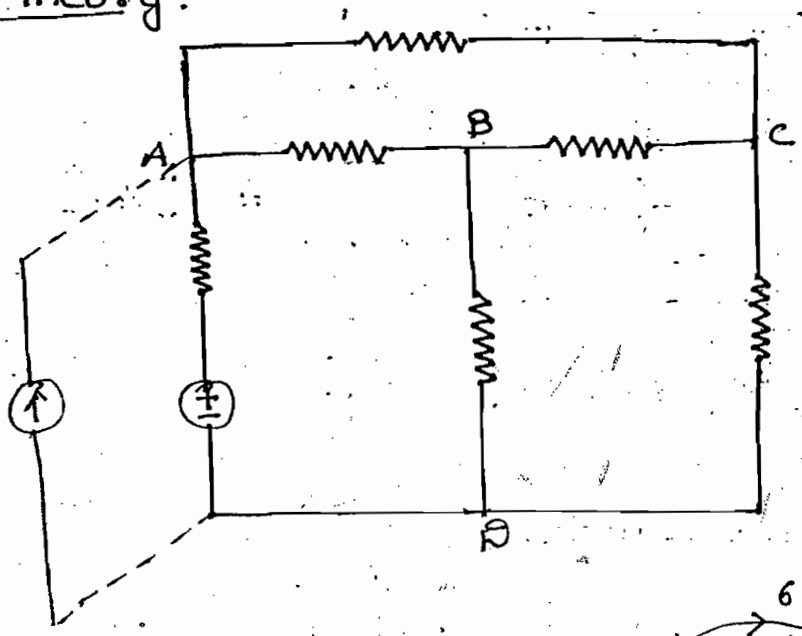
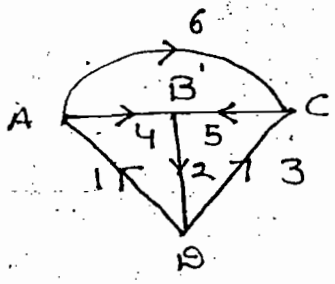
Lecture-15

Graph Theory :-





→ N/w topology is the steady of the N/w properties by investigating interconnection b/w branches and nodes, it mainly concentrate on the geometry of the N/w

→ In the N/w topology any N/w is replace by graph To develop graph ~~of~~ each element is replace by either straight line or arc of the semi-circle, voltage source is replace by S.C and current source is replace by O.C and graph retains all the nodes of original N/w

→

No. of branches of N/w	≥	No. of branch of graph.

Augmented Incidence Matrix :-

	1	2	3	4	5	6
A	-1			+1		+1
B		+1		-1	-1	
C			-1		+1	-1
D	+1	-1	+1			

$= [A_a]$

Reduced Incidence Matrix :-

	1	2	3	4	5	6
A	-1			+1		+1
B		+1		-1	-1	
C			-1		+1	-1
D						

$= [A]$

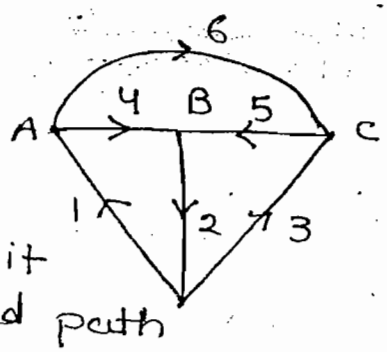
D → Ref - Node or Datum Node

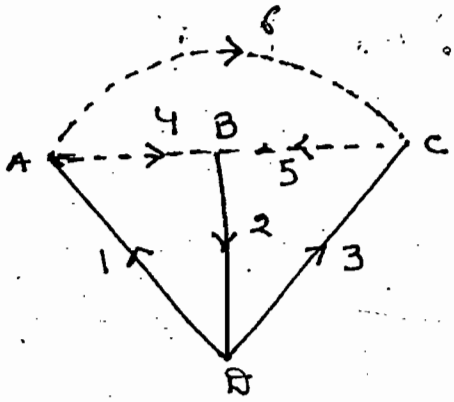
→ All the information regarding the graph can be represent mathematically in consize form is called as Augmented incidence matrix

→ For a given graph Augmented incidence matrix is unique

Tree :-

→ Tree is a connected sub-graph. It connects the all the nodes of the N/w but it doesn't consist of any closed path





Tree {1, 2, 3}

Total Tree Branches = N-1
= 4-1 = 3

→ The set of branches which are disconnected to form a tree is called as co-tree or complementary tree

