INVERSE TRIGONOMETRY FUNCTION

CHAPTER – 2

INVERSE TRIGONOMETRY FUCTION

INVERSE OF A FUNCTION:

Inverse of a function exists iff function is bijective. Let $y = f(x) : A \to B$ be a one-one and onto function. i.e. bijection, then there will always exist bijective function $x = g(y) : B \to A$ such that if (p, q) is an element of f, (q, p) will be an element of g and the functions f(x) and g(x) are said to be inverse of each other. g(x) is also denoted by $f^{-1}(x)$ and f(x) is denoted by $g^{-1}(x)$

Domain of f(x) = Range of g(x)

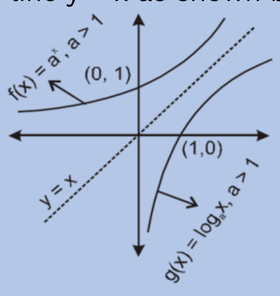
Range of f(x) = Domain of g(x)

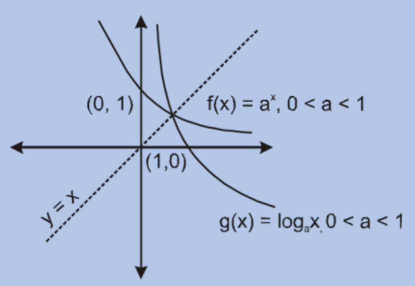
(i) Properties of Inverse Function:

(a) The graphs of f and g are the mirror images of each other in the line y = x.

Example

$f(x) = a^x$ and $g(x) = \log_a x$ are inverse of each other, and their graphs are mirror images of each other on the line y = x as shown below.





(b) Normally points of intersection of f and f^{-1} lie on the straight line y = x. However it must be noted that f(x) and $f^{-1}(x)$ may intersect otherwise also.

Example

f(x) = 1/x

(c) If f(x) and g(x) are inverse of each other then fog(x) = x and gof(x) = x

(d) If f and g are two bijections f:A → B, g : B → C, then the inverse of gof exists and $(gof)^{-1} = f^{-1} \circ g^{-1}$.

(e) If f(x) and g(x) are inverse function of each other, then $f'(g(x)) = \frac{1}{g'(x)}$

Formulas

The basic inverse trigonometric formulas are as follows:

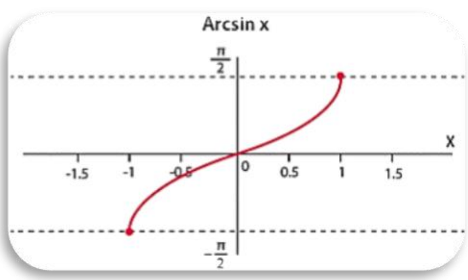
Inverse Trig Functions	Formulas
Arcsine	$\sin^{-1}(-x) = -\sin^{-1}(x)$, $x \in [-1, 1]$
Arccosine	$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$, $x \in [-1, 1]$
Arctangent	$\tan^{-1}(-x) = -\tan^{-1}(x)$, $x \in R$
Arccotangent	$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$, $x \in R$
Arcsecant	$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$, $\lvert x\rvert \geq 1$
Arccosecant	$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}(x)$, $\lvert x\rvert \geq 1$

INVERSE TRIGONOMETRY FUNCTIONS :

Six inverse trigonometric functions are $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\text{cosec}^{-1}x$, $\sec^{-1}x$ and $\cot^{-1}x$ which are described in detail as below

Arcsine Function

Arcsine function is an inverse of the sine function denoted by $\sin^{-1}x$. It is represented in the graph as shown below:



Domain	$-1 \leq x \leq 1$
Range	$-\pi/2 \leq y \leq \pi/2$

The branch with range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is called the principal value branch.

Note

$\sin^{-1} x$ is not equal to $(\sin x)^{-1}$ or $\frac{1}{\sin x}$

SOME OBSERVATIONS:

(i) Sin and sin^{-1} are increasing functions on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $[-1, 1]$ respectively.

$\therefore \theta_1 < \theta_2 \Rightarrow \sin\theta_1 < \sin\theta_2$ for all $\theta_1, \theta_2 \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

And , $x_1 < x_2 \Rightarrow \sin^{-1} x_1 < \sin^{-1} x_2$, for all $x_1, x_2 \in$ [-1 , 1]

(ii) The minimum and the maximum values of $\sin^{-1} x$ are $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ respectively.

(iii) $\sin^{-1} x$ attains the minimum value $\frac{-\pi}{2}$ at x = -1 and the maximum value $\frac{\pi}{2}$ at x = 1

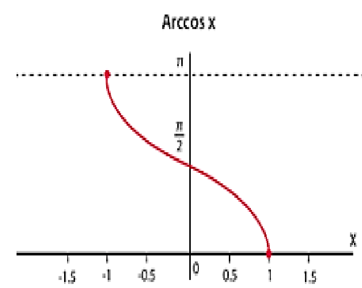
Example

Find the principal values of $\sin^{-1}\left(\frac{-1}{2}\right)$

Solution: $\sin^{-1}\left(\frac{-1}{2}\right) = -\frac{\pi}{6}$

Arccosine Function

Arccosine function is the inverse of the cosine function denoted by $\cos^{-1}x$. It is represented in the graph as shown below:



Therefore, the inverse of cos function can be expressed as; **y = $\cos^{-1}$x (arccosine *x*)**

Domain & Range of arcsine function:

Domain	$-1 \leq x \leq 1$
Range	$0 \leq y \leq \pi$

The branch of $cos^{-1}: [-1, 1] \rightarrow [0, \pi]$ is called the principal value branch and the value of $\cos^{-1} x$ lying in [0 , π] for a given value of x ∈ [-1,1] is called the principal value.

SOME OBSERVATIONS

(i) The domain and range of $\cos^{-1} x$ are [-1 ,1] and [0 , π] respectively.

(ii) Both cos and cos^{-1} are decreasing functions in their respective domains.

$\therefore\ \theta_1 < \theta_2 \Rightarrow \cos\theta_1 > \cos\theta_2$ for all $\theta_1, \theta_2\ \in$ [-1 ,1]

(iii) The minimum and maximum value of $\cos^{-1} x$ are 0 and π respectively which are attained at 1 and -1 respectively i.e., $\cos^{-1}(1) = 0$ and $\cos^{-1}(-1) = \pi$

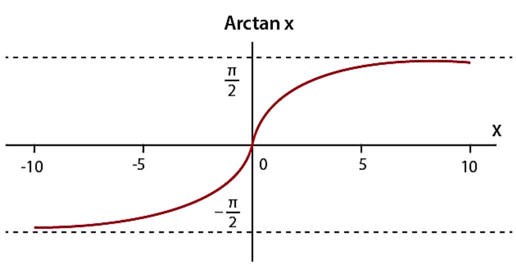
Example

Find the principal values of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Solution: $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

Arctangent (Arctan) Function

Arctangent function is the inverse of the tangent function denoted by $\tan^{-1}x$. It is represented in the graph as shown below:



Therefore, the inverse of tangent function can be expressed as; **y = $\tan^{-1}$x (arctangent *x*)**

Domain & Range of Arctangent:

Domain	-∞ < x < ∞
Range	-π/2 < y < π/2

The branch with range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is called the principal value branch of the function $\tan^{-1}$

SOME USEFUL OBSERVATIONS

It is evident from the graphs of tan : $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \rightarrow R$ and $tan^{-1}: R \rightarrow \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ i.e., the curves y = tan x and y = $\tan^{-1} x$ that

(i) $\frac{-\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$ for all x ∈ R i.e., $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ are minimum and maximum values of $\tan^{-1} x$ but it not does not attain these values.

(ii) Both tan and tan^{-1} are increasing functions in their respective domains.

$\therefore$ $\theta_1 < \theta_2 \Rightarrow \tan\theta_1 < \tan\theta_2$ for all $\theta_1, \theta_2\ \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

And , $x_1 < x_2 \Rightarrow\ \tan^{-1} x_1 < \tan^{-1} x_2$ for all $x_1, x_2 \in$ R.

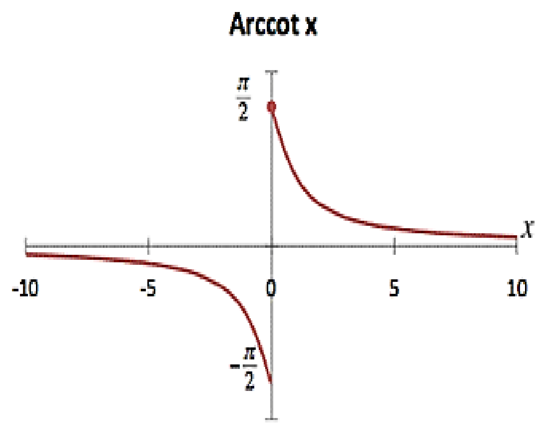
Example

Find the principal values of $\tan^{-1}(-\sqrt{3})$

Solution: $\tan^{-1}(-\sqrt{3}) = \frac{-\pi}{3}$

Arccotangent (Arccot) Function

Arccotangent function is the inverse of the cotangent function denoted by $\cot^{-1}x$. It is represented in the graph as shown below:



Therefore, the inverse of cotangent function can be expressed as; **y = $\cot^{-1}$x (arccotangent *x*)**

Domain & Range of Arccotangent:

Domain	-∞ < x < ∞
Range	0 < y < π

The branch with range $(0, \pi)$is called the principal value branch of the function $\cot^{-1}$

SOME USEFUL OBSERVATION

(i) Cot x is a decreasing function on (0 , π).

i.e., $\theta_1 < \theta_2 \Rightarrow \cot\theta_1 > \cot\theta_2$ for all $\theta_1, \theta_2\ \in (0, \pi)$

(ii) $\cot^{-1} x$ is a decreasing function on R.

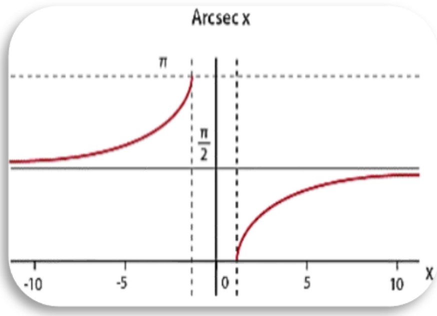
i.e., $x_1 < x_2 \Rightarrow\ \cot^{-1} x_1 > \cot^{-1} x_2$ for all $x_1, x_2 \in$ R.

