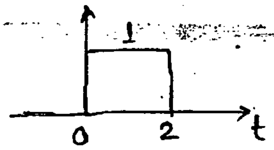
* Different operations on signal →

* Amplitude shifting
* Time shifting
* Time scaling
* Time reversal
* Amplitude Reversal
*

(1.) Time shifting →

$$x(t) \longrightarrow y(t) = x(t+k)$$

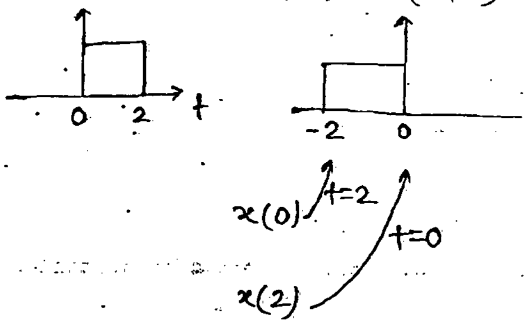


Case(1)

When $k>0$

Eg:- $k=2$

$$x(t) \longrightarrow y(t) = x(t+2)$$



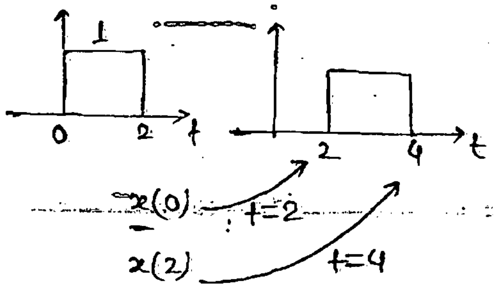
* It is a case of left shifting.

Case(2)

When $k<0$

Eg:- $k=-2$

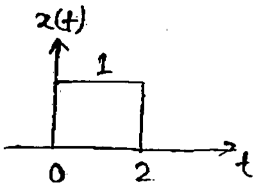
$$x(t) \longrightarrow y(t) = x(t-2)$$



* It is a case of right shifting

(2.) Amplitude shifting →

$$x(t) \longrightarrow y(t) = k + x(t)$$



$x(t) = \{ \; 0 \quad , \; t<0$

Case(1) → when $k<0$

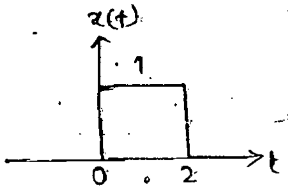
Eg:- $k=-1$

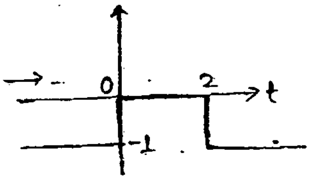
$x(t) \longrightarrow y(t) = -1 + x(t)$

$$y(t) = -1 + x(t)$$

$$= \begin{cases} -1+0 & , t<0 \\ -1+1 & ; 0 \le t \le 2 \\ -1+0 & ; t>2 \end{cases}$$

$$= \begin{cases} -1 & , t<0 \\ 0 & ; 0 \le t \le 2 \\ -1 & ; t>2 \end{cases}$$





* It is a case of downward shifting

Case(2) → When $k>0$

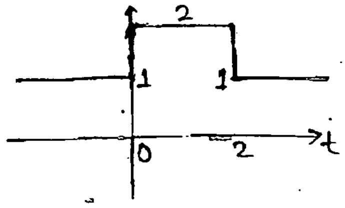
Eg:- $k=+1$

$x(t) \longrightarrow y(t) = 1 + x(t)$

$$y(t) = 1 + x(t)$$

$$= \begin{cases} 1+0 & , t<0 \\ 1+1 & ; 0 \le t \le 2 \\ 1+0 & ; t>2 \end{cases}$$

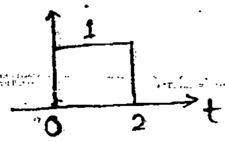
$$= \begin{cases} 1 & ; t<0 \\ 2 & ; 0 \le t \le 2 \\ 1 & ; t>2 \end{cases}$$



* It is a case of upward shifting

(3.) Time Scaling →

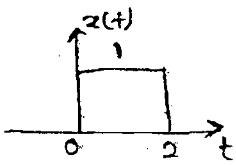
$x(t) \longrightarrow y(t) = x(at)$

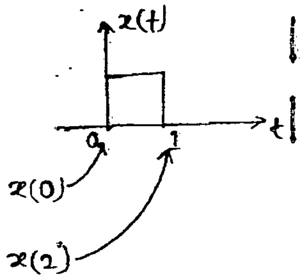


Case(1) → when $a>1$

Eg:- $a=2$

$x(t) = y(t) = x(2t)$



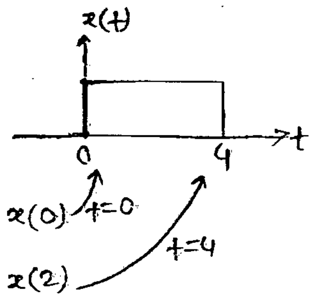


Time Compression

Case(2) → when $a<1$

Eg:- $a=0.5$

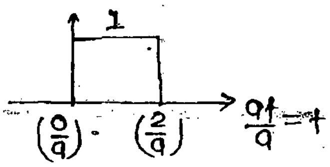
$x(t) = y(t) = x(0.5t)$



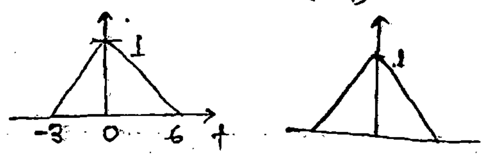
Time expansion

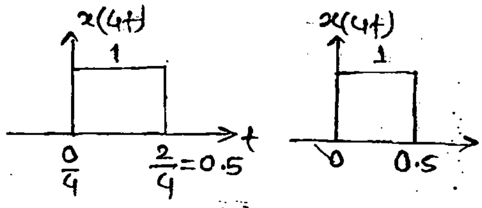
Rule General →

$x(at) \longrightarrow t$



Ex:- $x(t) \longrightarrow x(-3t)$



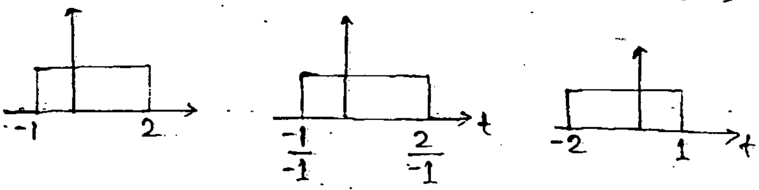


(4.) Time-reversal →

$$x(t) = y(t) = x(-t)$$

* Time reversal is a special case of time scaling in which signal folding will take palce around y-axis.

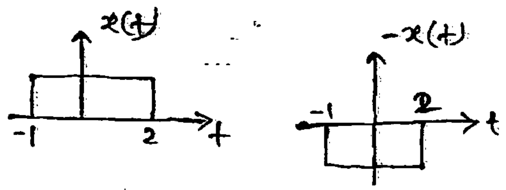
$x(-t) = a(=-1)$



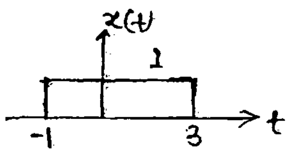
(5.) Amplitude Reversal →

$$x(t) \longrightarrow y(t) = -x(t)$$

* In this case, signal folding will take place about x-axis



Q→



Draw signal $y(t)$ if $y(t) = x(2t+1)$

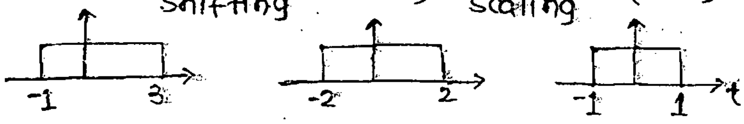
Solⁿ→ 1st method→

$$x(t) \xrightarrow[\text{scaling}]{\text{time}} x(2t) \xrightarrow[\text{shifting}]{\text{time}} x[2(t+0.5)]$$

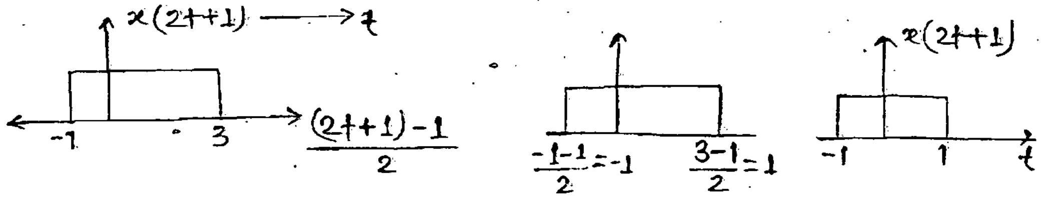
$y(t)$

2nd method →

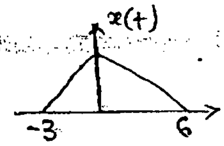
$x(t) \xrightarrow[\text{shifting}]{\text{time}} x(t+1) \xrightarrow[\text{scaling}]{\text{time}} x(2t+1)$



3rd method → (Trick)



Q →

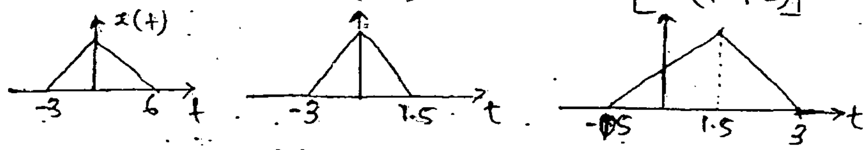


draw sig y(t) if $y(t) = x(-2t+3)$

Soln → 1st method →

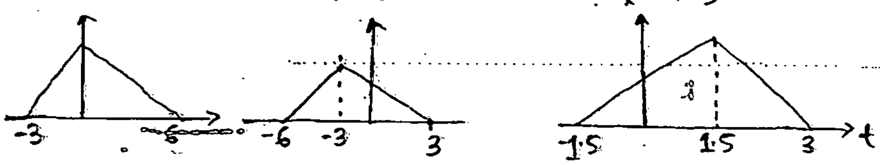
$$y(t) = x[-2(t-1.5)]$$

$x(t) \longrightarrow x(-2t) \longrightarrow x[-2(t-1.5)]$

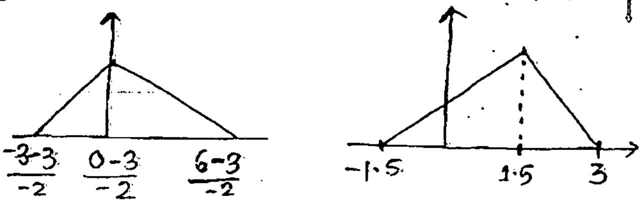


2nd method →

$x(t) \longrightarrow x(t+3) \longrightarrow x(2t+3)$



3rd method →



Chapter-01
Signal defination & Classifications

Signal → A signal is a f^n which contains some information.

