Section A

1. Let U be the universal containing 700 elements. If A and B are subsets of U such that n(A) = 200, n(B) = 300 and $n(A \cap B) = 100$ then $n(A' \cap B') = \ldots$.

   a. 400

   b. 300

   c. 500

   d. 800

2. If A = { 1,2,3,4 } , B = { 4,5,6,7 } , $A \cap B =$

   a. { 4 }

   b. { 1,2,3,4 }

   c. { 6 , 7 }.

   d. { 1, 2 }

3. If n (A) =3 and n (B) = 6 and A⊆ B , then $n(A \cup B) =$?

   a. 9

   b. 3

   c. 6

   d. none of these

4. The number of proper subsets of the set { 1, 2 , 3 } is :

a. 6

b. 7

c. 8

d. 5

5. If A class has 175 students . The following data shows the number of students offering one or more subjects. Mathematics 100 ; Physics 70 ; Chemistry 40 ; Mathematics and Physics 30 ; Mathematics and Chemistry 28 ; Physics and Chemistry 23 ; Mathematics , Physics and Chemistry 18 . How many students have offered Mathematics alone?

a. 35

b. 22.

c. 48

d. 60

6. Fill in the blanks:

   If A = {1, 3, 5, 7, 9} and B = {2, 3, 5, 7, 11}, then A$\triangle$B is _____.

7. Fill in the blanks:

   A set, consisting of a single element, is called a _____.

8. List all the elements of set {x : x is a month of a year not having 31 days}.

9. State whether the statement is true or false: {a, e, i, o, u) and {a, b, c, d} are disjoint sets.

10. If U = {a, b, c, d, e, f, g, h}, find the complement of the set: D = {f, g, h, a}

11. Let A = {1, 2, 4, 5} B = {2, 3, 5, 6} C = {4, 5, 6, 7}. Verify:
    $$A - (B \cup C) = (A - B) \cap (A - C)$$

12. If A is any set, prove that: $A \subseteq \phi \Leftrightarrow A = \phi$

13. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers.
Find: the number of people who read at least one of the newspaper.

14. For any two sets A and B prove that: $P(A \cap B) = P(A) \cap P(B)$.

15. If U = {a, b, c, d, e, f} , A = {a, b,c}, B = {c, d, e, f} , C = {c, d, e} and D = {d, e, f}, then tabulate the following sets:

   i. $A \cap D$
  ii. $A \cap C$
 iii. $U \cap D$
 iv. $A \cup \phi$
  v. $(U \cap \phi)'$
 vi. $(U \cup A)'$

Solution

Solution

Section A

1. (b) 300

   Explanation:

   Given n(A) = 200, n(B) =\ 300, $n(A \cap B) = 100$

   $n(A \cup B) = n(A) + n(B) - n(A \cap B)$= 200 + 300 - 100 = 400

   $n(A' \cap B') = n(A \cup B)' + n(U) - n(A \cup B)$= 700 - 400 = 300

   [By De morgans law]

2. (a) { 4 }

   Explanation: Given A=1,2,3,4 and B=4,5,6,7

   $(A \cap B) = \{4\}$

3. (c) 6

   Explanation:$A \subseteq B$

   $\Rightarrow n(A \cup B) = n(B) = 6$

4. (b) 7

   Explanation: The no of proper subsets=2^n-1=2^3-1=7

   Here n=no of elements of given set=3
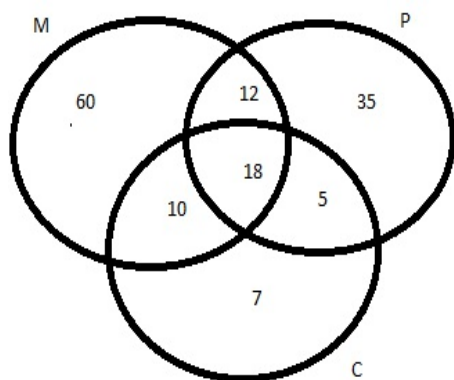
5. (d) 60

   Explanation:

   M -mathematics

   P - physics

   C - chemistry

   Venn Diagram

By Venn Diagram we can see that the students who offered mathematics alone are 60.

6. {1, 2, 9, 11}

7. singleton set

8. A month has either 28, 29, 30 or 31 days.

   Out of the 12 months in a year, the months that have 31 days are:

   January, March, May, July, August, October, December

   $\therefore$ Given set has elements {February, April, June, September, November}

9. Let A = {a, e, i, o, u} and B ={a, b, c, d}

   Now $A \cap B = \{a, e, i, o, u\} \cap \{a, b, c, d\}$= {a}

   Hence A and B are not disjoint. So the statement is false.