(iii) For all x ∈ R ,the values of $\cot^{-1}$ lies between 0 and π

(iv) $\cot^{-1} x$ does not attain its minimum value zero and maximum value π at points in R.

ARCSECANT FUNCTION

What is arcsecant (arcsec)function? Arcsecant function is the inverse of the secant function denoted by $\sec^{-1} x$. It is represented in the graph as shown below:



Therefore, the inverse of secant function can be expressed as; **y = sec^{-1}x (arcsecant *x*)**

Domain & Range of Arcsecant:

Domain	$-\infty \le x \le -1$ or $1 \le x \le \infty$
Range	$0 \le y \le \pi,\ y \ne \pi/2$

The value of $\sec^{-1} x$ in [0 , π/2)U$(\frac{\pi}{2},\pi]$ for given of x ∈ (-∞ , −1] U[1, ∞) is called the principal value.

SOME OBSERVATIONS:

(i) Sec x is an increasing function on the intervals [0 , π/2) and (π/2 ,π] but , it is neither increasing nor decreasing on [0 , π/2)U(π/2 , π] .

(ii) $\sec^{-1}$ x is an increasing function the intervals (- ∞ , −1] and [1, ∞) but , it is neither increasing nor decreasing on (- ∞ , −1] U[1, ∞)

(iii) The maximum value of $\sec^{-1} x$ is π which it attains at x = -1

(iv) The minimum value of $\sec^{-1} x$ is 0 which it attains at x = 1

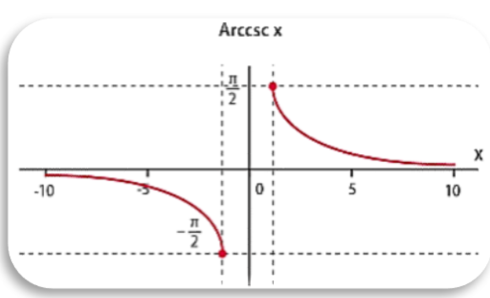
Example

Find the principal values $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

Solution: $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \frac{5\pi}{6}$

ARCCOSECANT FUNCTION

What is arccosecant (arccsc *x*) function? Arccosecant function is the inverse of the cosecant function denoted by cosec^{-1}*x*. It is represented in the graph as shown below:



Therefore, the inverse of cosecant function can be expressed as; **y = cosec^{-1}x (arccosecant *x*)**

Domain & Range of Arccosecant is:

Domain	$-\infty \le x \le -1$ **or** $1 \le x \le \infty$
Range	$-\pi/2 \le y \le \pi/2,\ y \ne 0$

The branch with range $[\frac{-\pi}{2},0)$ U$(0,\frac{\pi}{2}]$ is called the principal value branch of the function $cosec^{-1}$

SOME OBSERVATIONS:

(i) Cosec θ is a decreasing function on $[\frac{-\pi}{2},0)$ and (0 , $\frac{\pi}{2}]$ .But , it is neither decreasing nor increasing on $[\frac{-\pi}{2},0)$ U$(0,\frac{\pi}{2}]$.

(ii) $cosec^{-1}\ x$ is decreasing on (-∞ , −1] and [1 , ∞).But ,it is neither increasing nor decreasing on (-∞ , −1] U[1, ∞).

(iii) The maximum value of $cosec^{-1}\ x$ is $\frac{\pi}{2}$ which it attains at x = 1

(iv) The maximum value is $cosec^{-1}\ x$ is $\frac{-\pi}{2}$ which it attains at x = -1.

Example

Find the principal values of $cosec^{-1}\ 2$

Solutions: For x ∈ (-∞ , −1] U[1, ∞) , $cosec^{-1}\ x$ is an angle θ ∈ [-π\2 ,0)U$(0,\frac{\pi}{2}]$ such that cosec $\theta = x$

Therefore , $cosec^{-1}\ 2 = \frac{\pi}{6}$.

Properties of inverse trigonometric function

Property 1: If f : A → B is bijection , then $f^{-1}: B \to A$ exists such that $f^{-1}of(x) = x$, for all x ∈ A.

(i) $\sin^{-1}(\sin\theta) = \theta$ for all θ ∈ [-π/2 , π/2]

(ii) $\cos^{-1}(\cos\theta) = \theta$ for all θ ∈ [0 , π]

(iii) $\tan^{-1}(\tan\theta) = \theta$ for all θ ∈ $(-\frac{\pi}{2},\frac{\pi}{2})$

(iv) $\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$ for all θ ∈ $[-\frac{\pi}{2},0)$ U$(0,\frac{\pi}{2}]$

(v) $\sec^{-1}(\sec\theta) = \theta$ for all θ ∈ $[0,\pi/2)$ U$(\pi/2,\pi]$

(vi) $\cot^{-1}(\cot\theta) = \theta$ for all θ ∈ (0 , π).

Example

Evaluate $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

(i) As we know that , $\sin^{-1}(\sin\theta) = \theta$ for all θ ∈ [-π/2 , π/2]

Therefore , $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

Remark : In order to simplify trigonometrical expressions involving inverse trigonometrical functions , following substitution are very useful:

Expression	Substitution
a^2+x^2	$x = a\tan\theta$ or , $x = a\cot\theta$
a^2-x^2	$x = a\sin\theta$ or , $x = a\cos\theta$
x^2-a^2	$x = a\sec\theta$ or , $x = a\,cosec\,\theta$
$\sqrt{\frac{a-x}{a+x}}$ or , $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or , $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x = a^2\cos 2\theta$

Example

Write the following functions in the simplest form :

(i) $tan^{-1}\left(\sqrt{\frac{a-x}{a+x}}\right)$, -a < x< a

Solutions: Putting $x = a\cos\theta$, we obtain

$\tan^{-1}\sqrt{\frac{a-x}{a+x}} = \tan^{-1}\left(\sqrt{\frac{a-a\cos\theta}{a+a\cos\theta}}\right)$

$= \tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)$ $= \tan^{-1}\left(\sqrt{\frac{2sin^2\frac{\theta}{2}}{2\,cos^2\frac{\theta}{2}}}\right)$

$= \tan^{-1}\left(\left|\tan\frac{\theta}{2}\right|\right)$

[since -a < x< a $\Rightarrow 0 < \frac{\theta}{2} < \frac{\pi}{2}$ $\therefore$ $\left|\tan\frac{\theta}{2}\right| = \tan\frac{\theta}{2}$]

$= \frac{\theta}{2} = \frac{1}{2}\cos^{-1}\frac{x}{a}$

Property 2: If f : A → B is bijection , then $f^{-1}: B \to A$ exists such that $f^{-1}of(x) = x$, for all x ∈ A.

(i) $\sin(sin^{-1}x) = x$ for all $x \in$ [-1 , 1]

(ii) $\cos(\cos^{-1}x) = x$ for all $x \in$ [-1 , 1]

(iii) $\tan(\tan^{-1}x) = x$ for all $x \in$ R

(iv) $\csc(\csc^{-1}x) = x$ for all $x \in$ (-∞ , -1] U$[1,\infty)$

(v) $\sec(sec^{-1}x) = x$ for all $x \in$ (-∞ , -1] U$[1,\infty)$

(vi) $\cot(\cot^{-1}x) = x$ for all $x \in$ R

Example

Evaluate $\tan\left(sec^{-1}\left(\frac{13}{12}\right)\right)$

Solution: The right triangle with base b = 12 and hypotenuse h = 13 has perpendicular p =5

Therefore , $sec^{-1}\left(\frac{13}{12}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

Hence , $\tan\left(sec^{-1}\left(\frac{13}{12}\right)\right) = \tan\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{5}{12}$

Property 3

(i) $\sin^{-1}(-x) = -\sin^{-1}x$, for all $x \in$ [-1 , 1]

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$, for all $x \in$ [-1 , 1]

(iii) $\tan^{-1}(-x) = -\tan^{-1}x$, for all $x \in$ R

(iv) $\csc^{-1}(-x) = -\csc^{-1}x$, for all $x \in$ (-∞ , -1]U$[1,\infty)$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$, for all $x \in$ (-∞ , -1]U$[1,\infty)$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1}x$, for all $x \in$ R

Example

Prove that: $\sin^{-1}\left(\frac{-4}{5}\right) = \tan^{-1}\left(\frac{-4}{3}\right) = \cos^{-1}\left(\frac{-3}{5}\right) - \pi$

Solution: We find that:

$\sin^{-1}\left(\frac{-4}{5}\right) = -\sin^{-1}\left(\frac{4}{5}\right) = -\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{-4}{3}\right)$

And , $\cos^{-1}\left(\frac{-3}{5}\right) - \pi = \left(\pi - \cos^{-1}\left(\frac{3}{5}\right)\right) - \pi = -\cos^{-1}\left(\frac{3}{5}\right) = -\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{-4}{3}\right)$

Hence , $\sin^{-1}\left(\frac{-4}{5}\right) = \tan^{-1}\left(\frac{-4}{3}\right) = \cos^{-1}\left(\frac{-3}{5}\right) - \pi$

Property 4

(i) $\sin^{-1}\left(\frac{1}{x}\right) = \csc^{-1}x$, for all $x\in$ (-∞ , $-1]$ U$[1,\infty)$

(ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x$, for all $x\in$ (-∞ , $-1]$ U$[1,\infty)$

(iii) $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}x \ , for\ x > 0 \\ -\pi + \cot^{-1}x \ , for\ x < 0 \end{cases}$

Example

Prove that $\tan^{-1}x + \tan^{-1}\frac{1}{x} = \begin{cases} \pi\backslash 2\ , for\ x > 0 \\ -\pi\backslash 2\ , for\ x < 0 \end{cases}$

Solutions: We have ,

$\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}x\ , for\ x > 0 \\ -\pi + \cot^{-1}x\ , for\ x < 0 \end{cases}$

$\therefore$ $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\ , if\ x > 0 \\ \tan^{-1}x + \cot^{-1}x - \pi = \frac{\pi}{2} - \pi = -\frac{\pi}{2}\ , if\ x < 0 \end{cases}$

