Exercise 3.1

Chapter 3 Applications of Differentiation Exercise 3.1 1E

Suppose f is continuous on a closed interval $[a,b]$.

(a) The existence of an absolute maximum and absolute minimum value can be guaranteed by the "Extreme value theorem,"

The extreme value theorem states that,

If f is continuous on a closed interval $[a,b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a,b]$.

(b) An absolute maximum value must either occur at a local maximum or at an endpoint of the interval $[a,b]$. Similarly, an absolute minimum value must either occur at a local minimum or at an endpoint of the interval $[a,b]$.

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a,b]$:

1. Find the values of f at the critical numbers of f in (a,b).

2. Find the values of f at the endpoints of the interval.

3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Remember, for step 1, that the critical numbers are simply the numbers c where the derivative $f'(c)$ either is undefined or equal to 0. So, the process always begins with finding the derivative $f'(x)$.

Chapter 3 Applications of Differentiation Exercise 3.1 2E

Suppose f is continuous on a closed interval $[a,b]$.

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Remember, for step 1, that the critical numbers are simply the numbers c where the derivative $f'(c)$ either is undefined or equal to 0. So, the process always begins with finding the derivative $f'(x)$.

Chapter 3 Applications of Differentiation Exercise 3.1 3E

Consider the figure 1:

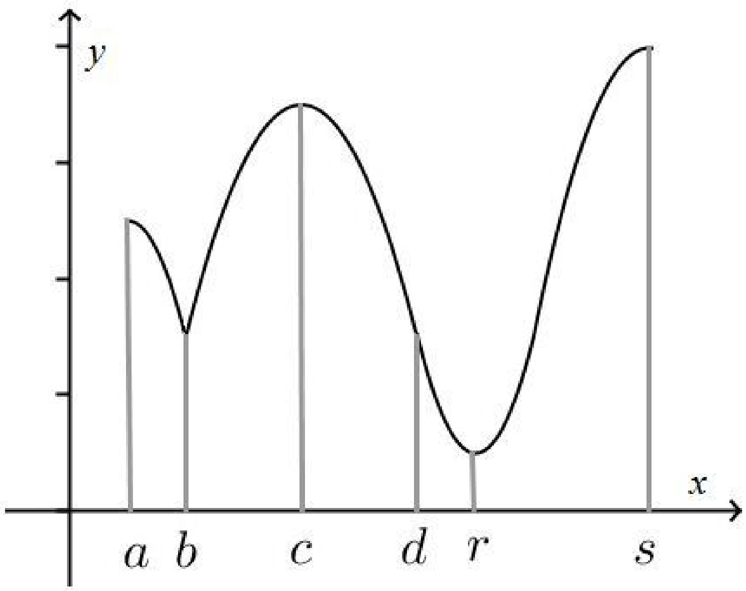


Figure 1

Go from left to right and inspect each point in turn on the graph.

Since *a* is an endpoint, it can't be considered a local maximum or minimum. It could be an absolute maximum or minimum, but it isn't since the function takes on both larger and smaller values elsewhere. so *a* is neither a maximum or a minimum.

The point *b* is a minimum. It is not an absolute minimum because other points on the function have lower *y*-values. So *b* is a local minimum.

The point *c* is a maximum, but not an absolute one since the function is larger elsewhere. Therefore *c* is a local maximum.

Since points immediately around *d* are both larger and smaller than it, *d* is neither a maximum nor a minimum.

The point *r* is a local minimum. It is also an absolute minimum since no other points, anywhere on the graph, are lower. So *r* is both a local and absolute minimum.

The point *s* cannot be considered a local maximum or minimum since it is an endpoint. However, since it is the highest point on the graph, *s* is an absolute maximum.

Chapter 3 Applications of Differentiation Exercise 3.1 4E

Consider the graph provided in the text book.

From the graph, point $(a, f(a))$ is the lowest point in the graph. So, the given graph will have an absolute minimum at $x = a$ and $x = a$ is an end-point of the graph, so it cannot be local minimum.

Point $(b, f(b))$ is the highest point in the neighborhood of $x = b$. So, $x = b$ is a point of local maximum.

Point $(c, f(c))$ is the neither the highest point nor the lowest point in the neighborhood of $x = c$. So, it is a point of neither maxima nor minima.

Point $(d, f(d))$ is the lowest point in the neighborhood of $x = d$. So, $x = d$ is a point of local minima.

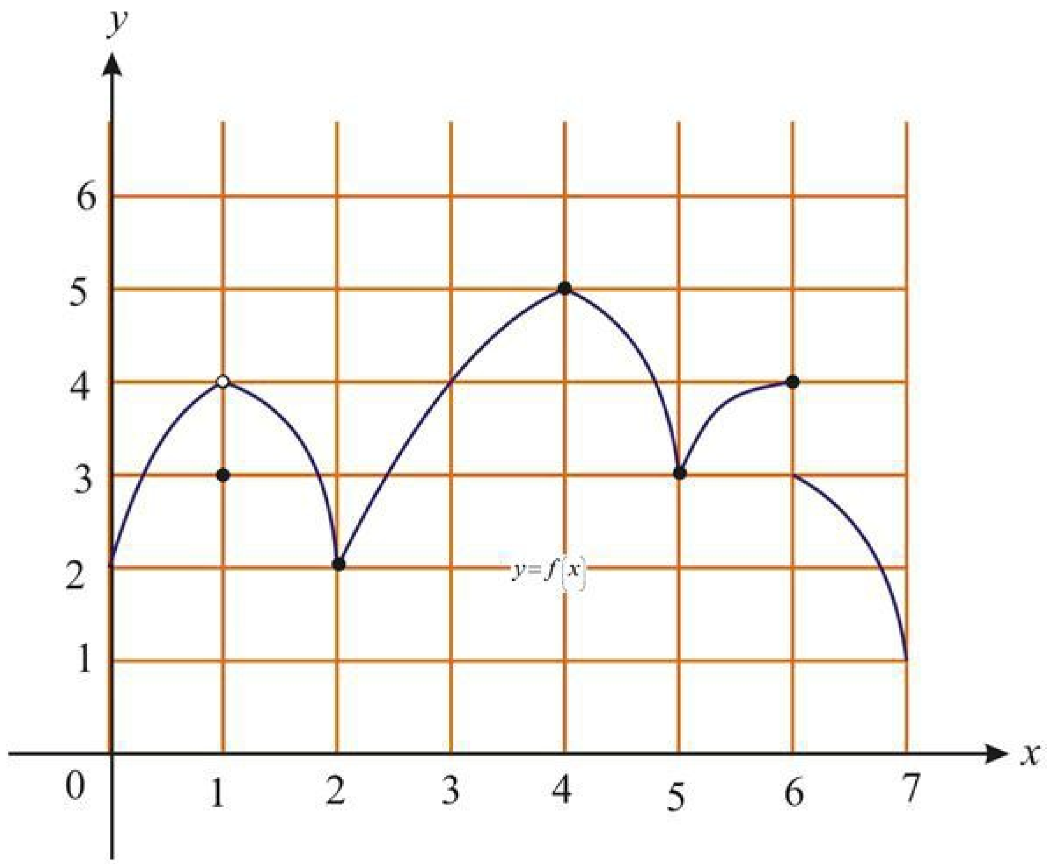
Point $(r, f(r))$ is the highest point in the graph. So, the given graph will have an absolute maximum at $x = r$ and $x = r$ is not an end-point of the graph, so the graph has a local maximum at point $x = r$.

Point $(s, f(s))$ is not lowest point in the graph. So, the given graph will not have an absolute minimum at $x = s$ and $x = s$ is an end-point of the graph, so the graph will not have a local minimum at point $x = s$.

Therefore, the given graph has an **absolute minimum at** $x = a$**, an absolute maximum at** $x = r$**, local maximum at** $x = b$ **and** $x = r$**, local minimum at** $x = d$**, neither a maximum nor a minimum at** $x = c$ **and** $x = s$.

Chapter 3 Applications of Differentiation Exercise 3.1 5E

Consider the graph of the function.



The objective is to identify the absolute, local maximum and minimum values of the function by verifying the graph.

Definition:

Let c be a number in the domain D of a function f. Then $f(c)$ is the absolute maximum value of f on D if $f(c) \geq f(x)$ for all x in D and absolute minimum value of f on D if $f(c) \leq f(x)$ for all x in D.

The number $f(c)$ is a local maximum value of f if $f(c) \geq f(x)$ when x is near c and

Local minimum value of f if $f(c) \leq f(x)$ when x is near c.

Observe from the graph that $f(4) = 5 \geq f(x)$ for all x in D.

Therefore $f(4)$ is the absolute maximum.

No absolute minimum exists for the function.

Observe that, $f(4) \geq f(x)$ when x is near 4, also $f(6) \geq f(x)$ when x is near 6.

Thus $f(4) = 5$ and $f(6) = 4$ are the local maximum values.

Also $f(1) \leq f(x)$ when x is near 2, $f(2) \leq f(x)$ when x is near 2 and $f(5) \leq f(x)$ when x is near 5.

Thus $f(1) = 3$, $f(2) = 2$ and $f(5) = 3$ are the local minimum values.

Therefore absolute maximum value is $f(4) = \boxed{5}$.

Local maximum values are $f(4) = \boxed{5}$, $f(6) = \boxed{4}$

Local minimum values are $f(1) = \boxed{3}$ $f(2) = \boxed{2}$ and $f(5) = \boxed{3}$

Chapter 3 Applications of Differentiation Exercise 3.1 6E

Absolute Miniumum: f(4) =1

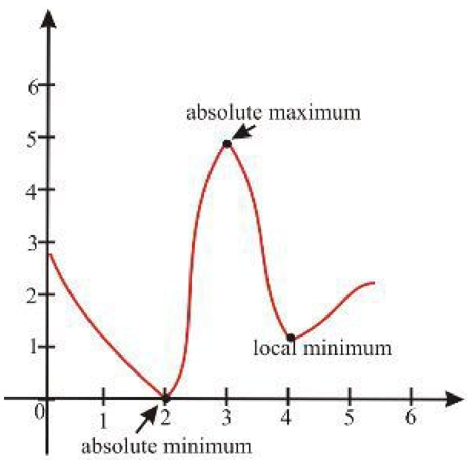
Absolute Maximum: None

Local Maximum: f(3)=4 and f(6)=3

Local Minimum: f(4)=1

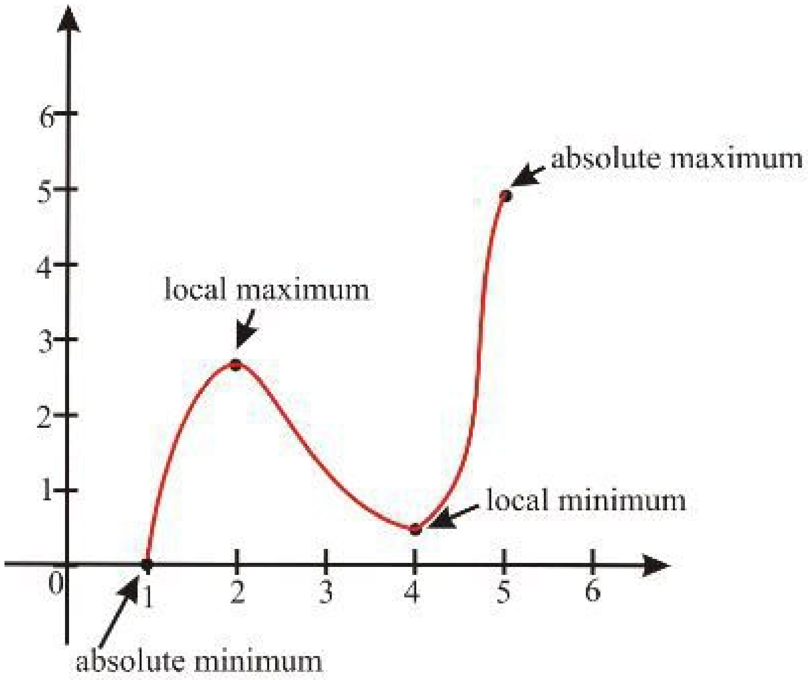
Chapter 3 Applications of Differentiation Exercise 3.1 7E

We need to draw the graph of a function f, which is continuous on [1, 5] and has an absolute minimum at 2, an absolute maximum at 3, and a local minimum at 4.



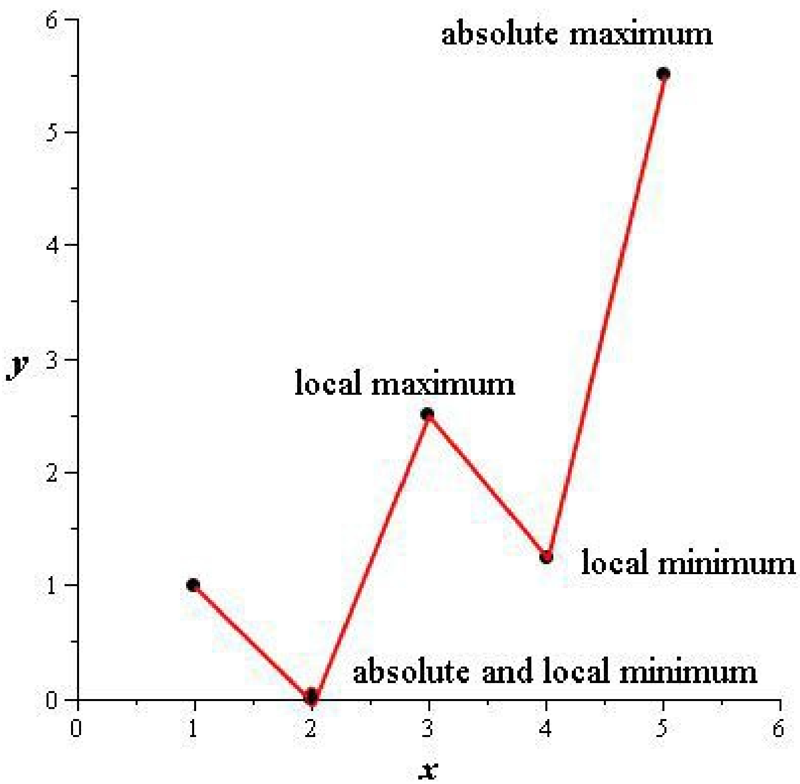
Chapter 3 Applications of Differentiation Exercise 3.1 8E

We need to draw the graph of a function f, which is continuous on [1, 5] and has an absolute minimum at 1, an absolute maximum at 5, local maximum at 2 and a local minimum at 4.



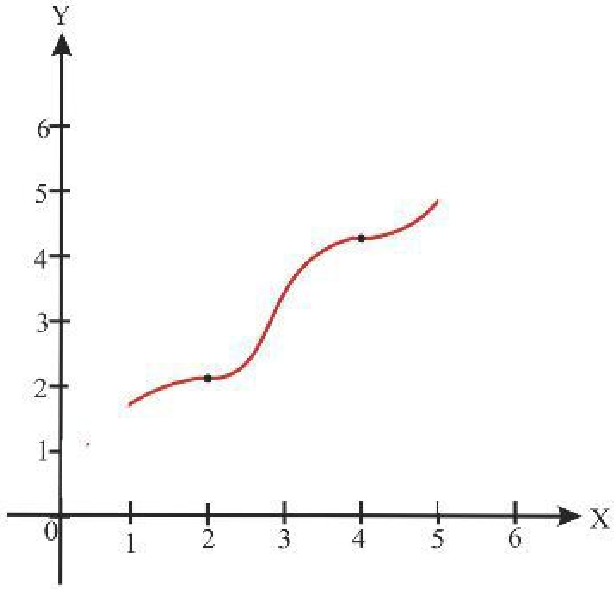
Chapter 3 Applications of Differentiation Exercise 3.1 9E

Graph of the function which satisfies the above conditions is:



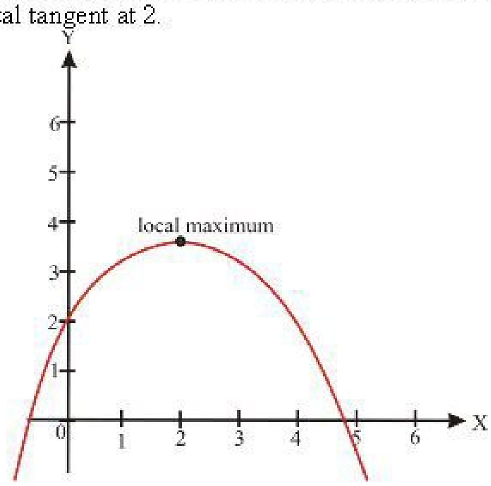
Chapter 3 Applications of Differentiation Exercise 3.1 10E

We have to sketch a graph of f that is continuous on [1, 5] and has no local maximum or minimum, but 2 and 4 are critical number,
For this we have to choose a graph of f such that f, has horizontal tangents at 2 and 4 but increasing for all x

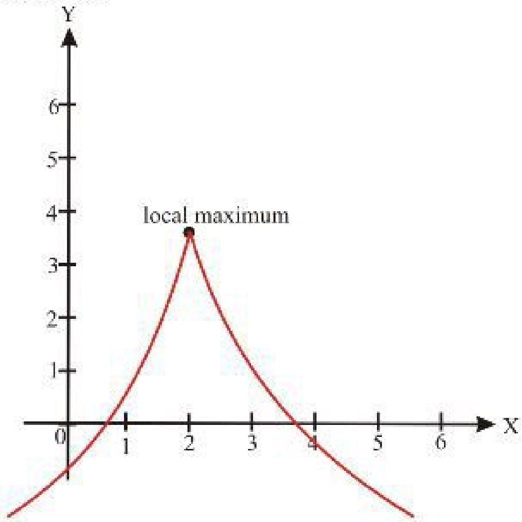


Chapter 3 Applications of Differentiation Exercise 3.1 11E

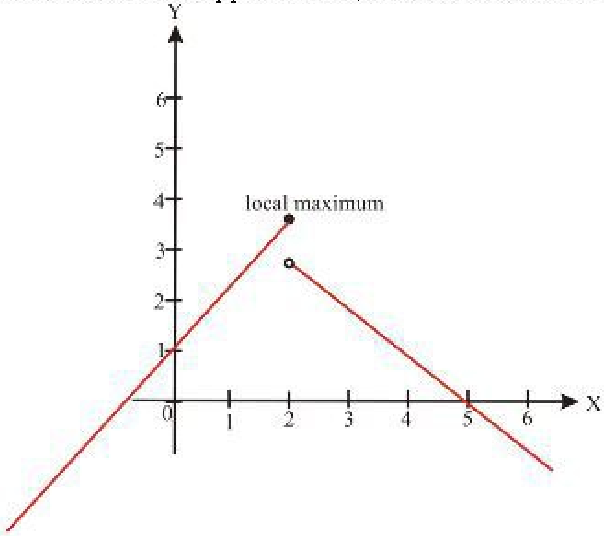
(A) The function has a local maximum at 2 and it is differentiable at 2. It means graph has a horizontal tangent at 2.



