CBSE Test Paper 03

Chapter 10 Vector Algebra

1. Let the vectors $\vec{a}\ and\ \vec{b}$ be such that $|\vec{a}| = 3\ and\ |b| = \frac{\sqrt{2}}{3},\ then\ \vec{a} \times \vec{b}$ is a unit vector if the angle between vectors $\vec{a}\ and\ \vec{b}$ is
   a. $\frac{\pi}{4}$
   b. $\frac{\pi}{3}$
   c. $\frac{\pi}{6}$
   d. $\frac{\pi}{2}$
2. Negative of a Vecto r$\vec{a}$ is a
   a. A vector whose magnitude is the same as that $\vec{a}$ of but direction is opposite to that of $\vec{a}$
   b. A vector whose magnitude is the same as that $\vec{a}$ of but direction is perpendicular to that of $\vec{a}$
   c. A vector whose magnitude is the same as that $\vec{a}$ of but direction is 120°to that of $\vec{a}$
   d. A scalar whose magnitude is the same as that $\vec{a}$
3. If $P_1\,(x_1,\ y_1,\ z_1)$ and $P_2\,(x_2,\ y_2,\ z_2)$ are any two points, then the vector joining P_1 and P_2 is the vector P_1P_2. Magnitude of the vector $\overrightarrow{P_1P_2}$ is
   a. $\sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2 + (z_2 - z_1)^2}$
   b. $\sqrt{(x_2 + x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
   c. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
   d. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 + z_1)^2}$
4. If λ is a real number $\lambda\vec{a}$ is a
   a. vector
   b. unit vector
   c. scalar
   d. inner product
5. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then the dot product $\vec{a}.\vec{b} =$.
   a. $a_1b_1 - a_2b_2 + a_3b_3$
   b. $a_1b_1 + a_2b_2 + a_3b_3$

c. $a_1b_1 - a_2b_2 - a_3b_3$

d. $a_1b_1 + a_2b_2 - a_3b_3$

6. The magnitude of the vector $6\hat{i}$ + $2\hat{j}$ + $3\hat{k}$ is _______.
7. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is _______.
8. If $\left|\overrightarrow{a} \times \overrightarrow{b}\right|^2$ + $\left|\overrightarrow{a} . \overrightarrow{b}\right|^2$ = 144 and $\left|\overrightarrow{a}\right|$ = 4, then $\left|\overrightarrow{b}\right|$ is equal to _______.
9. If A, B and C are the vertices of a $\triangle ABC$, then what is the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$?
10. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 3$,then find the projection of $\vec{b}$ on $\vec{a}$.
11. Find the angle between$\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively, when $|\vec{a} \times \vec{b}| = \sqrt{3}$.
12. Find the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.
13. Find the sum of the vectors: $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.
14. Find $\left|\vec{a} - \vec{b}\right|$ if $|\vec{a}| = 2, \left|\vec{b}\right| = 3$ and $\vec{a}.\vec{b} = 4$.
15. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy$.
16. Show that the points A (1, 2, 7), B (2, 6, 3) and C(3,10, -1) are collinear.
17. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, \left|\vec{b}\right| = 5, |\vec{c}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$.
18. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of the same magnitude, then prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

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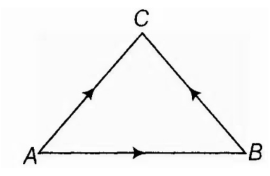
Solution

1. a. $\frac{\pi}{4}$, **Explanation:** It is given that $\overrightarrow{a} \times \overrightarrow{b}$ is a unit vector, then:
$\Rightarrow |\vec{a} \times \vec{b}| = 1 \Rightarrow |\vec{a}||\vec{b}|sin\theta = 1$
$\Rightarrow 3.\frac{\sqrt{2}}{3}sin\theta = 1 \Rightarrow sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$
2. a. A vector whose magnitude is the same as that $\vec{a}$of but direction is opposite to that of $\vec{a}$
Explanation: Negative of $\vec{a}$ is equal to -$\vec{a}$, i.e. A vector whose magnitude is the same as that of $\vec{a}$, but direction is opposite to that of $\vec{a}$.
3. c. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$,
Explanation: If $P_1(x_1,\ y_1,\ z_1)$ and $P_2(x_2,\ y_2,\ z_2)$ are any two points, then the vector joining P_1 and P_2 is the vector P_1P_2, then : $|\overrightarrow{P_1P_2}| =$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
4. a. vector, **Explanation:** If a vector is multiplied by any scalar then , the result is always a vector.
5. b. $a_1b_1 + a_2b_2 + a_3b_3$, **Explanation:** If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and
$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
then $\vec{a}.\vec{b} =a_1b_1 + a_2b_2 + a_3b_3$
$(\because \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ and $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0)$
6. 7
7. two
8. 3
9. Let ABC be the given triangle.