Property 5

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ **,** for all $x \in$ [-1 , 1]

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, for all $x \in$ R

(iii) $\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$, for all $x \in$ (-∞ , -1]U[1, ∞)

Example

If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, find $\cot^{-1} x + \cot^{-1} y$

Solutions: We have , $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$

$\Rightarrow \left(\frac{\pi}{2} - \cot^{-1} x\right) + \left(\frac{\pi}{2} - \cot^{-1} y\right) = \frac{4\pi}{5}$

[Since , $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ *and* $\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2}$]

$\Rightarrow \pi - (\cot^{-1} x + \cot^{-1} y) = \frac{4\pi}{5}$

$\Rightarrow \cot^{-1} x + \cot^{-1} y = \pi - \frac{4\pi}{5}$

$\Rightarrow \cot^{-1} x + \cot^{-1} y = \frac{\pi}{5}$

Property 6

(i) $$\tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), if\ xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), if\ x > 0, y > 0\ and\ xy > 1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), if\ x < 0, y < 0\ and\ xy > 1 \end{cases}$$

(ii) $$\tan^{-1} x - \tan^{-1} y = \begin{cases} \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), if\ x > 0, y < 0\ and\ xy < -1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), if\ x < 0, y > 0\ and\ xy < -1 \end{cases}$$

Remark

If $x_1, x_2, x_3, \ldots, x_n \in$ R ,then $\tan^{-1} x_1 + \tan^{-1} x_2 + \cdots + \tan^{-1} x_n = \tan^{-1}\left(\frac{S_1 - S_3 + S_5 - S_7 + \cdots}{1 - S_2 + S_4 - S_6 + \cdots}\right)$

Where , S_k denotes the sum of the products of $x_1, x_2, x_3, \ldots, x_n$ taken k at a time.

Example

Prove that : $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\frac{1}{2}$

Solutions: We have , $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$= \tan^{-1}\left\{\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}\right\}$ [since , $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if xy <1]

$= \tan^{-1}\left\{\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}\right\} = \tan^{-1}\left(\frac{125}{250}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

Property 7

(i) $$\sin^{-1} x + \sin^{-1} y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}; if\ -1 \le x, y \le 1\ and\ x^2 + y^2 \le 1 \\ \quad or\ if\ xy < 0\ and\ x^2 + y^2 > 1 \\ \pi - sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}, if\ 0 < x, y \le 1\ and\ x^2 + y^2 > 1 \\ -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}, if\ -1 \le x, y < 0\ and\ x^2 + y^2 > 1 \end{cases}$$

(ii) $\sin^{-1}x-\sin^{-1}y=\begin{cases}\sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}, if\ -1\le x, y\le 1\ and\ x^2+y^2\le 1\\ \quad or\\ \quad if\ xy>0\ and\ x^2+y^2>1\\ \pi-\sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}, if\ 0<x\le 1, -1\le y\le 0\ and\ x^2+y^2>1\\ -\pi-\sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}, if\ -1\le x<0, 0<y\le 1\ and\ x^2+y^2>1\end{cases}$

Example

Prove that : $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\frac{77}{85}=\tan^{-1}\frac{77}{36}$

Solutions: Using $\sin^{-1}x\pm\sin^{-1}y=\sin^{-1}\left\{x\sqrt{1-y^2}\pm y\sqrt{1-x^2}\right\}$, we obtain,

$=\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\left\{\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2}+\frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right\}$

$=\sin^{-1}\left\{\frac{3}{5}\times\frac{15}{17}+\frac{8}{17}\times\frac{4}{5}\right\}=\sin^{-1}\frac{77}{85}=\tan^{-1}\frac{77}{36}$

Property 8

(i) $\cos^{-1}x+\cos^{-1}y=\begin{cases}\cos^{-1}\left\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\right\}, if\ -1\le x, y\le 1\ and\ x+y\ge 0\\ 2\pi-\cos^{-1}\left\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\right\}, if\ -1\le x, y\le 1\ and\ x+y\le 0\end{cases}$

(ii) $\cos^{-1}x-\cos^{-1}y=\begin{cases}\cos^{-1}\left\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\right\}, if\ -1\le x, y\le 1\ and\ x\le y\\ -\cos^{-1}\left\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\right\}, if\ -1\le y\le 0, 0<x\le 1\ and\ x\ge y\end{cases}$

Example

Prove that: $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

Solutions: We have,

$\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\left\{\frac{4}{5}\times\frac{12}{13}-\sqrt{1-\left(\frac{4}{5}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right\}$

$=\cos^{-1}\left\{\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}\right\}=\cos^{-1}\left\{\frac{48}{65}-\frac{15}{65}\right\}=\cos^{-1}\frac{33}{65}$

Property 9

(i) $2\sin^{-1}x=\begin{cases}\sin^{-1}\left(2x\sqrt{1-x^2}\right), if\ \frac{-1}{\sqrt{2}}\le x\le\frac{1}{\sqrt{2}}\\ \pi-sin^{-1}\left(2x\sqrt{1-x^2}\right), if\ \frac{1}{\sqrt{2}}\le x\le 1\\ -\pi-\sin^{-1}\left(2x\sqrt{1-x^2}\right), if\ -1\le x\le\frac{-1}{\sqrt{2}}\end{cases}$

(ii) $3\sin^{-1}x=\begin{cases}\sin^{-1}(3x-4x^3), if\ \frac{-1}{2}\le x\le\frac{1}{2}\\ \pi-\sin^{-1}(3x-4x^3), if\ \frac{1}{2}<x\le 1\\ -\pi-\sin^{-1}(3x-4x^3), if\ -1\le x<\frac{-1}{2}\end{cases}$

Property 10

(i) $2cos^{-1}x=\begin{cases}\cos^{-1}(2x^2-1), if\ 0\le x\le 1\\ 2\pi-\cos^{-1}(2x^2-1), if\ -1\le x\le 0\end{cases}$

(ii) $3cos^{-1}x=\begin{cases}\cos^{-1}(4x^3-3x), if\ \frac{1}{2}\le x\le 1\\ 2\pi-\cos^{-1}(4x^3-3x), if\ \frac{-1}{2}\le x\le\frac{1}{2}\\ 2\pi+\cos^{-1}(4x^3-3x), if\ -1\le x\le\frac{-1}{2}\end{cases}$

Property 11

(i) $2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right), if -1 < x < 1 \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), if\ x > 1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), if\ x < -1 \end{cases}$

(ii) $3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), if -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), if\ x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), if\ x < -\frac{1}{\sqrt{3}} \end{cases}$

Example

Prove that : $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Solution: We have ,

$2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left\{\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right\} + \tan^{-1}\frac{1}{7}$ [since, $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), if -1 < x < 1$]

$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left\{\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right\} = \tan^{-1}\frac{31}{17}$

QUESTIONS

MCQ

Q1. $\cot^{-71}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = $ y, find sin (y)

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$

(c) tan α (d) cot $\left(\frac{\alpha}{2}\right)$

Q2. $\sin^{-1} x = 2\sin^{-1} y$, will have a solution for

(a) $\frac{1}{2} < |y| < \frac{1}{\sqrt{2}}$ (b) all real values of y

(c) $|y| < \frac{1}{2}$ (d) $|y| \le \frac{1}{\sqrt{2}}$

Q3. The upper $\left(\frac{3}{4}\right)^{th}$ "portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at point in the horizontal plane through its foot and at a distance 40m" from the foot. A possible height" of the vertical pole is

(a) 20m (b) 40m

(c) 60m (d) 80m

Q4. The principal value of $\text{cosec}^{-1}(-\sqrt{2})$ =?

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{4}$

Q5. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ =?

(a) $\frac{-3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{3\pi}{4}$ (d) $\frac{-\pi}{4}$

Q6. If $\sin^{-1}\alpha = \beta$, then

(a) $0 \le \beta \le \pi$ (b) $-\frac{\pi}{2} \le \beta \le \frac{\pi}{2}$

(c) $0 < \beta < \pi$ (d) $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$

Q7. $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ =?

(a) π (b) $\frac{-\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Q8. For $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$ to hold x should belong to

(a) $x \in \left[\frac{1}{2}, 1\right]$ (b) $x \in [-1,1]$

(c) $x \in [0,1]$ (d) $x \in \left[\frac{-1}{2}, -1\right]$

Q9. $\tan^{-1}\frac{\sqrt{1+\alpha^2}-1}{\alpha}$ =? , $\alpha \ne 0$

(a) $-\frac{1}{2}\tan^{-1}\alpha$ (b) $\tan^{-1}\alpha$

(c) $\frac{1}{2}\tan^{-1}\alpha$ (d) $-\tan^{-1}\alpha$

Q10. $\tan^{-1}\frac{1}{\sqrt{\alpha^2-1}}$ =? , $|\alpha| > 1$

(a) $\frac{\pi}{2} + \sec^{-1}\alpha$ (b) $-\pi/2 - \sec^{-1}\alpha$

(c) $\sec^{-1}\alpha - \pi/2$ (d) $\frac{\pi}{2} - \sec^{-1}\alpha$

Q11. $\tan^{-1}\left(\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\right)$ =? , $0 < \alpha < \pi$

(a) $\frac{-\alpha}{2}$ (b) $\frac{\alpha}{2}$

(c) α (d) $-\alpha$

Q12. $\cot(\tan^{-1} a + \cot^{-1} a)$=?

(a) 0 (b) 1

(c) -1 (d) ∞

Q13. If $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then x =?.

(a) $x = \pm\frac{1}{\sqrt{2}}$

(b) $x = \frac{1}{\sqrt{2}}$

(c) $x = -\frac{1}{\sqrt{2}}$

(d) Doesn't have any real values

Q14. $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ =?

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$

(c) $-\frac{\pi}{3}$ (d) $\frac{-2\pi}{3}$

Q15. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ =

(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Q16. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ =

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) 1

Q17. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ =

(a) π (b) $-\frac{\pi}{2}$

(c) 0 (d) $2\sqrt{3}$

Q18. $\cos^{-1}\frac{1-x}{1+x} =$; $x \in (0,1)$

(a) $\tan^{-1}\sqrt{x}$ (b) $2\tan^{-1}\sqrt{x}$

(c) $-\tan^{-1}\sqrt{x}$ (d) $-2\tan^{-1}\sqrt{x}$

Q19. $\sin(\tan^{-1} x)$ =? , $|x| < 1$

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

Q20. $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$ then x =?