(B) The function has a local maximum at 2 but not differentiable at 2. It means the graph has a corner at 2.

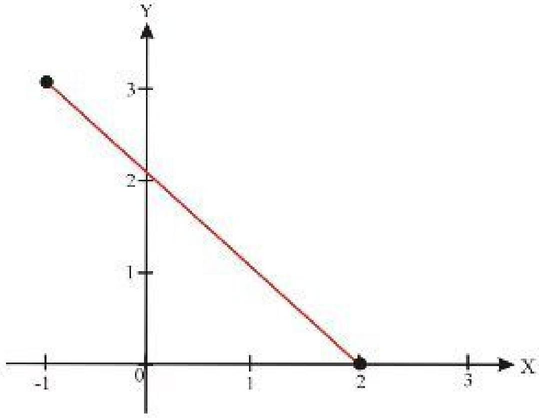


(C) The function has a local maximum at 2 but it is not continuous at 2. It means limits from both sides are not same as x approaches 2, but it is continuous from left.

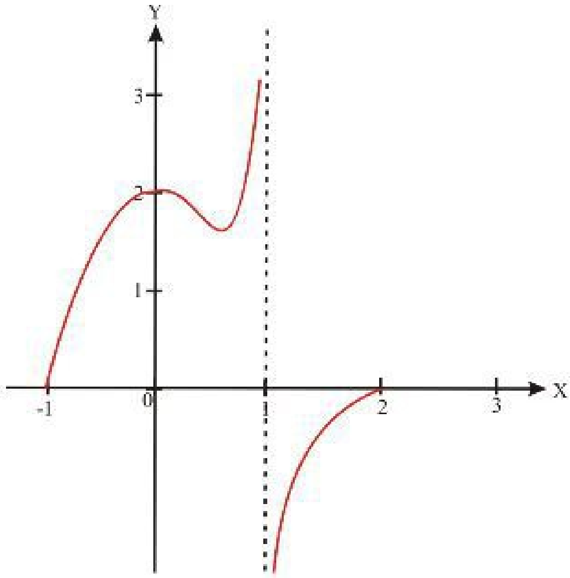


Chapter 3 Applications of Differentiation Exercise 3.1 12E

(A) The function has an absolute maximum but no local maximum it means the function is increasing or decreasing on its domain [-1, 2]



(B) By the Extreme value theorem, we know that if a function f is continuous on a closed interval then f attains an absolute maximum and an absolute minimum. But we have to draw the graph of the function which has a local maximum but not an absolute maximum. It means f must be discontinuous on the closed interval [-1, 2]



Chapter 3 Applications of Differentiation Exercise 3.1 13E

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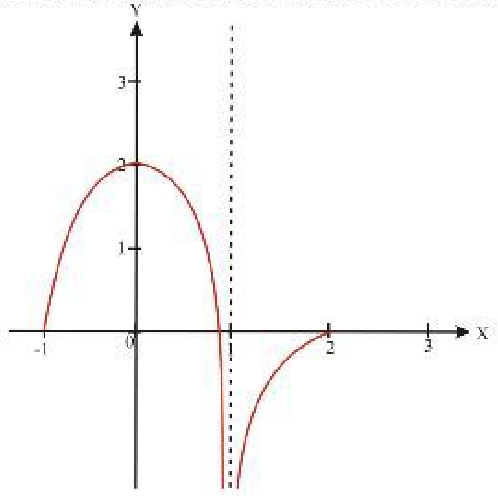
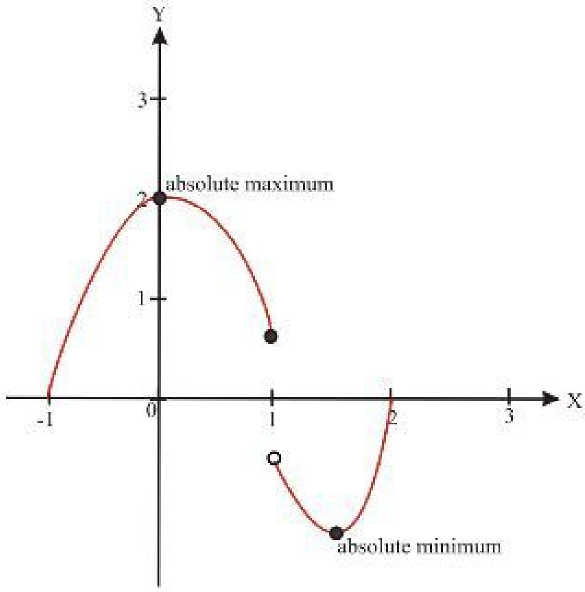
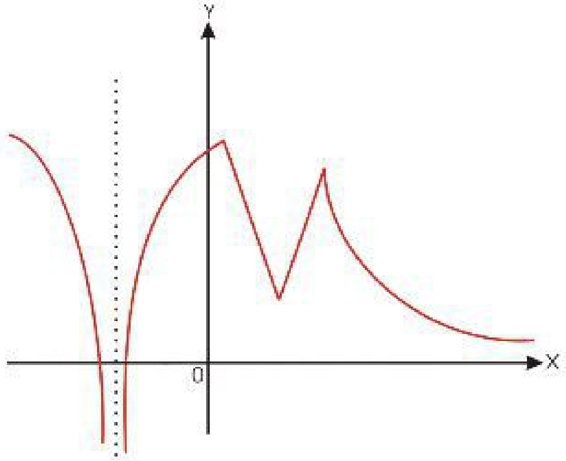


(B)

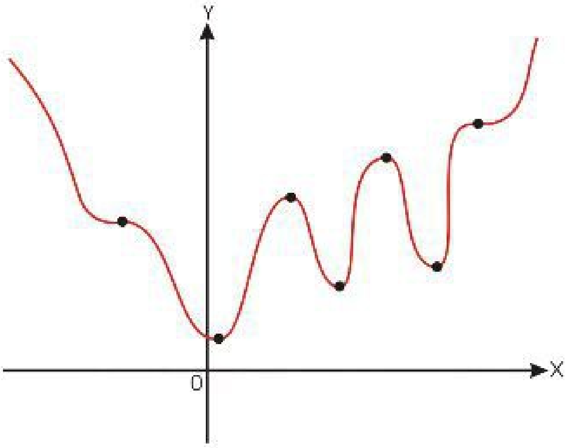


Chapter 3 Applications of Differentiation Exercise 3.1 14E

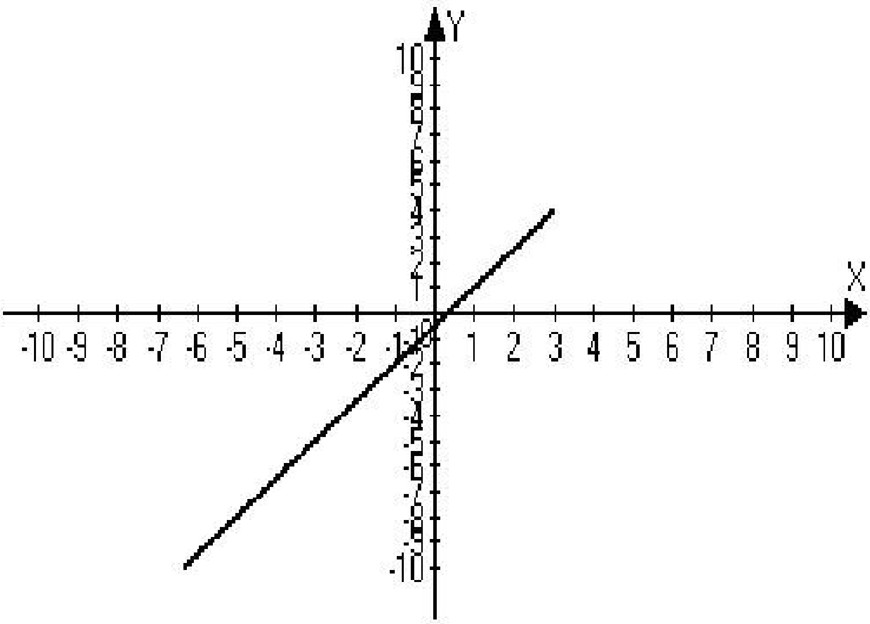
(A) We have to graph the function which has two local maxima and one local minimum and no absolute minimum.



(B) We have to draw the graph of the function that has three local minima, two local maxima, and seven critical numbers. It means at the two critical numbers there is no minimum or maximum but the graph has horizontal tangents.



Chapter 3 Applications of Differentiation Exercise 3.1 15E



From the graph absolute maximum is f(3)=4 and absolute minimum does not exist.

Chapter 3 Applications of Differentiation Exercise 3.1 16E

Take a function f defined on the domain *D*. For a number *b* in the domain *D*:

The absolute maximum value of the function is:

$$f(b) \geq f(x) \quad \forall x \text{ in } D$$

The absolute minimum value of the function is:

$$f(b) \leq f(x) \quad \forall x \text{ in } D$$

The local minimum value of the function is determined near the critical point *c* in the neighborhood:

$$f(c) \leq f(x)$$

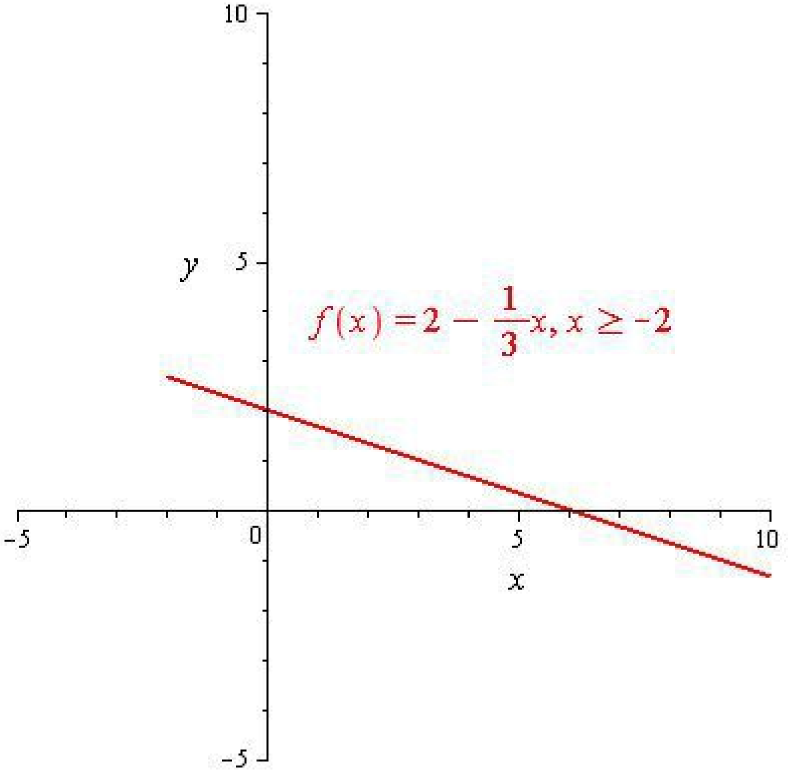
The local maximum value of the function is determined near the critical point *c* in the neighborhood:

$$f(c) \geq f(x)$$

Consider the function:

$$f(x)=2-\frac{1}{3}x \quad ,x\geq -2$$

Consider the sketch of the above function as shown below:



Observe the above graph to determine the absolute maximum value of $f(x)$.

Take the value of $x=-2$:

$$f(-2)=2-\frac{1}{3}\times(-2)$$
$$=2+\frac{2}{3}$$
$$=\frac{8}{3}$$

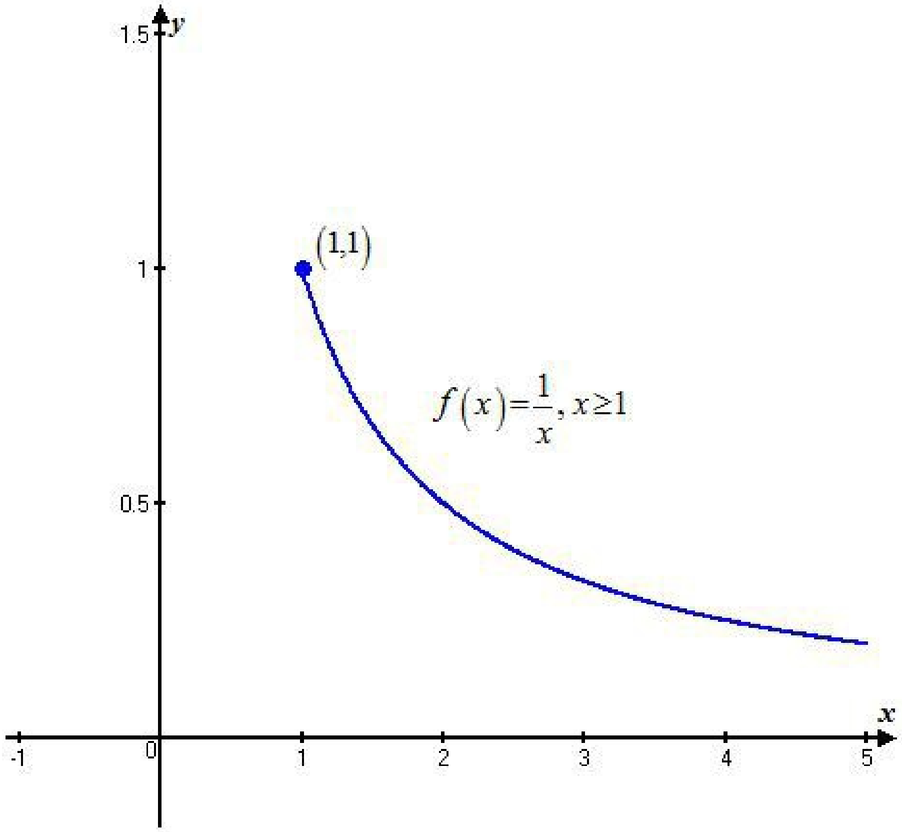
The absolute minimum value of the function does not exist.

Hence, the absolute maximum value of the function is $\frac{8}{3}$, the absolute minimum value does not exist and the function has no local minima or maxima.

Chapter 3 Applications of Differentiation Exercise 3.1 17E

Consider the function $f(x)=\frac{1}{x}, x\geq 1$

Sketch the graph of $f(x)=\frac{1}{x}, x\geq 1$ is shown below:



Observe the above graph, obtained that

There are no local maximum and minimum values of f

And the absolute maximum of f is $\boxed{f(1)=1}$.