System → A sys. is interconnection of devices (or) componants which converts signal from one form to another form.

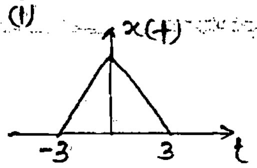
Classification of signals →

1) Even & odd signals →

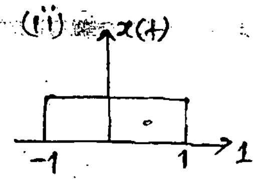
* Even → This are symmetrical (or) mirror image about y-axis.

i.e. → $\boxed{x(t) = x(-t)}$ → time reversal

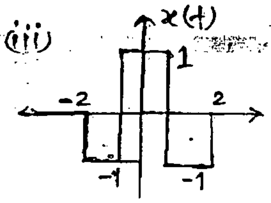
Eg:- (i)



(ii)



(iii)



(iv) $x(t) = \cos\omega_0 t$ (Even)

$t = -t$

$x(-t) = \cos\omega_0(-t)$

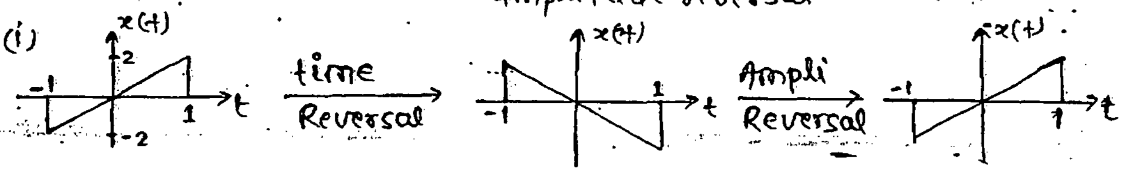
$= \cos\omega_0 t$

$x(-t) = x(t)$

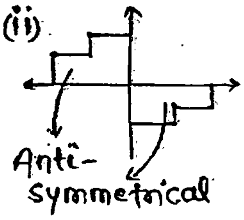
* Odd → This are antisymmetrical about y-axis.

i.e. $\boxed{\begin{array}{c} x(-t) = -x(t) \\ \text{(or)} \\ x(t) = -x(-t) \end{array}}$ — time reversal, amplitude reversal

Eg:- (i)



(ii)



(iii) $x(t) = \sin\omega_0 t$ → odd signal..

$(t = -t)$

$x(-t) = \sin\omega_0(-t)$

$x(-t) = -\sin\omega_0 t$

$\boxed{x(-t) = -x(t)}$

* The avg. value of an odd signal is 0; but converse of this statement is not true.

Important points →

Important points →

(1.) Even × Even = Even ; $t^2 \times t^4 = t^6$

(2) Even × odd = odd ; $t^2 \times t^3 = t^5$

(3.) Odd × odd = Even ; $t^3 \times t^5 = t^8$

(4.) Even ± Even = even

$x(t) = t^2 + \cos t$

$x(-t) = t^2 + \cos t = x(t)$

(5.) Odd + Odd = Odd

$x(t) = \sin t + t^3$

$x(-t) =$ [illegible]

$\boxed{x(-t) = -x(t)}$

(6.) Even + odd = Neither even nor odd.

$x(t) = t^2 + \sin t$

$x(-t) = t^2 - \sin t$

$\boxed{x(-t) \neq x(t)}$

* Any signal can be devided into 2 part in which one part will be even & the other part will be odd.

i.e. $\boxed{x(t) = x_E(t) + x_O(t)}$

where;

$x_E(t)$ = even part of $x(t)$ $= \dfrac{x(t) + x(-t)}{2}$

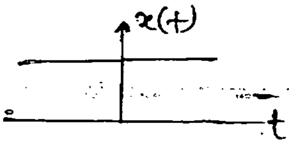
$x_O(t)$ = Odd part of $x(t)$ $= \dfrac{x(t) - x(-t)}{2}$

Eg → $x(t) = 2$ = dc signal

$\downarrow t = -t$

$x(-t) = 2 = x(t)$ [Even signal]

dc signal is a Even signal.



(2.) $f(k) = \sin(k^2)$

$\downarrow k = -k$

$f(-k) = \sin(k^2) = f(k)$ [Even signal]

(3.) $f(r) = \sin \pi/2$

$= 1$

$f(r) = f(-r)$ [Even signal]

(4) Find $x_E(t)$ & $x_O(t)$ of the signal.

$$x(t) = 3 - \frac{t^2}{\sin t} + \frac{\cos t}{t} - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t$$

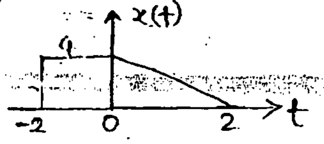
$$E - \frac{E}{O} + \frac{E}{O} - \frac{E}{E} + O \times O$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

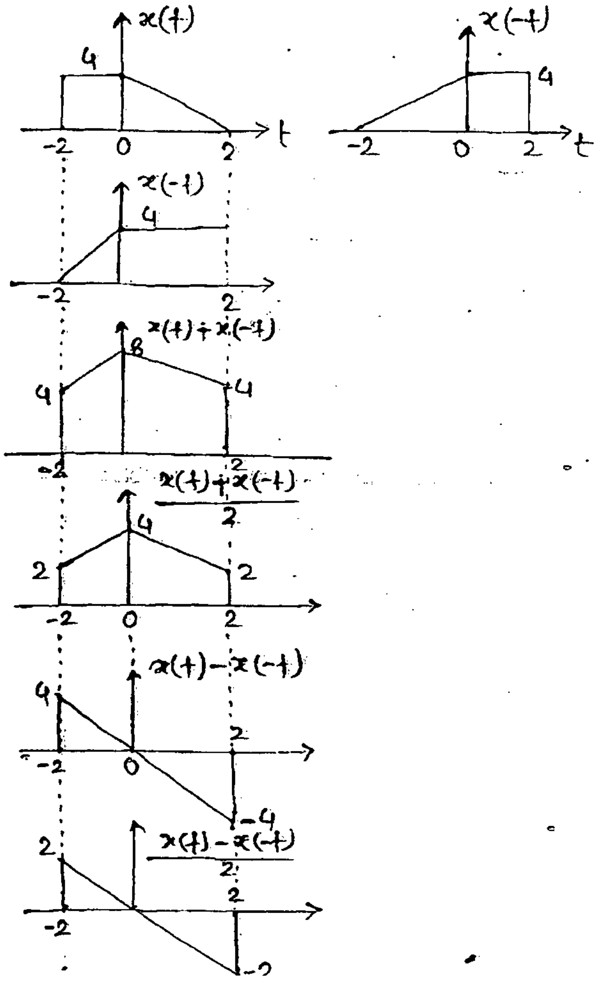
$$E \qquad O \qquad O \qquad E \qquad E$$

$$x_E(t) = 3 - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t \quad , \quad x_O(t) = \frac{-t^2}{\sin t} + \frac{\cos t}{t}$$

Que. → Draw $x_E(t)$ & $x_O(t)$ of



Sol^n → for even part of $x(t)$



(2) Conjugate Symmetric (CS) & Conjugate antisymmetric (CAS) signal →

★ Conjugate symmetric (CS)

$$\boxed{x(t) = x^*(-t)}$$

$$x(t) = a(t) + jb(t) \quad \text{---(i)}$$

$$(t = -t)$$

$$x(-t) = a(-t) + jb(-t)$$

$$x^*(-t) = a(-t) - jb(-t) \quad \text{---(ii)}$$

From eq^n (i) & (ii)

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$$a(t) = a(-t) \longrightarrow \text{Even}$$

$$b(t) = -b(-t) \longrightarrow \text{Odd}$$

Eg:- $x(t) = \underset{E}{\underset{\downarrow}{t^2}} + \underset{O}{\underset{\downarrow}{\sin t}}$

