CBSE Class 10 Mathematics Basic

Sample Paper - 08 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

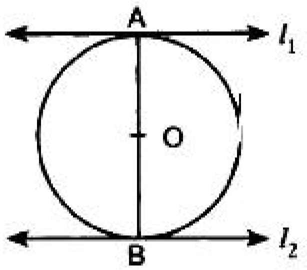
Part-A

1. Without actually performing the long division, Check whether $\frac{129}{2^2 5^7 7^5}$ will have terminating decimal expansion or non-terminating repeating decimal expansion.

OR

Show that $5 \times 11 \times 13 + 13$ is a composite number.

2. Find discriminant of the quadratic equation: $5x^2 + 5x + 6 = 0$.
3. Solve for x and y: $\frac{2x+5y}{xy} = 6; \frac{4x-5y}{xy} = -3$.
4. What is the distance between two parallel tangents of a circle of radius 7 cm?



5. Find the 35th term of the A.P 20, 17, 14, 11, ...

OR

Find the sum of the first 1000 positive integers.

6. Find the sum of first 24 terms of the AP 5,8,11,14,....
7. Solve for x: $x + \frac{1}{x} = 3$, $x \neq 0$

OR

Find the roots of the quadratic equation $2x^2 - \sqrt{5}x - 2 = 0$ using the quadratic formula.

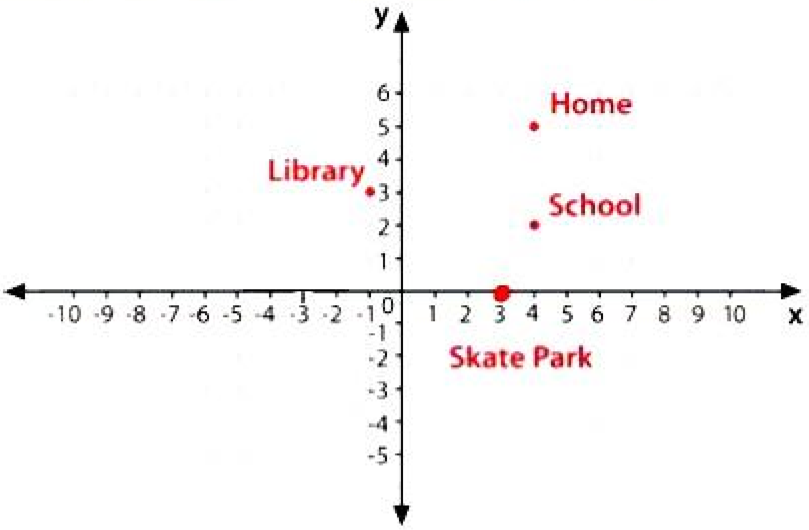
8. How many common tangents can be drawn to two circles touching internally?
9. From an external point C, k tangents can be drawn to the circle. Find the value of k.

OR

What will be the distance between two parallel tangents to a circle of radius 5 cm?

10. If ratio of corresponding sides of two similar triangles is 5 : 6, then find ratio of their areas.
11. Find the common difference of the A.P. and write the next two terms: 119, 136, 153, 170,.....
12. Find the value of x, if $\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$.
13. Prove that: $\tan^2\theta \cos^2\theta = 1 - \cos^2\theta$
14. 12 solid spheres of the same size are made by melting a solid metallic cone of base radius 1 cm and height of 48 cm. Find the radius of each sphere.
15. Find the 11th term from the end of the AP 10,7,4,....,-62.
16. A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is a ten.
17. Two brothers Ramesh and Pulkit were at home and have to reach School. Ramesh went to Library first to return a book and then reaches School directly whereas Pulkit went to

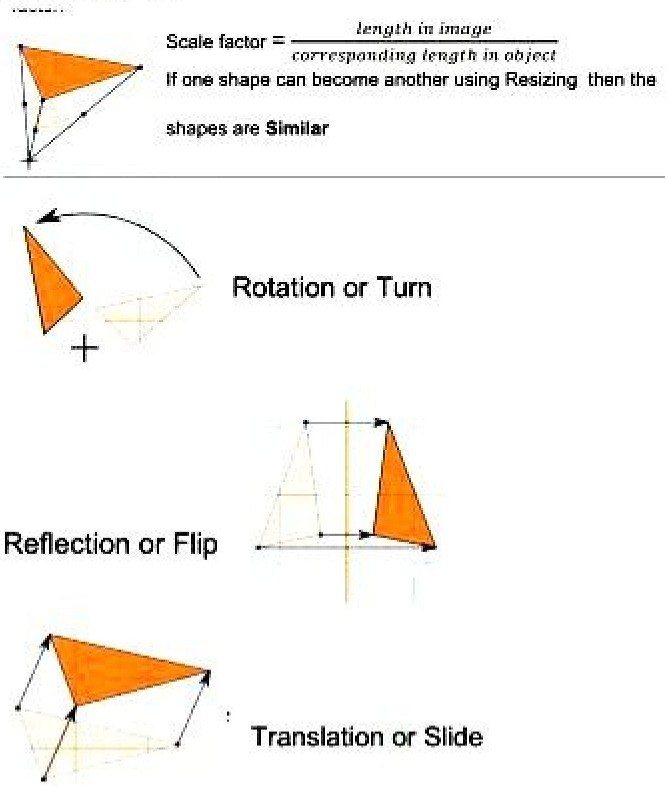
Skate Park first to meet his friend and then reaches School directly.



