Triangles

Exercise – 2.1

Solution 1:

$x° + 2x° + 3x° = 180°$
$\therefore 6x° = 180°$
$\therefore x° = 30°$
$2x° = 2 \times 30° = 60°$ and
$3x° = 3 \times 30° = 90°$.
So, the measures of the angles of the triangle are 30°, 60° and 90°.
The triangle is a right angled triangle.

Solution 2:

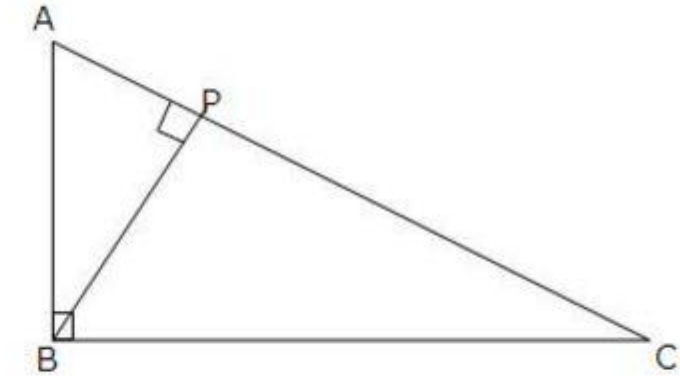
The three sides of an equilateral triangle are congruent.
$\therefore$ The perimeter of an equilateral triangle = 3 × side
$\therefore 3 \times \text{side} = 16.5$ cm
$\therefore \text{side} = \frac{16.5}{3}$
$\therefore \text{side} = 5.5$ cm
The length of the side of the equilateral triangle is 5.5 cm.

Solution 3:

The ratio of the angles of the triangle is 3 : 3 : 6.
Let the measures of the angles be 3x°, 3x° and 6x°.
Then $3x° + 3x° + 6x° = 180°$
$\therefore 12x° = 180°$
$\therefore x° = 15°$
$\therefore 3x° = 3 \times 15° = 45°$ and $6x° = 6 \times 15x° = 90x°$
Two angles of the triangle are equal and one angle is a right angle.
The triangle is an isosceles right angled triangle.

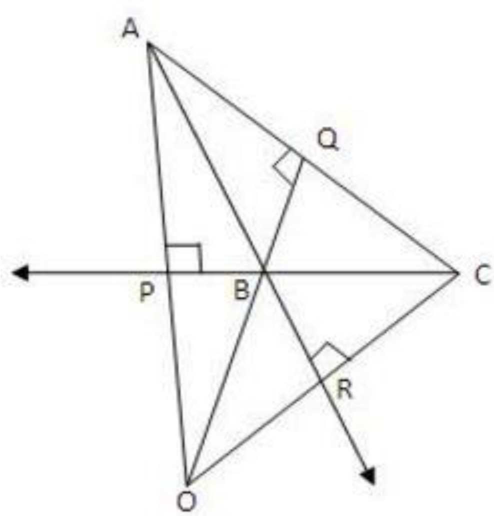
Solution 4:

In the ∆ABC, m∠B = 90°.
So, AB is the altitude on side BC.
BC is the altitude on side AB.
BP is the altitude on side AC.
All the altitudes meet at the point B.



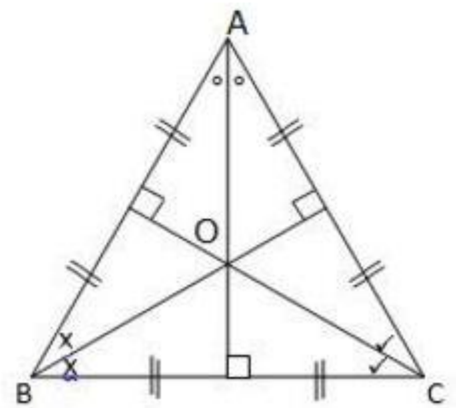
In a right angled triangle, the altitudes meet at the vertex forming a right angle.