(a) $0, \frac{1}{2}$ (b) $1, \frac{1}{2}$

(c) 0 (d) $\frac{1}{2}$

Q21. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = ?$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $-\frac{3\pi}{4}$

Q22. If $\sin^{-1}(\alpha^2 - 7\alpha + 12) = m\pi, \forall\, m \in I$, then $\alpha =$

(a) -2,1 (b) 4,3

(c) -3,3 (d) 5,-5

Q23. $\cot\left(\frac{\pi}{4} - 2\cot^{-1} 3\right) =$

(a) 7 (b) 6

(c) 5 (d) None

Q24. If $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$, then x=?

(a) $\pm\frac{1}{2}$ (b) $0, \frac{1}{2}$

(c) $0, \frac{-1}{2}$ (d) $0, \pm\frac{1}{2}$

Q25. $\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}] =$

(a) $\left(\sqrt{\frac{1+x^2}{2+x^2}}\right)$ (b) $\left(\sqrt{\frac{2+x^2}{1+x^2}}\right)$

(c) $\left(\sqrt{\frac{x^2-2}{x^2-1}}\right)$ (d) $\left(\sqrt{\frac{x^2-1}{x^2-2}}\right)$

Q26. If $y = \sec^{-1} x$ then

(a) $0 \le y \le \pi$ (b) $0 \le y \le \frac{\pi}{2}$

(c) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (d) None

Q27. If $\alpha + \frac{1}{\alpha} = 2$ then the principal value of $\sin^{-1} \alpha$ is α

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) $\frac{3\pi}{2}$

Q28. $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239} = ?$

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

Q29. Algebraic expression for $\sin(\cot^{-1} x)$ is

(a) $\frac{1}{1+x^2}$ (b) $\frac{1}{\sqrt{1+x^2}}$

(c) $\frac{x}{\sqrt{1+x^2}}$ (d) None

Q30. If $\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) + \sin^{-1}\frac{2\beta}{1+\beta^2} = 2\tan^{-1}c$ then c =?

(a) $\frac{\alpha-\beta}{1+\alpha\beta}$ (b) $\frac{\beta}{1+\alpha\beta}$

(c) $\frac{\beta}{1-\alpha\beta}$ (d) $\frac{\alpha+\beta}{1-\alpha\beta}$

SUBJECTIVE QUESTIONS

Q1. Find domain of $\sin^{-1}(2x^2 - 1)$

Q2. Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.

Q3. Find the value of $\tan\left\{\cot^{-1}\left(\frac{-2}{3}\right)\right\}$

Q4. Find the value of $\cos(2\cos^{-1}x + \sin^{-1}x)$ when $x = \frac{1}{5}$

Q5. Evaluate $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{4}{5} - \tan^{-1}\frac{63}{16}$

NUMERICAL TYPE QUESTIONS

Q1. $\cos^{-1}\left[\cos\left(-\frac{17}{15}\pi\right)\right]$ is equal to__________.

Q2. If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$, then x is_______.

Q3. The value of x satisfying equation $\cot^{-1} x + \tan^{-1} 3 = \frac{\pi}{2}$ is_________.

Q4. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is _______.

Q5. The value of $\sin\left[\frac{\pi}{6} + \sin^{-1}\left(-\frac{1}{2}\right)\right]$________.

TRUE AND FALSE

Q1. $\sin^{-1}\left(\frac{1}{x}\right) = \csc^{-1} x$, $for\ all\ x \in [-1, 1]$.

Q2. $\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$, $for\ all\ x \in (-\infty, -1] \cup [1, \infty)$

Q3. $2\cos^{-1} x = \begin{cases} \cos^{-1}(2x^2 - 1), & if\ 0 \le x \le 1 \\ 2\pi + \cos^{-1}(2x^2 - 1), & if\ -1 \le x \le 0 \end{cases}$

Q4. The number of solutions of the equation $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is two.

Q5. $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \csc^{-1}\left(\frac{1}{x}\right)$

ASSERTION AND REASONING

Directions (Q. No. 1 – 5) Each of these questions contains two statements : Assertion (A) and Reason(R).Each of these questions also has four alternative choices , any one of which is the correct answer .You have to select one of the codes (a) ,(b) ,(c) and (d) given below.

(a) A is true , R is true ; R is a correct explanation for A.

(b) A is true , R is true ; R is not a correct explanation for A.

(c) A is true ;R is false

(d) A is false ; R is true

Q1. **Assertion(A) :** We can write $\sin^{-1} x = (\sin x)^{-1}$
Reason (R) : Any value in the range of principal value branch is called principal value of that inverse trigonometric function.

Q2. **Assertion(A) :** Range of $\tan^{-1} x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.
Reason (R) : Domain of $\tan^{-1} x$ is R.

Q3. **Assertion (A) :** Principal value of $\cos^{-1}(1)$ is π
Reason (R) : Value of $\cos 0^o$ is 1

Q4. **Assertion(A) :** Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$
Reason (R) : Principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$

Q5. **Assertion (A) :** Function f : R → R given by f(x) = sin x is not a bijection .
Reason(R) : A function f : A → B is said to be bijection if it is one – one and onto .

HOMEWORK

MCQ

Q1. Domain of f(x) = $\cos^{-1} x + \cot^{-1} x + \text{cosec}^{-1} x$ is
(a) [– 1, 1] (b) R
(c) $(-\infty, -1] \cup [1, \infty)$ (d) {– 1, 1}

Q2. Range of f(x) = $\sin^{-1} x + \tan^{-1} x + \sec^{-1} x$ is
(a) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (b) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$
(c) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (d) $\left\{0, \frac{\pi}{4}\right\}$

Q3. $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \sin^{-1}\left(\frac{1}{3}\right)$ is equal to
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

Q4. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then $\cos^{-1}x + \cos^{-1}y$ =
(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

Q5. The value of $\sin\left[\cot^{-1}\left(\cot\frac{17\pi}{3}\right)\right]$ is-
(a) $-\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{2}{\sqrt{3}}$

Q6. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is -
(a) π (b) π/2
(c) π/3 (d) 4π/3

Q7. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is
(a) $\frac{3x-x^3}{1-3x^2}$ (b) $\frac{3x+x^3}{1-3x^2}$
(c) $\frac{3x-x^3}{1+3x^2}$ (d) $\frac{3x+x^3}{1+3x^2}$

Q8. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is-
(a) 1 (b) 3
(c) 4 (d) 5

Q9. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to-
(a) 2 sin 2α (b) 4
(c) $4\sin^2\alpha$ (d) $-4\sin^2\alpha$

Q10. The value of $\cot^{-1}(-\sqrt{3}) + \cos^{-1}(-1)$ =
(a) π (b) $\frac{5\pi}{6}$
(c) $\frac{11\pi}{6}$ (d) $\frac{2\pi}{3}$

SUBJECTIVE QUESTIONS

Q1. If $\sin\left(\cos^{-1}\frac{5}{13} + \sin^{-1}x\right) = 1$, then find the value of x .

Q2. For the principal values, evaluate $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$

Q3. Evaluate $\tan^{-1} 9 + \tan^{-1}\frac{5}{4}$.

Q4. Find the value of $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$

Q5. Find the value of $\sin^{-1}(\sin 7)$ and $\sin^{-1}(\sin(-5))$.

NUMERICAL TYPE QUESTIONS

Q1. Find the value of $\text{cosec}\left\{\cot\left(\cot^{-1}\frac{3\pi}{4}\right)\right\}$ is ______.

Q2. Find the value of $\sin\left(\tan^{-1}\frac{3}{4}\right)$ is ________.

Q3. If $\sin^{-1}(x^2 - 2x + 1) + \cos^{-1}(x^2 - x) = \frac{\pi}{2}$, then the value of x ____.

Q4. A value of x satisfying the equation $\sin[\cot^{-1}(1 + x)] = \cos[\tan^{-1}x]$, is________.

Q5. If $3\sin^{-1}x + \cos^{-1}x = \pi$,then find the value of x is ___.

TRUE AND FALSE

Q1. $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, for all $x \in R$

Q2. $\csc(\csc^{-1}x) = x$ **for all** $x \in (-\infty, -1] \cup [1, \infty)$

Q3. The domain and range of $\cos^{-1}x$ are $[-1, 1]$ and $[0, \frac{\pi}{2}]$ respectively.

Q4. $\sin^{-1}x$ attains the minimum value $\frac{-\pi}{2}$ at x = -1 and the maximum value $\frac{\pi}{2}$ at x = 1

Q5. $cosec^{-1}x$ is decreasing on $(-\infty, -1]$ and $[1, \infty)$.But ,it is neither increasing nor decreasing on $(-\infty, -1] \cap [1, \infty)$.

ASSERTION AND REASONING

Directions:(Q1-Q2) Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), **(b), (**c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

Q1. Assertions(A): $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{77}{85}\right)$

Reason (R) : $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Q2. Assertions(A) : If $f(x) = 2^x$ then $f^{-1}(x) = \log_2 x$

Reason (R) : $f(x) = a^x$ and $g(x) = \log_a x$ are inverse of each other

Q3. Assertions(A) : The value of $2\tan^{-1}\frac{1}{5}$ is $\tan^{-1}\frac{5}{12}$

Reason (R) :$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), if -1 < x < 1$

Q4. Assertions(A) : The principal value of $\sec^{-1}(2) = \frac{-\pi}{3}$

Reason (R) : The value of $\sec^{-1}x$ in $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ for given value of $x \in (-\infty, -1] \cup [1, \infty)$ is called the principal value.