★ Conjugate antisymmetric (CAS)

$$\boxed{x(t) = -x^*(-t)}$$

$$x(t) = a(t) + jb(t)$$

$$a(t) = -a(-t) \longrightarrow \text{Odd}$$

$$b(t) = b(-t) \longrightarrow \text{Even}$$

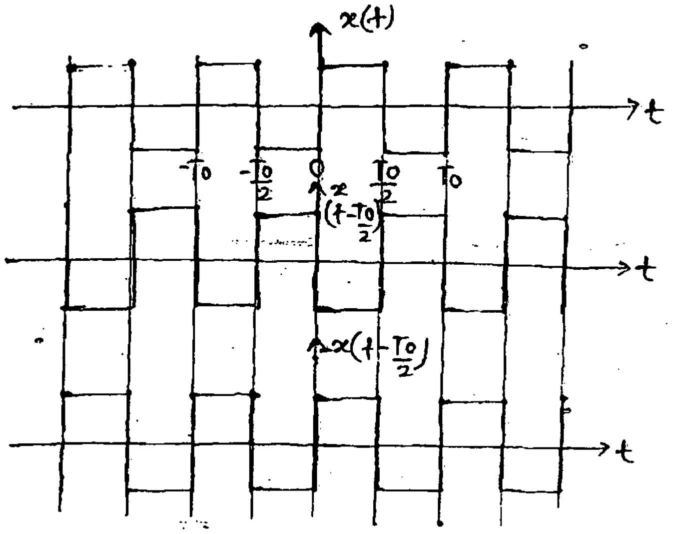
Eg:- $x(t) = \underset{O}{\underset{\downarrow}{\sin t}} + j\underset{E}{\underset{\downarrow}{t^2}}$

(3) Halfwave Symmetric signal (HWS) →

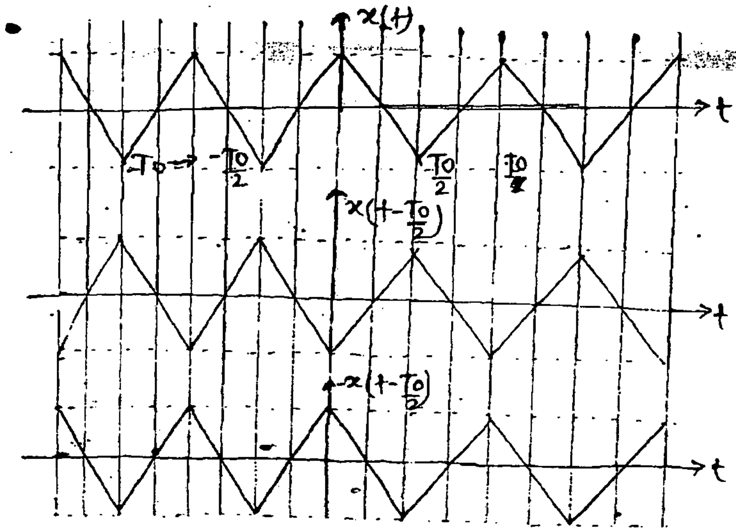
For Half wave symmetry (HWS)

$$\boxed{x(t) = \underset{\text{amp. reversal}}{-}x\left(t \underset{\text{time shifting}}{\pm} \frac{T_0}{2}\right)}$$

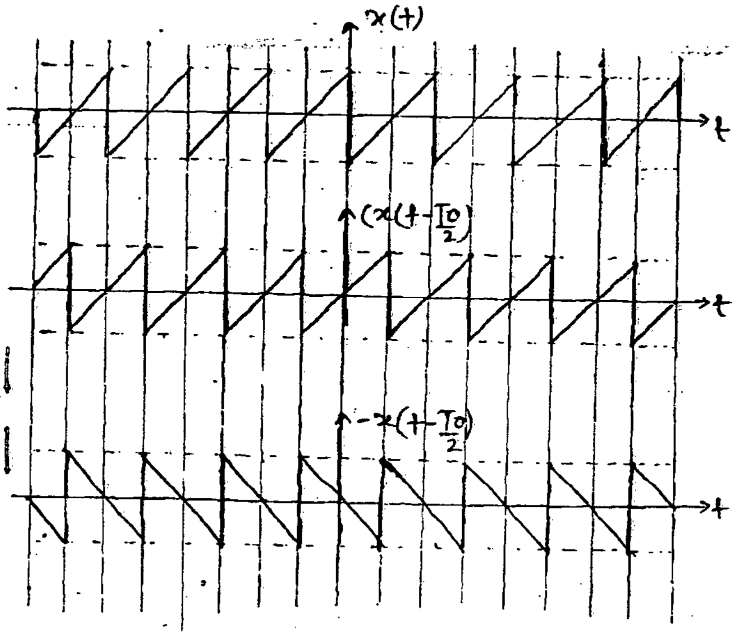
Eg→ (1.)



(2.)



(3.)



so; sawtooth wave doesn't follow the HWS.

* The avg. value of a HWS is 0. but converse of this statement is not true.

DATE-10/10/14

(4) Periodic & non-periodic signal →

Periodic → A signal repeats itself after some time period, the signal is said to be periodic.

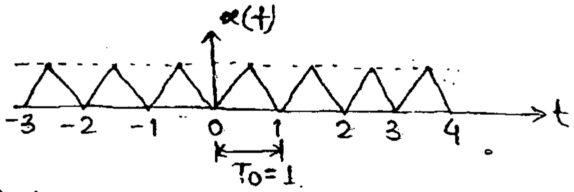
i.e. $\boxed{x(t) = x(t \pm nT_0)}$

where, n = an integer

T_0 = Fundamental time period. $\left\{ \begin{matrix} T_0 \neq 0 \\ T_0 \neq \infty \end{matrix} \right\}$

FTP → It is the smallest, +ve & fixed value of the time for which signal is periodic.

Eg →



FTP=1

Q → Find FTP of signal $x(t)$

$$x(t) = A_0 e^{j\omega_0 t}$$

Soln → Let 'T_0' be the FTP of the signal

i.e.

$$x(t) = x(t+T_0)$$

$$x(t+T_0) = A_0 e^{j\omega_0 (t+T_0)}$$

$$A_0 e^{j\omega_0 t} = A_0 e^{j\omega_0 (t+T_0)}$$

$$\cancel{A_0 e^{j\omega_0 t}} = \cancel{A_0 e^{j\omega_0 t}} e^{j\omega_0 T_0}$$

$$e^{j\omega_0 T_0} = 1 = e^{j2\pi K} \quad \text{(where } K = \text{an integer)}$$

$$j\omega_0 T_0 = j2\pi K$$

$$\underset{\text{smallest}}{T_0} = \frac{2\pi K}{\omega_0} \quad 1 \text{(least integer)}$$

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

Q.→ Find FTP of following signal →

(i) $x_1(t) = A_0 \sin(2\pi t)$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{2\pi} = 1$

(ii) $x_2(t) = A_0 \sin(2\pi t + 30^\circ)$

$\omega_0 = 2\pi$

$T_0 = 1$

(iii) $x_3(t) = -x_1(t)$

$= -A_0 \sin(2\pi t)$

$\omega_0 = 2\pi, \; T_0 = 1$

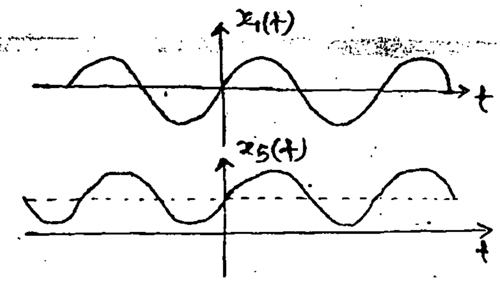
(iv) $x_4(t) = x_1(-t)$

$= -A_0 \sin 2\pi t$

$\omega_0 = 2\pi \; , T_0 = 2\pi$

(v) $x_5(t) = A_0 + x_1(t)$

$= A_0 + A_0 \sin(2\pi t)$



(vi) $x_6(t) = x_1(t - t_0)$

$= A_0 \sin[2\pi(t - t_0)]$

$\omega_0 = 2\pi$

$T_0 = 1$

* Time period of signal is unaffected by time shifting, time reversal, amp. reversal, amp. shifting & change in phase of signal.

(vii) $f(t) = \sin^2(4\pi t)$

$= \frac{1 - \cos 8\pi t}{2}$

$\omega_0 = 8\pi$

$T_0 = \frac{2\pi}{8\pi} = \frac{1}{4}$

* The sum of 2 (or) more than 2 periodic signal will be periodic if ratios of there fundamental time period (or) freq. are rational.

i.e. $x(t) = x_1(t) + x_2(t)$

$x_1(t) \downarrow T_1, f_1, \omega_1$; $x_2(t) \downarrow T_2, f_2, \omega_2$

→ $\frac{T_1}{T_2}$ (or) $\frac{\omega_1}{\omega_2}$ (or) $\frac{f_1}{f_2}$ (Rational no.)

→ $T_0 = LCM[T_1, T_2]$

→ $f_0 = HCF[f_1, f_2]$

Q→ Find FTP of signal if it is periodic :-

(i) $x(t) = \sin 2t + \cos 3\pi t$

Solⁿ→ $\omega_1 = 2$ $\quad \frac{\omega_1}{\omega_2} = \frac{2}{3\pi}$ (Irrational no.)

$\omega_2 = 3\pi$

Hence it is non-periodic

(ii) $x(t) = \sin 2\pi t + \cos\sqrt{2}\pi t$

Solⁿ→ $\omega_1 = 2\pi, \quad \omega_2 = \sqrt{2}\pi$

$$\frac{\omega_1}{\omega_2} = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2} \quad \text{(Irrational no.)}$$

Hence it is Non-periodic

(iii) $x(t) = \sin 4\pi t + \sin 7\pi t$

Solⁿ→ $\omega_1 = 4\pi, \quad \omega_2 = 7\pi$

$$\frac{\omega_1}{\omega_2} = \frac{4\pi}{7\pi} = \frac{4}{7} \quad \text{(Rational no.)}$$

Hence it is periodic. Then calculate T_0.