Solution 5:



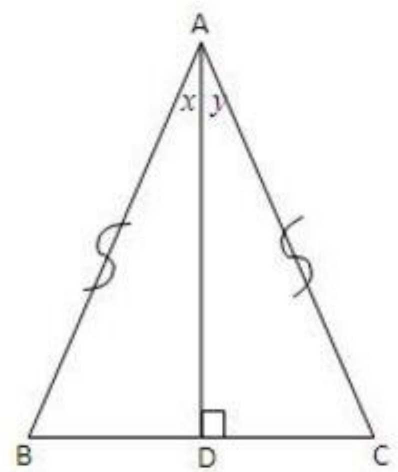
∆ABC is an obtuse angled triangle, ∠B being an obtuse angle.
The orthocenter O of the obtuse angled triangle lies outside the triangle.

Solution 6:



In an equilateral triangle, altitudes, medians and angle bisectors are one and the same.
So, in an equilateral triangle, the orthocenter, centroid and incentre lie at the same point.
In the above diagram, point O is the orthocenter, centroid and incentre of ∆ABC.

Solution 7:



∆ABC is an isosceles triangle in which AB = AC.
Side BC is its non-congruent side.
Seg AD is the median drawn on the side BC.
Median AD is the altitude and angle bisector of ∠A also.
[In an isosceles triangle, the centroid, orthocenter and incentre lie on the same line. In the given figure, all the three points lie on line AD.]

Solution 8:

The perpendicular bisectors of the sides of a triangle do not always pass through the opposite vertex except in an equilateral triangle. In the case of an isosceles triangle, the perpendicular bisector of the non-congruent side is a cevian and in a scalene triangle the perpendicular bisector of its sides are not cevians. Hence, in general, the perpendicular bisectors of the sides of a triangle are not cevians.
So, I agree with statement.

Exercise – 2.2

Solution 1(i):

The sum of the measures of the angles of a triangle is 180°.
$\therefore m\angle V + m\angle A + m\angle T = 180°$
$\therefore a° + a° + 40° = 180°$
$\therefore 2a° + 40° = 180°$
$\therefore 2a° = 180° - 40° = 140°$
$\therefore a° = 70°$

Solution 1(ii):

The sum of the measures of the angles of a triangle is 180°.
$\therefore m\angle P + m\angle U + m\angle X = 180°$
$\therefore 90° + 50° + a° = 180°$
$\therefore a° = 180° - 140° = 40°$
$\therefore a° = 40°$

Solution 1(iii):

The sum of the measures of the angles of a triangle is 180°.
$\therefore m\angle T + m\angle A + m\angle P = 180°$
$\therefore a° + 110° + 25° = 180°$
$\therefore a° = 180° - 135°$
$\therefore a° = 45°$

Solution 2:

The sum of the measures of the angles of a triangle is 180°.
$\therefore m\angle M + m\angle T + m\angle G = 180°$
$\therefore 76° + 48° + m\angle G = 180°$
$\therefore 124° + m\angle G = 180°$

$\therefore m\angle G = 180° - 124°$
$\therefore m\angle G = 56°$

Solution 3:

The ratio of the measures of the angles is 2 : 3 : 4.

Let the measures of the angles be $2x°, 3x°$ and $4x°$.

$$2x° + 3x° + 4x° = 180° \quad \ldots\begin{pmatrix}\text{The sume of the measures}\\ \text{of the angles of a triangle}\end{pmatrix}$$

$\therefore 9x° = 180°$

$\therefore x° = \dfrac{180°}{9}$

$\therefore x° = 20°$

$2x° = 2 \times 20° = 40°;$

$3x° = 3 \times 20° = 60°;$

$4x° = 4 \times 20° = 80°$

The measures of the angles of the triangle are $40°, 60°$ and $80°$ respectively.

Solution 4:

Let the measure of $\angle D$ be $x°$.

Then $4\angle S = 3\angle D = 3x°$.