Chapter 3 Applications of Differentiation Exercise 3.1 18E

Take a function f defined on the domain *D*. For a number *c* in the domain *D*:

The absolute maximum value of the function is:

$$f(c)\geq f(x) \quad \forall x \text{ in } D$$

The absolute minimum value of the function is:

$$f(c)\leq f(x) \quad \forall x \text{ in } D$$

The local minimum value of the function is determined near the critical point *c* in the neighborhood:

$$f(c)\leq f(x)$$

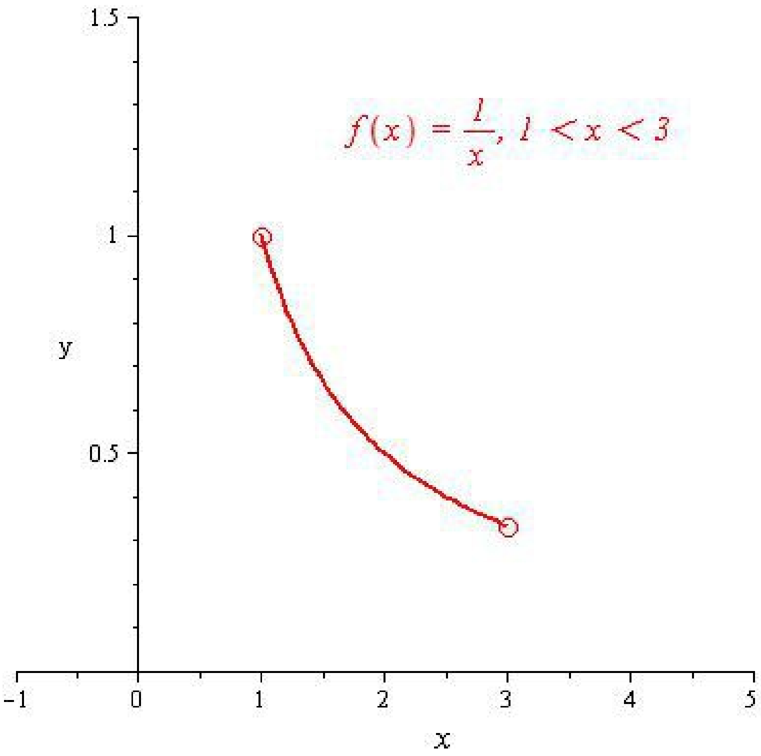
The local maximum value of the function is determined near the critical point *c* in the neighborhood:

$$f(c)\geq f(x)$$

Consider the function:

$$f(x) = \frac{1}{x} \quad ,1 < x < 3$$

Sketch the graph of the above function as shown below:



Consider the derivative of the function:

$$f(x) = \frac{1}{x}$$

$$f'(x) = \frac{-1}{x^2}$$

Since, there is no *c* in the domain for which the derivative is 0.

So, the function has no critical point.

Observe the above graph to obtain:

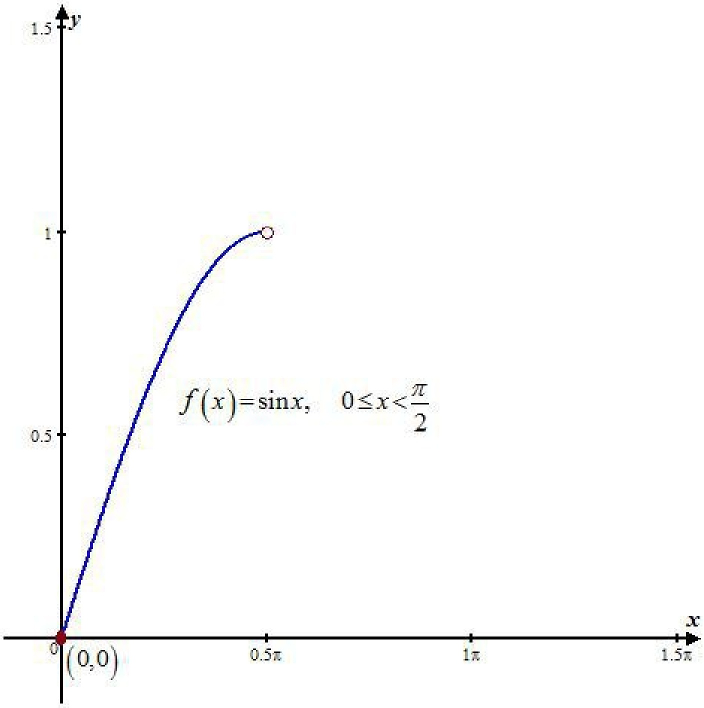
There are **no local maximum and minimum** values of the function f.

There are **no absolute maximum and absolute minimum** of the function f.

Chapter 3 Applications of Differentiation Exercise 3.1 19E

Consider the function $f(x)=\sin x, 0 \le x < \frac{\pi}{2}$

Sketch the graph of $f(x)=\sin x, 0 \le x < \frac{\pi}{2}$ is shown below:



Observe the above graph, obtain that

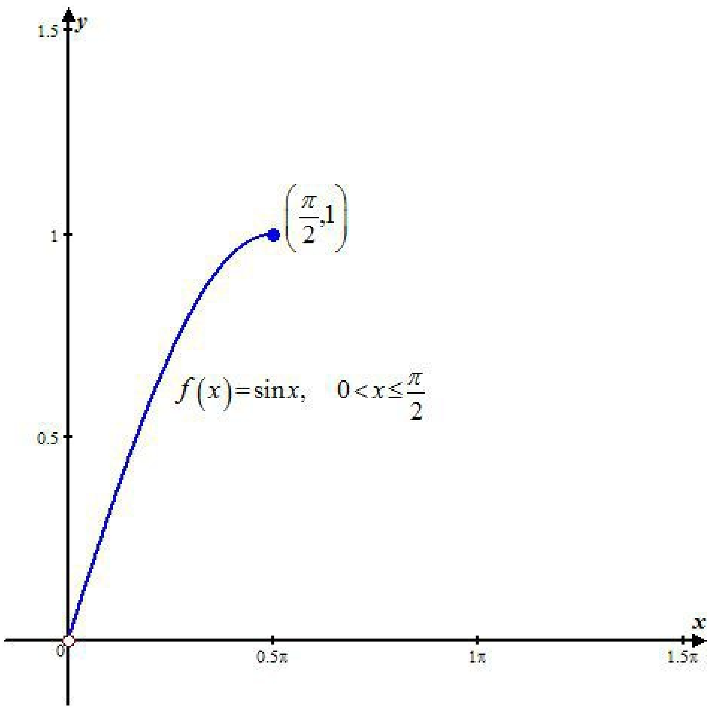
There are no local maximum and minimum values of f

And the absolute minimum of f is $\boxed{f(0)=0}$.

Chapter 3 Applications of Differentiation Exercise 3.1 20E

Consider the function $f(x)=\sin x, \quad 0 < x \le \frac{\pi}{2}$

Sketch the graph of $f(x)=\sin x, 0 < x \le \frac{\pi}{2}$ is shown below:



Observe the above graph,

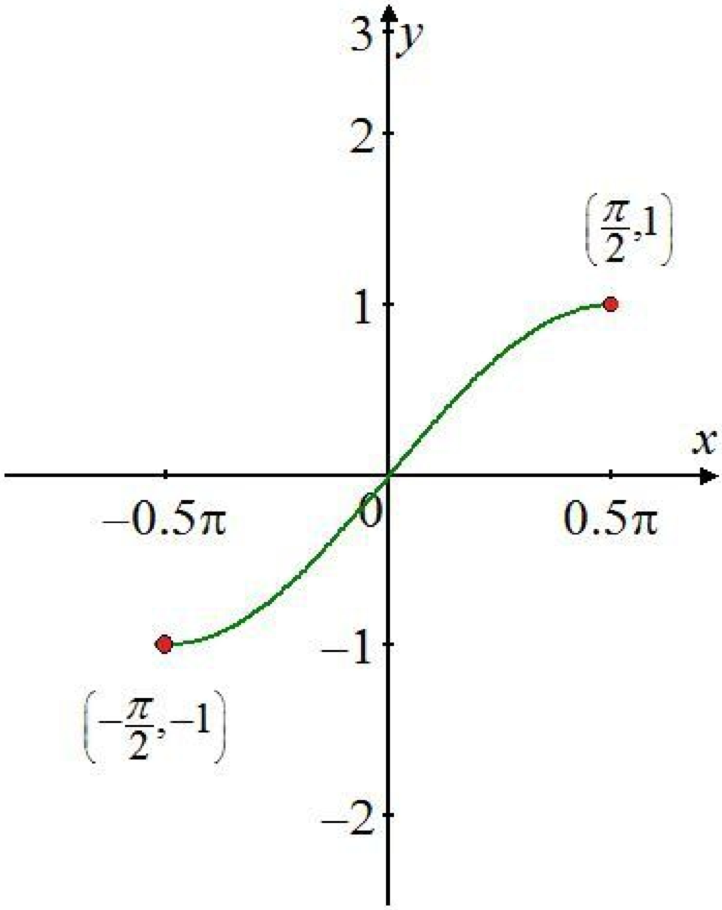
There are no local maximum and minimum values of f

And the absolute maximum of f is $\boxed{f\left(\frac{\pi}{2}\right)=1}$.

Chapter 3 Applications of Differentiation Exercise 3.1 21E

By use the graph of $f(x)=\sin x, -\frac{\pi}{2}\le x\le\frac{\pi}{2}$, to find the absolute and local maximum and minimum values of f.

The sketch of the graph of f in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ is shown below:



The absolute maximum or absolute minimum occurs at the end points of the interval.

And local maximum and local minimum occurs between the end points of the interval.

From the graph, observe that the absolute minimum of f is,

$$f\left(-\frac{\pi}{2}\right)=\boxed{-1}$$

And the absolute maximum of f is,

$$f\left(\frac{\pi}{2}\right)=\boxed{1}$$

And observe that, there is no minimum and maximum occurs between the end points.

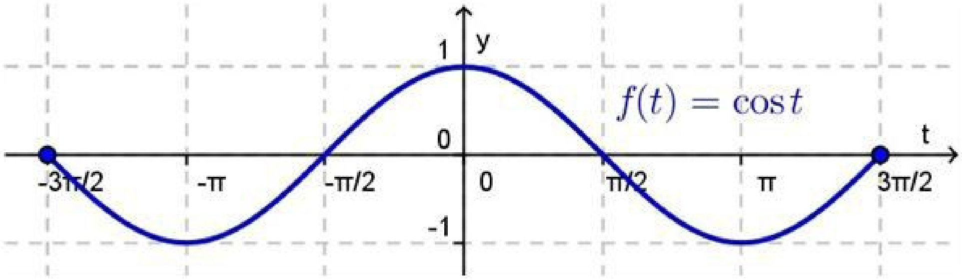
So, there are no local maximum, and local minimum for the function f.

Chapter 3 Applications of Differentiation Exercise 3.1 22E

Consider the function.

$f(t)=\cos t, \frac{-3\pi}{2}\le t\le\frac{3\pi}{2}$

Sketch the graph of the function as follows:



Since f is continuous on $\left[\frac{-3\pi}{2},\frac{3\pi}{2}\right]$, follow the closed Interval method.

According to the Closed Interval Method, find the absolute maximum and minimum values of a continuous function f on a closed interval, $[a,b]$.

Find the values of f at the critical number of f in (a,b).

Find the values of f at the end points of the interval.

The largest of the values from steps 1 and 2 is the absolute maximum value.

The smallest of these values is the absolute minimum value.

Find the derivative of the function.

$$f(t)=\cos t$$
$$f'(t)=-\sin t$$

Since $f'(t)$ exists for all t, the critical numbers of f occur when $f'(t)=0$.

$f'(t)=-\sin t=0$ When $t=-\pi$ and $t=\pi,\ t=0$

The critical points are $t=-\pi$ and $t=\pi$ lies in the given interval.

The value of f at the critical numbers is as follows:

$$f(-\pi)=\cos(-\pi)$$
$$=-1$$

$$f(\pi)=\cos\pi$$
$$=-1$$

$$f(0)=\cos 0$$
$$=1$$

The value of f at the end points of the interval is as follows:

$$f\left(\frac{-3\pi}{2}\right)=\cos\left(\frac{-3\pi}{2}\right)$$
$$=0$$

$$f\left(\frac{3\pi}{2}\right)=\cos\left(\frac{3\pi}{2}\right)$$
$$=0$$

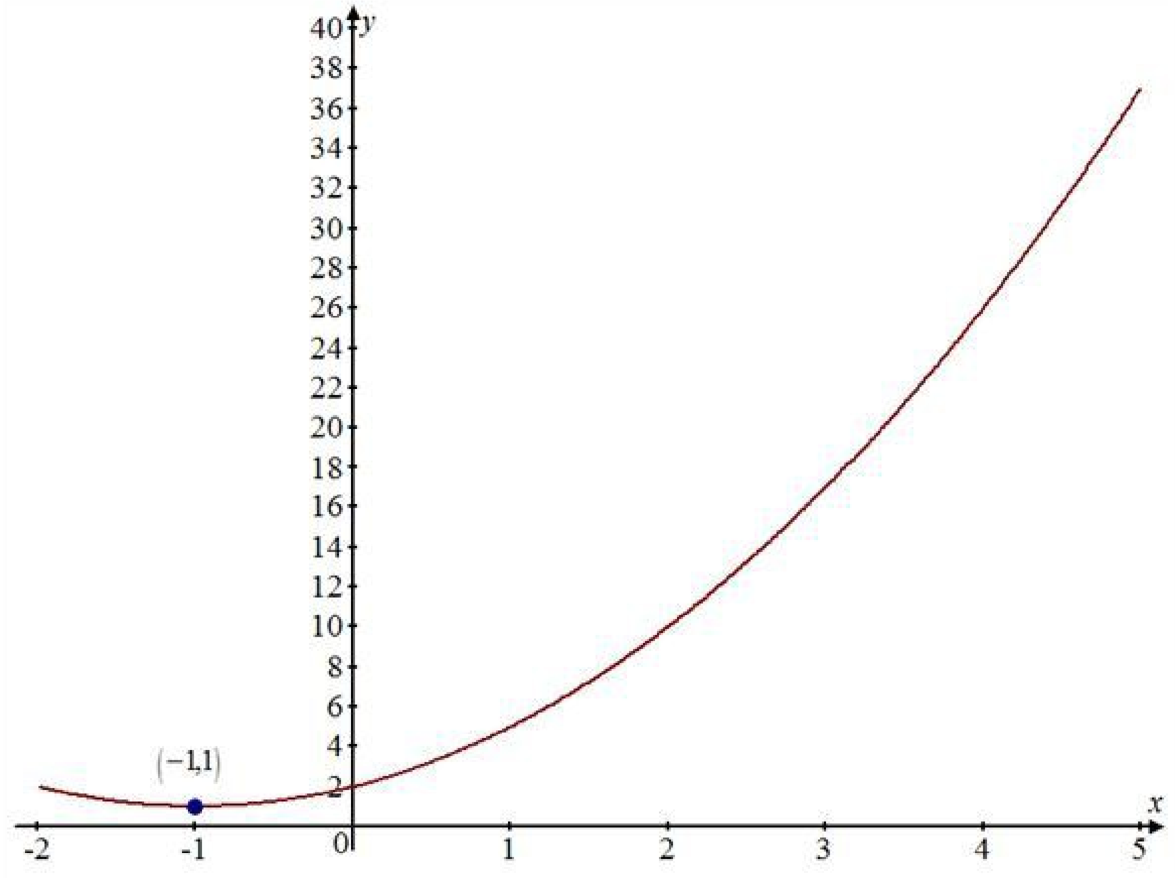
Compare these four numbers and observe that the absolute maximum value is $\boxed{f(0)=1}$ and the absolute minimum is $\boxed{f(-\pi)=-1}$ and $\boxed{f(\pi)=1}$.

From the graph, it clear that $\boxed{f(0)=1}$ is a local maximum and there is no local minimum.

Chapter 3 Applications of Differentiation Exercise 3.1 23E

By use the graph of $f(x)=1+(x+1)^2, -2 \le x < 5$, to find the absolute and local maximum and minimum values of f.

The sketch of the graph of f in the interval $[-2,5)$ is shown below:



The absolute maximum or absolute minimum occurs at the end points of the interval.

And local maximum and local minimum occurs between the end points of the interval.

But from the graph, observe that the minimum of f does not occurs at the end point of the interval.

So absolute minimum and local minimum are coincides.

