)}=\frac{P(E\cap F)}{P(F)},\ P(F)\neq 0$

   Explanation: The conditional probability of an event E, given the occurrence of the event F is given by :

   $$\mathrm{P(E\,|\,F)}=\frac{P(E\cap F)}{P(F)},\ P(F)\neq 0$$
4. c. P(A|B) = $\frac{16}{25}$

   Explanation: We have , P(B) = 0.5 and P (A ∩ B) = 0.32

   $$P\left(A/B\right)=\frac{P(A\cap B)}{P(B)}=\frac{0.32}{0.5}=\frac{16}{25}$$
5. c. 2

   Explanation: Let X be the random variable which denote the number obtained on the die. Therefore, X = 1, 2 or 5 Therefore, the probability distribution of X is:

X	1	2	5
P(X)	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$
XP(X)	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$

   Therefore, required mean = $\frac{3}{6}+\frac{4}{6}+\frac{5}{6}$ = 2

6. $\frac{4}{13}$
7. 0.5
8. event
9. S={12 good oranges,3 bad oranges}

   n(S)=15

   Probability that first orange drawn is good= $\frac{12}{15}$

   Probability that second orange is drawn is good= $\frac{11}{14}$

   Probability that third orange is good when both the first and second are good= $\frac{10}{13}$.

   P(a box is approved)= $\frac{C(12,3)}{C(15,3)} = \frac{12\times11\times10}{15\times14\times13} = \frac{44}{91}$
10. The probability distribution of x is

X	0	1	2	3	4
P(X)	0.1	K	2K	2K	K

   a. $\sum_{i=1}^{n} pi = 1$

      0.1 + K + 2K + 2K + K = 1

      K = 0.15

   b. p (study atleast two hours) = p (x ≥ 2)

      = 2K + 2K + K

      = 5K

      $= 5 \times 0.15$

      = 0.75

      p (Study exactly two hours) = p(x = 2)

      = 2K

      $= 2 \times 0.15$

      = 0.3

      p (Study at most two hours)= p(x$\leq$2) = p(x=0)+p(x=1)+p(x=2) = 0.1+k+2k = 0.1+3k = 0.1+3(0.15) = 0.1+0.45 = 0.55.

11. We have, P(A|B)=P(B|A)

   $\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$

   $\Rightarrow \frac{1}{P(B)} = \frac{1}{PB)}$

   $\Rightarrow$ P(A)=P(B)

12.

X	0	1
P(X)	$\frac{30}{100}$	$\frac{70}{100}$

E(X) = Mean = $\sum p_i x_i = \frac{70}{100} = 0.7$

Variance(X) = $\sum p_i x_i^2 - (\sum p_i x_i)^2 = \frac{70}{100} - \left(\frac{70}{100}\right)^2 = \frac{7}{10} - \frac{49}{100} = \frac{21}{100} = 0.21$

13. P (A) = $\frac{1}{4}$, P (B) = $\frac{1}{2}$, $P(A \cap B) = \frac{1}{8}$

    P (not A) = 1 – P (A) = $1 - \frac{1}{4} = \frac{3}{4}$

    P (not B) = 1 – P (B) = $1 - \frac{1}{2} = \frac{1}{2}$

    Now, P (A). P (B) $= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

    $\therefore P(A \cap B)$ = P (A).P (B)

    Thus, A and B are independent events.

    Therefore, ‘not A’ and ‘not B’ are independent events.

    Hence, P (not A and not B) = P (not A). P (not B)

    $= \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

14. S = {1, 2, 3, 4, 5, 6}$\Rightarrow n(S) = 6$

    A = {6} $\Rightarrow n(A) = 1$

    $p = \frac{n(A)}{n(S)} = \frac{1}{6}$ and $q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

    Since the 3rd six is obtained in the 6th throw of die, there are two sixes i.e., two successes in the first 5 throws.

    $\therefore$ Required probability = P (2 success in the first 5 throws) $\times$ P (success in the 6th throw)

    $= \left({}^5C_2 q^{5-2} p^2\right) p$

    = $\frac{5!}{3!2!}\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^3 = \frac{1250}{46656} = \frac{625}{23328}$

15. S = {10 balls with mark X, 15 balls with mark Y} $\Rightarrow n(S) = 25$

    Let a ball drawn with mark X be denoted by A.

    A = {10 balls with mark X} $\Rightarrow n(A) = 10$

    $p = \frac{n(A)}{n(S)} = \frac{10}{25} = \frac{2}{5}$ and q = 1 - p $= 1 - \frac{2}{5} = \frac{3}{5}$

    n = 6

    i. All will bear X mark, i.e., r = 6

       P (X = r) = ${}^nC_r p^r q^{n-r}$

       P (X = 6) = ${}^6C_6\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0 = \left(\frac{2}{5}\right)^6$

ii. Not more than 2 will bear mark Y.

