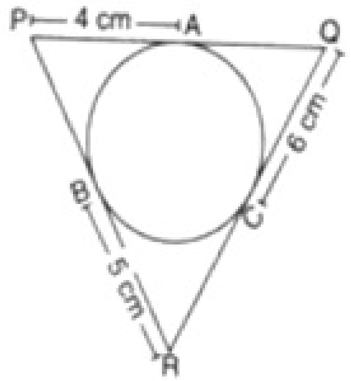
CBSE Test Paper 01

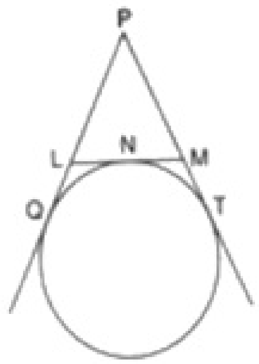
Chapter 10 Circles

1. The perimeter of $\triangle PQR$ in the given figure is **(1)**



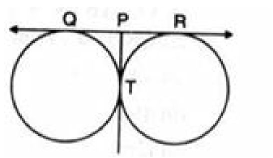
   a. 15 cm
   b. 60 cm
   c. 45 cm
   d. 30 cm.

2. If PQ = 28 cm, then the perimeter of $\triangle$PLM is **(1)**



   a. 48 cm
   b. 56 cm
   c. 42 cm
   d. 28 cm

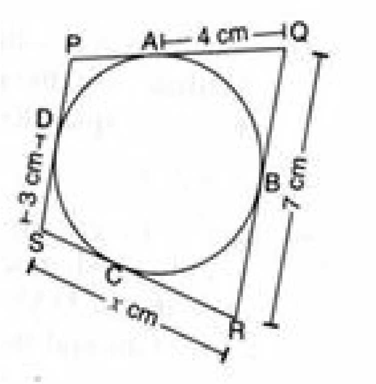
3. In the given figure if QP = 4.5 cm, then the measure of QR is equal to **(1)**



   a. 15 cm
   b. 9 cm

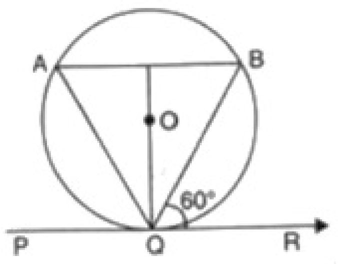
c. 18 cm
d. 13.5 cm

4. In the given figure, if AQ = 4 cm, QR = 7 cm, DS = 3 cm, then x is equal to **(1)**



a. 6 cm
b. 10 cm
c. 11 cm
d. 8 cm

5. If PQR is a tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 60^\circ$, then ∠AQB is equal to **(1)**



a. 60°
b. 30°
c. 90°
d. 45°

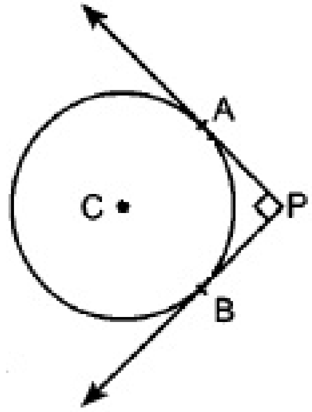
6. How many common tangents can be drawn to two circles touching internally? **(1)**

7. How many tangents can a circle have? **(1)**

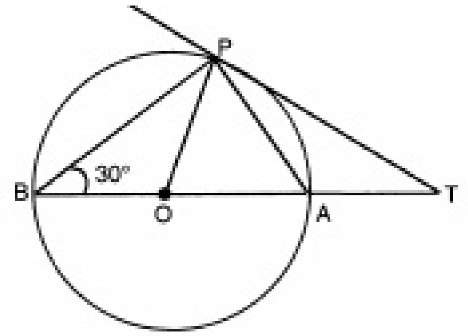
8. A quadrilateral ABCD is drawn to circumscribe a circle. If AB = 12 cm, BC = 15 cm and CD = 14 cm, find AD. **(1)**

9. How many tangents, parallel to a secant can a circle have? **(1)**

10. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, find the length of each tangent. **(1)**