i. How far is School from their Home?
   a. 5 m
   b. 3 m
   c. 2 m
   d. 4 m
ii. What is the extra distance travelled by Ramesh in reaching his School?
   a. 4.48 metres
   b. 6.48 metres
   c. 7.48 metres
   d. 8.48 metres
iii. What is the extra distance travelled by Pulkit in reaching his School? (All distances are measured in metres as straight lines)
   a. 6.33 metres
   b. 7.33 metres
   c. 5.33 metres
   d. 4.33 metres
iv. The location of the library is:
   a. (-1, 3)
   b. (1, 3)
   c. (3, 1)
   d. (3, -1)
v. The location of the Home is:
   a. (4, 2)

b. (1, 3)
c. (4, 5)
d. (5, 4)

18. SCALE FACTOR AND SIMILARITY SCALE FACTOR: A scale drawing of an object is the same shape as the object but a different size. The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio. SIMILAR FIGURES: The ratio of two corresponding sides in similar figures is called the scale factor.



Hence, two shapes are Similar when one can become the other after a resize, flip, slide, or turn.

i. A model of a boat is made on a scale of 1:4. The model is 120cm long. The full size of the boat has a width of 60cm. What is the width of the scale model?

a. 20 cm
b. 25 cm
c. 15 cm

d. 240 cm

ii. What will affect the similarity of any two polygons?

a. They are flipped horizontally

b. They are dilated by a scale factor

c. They are translated down

d. They are not the mirror image of one another

iii. If two similar triangles have a scale factor of a: b. Which statement regarding the two triangles is true?

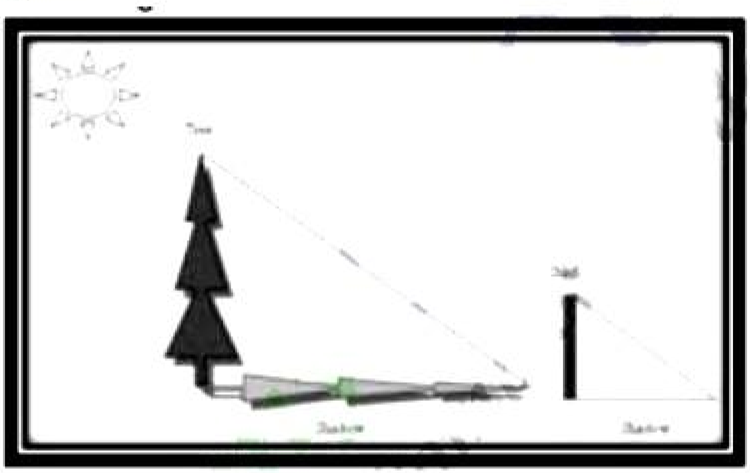
a. The ratio of their perimeters is 3a: b

b. Their altitudes have a ratio a: b

c. Their medians have a ratio $\frac{a}{2}$: b

d. Their angle bisectors have a ratio a^2: b^2

iv. The shadow of a stick 5m long is 2m. At the same time, the shadow of a tree 12.5m high is:



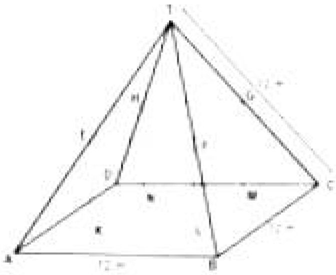
a. 3m

b. 3.5m

c. 4.5m

d. 5m

v. Below you see a student's mathematical model of a farmhouse roof with measurements. The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a rectangular prism, EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT, and H is the middle of DT. All the edges of the pyramid in the model have a length of 12 m.

What is the length of EF, where EF is one of the horizontal edges of the block?

a. 24m
b. 3m
c. 6m
d. 10m

19. A student noted the number of cars through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below:

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

i. While computing means of the grouped data, we assume that the frequencies are:
   a. evenly distributed over all the classes
   b. centered at the class marks of the classes
   c. centered at the upper limits of the classes
   d. centered at the lower limits of the classes

ii. The sum of the lower limits of the median class and modal class is:
   a. 40
   b. 60
   c. 80

d. 90

iii. Find the mode of the data.

