Long Answer Type Questions

[4 MARKS]

Que 1. Find the value of 'k', for which the points are collinear: (7, - 2), (5, 1), (3, k).

Sol. Let the given points be

A (x_1, y_1) = (7, - 2), B (x_2, y_2) = (5, 1) and C (x_3, y_3) = (3, k)

Since these points are collinear therefore area (ΔABC) = 0

⇒ $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

⇒ $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

⇒ 7 (1 – k) + 5 (k + 2) + 3 (- 2 – 1) = 0

⇒ 7 – 7k + 5k + 10 – 9 = 0

⇒ – 2 k + 8 = 0 ⇒ 2k = 8

⇒ k = 4

Hence, given points are collinear for k = 4.

Que 2. Find the area of the triangle formed by joining the mid-points of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

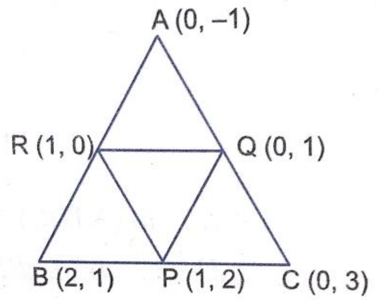

Fig. 6.22

Sol. Let A (x_1, y_1) = (0, - 1), B (x_2, y_2) = (2, 1), C (x_3, y_3) = (0, 3) be the vertices of ΔABC.

Now, let P, Q R be the mid-point of BC, CA and AB, respectively.

So, coordinates of P, Q, R are

P = $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ = (1, 2)

$Q = \left(\frac{0+0}{2}, \frac{3-1}{2}\right)$ = (0, 1)

$R = \left(\frac{2+0}{2}, \frac{1-1}{2}\right)$ = (1, 0)

Therefore, ar (ΔPQR) = $\frac{1}{2}$ [1(1 – 0) + 0(0 – 2) + 1(2 – 1)] = $\frac{1}{2}$ (1 + 1) = 1 sq. unit

Now, ar (ΔABC) = $\frac{1}{2}$ [0(1 – 3) + 2(3 + 1) + 0(- 1 – 1)]

= $\frac{1}{2}$ [0 + 8 + 0] = $\frac{8}{2}$ = 4 sq. units

∴ Ratio of ar (ΔPQR) to the ar (ΔABC) = 1: 4.

Que 3. Find the area of the quadrilateral whose vertices, taken in order, are (- 4, - 2), (- 3, - 5), (3, -2) and (2, 3).

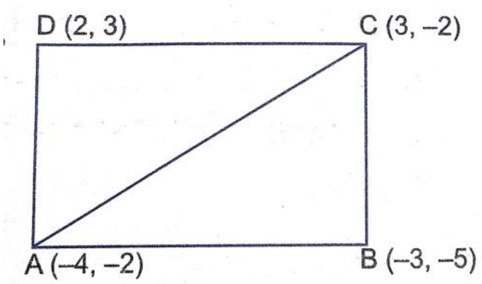

Fig. 6.23

Sol. Let A (- 4, - 2), B (- 3, - 5), C (3, -2) and D (2, 3) be the vertices of the quadrilateral ABCD.

Now, area of quadrilateral ABCD

= area of ΔABC + area of ΔADC

= $\frac{1}{2}$ [- 4 (- 5 + 2) – 3 (- 2 + 2) + 3 (- 2 + 5)]

+ $\frac{1}{2}$ [- 4 (- 2 – 3) + 3 (3 + 2) + 2 (- 2 + 2)]

= $\frac{1}{2}$ [12 – 0 + 9] + $\frac{1}{2}$ [20 + 15 + 0]

= $\frac{1}{2}$ [21 + 35] = $\frac{1}{2}$ x 56 = 28 sq. units.

Que 4. A median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, - 6), B (3, - 2), and C (5, 2).

Sol. Since AD is the median of ΔABC, therefore, D is the mid-point of BC.

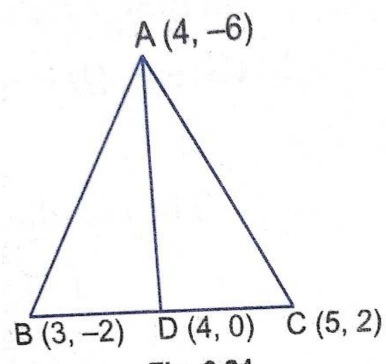

Fig. 6.24

Coordinates of D are $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ $i.e., (4, 0)$

Now, area of ΔABD

$$= \frac{1}{2}[4(-2-0) + 3(0+6) + 4(-6-2)]$$

$$= \frac{1}{2}(-8+18-16) = \frac{1}{2} \times (-6) = -3$$

Since area is a measure, it cannot be negative.