$\therefore$ For Y mark, r = 0, 1, 2 and For X mark, r = 6, 5, 4

P (not more than 2 will bear mark Y) = P (X = 4) + P (X = 5) + P (X = 6)

$$={}^6C_4\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2+{}^6C_5\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^1+{}^6C_6\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0$$

$$=15\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2+6\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^1+\left(\frac{2}{5}\right)^6$$

$$=\left(\frac{2}{5}\right)^4\left(\frac{27}{5}+\frac{36}{25}+\frac{4}{25}\right)$$

$$=\left(\frac{2}{5}\right)^4\left(\frac{175}{25}\right)=7\left(\frac{2}{5}\right)^4$$

iii. P (at least one ball will bear Y mark) = P (not more than 5 balls will bear mark X)

= 1 – P (6)

$$=1-{}^6C_0p^6q^0=1-\left(\frac{2}{5}\right)^6$$

iv. P (equal number of balls will bear mark X and Y) = P (3)

$$={}^6C_3p^3q^3=20\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^3=864\left(\frac{1}{5}\right)^3$$

16. Let 'A' be the event that the chosen student studies in class XII and B be the event that the chosen student is a girl.

There are 430 girls out of 1000 students

So, P(B) = P (Chosen student is girl) $=\frac{430}{1000}=\frac{43}{100}$

Since, 10% of the girls studies in class XII

So, total number of girls studies in class XII

$$=\frac{10}{100}\times 430=43$$

Then, P(A ∩ B) = P (Chosen student is a girl of class XII)

$$=\frac{43}{1000}$$

$\therefore$ Required probability = P(A / B)

$$=\frac{P(A\cap B)}{P(B)}\quad\left[\because P(A/B)=\frac{P(A\cap B)}{P(B)}\right]$$

$$=\frac{43/1000}{43/100}=\frac{1}{10}$$

17. Let the first observation be from the black die and second from the red die.

When two dice (one black die and another red) are rolled,the sample space S has 6×6 =36 number of elements.

Let A : obtaining a sum greater than 9 ={4,6),(5,5),(5,6),(6,4),(6,5),(6,6)}

and B: black die resulted in a 5 ={(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)}

$\therefore$ $A\cap B$={(5,5),(5,6)}

The conditional probability of obtaining a sum greater than 9,given that the black die resulted in a 5,is given by P(A/B)

$$\therefore P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{2}{36}}{\frac{6}{36}}=\frac{2}{6}=\frac{1}{3}$$

18. Let X be a random variable that denotes the number of bad oranges in a draw of 4 oranges.

Then, X can take values 0, 1, 2, 3 and 4.

Here, p = probability of getting a bad orange in a single draw $=\frac{5}{25}=\frac{1}{5}$

and q = probability of not getting a bad orange in a single draw

$=1-p=1-\frac{1}{5}=\frac{4}{5}$

Clearly, X follows binomial distribution with parameters $n=4, p=\frac{1}{5}$ and $q=\frac{4}{5}$

$\therefore P(X=r)={}^4C_r\left(\frac{1}{5}\right)^r\left(\frac{4}{5}\right)^{4-r}, r=0,1,2,3,4$

Now,$P(X=0)={}^4C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4=\left(\frac{4}{5}\right)^4=\frac{256}{625}$

$P(X=1)={}^4C_1\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^3=\left(\frac{4}{5}\right)^4=\frac{256}{625}$

$P(X=2)={}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2=\frac{96}{625}$

$P(X=3)={}^4C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)=\frac{16}{625}$

and $P(X=4)={}^4C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^0=\frac{1}{625}$

Thus, the probability distribution of X is

X	0	1	2	3	4
P (X=r)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Now, Mean=E(x)=$\Sigma x_i p_i$

$=0.\frac{256}{625}+1\cdot\frac{256}{625}+2\cdot\frac{96}{625}+3\cdot\frac{16}{625}+4\cdot\frac{1}{625}$

$=\frac{1}{625}[256+192+48+4]=\frac{500}{625}=\frac{20}{25}=\frac{4}{5}$

and variance= $E(X)^2-[E(x)]^2=\sum x_i^2\cdot p_i-\left(\frac{4}{5}\right)^2$

$=\left[0^2\cdot\frac{256}{625}+1^2\cdot\frac{256}{625}+2^2\cdot\frac{96}{625}+3^2\cdot\frac{16}{625}+4^2\cdot\frac{1}{625}\right]-\left(\frac{4}{5}\right)^2$

$=\frac{1}{625}[256+384+144+16]-\frac{16}{25}$

$=\frac{800-400}{625}=\frac{400}{625}=\frac{16}{25}$