CBSE Board
Class XII Mathematics
Board Paper 2012
Delhi Set – 2

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three Section A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Evaluate $\int 1 - x \ \sqrt{x}\, dx$.

2. Evaluate: $\int_{2}^{3} \frac{1}{x}\, dx$

3. If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$, write the value of x.

4. Find 'λ' when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

5. If a line has direction ratios 2,-1,-2 then what are its direction cosines?

6. Let * be a 'binary' operation on N given by a * b = LCM (a, b) for all a, b $\in$ N. Find 5 * 7.

7. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$.

8. Simplify: $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

9. Find the sum of the following vectors:
$\vec{a}=\hat{i}-2\hat{j},\ \vec{b}=2\hat{i}-3\hat{j},\ \vec{c}=2\hat{i}+3\hat{k}$.

10. If $\triangle=\begin{vmatrix}5 & 3 & 8\\ 2 & 0 & 1\\ 1 & 2 & 3\end{vmatrix}$, white the cofactor of the element a_{32}.

SECTION – B

11. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $|\vec{a}|=5$, $|\vec{b}|=12$ and $|\vec{c}|=13$ and $\vec{a}+\vec{b}+\vec{c}=0$ Find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

12. Solve the following differential equation:

$$2x^2\frac{dy}{dx}-2xy+y^2=0.$$

13. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

14. If $(\cos x)^y=(\cos y)^x$, find $\frac{dy}{dx}$.

OR

If $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{\sin^2 a+y}{\sin a}$.

15. Let A = R – {3} and B = R – {1}. Consider the function f : A → B defined by $f(x)=\left(\frac{x-2}{x-3}\right)$. Show that f is one-one and onto and hence find f^{-1}.

16. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{\pi}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

OR

Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$.

17. Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$.

OR

Using differentials, find the approximate value of $\sqrt{49.5}$.

18. Evaluate: $\int \sin x \sin 2x \sin 3x \, dx$

OR

Evaluate: $\int \frac{2}{1 - x \quad 1 + x^2} dx$

19. Using properties of determinants prove the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$$

20. If $y = 3 \cos (\log x) + 4 \sin (\log x)$, show that

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

21. Find the equation of the line passing through the point (-1,3,-2) and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

22. Find the particular solution of the following differential equation:

$$x + 1 \ \frac{dy}{dx} = 2e^{-y} - 1; \ y = 0 \text{ when } x = 0.$$

SECTION – C

23. Using matrices solve the following system of linear equations:

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12$$

OR

Using elementary operations, find the inverse of the following matrix:

$$\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

24. A manufacturer produces nuts and bolts. It takes 1 hours of work on machine A and 3 hours on machine B to product a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of `17.50 per package on nuts and `7 per package of bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day? From the above as a linear programming problem and solve it graphically.

25. Find the equation of the plane determined by the point A(3, - 1, 2), B(5, 2, 4) and C(-1, -1, 6) and hence find the distance between the plane and the point P(6, 5, 9).

26. Prove that $\int_0^{\frac{\pi}{4}} \sqrt{\tan x} + \sqrt{\cot x}\ dx = \sqrt{2} \cdot \frac{\pi}{2}$.

OR

Evaluate $\int_1^3 2x^2 + 5x\ \ dx$ as a limit of sum.

27. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

28. A girl throws a die. If she get a 5 OR 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 OR 4, she tosses a coin two times and notes the number of heads obtained. If she obtained exactly two heads, what is the probability that she threw 1, 2, 3 OR 4 with the die?

29. Using the method of method of integration, find the area of the region bounded by the following lines:

$3x - y - 3 = 0$,

$2x + y - 12 = 0$,

$x - 2y - 1 = 0$

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SECTION – A

1. $\int 1-x\ \sqrt{x}dx$

$$=\int\left(\sqrt{x}-x^{\frac{3}{2}}\right)dx$$

$$=\int\left(x^{\frac{1}{2}}-x^{\frac{3}{2}}\right)dx$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+C$$

$$=\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{3}x^{\frac{5}{2}}+C$$

2.

$$\int_2^3 \frac{1}{x}dx=\ \log x\ {}_2^3=\log 3-\log 2=\ \log\frac{3}{2}$$

3.