1st method :-

$$\omega_0 = \cancel{2\pi}\, HCF[\omega_1, \omega_2] = HCF[4\pi, 7\pi]$$

$$\omega_0 = \pi$$

$$T_0 = \frac{2\pi}{\omega_0} = 2$$

*** $HCF\left[\frac{P_1}{q_1}, \frac{P_2}{q_2}\right] = \frac{HCF[P_1, P_2]}{LCM[q_1, q_2]}$ $\quad LCM\left[\frac{P_1}{q_1}, \frac{P_2}{q_2}\right] = \frac{LCM[P_1, P_2]}{HCF[q_1, q_2]}$

2nd method →

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi} = \frac{1}{2} \qquad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7\pi} = \frac{2}{7}$$

$$T_0 = LCM[T_1, T_2] = LCM\left[\frac{1}{2}, \frac{2}{7}\right]$$

$$= \frac{LCM[1, 2]}{HCF[2, 7]} = \frac{2}{1} = 2$$

* Area & avg. value of signal →

Area of $x(t)$:-

$$\text{Area} = \int_{-\infty}^{\infty} x(\tau)\,d\tau$$

Area of $x(t)$ over Range (t_1, t_2)

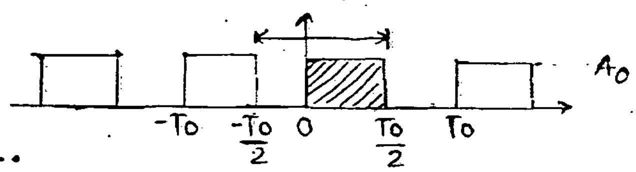
$$\text{Area} = \int_{t_1}^{t_2} x(\tau)\,d\tau$$

Avg. value of $x(t)$:

$$\text{Avg.} = \begin{cases} \dfrac{1}{T_0}\displaystyle\int_{-T_0/2}^{T_0/2} x(\tau)\,d\tau, & \text{For periodic sig.} \\ \displaystyle\lim_{T\to\infty} \dfrac{1}{T}\int_{-T/2}^{T/2} x(\tau)\,d\tau, & \text{for Non-periodic sig.} \end{cases}$$

Que.→ Find the avg. value of sig.

(i)



Sol^n →

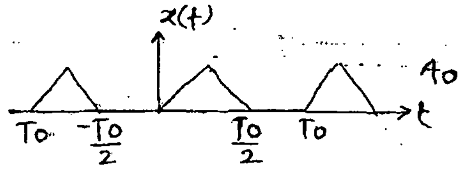
$$\text{avg.} = \frac{\displaystyle\int_{-T_0/2}^{T_0/2} x(\tau)\,d\tau}{T_0}$$

$$= \frac{\text{Area of } x(t) \text{ over } `T_0\text{'}}{T_0}$$

$$= \frac{A_0 \times \frac{T_0}{2}}{T_0}$$

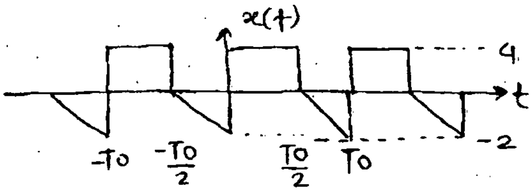
$$= \frac{A_0}{2}$$

(2.)



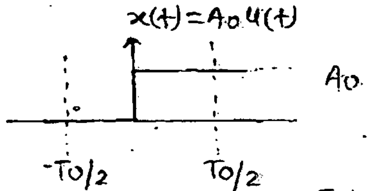
Solⁿ →
$$\text{avg} = \frac{\text{Area over } T_0}{T_0} = \frac{\frac{1}{2} \times A_0 \times \cancel{T_0}/2}{\cancel{T_0}} = \frac{A_0}{4}$$

(3.)



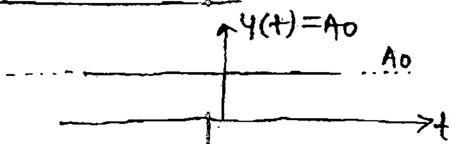
Solⁿ →
$$\text{Avg} = \frac{\text{Area over } T_0}{T_0} = \frac{-\frac{1}{2} \times \frac{T_0}{2} \times 2 + 4 \times \frac{T_0}{2}}{T_0} = \frac{3}{2}$$

(iv)



Solⁿ →
$$\text{Avg} = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} x(\tau)\,d\tau$$
$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_0^{T_0/2} A_0\,d\tau$$
$$= \lim_{T_0 \to \infty} \frac{A_0 \times T_0/2}{T_0}$$
$$= \frac{A_0}{2}$$

2nd method →



$$\text{avg } y(t) = A_0$$
$$\text{avg } x(t) = \frac{\text{avg } y(t)}{2}$$
$$= \frac{A_0}{2}$$

(5.) Energy & power signal →

* Energy of $x(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$\boxed{E = \int_{-\infty}^{\infty} |x(t)|^2 dt}$$

* Power of $x(t)$

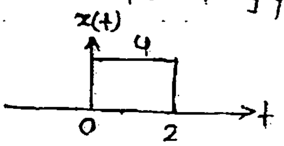
$$P = \begin{cases} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \; ; & \text{For periodic sig.} \\ \lim_{T_0 \to \infty} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \; ; & \text{Non periodic sig.} \end{cases}$$

* For an energy sig., energy should be finite & power should be zero.
* Energy signals are absoulately integrable signal.

i.e. $\boxed{\int_{-\infty}^{\infty} |x(t)|\, dt < \infty}$

Q.→ Calculate energy of sig.

(i)



Sol^n→ $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

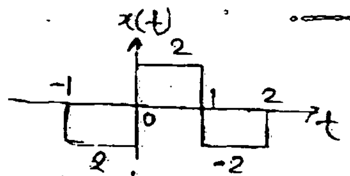
$= \int_0^2 4^2 dt = 32$

2nd method→

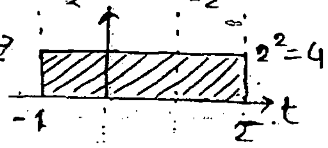
$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= 16 \times 2 = 32$

(ii)



Sol^n→

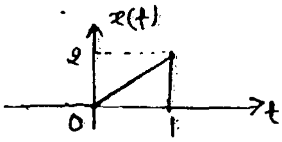


$E_{x(t)}$ = area of $|x(t)|^2$

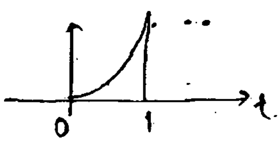
$= 4 \times 3$

$= 12$

(iii)



Sol^n→

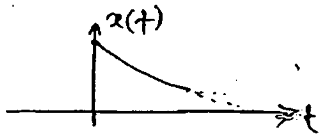


$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_0^1 (2t)^2 dt = \frac{4}{3}$

Q→ Cal. area & energy of signal:-

(i) $x(t) = e^{-at}u(t)$, $a>0$



Sol^n→ $\text{area} = \int_{-\infty}^{\infty} x(t)\,dt$

$= \int_{0}^{\infty} e^{-at}\,dt$

$= \left(\frac{e^{-at}}{-a}\right)_0^{\infty} = \frac{e^{-a\infty} - e^0}{-a}$

$\because e^{-a\infty} = 0,\ a>0 \quad (a=2)$

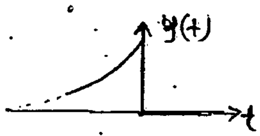
$e^{-2\infty} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$= \frac{0-1}{-a} = \frac{1}{a}$

$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2\,dt$

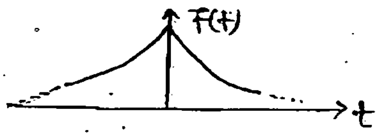
$= \int_0^{\infty} e^{-2at}\,dt = \left(\frac{e^{-2at}}{-2a}\right)_0^{\infty} = \frac{e^{-2a\infty} - e^0}{-2a} = \frac{1}{2a}$

(ii) $y(t) = x(-t) = e^{at}u(-t)$, $a>0$



Sol^n→ $\text{Area} = \frac{1}{a}$, $\text{Energy} = \frac{1}{2a}$

(iii) $f(t) = x(t) + y(t) = e^{-a|t|}$, $a>0$



Sol^n→ $f(t) = e^{-a|t|}$, $a>0$

$= \begin{cases} e^{at}, & t<0 \\ e^{-at}; & t>0 \end{cases}$

* $|t| = \begin{cases} -t, & t<0 \\ t, & t>0 \end{cases}$

$\text{area} = \frac{1}{a} + \frac{1}{a} = \frac{2}{a}$

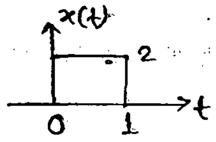
Energy = 1 . 1 . 1

2. → $x(t) \longrightarrow E$

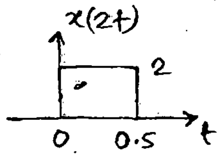
$x(2t) \longrightarrow ?$

(a.) $\frac{E}{4}$ (b.) $\frac{E}{2}$ (c) $2E$ (d) E

Sol^n →



$E \to 4$



$E \to 2 = \frac{E}{2}$

$x(t) \longrightarrow E$

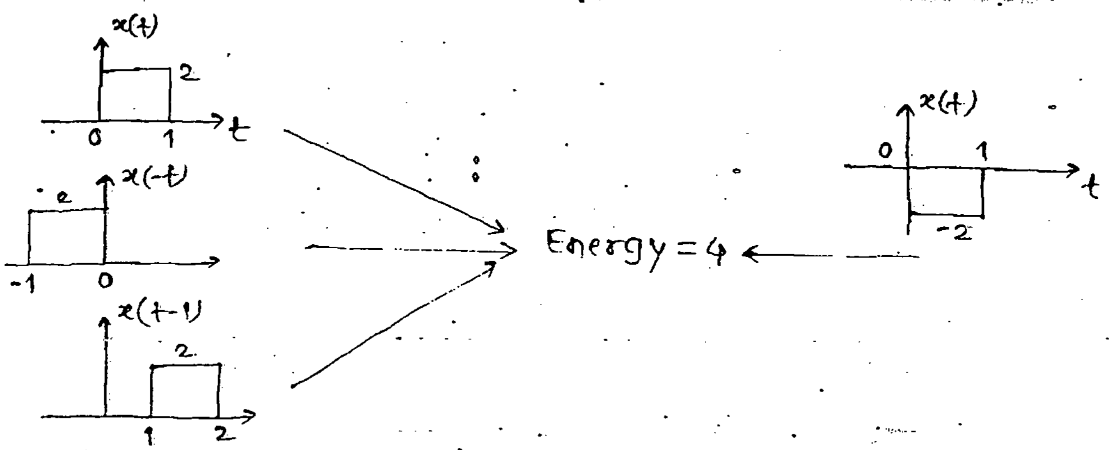
$x(2t) \longrightarrow \frac{E}{2}$

$x(-2t) \longrightarrow \frac{E}{2}$

***

$$\boxed{x(at)_{a \neq 0} \longrightarrow \frac{E}{|a|}}$$

*



* Energy of signal is independant of amp. reversal, time reversal, time shifting.