It is given as,

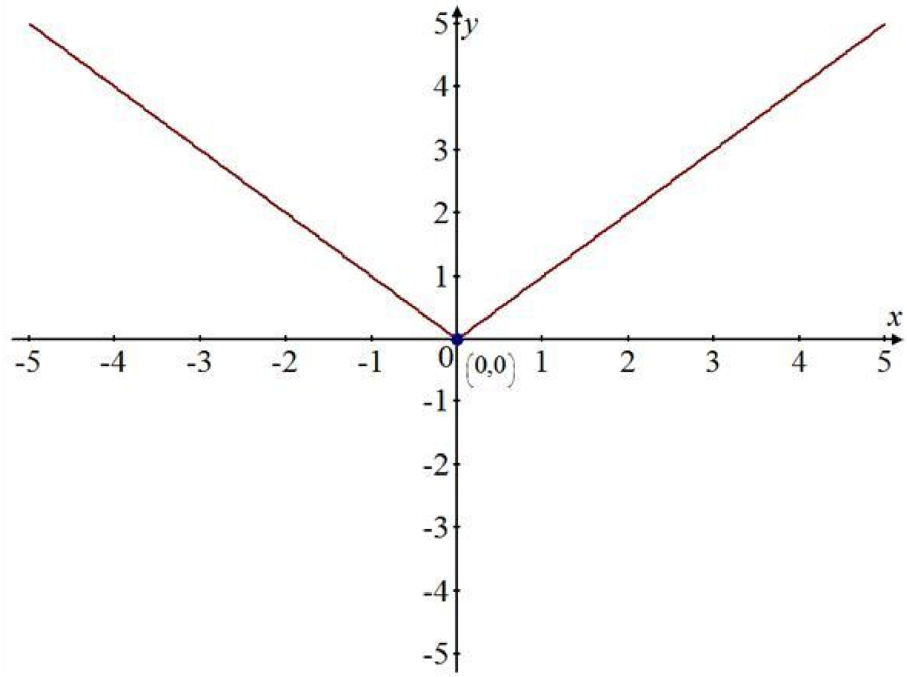
$$f(-1)=\boxed{1}$$

Also, from the graph, observe that the absolute maximum and local maximum does not exist.

Chapter 3 Applications of Differentiation Exercise 3.1 24E

By use the graph of $f(x)=|x|$, to find the absolute and local maximum and minimum values of f.

The sketch of the graph of f in the interval $[-5,5]$ is shown below:



The absolute maximum or absolute minimum occurs at the end points of the interval.

And local maximum and local minimum occurs between the end points of the interval.

But from the graph, observe that the minimum of f does not occurs at the end point of the interval $[-5,5]$

So the absolute minimum and the local minimum are coincides.

It is given as,

$f(0)=\boxed{0}$

Also, from the graph, observe that the absolute maximum and local maximum does not exist.

Chapter 3 Applications of Differentiation Exercise 3.1 25E

We have $f(x)=1-\sqrt{x}$

This function is defined for $x \geq 0$, so domain of the function is $[0, \infty)$.

Now we sketch the graph by reflecting the graph of $y=\sqrt{x}$ about x-axis and then shifting it 1 unit upward.

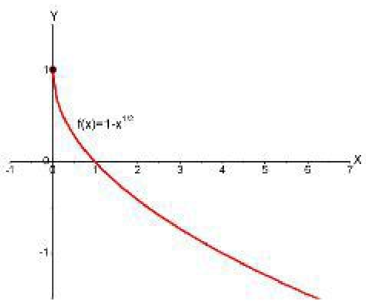


Fig.1

We see that is function is decreasing on its domain so we have Absolute maximum as $\boxed{f(0)=1}$. Also there is no local maximum.

And there is no local and absolute minimum.

Chapter 3 Applications of Differentiation Exercise 3.1 26E

We have $f(x) = 1 - x^3$

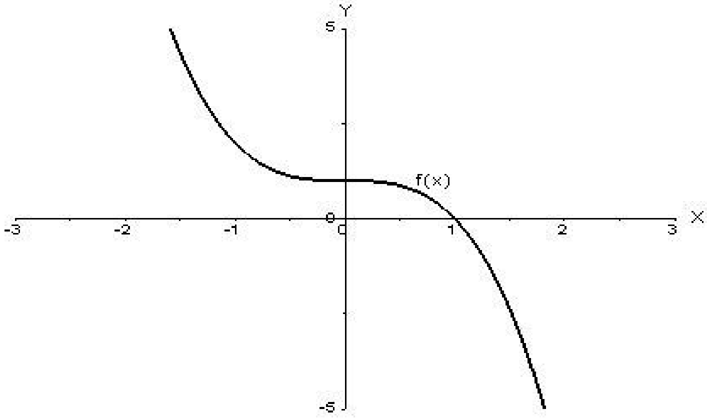


Fig. 1

We see that there is
No absolute and local maximum
And no absolute and local minimum

Chapter 3 Applications of Differentiation Exercise 3.1 27E

We sketch the graph of the function $f(x) = \begin{cases} 1-x, & \text{if } 0 \le x < 2 \\ 2x-4, & \text{if } 2 \le x \le 3 \end{cases}$

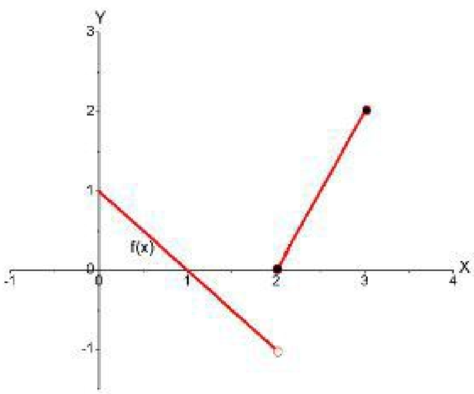


Fig.1

Absolute maximum is $\boxed{f(3) = 2}$

There is no local maximum
There is no local and absolute minimum

Chapter 3 Applications of Differentiation Exercise 3.1 28E

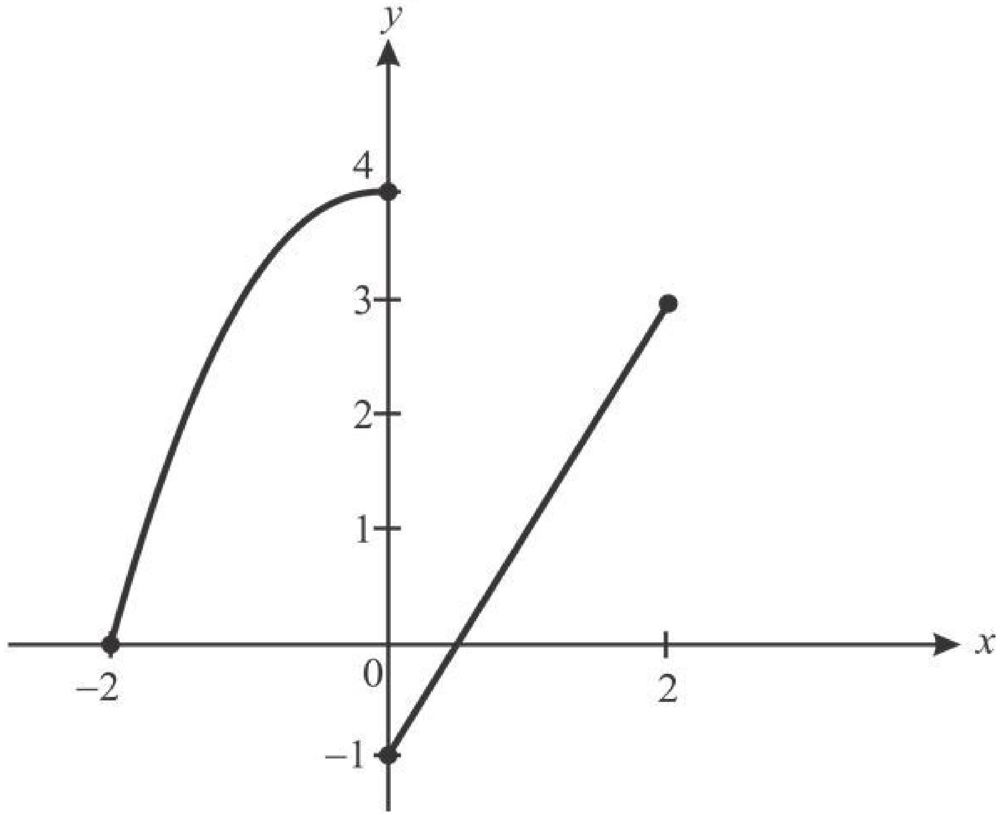
Local maxima and local minima are the largest and smallest values of function in which the value is either within a given range or on the entire domain of a function.

Consider:

$$f(x) = \begin{cases} 4 - x^2 & \text{if } -2 \le x < 0 \\ 2x - 1 & \text{if } 0 \le x \le 2 \end{cases}$$

Sketch the graph of *f* by hand and use your sketch to find the absolute and local maximum and minimum values of *f*.

The following is the graph of $f(x)=\begin{cases}4-x^2 & \text{if } -2\le x<0\\ 2x-1 & \text{if } 0\le x\le 2\end{cases}$



From the above graph, it is observed that the absolute minimum of the function is -1

At $x=0$

There is no absolute maximum, because the values get larger as *x* approaches 0 from the left, and the graph jumps down at 0.

Thus, there is no absolute maximum.

Therefore the required solution is $\boxed{-1,\text{noabsolutemaximum}}$

Chapter 3 Applications of Differentiation Exercise 3.1 29E

Given that the function is $f(x)=4+\frac{1}{3}x-\frac{1}{2}x^2$

Then $f'(x)=\frac{1}{3}-\frac{1}{2}(2x)$

$=\frac{1}{3}-x$

If $f'(x)=0$ then $\frac{1}{3}-x=0$

$\Rightarrow \quad x=\frac{1}{3}$

$\boxed{\text{Critical Point is } \frac{1}{3}}$

Chapter 3 Applications of Differentiation Exercise 3.1 30E

Consider the function $f(x)=x^3+6x^2-15x$

Now to find the critical numbers of the given function.

Critical number:

A number c in the domain of f is called a critical number of f if $f'(c)=0$ or $f'(c)$ does not exist.

Consider $f(x)=x^3+6x^2-15x$

Differentiate with respect to 'x', obtain that

$$f'(x)=3x^2+12x-15$$

Take $f'(x)=0$

$$\Rightarrow 3x^2+12x-15=0$$
$$\Rightarrow \quad x^2+4x-5=0$$
$$\Rightarrow \quad (x-1)(x+5)=0$$
$$\Rightarrow x-1=0 \quad \text{or} \quad x+5=0$$
$$\Rightarrow \quad x=1 \quad \text{or} \quad x=-5$$

Hence, the critical numbers of the given function are $\boxed{x=1,-5}$.

Chapter 3 Applications of Differentiation Exercise 3.1 31E

Consider the function,

$$f(x)=2x^3-3x^2-36x$$

Critical number:

A critical number a of a function is a number in the domain of the function at which either $f'(a)=0$ or $f'(a)=0$ does not exists.

Differentiate f and set to zero to identify the critical numbers.

$$f(x)=2x^3-3x^2-36x$$

$$f'(x)=\frac{d}{dx}\left(2x^3-3x^2-36x\right)$$
$$=2\left(3x^2\right)-3(2x)-36(1)$$
$$=6x^2-6x-36$$

Set $f'(x)=0$

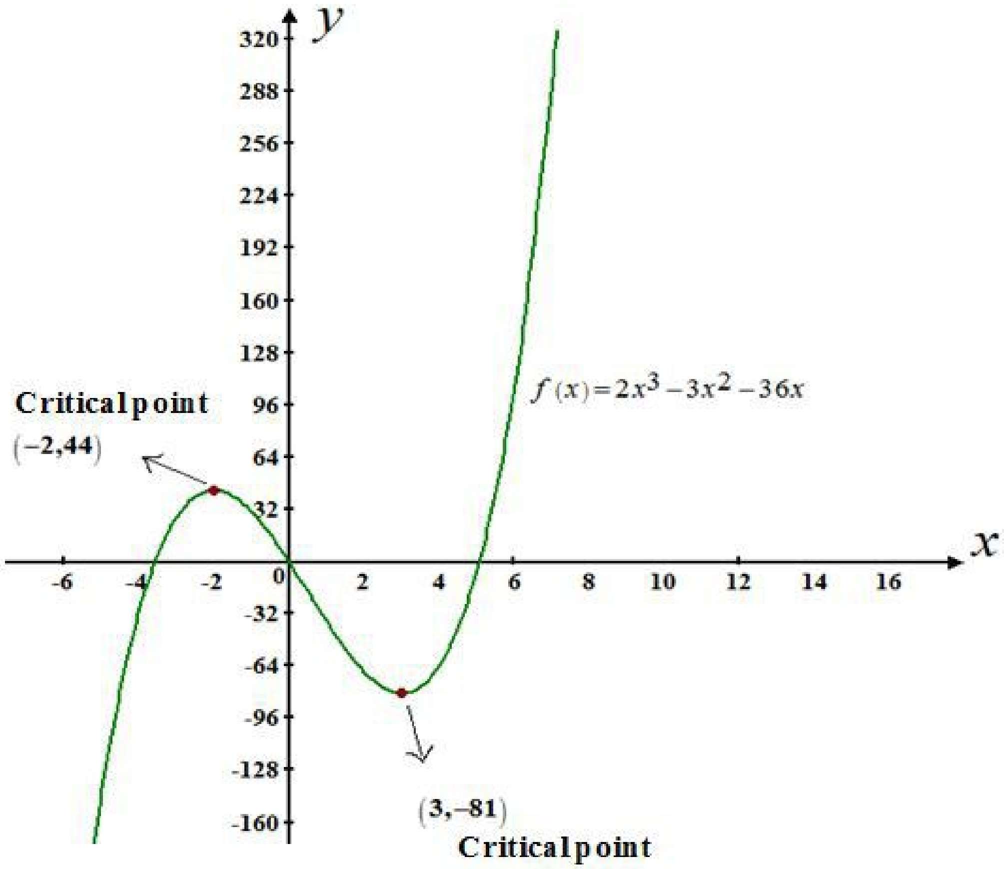
$$6x^2-6x-36=0$$
$$6\left(x^2-x-6\right)=0$$
$$\left(x^2-x-6\right)=0$$

Factor the quadratic equation and solve for x.

$$\left(x^2-x-6\right)=0$$
$$x^2-3x+2x-6=0$$
$$x(x-3)+2(x-6)=0$$
$$(x+2)(x-3)=0$$
$$x=-2 \text{ or } x=3$$

Therefore the critical numbers are $\boxed{x=-2}$ and $\boxed{x=3}$.

See the behavior of $f(x)$ at the critical points $x=-2$ and $x=3$.



Chapter 3 Applications of Differentiation Exercise 3.1 32E

Given that the function is $f(x)=2x^3+x^2+2x$

Then $f'(x)=6x^2+2x+2$

If $f'(x)=0$

Then $6x^2+2x+2=0$

$\Rightarrow \quad 3x^2+x+1=0$

$$x=\frac{-1\pm\sqrt{1-4(1)3}}{2}$$

$$=\frac{-1\pm\sqrt{1-12}}{2}$$

$$=\frac{-1\pm\sqrt{11}i}{2} \qquad \left[\because \sqrt{-1}=i\right]$$

Neither of these are linear. Thus, there are no critical points.

$\boxed{\text{No Critical Point}}$

Chapter 3 Applications of Differentiation Exercise 3.1 33E

Consider the function,

$g(t)=t^4+t^3+t^2+1$

Critical number:

A critical number a of a function is a number in the domain of the function at which either $f'(a)=0$ or $f'(a)=0$ does not exists.

To find the critical points, first differentiate g with respect to t and set to zero.

$$g(t)=t^4+t^3+t^2+1$$

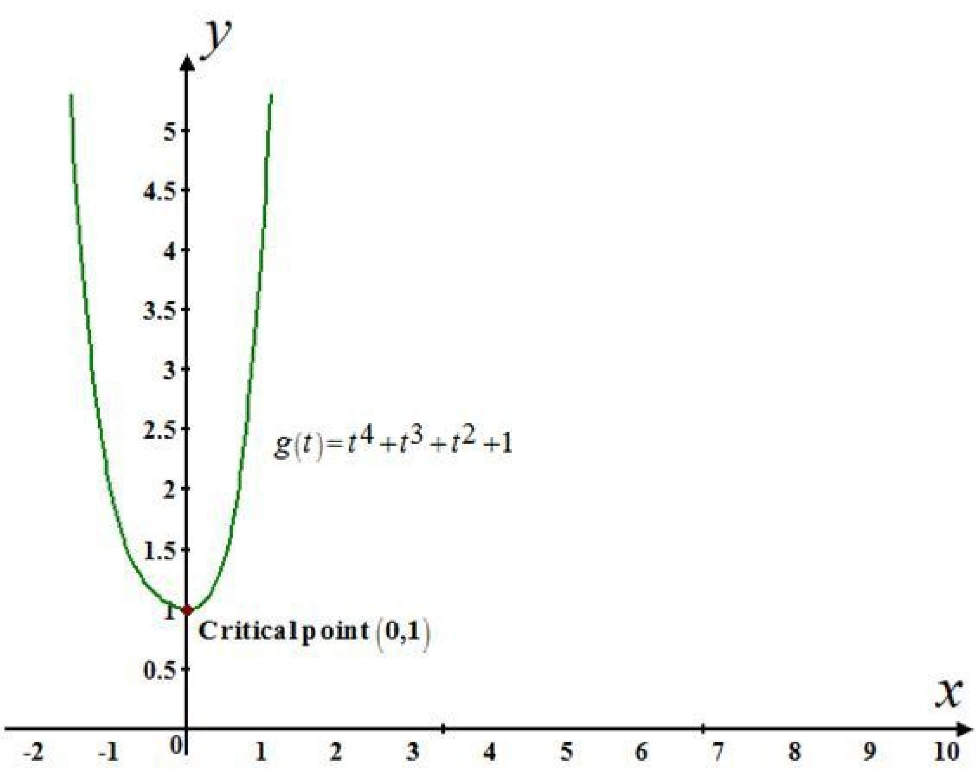
$$g'(t)=4t^3+3t^2+2t+0$$
$$=4t^3+3t^2+2t$$

Set $g(t)=0$, and solve for t.

$$4t^3+3t^2+2t=0$$
$$t(4t^2+3t+2)=0$$
$$t=0 \quad (4t^2+3t+2\neq 0 \text{ for any real value of } t)$$

Therefore the only critical point of the given function is $\boxed{t=0}$.