$$\begin{pmatrix}2 & 3\\ 5 & 7\end{pmatrix}\begin{pmatrix}1 & -3\\ -2 & 4\end{pmatrix}=\begin{pmatrix}-4 & 6\\ -9 & x\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}-4 & 6\\ -9 & 13\end{pmatrix}=\begin{pmatrix}-4 & 6\\ -9 & x\end{pmatrix}$$

by equality of matrices

$x=13$

4. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = 4$ (given)

$$\Rightarrow \frac{(\lambda\hat{i} + \hat{j} + 4\hat{k}).(2\hat{i} + 6\hat{j} + 3\hat{k})}{\sqrt{2^2 + 6^2 + 3^2}} = 4$$

$$\Rightarrow \frac{2\lambda + 6 + 12}{7} = 4$$

$$\Rightarrow \lambda = 5$$

5. The direction cosines are

$$\frac{2}{\sqrt{2^2 + (-1)^2 + (-2)^2}}, \frac{-1}{\sqrt{2^2 + (-1)^2 + (-2)^2}}, \frac{-2}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$$

$$= \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

6. According to the given operation

5 * 7 = L.C.M.(5, 7) = 35

7. Principal value of $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$

Principal value of $\sin^{-1}(\frac{-1}{2}) = \frac{-\pi}{6}$

Hence principal value of $\cos^{-1}(\frac{1}{2}) - 2\sin^{-1}(\frac{-1}{2})$

$$= \frac{\pi}{3} - 2\left(\frac{-\pi}{6}\right)$$

$$= \frac{2\pi}{3}$$

8. $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

$$=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\cos\theta\sin\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\quad\left[\because \cos^2\theta+\sin^2\theta=1\right]$$

9. Sum of the vectors,

$$\vec{a}+\vec{b}+\vec{c}=(\hat{i}-2\hat{j})+(2\hat{i}-3\hat{j})+(2\hat{i}+3k)$$
$$=5\hat{i}-5\hat{j}+3\hat{k}$$

10.

Minor of the element $a_{32}=\begin{vmatrix}5 & 8\\ 2 & 1\end{vmatrix}$

$$=5-16=-11$$

SECTION – B

11. Considering dot product on both sides,

$$\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)=0.0$$

$$\Rightarrow\quad |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$$

$$\Rightarrow\quad 5^2+12^2+13^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$$

$$\Rightarrow\quad 2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=-338$$

$$\Rightarrow\quad \left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=-\frac{338}{2}=-169$$

12. Here, $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$

$$\frac{dy}{dx} = \frac{2xy - y^2}{2x^2}$$

Hence the given equation is an homogeneous equation.

Let $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So, $v + x\frac{dv}{dx} = \frac{2x(vx) - (vx)^2}{2x^2}$

$$= \frac{2v - v^2}{2} = v - \frac{v^2}{2}$$

$\Rightarrow$ $x\frac{dv}{dx} = -\frac{v^2}{2}$

$\Rightarrow$ $2\int\frac{1}{v^2}dv = -\int\frac{dx}{x}$

$\Rightarrow$ $2\left(-\frac{1}{v}\right) = -\log|x| + c$

$\Rightarrow$ $\frac{2x}{y} = \log|x| + c$

13. Let the man toss the coin *n* times. The *n* tosses are *n* Bernoulli trials

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$

$\Rightarrow p = \frac{1}{2} \quad \Rightarrow q = \frac{1}{2}$

$$\therefore P(X=x) = {}^nC_x \, p^{n-x} \, q^x = {}^nC_x \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^x = {}^nC_x \left(\frac{1}{2}\right)^n$$

It is given that

$P(\text{getting at least one head}) > \frac{80}{100}$

$\Rightarrow P(X \geq 1) > 0.8$

$\Rightarrow 1 - P(X=0) > 0.8$

$\Rightarrow 1 - {}^nC_0 \frac{1}{2^n} > 0.8$

$\Rightarrow {}^nC_0 \frac{1}{2^n} < 0.2$

$\Rightarrow \frac{1}{2^n} < 0.2$

$\Rightarrow 2^n > \frac{1}{0.2} = 5$

$\Rightarrow 2^n > 5 \qquad ----(1)$

The minimum value of n which satisfies the given inequality is 3.

Thus, the man should toss the coin 3 or more than 3 times.

14. The given function is $(\cos x)^y = (\cos y)^x$

Taking logarithm on both the sides, we obtain

$y \log \cos x = x \log \cos y$

Differentiating both sides, we obtain

$$\log \cos x \times \frac{dy}{dx} + y \times \frac{d}{dx}(\log \cos x) = \log \cos y \times \frac{d}{dx}(x) + x \times \frac{d}{dx}(\log \cos y)$$

$$\Rightarrow \log \cos x \times \frac{dy}{dx} + y \times \frac{1}{\cos x} \times \frac{d}{dx}(\cos x) = \log \cos y \times 1 + x \times \frac{1}{\cos y} \times \frac{d}{dx}(\cos y)$$

$$\Rightarrow \log \cos x \times \frac{dy}{dx} + \frac{y}{\cos x}(-\sin x) = \log \cos y + \frac{x}{\cos y} \times (-\sin y) \times \frac{dy}{dx}$$

$$\Rightarrow \log \cos x \times \frac{dy}{dx} - y\tan x = \log\cos y - x\tan y \times \frac{dy}{dx}$$

$$\Rightarrow \log \cos x \times \frac{dy}{dx} + x\tan y \times \frac{dy}{dx} = \log\cos y + y\tan x$$

$$\Rightarrow \left(\log \cos x + x\tan y\right) \times \frac{dy}{dx} = \log\cos y + y\tan x$$

$$\therefore \frac{dy}{dx} = \frac{\log\cos y + y\tan x}{\log \cos x + x\tan y}$$

