Short Answer Type Questions – I

[2 marks]

Que 1. If x + 1 Is a factor of the polynomial $3x^2$ –kx, then find the value of k.

Sol. $Let\ p(x) = 3x^2 - kx, as\ (x+1)\ is\ a\ factor\ of\ p(x)$

$$So, p(-1) = 0\ i.e., \quad 3(-1)^2 - k(-1) = 0 \Rightarrow k = -3$$

Que 2. Find the value of k, if y+3 is a factor of $3y^2$ + ky + 6.

Sol. $Let\ p(y) = 3y^2 + ky + 6$

$$As\ y+3\ is\ a\ factor\ of\ p(y), so\ p(-3) = 0$$

$$i.e., \quad 3(-3)^2 + k(-3) + 6 = 0$$

$$\Rightarrow 27 - 3k + 6 = 0 \quad \Rightarrow 33 - 3k = 0$$

$$\Rightarrow -3k = -33 \quad \Rightarrow k = 11$$

Que 3. Find the value of a, if x – a is a factor of $x^3 - ax^2$ + 2x + a – 1.

Sol. $Let\ p(x) = x^3 - ax^2 + 2x + a - 1$

$$As\ (x-a)\ is\ a\ factor\ of\ p(x), so\ p(a) = 0, i.e., a^3 - a.a^2 + 2a + a - 1 = 0$$

$$\Rightarrow 3a - 1 = 0$$

$$\Rightarrow a = \frac{1}{3}$$

Que 4. If $\oint(\boldsymbol{z}) = \boldsymbol{z^2 - 3\sqrt{2}\,z - 1, then\ find}\ \oint\left(\boldsymbol{3\sqrt{2}}\right)$.

Sol. $\oint(z) = z^2 - 3\sqrt{2}\,z - 1, then\ find\ \oint\left(3\sqrt{2}\right)$.

$$\Rightarrow \oint\left(3\sqrt{2}\right) = \left(3\sqrt{2}\right)^2 - 3\sqrt{2}\left(3\sqrt{2}\right) - 1 = 9 \times 2 - 9 \times 2 - 1 = -1$$

Que 5. If $\boldsymbol{x^{11} + 101\ is\ divided\ by\ x + 1, what\ is\ the\ remainder}$?

Sol. $Let \quad p(x) = x^{11} + 101$

Using the remainder theorem, we have

$$Remainder = p(-1) = (-1)^{11} + 101$$

$$= -1 + 101 = 100$$

Que 6. Find the factors of (1 - x^3).

Sol. $1 - x^3 = 1^3 - x^3 = (1-x)(1+x+x^2)$

Que 7. Find the factors of $y^3 + y^2 + y + 1$.

Sol. $y^3 + y^2 + y + 1 = y^2(y+1) + 1(y+1) = (y+1)(y^2+1)$

Que 8. If $x + y = 9$ and $xy = 20$, then find the value of $x^2 + y^2$.

Sol. We know that $(x+y)^2 = x^2 + y^2 + 2xy$

$$\Rightarrow 9^2 = x^2 + y^2 + 2 \times 20$$

$$\Rightarrow x^2 + y^2 = 81 - 40 = 41$$

Que 9. If $x + \frac{1}{x} = 4$, then find the value of $x^2 + \frac{1}{x^2}$.

Sol. $x + \frac{1}{x} = 4$

$$\Rightarrow \left(x \frac{1}{x}\right)^2 = 4^2 \Rightarrow x^2 + \frac{1}{x^2} + 2x \times \frac{1}{x} = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 16 - 2 = 14$$

$$\therefore x^2 + \frac{1}{x^2} = 14$$

Que 10. Using factor theorem, show that (x- y) is a factor of $x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$.

Sol. $Let\ p(x) = x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$

Putting x = y in given polynomial p(x), we get

$$p(y) = y(y^2 - z^2) + y(z^2 - y^2) + z(y^2 - y^2)$$

$$= y(y^2 - z^2) - y(y^2 - z^2) = 0$$

$\therefore$ $(x - y)$ is a factor of given polynomial $p(x)$.

Que 11. If $x^2 - 1$ is a factor of $ax^3 + bx^2 + cx + d$, show that a + c = 0.

Sol. $Since\ x^2 - 1 = (x+1)(x-1)\ is\ a\ factor\ of\ p(x) = ax^3 + bx^2 + cx + d$

$$\therefore p(1) = p(-1) = 0$$

$$\Rightarrow a + b + c + d = -a + b - c + d = 0$$

$$\Rightarrow 2a + 2c = 0 \quad \Rightarrow 2(a + c) = 0$$

$$\Rightarrow a + c = 0$$

Que 12. If $\boldsymbol{x + 2k\ is\ a\ factor\ of\ \oint(x) = x^5 - 4k^2x^3 + 2x + 2k + 3, find\ k}$.

Sol. $Here, \oint(x) = x^5 - 4k^2x^3 + 2x + 2k + 3$

$$Since\ x + 2k\ is\ a\ factor\ of\ \oint(x), so\ by\ factor\ theorem,$$

$$\oint(-2k) = 0$$

$$(-2k)^5 - 4k^2(-2k)^3 + 2(-2k) + 2k + 3 = 0$$

$$-32k^5 + 32k^5 - 4k + 2k + 3 = 0$$

$$\Rightarrow \quad -2k + 3 = 0 \Rightarrow -2k = -3 \Rightarrow k = \frac{3}{2}$$

Que 13. Find the remainder when $\boldsymbol{\oint(x) = 4x^3 - 12x + 14x - 3}$ is divided by $\boldsymbol{g(x) = (2x - 1)}$.