$\therefore \angle S = \dfrac{3x°}{4}$

Also, $6\angle R = 3\angle D = 3x°$

$\therefore \angle R = \dfrac{3x°}{6} = \dfrac{x°}{2}$

$$m\angle D + m\angle S + m\angle R = 180° \quad \ldots.\begin{pmatrix}\text{The sum of the measures}\\ \text{of the angles of a triangle}\end{pmatrix}$$

$\therefore x° + \dfrac{3x°}{4} + \dfrac{x°}{2} = 180°$

$\therefore 4x° + 3x° + 2x° = 180° \times 4$

$\therefore 9x° = 180° \times 4$

$\therefore x° = \dfrac{180° \times 4}{9}$

$\therefore x° = 20° \times 4 = 80°$

$m\angle S = \dfrac{3x°}{2} = \dfrac{3 \times 80°}{4} = 3 \times 20° = 60°$

$m\angle R = \dfrac{x°}{2} = \dfrac{80°}{2} = 40°.$

$m\angle D = 80°; m\angle S = 60°; m\angle R = 40°.$

Solution 5:

$m\angle M + m\angle N + m\angle K = 180^\circ$
…(The sum of the measures of the angles of a triangle) ….(1)
$m\angle M + m\angle N = 125^\circ$ …(Given)..(2)
From (1) and (2),
$125^\circ + m\angle K = 180^\circ$
$\therefore m\angle K = 55^\circ$ …(3)
$m\angle M + m\angle K = 113^\circ$ ….(Given)…(4)
$m\angle M + 55^\circ = 113^\circ$ …[From (3)]
$\therefore m\angle M = 58^\circ$
From (1) and (4),
$m\angle N + 113^\circ = 180^\circ$
$\therefore m\angle N = 67^\circ$
$m\angle M = 58^\circ$; $m\angle N = 67^\circ$; $m\angle K = 55^\circ$.

Solution 6:

Let the measure of the smallest angle be x°.
Then the measures of the other two amgles are $2x^\circ$
and $3x^\circ$ respectively.
The sum of the measures of the angles of a triangle is 180°.
$\therefore x^\circ + 2x^\circ + 3x^\circ = 180^\circ$
$\therefore 6x^\circ = 180^\circ$
$\therefore x^\circ = 30^\circ$.
$2x^\circ = 2 \times 30^\circ = 60^\circ$ and $3x^\circ = 3 \times 30^\circ = 90^\circ$.
The measures of the angles of the triangle are 30°, 60°
and 90° respectively.

Solution 7(i):

$m\angle NME + m\angle EMR = 180^\circ$
…(Angles in a linear pair)
$\therefore m\angle NME + 140^\circ = 180^\circ$
$\therefore m\angle NME = 180^\circ - 140^\circ$
$\therefore m\angle NME = 40^\circ$ …(1)
$\angle TEN = \angle ENM + \angle NME$
….(Remote interior angle theorem)
$\therefore 100^\circ = x + 40^\circ$
…[Given and from(1)]

∴x = 100° – 40°
∴x = 60°.

Solution 7(ii):

∠RQS = ∠PQX
…(Vertically opposite angles)
m∠PQX = 100° …(Given)
∴m∠RQS = 100°
∠QST = ∠QRS + ∠RQS
…(Remote interior angle theorem)
∴ x = 50° + 100°
∴ x = 150°

Solution 7(iii):

In ∆NYX, m∠NYX = 90° …(Given)
∴m∠N + m∠X = 90°
…(Acute angles of a right angled triangle)
∴m∠N + 45° = 90°
…(Given : m∠X = 45°)
∴m∠N = 45°
Consider the DNMZ.
Now, m∠N + m∠NMZ + m∠Z = 180°
…(The sum of the measures of the angles of a triangle)
∴45° + 110° + x = 180°
…[From (1) and given]
∴155° + x = 180°
∴x = 180° – 155°
∴x = 25°

Solution 7(iv):

AB∥DE …(Given)
AD is the transversal.
∴∠BAD = ∠ADE
…(Alternate angles)
∠BAD = 70°
∴∠ADE = 70° i.e.∠EDC = 70° …(1)
Now consider the DCDE:
∠CED + ∠ECD + ∠EDC = 180°
…(The sum of the measures of the angles of a triangle)
∴35° + x + 70° = 180°
…[Given and from (1)]