See the behavior of g at the critical point $t=0$:



Chapter 3 Applications of Differentiation Exercise 3.1 34E

Consider the function $g(t)=|3t-4|$.

Rewrite the above function as,

$$g(t)=\begin{cases}3t-4, & 3t-4>0\\ -(3t-4), & 3t-4<0\end{cases}$$
$$=\begin{cases}3t-4, & t>4/3\\ -3t+4, & t<4/3\end{cases}$$

Find the critical numbers of the function .

The critical numbers of a function are the numbers, where the derivative is either undefined or equal to 0.

Now, in each of the two cases, the derivative can be taken separately.

$$g'(t)=\begin{cases}3, & t>4/3\\ -3, & t<4/3\end{cases}$$

Therefore, the derivative of a function never equals to 0.

Find the derivative at $t = \frac{4}{3}$.

To find right derivative, consider $t = \frac{4}{3} + h$,

If $h \to 0^+$, then $t \to \frac{4}{3}$.

$$\lim_{h\to 0^+} \frac{g\left(\frac{4}{3}+h\right) - g\left(\frac{4}{3}\right)}{h} = \lim_{h\to 0^+} \frac{\left|3\left(\frac{4}{3}+h\right)-4\right| - \left|3\left(\frac{4}{3}\right)-4\right|}{h}$$
$$= \lim_{h\to 0^+} \frac{|4+3h-4| - |0|}{h}$$

$$= \lim_{h\to 0^+} \frac{|3h|}{h}$$
$$= \lim_{h\to 0^+} \frac{3h}{h}$$
$$= 3$$

To find left derivative, consider $t = \frac{4}{3} - h$

If $h \to 0^-$, then $t \to \frac{4}{3}$

$$\lim_{h\to 0^-} \frac{g\left(\frac{4}{3}-h\right) - g\left(\frac{4}{3}\right)}{h} = \lim_{h\to 0^-} \frac{\left|3\left(\frac{4}{3}-h\right)-4\right| - \left|3\left(\frac{4}{3}\right)-4\right|}{h}$$
$$= \lim_{h\to 0^-} \frac{|4-3h-4| - |0|}{h}$$

$$= \lim_{h\to 0^-} \frac{|-3h|}{h}$$
$$= |-3| \lim_{h\to 0^-} \frac{|h|}{h}$$
$$= 3\left(\frac{-1}{1}\right)$$
$$= -3$$

Since, the limits are different from the left and right hand sides, $g'\left(\frac{4}{3}\right)$ does not exist.

Therefore, the only critical number is $\boxed{t = \frac{4}{3}}$.

Chapter 3 Applications of Differentiation Exercise 3.1 35E

Consider:

$$g(y) = \frac{y-1}{y^2 - y + 1}.$$

Find the critical numbers of the function.

Critical number:

A critical number of a function *f* is a number *c* in the domain of *f* such that $f'(c) = 0$ or does not exist.

Now find $g'(y)$ to find critical numbers of the function.

$$g(y)=\frac{y-1}{y^2-y+1}$$

$$g'(y)=\frac{(y^2-y+1)(1)-(y-1)(2y-1)}{(y^2-y+1)^2} \qquad \frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vdu-udv}{v^2}$$

$$g'(y)=\frac{y^2-y+1-(2y^2-2y-y+1)}{(y^2-y+1)^2}$$

$$g'(y)=\frac{y^2-y+1-(2y^2-3y+1)}{(y^2-y+1)^2}$$

The above is simplified to,

$$g'(y)=\frac{y^2-y+1-2y^2+3y-1}{(y^2-y+1)^2}$$

$$g'(y)=\frac{-y^2+2y}{(y^2-y+1)^2}$$

$$g'(y)=\frac{y(2-y)}{(y^2-y+1)^2}$$

Here, $g'(y)=0$ when, $y(2-y)=0$.

Then,

$$y=0 \text{ or } 2-y=0$$
$$y=0 \text{ or } y=2$$

The roots of $(y^2-y+1)^2$ in $g'(y)=\frac{y(2-y)}{(y^2-y+1)^2}$ are not real.

Therefore, the critical numbers are $\boxed{y=0,2}$.

Chapter 3 Applications of Differentiation Exercise 3.1 36E

Consider the function

$$h(p)=\frac{p-1}{p^2+4}$$

Need to find the critical numbers.

In general,

If a function has a critical number at *c*, then $f'(c)=0$ or $f'(c)$ does not exists.

So, first find the derivative of the function $h(p)=\frac{p-1}{p^2+4}$.

The function is a fraction. So use the quotient rule of derivatives to find the derivative of the function.

The quotient rule of derivatives recollects that

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-g'(x)f(x)}{\left[g(x)\right]^2}$$

From the original function,

$$f(x)=p-1$$
$$g(x)=p^2+4$$

Differentiation as follows

$$h(p)=\frac{p-1}{p^2+4}$$

Differentiate both sides with respect to p

$$h'(p)=\frac{(p^2+4)\left[\frac{d}{dx}(p-1)\right]-(p-1)\left[\frac{d}{dx}(p^2+4)\right]}{(p^2+4)^2}$$ Use the quotient rule

$$=\frac{(p^2+4)(1)-(p-1)(2p)}{(p^2+4)^2}$$ Simplify

$$=\frac{p^2+4-(2p^2-2p)}{(p^2+4)^2}$$

$$=\frac{p^2+4-2p^2+2p}{(p^2+4)^2}$$

$$=\frac{-p^2+2p+4}{(p^2+4)^2}$$

The derivative function is defined for all real numbers. Because the denominator is not zero for any real number

To find the critical points, equate the differential equation to zero and solve for p.

$$h'(p)=0$$

$$\frac{-p^2+2p+4}{(p^2+4)^2}=0$$

$$-p^2+2p+4=0$$
$$p^2-2p-4=0$$

Use the quadratic roots formula to find the root s of the equation $p^2-2p-4=0$.

The quadratic roots formula recollects that

The roots for $ax^2+bx+c=0$ are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

From our equation $p^2-2p-4=0$,

$$a=1, b=-2, c=-4$$

Substitute these values in $a=1, b=-2, c=-4$

$$p=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$$
$$=\frac{2\pm\sqrt{(4+16)}}{2}$$
$$=\frac{2\pm\sqrt{20}}{2}$$
$$=\frac{2\pm 2\sqrt{5}}{2}$$
$$=1\pm\sqrt{5}$$

From the definition of critical numbers, $1\pm\sqrt{5}$ are the critical numbers to the function.

Therefore, the critical numbers for the function $h(p)=\frac{p-1}{p^2+4}$ are

$\boxed{1+\sqrt{5}=3.24}, \boxed{1-\sqrt{5}=-1.24}$

Chapter 3 Applications of Differentiation Exercise 3.1 37E

A critical number of a function $h(t)$ is a number c in the domain of $h(t)$ such that either $h'(c)=0 \text{ or } h'(c)$ does not exist.

Consider the function:

$$h(t)=t^{\frac{3}{4}}-2t^{\frac{1}{4}}$$

Find the derivative of the above function:

$$h'(t)=\frac{3}{4}t^{\frac{3}{4}-1}-\frac{1}{4}\times 2t^{\frac{1}{4}-1}$$
$$=\frac{3}{4}t^{-\frac{1}{4}}-\frac{1}{2}t^{-\frac{3}{4}}$$
$$=\frac{3t^{-\frac{1}{4}}-2t^{-\frac{3}{4}}}{4}$$
$$=\frac{3}{4t^{\frac{1}{4}}}-\frac{1}{2t^{\frac{3}{4}}}$$

Use the definition of critical number and put $h'(t)=0$:

$$h'(t)=0$$

$$\frac{3t^{-\frac{1}{4}}-2t^{-\frac{3}{4}}}{4}=0$$

$$3t^{-\frac{1}{4}}-2t^{-\frac{3}{4}}=0$$

$$3t^{-\frac{1}{4}}=2t^{-\frac{3}{4}}$$

Solve the above equation for the value of t:

$$\frac{t^{-\frac{1}{4}}}{t^{-\frac{3}{4}}}=\frac{2}{3}$$

$$t^{-\frac{1}{4}+\frac{3}{4}}=\frac{2}{3}$$

$$t^{\frac{1}{2}}=\frac{2}{3}$$

Also, consider that the derivative does not exist for:

$$t=0$$

Square on both sides and continue further:

$$\left(t^{\frac{1}{2}}\right)^2=\left(\frac{2}{3}\right)^2$$

$$t=\frac{4}{9}$$

Hence, the critical numbers of the function are $\boxed{0,\frac{4}{9}}$.

Chapter 3 Applications of Differentiation Exercise 3.1 38E

We have $g(x)=x^{1/3}-x^{-2/3}$

Then by using power rule differentiate the function $g(x)$

$$g'(x)=\frac{1}{3}x^{1/3-1}+\frac{2}{3}x^{-2/3-1}$$

$$=\frac{1}{3}x^{-2/3}+\frac{2}{3}x^{-5/3}$$

$$=\frac{1}{3x^{2/3}}+\frac{2}{3x^{5/3}}$$

$$g'(x)=\frac{x+2}{3x^{5/3}}$$

$g'(x)=0$ When $x+2=0$ or $x=-2$

$g'(x)$ Does not exist when $x=0$ but $x=0$ is not in domain of $g(x)$

So the Critical numbers is $\boxed{-2}$

Chapter 3 Applications of Differentiation Exercise 3.1 39E

Consider the function $f(x)=x^{4/5}(x-4)^2$.

To find the critical points, differentiate the function and equate it to zero.

The derivative of the function is,

$$f(x)=x^{4/5}(x-4)^2$$

Apply the product rule and power rule to the function,

$$\begin{aligned} f'(x) &= \left(x^{4/5}(x-4)^2\right)' \\ &= x^{4/5}\left((x-4)^2\right)' + (x-4)^2\left(x^{4/5}\right)' \\ &= x^{4/5}\left(2(x-4)\right) + (x-4)^2\left(\frac{4}{5}x^{\frac{4}{5}-1}\right) \\ &= (x-4)\left(2x^{4/5} + (x-4)\left(\frac{4}{5x^{\frac{1}{5}}}\right)\right) \quad \text{(Take out the common terms)} \end{aligned}$$

">

Continuous from the previous step,

$$\begin{aligned} f'(x) &= (x-4)\left(\frac{10x^{\frac{4}{5}}\cdot x^{\frac{1}{5}} + (x-4)(4)}{5x^{\frac{1}{5}}}\right) \\ &= (x-4)\left(\frac{10x+4x-16}{5x^{\frac{1}{5}}}\right) \\ &= (x-4)\left(\frac{14x-16}{5x^{\frac{1}{5}}}\right) \\ &= \frac{2(7x-8)(x-4)}{5x^{\frac{1}{5}}} \end{aligned}$$

Hence, the derivative of the function is $\boxed{f'(x)=\frac{2(7x-8)(x-4)}{5x^{\frac{1}{5}}}}$.

The critical points of the function is,

$$\begin{aligned} f'(x) &= 0 \\ \frac{2(7x-8)(x-4)}{5x^{\frac{1}{5}}} &= 0 \\ 2(7x-8)(x-4) &= 0 \\ x &= \frac{8}{7} \text{ and } 4 \end{aligned}$$

But, the derivative does not exist at $x=0$. So, the value $x=0$ will include in the critical points.

Hence, the critical points of the function $f(x)=x^{4/5}(x-4)^2$ are $\boxed{x=0,\frac{8}{7} \text{ and } 4}$.

Chapter 3 Applications of Differentiation Exercise 3.1 40E

Now, for the critical points set $g'(x)=0$:

$$\frac{x+2}{3x^{5/3}}=0$$

The above function can be zero when,

$$x+2=0$$

Or

$$x=-2$$

Also, $g'(x)$ Does not exist when x = 0but x =0 is not in domain of g(x).

Hence the critical number of the given function is $\boxed{-2}$.

Chapter 3 Applications of Differentiation Exercise 3.1 41E

We have $f(\theta) = 2\cos\theta + \sin^2\theta$

Then $f'(\theta) = -2\sin\theta + 2\sin\theta\cos\theta$

Or $f'(\theta) = 2\sin\theta(\cos\theta - 1)$

Now $f'(\theta) = 0$ when $\sin\theta = 0$ or $(\cos\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$ or $\cos\theta = 1$

$\Rightarrow \theta = n\pi$ or $\theta = 2n\pi$

Thus critical points are $\boxed{\theta = n\pi}$

Where n is an integer.

Chapter 3 Applications of Differentiation Exercise 3.1 42E

Consider the function:

$$g(x) = \sqrt{1-x^2}$$

Determine the derivative of the above function:

$$g'(x) = \frac{1}{2}(1-x^2)^{-1/2}(-2x)$$
$$= -x(1-x^2)^{-1/2}$$
$$= \frac{-x}{\sqrt{1-x^2}}$$

Substitute the value $g'(x) = 0$ as shown below:

$$g'(x) = 0$$
$$\frac{-x}{\sqrt{1-x^2}} = 0$$
$$-x = 0$$
$$x = 0$$

Also, consider the equation:

$$1 - x^2 = 0$$
$$x^2 = 1$$
$$x = -1, 1$$

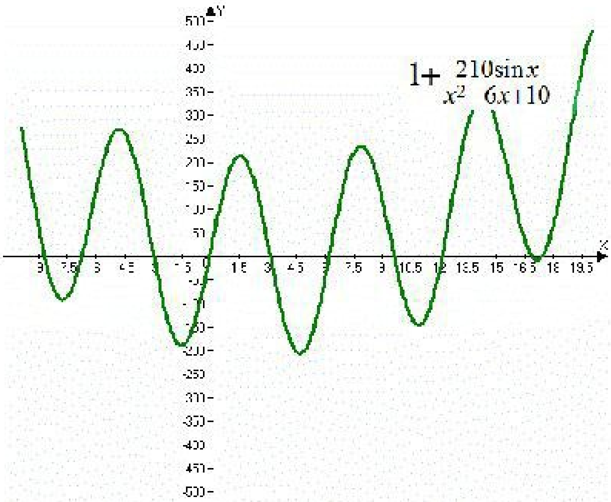
Hence, the critical number of the function is $\boxed{-1, 0 \text{ and } 1}$.

Chapter 3 Applications of Differentiation Exercise 3.1 43E

If f has a local maximum or minimum at c, then c is a critical number of f

Given the derivative is $f'(x) = 1 + \dfrac{210\sin x}{x^2 - 6x + 10}$

The Graph of the above function is



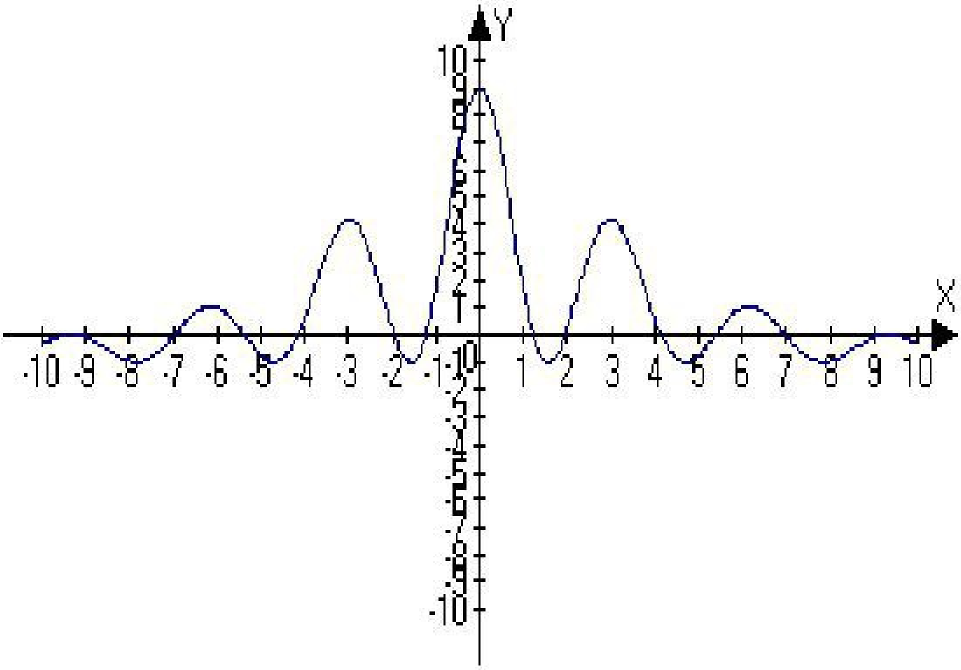
Chapter 3 Applications of Differentiation Exercise 3.1 44E

if f ' (a) = 0, then we follow that 'a' is the critical value of f.

so, from the given f ' (x) , we observe that howmany times f ' crosses x axis or f ' image is becoming 0 and at what values on x axis.

we can say that those are the critical values of the given function f.

for, we use the following graph to detect the critical values of f.



there are 14 times the curve crossed x axis.

i.e. f ' (x) = 0 for 14 times in the radian interval [-10,10].