a. 44.7

b. 47.7

c. 54.5

d. 54.3

iv. Half of (upper-class limit + lower class limit) is:

a. Class interval

b. Classmark

c. Class value

d. Class size

v. The median of data is:

a. 44

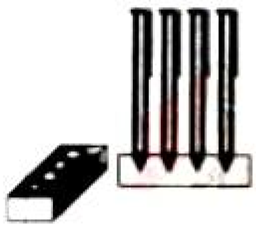
b. 43

c. 41

d. 42

20. A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:

A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



By using the above-given information, find the following:

i. The volume of the cuboid is:

a. 552 cm^3

b. 252 cm^3

c. 525 cm^3

d. 225 cm^3

ii. Volume of four conical depressions is:

a. $\frac{15}{22}$ cm^3

b. $\frac{22}{15}$ cm^3

c. $\frac{22}{30}$ cm^3

d. $\frac{11}{30}$ cm^3

iii. The volume of wood in the entire stand is:

a. 523.53 cm^3

b. 532.53 cm^3

c. 325.53 cm^3

d. 552.53 cm^3

iv. The formula of TSA of the cone is given by:

a. $2\pi rl + \pi r^2$

b. $\pi r^2 l + \pi r^2$

c. $\pi rl + 2\pi r^2$

d. $\pi rl + \pi r^2$

v. During the conversion of a solid from one shape to another the volume of the new shape will

a. increase

b. decrease

c. remain unaltered

d. be double

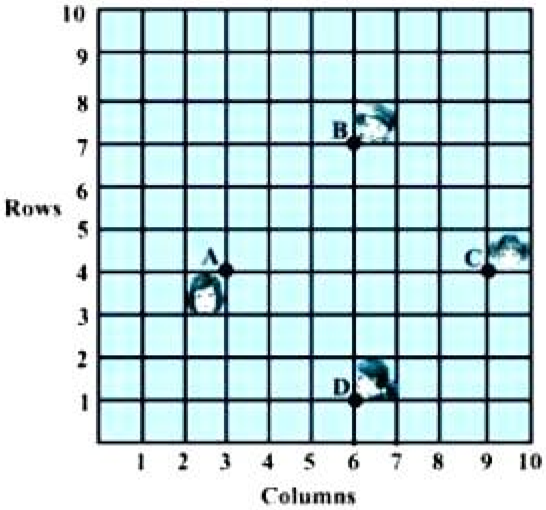
Part-B

21. The decimal expansion of $\frac{51}{2^3 \times 5^2}$ will terminate after how many decimal places?

22. Find the distance of the point P(6, -6) from the origin.

OR

In a classroom, 4 friends are seated at the four points A, B, C and D as shown in Fig. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, Don't you think ABCD is a square? Chameli disagrees. Using distance formula, find which of them is correct.



23. Find the condition which must be satisfied by the coefficients of the polynomial f(x) = $x^3 - px^2 + qx - r$ when the sum of its two zeros is zero.
24. Construct a quadrilateral similar to a given quadrilateral with sides $\frac{4}{7}$ of the corresponding sides of ABCD.
25. If $\tan^2\theta = 1 - a^2$, prove that $\sec\theta + \tan^3\theta \operatorname{cosec}\theta = (2 - a^2)^{3/2}$

OR

Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2cosec\theta$

26. If PA and PB are tangents from an outside point P. such that PA = 10 cm and $\angle APB = 60^o$. Find the length of chord AB.
27. Find the LCM of the following polynomials:$a^8 - b^8$ and $\left(a^4 - b^4\right)(a + b)$
28. Solve the quadratic equation by factorization:
$\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}, x \neq 5, 7.$

OR

Solve: $\frac{2}{(x+1)} + \frac{3}{2(x-2)} = \frac{23}{5x}, x \neq 0, -1, 2.$

29. If one root of the quadratic polynomial $2x^2 - 3x + p$ is 3, find the other root. Also, find the value of p.
30. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

OR

E is a point on the side AD produced of a || gm ABCD and BE intersects CD at F. Show that $AB \times BC = AE \times CF$.

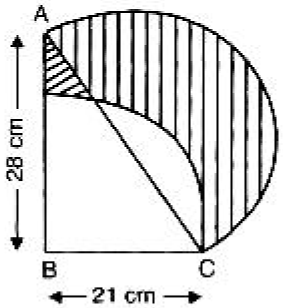
31. Cards numbered 1 to 30 are put in a bag. A card is drawn at random. Find the probability that the drawn card is
    i. prime number > 7
    ii. not a perfect square

32. If a 1.5-m-tall girl stands at a distance of 3m from a lamp-post and casts a shadow of length 4.5m on the ground then find the height of the lamp-post.

33. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Age in years	0 - 10	10 -20	20 - 30	30 - 40	40 - 50
No. of persons	5	25	?	18	7

34. In the fig., ABC is a right-angled triangle, $\angle B = 90^{\circ}$, AB = 28 cm and BC = 21 cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region correct to two decimal places.



35. Solve the following equations for x and y :

$mx - ny = m^2 + n^2$

$x + y = 2m$

36. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

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Solution

Part-A

1. The given number is $\frac{129}{2^2 5^7 7^5}$

   Since, the denominator is not in the form of $2^m \times 5^n$, as it has 7 in denominator.

   So, the decimal expansion of $\frac{129}{2^2 5^7 7^5}$ is non-terminating repeating.