Co-Tree {4, 5, 6}

→ The branch which form a tree is called as tree branch (Twig)

→ Total no. of tree branches = N-1

→ The branch which is disconnected to form a tree is called as link (chord)

→ Total no. of links (l) = b - (N-1)

where b = total No. of branches of the N/w

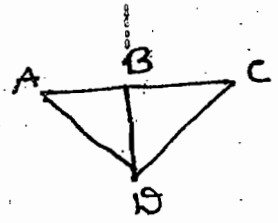
→ Tree is not unique

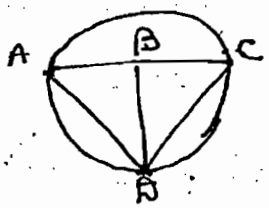
Total no. of tree = $N^{N-2} = 4^{4-2} = 16$

→ Total No. of possible tree = N^{N-2}

Note:-

→ The above formula can be applied
(a) When connection is present b/w all the node
(b) When no repeated branch is present b/w the node





→ Total no. of possible tree for any graph

$$= \text{Det}(AA^T)$$

where A = Reduced incidence matrix

Node Pair Voltages :—

Total no. of node pair voltages $= \frac{N(N-1)}{2}$

eg :- $\underline{V_{AB}, V_{AC}, V_{AD}, V_{BC}, V_{BD}, V_{CD}}$

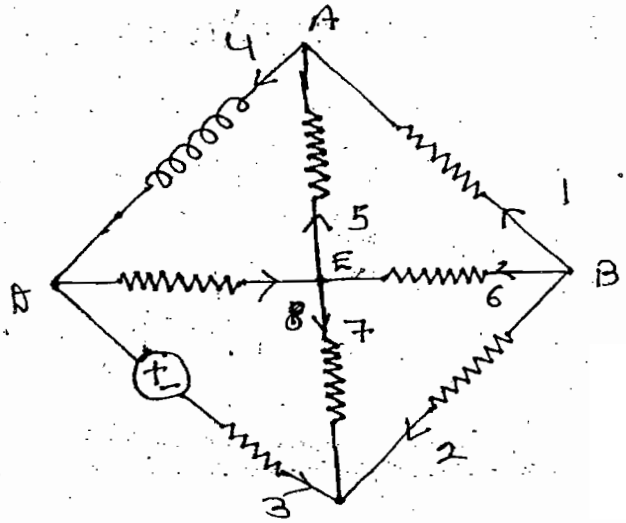
6

→ Edges = Branches

→ Total no. of edges (branches) $= \frac{N(N-1)}{2} = \frac{4\times3}{2}$

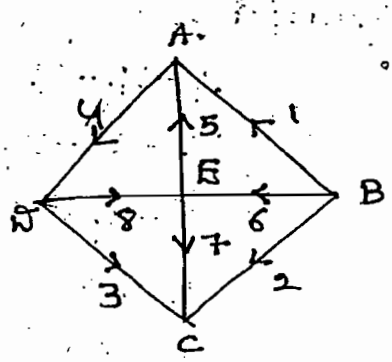
$= 6$

Ques :- Develop Tie-set matrix of the N/w shown



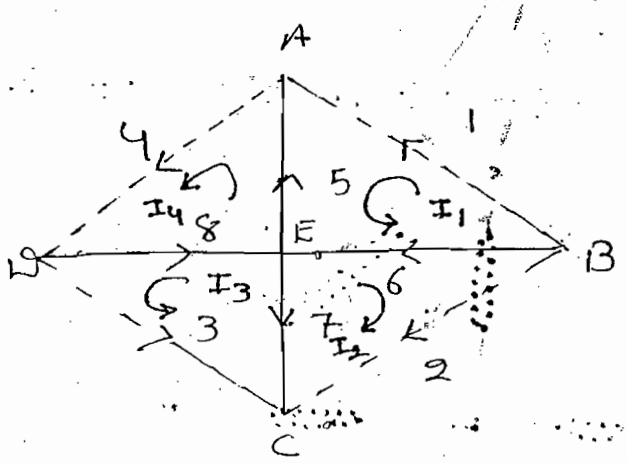
Step-1 :—

Develop the graph for the given N/w



Step-(II):-

Develop a tree for the graph



Step-(III):-

Identify total no. of basic loop/ fundamental loop (f-loop) or independent loops

⇒ Basic loop should consist of only one link

⇒ Total no. of basic loop = total no. of links

i.e $l = b-(N-1)$

⇒ Basic loop direction is same as link current direction

C_b

	V_1 1	V_2 2	V_3 3	V_4 4	V_5 5	V_6 6	V_7 7	V_8 8
I_1	+1			-	-1	-1	-	-
I_2		+1		-		-1	-1	
I_3			+1				-1	-1
I_4				+1	+1			+1

