INTEGRALS

Fill in the Blanks

1) $\int_0^3 dx =$_____

2) $\int \sec^2 x\, dx =$_____

3) $\int_0^3 3x^2 dx =$_____

4) $\int_0^5 2x dx =$_____

5) Integration is _____ process of differentiation.

6) Definite integrals may also be calculated as _____ of a sum.

7) $\int_{-a}^{a} f(x)\, dx = 0$ if f is _____function.

8) $\int \frac{dx}{1+x^2} =$_____

9) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx =$_____

10) $\int_a^b f(x)dx =$_____$[f(a) + f(a+h) + f(a+2h) + \cdots \ldots\ldots\ldots + f(a+(n-1)h)]$.

Multiple Choice Questions

1 $\int_0^{\pi/2} \frac{\sin^{1/2} x}{\sin^{1/2} x+\cos^{1/2} x} dx$ is equal to :

(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

2 $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x} dx$ is equal to :

(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

3 $\int \frac{dx}{2x+3}$ equals:

(a)$\log|2x+3|+c$ (b)$\log|2x-3|+c$ (c)$\frac{\log|2x+3|}{3}+c$ (d)$\frac{\log|2x+3|}{2}+c$

4 $\int \frac{dx}{2x-5}$ equals:

(a)$\log|2x-5|+c$ (b)$\log|2x+5|+c$ (c)$\frac{\log|2x-5|}{5}+c$ (d)$\frac{\log|2x-5|}{2}+c$

5 $\int_{-1}^{1} x^3 \cos x\, dx$ equals:

(a)0 (b)1/4 (c)π (d)none of these

6 $\int_0^{\pi/2} \frac{\sin^{1/2} x}{\sin^{1/2} x+\cos^{1/2} x} dx$ is equal to :

(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

7 $\int_{-2}^{2} x^3 dx$ is equal to :

(a)0 (b)4 (c)16/3 (d) $\frac{\pi}{4}$

8 $\int_{-1}^{1} x \sin^2 x\, dx$ is equal to

(a)0 (b)$\frac{1}{2}$ (c)$\frac{1}{3}$ (d)-1

9 $\int_0^1 \frac{dx}{1+x^2}$ is

(a)$\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

10 $\int_{\pi/6}^{\pi/3} \frac{\cos^4 x}{\sin^4 x+\cos^4 x}\,dx$ **is equal to**

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{6}$ **(c)** $\frac{\pi}{12}$ **(d)** $\frac{\pi}{2}$

2 & 4 Marks Questions

1. Evaluate the following :

(a) $\int \sqrt{1-\cos 2x}\,dx$ **(b)** $\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}\,dx$ **(c)** $\int (x^6-\cos x+\sec^2 x+\sqrt{x})\,dx$

(d) $\int \left(e^x+3\cos x-4x^3+2\right)dx$ **(e)** $\int \frac{2^x+3^x}{5^x}\,dx$ **(f)** $\int \frac{x^2}{1+x}\,dx$

(g) $\int \frac{(x-4)^3}{x^2}\,dx$ **(h)** $\int \frac{dx}{1-\sin x}$ **(i)** $\int \frac{dx}{1+\cos x}$

2. Compute the following :

(a) $\int \frac{\log x}{x}\,dx$ **(b)** $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$ **(c)** $\int \frac{2x}{1+x^2}\,dx$ **(d)** $\int \frac{x^2}{1+x^3}\,dx$ **(e)** $\int \frac{x^2+4x}{x^3+6x^2+5}\,dx$

(f) $\int \frac{\sec x\,dx}{\log(\sec x+\tan x)}$ **(g)** $\int \sqrt{\sin 2x}\cos 2x\,dx$ **(h)** $\int \frac{6x-8}{3x^2-8x+5}\,dx$ **(i)** $\int \frac{2x+9}{x^2+9x+20}\,dx$ **(j)** $\int \frac{\cos x-\sin x}{\cos x+\sin x}\,dx$

(k) $\int (2x+4)\sqrt{x^2+4x+3}\,dx$ **(l)** $\int \frac{dx}{\sqrt{x+2}-\sqrt{x+3}}$ **(m)** $\int \frac{dx}{\sqrt{x+1}+\sqrt{x+2}}$ **(n)** $\int \frac{dx}{\sqrt{x-5}-\sqrt{x+4}}$

3. Integrate the following :

(a) $\sin^2 x\cos^3 x$ **(b)** $\cos^2 x\sin^3 x$ **(c)** $\tan^3 x$ **(d)** $\tan^4 x$ **(e)** $\tan^4 x\sec^2 x$

(f) $\sec^4 x\tan x$ **(g)** $\frac{1}{1-\cot x}$ **(h)** $\frac{1+\tan x}{x+\log\sec x}$ **(i)** $\frac{1-\tan x}{x+\log\cos x}$ **(j)** $\frac{1}{\sqrt{\sin^3 x\cos x}}$

(k) $\frac{\sin x}{\sin(x-\alpha)}$ **(l)** $\frac{\cos x}{\cos(x-\alpha)}$ **(m)** $\frac{\sin(x-\alpha)}{\sin(x+\alpha)}$ **(n)** $\frac{1}{\sin(x-a)\cos(x-b)}$

4. Evaluate the following integrals :

(a) $\int \sec^4 x\,dx$ **(b)** $\int \operatorname{cosec}^4 x\,dx$ **(c)** $\int \frac{1-\tan x}{1+\tan x}\,dx$ **(d)** $\int \frac{1+\tan x}{1-\tan x}\,dx$ **(e)** $\int x^2 e^{x^3}\cos\left(2e^{x^3}\right)dx$