Q5. Assertions(A) : The value of $\cot\left(\cot^{-1}\frac{4}{3}\right) = \frac{4}{3}$

Reason (R) : $\cot(\cot^{-1}x) = -x$ for all $x \in \boldsymbol{R}$

SOLUTIONS

MCQ

S1. **(a)** Given,

$\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = y$

$\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = y$ $[\because \cot^{-1}x = \tan^{-1}\frac{1}{x}]$

$\Rightarrow \tan^{-1}\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1+\frac{1}{\sqrt{\cos\alpha}}\cdot\sqrt{\cos\alpha}} = y$ $\quad [\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1-xy}]$

$\Rightarrow \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = y$

$\Rightarrow \tan y = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} \Rightarrow \cot y = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha}$

$\Rightarrow \sin y = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{1-\left(1-\frac{2\sin^2\alpha}{2}\right)}{1+\frac{2\cos^2\alpha}{2}-1} = \left(\tan\frac{\alpha}{2}\right)^2$

S2. **(d)** The given equation $\sin^{-1}x =$ $2\sin^{-1}y$ will have a solution if

$2\sin^{-1}y \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]\left(\because \sin^{-1}x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]\ by\ definition\ of\ \sin^{-1}x\right)$

$\Rightarrow \sin^{-1}y \in \left[-\frac{\pi}{4},\frac{\pi}{4}\right]$

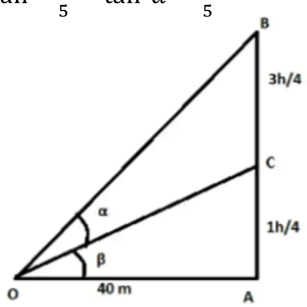
$\Rightarrow y \in \left[-\sin\frac{\pi}{4}, \sin\frac{\pi}{4}\right]$

$\Rightarrow y \in \left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$

$\Rightarrow |y| \le \frac{1}{\sqrt{2}}$

S3. **(b)** Given, that

$\alpha = \tan^{-1}\frac{3}{5} = \tan\alpha = \frac{3}{5}$



$\tan\beta = \frac{AC}{OA} = \frac{h}{4\times 40} = \frac{h}{160}$

Now, In $\triangle AOB$,

$\tan(\alpha+\beta) = \frac{\mathbf{AB}}{\mathbf{OA}} = \frac{h}{40}$

$\& \Rightarrow \tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\& \Rightarrow \frac{h}{40} = \frac{\frac{h}{160}+\frac{3}{5}}{1-\frac{h}{160}\times\frac{3}{5}}$

$\Rightarrow \frac{h}{40} = \frac{\frac{h}{160}+\frac{3}{5}}{\frac{800-3h}{800}}$

$\Rightarrow \frac{h}{40} = \frac{h+96}{160}\times\frac{800}{800-3h}$

$\Rightarrow \frac{h}{40} = \frac{5(h+96)}{800-3h}$

$\& \Rightarrow 800h - 3h^2 = 200(h+96)$

$\Rightarrow 800h - 3h^2 = 200h + 19200$

$\Rightarrow 800h - 3h^2 = 200h + 19200$

$\Rightarrow 3h^2 - 600h + 19200 = 0$

$\Rightarrow h^2 - 200h + 6400 = 0$

$\Rightarrow (h-160)(h-40) = 0$

$\Rightarrow h - 160 = 0, h - 40 = 0$

$\Rightarrow h = 160$ and $h = 40$

Thus, $h =$ 40m as the height of the pole is less than 100m.

S4. **(a)** Ley $x = \text{cosec}^{-1}(-\sqrt{2})$

$\Rightarrow \text{cosec}\ x = -\sqrt{2}$

$\Rightarrow \text{cosec}\ x = \text{cosec}\ \frac{-\pi}{4}$

As principle value of cosec^{-1} is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right] - \{0\}$

Thus, Principal value of $\text{cosec}^{-1}(-\sqrt{2})$ is $\frac{-\pi}{4}$.

S5. **(c)** $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \cos^{-1}\left(-\cos\frac{\pi}{3}\right) + \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$ $[\because \tan\frac{\pi}{4} = 1, \cos\frac{\pi}{3} = \frac{1}{2}, \sin\frac{\pi}{6} = \frac{1}{2}]$

$= \frac{\pi}{4} + \cos\left(\pi - \frac{\pi}{3}\right) + \sin^{-1}\ \sin\left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi+8\pi-2\pi}{12}$

$= \frac{9\pi}{12} = \frac{3\pi}{4}$

S6. **(b)** Given $\sin^{-1}\alpha = \beta$,

As the range of the principle value of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Thus, $-\frac{\pi}{2} \le \beta \le \frac{\pi}{2}$

S7. **(b)** $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \tan^{-1}(\tan\pi/3) - \sec^{-1}(-\sec\pi/3)$

$= \pi/3 - \sec^{-1}(\sec(\pi - \pi/3))$

$= \pi/3 - 2\pi/3 = -\pi/3$

S8. (a) First take R.H.S $\cos^{-1}(4x^3-3x)$

Let us take $x=\cos\theta$, we know that $\cos 3\theta = 4cos^3\theta - 3\cos\theta$

$\Rightarrow \cos^{-1}(4x^3-3x) = \cos^{-1}(4cos^3\theta - 3\cos\theta) = \cos^{-1}(\cos 3\theta) = 3\theta$

$\because \theta = \cos^{-1} x \Rightarrow \cos^{-1}(4x^3-3x) = 3\cos^{-1} x$

∴LHS=RHS

$0 \le 3\cos^{-1} x \le \pi$

$0 \le \cos^{-1} x \le \pi/3$

$\therefore x \in \left[\frac{1}{2}, 1\right]$

S9. (c) Let $\alpha = \tan\theta \Rightarrow \theta = \tan^{-1}\alpha$

Now,

$\tan^{-1}\frac{\sqrt{1+\alpha^2}-1}{\alpha} = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}\alpha$

S10. (d) Let $\alpha = \sec\theta$, then $\theta = \sec^{-1}\alpha$

Which will imply

$\tan^{-1}\frac{1}{\sqrt{\alpha^2-1}} = \tan^{-1}\frac{1}{\sqrt{\sec^2\theta-1}}$

$= \tan^{-1}\frac{1}{\sqrt{\tan^2\theta}}$

$= \tan^{-1}\left(\frac{1}{\tan\theta}\right)$

$= \tan^{-1}(\cot\theta)$

$= \tan^{-1}\tan(\pi/2-\theta)$

$= (\pi/2-\theta)$

$= \frac{\pi}{2} - \sec^{-1}\alpha$

Hence, (d) is the answer.

S11. (b) $\tan^{-1}\left(\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\right) = \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}}\right)$ $[\because 1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$ *and* $1+\cos\alpha = 2\cos^2\frac{\alpha}{2}]$

$= \tan^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{\alpha}{2}\right) = \frac{\alpha}{2}$

Hence, (b) is the answer.

S12. (a) We know the identity: $\tan^{-1}\alpha + \cot^{-1}\alpha = \pi/2$.

Taking cot both sides we get,

$\cot(\tan^{-1}\alpha + \cot^{-1}\alpha) = \cot\pi/2 = 0$

S13. (a) We know $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy} \ldots (i)$

$\tan^{-1}\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)} = \frac{\pi}{4}$ (Using (i))

Simplifying we get,

$\Rightarrow \tan^{-1}\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)-(x-1)(x+1)} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{x^2+2x-x-2+x^2-2x+x-2}{x^2-4-(x^2-1)} = \frac{\pi}{4}$

$\Rightarrow \frac{2x^2-4}{x^2-4-x^2+1} = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow (2x^2-4)/-3 = 1$

$\Rightarrow 2x^2 = 1$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

S14. (b) Given, $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

Now we write $\frac{2\pi}{3}$ as $\pi - \frac{\pi}{3} = \frac{(3\pi-\pi)}{3}$

Substituting this, we get

$\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

$\Rightarrow \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ is $\frac{\pi}{3}$

Hence, (b) is the answer.

S15. (b) Given, $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi-\frac{7\pi}{6}\right)\right)$

$(\because \cos(2\pi-x) = \cos x)$

Also $2\pi - \frac{7\pi}{6} = \frac{12\pi-7\pi}{6} = \frac{5\pi}{6}$

Hence, (b) is the answer.

S16. (d) Given, $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right) = \sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$

$= -\pi/6$

Also,

$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

$= \sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right]$

$= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$

$= \sin(\pi/2)$

$= 1$

S17. (b) $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$= \tan^{-1}\tan\frac{\pi}{3} - \cot^{-1}\left(-\cot\frac{\pi}{6}\right)$ $[\because \tan\frac{\pi}{3} = \sqrt{3}$ *and* $\cot\frac{\pi}{6} = \sqrt{3}]$

$= \frac{\pi}{3} - \cot^{-1}\left[\cot\left(\pi-\frac{\pi}{6}\right)\right]$

$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$

$= \frac{\pi}{3} - \frac{5\pi}{6}$

$= \frac{-3\pi}{6}$

$= -\pi/2$

S18. (b) Let $\tan^{-1}\sqrt{x} = \theta \Rightarrow \sqrt{x} = \tan\theta$

Squaring both the sides, we get

$\tan^2\theta = x \ldots (i)$

Now, using (i) in $\frac{1}{2}\cos^{-1}\frac{1-x}{1+x}$, we get

$= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \frac{1}{2}\cos^{-1}(\cos 2\theta)$

$= \frac{1}{2}(2\theta)$

$= \theta$

$= \tan^{-1}\sqrt{x}$

$\Rightarrow \cos^{-1}\frac{1-x}{1+x} = 2\tan^{-1}\sqrt{x}$

S19. (d) Let $\theta = \tan^{-1}x \ldots (i)$

$\Rightarrow x = \tan\theta$

Then we have,

$\sin(\tan^{-1}x) = \sin\theta$(Using (i))

$\Rightarrow \sin(\tan^{-1}x) = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{1+\cot^2\theta}}$

Also, $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{x}$

$\Rightarrow \sin(\tan^{-1}x) = \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{x}{\sqrt{x^2+1}}$

S20. (c) Let $\sin^{-1}x = \theta \ldots (i)$

$\Rightarrow x = \sin\theta$

Then, $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$(Using (i))

$\sin^{-1}(1-x) - 2\theta = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\theta$

$\Rightarrow 1 - x = \sin\left(\frac{\pi}{2} + 2\theta\right)$

$\Rightarrow 1 - x = \cos 2\theta = 1 - 2\sin^2\theta$

$\Rightarrow 1 - x = 1 - 2x^2$

(Using (i))

simplifying, we get

$x(2x-1) = 0$

$\Rightarrow x = 0$ or $2x - 1 = 0$

$\Rightarrow x = 0$ or $x = \frac{1}{2}$

Equation doesn't hold true for $x = \frac{1}{2}$. Thus the only answer is $x = 0$.