By law of vectors, we have $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{CA}+\overrightarrow{AC}$........(1) [adding $\overrightarrow{CA}$on both sides] (using (1))
$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{CA}-\overrightarrow{CA}[\because \overrightarrow{AC}=-\overrightarrow{CA}]$
$\therefore \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

10. If $|\vec{a}|=2, |\vec{b}|=3$ and $\vec{a}\cdot\vec{b}=3$,then we have to find the projection of $\vec{b}\ on\ \vec{a}$.
Projection of $\vec{b}\ on\ \vec{a}=\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ = $\frac{3}{2}$

11. Here we are given that, $|\vec{a}|=1,\ |\overrightarrow{b}|=2$ and $|\vec{a}\times\vec{b}|=\sqrt{3}$
$\Rightarrow |\vec{a}||\vec{b}|\sin\theta=\sqrt{3}\ [\because \vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\hat{n}$ and $|\hat{n}|=1]$
$\Rightarrow \quad 1\times 2\times\sin\theta=\sqrt{3}$
$\Rightarrow \quad \sin\theta=\frac{\sqrt{3}}{2}=\sin\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{3}$
Hence, the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$.

12. Let $\vec{a}=5\hat{i}-\hat{j}+2\hat{k}$
A vector in the direction of vector $\vec{a}$ which has magnitude 8 units $=8\hat{a}$
$=8.\frac{\vec{a}}{|\vec{a}|}=\frac{8\left(5\hat{i}-\hat{j}+2\hat{k}\right)}{\sqrt{(5)^2+(-1)^2+(2)^2}}$
$=\frac{8\left(5\hat{i}-\hat{j}+2\hat{k}\right)}{\sqrt{25+1+4}}=\frac{8}{\sqrt{30}}\left(5\hat{i}-\hat{j}+2\hat{k}\right)$
$=\frac{40}{\sqrt{30}}\hat{i}-\frac{8}{\sqrt{30}}\hat{j}+\frac{16}{\sqrt{30}}\hat{k}$

13. Given: $\vec{a}=\hat{i}-2\hat{j}+\hat{k},\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$
Adding, $\vec{a}+\vec{b}+\vec{c}=\hat{i}-2\hat{j}+\hat{k}-2\hat{i}+4\hat{j}+5\hat{k}+\hat{i}-6\hat{j}-7\hat{k}$
$=0\hat{i}-4\hat{j}-\hat{k}=-4\hat{j}-\hat{k}$

14. $\left|\vec{a}-\vec{b}\right|^2=\left(\vec{a}-\vec{b}\right).\left(\vec{a}-\vec{b}\right)$
$=\vec{a}.\vec{a}-\vec{a}.\vec{b}-\vec{b}.\vec{a}+\vec{b}.\vec{b}$
$=|\vec{a}|^2-2\vec{a}.\vec{b}+\left|\vec{b}\right|^2.[\because \vec{a}.\vec{b}=\vec{b}.\vec{a}]$
$=4-2\times 4+9$ = 5
Therefore, $\left|\vec{a}-\vec{b}\right|=\sqrt{5}$

15. According to the question,

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Now, $\vec{r} \times \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i}$

$= x(\hat{i} \times \hat{i}) + y(\hat{j} \times \hat{i}) + z(\hat{k} \times \hat{i})$

$= x(0) + y(-\hat{k}) + z(\hat{j})$

$= -y\hat{k} + z\hat{j}$

Now, $(\vec{r} \times \hat{j}) = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{j}$

$= x(\hat{i} \times \hat{j}) + y(\hat{j} \times \hat{j}) + z(\hat{k} \times \hat{j})$

$= x\hat{k} + y(0) + z(-\hat{i}) = x\hat{k} - z\hat{i}$

$\therefore \quad (\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) = (-y\hat{k} + z\hat{j}) \cdot (x\hat{k} - z\hat{i})$