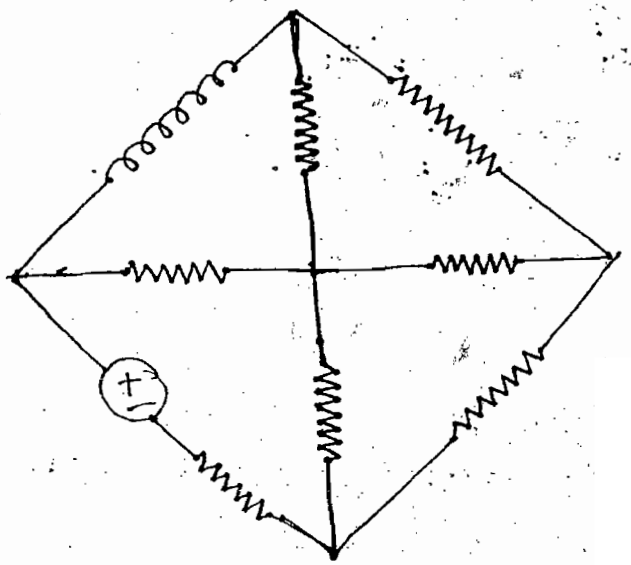
$= [C]$

KVL Equations:-

$$V_1 - V_5 - V_6 = 0$$
$$V_2 - V_6 - V_7 = 0$$
$$V_3 - V_7 - V_8 = 0$$
$$V_4 + V_5 + V_8 = 0$$

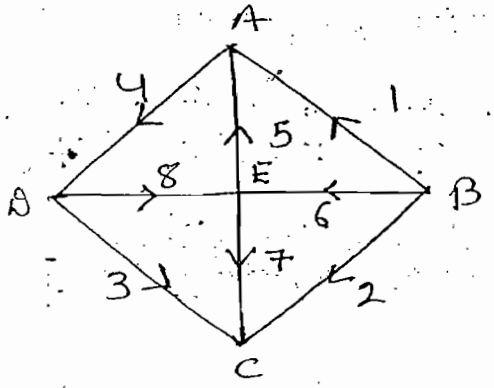
$$[C] = [U : C_b]$$

Ques:- Develop cut-set matrix of the N/w shown



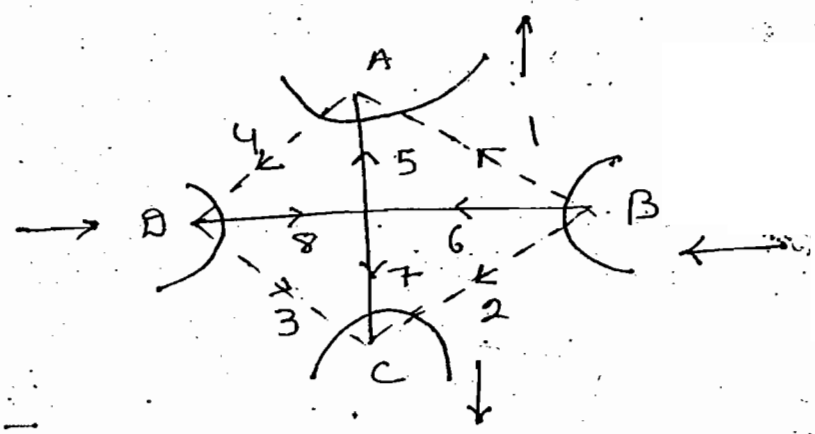
Step-(I) :-

Develop a graph for the given N/w



Step-(II) :-

Develop a tree for a graph



Step-(III)!-

Identify total no. of basic cut-sets or fundamental cut-sets / f-cut-sets

→ Basic cut-sets should consist of only one tree branch

→ Total No. of basic cut-sets = total No. of tree branches

i.e $= N-1$

→ Basic cut-set direction as same as a tree branch current direction

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8
	1	2	3	4	5	6	7	8
A	+1			-1	+1			
B	+1	+1				+1		
C		+1	+1				+1	
D			+1	-1				+1