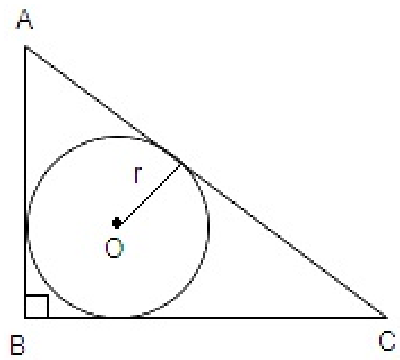


11. In the given figure,line BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle$PBO = 30^0 what is the measure of PTA? **(2)**



12. Two concentric circles are of radii 7 cm and r cm respectively where r > 7. A chord of the larger circle of the length 48 cm, touches the smaller circle. Find the value of r. **(2)**

13. In the adjoining figure, a right angled $\triangle ABC$, circumscribes a circle of radius r. If AB and BC are of lengths 8 cm and 6 cm respectively, then find the value of r. **(2)**



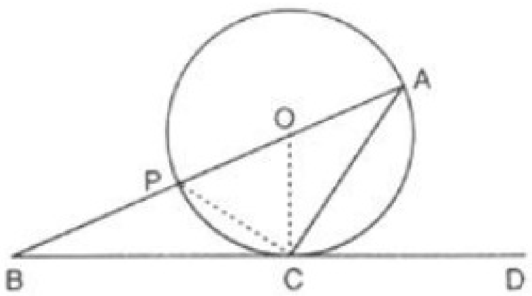
14. PQR is a right angled triangle right angled at Q. PQ = 5 cm, QR = 12 cm. A circle with centre O is inscribed in $\triangle$PQR, touching its all sides. Find the radius of the circle. **(3)**

15. ABC is a right-angled triangle, right angled at A. A circle is inscribed in it. The lengths of two sides containing the right angle are 24 cm and 10 cm. Find the radius of the incircle. **(3)**

16. Two concentric circles are of radii 5 cm and 3 cm, find the length of the chord of the larger circle which touches the smaller circle. **(3)**

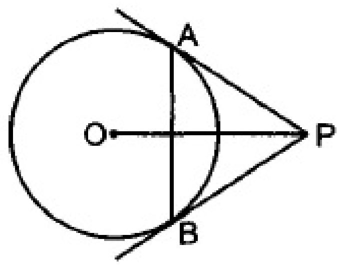
17. The common tangents AB and CD to two circles with centres O and O' intersect at E between their centres. Prove that the points O, E and O' are collinear. **(3)**

18. In fig O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90^{\circ}$. **(4)**



19. If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that at the radius r of the circle which touches the sides of the triangle is given by r = $\frac{a+b-c}{2}$ **(4)**

20. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre O. Prove that OP is the right bisector of line segment AB. **(4)**



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Solution

1. d. 30 cm.

   Explanation Since Tangents from an external point to a circle are equal.

   $\therefore$ PA = PB = 4 cm,

   BR = CR = 5 cm

   CQ = AQ = 6 cm

   Perimeter of $\triangle$PQR = PQ + QR + RP

   = PA + AQ + QC + CR + BR + PB

   = 4 + 6 + 6 + 5 + 5 + 4 = 30 cm

2. b. 56 cm

   Explanation: We know that, PQ = $\frac{1}{2}$ (Perimeter of $\triangle$ PLM)

   $\Rightarrow$ 28 = $\frac{1}{2}$ (Perimeter of $\triangle$PLM)

   $\Rightarrow$(Perimeter of $\triangle$PLM) = 28 $\times$ 2 = 56 cm

3. b. 9 cm

   Explanation: Here QP = PT = 4.5 cm [Tangents to a circle from an external point P]

   Also PT = PR = 4.5 cm [Tangents to a circle from an external point P]

   $\therefore$ QR = QP + PQ= 4.5 + 4.5 = 9 cm

4. a. 6 cm

   Explanation: Here AQ = 4 cm

   $\therefore$ QB = AQ = 4 cm [Tangents from an external point]

   $\therefore$ BR = 7 - 4 = 3 cm

   $\therefore$ BR = CR = 3 cm [Tangents from an external point]

   Also SD = SC = 3 cm [Tangents from an external point]

   Therefore, x = CS + CR = 3 + 3 = 6 units

5. a. 60°

   Explanation: Since AB $\parallel$ PR and BQ is intersecting them.