   OR

   We have

   $5 \times 11 \times 13 + 13 = 13 \times (5 \times 11 + 1) = 13 \times 56$

   Clearly, it shows that the given number has more than two factors. Hence, given number is a composite number.

2. Given equation is $5x^2 + 5x + 6 = 0$

   Here a = 5, b = 5, c = 6

   $D = b^2 - 4ac = (5)^2 - 4 \times 5 \times 6 = -95$

3. $\frac{2x+5y}{xy} = 6$..........(i)

   $\frac{4x-5y}{xy} = -3$.......(ii)

   Adding (i) and (ii),

   $\Rightarrow \frac{6}{y} = 3$

   $\Rightarrow y = 2$

   Substituting in (i),we get $x = 1$

   $\therefore x = 1 \; and \; y = 2$

4. Two parallel tangents of a circle can be drawn only at the end points of the diameter

   $\Rightarrow l_1 || l_2$

   $\Rightarrow$ Distance between l_1 and l_2 = AB = Diameter of the circle

   $= 2r = 2 \times 7cm = 14cm$

5. The given A.P is 20, 17, 14, 11, ...

   First term, a = 20

   Common difference, d = - 3

nth term of the A.P, $a_n = a + (n - 1)d = 20 + (n - 1) \times (-3)$

$\therefore$ 35th term of the A.P, $a_{35} = 20 + (35 - 1) \times (-3) = 20 - 102 = -82$

OR

$S_n = \frac{n}{2}[a + l]$

$\Rightarrow S_{1000} = \frac{1000}{2}[1 + 1000] = 500500$

6. Here a = 5, d = (8 - 5) = 3 and n = 24.

Using the formula, $S_n = \frac{n}{2} \times \{2a + (n - 1)d\}$, we get

$S_{24} = \frac{24}{2} \times \{2 \times 5 + (24 - 1) \times 3\}$ [$\because$ a = 5, d = 3 and n = 24]

$= 12 \times (10 + 69) = 948.$

Hence, the sum of first 24 terms of the given AP is 948.

7. The given equation is: $x + \frac{1}{x} = 3$

$\Rightarrow x^2 - 3x + 1 = 0$

Now Discriminant $D = (-3)^2 - 4 \times 1 \times 1 = 9 - 4 = 5$

So roots of the given equation = $\frac{-(-3)\pm\sqrt{D}}{2\times1} = \frac{3\pm\sqrt{5}}{2}$

$x = \frac{3\pm\sqrt{5}}{2}$ are the roots of the equation.

OR

As per the formula to find out value of x :

$x = -\frac{b\pm\sqrt{b^2-4ac}}{2a}$

According to the equation ,

a = 2

$b = -\sqrt{5}$

c = -2

Put the values

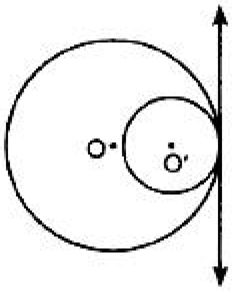
we get

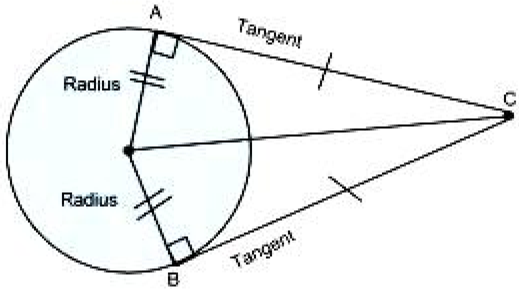
$x = \frac{+\sqrt{5}\pm\sqrt{5-4\times2\times-2}}{2\times2}$

$x = \frac{\sqrt{5}\pm\sqrt{21}}{4}$

8. 1 common tangent can be drawn to two circles touching internally

Figure:



9. 

Consider C is an external point as shown in figure.

According to theorem, from an external point only two tangents can be drawn to a circle.

So, value of k = 2

OR

Two parallel tangents can exist at the two ends of the diameter of the circle. Therefore, the distance between the two parallel tangents will be equal to the diameter of the circle.

In the problem the radius of the circle is given as 5 cm. Therefore,

Diameter = 5×2

Diameter = 10 cm

Hence, the distance between the two parallel tangents is 10 cm.

10. Let the triangles be $\triangle$ABC and $\triangle$DEF

Then the ratio of their area is =

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)}$$

$$= \frac{5^2}{6^2} = \frac{25}{36}$$

11. Given A.P is

119, 136, 153, 170........

We know that common difference is the difference between any two consecutive terms of an A.P.

So, common difference = 136 - 119 = 17

5th term = 170 + 17 = 187 ($a_5 = a + 4d$)

6th term = 187 + 17 = 204. (a_6 = a + 5d)

12. Given ,

$\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$

$\Rightarrow \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\Rightarrow \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \sqrt{3}\tan 2x = 1$

$\Rightarrow \tan 2x = \frac{1}{\sqrt{3}}$

$\Rightarrow$ tan 2x = tan30 ($Since, \tan 30^\circ = \sqrt{\frac{1}{3}}$)

$\Rightarrow 2x = 30^\circ$

$\Rightarrow x = 15^\circ$

13. We have,

$\text{LHS} = \tan^2\theta \cos^2\theta \left[\because\ \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\right]$

$= \sin^2\theta$

$= 1 - \cos^2\theta \left[\because \sin^2\theta = 1 - \cos^2\theta\right]$

= RHS

Hence proved.