∴ in this interval the given function f has 10 critical values.

but , we can see that the graph is becoming flatter and parallel to x axis beyond this interval.

∴ the given function has few more critical values beyond this interval.

Chapter 3 Applications of Differentiation Exercise 3.1 45E

Given that the function is $f(x)=12+4x-x^2,\quad [0,5]$

Then $f'(x)=4-2x$

If $f'(x)=0$

Then $4-2x=0$

$\Rightarrow \quad x=2$

$$f(2)=12+4(2)-2^2$$
$$=12+8-4$$
$$=12+4$$
$$=16$$

Applying the closed interval method,

We find that $f(0)=12$,

$$\begin{aligned} f(5) &= 12+4(5)-5^2 \\ &= 12+20-25 \\ &= 12-5 \\ &= 7 \end{aligned}$$

$$\boxed{\begin{array}{l} f(2)=16 \text{ is the absolute maximum value.} \\ f(5)=7 \text{ is the absolute minimum value.} \end{array}}$$

Chapter 3 Applications of Differentiation Exercise 3.1 46E

Consider the function $f(x)=5+54x-2x^3$.

The objective is to find the absolute maximum and absolute minimum of *f* on the interval $[0,4]$.

The steps to find the absolute maximum and minimum values of a continuous function f on a closed interval $[a,b]$:

1. Find the values of f at the critical numbers of f in (a,b).

2. Find the values of f at the endpoints of the interval.

3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Critical numbers are numbers where the derivative is either undefined or equal to 0.

Differentiate $f(x)$ with respect to *x* to get,

$$f'(x)=54-6x^2$$

For critical points, set $f'(x)=0$.

$$\begin{aligned} 54-6x^2 &= 0 \\ 6x^2 &= 54 \\ x^2 &= 9 \\ x &= \pm 3 \\ x &= 3 \in (0,4) \end{aligned}$$

Only the critical number $x=3$ is inside of the interval $(0,4)$.

The value of *f* at the critical number is,

$$\begin{aligned} f(3) &= 5+54(3)-2(3)^3 \\ &= 5+162-54 \\ &= 113 \end{aligned}$$

The values of f at the endpoints are,

$$\begin{aligned} f(0) &= 5+54(0)-2(0)^3 \\ &= 5+0-0 \\ &= 5 \end{aligned}$$

$$\begin{aligned} f(4) &= 5+54(4)-2(4)^3 \\ &= 5+216-128 \\ &= 93 \end{aligned}$$

Therefore, the absolute maximum value is $\boxed{f(3)=113}$ and the absolute minimum value is $\boxed{f(0)=5}$.

Chapter 3 Applications of Differentiation Exercise 3.1 47E

We have $f(x) = 2x^3 - 3x^2 - 12x + 1$

Then $f'(x) = 6x^2 - 6x - 12$

For critical points, $f'(x) = 0$

$\Rightarrow \quad 6(x^2 - x - 2) = 0$

$\Rightarrow \quad 6(x-2)(x+1) = 0$

$\Rightarrow \quad x = 2, -1$ are the critical numbers.

Since the function is a polynomial so it is continuous everywhere
So we can use closed Interval method on the interval [-2, 3]

At $x = 2 \Rightarrow f(2) = 16 - 12 - 24 + 1 = -19$

At $x = -1 \Rightarrow f(-1) = -2 - 3 + 12 + 1 = 8$

At $x = -2 \Rightarrow f(-2) = -16 - 12 + 24 + 1 = -3$

At $x = 3 \Rightarrow f(3) = 54 - 27 - 36 + 1 = -8$

So,

Absolute maximum $\Rightarrow$ $\boxed{f(-1) = 8}$

Absolute minimum $\Rightarrow$ $\boxed{f(2) = -19}$

Chapter 3 Applications of Differentiation Exercise 3.1 48E

Given that the function is $f(x) = x^3 - 6x^2 + 5 \quad [-3,5]$

Then $f'(x) = 3x^2 - 12x$

If $f'(x) = 0$

Then $3x^2 - 12x = 0$

$\Rightarrow 3x(x-4) = 0$

$\Rightarrow \quad x = 0, 4.$

Applying the closed Interval method, we find that

$f(0) = 5,$

$f(4) = 4^3 - 6 \cdot 4^2 + 5$
$= 64 - 96 + 5$
$= -27$

$f(-3) = (-3)^3 - 6(-3)^2 + 5$
$= -27 - 54 + 5$
$= -76$

And

$f(5) = 125 - 150 + 5$
$= -25 + 5$
$= -20$

$\boxed{f(0) = 5 \text{ is the absolute maximum value and } f(-3) = -76 \text{ is the absolute minimum.}}$

Chapter 3 Applications of Differentiation Exercise 3.1 49E

Given that the function is $f(x) = 3x^4 - 4x^3 - 12x^2 + 1, \quad [-2,3]$

Then $f'(x) = 12x^3 - 12x^2 - 24x$

If $f'(x) = 0$

Then $12x^3 - 12x^2 - 24x = 0$

$x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

$x = 0, (x-2)(x+1) = 0$

$\Rightarrow \quad x = 0, 2, -1$

If $x=0$ then $f(0)=1$

If $x=2$ then $f(2)=48-32-48+1$
$=-31$

If $x=-1$ then $f(-1)=3+4-12+1$
$=8-12$
$=-4$

If $x=-2$ then $f(-2)=48+32-48+1$
$=33$

If $x=3$ then $f(3)=243-108-108+1$
$=28$

$\boxed{f(-2)=33 \text{ is the absolute maximum and } f(2)=-31 \text{ is the absolute minimum.}}$

Chapter 3 Applications of Differentiation Exercise 3.1 50E

We have $f(x)=(x^2-1)^3$

Then $f'(x)=3(x^2-1)^2(2x)$ [Chain rule]

$= 6x(x^2-1)^2$

Now $f'(x)=0$ when $x=0$ or $x=\pm1$

Critical numbers are $\{-1,0,1\}$

Since $f(x)=(x^2-1)^3$ is continuous on [-1, 2], so we can use Closed Interval Method

$f(0)=-1$

$f(-1)=0$

$f(1)=0$

$f(2)=27$

Thus

Absolute maximum $\Rightarrow$ $\boxed{f(2)=27}$

Absolute minimum $\Rightarrow$ $\boxed{f(0)=-1}$

Chapter 3 Applications of Differentiation Exercise 3.1 51E

Given that the function is $f(x)=x+\frac{1}{x}$, $[0.2, 4]$

Then $f'(x)=1-\frac{1}{x^2}$

If $f'(x)=0$

Then $1-\frac{1}{x^2}=0$

$\Rightarrow \quad x^2=1$

$\Rightarrow \quad x=\pm1$

Since $-1\notin[0.2,4]$, so $f(1)=1+1=2$,

$f(0.2)=0.2+\frac{1}{0.2}$
$=0.2+5$
$=5.2$

$f(4)=4+\frac{1}{4}$
$=4.25$

$\boxed{f(0.2)=5.2 \text{ is the absolute maximum and } f(1)=2 \text{ is the absolute minimum.}}$

Chapter 3 Applications of Differentiation Exercise 3.1 52E

Given that the function is $f(x)=\frac{x}{x^2-x+1}$, $[0,3]$

Then $f'(x)=\frac{(x^2-x+1)\frac{d}{dx}(x)-x\frac{d}{dx}(x^2-x+1)}{(x^2-x+1)^2}$

$=\frac{(x^2-x+1)-x(2x-1)}{(x^2-x+1)^2}$

$=\frac{x^2-x+1-2x^2+x}{(x^2-x+1)}$

$=\frac{-x^2+1}{(x^2-x+1)^2}$

if $f'(x)=0$ then $\frac{-x^2+1}{(x^2-x+1)^2}=0$

$\Rightarrow 1-x^2=0$

$\Rightarrow \quad x=\pm 1$

Since $-1\notin[0,3]$,

So $f(1)=\frac{1}{1^2-1+1}$

$=1$,

$f(0)=\frac{0}{1}$

$=0$

$f(3)=\frac{3}{3^2-3+1}$

$=\frac{3}{9-3+1}$

$=\frac{3}{7}$

$f(1)=1$ is the absolute maximum and $f(0)=0$ is the absolute minimum.

Chapter 3 Applications of Differentiation Exercise 3.1 53E

Given $f(t)=t\sqrt{4-t^2}$

Or $f(t)=t(4-t^2)^{1/2}$

Then $f'(t)=t\cdot\frac{1}{2}(4-t^2)^{-1/2}(-2t)+(4-t^2)^{1/2}$ [Product and chain rules]

Or $f'(t)=-t^2(4-t^2)^{-1/2}+(4-t^2)^{1/2}$

Or $f'(t)=(4-t^2)^{-1/2}\left[-t^2+(4-t^2)\right]$

Or $f'(t)=\frac{2(2-t^2)}{\sqrt{4-t^2}}$

Now $f'(t)=0$, when $(2-t^2)=0$

$\Rightarrow \quad t=\pm\sqrt{2}$

And $f'(t)$ does not exist when $4-t^2=0$.

$\Rightarrow \quad t=\pm 2$

But $t=-2$ and $t=-\sqrt{2}$ are not in the given interval [-1, 2].

So the critical number is $t=\sqrt{2}$, as it lies in the interval (-1, 2).

Since $f(t) = t\sqrt{4-t^2}$ is continuous on [-1, 2], so we can use Closed Interval method.

Now, $f(-1) = -\sqrt{3}$

$f(\sqrt{2}) = 2$

$f(2) = 0$

Thus,

Absolute maximum $\Rightarrow$ $\boxed{f(\sqrt{2}) = 2}$

Absolute minimum $\Rightarrow$ $\boxed{f(-1) = -\sqrt{3}}$

Chapter 3 Applications of Differentiation Exercise 3.1 54E

We have $f(t) = \sqrt[3]{t}(8-t)$, $[0,8]$

Or $f(t) = t^{1/3}(8-t)$

Then $f'(t) = t^{1/3}(-1) + (8-t)\frac{1}{3}t^{-2/3}$ [Product rule]

Or $f'(t) = -t^{1/3} + (8-t)\frac{1}{3}t^{-2/3}$

Or $f'(t) = \frac{1}{3}t^{-2/3}\left[-3t + (8-t)\right]$

Or $f'(t) = \frac{4(2-t)}{3t^{2/3}}$

Now $f'(t) = \frac{4(2-t)}{3t^{2/3}} = 0$ when $(2-t) = 0 \Rightarrow t = 2$

And $f'(t) = \frac{4(2-t)}{3t^{2/3}}$ does not exist when t = 0

But 0 is the end point of the given interval [0, 8]

So $t = 2$ is a critical number in (0, 8)

Since $f(t) = t^{1/3}(8-t)$ is continuous on [0, 8], so we can use Closed Interval Method

$f(0) = 0$

$f(2) = 6.2^{1/3} = 6\sqrt[3]{2}$

$f(8) = 0$

Thus

Absolute maximum $\Rightarrow$ $\boxed{f(2) = 6\sqrt[3]{2}}$

Absolute minimum $\Rightarrow$ $\boxed{f(0) = f(8) = 0}$

Chapter 3 Applications of Differentiation Exercise 3.1 55E

Consider the function:

$$f(t) = 2\cos t + \sin 2t,\ [0, \pi/2]$$

Determine the absolute maximum and minimum of the function in the given interval.

Find the critical numbers in the interval $[0, \pi/2]$:

To find the critical points, find $f'(t)$ and set it equal to zero.

Differentiate $f(t) = 2\cos t + \sin 2t$ with respect to *t*:

$$\begin{aligned} f'(t) &= \frac{d}{dt}(2\cos t + \sin 2t) \\ &= 2\frac{d}{dt}(\cos t) + \frac{d}{dt}(\sin 2t) \\ &= 2(-\sin t) + 2\cos 2t \\ &= 2(\cos 2t - \sin t) \end{aligned}$$

To find the critical numbers, solve $f'(t)=0$:

$$\begin{aligned} 2(\cos 2t-\sin t)&=0 \\ \cos 2t-\sin t&=0 && \text{Simplify} \\ 1-2\sin^2 t-\sin t&=0 && \text{Use } \cos 2t=1-2\sin^2 t \\ 2\sin^2 t+\sin t-1&=0 && \text{Rewrite} \end{aligned}$$

$$\begin{aligned} 2\sin^2 t+2\sin t-\sin t-1&=0 \\ 2\sin t(\sin t+1)-1(\sin t+1)&=0 \quad \text{Factor} \\ (\sin t+1)(2\sin t-1)&=0 \end{aligned}$$

$$\sin t+1=0 \quad \text{or} \quad 2\sin t-1=0$$

$$\sin t=-1 \quad \text{or} \quad \sin t=\frac{1}{2}$$

$$t=-\frac{\pi}{2} \quad \text{or} \quad t=\frac{\pi}{6}$$

Observe that $-\frac{\pi}{2}$ is not included in the interval $[0,\pi/2]$.

Thus, the only critical number of the function $f(t)=2\cos t+\sin 2t$ in the interval $[0,\pi/2]$ is $\boxed{\frac{\pi}{6}}$.

Recollect, the **Closed Interval Method**

Suppose f is a continuous function on a closed interval $[a,b]$.

Then to find the absolute maximum and minimum values of f:

1. Evaluate the value of $f(x)$ at the critical points of f in (a,b).

2. Evaluate the value of $f(x)$ at the endpoints of interval.

3. Finally, conclude that the largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Evaluate the function $f(t)=2\cos t+\sin 2t$ at the critical number $\pi/6$ and at the end points of the interval $[0,\pi/2]$.

At the critical number $\pi/6$:

Substitute $\pi/6$ for t in $f(t)=2\cos t+\sin 2t$, then

$$\begin{aligned} f(\pi/6)&=2\cos(\pi/6)+\sin 2(\pi/6) \\ &=2\left(\frac{\sqrt{3}}{2}\right)+\frac{\sqrt{3}}{2} \\ &=\frac{3\sqrt{3}}{2} \end{aligned}$$

At the left endpoint 0:

Substitute 0 for t in $f(t)=2\cos t+\sin 2t$, then

$$\begin{aligned} f(0)&=2\cos(0)+\sin 2(0) \\ &=2(1)+0 \\ &=2 \end{aligned}$$

At the right endpoint $\pi/2$:

Substitute $\pi/2$ for t in $f(t)=2\cos t+\sin 2t$, then

$$\begin{aligned} f(\pi/2)&=2\cos(\pi/2)+\sin 2(\pi/2) \\ &=2(0)+0 \\ &=0 \end{aligned}$$

The above obtained results are tabulated as shown below:

Left endpoint	Critical number	Right endpoint
$f(0)=2$	$f(\pi/6)=\frac{3\sqrt{3}}{2}$ $=2.59$ **Maximum**	$f(\pi/2)=0$ **Minimum**

From the above table, observe that

1. The **absolute minimum** is obtained at the right endpoint $\pi/2$ of the interval $[0,\pi/2]$ and the minimum is $\boxed{0}$.

2. The **absolute maximum** is obtained at the critical point $\pi/6$ and the maximum is $\boxed{\frac{3\sqrt{3}}{2}}$.

Chapter 3 Applications of Differentiation Exercise 3.1 56E

Take a function f defined on the domain *D*. For a number *b* in the domain *D*:

The absolute maximum value of the function is:

$$f(b)\geq f(x) \quad \forall x \text{ in } D$$

The absolute minimum value of the function is:

$$f(b)\leq f(x) \quad \forall x \text{ in } D$$

Consider the function:

$$f(t)=t+\cot(t/2), \quad [\pi/4,7\pi/4]$$

Since f is continuous on $[\pi/4,7\pi/4]$, so follow the closed Interval Method:

$$f(t)=t+\cot(t/2)$$
$$f'(t)=1-\frac{1}{2}\times\operatorname{cosec}^2(t/2)$$

Since $f'(t)$ exists for all t, so critical points of f occur when $f'(t)=0$.