$= -yx + 0 + 0 + 0 = -xy$

$\therefore \quad (\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy = 0$

16. The given points are A (1, 2, 7), B (2, 6, 3) and C(3,10, - 1) respectively.

$\therefore$ Position vector of point $A = \overrightarrow{OA} = \hat{i} + 2\hat{j} + 7\hat{k}$

Position vector of point $B = \overrightarrow{OB} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

Position vector of point $C = \overrightarrow{OC} = 3\hat{i} + 10\hat{j} - \hat{k}$

Now $\overrightarrow{AB}$ = Position vector of point B – Position vector of point A

$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \left(\hat{i} + 2\hat{j} + 7\hat{k}\right)$

$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \hat{i} - 2\hat{j} - 7\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$...(i)

And $\overrightarrow{AC}$ = Position vector of point C – Position vector of point A

$= 3\hat{i} + 10\hat{j} - \hat{k} - (\hat{i} + 2\hat{j} + 7\hat{k})$

$= 3\hat{i} + 10\hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 7\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k} = 2(\hat{i} + 4\hat{j} - 4\hat{k})$...(ii)

$\Rightarrow \overrightarrow{AC} = 2.\overrightarrow{AB}$ [Using eq. (i)]

$\Rightarrow$ Vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel. $[\because \vec{a} = mb]$

But $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have a common point A and hence they can't be parallel.Thus, the points A,B,C are collinear.

17. $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} + \vec{b} = -\vec{c}$

$\left(\vec{a} + \vec{b}\right).(-\vec{c}) = -\vec{c}.(-\vec{c})$

$\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)=\vec{c}.\vec{c}$

$|\vec{a}|^2+2ab+\left|\vec{b}\right|=|\vec{c}|^2$

$\vec{a}.\vec{b}=\frac{|\vec{c}|^2-|\vec{a}|^2-|\vec{b}|^2}{2}$

$\vec{a}.\vec{b}=\frac{49-9-25}{2}=\frac{15}{2}$

$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\frac{1}{2}$

$\theta=60$

18. According to the question, $|\vec{a}|=|\vec{b}|=|\vec{c}|=\lambda$(say) ...(i)

and $\vec{a}\cdot\vec{b}=0,\vec{b}.\vec{c}=0$and $\vec{a}.\vec{c}=0$ ...(ii)

Now, $|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

$=\lambda^2+\lambda^2+\lambda^2+2(0+0+0)=3\lambda^2$

$\Rightarrow|\vec{a}+\vec{b}+\vec{c}|=\sqrt{3}\lambda$ [length cannot be negative]

Let $(\vec{a}+\vec{b}+\vec{c})$ is inclined at angles θ_1,θ_2

and θ_3 respectively with vector $\vec{a},\vec{b}$ and $\vec{c}$,

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}=|\vec{a}+\vec{b}+\vec{c}||\vec{a}|cos\,\theta_1$

$[\because\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta]$

$\Rightarrow|\vec{a}|^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=\sqrt{3}\lambda\times\lambda\cos\theta_1$

$\Rightarrow\lambda^2+0+0=\sqrt{3}\lambda^2\cos\theta_1$

[from Equations (i) and (ii)]

$\therefore\quad\cos\theta_1=\frac{1}{\sqrt{3}}$

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}=\left|\vec{a}+\vec{b}+\vec{c}\right|\left|\vec{b}\right|\cos\theta_2$

$\Rightarrow\quad\vec{a}\cdot\vec{b}+|\vec{b}|^2+\vec{c}\cdot\vec{b}=\sqrt{3}\lambda\cdot\lambda\cos\theta_2$

$\Rightarrow\quad\theta+\lambda+\theta=\sqrt{3}\lambda_2\cos\theta_2$

[from Equation. (i) and (ii)]

$\Rightarrow\cos\theta_2=\frac{1}{\sqrt{3}}$

Similarly, $(a+\vec{b}+\vec{c})\cdot\vec{c}=|\vec{a}+\vec{b}+\vec{c}|\vec{c}|\cos\theta_3|$.

$\Rightarrow\quad\cos\theta_1=\frac{1}{\sqrt{3}}$

Thus, $\cos\theta_1=\cos\theta_2=\cos\theta_3=\frac{1}{\sqrt{3}}$

$\therefore\ \overrightarrow{(a+\vec{b}+\vec{c})}$ is equally inclined with the vectors $\vec{a},\vec{b}$ and $\vec{c}$.