S21. (c) Given, $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

$= \tan^{-1}\left[\frac{\frac{x}{y} - \left(\frac{x-y}{x+y}\right)}{1 + \frac{x}{y}\left(\frac{x-y}{x+y}\right)}\right]\left[\because \tan^{-1}\alpha - \tan^{-1}\beta = \tan^{-1}\frac{\alpha-\beta}{1+\alpha\beta}\right]$

Simplifying,

$= \tan^{-1}\left[\frac{x(x+y) - y(x-y)}{y(x+y) + x(x-y)}\right]$

$= \tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$

$= \tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

S22. (b) If $\sin^{-1}(\alpha^2 - 7\alpha + 12) = m\pi$

$\Rightarrow \alpha^2 - 7\alpha + 12 = \sin m\pi$

$\Rightarrow \alpha^2 - 7\alpha + 12 = 0 (\because \sin m\pi = 0 \ \forall \ m \in I)$

$\Rightarrow (\alpha - 4)(\alpha - 3) = 0$

$\Rightarrow \alpha = 4,3$

Hence, (b) is the answer.

S23. (a) $\cot\left\{\frac{\pi}{4} - 2\cot^{-1}3\right\} = \cot\left\{\frac{\pi}{4} - 2\tan^{-1}\frac{1}{3}\right\}\left[\because \tan^{-1}x = \cot^{-1}\frac{1}{x}\right]$

$= \cot\left\{\frac{\pi}{4} - \tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right)\right\}$

$= \cot\left\{\frac{\pi}{4} - \tan^{-1}\left(\frac{3}{4}\right)\right\} = \frac{1}{\tan\left\{\frac{\pi}{4} - \tan^{-1}\left(\frac{3}{4}\right)\right\}}$

$= \frac{1+\frac{3}{4}}{1-\frac{3}{4}} = 7$

S24. (d) Using $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{(x+y)}{(1-xy)}$ when $xy < 1$

Hence, (d) is the answer.

S25. (a) $\sin[\cot^{-1}(\cos(\tan^{-1}x))]$

let $\tan^{-1}x = \theta$

$\Rightarrow \tan\theta = x$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{1+x^2}} = a$ $\left[\because \tan\alpha = \frac{P}{B} \text{ and } \cos\alpha = \frac{B}{H}\right]$

the function is reduced to $\sin[\cot^{-1}a]$

let $\cot^{-1}a = \varphi$

$\cot\varphi = a = \frac{1}{\sqrt{1+x^2}}$

$\tan\varphi = \sqrt{1+x^2}$

$\sin\varphi = \sqrt{\frac{1+x^2}{2+x^2}}$

$\sin[\cot^{-1}(\cos(\tan^{-1}x))] = \sqrt{\frac{1+x^2}{2+x^2}}$

S26. (d) $\Rightarrow$ $y = \sec^{-1}x$

$x = \sec y$

Here, $x \geq 1$ or $x \leq 1$

The domain is $(-\infty, \infty) - [-1, 1]$

$\therefore$ Range of $\sec^{-1}x$ is all the angles between $[0,\pi] - \frac{\pi}{2}$

$\therefore$ Range $= [0,\pi] - \left[\frac{\pi}{2}\right]$

S27. (d) Given,

$\alpha + \left(\frac{1}{\alpha}\right) = 2$

$\Rightarrow \alpha = 1$

Thus ,the principal value of $\sin^{-1}\alpha$ *is* $\frac{\pi}{2}$.

S28. (d) Given,

$4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)$

$= 2\left(2\tan^{-1}\frac{1}{5}\right) - \tan^{-1}\frac{1}{239}$

$= 2\tan^{-1} \frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2} - \tan^{-1} \frac{1}{239}$

$= 2\tan^{-1} \frac{2/5}{24/25} - \tan^{-1} \frac{1}{239}$

$= 2\tan^{-1} \frac{5}{12} - \tan^{-1} \frac{1}{239}$

$= 2\tan^{-1} \frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2} - \tan^{-1} \frac{1}{239}$

$= 2\tan^{-1} \frac{2/5}{24/25} - \tan^{-1} \frac{1}{239} \left(\because 2\tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}\right)$

$= 2\tan^{-1} \frac{5}{12} - \tan^{-1} \frac{1}{239}$

$= \tan^{-1} \frac{2\cdot\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2} - \tan^{-1} \frac{1}{239} \left(\because 2\tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}\right)$

$= \tan^{-1} \frac{144\times5}{119\times6} - \tan^{-1} \frac{1}{239}$

$= \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{1}{239}$

$= \tan^{-1} \frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\cdot\frac{1}{239}}$

$\left(\because \tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}\right)$

$= \tan^{-1} \frac{120\times239-119}{119\times239+120} = \tan^{-1} \frac{28680-119}{28441+120}$

$= \tan^{-1} \frac{28561}{28561} = \tan^{-1} 1 = \frac{\pi}{4}$

S29. (b) Let $\cot^{-1} x = \theta$

$x = \cot\theta$

$\operatorname{cosec}\theta = \sqrt{1 - \cot^2\theta} = \sqrt{1 + x^2}$

$\sin\theta = 1 / \operatorname{cosec}\theta = 1 / \sqrt{1 + x^2}$

$\theta = \sin^{-1} 1 / \sqrt{1 + x^2}$

$\sin(\cot^{-1} x) = \sin[\sin^{-1} 1 / (\sqrt{1 + x^2})]$

$= 1 / \sqrt{1 + x^2}$

$= (1 + x^2)^{-½}$

S30. (d) Given,

$\sin^{-1} \frac{2\alpha}{1+\alpha^2} + \sin^{-1} \frac{2\beta}{1+\beta^2} = 2\tan^{-1} c$

$\Rightarrow 2\tan^{-1} \alpha + 2\tan^{-1} \beta =$

$2\tan^{-1} c \left[\because 2\tan^{-1} a = \sin^{-1} \frac{2a}{1+a^2}\right]$

$\& \Rightarrow \tan^{-1} \alpha + \tan^{-1} \beta = \tan^{-1} c$

$\Rightarrow \tan^{-1} \frac{\alpha+\beta}{1-\alpha\beta} = \tan^{-1} c$

$\& \Rightarrow c = \frac{\alpha+\beta}{1-\alpha\beta}$

SUBJECTIVE QUESTIONS

S1. Let $y = \sin^{-1}(2x^2 - 1)$

For y to be defined $-1 \le (2x^2 - 1) \le 1$

$\Rightarrow 0 \le 2x^2 \le 2$

$\Rightarrow 0 \le x^2 \le 1$

$\Rightarrow x \in [-1, 1]$.

S2. Since $\tan^{-1}(\tan x) = x$

if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1}\left(\tan \frac{3\pi}{4}\right) = \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right) = \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$\tan^{-1}\left(-\tan\frac{\pi}{4}\right) = -\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$\therefore -\tan^{-1}\left(\tan\frac{\pi}{4}\right) = -\frac{\pi}{4}$

S3. Let $y = \tan\left\{\cot^{-1}\left(\frac{-2}{3}\right)\right\}$ ....(i)

Since, $\cot^{-1}(-x) = \pi - \cot^{-1}x, x \in R$

(i) can be written as

$y = \tan\left\{\pi - \cot^{-1}\left(\frac{2}{3}\right)\right\}$

$y = -\tan\left(\cot^{-1}\frac{2}{3}\right)$

Since $\cot^{-1} x = \tan^{-1}\frac{1}{x}$ if $x > 0$

$\therefore y = -\tan\left(\tan^{-1}\frac{3}{2}\right)$

$\Rightarrow y = -\frac{3}{2}$

S4. $\cos\left(2\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5}\right) = \cos\left(\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5} + \cos^{-1}\frac{1}{5}\right)$

$= \cos\left(\frac{\pi}{2} + \cos^{-1}\frac{1}{5}\right) = -\sin\left(\cos^{-1}\left(\frac{1}{5}\right)\right)$ ....(i)

$= -\sqrt{1-\left(\frac{1}{5}\right)^2} = -\frac{\sqrt{24}}{5}$.

Alter : Let $\cos^{-1}\frac{1}{5} = \theta$

$\Rightarrow \cos\theta = \frac{1}{5}$ and $\theta \in \left(0, \rightleftharpoons \frac{\pi}{2}\right)$

$\therefore \sin\theta = \frac{\sqrt{24}}{5}$

$\therefore \sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{\sqrt{24}}{5}\right)$ ....(ii)

Since, $\theta \in \left(0, \rightleftharpoons \frac{\pi}{2}\right)$

$\Rightarrow \sin^{-1}(\sin\theta) = \theta$

$\therefore$ equation (ii) can be written as

$\theta = \sin^{-1}\left(\frac{\sqrt{24}}{5}\right)$

Since $\theta = \cos^{-1}\left(\frac{1}{5}\right)$

$\Rightarrow \cos^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{\sqrt{24}}{5}\right)$

Now equation (i) can be written as

$y = -\sin\left\{\sin^{-1}\left(\frac{\sqrt{24}}{5}\right)\right\}$ ....(iii)

Since $\frac{\sqrt{24}}{5} \in [-1, 1]$

$\therefore \sin\left\{\sin^{-1}\left(\frac{\sqrt{24}}{5}\right)\right\} = \frac{\sqrt{24}}{5}$

$\therefore$ from equation (iii), we get

$y = -\frac{\sqrt{24}}{5}$

S5. Let $z = \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{4}{5} - \tan^{-1}\frac{63}{16}$

Since $\sin^{-1}\frac{4}{5} = \frac{\pi}{2} - \cos^{-1}\frac{4}{5}$

$\therefore z = \cos^{-1}\frac{12}{13} + \left(\frac{\pi}{2} - \cos^{-1}\frac{4}{5}\right) - \tan^{-1}\frac{63}{16}$

$z = \frac{\pi}{2} - \left(\cos^{-1}\frac{4}{5} - \cos^{-1}\frac{12}{13}\right) - \tan^{-1}\frac{63}{16}$