Substitute the function $f'(t)=0$:

$$f'(t)=0$$
$$1-\frac{1}{2}\times\operatorname{cosec}^2(t/2)=0$$
$$\operatorname{cosec}^2(t/2)=2$$

So, the value of *t* in the interval $[\pi/4,7\pi/4]$ is:

$$t=\frac{\pi}{2}$$
$$\approx 1.57$$
$$t=\frac{3\pi}{2}$$
$$\approx 4.71$$

The critical points are $t=\frac{\pi}{2}$ and $t=\frac{3\pi}{2}$ in the interval.

Determine the values of f at the critical number:

$$f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}+\cot\left(\frac{\pi/2}{2}\right)$$
$$=2.57$$
$$f\left(\frac{3\pi}{2}\right)=\frac{3\pi}{2}+\cot\left(\frac{3\pi}{4}\right)$$
$$=3.71$$

Determine the value of f at the endpoints of the interval:

$$f\left(\frac{\pi}{4}\right)=\frac{\pi}{4}+\cot\left(\frac{\pi}{8}\right)$$
$$=3.195$$
$$f\left(\frac{7\pi}{4}\right)=\frac{7\pi}{4}+\cot\left(\frac{7\pi}{6}\right)$$
$$=3.08$$

Comparing the above four values.

Observe that the absolute maximum value is:

$$f\left(\frac{3\pi}{2}\right)=3.71$$

Observe that the absolute minimum value is:

$$f\left(\frac{\pi}{2}\right)=2.57$$

Hence, the **absolute maximum** value is $\boxed{3.71}$ and the **absolute minimum** value is $\boxed{2.57}$.

Chapter 3 Applications of Differentiation Exercise 3.1 57E

We have $f(x)=x^a(1-x)^b$; $\quad 0\le x\le 1 \quad$ where $a,b>0$

Then $f'(x)=x^a.b(1-x)^{b-1}(-1)+(1-x)^b.ax^{a-1}$ [Product rule]

Or $f'(x)=x^{a-1}(1-x)^{b-1}\left[-bx+(1-x)a\right]$

Or $f'(x)=x^{a-1}(1-x)^{b-1}\left[-x(a+b)+a\right]$

For critical numbers, we must have $f'(c)=0$

Or, $c^{a-1}(1-c)^{b-1}\left[a-c(a+b)\right]=0$

Or, $c=0,\ 1,\ \text{or}\ \frac{a}{a+b}$.

There is only one critical number in the interval (0, 1)

Now we use Closed Interval Method for the interval [0, 1]

$$f(0)=0,\qquad f(1)=0$$

$$f\left(\frac{a}{a+b}\right)=\left(\frac{a}{a+b}\right)^a\left(1-\frac{a}{a+b}\right)^b$$
$$=\frac{a^a}{(a+b)^a}\cdot\frac{b^b}{(a+b)^b}$$
$$=\frac{a^a.b^b}{(a+b)^{a+b}}\qquad \text{where } a,b>0$$

So the maximum value is $\boxed{f\left(\frac{a}{a+b}\right)=\frac{a^a.b^b}{(a+b)^{a+b}}}$

Chapter 3 Applications of Differentiation Exercise 3.1 58E

We have $f(x) = |x^3 - 3x^2 + 2|$

On drawing the graph of $f(x)$, we see the following:

At $x \approx 0, 2$ the tangent lines are horizontal so at $x = 0$ and $x=2$, $f'(x) = 0$.

At about $x \approx -0.7, 1,$ and 2.7, the curve has the corners. So here $f(x)$ is not differentiable. In other words at $x \approx -0.7, 1,$ and 2.7, $f'(x)$ does not exist.

So the critical numbers are

$\boxed{-0.7, 0, 1, 2, 2.7}$

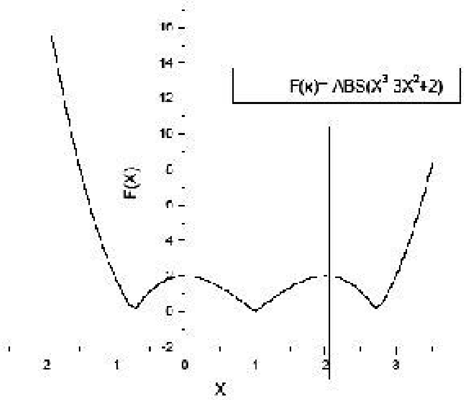


FIGURE - 1

Chapter 3 Applications of Differentiation Exercise 3.1 59E

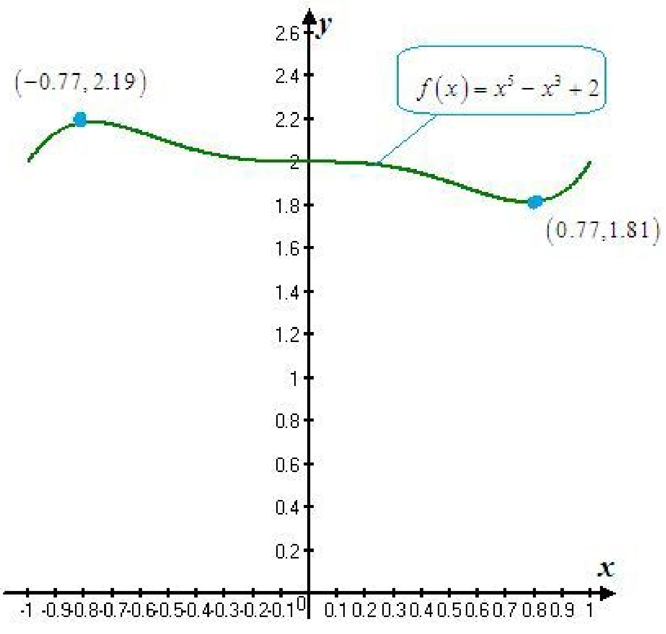
Consider:

$f(x) = x^5 - x^3 + 2, -1 \le x \le 1$.

(a)

Use a graph to estimate absolute maximum and minimum values of the function to two decimal places.

The graph of the function $f(x) = x^5 - x^3 + 2$ on the interval $[-1,1]$ is shown below.



From the above graph, the absolute maximum value of the function is approximately $\boxed{2.19}$, and the absolute minimum value of the function is approximately $\boxed{1.81}$.

(b)

Use calculus to find the exact maximum and minimum values of the function.

To find the absolute extreme values of a continuous function f on a closed interval $[a,b]$, use the closed interval method:

1. Find the values of f at the critical numbers of f in (a,b).

2. Find the values of f at the endpoints of the interval.

3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

To find the critical numbers, take the derivative and see where it is equal to zero or undefined.

$$\begin{aligned} f'(x) &= 5x^4 - 3x^2 \\ 0 &= 5x^4 - 3x^2 \\ 0 &= x^2(5x^2 - 3) \\ x^2 = 0 &\text{ or } 5x^2 - 3 = 0 \end{aligned}$$

$$x = 0 \text{ or } 5x^2 = 3$$

$$x = 0 \text{ or } x^2 = \frac{3}{5}$$

$$x = 0, \sqrt{\frac{3}{5}}, -\sqrt{\frac{3}{5}}$$

The derivative is defined everywhere, since it is a polynomial, so these are the only critical numbers, and they all lie in $(-1,1)$.

The values of the function at these numbers are

$$f(x) = x^5 - x^3 + 2$$

$$\begin{aligned} f(0) &= 2, \\ f\left(\sqrt{\frac{3}{5}}\right) &= \frac{9}{25}\sqrt{\frac{3}{5}} - \frac{3}{5}\sqrt{\frac{3}{5}} + 2 \\ &= \left(\frac{9}{25} - \frac{3}{5}\right)\sqrt{\frac{3}{5}} + 2 \\ &= -\frac{6}{25}\sqrt{\frac{3}{5}} + 2 \end{aligned}$$

$$\begin{aligned} f\left(-\sqrt{\frac{3}{5}}\right) &= -\frac{9}{25}\sqrt{\frac{3}{5}} + \frac{3}{5}\sqrt{\frac{3}{5}} + 2 \\ &= \left(-\frac{9}{25} + \frac{3}{5}\right)\sqrt{\frac{3}{5}} + 2 \\ &= \frac{6}{25}\sqrt{\frac{3}{5}} + 2 \end{aligned}$$

The values at the endpoints are

$$\begin{aligned} f(1) &= 1 - 1 + 2 \\ &= 2, \\ f(-1) &= -1 - (-1) + 2 \\ &= 2 \end{aligned}$$

Therefore, the exact maximum value is $\boxed{2 + \frac{6}{25}\sqrt{\frac{3}{5}}}$ and the exact minimum value is $\boxed{2 - \frac{6}{25}\sqrt{\frac{3}{5}}}$.

Chapter 3 Applications of Differentiation Exercise 3.1 60E

(A)

We draw the graph of the function $f(x) = x^4 - 3x^3 + 3x^2 - x$

By the computers and move the cursor at the lowest point, this point has the co-ordinates $\approx (0.244, -0.101)$

So the absolute minimum is about $\boxed{-0.101}$ *or* ≈ -0.10

And repeat the process for highest point that has the co-ordinates $\approx (2,2)$

So the absolute $\boxed{\text{maximum is } = 2}$

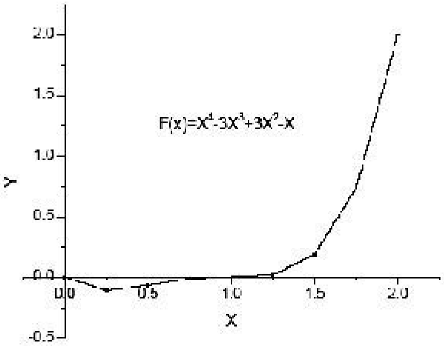


FIGURE - 1

(B) Calculate the critical numbers

$$f(x) = x^4 - 3x^3 + 3x^2 - x$$

Then $f'(x) = 4x^3 - 9x^2 + 6x - 1$

$f'(x) = 0$ When $4x^3 - 9x^2 + 6x - 1 = 0$

Or $4x^2(x-1) - 5x(x-1) + (x-1) = 0$

$$\Rightarrow (x-1)(4x^2 - 5x + 1) = 0$$
$$\Rightarrow (x-1)(4x^2 - 4x - x + 1) = 0$$
$$\Rightarrow (x-1)(4x(x-1) - 1(x-1)) = 0$$
$$\Rightarrow (x-1)(x-1)(4x-1) = 0$$

So $x = 1, \frac{1}{4}$

Critical numbers are 1 and $\frac{1}{4}$

Now $f(1) = 1^4 - 31^3 + 31^2 - 1 = 1 - 3 + 3 - 1 = 0$

$$f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^4 - 3\left(\frac{1}{4}\right)^3 + 3\left(\frac{1}{4}\right)^2 - \frac{1}{4} = -\ 0.1055 \text{ or } \boxed{-\frac{27}{256}}$$

To get the value of at the end points of the interval [-3, 3]

$f(0) = 10$

And $f(2) = 2^4 - 3 \times 2^3 + 3 \times 2^2 - 2$

$= 16 - 24 + 12 - 2 = 28 - 26$

Or $f(2) = 2$

Now the largest number from step 2 and step 3 is the exact maximum $\boxed{=2}$ and smallest number form step 2 and step 3 is the exact

Minimum $= -0.1055$ $\boxed{\approx -0.106}$ *or* $\boxed{= -\frac{27}{256}}$

Chapter 3 Applications of Differentiation Exercise 3.1 61E

(A)

We draw the graph of the function $f(x) = x\sqrt{x - x^2}$.

We note the value of coordinates corresponding to highest point on graph. So the value of coordinate is (0.7511, 0.32409) thus the absolute maximum is at $\boxed{\approx 0.32}$.

Now note the value of coordinates corresponding to lowest point on graph. We see that Absolute minimum $\boxed{\text{is } 0}$.

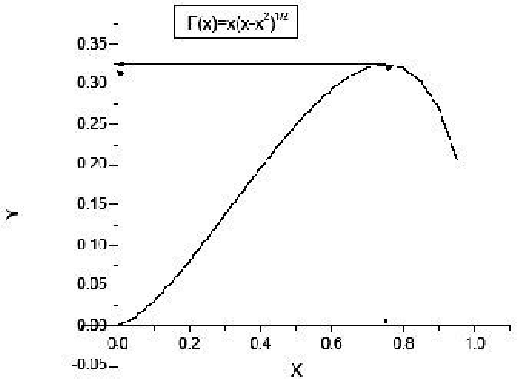


FIGURE - 1

(B) We have $f(x) = x\sqrt{x-x^2}$

Thus function is defined when $x - x^2 \geq 0$

Or $x(1-x) \geq 0$

i.e., $x \geq 0$ and $1 - x \geq 0$ (or $1 \geq x$).

Hence, $f(x)$ is defined when $0 \leq x \leq 1$.

Now we calculate the critical number as follows:

$$f'(x) = \sqrt{x-x^2} + x.\frac{1}{2}\left(x-x^2\right)^{-\frac{1}{2}}.(1-2x) \quad \text{[By chain and product rule]}$$

$$= \frac{2\left(x-x^2\right)+x(1-2x)}{2\sqrt{x-x^2}}$$

$$= \frac{2x-2x^2+x-2x^2}{2\sqrt{x-x^2}} = \frac{3x-4x^2}{2\sqrt{x-x^2}}$$

$f'(x) = 0$, when $3x - 4x^2 = 0$ or $x(3-4x) = 0$

We get $x = 0$ or $3 - 4x = 0$

$$\Rightarrow \boxed{x=0} \text{ or } \boxed{x = \frac{3}{4}}$$

And $f'(x)$ is not defined when $x - x^2 \geq 0$

Or $x(1-x) \geq 0$

i.e., $x \geq 0$ and $1 - x \geq 0$ (or $1 \geq x$)

i.e., for $x=0$ or $x=1$

Now we calculate the value of $f(x)$ at $x = 0, \frac{3}{4}$, and 1.

$$f(0) = 0$$

$$f\left(\frac{3}{4}\right) = \frac{3}{4}\sqrt{\frac{3}{4}-\frac{9}{16}} = \frac{3}{4}\sqrt{\frac{12-9}{16}} = \frac{3}{4}\sqrt{\frac{3}{16}}$$

$$= \frac{3}{16}\sqrt{3} \approx 0.325$$

$$f(1) = 0$$

By the closed interval method, we have

$$\boxed{\text{Absolute maximum} = \frac{3\sqrt{3}}{16} \approx 0.325}$$

And $\boxed{\text{Absolute minimum} = 0}$

Chapter 3 Applications of Differentiation Exercise 3.1 62E

1. Let c be a number in the domain D of a function f. Then $f(c)$ is the

   Absolute maximum value of f on D if $f(c) \geq f(x)$ for all x in D.

   Absolute minimum value of f on D if $f(c) \leq f(x)$ for all x in D.

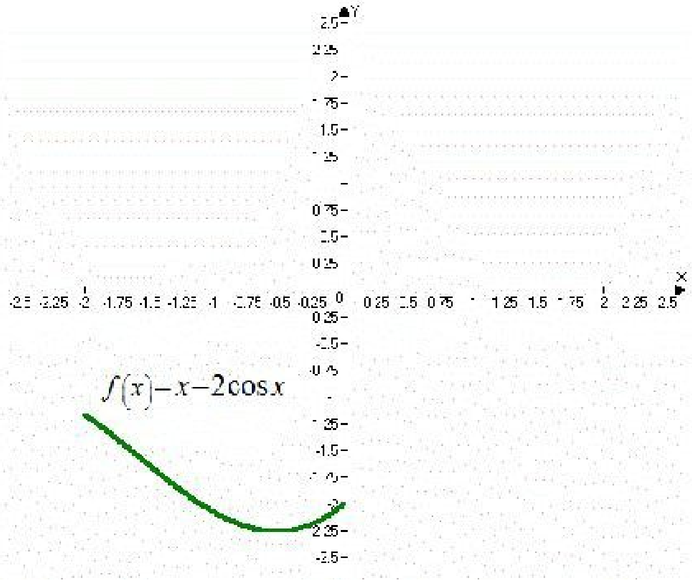
2. The number $f(c)$ is a

   Local maximum value of f if $f(c) \geq f(x)$ when x is near c.

   Local minimum value of f if $f(c) \leq f(x)$ when x is near c.

(a) Given $f(x) = x - 2\cos x, -2 \le x \le 0$

The Graph of the function is



From the graph $\boxed{-0.52}$ is the local minimum and there is no local maximum.