10. $D' = U - D = \{a, b, c, d, e, f, g, h\} - \{f, g, h, a\}$={b, c, d, e}

11. A = {1,2,4,5}, B = {2,3,5,6}, C = {4,5,6,7}

    $B \cup C$ = {2,3,4,5,6,7}

    $A - (B \cup C)$= {1} .....(i)

    (A - B) = { 1, 4}

    (A - C) = {1, 2}

    $(A - B) \cap (A - C)$= {1} ....(ii)

    From eq^n (i) and eq^n (ii), we get

    $A - (B \cup C) = (A - B) \cap (A - C)$

12. The symbol '$\Leftrightarrow$' stands for if and only if (in short if).

    In order to show that two sets A and B are equal, we show that $A \subseteq B$ and $B \subseteq A$.

    We have $A \subseteq \phi, \because \phi$is a subset of every set,

$$\therefore \quad \phi \subseteq A$$

Hence A = ϕ

To show the backward implication, suppose that $A = \phi$.

$\because$ every set is a subset of itself

$$\therefore \quad \phi = A \subseteq \phi$$

Hence, proved.

13. Here

n(U) = a + b + c + d + e + f + g + h = 60 .... (i)

n (H) = a + b + c +d = 25 .... (ii)

n(T) = b + c + f + g = 26 .... (iii)

n(I) = c + d + e + f = 26 .... (iv)

$n(H \cap I) = c + d = 9$....... (v)

$n(H \cap T) = b + c = 11$....... (vi)

$n(T \cap I) = c + f = 8$....... (vii)

$n(H \cap T \cap I) = c = 3$....... (viii)
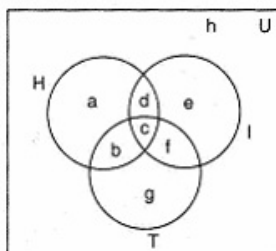
Putting value of c in (vii),

3+ f = 8 $\Rightarrow$ f = 5

Putting value of c in (vi),

3 +b = 11 $\Rightarrow$ b = 8

Putting values of c in (v),

3 + d = 9 $\Rightarrow$ d = 6

Putting value of c, d, f in (iv),

3 + 6 + e + 5 = 26 $\Rightarrow$ e = 26 - 14 = 12

Putting value of b, c, f in (iii),

8 + 3 + 5 + g = 26 $\Rightarrow$ g = 26 - 16 = 10

Putting value of b, c, d in (ii)

a + 8 + 3 + 6 = 25 $\Rightarrow$ a = 25 - 17 = 8

Number of people who read at least one of the three newspapers

= a + b + c + d +e + f + g

= 8 + 8 + 3 + 6 + 12 + 5 + 10 = 52

14. Let $x \in P(A \cap B)$

$\Rightarrow x \subset (A \cap B)$

$\Rightarrow x \subset A$ and $x \subset B$

$\Rightarrow x \in P(A)$ and $x \in P(B)$

$\Rightarrow x \in P(A) \cap P(B)$

$\Rightarrow x \subset P(A) \cap P(B)$

$\therefore P(A \cap B) \subset P(A) \cap P(B) \ldots$ (i)

Let $x \in P(A) \cap P(B)$

$\Rightarrow x \in P(A)$ and $x \in P(B)$

$\Rightarrow x \subset A$ and $\Rightarrow x \subset B$

$\Rightarrow x \subset A \cap B$

$\Rightarrow x \subset P(A \cap B)$

$\therefore P(A) \cap P(B) \subset P(A \cap B) \ldots$ (ii)

From (i) and (ii), we have

$$P(A \cup B) = P(A) \cap P(B)$$

15. According to the question, we are given that,

U = {a, b, c, d, e, f} , A = {a, b,c}, B = {c, d, e, f} , C = {c, d, e} and D = {d, e, f}

i. A $\cap$ D = {a, b,c} $\cap$ {d, e, f} = ϕ

ii. A $\cap$ C = {a, b, c} $\cap$ {c, d, e} = {c}

iii. U $\cap$ D = {a, b, c, d, e, f} $\cap$ {d, e, f} = {d, e, f}

iv. $A \cup \phi$ = {a, b, c} $\cup$ {} = {a,b,c}

v. U $\cap$ ϕ = {a, b, c, d, e, f} $\cap$ {} = ϕ

$(U \cap \phi)'$

= ϕ'

= U

vi. U $\cup$ A = {a, b, c, d, e, f} $\cup$ {a, ,b, c}

= {a, b, c, d, e, f}

= U

$\therefore$ (U $\cup$ A)' = ϕ