Sol. $Taking\ g(x) = 0\ we\ have,$

$$2x - 1 = 0$$

$$\Rightarrow x = \frac{1}{2}$$

By remainder theorem when $\oint(x)$ is divided by $g(x)$, the remainder is equal to $\oint\left(\frac{1}{2}\right)$

Now, $\oint(x) = 4x^3 - 12x^2 + 14x - 3$

$$\oint\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$$

$$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 7 - 3$$

$$= \frac{1}{2} - 3 + 7 - 3 = \frac{1}{2} - 6 + 7 = 1 + \frac{1}{2}$$

$$\oint\left(\frac{1}{2}\right) = \frac{3}{2}$$

Hence, required remainder = $\frac{3}{2}$.

Que 14. $\boldsymbol{If\ (x + 1)\ is\ a\ factor\ of\ ax^3 + x^2 - 2x + 4a - 9, find\ the\ value\ of\ a}$.

Sol. $Let\ \oint(x) = ax^3 + x^2 - 2x + 4a - 9$

$$As\ (x + 1)\ is\ a\ factor\ of\ f(x)$$

$$\therefore \oint(-1) = 0$$

$$\Rightarrow\ a(-1)^3\ +(-1)^2\ -2(-1)+4a-9=0$$

$$\Rightarrow\ -a+1+2+4a-9=0$$

$$3a-6=0\ \Rightarrow\ 3a\ =\ 6$$

$$\Rightarrow\ a=\frac{6}{3}\ \Rightarrow a=2$$

Que 15. For what value of $\boldsymbol{k}$, $(\boldsymbol{x+1})$ $\boldsymbol{is\ a\ factor\ of\ p(x)\ =\ kx^2\ -x\ -4^2}$

Sol. $As\ x+1\ is\ a\ factor\ of\ p(x), so\ p(-1)\ =\ 0,$

$$i.e.,\quad k(-1)^2\ -(-1)\ -4\ =\ 0$$

$$k+1-4\ =4$$

$$\Rightarrow\ k-3\ =\ 0\ \ \Rightarrow k=3$$

Que 16. Expand using suitable identity (-2x+5y-3z)^{2}.

Sol. $(-2x+5y-3z)^2$

$$=(-2x)^2\ +(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)\ (-3z)+2(-3z)(-2x)$$

$$=\ 4x^2\ +25y^2\ +9z^2\ -20xy-30yz+12zx$$

Que 17. Find: $\boldsymbol{x+\frac{1}{x}, if\ x^2\ +\frac{1}{x^2}=62}$.

Sol.

$$\left(x+\frac{1}{x}\right)^2\ =x^2\ +\frac{1}{x^2}+2.x.\frac{1}{x}$$

$$=x^2+\frac{1}{x^2}+2=62+2\ =\ 64\quad \Rightarrow\left(x+\frac{1}{x}\right)^2\ =\ 64$$

Taking square root on both sides, we get $x+\frac{1}{x}$ = 8.

Que 18. Factorise: $\boldsymbol{\frac{25}{4}x^2\ -\frac{y^2}{9}}$.

$$Sol.\frac{25x^2}{4}-\frac{y^2}{9}=\left(\frac{5}{2}x\right)^2\ -\left(\frac{y}{3}\right)^2\ =\left(\frac{5}{2}x+\frac{y}{3}\right)\left(\frac{5}{2}x-\frac{y}{3}\right)$$

Que 19. Find the value of k if (x – 2) is a factor of polynomial $\boldsymbol{p(x)\ =\ 2x^3\ -6x^2\ +5x+k}$.

Sol. As (x – 2) is a factor of polynomial $p(x)\ =\ 2x^3\ -6x^2\ +5x+k, so, p(2)\ =\ 0$

$$\Rightarrow 2(2)^3\ -6(2)^2\ +5\times 2\ +k\ =\ 0$$

$$\Rightarrow\ 16 - 24 + 10 + k = 0$$

$$26 - 24 + k = 0 \quad \Rightarrow\ k + 2 = 0\ \Rightarrow\ k = -2$$

Que 20. Factorise: $\boldsymbol{a^2 + b^2 - 2bc + 2bc - 2ca}$

Sol. $a^2 + b^2 - 2bc + 2bc - 2ca = (a-b)^2 + 2c(b-a) = (a-b)^2 - 2c(a-b)$

$$= (a-b)(a-b-2c) \qquad [Taking\ common\ (a-b)]$$

Que 21. $\boldsymbol{Evaluate\ 185 \times 185 - 15 \times 15}$

Sol. $185 \times 185 - 15 \times 15$

$\Rightarrow (185)^2 - (15)^2 \qquad [\,Using\ a^2 - b^2 = (a-b)(a+b)]$

$$\Rightarrow (185 + 15)\ (185 - 15)$$

$$\Rightarrow 200 \times 170 = 34000$$