* Power Signal → * For this signal power should be finite & energy should be ∞.

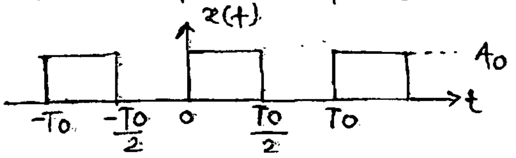
* Periodic power signals are absolutely integrable over there time period.

i.e. $\boxed{\int_{T_0} |x(t)|\,dt < \infty}$ periodic power sig.

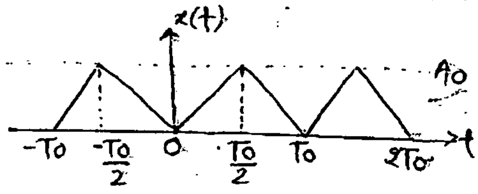
$$P = \begin{cases} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & ; \text{ For periodic signal.} \\ \lim_{T_0 \to \infty} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & ; \text{ For Non-periodic} \end{cases}$$

Q→ Calculate power of signal :-

(i.)



(ii)



Soln →

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{0}^{T_0/2} A_0^2 dt$$

$$\boxed{P = \frac{A_0^2}{2}}$$

Soln →

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} |x(t)|^2 dt$$

$$x(t) = mt = \left(\frac{2A_0}{T_0}\right)t \quad \left(\because m = \frac{A_0}{T_0/2}\right)$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2}\left(\frac{2A_0}{T_0}\right)^2 t^2 dt$$

$$= \frac{8 \times A_0^2}{T_0^3}\int_{0}^{T_0/2} t^2 dt$$

$$= \frac{8A_0^2}{T_0^3} \times \frac{T_0^3}{8 \times 3}$$

$$\boxed{P = \frac{A_0^2}{3}}$$

(iii) $x(t) = A_0 \sin \omega_0 t$

Soln →

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A_0 \sin \omega_0 t)^2 dt$$

$$P = \frac{A_0^2}{T_0}\int_{-T_0/2}^{T_0/2}\left(\frac{1-\cos 2\omega_0 t}{2}\right) dt$$

$$P = \frac{2A_0^2}{2T_0}\int_{0}^{T_0/2} (1-\cos 2\omega_0 t)\, dt$$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \left(\frac{\sin 2\omega_0 t}{2\omega_0}\right)_0^{T_0/2}\right]$$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \frac{\sin 2\omega_0 \frac{T_0}{2}}{2\omega_0}\right]$$

$$(\because \omega_0 T_0 = 2\pi) \qquad = \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \frac{\sin \omega_0 T_0}{2\omega_0}\right]$$

$$= \frac{A_0^2}{T_0} \times \frac{T_0}{2}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

$\because$ RMS of the Given signal is $\frac{A_0}{\sqrt{2}}$

$$\boxed{RMS^2 = \frac{A_0^2}{2} = P}$$

* Power is also known as mean square value of signal.

Q→ Calculate power of signal

(i.) $x_1(t) = A_0 \sin \omega_0 t$

(ii) $x_2(t) = x_1(t-t_0) = A_0 \sin[\omega_0(t-t_0)]$

(iii) $x_3(t) = x_1(2t) = A_0 \sin 2\omega_0 t$

(iv) $x_4(t) = A_0 \sin(\omega_0 t + \phi)$

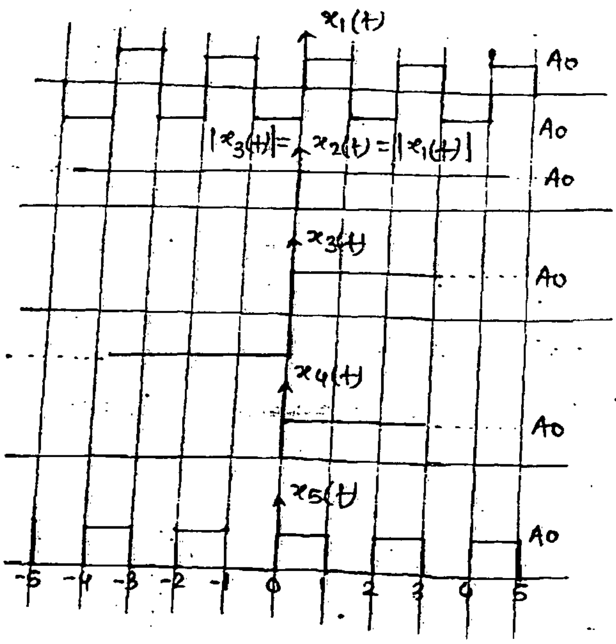
Soln→ For above all signals

$$Rms = \frac{A_0}{\sqrt{2}}$$

$$power = \frac{A_0^2}{2}$$

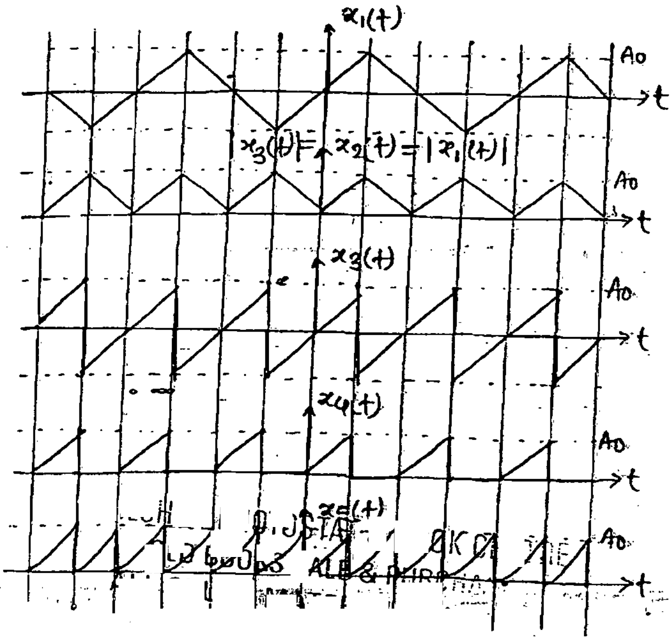
* Power calculation is independant of time shifting, time scaling, change in freq. (or) time period & change in phase of signals.

Q→



Power	Avg.
$P_1 = A_0^2$	0
$P_2 = A_0^2$	A_0
$P_3 = A_0^2$	0
$P_4 = \frac{P_2}{2} = \frac{A_0^2}{2}$	$\frac{A_0}{2}$
$P_5 = \frac{P_2}{2} = \frac{A_0^2}{2}$	$\frac{A_0}{2}$

Q →



	Power	Avg.
	$P_1 = \frac{A_0^2}{3}$	0
	$P_2 = \frac{A_0^2}{3}$	A_0
	$P_3 = \frac{A_0^2}{3}$	0
	$P_4 = \frac{A_0^2}{6}$	$\frac{A_0}{4}$
	$P_5 = 2P_4 = \frac{A_0^2}{3}$	$\frac{A_0}{2}$

***Concept of Orthogonality →** 2 signals $x_1(t)$ & $x_2(t)$ are said to be orthogonal if

* $\int_{-\infty}^{\infty} x_1(t)\cdot x_2(t)\,dt = 0$, For non-periodic sig.

* $\int_{T_0} x_1(t)\cdot x_2(t)\,dt = 0$; For periodic sig.