$\therefore \angle BQR = \angle QBA = 60^\circ$ [Alternate angles]

And $\angle BQR = \angle QAB = 60^\circ$ [Alternate segment theorem]

Now, in triangle AQB,

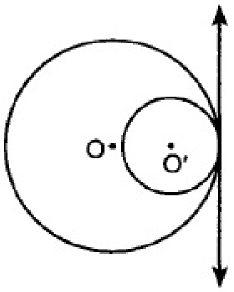
$\angle AQB + \angle QBA + \angle BAQ = 180^\circ$

$\Rightarrow \angle AQB + 60° + 60° = 180°$

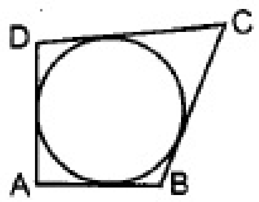
$\Rightarrow \angle AQB = 60$

6. One common tangent can be drawn to two circles touching internally

   Figure:



7. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

8. 

   Now,

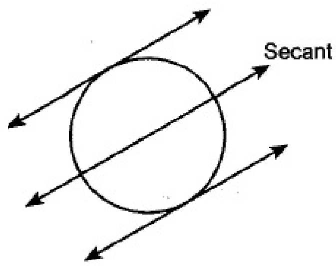
   $AB + CD = BC + AD$

   $\Rightarrow \ 12 + 14 = 15 + AD$

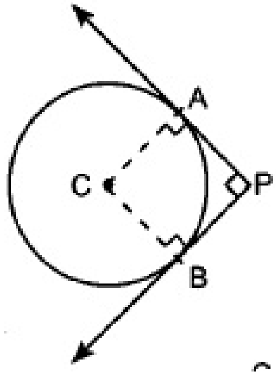
   $\Rightarrow AD = 11cm$

9. A circle can have 2 tangents parallel to a secant.

   Diagram:



10. PA and PB are two tangents drawn from an external point P to a circle.



CA $\perp$ AP

CB $\perp$ BP

PA $\perp$ PB

$\therefore$ BPAC is a square.

$\Rightarrow AP = PB = BC = 4cm$

11. $\angle AOP \ = \ 2\angle ABP$ (Angle subtended by an arc is twice angle subtended by same arc at any other point on the circle)

$\Rightarrow \angle AOP \ = \ 2 \ \times \ 30 \ = \ 60^\circ$

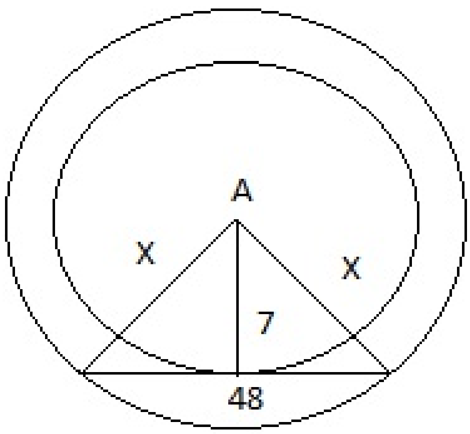
$\angle OPT \ = 90^\circ$ (Radius and Tangent are perpendicular to each other)

In $\triangle \ OTP$

$90^\circ + 60^\circ + \angle T = 180^\circ$ (ASP)

$\Rightarrow \angle ATP = 30^\circ$

12.



Let us take r = x

Now using Pythagoras theorem

$(x)^2 = 24^2 + 7^2$

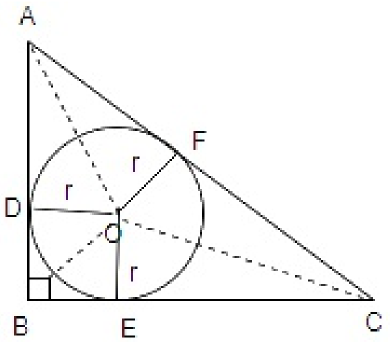
$(x)^2 = 576 + 49$

$(x)^2 = 625$

Therefore, x = 25 cm.

r = 25 cm.