14. No. of spheres = 12

Radius of given cone, r = 1 cm

Height of the given cone = 48 cm

Let the radius of sphere be R cm

According to question,

12 (Volume of one sphere) = Volume of the cone

$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$

$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (1)^2 \times 48$

$16R^3 = 16$

$R^3 = 1$

R = 1cm

Radius of each sphere = 1cm

15. We have

a = 10, d = (7-10) = -3, l= -62 and n = 11.

$\therefore$ 11th term from the end = [l - (n -1) $\times$ d]

= {-62 - (11 -1) $\times$ (-3)}

= (-62 + 30) = -32.

Hence, the 11th term from the end of the given AP is -32.

16. Given: A card is drawn at random from a pack of 52 cards

TO FIND: Probability of the following

Total number of cards = 52

Total number of ten is 4

We know that PROBABILITY $= \frac{\text{Number of favourable event}}{\text{Total number of event}}$

Hence probability of getting a ten is $\frac{4}{52} = \frac{1}{13}$

17. Let Home represented by point H(4, 5), Library by point L(-1, 3), Skate Park by point P(3, 0) and School by S(4, 2).

i. (b) Distance between Home and School, HS = $\sqrt{(4-4)^2 + (2-5)^2}$ = 3 metres

ii. (c) Now, HL = $\sqrt{(-1-4)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29}$

LS = $\sqrt{[4-(-1)]^2 + (2-3)^2} = \sqrt{25+1} = \sqrt{26}$

Thus, HL + LS = $\sqrt{29} + \sqrt{26}$ = 10.48 metres

So, extra distance covered by Ramesh is = HL + LS - HS = 10.48 - 3 = 7.48 metres

iii. (d) Now, HP = $\sqrt{(3-4)^2 + (0-5)^2} = \sqrt{1+25} = \sqrt{26}$

PS = $\sqrt{[4-3]^2 + (2-0)^2} = \sqrt{1+4} = \sqrt{5}$

Thus, HP + PS = $\sqrt{26} + \sqrt{5}$ = 7.33 metres

So, extra distance covered by Pulkit is = HP + PS - HS = 7.33 - 3 = 4.33 metres

iv. (a) (-1, 3)

v. (c) (4, 5)

18. i. (c) 15 cm

ii. (d) They are not the mirror image of one another

iii. (b) Their altitudes have a ratio a:b

iv. (d) 5 m

v. (c) 6 m

19. i. (b) Centered at the class marks of the classes

ii. (c) 80

iii. (a) 44.7

iv. (b) Classmark

v. (d) 42

20. i. (c) Volume of the cuboid

$= 15 \times 10 \times 3.5 = 525\text{cm}^3$

ii. (b) Volume of a conical depression

$= \frac{1}{3}\pi(0.5)^2(1.4)$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times \frac{14}{10} = \frac{11}{30}\text{cm}^3$

Volume of four conical depressions

$= 4 \times \frac{11}{30}\text{cm}^3 = \frac{22}{15}\text{cm}^3$

iii. (a) Volume of the wood in the entire stand

$= 525 - 1.47 = 523.53\text{cm}^3$

iv. (d) The formula of TSA of cone is given by:

$\pi rl + \pi r^2$

v. (c) Remain unaltered

Part-B

21. Given term is $\frac{51}{2^3 \times 5^2}$

It can be written as $\frac{51}{8 \times 25}$

= $\frac{51}{200}$

= 0.255

Therefore decimal expansion terminates after 3 points.

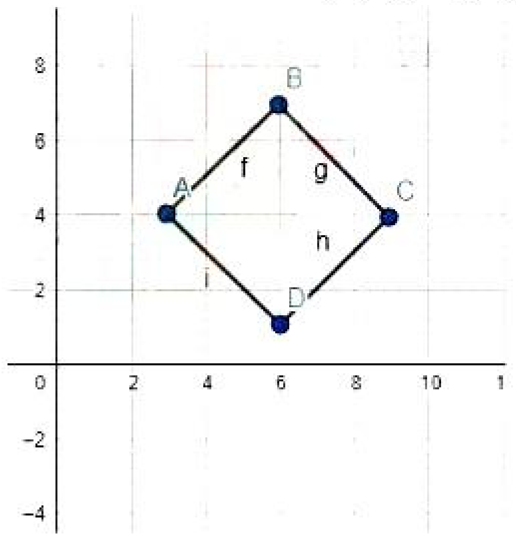
22. Let P(6, -6) be the given point and O(0, 0) be the origin.