Use of orthogonality for energy & power calculation →

If $x_1(t)$ & $x_2(t)$ are orthogonal & $z(t) = x_1(t) \pm x_2(t)$ then;

$$P_z = P_{x_1} + P_{x_2} \quad \{\text{If } x_1 \text{ \& } x_2 \text{ are power signal}\}$$

(or)

$$E_z = E_{x_1} + E_{x_2} \quad (\text{If } x_1 \text{ \& } x_2 \text{ are energy signals})$$

Important trignomatrical results →

(1) $\int_{T_0} \sin(m\omega_0 t + \phi)\,dt = 0$, (m = an integer, $T_0 = \frac{2\pi}{\omega_0}$)

(2) $\int_{T_0} \cos(m\omega_0 t + \phi)\,dt = 0$

(3) $\int_{T_0} \sin^2(m\omega_0 t + \phi)\,dt = \frac{T_0}{2}$

* (4.) $\int_{T_0} \cos^2(m\omega_0 t + \phi)\, dt = \frac{T_0}{2}$

* (5.) $\int_{T_0} \sin(m\omega_0 t + \phi_1) \cdot \sin(n\omega_0 t + \phi_2)\, dt = 0$; ($m \neq n$ & both are integer)

Q→ $z(t) = 2\sin(3\pi t + 30^\circ) - 4\sin(7\pi t + 40^\circ)$

Sol^n→ In the above sig. the freq. of the signals are diff. ($m \neq n$). So that they are orthognal.

$$P_z = P_{x_1} + P_{x_2}$$

$$P_{x_1} = \frac{2^2}{2} = 2 \qquad P_{x_2} = \frac{4^2}{2} = 8$$

$$P_z = 10$$

Q→ $z(t) = 2\sin 3\pi t + 4\sin(7\pi t + 30^\circ) + 5\sin(10\pi t + 45^\circ)$

Sol^n→

$$P_z = P_1 + P_2 + P_3$$

$$= \frac{2^2}{2} + \frac{4^2}{2} + \frac{5^2}{2}$$

* (6.) $\int_{T_0} \cos(m\omega_0 t + \phi_1) \cdot \cos(n\omega_0 t + \phi_2) = 0$ $\{m \neq n\}$

Q.→ $z(t) = 3\cos(3\pi t + 70^\circ) + 4\cos(7\pi t + 85^\circ)$

Sol^n→

$$P_z = \frac{3^2}{2} + \frac{4^2}{2}$$

* (7.) $\int_{T_0} \cos(m\omega_0 t + \phi_1) \cdot \sin(n\omega_0 t + \phi_2)\, dt = 0$ → ($m \neq n$) ; → ($m = n$, $\phi_1 = \phi_2$)

Q→ $z(t) = 2\sin(3\pi t + 40^\circ) + 3\cos(7\pi t)$

Sol^n→

$$P = \frac{2^2}{2} + \frac{3^2}{2}$$

Q→ $z(t) = 2\sin(2\pi t + 46^\circ) + 3\cos(2\pi t + 45^\circ)$

Sol^n→

$$P = \frac{2^2}{2} + \frac{3^2}{2}$$

* (8.) $\int_{T_0} A_0 \sin(m\omega_0 t + \phi) = 0$

$\int_{T_0}$ ↓ AC $\sin(m\omega_0 t+\phi)$ ↓ Sinusoidal (sin, cos)

Q→ $z(t) = 2 + 4\sin(3\pi t + 45^\circ)$

Soln→ $P_z = P_1 + P_2$

$= 2^2 + \frac{4^2}{2}$

* Harmonics of diff. freq. are orthogonal.
* Sine & cosine f^n of same freq. & same phase are also orthogonal.
* DC & sinusoidal f^n are also orthogonal.

Q→ $z(t) = A_1\sin(\omega_0 t + \phi_1) + A_2\sin(\omega_0 t + \phi_2)$ where $\phi_1 - \phi_2 \neq \frac{n\pi}{2}$; (n = integer)

Soln→ $P = \frac{1}{T_0}\int_{T_0} z^2(t)\,dt$

$$P = \frac{1}{T_0}\int_{T_0}\left[A_1\sin(\omega_0 t + \phi_1) + A_2\sin(\omega_0 t + \phi_2)\right]^2 dt$$

$$= \frac{1}{T_0}\int_{T_0}\left[A_1^2\sin^2(\omega_0 t + \phi_1) + \frac{A_2^2}{2}\sin^2(\omega_0 t + \phi_2) + 2A_1A_2\sin(\omega_0 t + \phi_1)\cdot\sin(\omega_0 t + \phi_2)\right]$$

$$= \frac{1}{T_0}\int_{T_0}\left[A_1'\left(\frac{1-\cos 2\omega_0 t}{}\right.\right. \ldots$$

$$\boxed{P = \frac{A_0^2}{2},\quad \text{Rms} = \frac{A_0}{\sqrt{2}} \qquad A_0 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2)}}$$

Q→ $z(t) = 2\sin 3\pi t + 3\cos\left(3\pi t + \frac{\pi}{3}\right)$

Soln→ $A_0 = \sqrt{2^2 + 3^2 + 2\times 2\times 3\cos(0 - \pi/3)}$

$= \sqrt{13 + 12\times\frac{1}{2}}$

$= \frac{\sqrt{19}}{\sqrt{2}}$

Above calculation is wrong because sin & cos is present.

$z(t) = 2\sin 3\pi t + 3\sin\left(3\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

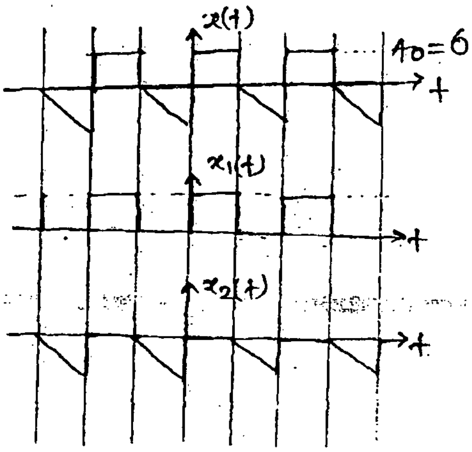
$= 2\sin 3\pi t + 3\sin(\ldots)$

$$\phi_1-\phi_2=150^\circ$$

$$A_0=\sqrt{2^2+3^2+2\times2\times3\cos(150^\circ)}$$

$$Rms=\frac{A_0}{\sqrt{2}}=1.14$$

Q→



Rms=?

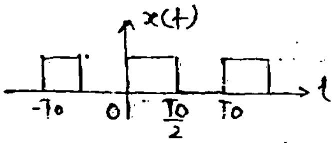
Sol^n→

For this 1st check that are they orthogonal (or) not.

$$P_z=P_1+P_2=\frac{A_0^2}{2}+\frac{A_0^2}{6}=\frac{6^2}{2}+\frac{6^2}{6}=24$$

$$Rms=\sqrt{24}=2\sqrt{6}$$

Q→



Sol^n→ $E=\int_{-\infty}^{\infty}|x(t)|^2dt$

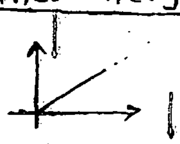
$$=\text{no. of pulses}^{\uparrow\infty}\times\int_{T_0}|x(t)|^2dt^{\uparrow\text{finite}}=\infty$$

Note→

* Periodic signals are not energy signals because ~~there~~ energy content is ∞.

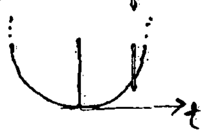
* (1.) If magnitude of sig. is ∞ at any instant of time then signal will be <u>neither energy nor power.</u>

eg.→ (a.)



$x(t)=r(t)$ [Ramp sig.]

(b.)

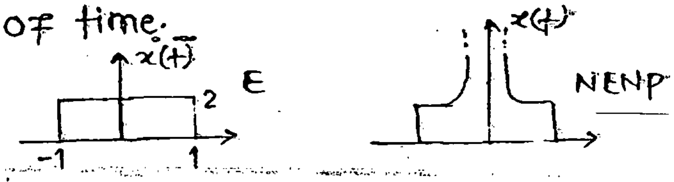


(c.) $x(t)=\tan t$

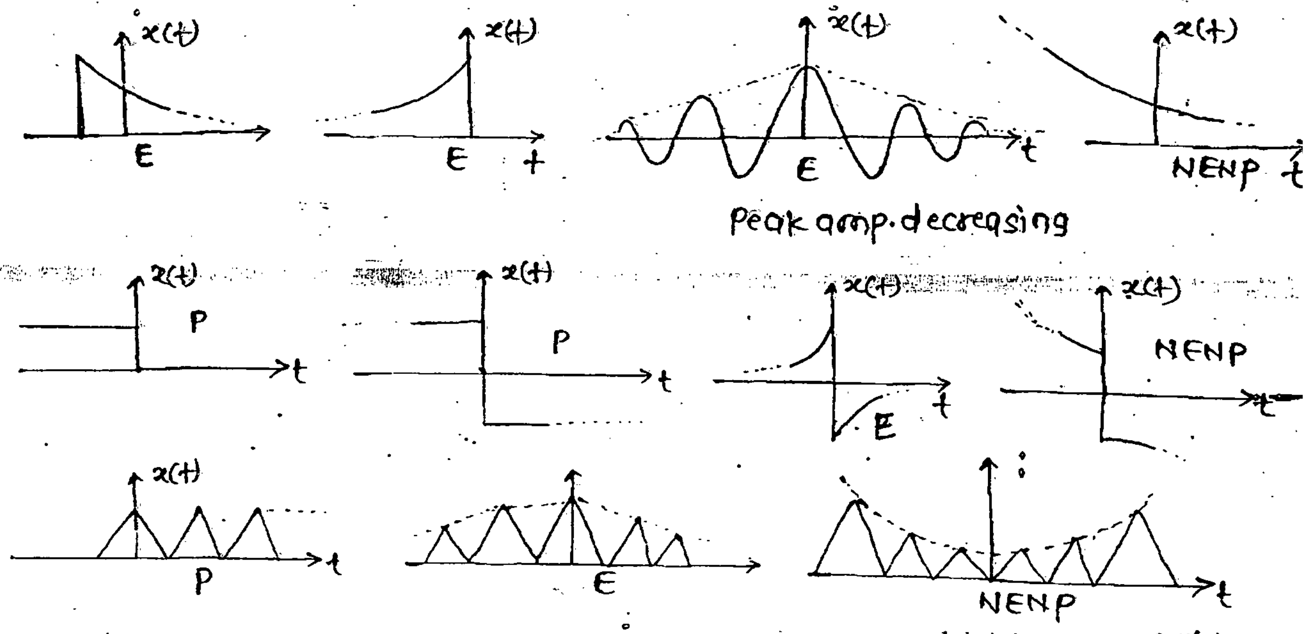
(d.) $x(t)=\frac{1}{t}$ (because $t=0$, $x(t)=\infty$)

*(2) Energy signals are:-

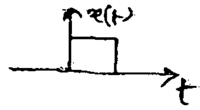
(i) Finite duration signals having finite amp. for each & every instant of time.