13. Let D, E and F are points where the in-circle touches the sides AB, BC and CA respectively. Join OA, OB and OC.



In $\triangle ABC, AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$= 8^2 + 6^2$

$= 64 + 36$

= 100

$\therefore AC = \sqrt{100} = 10\ cm$ [taking positive square root , as length cannot be negative]

Now, $ar(\triangle OAB) = \frac{1}{2} \times OD \times AB = \frac{1}{2} \times r \times 8 = \frac{8r}{2} = 4rcm^2$

$ar(\triangle OBC) = \frac{1}{2} \times OE \times BC = \frac{1}{2} \times r \times 6 = \frac{6r}{2} = 3rcm^2$

and $ar(\triangle OAC) = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times r \times 10 = \frac{10r}{2} = 5rcm^2$

$\therefore ar(\triangle ABC) = ar(\triangle OAB) + ar(\triangle OBC) + ar(\triangle OAC)$

$\Rightarrow \frac{1}{2} \times AB \times BC = 4r + 3r + 5r = 12r$

$\Rightarrow \frac{1}{2} \times 8 \times 6 = 12r$

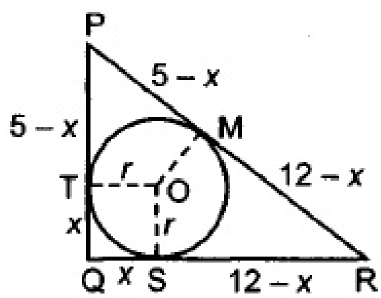
$\Rightarrow 24 = 12r$

$\Rightarrow r = \frac{24}{12}$

$\Rightarrow r = 2\ cm$

The value of r is 2 cm.

14.



Let QS = x; SR = 12 - x

$\therefore$ PT = 5 - x, PM = PT

$\therefore$ PM = 5 - x

Also SR = MR $\Rightarrow$ MR = 12 - x

Also $PQ^2 + QR^2 = PR^2$

$\Rightarrow$ PR = 13 $\Rightarrow$ PM + MR = 13

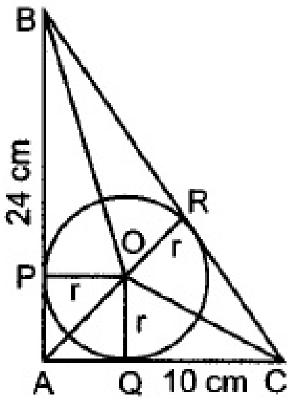
$\Rightarrow 5 - x + 12 - x = 13 \Rightarrow$ 2x = 4 $\Rightarrow$ x = 2

Also OSQT is a square

$OS = QS$

$\Rightarrow OS = 2cm$

15. Given,



$AB = 24cm, AC = 10cm$

In right-angled $\triangle$ABC

$\mathrm{BC}^2 = \mathrm{AB}^2 + \mathrm{AC}^2$

$= 24^2 + 10^2$

$= 676$

$\Rightarrow \mathbf{BC} = 26\mathrm{cm}$

Let r be the radius of the incircle

$\Rightarrow$ OP $\perp$ AB, OQ $\perp$ AC and OR $\perp$ BC

OP = OQ = OR [Incentre of a triangle is equidistant from its sides]

ar($\triangle$ABC) = ar($\triangle$AOB) + ar($\triangle$BOC) + ar($\triangle$AOC)

$\frac{1}{2}\mathrm{AB} \times \mathrm{AC} = \frac{1}{2}\mathrm{AB} \times \mathrm{OP} + \frac{1}{2}\mathrm{AC} \times \mathrm{OQ} + \frac{1}{2} \times \mathrm{BC} \times \mathrm{OR}$

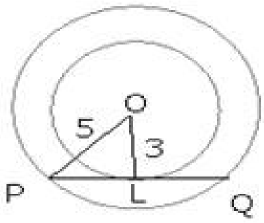
$\frac{1}{2} \times 24 \times 10 = \frac{1}{2}[24 \times r + 10 \times r + 26 \times r]$

$\Rightarrow\ 120 = r[24 + 10 + 26]$

$\Rightarrow$ 120 = r[24+ 10 + 26]

$\Rightarrow$ 120 = 30r $\Rightarrow$ r = 4 cm

16. $\because$ PQ is the chord of the larger circle which touches the smaller circle at the point L.
Since PQ is tangent at the point L to the smaller circle with centre O.