Then, $OP = \sqrt{(6-0)^2 + (-6-0)^2} = \sqrt{6^2 + (-6)^2}$

$= \sqrt{36+36} = \sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}\ units.$

OR

It can be seen that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of 4 friends



Distance between two points A(x_1, y_1) and B(x_2, y_2) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Hence,

AB = $[(3-6)^2 + (4-7)^2]^{1/2}$

= $\sqrt{9+9} = \sqrt{18}$

= $3\sqrt{2}$

BC = $[(6-9)^2 + (7-4)^2]^{1/2}$

= $\sqrt{9+9} = \sqrt{18}$

= $3\sqrt{2}$

CD = $[(9-6)^2 + (4-1)^2]^{1/2}$

= $\sqrt{9+9} = \sqrt{18}$

= $3\sqrt{2}$

AD = $[(3-6)^2 + (4-1)^2]^{1/2}$

= $\sqrt{9+9} = \sqrt{18}$

= $3\sqrt{2}$

Diagonal AC = $[(3-9)^2 + (4-4)^2]^{1/2}$

= $\sqrt{36+0}$ = 6

Diagonal BD = $[(6-6)^2 + (7-1)^2]^{1/2}$

= $\sqrt{36+0}$ = 6

It can be seen that all sides of quadrilateral ABCD are of the same length and diagonals are of the same length

Therefore, ABCD is a square and hence, Champa was correct.

23. Let α, β and γ be the zeroes of the polynomial f(x) such that $\alpha + \beta$ = 0

Now, Sum of the zeroes$= -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\alpha + \beta + \gamma = -\left(\frac{-\text{p}}{1}\right)$

$\Rightarrow \quad \alpha + \beta + \gamma = p$

$\Rightarrow \quad 0 + \gamma = p$

$\Rightarrow \quad \gamma = p$

Now since γ is a zero of f(x)

and γ = p

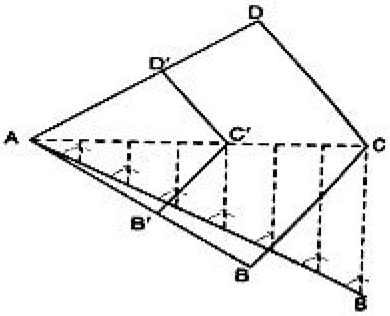
Hence f(p)=0

$p^3 - p \times p^2 + pq - r = 0$

pq - r = 0

Hence pq = r is the condition

24. Steps of construction:
    i. Draw a quadrilateral ABCD with the given data.
    ii. Join AC and divide it into 7 equal parts.
    iii. Let C' be the point which divides AC in the ratio 4 : 3, i.e.
    $AC' = \frac{4}{7}AC$
    iv. From C', draw lines parallel to CB and CD meeting AB and AD at B' and D' respectively.
    AB'C'D' is the required quadrilateral



25. We have,

L.H.S $= \sec\theta + \tan^3\theta \operatorname{cosec}\theta$

$= \sec\theta[\frac{\sec\theta + \tan^3\theta . cosec\theta}{\sec\theta}]$ [Multiplying numerator and denominator by $\sec\theta$]

$= \sec\theta[1 + \tan^3\theta \cdot \frac{\cos\theta}{\sin\theta}]$

$= \sec\theta\left\{1 + \tan^3\theta \times \cot\theta\right\} \left[\because \frac{\cos\theta}{\sin\theta} = cot\theta\right]$

$= \sqrt{1+\tan^2\theta}[1 + \tan^2\theta] = \left(1 + \tan^2\theta\right)^{\frac{1}{2}} . \left\{1 + \tan^2\theta\right\}$

$= (1+ \tan^2\theta)^{3/2} = [1 + (1 - a^2)]^{3/2} = (2 - a^2)^{\frac{3}{2}}$ [$\because \tan^2\theta = 1 - a^2$]

Therefore, $\sec\theta + \tan^3\theta \operatorname{cosec}\theta = (2 - a^2)^{\frac{3}{2}}$

OR

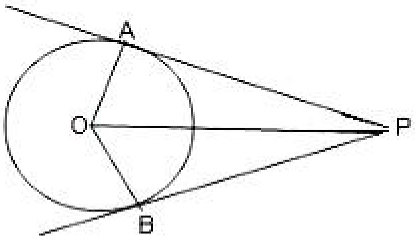
We have, $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= 2 \operatorname{cosec}\theta$

26. 