.........(i)

Since $\frac{4}{5} > 0$, $\frac{12}{13} > 0$ and $\frac{4}{5} < \frac{12}{13}$

$\therefore \cos^{-1}\frac{4}{5} - \cos^{-1}\frac{12}{13} = \cos^{-1}\left[\frac{4}{5}\times\frac{12}{13} + \sqrt{1-\frac{16}{25}}\ \sqrt{1-\frac{144}{169}}\right] = \cos^{-1}\left(\frac{63}{65}\right)$

$\therefore$ equation (i) can be written as

$z = \frac{\pi}{2} - \cos^{-1}\left(\frac{63}{65}\right) - \tan^{-1}\left(\frac{63}{16}\right)$

$z = \sin^{-1}\left(\frac{63}{65}\right) - \tan^{-1}\left(\frac{63}{16}\right)$ ...(ii)

Since $\sin^{-1}\left(\frac{63}{65}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

$\therefore$ from equation (ii), we get

$\therefore z = \tan^{-1}\left(\frac{63}{16}\right) - \tan^{-1}\left(\frac{63}{16}\right)$

$\Rightarrow z = 0$

NUMERICAL TYPE QUESTIONS

S1. $\left(\frac{13\pi}{15}\right)$ $\cos^{-1}\left[\cos\left(-\frac{17}{15}\pi\right)\right]$

$\Rightarrow \cos^{-1}\left[\cos\left(-\pi - \frac{2\pi}{15}\right)\right]$

$\Rightarrow \cos^{-1}\left[\cos\left\{-\left(\pi + \frac{2\pi}{15}\right)\right\}\right]$

$\Rightarrow \cos^{-1}\left[\cos\left\{-\left(\pi + \frac{2\pi}{15}\right)\right\}\right]$

[since, $\cos(-\theta) = \cos(\theta)$]

$\Rightarrow \cos^{-1}\left[-\cos\frac{2\pi}{15}\right]$

[since , $\cos(\pi + \theta) = -\cos(\theta)$]

$\Rightarrow \pi - \cos^{-1}\left(\cos\frac{2\pi}{15}\right)$

[since, $\cos^{-1}(-x) = \pi - \cos^{-1}x$]

$\Rightarrow \pi - \frac{2\pi}{15}$

$\Rightarrow \frac{13\pi}{15}$

S2. $\left(\frac{\sqrt{3}}{2}\right)$ If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$, then x is-

$\Rightarrow \frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow 2\cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\Rightarrow x = \cos\frac{\pi}{6}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

S3. (3) The value of x satisfying equation $\cot^{-1}x + \tan^{-1}3 = \frac{\pi}{2}$ is

Given,

$\cot^{-1}(x) + \tan^{-1}(3) = \pi/2$

We know that $\tan^{-1}(x) + \cot^{-1}(x) = \pi/2$

Therefore, x = 3

S4. $\left(\frac{6}{17}\right)$ The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

In right angled triangle , perpendicular = 3 , Hypotenuse = 5 , base = 4

$\Rightarrow \cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$

$\Rightarrow \cot\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right)$

$\Rightarrow \cot\left(\tan^{-1}\left(\frac{17}{6}\right)\right)$

$\Rightarrow \cot\left(\cot^{-1}\left(\frac{6}{17}\right)\right)$

$\Rightarrow \frac{6}{17}$

S5. (0) $\sin\left[\frac{\pi}{6} + \sin^{-1}\left(-\frac{1}{2}\right)\right] =$

$\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$

[*since* $\sin^{-1}(-x) = -\sin^{-1}x$,for all $x \in [-1, 1]$]

$\sin\left[\frac{\pi}{6} - \frac{\pi}{6}\right] = 0$

TRUE AND FALSE

S1. (False)

$\sin^{-1}\left(\frac{1}{x}\right) = \csc^{-1}x$, *for all* $x \in (-\infty, -1] \cup [1, \infty)$

S2. (True)

$\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$, $for\ all\ x \in (-\infty, -1] \cup [1, \infty)$

S3. (False)

$2\cos^{-1} x = \begin{cases} \cos^{-1}(2x^2 - 1), if\ 0 \le x \le 1 \\ 2\pi - \cos^{-1}(2x^2 - 1), if - 1 \le x \le 0 \end{cases}$

S4. (True)

The number of solutions of the equation $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is

$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 5x = 1 - 6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow 6x^2 + 6x - x - 1 = 0$

$\Rightarrow 6x(x + 1) - (x + 1) = 0$

$\Rightarrow (6x - 1)(x + 1) = 0$

$\Rightarrow x = \frac{1}{6}, x = -1$

S5. (True) $\sin^{-1} x = \cos^{-1} \sqrt{1 - x^2} = \tan^{-1} \frac{x}{\sqrt{1-x^2}} = \cot^{-1} \frac{\sqrt{1-x^2}}{x} = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \csc^{-1}\left(\frac{1}{x}\right)$

ASSERTION AND REASONING

S1. (d) We can write $\sin^{-1} x \neq (\sin x)^{-1}$

Any value in the range of principal value branch is called principal value of that inverse trigonometric function.

Therefore , A is false ; R is true.

S2. (b) Domain of $\tan x$ is the set $\left\{x : x \in R\ and\ x \neq (2n+1)\frac{\pi}{2}, n \in Z\right\}$ and range R.

$\Rightarrow \tan x$ is not defined for odd multiples of $\frac{\pi}{2}$

If we restrict the domain of tangent function to $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, then it is one -one and onto with its range as R.

Actually tan x restricted to any of the intervals $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right)$, $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ etc. is bijective and its range is R.

Thus $\tan^{-1} x$ can be defined as a function whose domain is R and range could be any of the intervals

$\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right)$, $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and soon.

Both A is true , R is true ; R is not a correct explanation for A.

S3 . (d) $\Rightarrow \cos^{-1}(1) = y$

$\Rightarrow \cos y = 1$

$\Rightarrow \cos y = \cos 0^o$

$\Rightarrow y = 0$

$\Rightarrow$ Principal value of $\cos^{-1}(1)$ is 0

Hence Assertion is incorrect.

Reason is correct.

S4 . (c) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$

$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$

$\cot y = \frac{-1}{\sqrt{3}}$

$\cot y = -\cot\left(\frac{\pi}{3}\right)$

$\cot y = \cot\left(\pi - \frac{\pi}{3}\right)$

$\cot y = \cot\left(\frac{2\pi}{3}\right)$

$\Rightarrow \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{2\pi}{3}$

Hence , Assertion is correct and Reason is incorrect.

S5 . (a) **Assertion (A) :** Function f : R $\rightarrow R$ given by f(x) = sin x is not a bijection .

Because f(0) = sin 0 = 0

f(π) = sin π = 0

f(0) = f(π) but $0 \neq \pi$

Reason(R) : A function f : A $\rightarrow B$ is said to be bijection if it is one – one and onto .

HOMEWORK

MCQ

S1. (d) f(x) = $\cot^{-1}x + \cos^{-1}x + \csc^{-1}x$
Domain of $\cot^{-1}x = (-\infty, \infty)$
Domain of $\cos^{-1}x = [-1, 1]$
Domain of $\csc^{-1}x = (-\infty, -1] \cup [1, \infty)$
These function are in addition.
So, we have to take the intersection of all domains .
So , answer is { -1 , 1}

S2. (c) Domain for $\sin^{-1}x$ is [- 1, 1] and for $\sec^{-1}x$, $x \in (-\infty, -1] \cup [1, \infty)$
∴ Common domain is $x \in \{-1, 1\}$
At $x = -1, f(x) = \frac{-\pi}{2} - \frac{\pi}{4} + \pi = \frac{\pi}{4}$
Similarly , at x= 1 , f(x) = $\frac{\pi}{2} + \frac{\pi}{4} + 0 = \frac{3\pi}{4}$

S3. (b) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \sin^{-1}\frac{1}{3}$
$= \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \cos^{-1}\sqrt{1 - \left(\frac{1}{3}\right)^2}$
$= \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$
$= \frac{\pi}{2}$
[since, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$]

S4. (d) Given that $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$
Therefore , $\left(\frac{\pi}{2} - \cos^{-1}x\right) + \left(\frac{\pi}{2} - \cos^{-1}y\right) = \frac{\pi}{2}$
$\Rightarrow \cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$

S5. (b) $\Rightarrow \sin\left[\cot^{-1}\left(\cot\frac{17\pi}{3}\right)\right]$
$= \sin\left[\cot^{-1}\left(\cot\left(5\pi + \frac{2\pi}{3}\right)\right)\right]$
$= \sin\left[\cot^{-1}\left(\cot\left(\frac{2\pi}{3}\right)\right)\right]$
[Since , $\cot^{-1}(\cot x) = x$; if $x \in [0, \pi]$]
$= \sin\left(\frac{2\pi}{3}\right)$
$= \sin\left(\pi - \frac{\pi}{3}\right)$
$= \sin\frac{\pi}{3}$
$= \frac{\sqrt{3}}{2}$

S6. (a) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$
$\Rightarrow \cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$
$\Rightarrow \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
$\Rightarrow \cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{2\pi}{3} + \frac{\pi}{3} = \pi$

S7. (a) Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is
$\Rightarrow \tan^{-1}y = \tan^{-1}\left(\frac{x + \frac{2x}{1-x^2}}{1 - \frac{2x^2}{1-x^2}}\right)$
$\Rightarrow \tan^{-1}y = \tan^{-1}\left(\frac{x - x^3 + 2x}{1 - x^2 - 2x^2}\right)$
$\Rightarrow \tan^{-1}y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$
$\Rightarrow y = \frac{3x - x^3}{1 - 3x^2}$

S8. (b) $\sin^{-1}(x / 5) + \text{cosec}^{-1}(5 / 4) = \pi / 2$
$\sin^{-1}(x / 5) + \sin^{-1}(4 / 5) = \pi / 2$
$\sin^{-1}(x / 5) = (\pi / 2) - \sin^{-1}(4 / 5)$
$(x / 5) = \sin[(\pi / 2) - \sin^{-1}(4 / 5)]$
$(x / 5) = \cos(\sin^{-1}(4 / 5)) = \cos(\cos^{-1}(3 / 5))$
= 3 / 5
x = 3