$\therefore$ OL $\perp$ PQ

$\therefore$ PQ is a chord of the bigger circle and OL $\perp$ PQ

$\therefore$ OL bisects PQ

$\therefore$ PQ = 2 PL

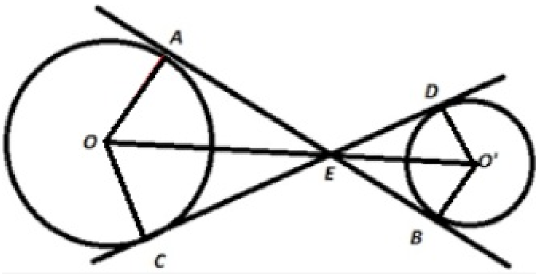
In $\triangle$OPL,

$$PL = \sqrt{OP^2 - OL^2} = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = 4$$

$\therefore$ Chord PQ = 2PL =8 cm

$\therefore$ Length of chord PQ = 8 cm

17. Construction: Join OA and OC.



$\angle AEC = \angle DEB$ ....(vertically opposite angles)

In ΔOAE and ΔOCE,

OA = OC ...(Radii of the same circle)

OE = OE ...(Common side)

$\angle OAE = \angle OCE$ ....(each is 90°)

$\Rightarrow \Delta OAE \cong \Delta OCE$ ....(RHS congruence criterion)

$\Rightarrow \angle AEO = \angle CEO$ ....(cpct)

Similarly, for the circle with centre O',

$\angle DEO' = \angle BEO'$

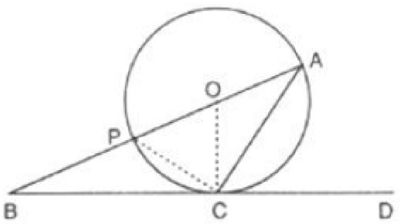
Now, $\angle AEC = \angle DEB$

$\Rightarrow \frac{1}{2}\angle AEC = \frac{1}{2}\angle DEB$

$\Rightarrow \angle AEO = \angle CEO = \angle DEO' = \angle BEO'$

Hence, all the fours angles are equal and bisected by OE and O'E.

So, O, E and O' are collinear.

18. 

$\angle OCD = 90^\circ$ (tangent and radii are $\perp$ to one another at the point of contact)

In $\triangle$OCA,

OC = OA (radii of circle)

Hence, $\angle OCA = \angle OAC$ (angles opposite to equal sides are equal)

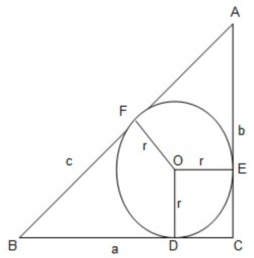
Also, $\angle OCD = \angle OCA + \angle ACD$

$90^\circ = \angle OAC + \angle ACD \; (\because \angle OCA = \angle OAC)$

$90^\circ = \angle BAC + \angle ACD$

Hence, $\angle BAC + \angle ACD = 90^\circ$

Hence proved.

19. 

The circle touches the sides BC, CA, AB of the right triangle ABC at D, E and F respectively. Let BC = a, CA = b and AB = c

Now, AF = AE and BD = BF

⇒ AF = AE = AC - CE and BF = BD = BC - CD

⇒ AF = b - r and BF = a - r (∵ OEDC is a square)

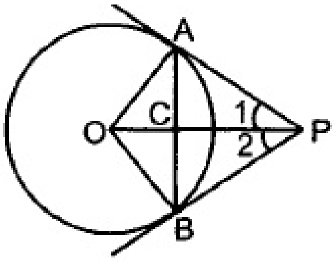
⇒ AF + BF = (b - r) + (a - r)

⇒ AB = a + b - 2r

⇒ c = a + b - 2 r

⇒ r = $\frac{a+b-c}{2}$

20. Given, PA and PB are two tangents.



Construction: Join $OA\ and\ OB.$

In $\triangle$PAO and $\triangle PBO, OA = OB$ [Radii]

$OP = OP$ [Common]

and $AP = BP$ [Tangents from P]

$\therefore\triangle$PAO = $\triangle$PBO (SSS)

$\Rightarrow \angle$1 = $\angle$2

In $\triangle$APC and $\triangle$BPC, $\angle$1 = $\angle$2 [Proved]

$AP = BP\ and\ PC = PC,$

$\triangle$APC $\cong$ $\triangle$BPC [SAS]

AC = BC

and $\angle$ACP = $\angle$BCP

Also, $\angle$ACP + $\angle$BCP = 180°[by linear pair axiom]

$\angle$ACP + 90° = 180°

$\Rightarrow \angle$ACP = 90°

$\therefore$OP is right bisector of AB.