$\because \angle APB = 60^o$

$\angle AOB = 120^o$[O is centre of circle]

$\angle OAB = \angle OBA = 30^o$

$\therefore \angle PAB = 60^o, \angle PBA = 60^o$

$\therefore \triangle PAB$ is equilateral triangle

$\therefore AB = PA = 10$ cm

27. $P(x) = a^8 - b^8 = (a^4 + b^4)(a^4 - b^4)$
$= (a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$
$= (a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$ $Q(x) = (a + b)(a^4 - b^4)$
$= (a + b)(a^2 + b^2)(a^2 - b^2)$
$= (a + b)(a^2 + b^2)(a + b)(a - b)$ {Using Identity $a^2 - b^2 = (a + b)(a - b)$}
Common factors: $(a^2 + b^2), (a - b), (a + b)$
Uncommon factors: $(a^4 + b^4), (a + b)$
$\therefore LCM$ of $P(x)$ and $Q(x)$
$= (a^2 + b^2)(a - b)(a + b) \times (a^4 + b^4)(a + b)$
$= (a^4 + b^4)(a^2 + b^2)(a + b)^2(a - b)$

28. The given equation is $\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}$, $x \neq 5, 7$

$\Rightarrow \frac{(x-4)(x-7)+(x-6)(x-5)}{(x-5)(x-7)} = \frac{10}{3}$

$= \frac{x^2-11x+28+x^2-11x+30}{x^2-12x+35} = \frac{10}{3}$

$\Rightarrow 3[2x^2 - 22x + 58] = 10[x^2 - 12x + 35]$

$\Rightarrow 6x^2 - 66x + 174 = 10x^2 - 120x + 350$

$\Rightarrow 4x^2 - 54x + 176 = 0$

$\Rightarrow 2x^2 - 27x + 88 = 0$

$\Rightarrow 2x^2 - 16x - 11x + 88 = 0$

$\Rightarrow 2x(x - 8) - 11(x - 8) = 0$

$\Rightarrow (2x - 11)(x - 8) = 0$

$\Rightarrow x = \frac{11}{2}, 8$

OR

Given,

$\frac{2}{(x+1)} + \frac{3}{2(x-2)} = \frac{23}{5x}$

Taking LCM, we get

$\Rightarrow \frac{4(x-2)+3(x+1)}{2(x+1)(x-2)} = \frac{23}{5x}$

$\Rightarrow \frac{7x-5}{2(x^2-x-2)} = \frac{23}{5x}$

By cross multiplication

$\Rightarrow 5x(7x - 5) = 46(x^2 - x - 2)$

$\Rightarrow 35x^2 -25x = 46x^2 - 46x - 92$

$\Rightarrow 46x^2 - 35x^2 - 46x + 25x - 92 = 0$

$\Rightarrow 11x^2 - 21x - 92 = 0$

$\Rightarrow 11x^2 - 44x + 23x - 92 = 0$

$\Rightarrow 11x(x - 4) + 23(x - 4) = 0$

$\Rightarrow (x - 4)(11x + 23) = 0$

$\Rightarrow x - 4 = 0$ or $11x + 23 = 0$

$\Rightarrow x = 4$ or $x = \frac{-23}{11}$

Therefore, 4 or $\frac{-23}{11}$ are the roots of the given equation.

29. The given quadratic polynomial is $p(x) = 2x^2 - 3x + p$

Since, 3 is a root (zero) of p(x)

$\Rightarrow 2(3)^2 - 3 \times 3 + p = 0$

$\Rightarrow 18 - 9 + p = 0$

$\Rightarrow 9 + p = 0$

$\Rightarrow p = - 9$

Now $p(x) = 2x^2 - 3x - 9$

$= 2x^2- 6x + 3x - 9$

$= 2x (x - 3) + 3 (x - 3)$

$= (x - 3) (2x + 3)$

For roots of polynomial, $p(x) = 0$

$\Rightarrow (x - 3) (2x + 3) = 0$

$\Rightarrow x = 3$ or $x = -\frac{3}{2}$

Hence the other root is $-\frac{3}{2}$.

30. Let ABCD is a rhombus with AC and BD as its diagonals.

We know that the diagonals of a rhombus bisect each other at right angles.

Let O be the intersecting point of both the diagonals.

Let AC = 30 cm and BD = 40 cm

OA = AC/2

OA = 30/2

= 15 cm

OB = BD/12

OB = 40/2

= 20 cm

In right $\triangle$AOB by pythagoras theorem we have

$AB^2 = OA^2 + OB^2$

$= (15)^2 + (20)^2$

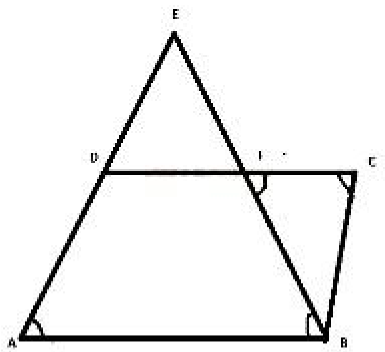
= 225 + 400

= 625

AB = 25 cm

Hence, each side of the rhombus is length 25 cm.

OR



Given: ABCD is a parallelogram

To prove: AB $\times$ BC = AE $\times$ CF

Proof : In ΔABE and ΔCFB,

$\angle$A = $\angle$C (Opposite angles of a parallelogram)

$\angle$AEB = $\angle$CBF (Alternate interior angles as AE || BC)

$\Delta ABE \sim \Delta CFB$ (By AA similarity criterion)

Since corresponding sides of similar triangles are proportional,

$\therefore$ AB/CF=AE/BC

or AB $\times$ BC = AE $\times$ CF

Hence proved.