Therefore, ar (ΔABD) = 3 sq. units

And area of ΔADC $= \frac{1}{2}[4(0-2) + 4(2+6) + 5(-6-0)]$

$$= \frac{1}{2}(-8+32-30)$$

$$= \frac{1}{2}(-6) = -3, \textit{which cannot be negative.}$$

∴ ar (ΔADC = 3 sq. units

Here, ar (ΔABD) = ar (ΔADC)
Hence, the median divides it into two triangles of areas.

Que 5. Find the ratio in which the point P (x, 2), divides the line segment joining the points A (12, 5) and B (4, 3). Also find the value of x.

Sol.

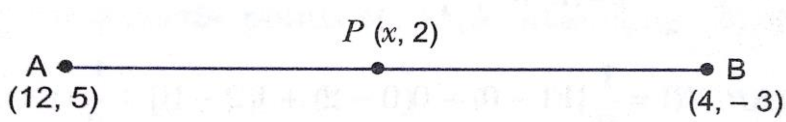

Fig. 6.25

Let the ratio in which point P divides the line segment be k: 1.

Then, coordinates of P: $\left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right)$

Given, the coordinates of P as (x, 2)

$\therefore \qquad \frac{4k+12}{k+1} = x \qquad$ …… (i)

And $\qquad \frac{-3k+5}{k+1} = 2 \qquad$ ……. (ii)

-3k + 5 = 2k + 2

5k = 3 $\Rightarrow$ k = $\frac{3}{5}$

Putting the value of k in (i), we have

$$\frac{4 \times \frac{3}{5} + 12}{\frac{3}{5} + 1} = x \qquad \Rightarrow \qquad \frac{12 + 60}{3 + 5} = x$$

$$x = \frac{72}{8} \qquad \Rightarrow \qquad x = 9$$

The ratio in which p divides the line segment is $\frac{3}{5}, i.e., 3:5$.

Que 6. If A (4, 2), B (7, 6) and C (1, 4) are the vertices of a ∆ABC and AD is its median, prove that the median AD divides ∆ABC into two triangles of equal areas.

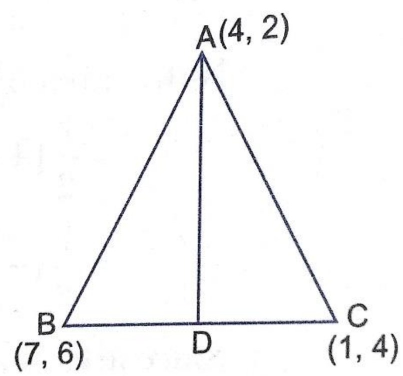

Fig. 6.26

Sol. Given: AD is the median on BC.

$\Rightarrow$ BD = DC

The coordinates of mid-point D are given by.

$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right) \quad i.e., \quad \left(\frac{1+7}{2}, \frac{4+6}{2}\right)$

Coordinates of D are (4, 5).

Now, Area of triangle ABD = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$=\frac{1}{2}[4(6-5) + 7(5-2) + 4(2-6)] = \frac{1}{2}[4+21-16] = \frac{9}{2} \text{ sq. units}$$

Area of ΔACD = $\frac{1}{2}[4(4-5) + 1(5-2) + 4(2-4)]$

$$= \frac{1}{2}[-4+3-8] = -\frac{9}{2} = \frac{9}{2} \text{ sq. units}$$

Hence, AD divides ΔABC into two equal areas.

Que 7. If the point A (2, 4) is equidistant from P (3, 8) and Q (-10, y), find the values of y. Also find distance PQ.

Sol. Given points are A (2, - 4), P (3, 8) and Q (-10, y)

According to the question,

$$PA = QA$$
$$\sqrt{(2-3)^2 + (-4-8)^2} = \sqrt{(2+10)^2 + (-4-y)^2}$$
$$\sqrt{(-1)^2 + (-12)^2} = \sqrt{(12)^2 + (4+y)^2}$$
$$\sqrt{1+144} = \sqrt{144+16+y^2+8y}$$
$$\sqrt{145} = \sqrt{160+y^2+8y}$$

On squaring both sides, we get

$$145 = 160 + y^2 + 8y$$
$$y^2 + 8y + 160 - 145 = 0$$
$$y^2 + 8y + 15 = 0$$
$$y^2 + 5y + 3y + 15 = 0$$
$$y(y+5) + 3(y+5) = 0$$

⇒ $(y + 5)(y + 3) = 0$

⇒ $y + 5 = 0 \quad ⇒ y = -5$

and $y + 3 = 0 \quad ⇒ y = -3$

∴ $y = -3, -5$

Now, $PQ = \sqrt{(-10-3)^2 + (y-8)^2}$

For y = - 3 $\quad PQ = \sqrt{(-13)^2 + (-3-8)^2} = \sqrt{169+121} = 290\ units$

And for y = - 5 $\quad PQ = \sqrt{(-13)^2 + (-5-8)^2} = \sqrt{169+169} = \sqrt{338}\ units$

Hence, values of y are – 3 and – 5, PQ = $\sqrt{290}\ and\ \sqrt{338}$

Que 8. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, - 3). The origin is the mid-point of the base. Find the coordinates

of the points A and B. Also find the coordinates of another point D such that BACD is a rhombus.