(f) $\int \frac{e^x-1}{e^x+1}\,dx$ **(g)** $\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx$ **(h)** $\int \frac{(\tan^{-1}x)^2}{1+x^2}\,dx$ **(i)** $\int \frac{\sec^2(2\tan^{-1}x)}{1+x^2}\,dx$ **(j)** $\int \frac{\sin(\tan^{-1}x)}{1+x^2}\,dx$

(k) $\int \sin 4x\cos 7x\,dx$ **(l)** $\int \sin 7x\sin 5x\,dx$ **(m)** $\int \cos 3x\cos 5x\,dx$

5. Evaluate the following integrals :

(i) $\int \frac{\sin 2x}{\sin 5x\sin 3x}\,dx$ **(ii)** $\int \frac{1+\sin x}{1-\sin x}\,dx$ **(iii)** $\int \frac{dx}{9x^2+6x+5}$ **(iv)** $\int \frac{dx}{\sqrt{x^2-5x+7}}$ **(v)** $\int \frac{2^x dx}{\sqrt{1-4^x}}$

(vi) $\int \frac{dx}{4\sin^2 x+5\cos^2 x}$ **(vii)** $\int \frac{dx}{1+3\sin^2 x}$ **(viii)** $\int \frac{dx}{1+3\sin^2 x+8\cos^2 x}$ **(ix)** $\int \sqrt{\frac{a-x}{a+x}}\,dx$ **(x)** $\int \sqrt{\frac{5-x}{5+x}}\,dx$

5. Integrate the following functions:

(a) $x\sec^2 x$ (b) $x\cos 2x\sin 4x$ (c)$\sec^3 x$ (d)$\text{cosec}^3 x$ (e) x^2e^x (f) $x^2\cos 3x$

(g) $\log(4+x^2)$ (h) $(\log x)^2$ (i) $x\cot^{-1}x$ (j) $\sin^{-1}x$ (k) $\tan^{-1}x$ (l) $\tan^{-1}\sqrt{x}$

7. Integrate the following functions :

(a) $e^x\sin 2x$ (b) $e^{3x}\cos 5x$ (c)$e^x\cos^2 x$ (d) $\cos(\log x)$ (e) $e^x\left(\frac{1-\sin x}{1-\cos x}\right)$

(f) $e^x(\cot x+\log\sin x)$ (g) $e^x\frac{x-1}{(x+1)^3}$ (h) $\sin(\log x)+\cos(\log x)$ (i)$\frac{\log x}{(1+\log x)^2}$

8. Integrate the following functions :

(a) $\frac{1}{5x^2-20x+1}$ (b) $\frac{\cos x}{\cos 3x}$ (c)$\frac{x+1}{x^2-4x+6}$ (d) $\frac{x^2}{x^2+6x+12}$ (e) $\frac{1}{(x+1)(2x+1)}$

(f) $\frac{1}{(x+1)(x+2)(x+3)}$ (g) $\frac{x^2}{x(x-1)(x-2)}$ (h) $\frac{1}{x^3-1}$ (i) $\frac{1}{(1-x)(1+x^2)}$ (j) $\frac{x}{(x-2)(x^2+4)}$

9. Integrate the following functions :

(a) $\frac{2x}{\sqrt{(x+1)(x-2)}}$ (b) $\frac{5-2x}{\sqrt{6+x-x^2}}$ (c)$\frac{4x+5}{\sqrt{2x^2+x-3}}$ (d) $\frac{3x+5}{\sqrt{x^2-8x+7}}$ (e) $\sqrt{8+2x-x^2}$

(f) $(x+3)\sqrt{x^2-5x+1}$ (g)$(3x-2)\sqrt{x^2+x+1}$ (h) $\frac{x^2-1}{x^4+x^2+1}$ (i)$\frac{x^2+1}{x^4-x^2+1}$

(j) $\frac{x^2}{x^4+x^2+1}$ (k) $\frac{x^2-1}{x^4+1}$ (l) $\frac{1}{x^4+x^2+1}$ (m) $\frac{1}{x(x^5+1)}$ (n) $\frac{1}{x(x^7+1)}$

10 Evaluate the following as limit of a sum :

(a) $\int_0^3(x^2+1)dx$ (b) $\int_2^5(x^2+2x)dx$ (c) $\int_1^3(x^2+x)dx$

(d)$\int_0^4(x^2+3x)dx$ (e) $\int_{-2}^2 e^x dx$ (f) $\int_2^4 3^x dx$

11 Evaluate the following :

(a) $\int_0^{\pi/2}\frac{dx}{4\cos x+3\sin x}$ (b) $\int_0^{\pi/2}\frac{dx}{3+2\sin x+\cos x}$ (c) $\int_0^{\pi/2}\frac{\sin 2xdx}{\sin^4 x+\cos^4 x}$ (d) $\int_0^{\pi/2}\frac{dx}{4\sin^2 x+5\cos^2 x}$

12 Evaluate the following integrals :

(a) $\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ (b) $\int_0^{\pi/2}\frac{1}{1+\sqrt{\cot x}}dx$

(c)$\int_0^{\pi/2}\frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$ (d) $\int_0^{\pi/2}\frac{dx}{1+\tan^3 x}$

(e) $\int_0^{\pi}\frac{x}{4-\cos^2 x}dx$ (f) $\int_0^{\pi}\frac{xdx}{1+\sin^2 x}$

(g) $\int_0^1|x-5|dx$ (h) $\int_{-6}^6|x+2|dx$

(i) $\int_0^{\pi/2}\log\tan x\,dx$ (j) $\int_0^{\pi/2}\sin 2x\log\tan x\,dx$

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