11. Statistics

Questions Pg-249

1. Question

The distance covered by an athlete in long jump practice are

6.10, 6.20, 6.18, 6.20, 6.25, 6.21, 6.15, 6.10

in metres. Find the mean and median. Why is it that there is not much difference between these?

Answer

Mean is the average value

$$\text{mean} = \frac{6.10 + 6.20 + 6.18 + 6.20 + 6.25 + 6.21 + 6.15 + 6.10}{8}$$

⇒ mean = 6.17

Median is the central value of a data set.

in this question we have 8 different number, and we have to find the median of these numbers.

Let's write them in ascending order

6.10, 6.10, 6.15, 6.18, 6.20, 6.20, 6.21, 6.25

the two central values are $\frac{n}{2}$ and $\frac{n+1}{2}$, i.e., 4^{th} and 5^{th} numbers

which are 6.18 and 6.20

$$\Rightarrow \text{median} = \frac{6.18 + 6.20}{2}$$

⇒ median = 6.19

The mean and median are approximately same because all of the data are very near to each other.

2. Question

The table below gives the rainfall during one week of September 2015 in various districts of Kerala.

District	Rainfall (mm)
Kasaragod	66.7
Kannur	56.9
Kozhikode	33.5
Wayanad	20.5
Malappuram	13.5
Palakkad	56.9
Thrissur	53.4
Emakulam	70.6
Kottayam	50.3
Idukki	30.5
Pathanamthitta	56.4
Alapuzha	45.5
Kollam	56.3
Thiruvananthapuram	89.0

Calculate the mean and median rainfall in Kerala during this week. Why is the median less than mean?

Answer

let's first write the data in ascending order

13.5,20.5,30.5,33.5,45.5,50.3,53.4,56.3,56.4,56.9,56.9,66.7,

70.6,89.0

there are total 14 values, so the two central values are 7^{th} and 8^{th}

term.

$\Rightarrow \text{median} = \frac{53.4 + 56.3}{2}$

⇒ median = 54.85

mean is the average of all the values

$\Rightarrow \text{mean} = \frac{13.5 + 20.5 + 30.5 + 33.5 + 45.5 + 50.3 + 53.4 + 56.3 + 56.4 + 56.9 + 56.9 + 66.7 + 70.6 + 89.0}{14}$

⇒ 41.9

3. Question

Prove that for a set of n numbers in arithmetic sequence, the mean and median are equal.

Answer

let the first value be 'a' and the common difference be 'd'.

$\text{Sum of AP} = \text{na} + \frac{\text{n}(\text{n} + 1)}{2}\text{d}$

$\Rightarrow \text{mean} = \text{a} + \frac{\text{n}+1}{2}\text{d}$

Now we know that any k^{th} term of the AP is given by

a + kd

n is the total number of terms, so the middle term would be $\frac{\text{n}+1}{2}\text{th term}$.

and its value would be $\text{a} + \frac{\text{n}+1}{2}\text{d}$ which is equal to the mean.

Questions Pg-252

1. Question

15 households in a neighbourhood are sorted a according to their monthly income in the table below.

Monthly in income (Rs)	Number of households
4000	3
5000	7
6000	8
7000	5
8000	5
9000	4
10000	3

Calculate the median income.

Answer

There are total 35 households.

the median monthly income of these houses will be Monthly Income of $\frac{35+1}{2}$ which is 18^{th} term.

Monthly in income (Rs)	Number of households
4000	3
5000 or less	10
6000 or less	18
7000 or less	23
8000 or less	28
9000 or less	32
10000 or less	35

we can see that the 18^{th} value is Rs. 6000 which is the also the monthly income and 8 other households and the other 10 have low income than Rs.6000.

∴ median = Rs.6000

2. Question

The table below shows the workers in a factory sorted according to their daily wages:

Daily wages (Rs)	Number of workers
400	2
500	4
600	5
700	7
800	5
900	4
1000	3

Calculate the median daily wage

Answer

Daily wages (Rs)	Number of workers
400	2
500 or less	6
600 or less	13
700 or less	20
800 or less	25
900 or less	29
1000 or less	32

total no. of workers = 32

the median would be the wage of = mean of 16^{th} and 17^{th} worker's wage

⇒ In this question both the 16^{th} and 17^{th} worker have a wage of Rs. 700.

