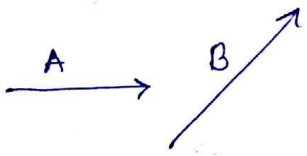
Vector Algebra :-

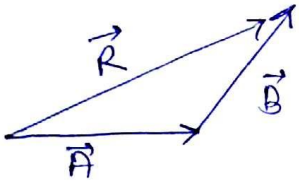
Vector :- Those physical quantities which have magnitude as well as direction & follow triangle law of vector addition are called vector quantity.

Ex → $\vec{V}, \vec{a}, \vec{F}, \vec{P}, \vec{L}, \vec{T}$ etc.

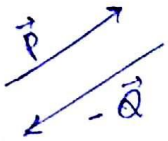
Triangle law of vector addition :-

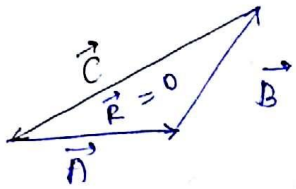
$$\vec{R} = \vec{A} + \vec{B}$$





$$\Rightarrow \vec{A} + \vec{B} + \vec{C} = 0$$



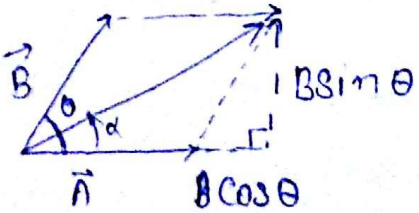


$$P = Q$$

$$\vec{P} = -\vec{Q}$$

$$\vec{P} + \vec{Q} = 0$$

Parallelogram law :-



Vector = mag (dir^n)

Unit Vector = 1 (dir^n) ≡ dir^n

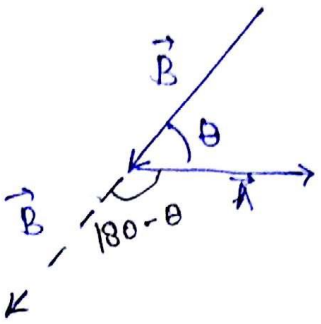
$$\vec{A} = A_x i + A_y j + A_z k$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$(R)^2 = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$

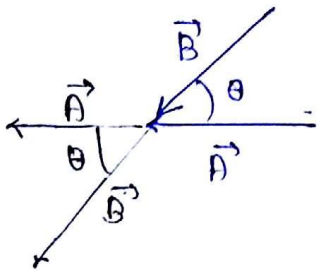
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$



$$R = \sqrt{A^2 + B^2 + 2AB\cos(180 - \theta)}$$

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$



$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Question: Resultent of two vectors $\vec{A}$ and $\vec{B}$ is $\vec{R}$ if B is doubled then the new resultand $\perp$ to A then

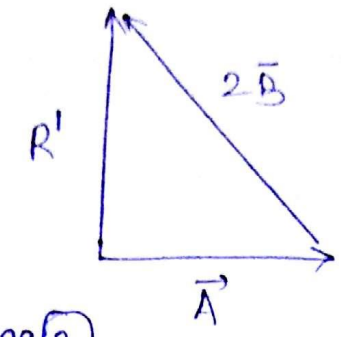
a) A = B b) A = R c) B = R c) None of the

Soln

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad -(1)$$

New resultand No change in dirn $\theta = \theta$

$$R' = A^2 + (2B)^2 + 2A(2B)\cos\theta \quad -(2)$$



$$A^2 + (R')^2 = 4B^2 \quad -(3)$$

From eq (2)

$$A^2 + 4B^2 + 4AB\cos\theta = 4B^2 - A^2$$

$$2AB\cos\theta = -A^2$$

Put in eqn (1)

$$A^2 + B^2 - A^2 = R^2 \Rightarrow \boxed{B = R}$$

Question:- When two forces act $\perp$ to each other their Resultant is 10 kN. if they act at 60° their resutant is $5\sqrt{6}$ kN find A & B (magnitude)

Soln

A & B

$\theta = 90°$ $R = 10$ kN

$\theta = 60°$ $R = 5\sqrt{6}$ kN

$$A^2 + B^2 = 100 \quad -(1)$$

$$A^2 + B^2 + AB = 150 \quad -(2)$$

From (1) & (2)

$$AB = 50$$

$$(A+B)^2 = A^2 + B^2 + 2AB$$

$$(A+B)^2 = 100 + 2 \times 50$$

$$(A+B)^2 = \sqrt{200} = 10\sqrt{2}$$

$$(A-B)^2 = 100 - 100$$

$$A = B$$

Now $A + A = 10\sqrt{2}$

$$2A = 10\sqrt{2}$$

~~$A = 5\sqrt{2}$~~

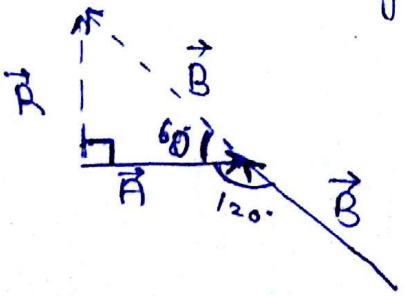
$$A = 5\sqrt{2} \text{ kN}$$

$$B = 5\sqrt{2} \text{ kN}$$

Question: Two forces are acting at a point making an angle of 120° with each other if resultant force is ⊥ to smaller force ~~equal~~ which equal to 50 N. The larger force is ?

Soln

$\vec{R} = 50$ N



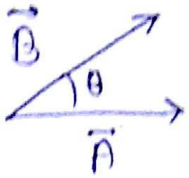
Product of Two vectors :-

$\vec{A}$ & $\vec{B}$

(i) Dot product (scalar Product)

$\vec{A} \cdot \vec{B} = C$

$C = AB\cos\theta$



$\vec{B} \cdot \vec{A} = D$

$D = BA\cos\theta$

$D = C$

$\Rightarrow \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

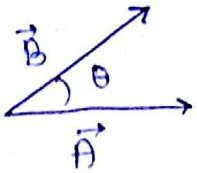
(ii) Cross Product (Vector Product)

$\vec{A} \times \vec{B} = \vec{C}$

$\vec{C} = |\vec{C}|\,\hat{C}$

$\vec{C} = AB\sin\theta$

$\hat{C}$ { Dirn of vector C will be ⊥ to plane containing A & B and will be given by Right Hand thumb Rule.



$\theta < 180°$

$\hat{C} \rightarrow$ ⊥ to the plane & outward

$\vec{B} \times \vec{A} = \vec{D}$

$D = BA\sin\theta$

$\hat{D} \rightarrow$ ⊥ to the plane & inward

$\vec{C} = -\vec{D}$

$(\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\Rightarrow \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\Rightarrow \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$