31. No. of possible outcomes = 30

i. P(prime no. > 7) = 11,13,17,19,23,29 so m=6

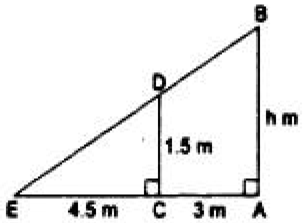
$P(E_1)=\frac{m}{n}=\frac{6}{30}=\frac{1}{5}$

No. of perfect square are 1,4,9,16,25, = 5

ii. No. of non perfect square = 30 - 5 = 25 so m=25

iii. P(not a perfect square) = $\frac{m}{n}=\frac{25}{30}=\frac{5}{6}$

32. Let AB be the lamp-post and CD be the girl.



Let CE be the shadow of CD. Then,

CD = 1.5m, CE = 4.5m and AC = 3m.

Let AB = h m.

Now, ΔAEB and ΔCED are similar.

$\therefore \quad \frac{AB}{AE}=\frac{CD}{CE} \Rightarrow \frac{h}{(3+4.5)}=\frac{1.5}{4.5}=\frac{1}{3}$

$\Rightarrow h=\frac{1}{3}\times 7.5=2.5$

33.

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	25	30
20-30	x	30 + x
30-40	18	48 + x
40-50	7	55 + x
	N = 55 + x	

Let the missing frequency be x

Given, Median = 24......(1)

From table, total frequency N = 55 + x Or, (N/2) = 27.5 +(x/2)

Hence, c.f. just greater than (N/2) is (30+x), which corresponding class is 20 - 30.

Then, median class = 20 - 30

∴ l = 20, h = 30 - 20 =10, f =x, F = 30

$$\therefore Median = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow \quad 24 = 20 + \frac{\frac{55+x}{2} - 30}{x} \times 10$$

$$\Rightarrow \quad 24 - 20 = \frac{\frac{55+x}{2} - 30}{x} \times 10$$

$$\Rightarrow \quad 4x = \left(\frac{55+x}{2} - 30\right) \times 10$$

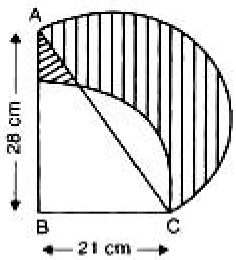
⇒ 4x = 5 (55 + x) - 300

⇒ 4x - 5x = -25

⇒ -x = -25

⇒ x = 25

∴ Missing frequency = 25.

34. 

In △ABC, ∠B = 90°

$AC^2 = AB^2 + BC^2$

$AC^2 = 28^2 + 21^2$

$AC^2 = 784 + 441$

$AC^2 = 1225$

AC = 35 cm

Area of shaded region = Area of triangle + Area of semi-circle with diameter AC - area of quadrant with radius BC

$$= \frac{1}{2} \times (21 \times 28) + \frac{1}{2} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} - \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$$
$$= (7 \times 28) + 11 \times \frac{5}{2} \times \frac{35}{2} - \frac{1}{2} \times \frac{11}{1} \times 3 \times 21$$
$$= 294 + 481.25 - 346.5$$

$= 775.25 - 346.5$

$= 428.75\ cm^2.$

35. $mx - ny = m^2 + n^2$ ...(i)

$x + y = 2m$ ...(ii)

Multiplying equation (ii) by m and subtracting from (i), we get

$$mx - ny = m^2 + n^2$$
$$mx + my = 2m^2$$
$$- \quad - \quad -$$
$$-(m+n)y = m^2 + n^2 - 2m^2$$

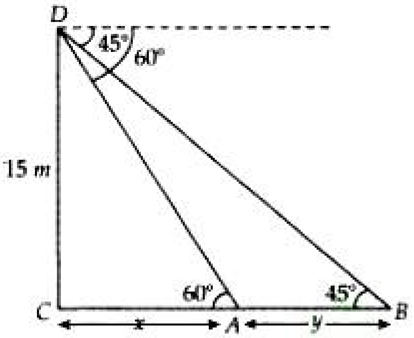
$y = \frac{n^2 - m^2}{-(m+n)} = -\frac{(n-m)(n+m)}{m+n}$

$y = m - n$

Putting value of y in (i), we get

$$mx - n(m-n) = m^2 + n^2$$
$$mx - nm + n^2 = m^2 + n^2$$
$$mx = m^2 + nm = \frac{m(m+n)}{m}$$

$x = m + n$

36. 

In $\triangle DCA\ \frac{DC}{CA} = \tan 60°$

$\Rightarrow \quad \frac{15}{x} = \sqrt{3}$

$\Rightarrow \quad x = \frac{15}{\sqrt{3}}$

$\Rightarrow \quad x = 5\sqrt{3}$

In $\triangle$ BCD

$\frac{DC}{BC} = \frac{15}{x+y} = tan45^o = 1$

$x + y = 15$

$5\sqrt{3} + y = 15$

$y = 15 - 5\sqrt{3} = 5(3 - \sqrt{3}) = 6.35$ meter

The distance between the two points is 6.35 meter