$\therefore x + 105° = 180°$
$\therefore x = 180° – 105°$
$\therefore x = 75°$

Solution 8:

m∠TPR + m∠RPQ = 180°
…(Angles in a linear pair) …(1)
Similarly, m∠PRQ + m∠SRQ = 180° …(2)
and m∠RQP + m∠PQU = 180° …(3)
From (1), (2) and (3),
m∠TPR + m∠RPQ + m∠PRQ + m∠SRQ + m∠RQP + m∠PQU = 540°
∴(m∠TPR + m∠SRQ + m∠PQU)
+ (m∠RPQ + m∠PRQ + m∠RQP) = 540° …(4)
But, m∠RPQ + m∠PRQ + m∠RQP = 180°
…(The sum of the measures of the angles of a triangle) …(5)
From (4) and (5),
(m∠TPR + m∠SRQ + m∠PQU) + 180° = 540°
∴m∠TPR + m∠SRQ + m∠PQU = 540° – 180°
= 360°
∴∠TPR + ∠SRQ + ∠PQU = 4 right angles.

Solution 9:

seg BO and seg CO are the angle bisectors of ∠B and ∠C respectively.
...(Given)
$\therefore \angle CBO = \frac{1}{2}\angle CBA$ and $\angle BCO = \frac{1}{2}\angle BCA$ ...(1)
Consider the triangle ABC:
The sum of the measures of the angles of a triangle is 180°
$\therefore m\angle A + m\angle CBA + m\angle BCA = 180°$
$\therefore 70° + m\angle CBA + m\angle BCA = 180°$
....(Given: m∠A = 70°)
$\therefore m\angle CBA + m\angle BCA = 180° - 70° = 110°$ ....(2)

In ΔBOC, m∠CBO + m∠BCO + m∠BOC = 180°
$\therefore \frac{1}{2}m\angle CBA + \frac{1}{2}m\angle BCA + m\angle BOC = 180°$ ...[From(1)] ...(3)
$\therefore \frac{1}{2}(m\angle CBA + m\angle BCA) + m\angle BOC = 180°$
$\therefore \frac{1}{2}(110°) + m\angle BOC = 180°$ ...[From (2)]
$\therefore 55° + m\angle BOC = 180°$
$\therefore m\angle BOC = 180° - 55°$
$\therefore m\angle BOC = 125°$

Solution 10:

$\angle R : \angle A : \angle P = 3:2:1$ ...(Given)

Let $\angle R$ be $3x^\circ$.

Then $\angle A = 2x^\circ$ and $\angle P = x^\circ$

The sum of the measures of th angles of a triangle is 180°

$\therefore 3x^\circ + 2x^\circ + x^\circ = 180^\circ$

$\therefore 6x^\circ = 180^\circ$

$\therefore x^\circ = \dfrac{180^\circ}{6} = 30^\circ$

$2x^\circ = 2 \times 30^\circ = 60^\circ$ and $3x^\circ = 3 \times 30^\circ = 90^\circ$

$\therefore m\angle R = 90^\circ, \angle A = 60^\circ$ and $\angle P = 30^\circ$.

$m\angle R = 90^\circ = \angle RPS$

$\therefore AR \parallel PS$ ...(Alternate angles test)

Now, $AR \parallel PS$ and AW is the transversal

$\therefore \angle SPW = \angle RAP$ ...(Corresponding angles)

But $m\angle RAP = 60^\circ$

$\therefore m\angle SPW = 60^\circ$

The measure of $\angle SPW = 60^\circ$.

Exercise – 2.3

Solution 1:

i. $\angle R \cong \underline{\angle X}, \angle S \cong \underline{\angle Y}, \angle T \cong \underline{\angle Z}$.

ii. $\dfrac{RT}{XZ} = \dfrac{\underline{ST}}{YZ}, \dfrac{RS}{XY} = \dfrac{ST}{\underline{YZ}}, \dfrac{XY}{\underline{RS}} = \dfrac{YZ}{ST}$.

Solution 2:

$\Delta CAB \sim \Delta FDE$ ...(Given)

$\therefore \frac{CA}{FD} = \frac{AB}{DE} = \frac{CB}{FE}$

....(Corresponding sides are proportional)

$\therefore \frac{13}{n} = \frac{12}{m} = \frac{5}{2}$

....(Substituting the given values)

$\frac{13}{n} = \frac{5}{2}$

$\therefore 13 \times 2 = 5 \times n$

$\therefore n = \frac{13 \times 2}{5} = \frac{26}{5}$

$\therefore n = 5.2$

$\frac{12}{m} = \frac{5}{2}$

$\therefore 12 \times 2 = 5 \times m$

$\therefore m = \frac{12 \times 2}{5} = \frac{24}{5}$

$\therefore m = 4.8$

$\therefore$ m = 4.8, n = 5.2

Solution 3:

$\Delta ABC \sim \Delta DEF$ .....(Given)

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ....(Corresponding sides are proportional)

$\therefore \frac{12}{18} = \frac{8}{EF} = \frac{15}{DF}$ .....(Substituting the given values)

$\frac{12}{18} = \frac{8}{EF}$ $\therefore 12 \times EF = 8 \times 18$

$\therefore EF = \frac{8 \times 18}{12}$

$\therefore EF = 12$

$\frac{12}{18} = \frac{15}{DF}$

$\therefore 12 \times DF = 15 \times 18$

$\therefore DF = \frac{15 \times 18}{12}$

$\therefore DF = 22.5$

$EF = 12,\ DF = 22.5$

Solution 4:

$\Delta GHK \sim \Delta PHS$ .....(Given)

$\therefore \frac{GH}{PH} = \frac{KH}{SH}$ ....(Corresponding sides are proportional)

$\therefore \frac{6}{5} = \frac{18}{SH}$ ....(Substituting the given values)

$\therefore SH = \frac{18 \times 5}{6}$

$\therefore SH = 15$ .....(1)

$KS = KH + HS$ ....(K-H-S)

$= 18 + 15$ .....[Given and from (1)]

$\therefore KS = 33$

Solution 5:

i. $\Delta MAP \sim \Delta NBP$. .....(Given)

$\therefore \frac{MA}{NB} = \frac{PM}{PN}$ ....(Corresponding sides are propotional)

$\therefore \frac{2}{3} = \frac{6}{PN}$ ...(Substituting the given values)

$\therefore 2 \times PN = 6 \times 3$

$\therefore PN = 9$

ii. $\Delta MAN \sim \Delta NBM$. .....(Given)

$\therefore \frac{MA}{NB} = \frac{AN}{BM}$ ....(Corresponding sides are propotional)

$\therefore \frac{2}{3} = \frac{AN}{3}$ ...(Substituting the given values)

$\therefore AN = 2.$

Solution 6:

$\Delta BLD \sim \Delta PST$. .....(Given)

$\therefore \frac{BL}{PS} = \frac{LD}{ST}$ ....(Corresponding sides are propotional)

$\therefore \frac{BL}{2} = \frac{24}{4}$ ...(Substituting the given values)

$\therefore BL = \frac{24 \times 2}{4}$

$\therefore BL = 12.$

The height of the building is 12 m.

Solution 7:

In ΔDEM and ΔKNM,

$\angle DME \cong \angle KMN$ ....(Common angle)

$\angle DEM \cong \angle KNM$ .....(Each is 90°)

$\therefore \Delta DEM \sim \Delta KNM$ ....(AA test for similarity)

$\Delta DEM \sim \Delta KNM$ .....(Proved)

$\therefore \frac{DM}{KM} = \frac{ED}{NK}$ .....(Corresponding sides are proportional)

$\therefore \frac{5}{3} = \frac{ED}{4.5}$ .....(Substituting the given values)

$\therefore ED = \frac{5 \times 4.5}{3}$

$\therefore ED = 7.5$

Solution 8:

$\Delta SMA \sim \Delta RTQ$

$\therefore \angle M \cong \angle T, \angle A \cong \angle Q$ and $\angle S \cong \angle R$

.....(Corresponding angles are congruent)

$\therefore 110^\circ = m\angle T,\ m\angle A = 27^\circ$ .....(Given)

Now, in ΔSMA, $m\angle S + m\angle M + m\angle A = 180^\circ$

$\therefore m\angle S + 110^\circ + 27^\circ = 180^\circ$

$\therefore m\angle S + 137^\circ = 180^\circ$

$\therefore m\angle S = 180^\circ - 137^\circ = 43^\circ$

$\angle S \cong \angle R$

$\therefore m\angle R = 43^\circ$

Now, $\frac{SM}{RT} = \frac{MA}{TQ}$ ......(Corresponding sides are proportional)