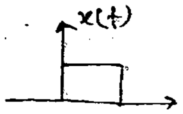
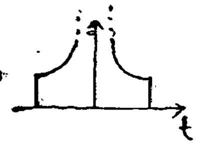


(ii) ∞ extension signals with amp. or peak amp decreasing in nature.



* Periodic signals
  - P → sin t
  - NENP → tan t

* Non-Periodic
  - E → 
  - P → u(t)
  - NENP → r(t) = t u(t)

* Finite duration
  - E 
  - NENP 

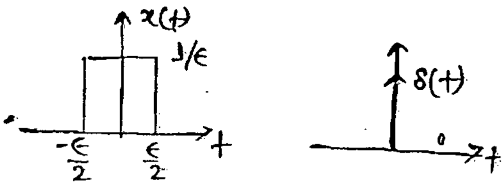
* ∞ extension
  - E 
  - P → u(t)
  - NENP → r(t)

*Basic Signals →

(1) Unit-impulse :- $\delta(t)$

$$\delta(t) = \lim_{\epsilon \to 0} x(t)$$

$$= \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



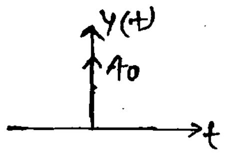
Properties →

* (1.) $\delta(t)$ is an even signal.
* (2.) It is a NENP signal.
* (3.) Area under impulse :-

$$= \int_{-\infty}^{\infty} \delta(t)dt = \int_{-\infty}^{\infty} \left[\lim_{\epsilon \to 0} x(t)\right] dt = \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} x(t) \cdot dt = 1$$

* (4.) Weight/ strength of impulse :-

$$y(t) = A_0 \delta_0(t)$$



Area of weighted impulse $y(t)$

$$= \int_{-\infty}^{\infty} y(t)dt = A_0 \int_{-\infty}^{\infty} \delta(t) \cdot dt = A_0 = \text{weight of impulse}$$

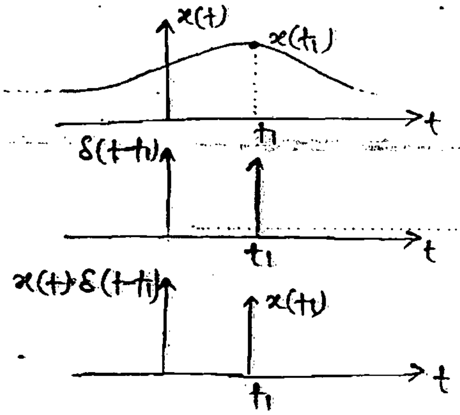
* (5.) Scaling property of impulse :-

$$\delta[a(t-t_1)]_{a \neq 0} = \frac{1}{|a|} \delta(t)$$

Eg. → (1.) $\delta(-2t) = \frac{1}{2}\delta(t)$

(2.) $\delta(2t-3) = \delta[2(t-3/2)] = \frac{1}{2}\delta(t-3/2)$

* (6.) $x(t) * \delta(t-t_1) = ? = x(t_1) \cdot \delta(t-t_1)$



Eg :- (1.) $y(t) = 2\sin t \cdot \delta(t - \frac{\pi}{2})$

$$= 2\sin(\tfrac{\pi}{2})\, \delta(t - \tfrac{\pi}{2})$$

$$= 2\delta(t - \pi/2)$$

(2.) $y(t) = e^{-2t^2} \cdot \delta(2t-1)$

$$= e^{-2t^2} \delta[2(t - \tfrac{1}{2})]$$

$$= e^{-2t^2} \cdot \tfrac{1}{2}\delta(t - \tfrac{1}{2})$$

$$= \tfrac{1}{2} \cdot e^{-2 \times \frac{1}{4}} \delta(t - \tfrac{1}{2})$$

$$= \tfrac{1}{2} e^{-1/2} \delta(t - \tfrac{1}{2})$$

(7.) $\int_{-\infty}^{\infty} x(t)\cdot\delta(t-t_1)\,dt = ?$

$$= \int_{-\infty}^{\infty} x(t_1)\,\delta(t-t_1)\,dt$$

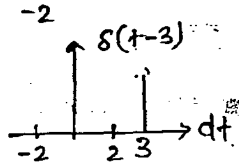
$$= x(t_1)\underbrace{\int_{-\infty}^{\infty} \delta(t-t_1)\,dt}_{1}$$

$$= x(t_1)$$

Q.→ calculate the value of

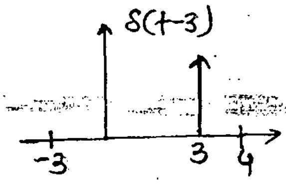
(i) $I = \int_{-2}^{2} \delta(t-3)\,dt$ (ii) $I = \int_{-3}^{4} \delta(t-3)\,dt$

Sol^n→



$\boxed{I=0}$

Sol^n→



$\boxed{I=0}$

(iii) $I = \int_{-\infty}^{\infty}\left[2\cos\left(\frac{t}{2}\right)+t^2\right]\cdot\delta(t-\pi)\,dt$

Sol^n→ $I = \int_{-\infty}^{\infty}\underbrace{\left[2\cos\left(\frac{t}{2}\right)+t^2\right]}_{x(t)}\ \underbrace{\delta(t-\pi)}_{\delta(t-t_1)}$

$$= x(t_1)$$

$$= \left[2\cos\frac{\pi}{2}+\pi^2\right]$$

$$= \pi^2$$

*(8.) $\int_{-\infty}^{\infty} x(t)\cdot\frac{d^n\delta(t-t_1)}{dt^n}\cdot dt = (-1)^n\,\frac{d^n x(t)}{dt^n}\Big|_{t=t_1}$

Sol^n→

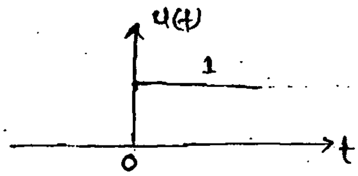
Eg.→ $\int_{-\infty}^{\infty}(t^2+3t)\,\delta'(t-2)\,dt$

$$= (-1)^1\frac{d}{dt}(t^2+3t)\Big|_{t=2}$$

$$= -(2t+3)\Big|_{t=2}$$

$$= -(4+3) = -7$$

(2) Unit-step signals → $u(t)$



* $u(t)$ is discontineous at $t=0$.

Gibb's phenomenon → At the point of discontinuity signal value is given by the avg. of signal value taking just before & after the point of discontinuity.

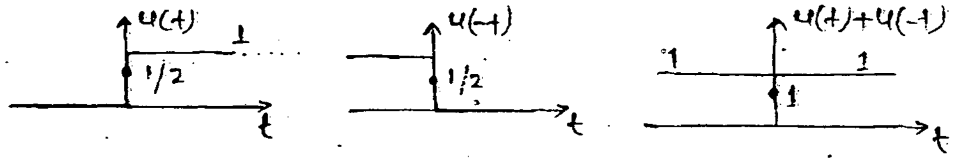
$$u(0) = \frac{u(0^-)+u(0^+)}{2}$$

$$= \frac{0+1}{2}$$

$$\boxed{u(0) = \frac{1}{2}}$$

*Properties →

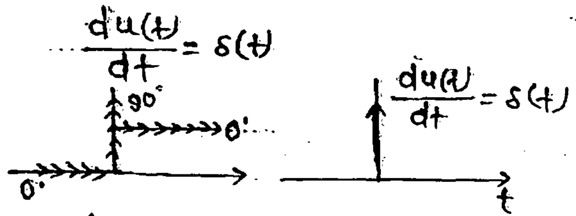
(1) $u(t) + u(-t) = 1$



(2) $U(t)$ is a power signal.

Power $= \frac{1}{2}$, Rms $= \frac{1}{\sqrt{2}}$; avg. $= \frac{1}{2}$

(3) Derivative of $u(t)$

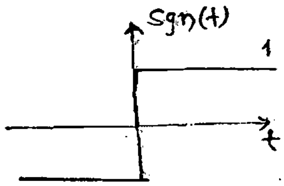


$$\left\{ \frac{dx(t)}{dt} = \text{slope of } x(t) \text{ wrt 't'} \right.$$

* And $\int_{-\infty}^{t} \delta(t)\,dt = u(t)$

$$\begin{tikzcd} \delta(t) \arrow[r, "I"] & u(t) \arrow[r, "I"] \arrow[l, bend right, "D"'] & r(t) \arrow[r, "I"] \arrow[l, bend right, "D"'] & p(t) \arrow[l, bend right, "D"'] \end{tikzcd}$$

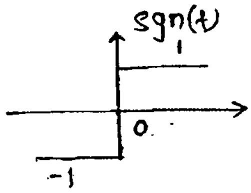
(3.) Signum function →

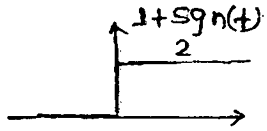


* This is a power signal.

$P = 1, \quad RMS = 1, \quad Avg = 0$

*



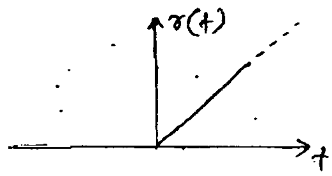


$1 + sgn(t) = 2u(t)$

*** $\boxed{u(t) = \frac{1 + sgn(t)}{2}}$

(4.) Ramp signal → $r(t)$

$$r(t) = \int_{-\infty}^{t} u(t)\,dt = t\,u(t)$$



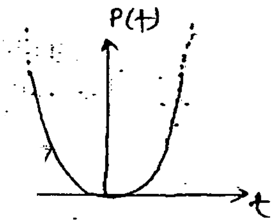
* This is NENP sig.