S9. (c) Given , $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$
$\Rightarrow \cos^{-1}\left(\frac{xy}{2} + \sqrt{1-x^2}\sqrt{1 - \frac{y^2}{4}}\right) = \alpha$
$\Rightarrow \frac{xy}{2} + \sqrt{1-x^2}\sqrt{1 - \frac{y^2}{4}} = \cos\alpha$
$\Rightarrow 2\sqrt{1-x^2}\sqrt{1 - \frac{y^2}{4}} = 2\cos\alpha - xy$
On squaring both sides , we get
$4(1-x^2)\left(\frac{4-y^2}{4}\right) = 4cos^2\alpha + x^2y^2 - 4xy\cos\alpha$
$\Rightarrow (1-x^2)(4-y^2) = 4cos^2\alpha + x^2y^2 - 4xy\cos\alpha$
$\Rightarrow 4x^2 + y^2 - 4xy\cos\alpha = 4 - 4cos^2\alpha$
$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4sin^2\alpha$

S10. (a) As we know that :
$\cot^{-1}(-x) = \pi - \cot^{-1}x$, for all $x \in$ R
$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$
$\cos^{-1}(-x) = \pi - \cos^{-1}x$, for all $x \in$ [-1 , 1]
$\cos^{-1}(-1) = \pi - \cos^{-1}(1) = \pi - 0 = \pi$

SUBJECTIVE QUESTIONS

S1. We have , $\sin\left(\cos^{-1}\frac{5}{13} + \sin^{-1}x\right) = 1$
$\Rightarrow \cos^{-1}\left(\frac{5}{13}\right) + \sin^{-1}x = \sin^{-1}1$
$\Rightarrow \cos^{-1}\left(\frac{5}{13}\right) + \sin^{-1}x = \frac{\pi}{2}$
$\Rightarrow \sin^{-1}x = \frac{\pi}{2} - \cos^{-1}\frac{5}{13}$
$\Rightarrow \sin^{-1}x = \sin^{-1}\frac{5}{13}$

S2. We know that $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$
$\therefore \tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\} = \tan^{-1}\left\{2\cos\left(2 \times \frac{\pi}{6}\right)\right\} = \tan^{-1}\left(2\cos\frac{\pi}{3}\right) = \tan^{-1}\left(2 \times \frac{1}{2}\right) = \frac{\pi}{4}$

S3. Since , $9 > 0$, $\frac{5}{4} > 0$ and $\left(9 \times \frac{5}{4}\right) > 1$

$\therefore \tan^{-1} 9 + \tan^{-1}\frac{5}{4} = \pi + \tan^{-1}\left(\frac{9+\frac{5}{4}}{1-9.\frac{5}{4}}\right) =$

$\pi + \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

S4. Let $y = \tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$ ....(i)

Let $\cos^{-1}\frac{\sqrt{5}}{3} = \theta \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$ and

$\cos\theta = \frac{\sqrt{5}}{3}$

$\therefore$ (i) becomes $y = \tan\left(\frac{\theta}{2}\right)$ ....(ii)

Since $\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\frac{\sqrt{5}}{3}}{1+\frac{\sqrt{5}}{3}} = \frac{3-\sqrt{5}}{3+\sqrt{5}} = \frac{(3-\sqrt{5})^2}{4}$

$\tan\frac{\theta}{2} = \pm\left(\frac{3-\sqrt{5}}{2}\right)$ ....(iii)

$\frac{\theta}{2} \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan\frac{\theta}{2} > 0$

$\therefore$ from (iii), we get $y = \tan\frac{\theta}{2} = \left(\frac{3-\sqrt{5}}{2}\right)$

S5. Let $y = \sin^{-1}(\sin 7)$

$\sin^{-1}(\sin 7) \neq 7$ as $7 \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}(\sin 7) = \sin^{-1}\sin(7-2\pi)$

$\therefore \sin^{-1}\sin(7-2\pi) = 7-2\pi$ (since $7 - 2\pi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$) (using property 3)

Similarly if we have to find $\sin^{-1}(\sin(-5))$ then

Let $y = \sin^{-1}(\sin - 5)$

$\sin^{-1}(\sin -5) \neq -5$ as $-5 \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}(\sin - 5) = -\sin^{-1}\sin 5$ (using property 1)

$-\sin^{-1}\sin 5 = -\sin^{-1}\sin(5 - 2\pi)$

$-\sin^{-1}\sin(5 - 2\pi) = -(5 - 2\pi)$ (Since $5 - 2\pi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)

NUMERICAL TYPE QUESTIONS

S1. **($\sqrt{2}$)** Since $\cot(\cot^{-1} x) = x, \ \forall x \in R$

$\therefore \cot\left(\cot^{-1}\frac{3\pi}{4}\right) = \frac{3\pi}{4}$

$\text{cosec}\left\{\cot\left(\cot^{-1}\frac{3\pi}{4}\right)\right\} = \text{cosec}\left(\frac{3\pi}{4}\right) = \sqrt{2}$.

S2. **($\frac{3}{5}$)** $\sin\left(\tan^{-1}\frac{3}{4}\right) = \sin\left(\sin^{-1}\frac{3}{5}\right) = \frac{3}{5}$

S3. **(1)** $\sin^{-1}(f(x)) + \cos^{-1}(g(x)) = \frac{\pi}{2} \Leftrightarrow f(x) = g(x)$ and $-1 \le f(x), g(x) \le 1$

$x^2 - 2x + 1 = x^2 - x \Leftrightarrow x = 1$, accepted as a solution

S4. **($\frac{-1}{2}$)**

Given equation is: $\sin[\cot^{-1}(1 + x)] = \cos[\tan^{-1}x]$

Let $\cot^{-1}(1 + x)] = a$

$\Rightarrow \cot a = 1 + x$ ...(1)

And $\tan^{-1}x = b$

$\Rightarrow x = \tan b$ ...(2)

Solve (1) for a in terms of sin function:

$\cot a = 1 + x$

We know, $\text{cosec } a = \sqrt{(1+\cot^2 a)} = \sqrt{(1 + (1 + x)^2} = \sqrt{(x^2 + 2x + 2)}$

Also, $\sin a = 1/\text{cosec } a$

$\Rightarrow \sin a = 1/\sqrt{(x^2 + 2x + 2)}$

Or $a = \sin^{-1}[1/\sqrt{(x^2 + 2x + 2)}]$

Solve (2) for b in terms of cos function:

$x = \tan b$

We know, $\sec b = \sqrt{(1+\tan^2 b)} = \sqrt{(1+x^2)}$

Also, $\cos b = 1/\sec b = 1/\sqrt{(1+x^2)}$

Or $b = \cos^{-1}[1/\sqrt{(1+x^2)}]$

Given equation

$\Rightarrow \sin(\sin^{-1}[1/\sqrt{(x^2 + 2x + 2)}]) = \cos[\cos^{-1}(1/\sqrt{(1+x^2)})]$

$\Rightarrow 1/\sqrt{(x^2 + 2x + 2)} = 1/\sqrt{(1+x^2)}$

Squaring both sides and solving, we get

$2x = -1$

Or $x = -1/2$

S5. ($\frac{1}{\sqrt{2}}$) Given :

$3\sin^{-1}x + \cos^{-1}x = \pi$

$\Rightarrow 3\sin^{-1}x + \cos^{-1}x = 2\sin^{-1}x + [\sin^{-1}x + \cos^{-1}x] = \pi$

As we know that, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $x \in [-1, 1]$

$\Rightarrow 2\sin^{-1}x + \frac{\pi}{2} = \pi$

$\Rightarrow x = \frac{1}{\sqrt{2}}$

TRUE AND FALSE

S1. (True)

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, for all $x \in R$

S2. (True)

$\csc(\csc^{-1}x) = x$ for all $x \in (-\infty, -1] \cup [1, \infty)$

S3. (False)

The domain and range of $\cos^{-1}x$ are [-1 ,1] and [0 , π] respectively.

S4. (True)

$\sin^{-1}x$ attains the minimum value $\frac{-\pi}{2}$ at x = -1 and the maximum value $\frac{\pi}{2}$ at x = 1

S5. (True)

$cosec^{-1}x$ is decreasing on (-∞ , $-1]$ and [1 , ∞).But ,it is neither increasing nor decreasing on ($-\infty, -1] \cup [1, \infty)$.

ASSERTION AND REASONING

S1. (b) We have ,

$\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$

$= \sin^{-1}\left\{\frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2} + \frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2}\right\}$

$= \sin^{-1}\left\{\frac{8}{17}\times\frac{4}{5} + \frac{3}{5}\times\frac{15}{17}\right\} = \sin^{-1}\left\{\frac{77}{85}\right\}$

Hence , Assertion is correct and Reason is correct but Reason is not the correct explanation of assertion.

S2. (a) If f(x) = 2^x then $f^{-1}(x) = \log_2 x$

f(x) = a^x and g(x) = $\log_a x$ are inverse of each other

Hence , Assertion is correct , reason is correct ; reason is a correct explanation for assertion.

S3. (a) The value of $2\tan^{-1}\frac{1}{5}$

As we know that

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), if -1 < x < 1$

$2\tan^{-1}\frac{1}{5} = \tan^{-1}\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}\right)$

$2\tan^{-1}\frac{1}{5} = \tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right)$

$2\tan^{-1}\frac{1}{5} = \tan^{-1}\left(\frac{2\times25}{5\times24}\right) = \tan^{-1}\frac{5}{12}$

Hence , Assertion is correct , reason is correct ; reason is a correct explanation for assertion.

S4. (d) For any $x \in (-\infty, -1] \cup [1, \infty)$, $\sec^{-1}x$ is an angle $\theta \in \left[0, \frac{\pi}{2}\right) \cup (\frac{\pi}{2}, \pi]$ whose secant is x i.e.,

$\sec\theta = x$

$\therefore \sec^{-1}2 = \frac{\pi}{3}$

Assertion is incorrect, reason is correct.

S5. (c) The value of $\cot\left(\cot^{-1}\frac{4}{3}\right) = \frac{4}{3}$

Since, $\cot(\cot^{-1}x) = x$ $for\ all\ x \in \boldsymbol{R}$.

Assertion is correct ,reason is incorrect.