Sol. ∵ O is the mid-point of the base BC.

∵ Coordinates of point B are (0, 3).

So, BC = 6 units

Let the coordinates of point A be (x, 0).

Using distance formula,

$$AB = \sqrt{(0-x)^2 + (3-0)^2} = \sqrt{x^2+9}$$

$$BC = \sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{36}$$

Also, $Ab = BC$ (∴ Δ ABC is an equilateral triangle)

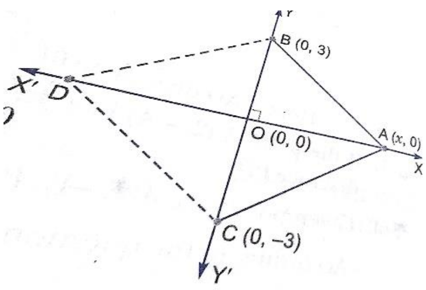

Fig. 6.27

$$\sqrt{x^2+9} = \sqrt{36}$$

$x^2 + 9 = 36$

$x^2 = 27$ ⇒ $x^2 - 27 = 0$

$$x^2 - \left(3\sqrt{3}\right)^2 = 0 \quad \Rightarrow \quad \left(x+3\sqrt{3}\right)\left(x-3\sqrt{3}\right) = 0$$

$$x = -3\sqrt{3} \; or \; x = 3\sqrt{3}$$

⇒ $x = \mp 3\sqrt{3}$

∴ Coordinates of point D = $\left(-3\sqrt{3},\ 0\right)$

Que 9. Prove that the area of a triangle with vertices (t, t – 2), (t + 2, t + 2) and (t + 3, t) is independent of t.

Sol. Area of a triangle = $\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

Area of the triangle = $\frac{1}{2}|t(t+2-t) + (t+2)(t-t+2) + (t+3)(t-2-t-2)|$

$= \frac{1}{2}|2t+2t+4-4t-12|$

$= 4$ Sq. units

Which is independent of t. Hence proved.

Que 10. The coordinates of the points A, B and c are (6, 3), (-3, 5) and (4, -2) respectively. P(x, y) is any point in the plane. Show that $\frac{ar(\Delta PBC)}{ar(\Delta ABC)} = \frac{x+y-2}{7}$

Sol. P(x, y), B (-3, 5), C (4, -2), A (6, 3)

$\therefore\ ar(\Delta PBC) = \frac{1}{2}|x\,(7) - 3\,(-2 - y) + 4\,(y - 5)| = \frac{1}{2}|7x + 7y - 14|$

$ar(\Delta ABC) = \frac{1}{2}|6 \times 7 - 3(-5) + 4(3 - 5)| = \frac{1}{2}|42 + 15 - 8| = \frac{49}{2}$

$LHS = \left|\frac{ar(\Delta PBC)}{ar(\Delta ABC)}\right| = \frac{\frac{1}{2}|7x + 7y - 14|}{\frac{1}{2} \times 49}\left|\frac{x + y + -2}{7}\right| = \frac{x + y + -2}{7} = RHS$

Que 11. In fig. 6.28, the vertices of Δ ABC are A (4, 6), B (1, 5) and C (7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.

Sol. Since $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

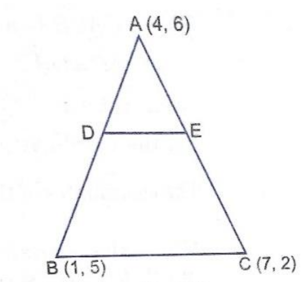

Fig. 6.28

⇒ D and E divide AB and AC respectively in the ratio 1: 2

Coordinates of D are $\left(\frac{1(1)+2(4)}{3}\right), \left(\frac{1(5)+2(6)}{3}\right)\ i.e., \left(3, \frac{17}{3}\right)$

Coordinates of E are $\left(\frac{1(7)+2(4)}{3}, \frac{1(2+2(6))}{3}\right)\ i.e., \left(5, \frac{14}{3}\right)$

Area of ΔADE = $\frac{1}{2}\left[4\left(\frac{17}{3} - \frac{14}{3}\right) + 3\left(\frac{14}{3} - 6\right) + 5\left(6 - \frac{17}{3}\right)\right]$

$= \frac{1}{2}\left(4 + (-4) + \frac{5}{3}\right) = \frac{5}{6}\ sq\ unit$

Area ΔABC $= \frac{1}{2}[4(3) + 1(-4) + 7(1)] = \frac{15}{2}\ sq\ unit$

Area ΔADE: area ΔABC = $\frac{5}{6} : \frac{15}{2}\ or\ 1: 9$