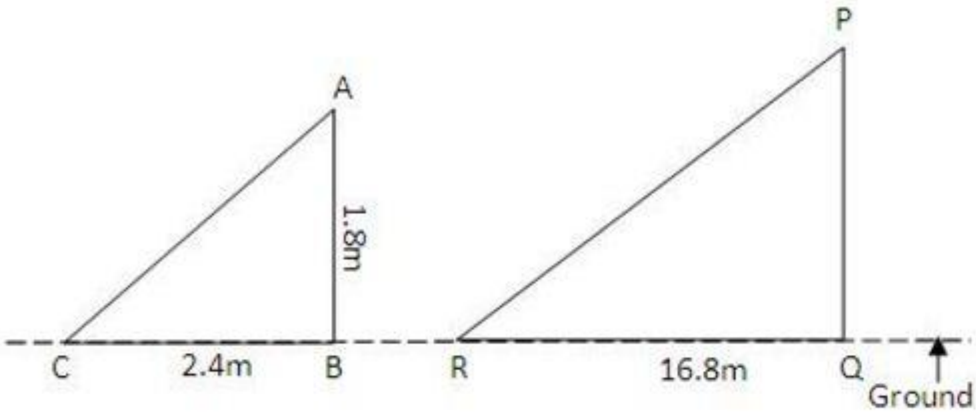
$\therefore \frac{8}{6} = \frac{10}{TQ}$ ......(Substituting the given values)

$\therefore TQ = \frac{10 \times 6}{8}$

$\therefore TQ = 7.5$

$m\angle S = 43^\circ, m\angle A = 27^\circ,\ m\angle T = 110^\circ,\ m\angle R = 43^\circ,\ TQ = 7.5$

Solution 9:



Let PQ represent the flagpost and AB a person of height 1.8 m. QR and BC be the shadows of the flagpost and the person respectively.

The triangles determinded by the flagspot with its shadow and the person with its shadow are similar triangles.

$\therefore \Delta PQR \sim \Delta ABC$

$\therefore \frac{PQ}{AB} = \frac{QR}{BC}$ .... (Corresponding sides are proportional)

$\therefore \frac{PQ}{1.8} = \frac{16.8}{2.4}$ .... (Substituting the given values)

$\therefore PQ = \frac{16.8 \times 1.8}{2.4} = 7 \times 1.8 = 12.6$

The height of the flagpost is 12.6 m.

Solution 10:

Seg ST || seg MP and MN is the transversal. ....(Given)

$\therefore \angle NST \cong \angle NMP$ .....(Corresponding angles)

In ΔNST and ΔNMP,

$\angle NST \cong \angle NMP$ .....(Proved)

$\angle SNT \cong \angle MNP$ .....(Common angle)

$\therefore \Delta NST \sim \Delta NMP$ .....(AA test for similarity)

$\therefore \frac{NS}{NM} = \frac{ST}{MP}$ ......(Corresponding sides are proportional)

$\therefore \frac{6}{10} = \frac{4}{MP}$ ......(Substituting the given values)

$\therefore 6 \times MP = 4 \times 10$

$\therefore MP = \frac{4 \times 10}{6}$

$\therefore MP = \frac{20}{3} = 6\frac{2}{3}$

Solution 11:

Seg PN || seg LM and LN is the transversal ....(Given)

$\therefore \angle MLN (i.e.\ \angle MLA) \cong \angle LNP (i.e.,\ \angle ANP)$ .....(Alternate angles)

In ΔLAM and ΔNAP,

$\angle MLA \cong \angle ANP$ .....(Proved)

$\angle LAM \cong \angle NAP$ .....(Vertically opposite angles)

$\therefore \Delta LAM \sim \Delta NAP$ .....(AA test for similarity)

$\Delta LAM \sim \Delta NAP$ .....(Proved)

$\therefore \frac{LM}{NP} = \frac{LA}{NA}$ .....(Corresponding sides are proportional)

$\therefore \frac{8}{12} = \frac{5}{NA}$ .....(substituting the given values)

$\therefore NA = \frac{5 \times 12}{8}$

$\therefore NA = AN = 7.5$

Solution 12:

In ΔXYZ and ΔDEN,

$\angle ZXY = \angle NDE$ .....(Given)....(1)

