Long Answer Type Questions

[4 marks]

Que 1. Use Euclid's division Lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Sol. Let a be an arbitrary positive integer.

Then by Euclid's Division algorithm, corresponding to the positive integers a and 3 there exist non-negative integers q and r such that

$a = 3q + r$ where $0 < r < 3$

$\Rightarrow$ $a^2 = 9q^2 + 6qr + r^2$ …. (i) $0 \leq r < 3$

Case-**I:** When r = 0 putting on (i)

$a^2 = 9q^2 = 3(3q^2) = 3m$ where $m = 3q^2$

Case-**II:** r = 1

$A^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$ where $m = 3q^2 + 2q$

Case-**III:** r = 2

A2 = 9q2 + 12q + 4 = 3(3q2 + 4q + 1) + 1 = 3m + 1 where m = (3q2 + 4q + 1)

Hence, square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Que 2. Show that one and only one out of n, n+2, n+4 is divisible by 3, where n is any positive integer.

Sol. Let q be the quotient and r be the remainder when n is divided by 3.

Therefore, $n = 3q + r$ where r = 0, 1,2

$\Rightarrow$ $n = 3q$ or $n = 3q + 1$ or $n = 3q + 2$

Case (i) if n = 3q, then n is divisible by 3, n + 2 and n + 4 are not divisible by 3.

Case (ii) if n = 3q + 1 then n + 2 = 3q + 3(q + 1), which is not divisible by 3 and n + 4 = 3q + 5, which is not divisible by 3.

So, only (n + 4) is divisible by 3.

Hence one and only one out of n, (n+2), (n+4) is divisible by 3.

Que 3. Use Euclid's division algorithm to find the HCF of:

(i) 960 and 432
(ii) 4052 and 12576

Sol. (i) since 960 > 432, we apply the division Lemma to 960 and 432.

We have, $960 = 432 \times 2 + 96$

Since the remainder $96 \neq 0$, so we apply the division lemma to 432 and 96.

We have, $960 = 432 \times 4 + 48$

Again remainder $48 \neq 0$, so we again apply division Lemma to 96 and 48.

We have, $96 = 48 \times 2 + 0$

The remainder has now become zero. So our procedure stops.

Since the divisor at this stage is 48.

Hence, HCF of 960 and 432 is 48

i.e., HCF (960, 432) = 48

(ii) Since 12576 > 4052, we apply the division lemma to 12576 and 4052 to get

$$12576 = 4052 \times 3 + 420$$

Since the remainder $420 \neq 0$, we apply the division lemma to 4052 and 420, to get

$$4052 = 420 \times 9 + 272$$

We consider the new divisor 272 and the new remainder 148, and apply the division lemma to get

$$420 = 272 \times 1 + 148$$

We consider the new divisor 272 and the new remainder 148, and apply the division lemma to get

$$148 = 124 \times 1 + 24$$

We consider the new divisor 124 and the new remainder 4, and apply the division lemma to get

$$24 = 4 \times 6 + 0$$

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 4, the HCF of 12576 and 4052 is 4.

Que 4. Using prime factorization method, find the HCF and LCM of 30, 72 and 432. Also show that HCF × LCM ≠ Product of the three numbers.

Sol. Given numbers = 30, 72, 432

$30 = 2 \times 3 \times 5$; $72 = 2^3 \times 3^2$ and $432 = 2^4 \times 3^3$

Here, 21 and 31 are the smallest powers of the common factors 2 and 3 respectively.

So, HCF (30, 72, 432) = $2^1 \times 3^1 = 2 \times 3 = 6$

Again, 2^4, 3^3 and 5^1 are the greatest powers of the prime factors 2,3 and 5 respectively.

So, LCM (30, 72, 432) = $2^4 \times 3^3 \times 5^1 = 2160$

HCF × LCM = 6 × 2160 = 12960

Product of numbers = 30 × 72 × 432 = 933120

Therefore, HCF × LCM ≠ Product of the numbers.

Que 5. Prove that $\sqrt{7}$ is an irrational number.

Sol. Let us assume, to the contrary, that $\sqrt{7}$ is a rational number.

Then, there exist co-prome positive integers a and b such that

$\sqrt{7} = \frac{a}{b}$, $b \neq 0$

So, $a = \sqrt{7}b$

Squaring both sides, we have

$a^2 = 7b^2$

⇒ 7 divides a^2 ⇒ 7 divides a

So, we can write

a = 7c, (where c is any integer)

Putting the value of a = 7c in (i), we have

$49c^2 = 7b^2$ ⇒ $7c^2 = b^2$

It means 7 divides b^2 and so 7 divides b.

So, 7 is a common factor of both a and b which is a contradiction.

So, our assumption that $\sqrt{7}$ is rational is wrong.

Hence, we conclude that $\sqrt{7}$ is an irrational number.

Que 6. Show that 5-$\sqrt{3}$ is an irrational number.

Sol. Let us assume that 5-$\sqrt{3}$ is rational.

So, 5-$\sqrt{3}$ may be written as

5-$\sqrt{3}$ = $\frac{p}{q}$, where p and q are integers, having no common factor except 1 and q ≠ 0.

$$\Rightarrow 5 - \frac{p}{q} = \sqrt{3} \quad \Rightarrow \quad \sqrt{3} = \frac{5q-p}{q}$$

Since $\frac{5q-p}{q}$ is a rational number which is a contradiction.

∴ $\sqrt{3}$ is also a rational number which is a contradiction.

Thus, our assumption is wrong.

Hence, 5 - $\sqrt{3}$ is an irrational number.

Que 7. Using Euclid's division algorithm, find whether the pair of numbers 847, 2160 are co-prime or not.

Sol. Since 2160 > 847 we apply the division lemma to 2160 and 847

We have, $2160 = 847 \times 2 + 466$

Since remainder 466 ≠ 0. So we apply the division lemma to 847 and 466

$$847 = 466 \times 1 + 381$$

Again remainder 381 ≠ 0. So we again apply the division lemma to 466 and 381.

$$466 = 381 \times 1 + 85$$

Again remainder 85 ≠ 0. So we again apply the division lemma to 381 and 85.

$$381 = 85 \times 4 + 41$$

Again remainder 41 ≠ 0. So we again apply the division lemma to 85 and 41.

$85 = 41 \times 2 + 3$

Again remainder $3 \neq 0$. So we again apply the division lemma to 41 and 3.

$41 = 3 \times 13 + 2$

Again remainder $2 \neq 0$. So we again apply the division lemma to 3 and 2.

$3 = 2 \times 1 + 1$

Again remainder $1 \neq 0$. So we again apply the division lemma to 2 and 1.

$2 = 1 \times 4 + 0$

The remainder now becomes 0. So, our procedure stops.

Since the divisor at this stage is 1.

Hence, HCF of 847 and 2160 is 1 and numbers are co-prime.

Que 8. Check whether 6^n can end with the digit 0 for any natural number n.

Sol. If the number 6^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of 6^n would contain the prime 5. But $6^n = (2 \times 3)^n = 2^n \times 3^n$ So the primes in factorisation of 6^n are2 and 3. So the uniqueness of the fundamental theorem of arithmetic guarantees that there are no other primes except 2 and 3 in the factrosation of 6^n. So there is no natural number n for which 6^n ends with digit zero.