$= [B]$

KCL Equations!-

$$I_1 - I_4 + I_5 = 0$$
$$I_1 + I_2 + I_6 = 0$$
$$I_2 + I_3 + I_7 = 0$$
$$I_3 - I_4 + I_8 = 0$$

$$[B] = [B_l : U]$$

$$[C] = [U : C_b]$$

↓

Tie-set

$$[B] = [B_l : U]$$

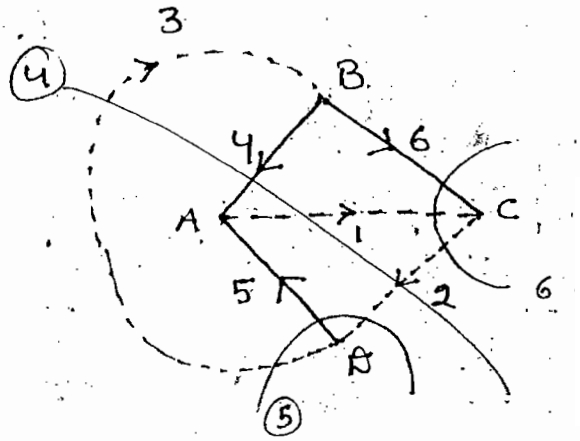
↓

cut-set

$$[C_b] = -[B_l]^T$$

$$[B_l] = -[C_b]^T$$

Ques:- Develop cut-set matrix of the graph shown



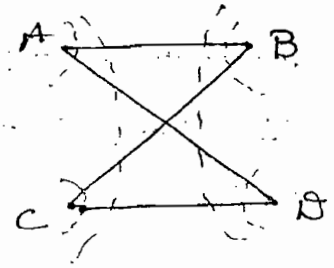
Soln:-

	1	2	3	4	5	6
** 4	-1	+1	-1	+1	.	
5	.	-1	+1	.	+1	.
6	+1	-1	.	.	.	+1

Ques:- Identify total no. of cut-sets of the graph shown

(a) 3 (b) 4

(c) 5 (d) 6



Note:-

→ cut-sets may consist of one tree branch or more than one tree branch

→ Basic cut-sets consist of only one tree branch

Conclusion:-

→ Total no. of possible tree = N^{N-2}

→ Tie-set matrix is not unique, total no. of possible tie-set matrix = N^{N-2}

→ cut-set matrix is not unique, Total no. of possible cut-set matrix = N^{N-2}

→ Rank of Tie-set matrix = total no. of links

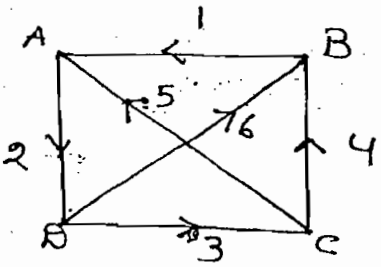
i.e. $l = b-(N-1)$

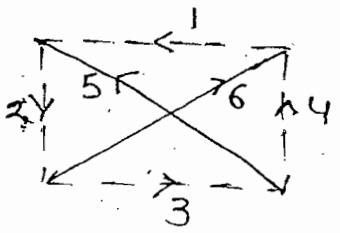
→ Rank of cut-set matrix = total no. of tree branches

$= N-1$

→ Rank of Incidence Matrix = $N-1$

→ The given figure is invalid tree. Since no connection is present b/w tree branches