$\frac{XY}{DE} = \frac{6}{12} = \frac{1}{2}$ ....(2)

$\frac{XZ}{DN} = \frac{4}{8} = \frac{1}{2}$ ....(3)

From (2) and (3), $\frac{XY}{DE} = \frac{XZ}{DN}$ ....(4)

From (1) and (4)

$\Delta XYZ \sim \Delta DEN$ .....(SAS test for similarity)

Solution 13:

seg SK ∥ seg MP ∥ seg TN.
and AT is the transversal. ….(Given)
∴∠ASK begin mathsize 12px style approximately equal to end style ∠AMP begin mathsize 12px style approximately equal to end style ∠ATN …(Corresponding angles) ….(1)
∠A is common to the triangles, ∆ASK,
∆AMP and ∆ATN.
∴∆ASK~∆MAP~∆TAN ….(AA test for similarity)

Solution 14:

$\frac{PQ}{AD} = \frac{4}{8} = \frac{1}{2}$ ....(1)

$\frac{PR}{AE} = \frac{5}{10} = \frac{1}{2}$ ....(2)

$\frac{QR}{DE} = \frac{6}{12} = \frac{1}{2}$ ....(3)

From (1),(2) and (3) $\frac{PQ}{AD} = \frac{PR}{AE} = \frac{QR}{DE}$

$\therefore \Delta PRQ \sim \Delta AED$ ....(SSS test for similarity)

Solution 15:

Seg RV ⊥ seg PS
∴ m∠RVP = m∠SVR = 90°
In ΔSRP and ΔSVR,
m∠SRP = m∠SVR = 90° …[Given and from(1)]
∠PSR = ∠VSR …(Common angle)
∴ΔSRP ~ ΔSVR …(AA test for similarity)
In ΔSRP and ΔRVP,
m∠SRP = m∠RVP = 90° …[Given and from(1)]
∠SPR = ∠RPV …(Common angle)
∴ΔSRP ~ ΔRVP …(AA test for similarity)
In ΔSPR, m∠S + m∠P = 90°
In ΔRVP, m∠P + m∠VRP = 90° …(2) …(Acute angles of a right angled triangle)
From (2), ∠RSV = ∠VRP …(3)
In ΔSVR and ΔRVP
m∠SVR = m∠RVP = 90°
and ∠RSV = ∠VRP …[From (3)]
∴ΔSVR ~ ΔRVP …(AA test for similarity)
The pairs of the given triangles are similar.

Solution 16:

In ΔAOB and ΔMOP,

$\frac{OB}{OP} = \frac{6}{9} = \frac{2}{3}, \frac{OA}{OM} = \frac{8}{12} = \frac{2}{3},$

$\therefore \frac{OB}{OP} = \frac{OA}{OM}$

and ∠BOA ≅ ∠POM ....(Vertivally opposote angles)

∴ ΔAOB ~ ΔMOP .....(SAS test for similarity)

Solution 17:

In ΔRYX and ΔRPQ,

$\angle YRX \cong \angle PRQ$ ....(Common angle)

$\therefore$ To prove $\Delta RYX \sim \Delta RPQ$ by SAS test, their sides conatining $\angle R$ should be in proportion.

(i) $\frac{RY}{RP} = \frac{6}{14} = \frac{3}{7}$ and $\frac{RX}{RQ} = \frac{3}{7}$

$\therefore \frac{RY}{RP} = \frac{RX}{RQ}$.

The information given in (i) is sufficient.

(ii) $RP = RY + YP = 6 + 5 = 11$

$RQ = RX + XQ = 9 + 8 = 17$

Now, $\frac{RY}{RP} = \frac{6}{11}$ and $\frac{RX}{RQ} = \frac{9}{17}$

$\therefore \frac{RY}{RP} \neq \frac{RX}{RQ}$

The information given in (ii) is insufficient.

(iii) $RP = RY + YP$

$\therefore 12 = RY + 3$ $\therefore RY = 9$

Now, $\frac{RY}{RP} = \frac{9}{12} = \frac{3}{4}$ and $\frac{RX}{RQ} = \frac{6}{8} = \frac{3}{4}$

$\therefore \frac{RY}{RP} = \frac{RX}{RQ}$.

The information given in (iii) is sufficient.