(5.) Parabolic signal →

$$P(t) = \int_{-\infty}^{t} r(t)\,dt$$

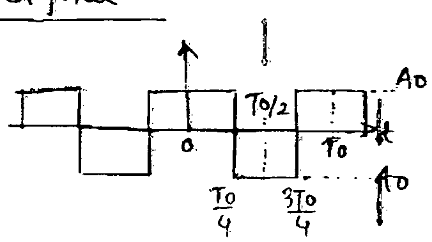
$$= \int_{-\infty}^{t} t\,u(t)\,dt$$

$$= \frac{t^2}{2} u(t)$$



* This is NENP signal.

(6.) Square signal →

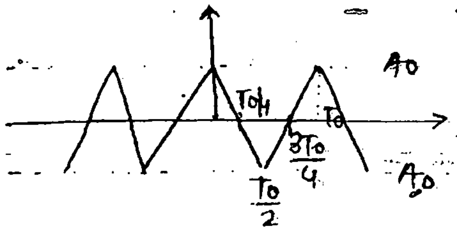


$P = A_0^2$

$RMS = A_0$

$Avg = 0$

(7.) Triangular wave →



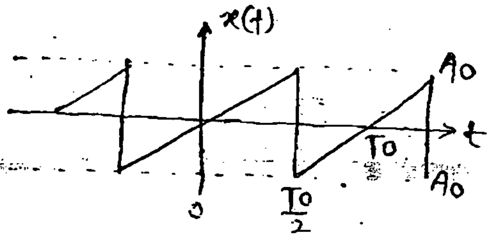
$P = A_0^2/3$

$Rms = A_0/\sqrt{3}$

$avg. = 0$

HWS = yes.

(8.) Sawtooth wave →



$P = A_0^2/3$

$Rms = A_0/\sqrt{3}$

$Avg. = 0$

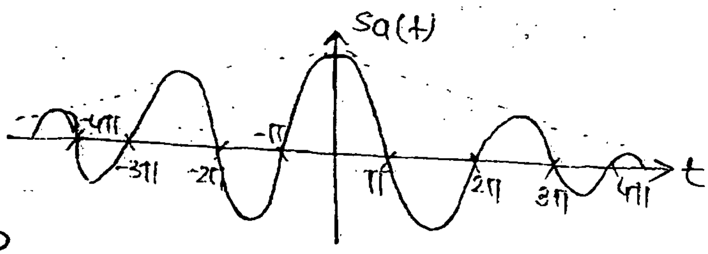
HWS = No.

(9.) Sampling signal →

$$Sa(t) = \frac{\sin t}{t}$$

* $Sa(0) = \lim_{t\to 0} \frac{\sin t}{t} = 1$

* $Sa(\infty) = \frac{\sin\infty}{\infty} = \frac{(-1,1)}{\infty} = 0$



* If $Sa(t) = 0$, then $\frac{\sin t}{t} = 0$

  so $\sin t = 0$, $\boxed{t = n\pi, n \neq 0}$

* This is a energy signal.

$$E = x(t) = \int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2} dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{1-\cos 2t}{2t^2}\right) dt$$

$$= \frac{1}{2}\left[\left(\frac{1}{-2t^2}\right)_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{\cos 2t}{t^2} dt\right]$$

$\boxed{E = \pi}$

(10.) Sinc function →

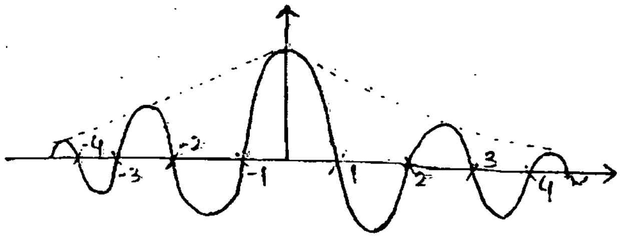
$$\text{sinc}\,t = \frac{\sin \pi t}{\pi t} = Sa(\pi t)$$

* $\text{sinc}(0) = \frac{\sin \pi t}{\pi t} = 1$
* $\text{sinc}(\infty) = 0$
* If $\text{sinc}(t) = 0$, $\frac{\sin(\pi t)}{\pi t} = 0$

$\sin \pi t = 0$

$\cancel{\pi} t = n \cancel{\pi}$, $n \neq 0$

$\boxed{t = n,\ n \neq 0}$

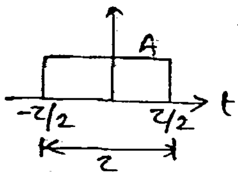


* Energy = 1

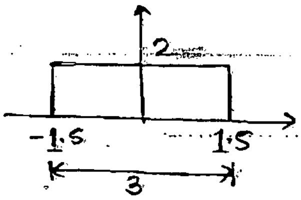
$x(t) \rightarrow E$	$Sa(t) = \pi$
$x(at) \rightarrow \frac{E}{\lvert a \rvert}$	$\text{sinc}(t) = Sa(\pi t) = 1$

(11.) Rect function →

$x(t) = A\,\text{rect}\left(\frac{t}{2}\right)$

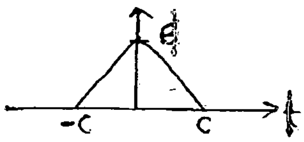


$x(t) = 2\,\text{rect}\left(\frac{t}{3}\right)$

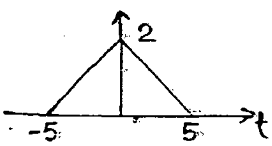


(12) Tri-function →

$x(t) = B\,\text{tri}\left(\frac{t}{c}\right)$



$x(t) = 2\,\text{tri}\left(\frac{t}{5}\right)$



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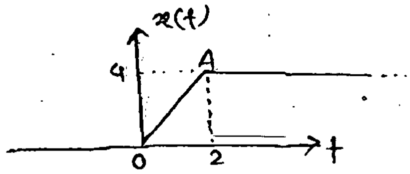
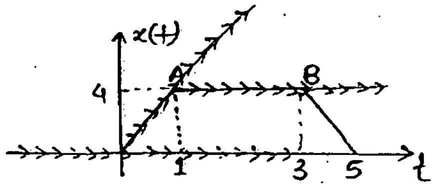
Mathematical representation of waveform →

(1.)



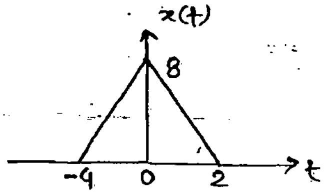
$$* \; x(t) = 0 + 2r(t-0) - 2r(t-2)$$

(2.)



$$* \; x(t) = 0 + 4r(t-0) \cancel{+} - 4r(t-1) - 2r(t-3) + 2r(t-5)$$

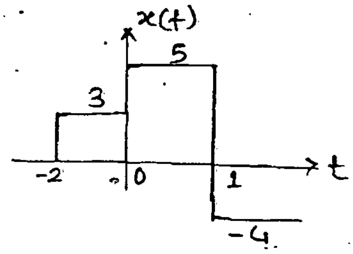
(3.)



$$-2r(t-0)$$

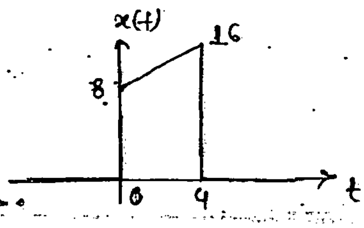
$$* \; x(t) = 0 + (+2)r(t+4) \cancel{-4r(t-0)} + 4r(t-2)$$

(4.)



$$* \; x(t) = 0 + 3u(t+2) + 2u(t-0) - 9u(t-1)$$

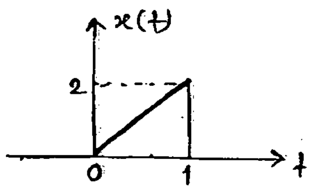
(5.)



$$* \; x(t) = 0 + 8u(t-0) + 2r(t-0) - 2r(t-4) \cancel{+} - 16u(t-4)$$

$$= 8u(t) + 2r(t) - 2r(t-4) - 16u(t-4)$$

(6.)



$$* \; x(t) = 2r(t) - 2r(t-1) - 2u(t-1)$$

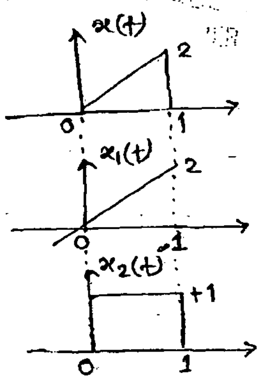
$$= 2tu(t) - 2(t-1)u(t-1) - 2(\cancel{t-t+1})u(t-1)$$

$$= 2tu(t) - 2tu(t-1) + \cancel{2u(t-1)} - \cancel{2u(t-1)}$$

$$= 2t[u(t) - u(t-1)]$$

$$= 2t[u(t) - u(t-1)]$$

2nd method →

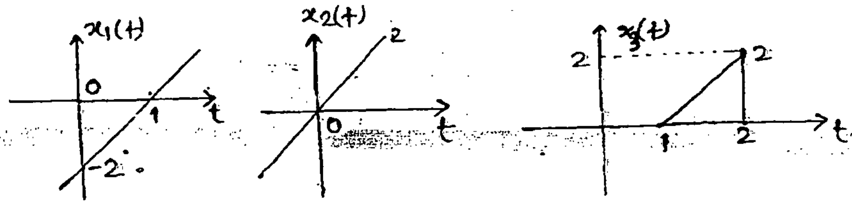


$\longrightarrow 2t[u(t) - u(t-1)]$

$\because x(t) = x_1(t) \cdot x_2(t)$

$= 2t[u(t) - u(t-1)]$

Q.→



Ans.→ $x_2(t) = 2t$

$\therefore x_1(t) = 2(t-1)$

$x_3(t) = 2(t-1)[u(t-1) - u(t-2)]$