Duality:-

R ⟷ G

L ⟷ C

V ⟷ I

Series ⟷ Parallel

O.C ⟷ S.C

KVL ⟷ KCL

Loop (Mesh) $\longleftrightarrow$ Node

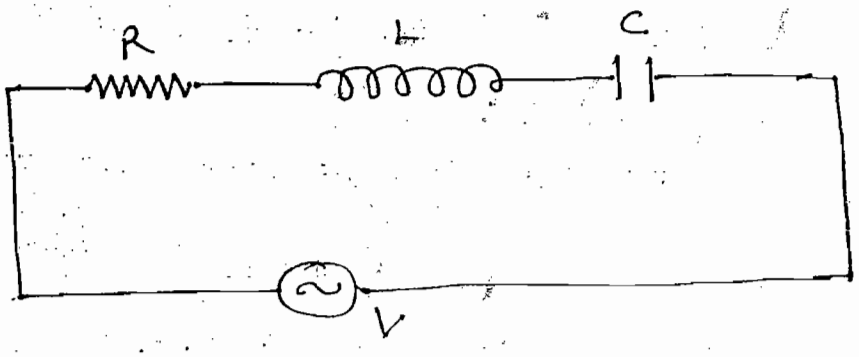
Tie-set $\longleftrightarrow$ Cut-set

Thevenin's $\longleftrightarrow$ Norton's

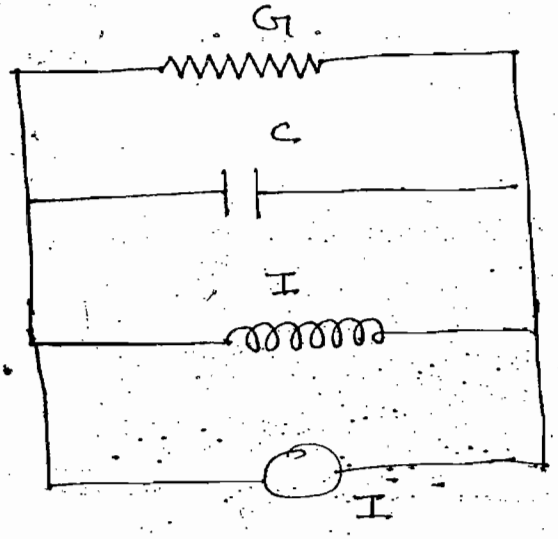
Foster-I form (series) $\longleftrightarrow$ Foster-II form (Parallel)

$\frac{dV}{dt} \longleftrightarrow \frac{dI}{dt}$

$\int V dt \longleftrightarrow \int i dt$



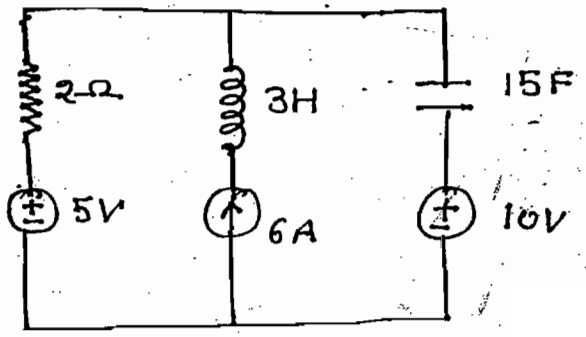
$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \longrightarrow \text{KVL}$$



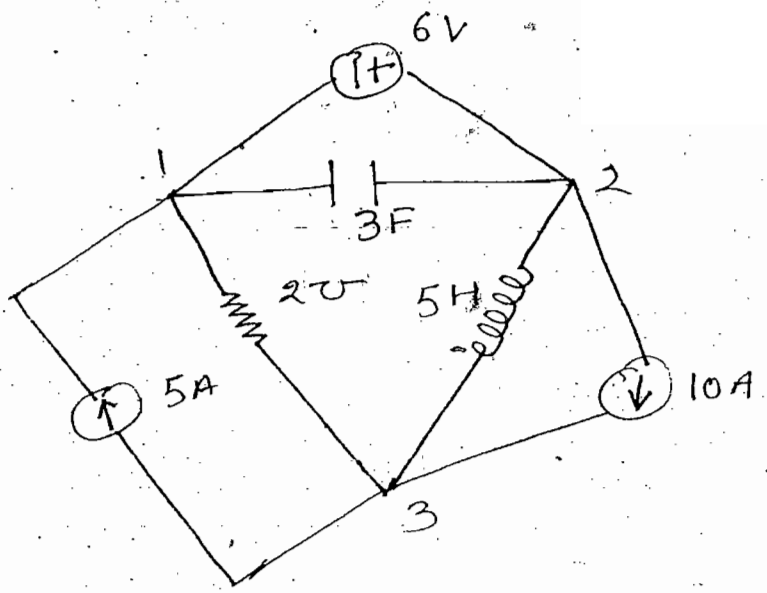
$$I = VG + C\frac{dV}{dt} + \frac{1}{L}\int V dt \longrightarrow \text{KCL}$$

Duality does not means the equivalence but it means that mathematical representation of both N/w are identicals.

Ques:- Develop dual of the N/w shown



Soln:-



Note:-

→ When voltage source circulate a current in clockwise direction arrowmark of the current source is indicated towards respective node

→ When current source circulate a current in clockwise direction +ve sign is assign to respecti node