∴ median = Rs. 700

3. Question

The table below gives the number of babies born in a hospital during a week, sorted according to their birth weight.

Weight (kg)	Number of babies
2.500	4
2.600	6
2.750	8
2.800	10
3.000	12
3.150	10
3.250	8
3.300	7
3.500	5

Calculate the median birth-weight.

Answer

Weight (kg)	Number of babies
2.500	4
2.600 or less	10
2.750 or less	18
2.800 or less	28
3.000 or less	40
3.150 or less	50
3.250 or less	58
3.300 or less	65
3.500 or less	70

Sol. total no. of babies = 70

the median weight of babies would be the average weight of 35^{th} and 36^{th} baby.

both babies are having a weight of 3.000 kg

∴ the median weight of all the babies = 3.000 kg

Questions Pg-257

1. Question

Some households in a locality are sorted according to their electricity usage and given in the table below:

Usage of electricity (units)	Number of households
80 – 90	3
90 – 100	6
100 – 110	5
110 – 120	8
120 – 130	9
130 – 140	4

Calculate the median usage.

Answer

Usage of electricity (units)	Number of households
Below 90	3
Below 100	9
Below 110	14
Below 120	22
Below 130	31
Below 140	35

total number of households = 35

∴ the median is the usage of 18^{th} household.

which lies between 110-120 and in this range there are 8 households.

let's assume a uniform distribution.

i.e., 14^{th} household's usage is 110 and 15^{th} household's usage is $110 + \frac{120-110}{8}$

$= 111\frac{1}{4}$

⇒ usage of 18^{th} household = $111\frac{1}{4} + 3 \times \frac{10}{8}$

⇒ usage of 18^{th} household = 115

∴ 115 is the median value

2. Question

The table below shows children in a class sorted according to their marks in a maths exam:

Makrs	Number of children
0 – 10	4
10 – 20	10
20 – 30	12
30 – 40	9
40 – 50	5

Calculate the median mark.

Answer

Makrs	Number of children
Below 10	4
Below 20	14
Below 30	26
Below 40	35
Below 50	40

total no. of children = 40

∴ the median is the mean of marks of 20^{th} and 21^{st} child.

both lies in the range of 20-30.

let's assume a uniform distribution of marks.

i.e., 14^{th} child has 20 marks and 15^{th} child has $20 + \frac{10}{12}$ marks.

⇒ marks of 15^{th} child = $20\frac{10}{12}$

⇒ marks of 20^{th} child = $20\frac{10}{12} + 5 \times \frac{10}{12}$

⇒ marks of 20^{th} child = 25

also, marks of 21^{st} child = $25 + \frac{10}{12} = \frac{155}{6}$

median = $\frac{25 + \frac{155}{6}}{2}$

median = $\frac{305}{12}$

3. Question

The income tax paid by the workers in an office is shown below:

Income tax (Rs)	Number of workers
1000 – 2000	8
2000 –3000	10
3000 – 4000	15
4000 – 5000	18
5000 – 6000	22
6000 – 7000	8
7000 – 8000	6
8000 – 9000	3

Calculate the median tax paid.

Answer

Income tax (Rs)	Number of workers
Below 2000	8
Below 3000	18
Below 4000	33
Below 5000	51
Below 6000	73
Below 7000	81
Below 8000	87
Below 9000	90

the total number of workers = 90

⇒ median is the mean of 45^{th} and 46^{th} worker's Income tax

both lies in the range of 4000-5000

let's assume an equal distribution

33^{rd} worker → 4000

34^{th} worker → $4000 + \frac{1000}{18}$

45^{th} worker → $4000 + 7 \times \frac{1000}{18} = \frac{79000}{18}$

45^{th} worker → $4000 + 8 \times \frac{1000}{18} = \frac{80000}{18}$

median = $\frac{79000 + 80000}{36}$

⇒ median = $\frac{26500}{6}$