Therefore the absolute minimum value

$\boxed{f(-0.52) = -2.25}$ and there is no absolute maximum.

(b) Given $f(x) = x - 2\cos x, -2 \le x \le 0$

The derivative of the given function is

$$f'(x) = 1 + 2\sin x$$
$$= 0$$
$$2\sin x = -1$$
$$\sin x = \frac{-1}{2}$$
$$\boxed{x = -0.523}$$

Therefore the only critical number is -0.523

(The critical numbers of f occur when $f'(t) = 0$)

The value of f at the critical number is

$$f(-0.523) = -0.523 - 2\cos(-0.523)$$
$$= -2.25$$

The value of f at the endpoints

$$f(0) = 0 - 2\cos 0$$
$$= -2$$
$$f(-2) = -2 - 2\cos(-2)$$
$$= -1.17$$

Therefore the absolute minimum value is

$\boxed{f(-0.523) = -2.25}$ and there is $\boxed{\text{no absolute maximum}}$ value.

Chapter 3 Applications of Differentiation Exercise 3.1 63E

We have volume

$$V = 999.87 - 0.06426\,T + 0.0085043\,T^2 - 0.0000679\,T^3$$

Where 'T' is temperature and $0°c \le T \le 30°c$

Now The density of water $= \frac{\text{mass}}{\text{volume}}$

Mass of water is 1 kg = 1000 gr.

So Density $D = \dfrac{1000}{999.87 - 0.06426\,T + 0.0085043\,T^2 - 0.0000679\,T^3}$ gr/cm^3

Now we graph the function density 'D' by the computer with respect to temperature 'T' where 'T' is in the interval $[0°, 30°c]$ and move the cursor at the highest point of the graph that has the x-coordinate $\approx 3.9665°c$

So at temperature about $\boxed{3.9665°c}$ water has maximum density.

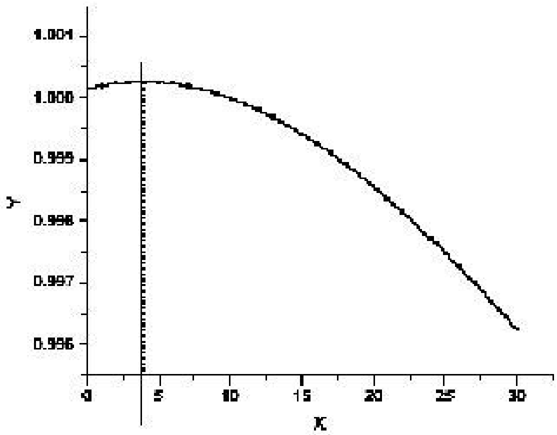


FIGURE - 1

Chapter 3 Applications of Differentiation Exercise 3.1 64E

Multiply and divide the given function by $\cos\theta$:

$$F = \frac{\mu W / \cos\theta}{(\mu \sin\theta / \cos\theta) + 1}$$

$$= \frac{\mu W \sec\theta}{\mu \tan\theta + 1}$$

$$= \frac{\mu W \sqrt{1 + \tan^2\theta}}{\mu \tan\theta + 1}$$

The value of function when $\theta = \tan^{-1}\mu$ or $\mu = \tan\theta$:

$$F = \frac{\mu W \sqrt{1+\mu^2}}{\mu^2 + 1}$$

Or

$$\boxed{F = \frac{\mu W}{\sqrt{1+\mu^2}}}$$

Now, find the value of F at the end points of the interval $\left[0, \frac{\pi}{2}\right]$:

At $\theta = 0$,

$$F = \frac{\mu W}{\mu \sin 0 + \cos 0}$$

$$= \mu W$$

And at $\theta = \frac{\pi}{2}$

$$F = \frac{\mu W}{\mu \sin\frac{\pi}{2} + \cos\frac{\pi}{2}}$$

$$= \frac{\mu W}{\mu}$$

$$= W$$

Compare the values of the function F at $\theta = \tan^{-1}\mu$, $\theta = 0$ and $\theta = \frac{\pi}{2}$, then:

$$\frac{\mu W\sqrt{1+\mu^2}}{\left(\mu^2+1\right)} < \mu W < W$$

This is because the coefficient of friction $\mu < 1$.

Hence it has been shown that the function F is minimum, when $\boxed{\tan\theta = \mu}$.

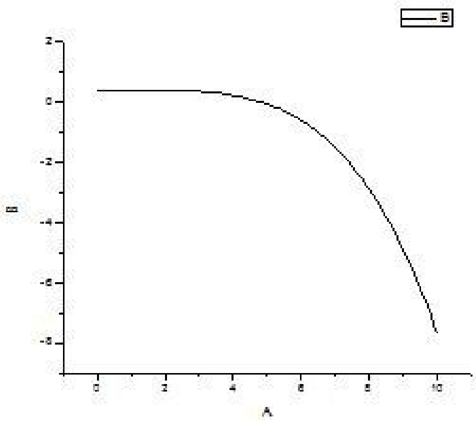
Proved

Chapter 3 Applications of Differentiation Exercise 3.1 65E

a pound of white sugar price has fluctuation given by the function

S(t) = -0.00003237 t5 + 0.0009037 t4 - 0.008956t3 +0.03629t2 -0.04458t +0.4074 where t is measured in years

we graph this function between 1993 and 2003 to find the time at which the price of the sugar is maximum and minimum as follows :



from the graph and the relevant data , we can follow that when t = 0.855, the price of the sugar is cheap and when t = 4.618, the price is maximum.

i.e. in june 1994, the price of the sugar is minimum and in march, 1998 the price ismaximum.

Chapter 3 Applications of Differentiation Exercise 3.1 66E

(A)

The given data is

0	10	15	20	32	59	62	125	Time (S)
0	185	319	447	742	1325	1445	4154	Velocity (ft/s)

Now we graph this data by the computer and get the cubic polynomial that best models the velocity of the shuttle for the time interval t $\in [0,125]$

The cubic polynomial is

$$\boxed{V = 0.00146\,t^3 - 0.11553\,t^2 + 24.98169\,t - 21.26872} \quad \text{--- (1)}$$

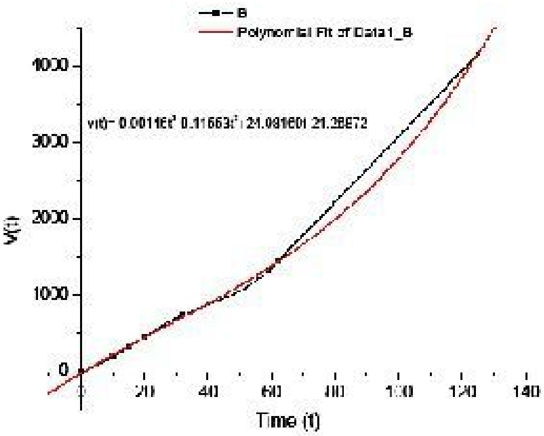


FIGURE - 1

(B)

Acceleration of the shuttle $= \dfrac{dV}{dt} = a$

So differentiate the equation (1) with respect to t

$$\boxed{a(t) = V' = 0.00438\,t^2 - 0.23106\,t + 24.98169}$$

Now differentiate the function a(t) with respect to t for getting critical values (numbers)

$$\frac{da}{dt} = 0.00876\,t - 0.23106$$

$$\frac{da}{dt} = 0 \text{ when } 0.00876\,t - 0.23106 = 0$$

Or $\quad t = \dfrac{0.23106}{0.00876}$

Or $\quad t \approx 26.37$ seconds

Calculating the value of a(t) at $t \approx 26.37$ seconds

$$a(26.37) = 0.00438(26.37)^2 - 0.23106(26.37) + 24.98169$$

$\boxed{a \approx 21.93}$ ft^2/s

Now calculate the value of a(t) at the end points of the interval [0, 125] at t = 0

$$a(0) = 0.00438(0)^2 - 0.23106(0) + 24.98169$$

$\boxed{a = 24.98169}$ ft^2/s

At t = 125

$$a = 0.00438(125)^2 - 0.23106(125) + 24.98169$$

Or $\boxed{a = 64.53\,\text{ft}^2/\text{s}}$

Now the smallest number form step 2 and step 3 is $21.93\,\text{ft}^2/\text{s}$

So the minimum value of acceleration is about $\boxed{21.93\ \ \text{ft}^2/\text{s}}$

The greatest number from step 2 and 3 is $64.53\ \ \text{ft}^2/\text{s}$

So the maximum value of acceleration is about $\boxed{64.53\,\text{ft}^2/\text{s}}$

Chapter 3 Applications of Differentiation Exercise 3.1 67E

(A) The equation is given as $v(r)=k(r_0-r)r^2;\quad \frac{r_0}{2}\le r\le r_0$

Then $v'(r)=2kr(r_0-r)-kr^2$ [Product rule]

For max or min, $v'(r)=0$

$\Rightarrow\quad 2kr(r_0-r)=kr^2$

$\Rightarrow\quad 2kr_0r-2kr^2=kr^2$

$\Rightarrow\quad 3kr^2-2kr_0r=0$

$\Rightarrow\quad rk(3r-2r_0)=0$

$\Rightarrow\quad r=0\quad$ Or, $\quad r=\frac{2}{3}r_0$

But $r=0$ is not in the interval $\left[\frac{r_0}{2},\ r_0\right]$ so we take only $r=\frac{2}{3}r_0$

Now we use closed interval method

$$v(2r_0/3)=k(r_0-2r_0/3)(2r_0/3)^2=\frac{4kr_0^3}{27}$$

$$v(r_0/2)=k(r_0-r_0/2)(r_0/2)^2=\frac{kr_0^3}{8}$$

And $\quad v(r_0)=k(r_0-r_0)(r_0)^2=0$

Since $\frac{4kr_0^3}{27}>\frac{kr_0^3}{8}$, so $v(r)$ has an absolute maximum at $\boxed{r=\frac{2}{3}r_0}$

This supports the experimental evidence.

(C)

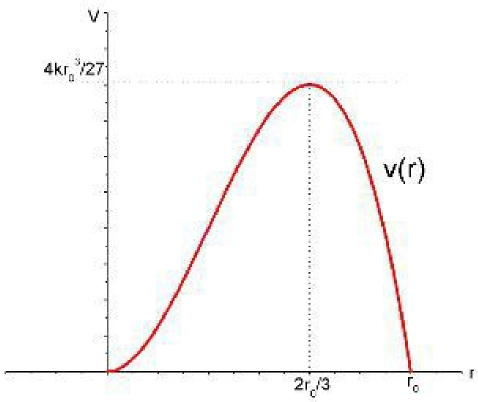


Fig.1

Chapter 3 Applications of Differentiation Exercise 3.1 68E

We have

$$g(x)=2+(x-5)^3$$

Then $\quad g'(x)=3(x-5)^2$

For critical numbers, we must have $g'(x)=3(x-5)^2=0$

$$\Rightarrow (x-5)=0$$

$$x=5$$

Thus, 5 is a critical number of the function $g(x)=2+(x-5)^3$.

But we see that $g(5)=2$

We take an open interval $(4,\ 6)$ containing 5.

We see that $g(4.9)=2+(-0.1)^3=1.999<2$

And $\quad g(5.1)=2+(0.1)^3=2.001>2$

Thus

$g(5)>g(x)$ for $x<5$ and $g(5)<g(x)$ for $x>5$

So the function $g(x)=2+(x-5)^3$ does not have a local extreme value at $x=5$.

Chapter 3 Applications of Differentiation Exercise 3.1 69E

We have $f(x) = x^{101} + x^{51} + x + 1$

Then $f'(x) = 101x^{100} + 51x^{50} + 1$

For finding the critical number, we must have $f'(x) = 0$

Or $101x^{100} + 51x^{50} + 1 = 0$

But $101x^{100} + 51x^{50} + 1 \geq 1$ for all x.

Thus the equation $101x^{100} + 51x^{50} + 1 = 0$ has no real root, and so there is no critical number.

Therefore the function $f(x) = x^{101} + x^{51} + x + 1$ has neither a local maximum nor a local minimum.

Chapter 3 Applications of Differentiation Exercise 3.1 70E

Function f has a minimum value of c, then by the definition we have

$$f(c) \leq f(x) \quad \text{for all } x \text{ near } c.$$

Then

$$-f(c) \geq -f(x) \qquad [\text{if } a > b \quad \text{then} \quad -a < -b]$$

We have

$$g(x) = -f(x) \text{ so } g(c) = -f(c)$$

Hence,

$$g(c) \geq g(x)$$

Thus by the definition of the maximum value, $g(x)$ has a maximum value at c.

Proved.

Chapter 3 Applications of Differentiation Exercise 3.1 71E

We have the statement of Fermat's theorem such that " if f has a local maximum or minimum at c and if $f'(c)$ exists then $f'(c) = 0$

Proof

Suppose that f has a local minimum at c then by the definition of minimum we have $f'(c) \leq f(x)$ if x is very close to c.

Let h be a positive or negative number such that $h \to 0$ then

$$f'(c) \leq f(c+h)$$

And so $f(c+h) - f(c) \geq 0$

Now if h > 0 and h is so small, we have

$$\frac{f(c+h) - f(c)}{h} \geq 0$$

Now taking right hand limit of both sides, we have

$$\lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h} \geq \lim_{h \to 0^+} 0 = 0$$

Since $f'(c)$ exists, we have

$$f'(c) = \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$$

So $f'(c) \geq 0$

Now if h < 0 and h is so small we have

$$\frac{f(c+h) - f(c)}{h} \leq 0$$

Taking the left hand limit, we have

$$\lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h} \leq \lim_{h \to 0^-} 0 = 0$$

Since $f'(c)$ exists we have

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f'(c)}{h} = \lim_{h \to 0} \frac{f(c+h) - f'(c)}{h} \leq 0$$

So $\boxed{f'(c) \leq 0}$

So we have $f'(c) \geq 0$ and $f'(c) \leq 0$ must exist or true

It is possible only when $\boxed{f'(c) = 0}$

Thus

If f has a local minimum at c and if $f'(c)$ exists

Then $f'(c) = 0$ proved

Chapter 3 Applications of Differentiation Exercise 3.1 72E

(A) The form of the cubic polynomial is $f(x) = ax^3 + bx^2 + cx + d$.

Then $f'(x) = 3ax^2 + 2bx + c$

For critical numbers $f'(x) = 0$

$\Rightarrow$ $3ax^2 + 2bx + c = 0$

This is a quadratic equation which can have one, two, or no real roots.

Therefore, the function $f(x) = ax^3 + bx^2 + cx + d$ can have one, two, or no critical numbers.

(i) Example of function having two critical points is as follows:

$f(x) = x^3 - x$.

The critical numbers are $x = \pm 1/\sqrt{3}$.

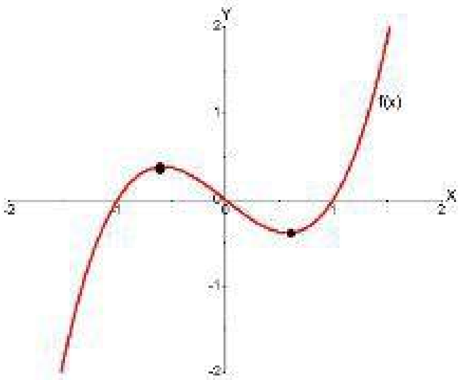


Fig.1

(ii) Example of function having one critical points:

$f(x) = x^3$.

The critical number is $x = 0$.

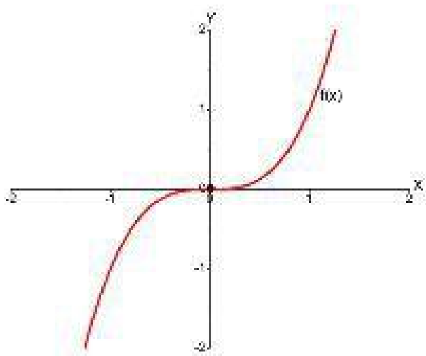


Fig.2

(iii) Example of function having no critical number:

$f(x) = x^3 + x$.

Since $f'(x) = 3x^2 + 1 = 0$ has no real root.

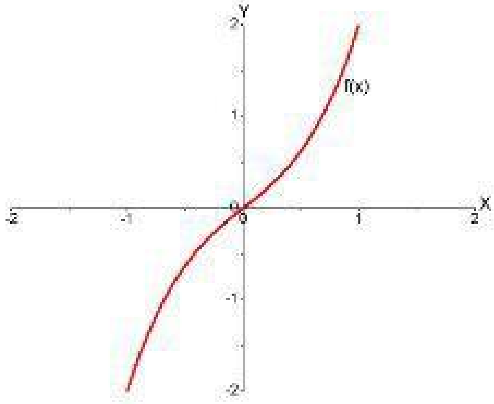


Fig.3

(B) From part (A), we see that a cubic function can have at most two local extreme values. If the cubic function has only one critical number or no critical number then there is no local extreme value.