OR

We have,

$\sin y = x\sin\left(a+y\right)$

$$\Rightarrow x = \frac{\sin y}{\sin\left(a+y\right)}$$

Differentiating the above function we have,

$$1 = \frac{\sin\left(a+y\right)\times \cos y\frac{dy}{dx} - \sin y \times \cos\left(a+y\right)\frac{dy}{dx}}{\sin^2\left(a+y\right)}$$

$$\Rightarrow \sin^2\left(a+y\right) = \left[\sin\left(a+y\right)\times\cos y - \sin y\times\cos\left(a+y\right)\right]\frac{dy}{dx}$$

$$\Rightarrow \frac{\sin^2\left(a+y\right)}{\left[\sin\left(a+y\right)\times\cos y - \sin y\times\cos\left(a+y\right)\right]} = \frac{dy}{dx}$$

$$\Rightarrow \frac{\sin^2\left(a+y\right)}{\sin\left(a+y-y\right)} = \frac{dy}{dx}$$

$$\Rightarrow \frac{\sin^2\left(a+y\right)}{\sin a} = \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2\left(a+y\right)}{\sin a}$$

15. Given that $A = R - \{3\}$, $B = R - \{1\}$

Consider the function

f: $A \rightarrow B$ defined by $f\left(x\right) = \left(\frac{x-2}{x-3}\right)$

Let $x, y \in A$ such that $f\left(x\right) = f\left(y\right)$

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$\Rightarrow (x-2)(y-3)=(y-2)(x-3)$

$\Rightarrow xy-3x-2y+6=xy-3y-2x+6$

$\Rightarrow -3x-2y=-3y-2x$

$\Rightarrow 3x-2x=3y-2y$

$\Rightarrow x=y$

$\therefore$ f is one-one.

Let $y \in B = R-\{1\}$

Then, $y \neq 1$. The function f is onto if there exists $x \in A$ such that $f(x)=y$.

Now, $f(x)=y$

$\Rightarrow \frac{x-2}{x-3}=y$

$\Rightarrow x-2=y(x-3)$

$\Rightarrow x-2=xy-3y$

$\Rightarrow x-xy=2-3y$

$\Rightarrow x(1-y)=2-3y$

$\Rightarrow x=\frac{2-3y}{1-y} \in A \quad [y \neq 1] \ldots(1)$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right)=\frac{\frac{2-3y}{1-y}-2}{\frac{2-3y}{1-y}-3}$$

$$=\frac{2-3y-2+2y}{2-3y-3+3y}$$

$$=\frac{-y}{-1}$$

$$=y$$

$\therefore$ f is onto.

Hence, the function is one-one and onto.

Therefore, f^{-1} exists.

Consider equation (1).

$$x = \frac{2-3y}{1-y} \in A \quad [y \neq 1]$$

Replace y by x and x by $f^{-1}(x)$ in the above equation,

we have,

$$f^{-1}(x) = \frac{2-3x}{1-x},\ x \neq 1$$

16. $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$= \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}\right]$$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right] \qquad \left[\because \sin\theta = 2\sin(\theta/2)\cos(\theta/2) \text{ and } 1+\cos\theta = 2\cos^2(\theta/2)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] = \left(\frac{\pi}{4}-\frac{x}{2}\right) \qquad \text{(proved)}$$

OR

Let $\sin^{-1}\frac{8}{17} = x$.

Then, $\sin x = \frac{8}{17}$; $\cos x = \sqrt{1-x^2}$

$$\Rightarrow \cos x = \sqrt{1-\left(\frac{8}{17}\right)^2}$$

$$\Rightarrow \cos x = \sqrt{\frac{225}{289}}$$

$$\Rightarrow \cos x = \frac{15}{17}$$

$$\therefore \tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \tan x = \frac{\frac{8}{17}}{\frac{15}{17}}$$

$$\Rightarrow \tan x = \frac{8}{15}$$

$$\Rightarrow x = \tan^{-1}\left(\frac{8}{15}\right) \ldots.(1)$$

Let $\sin^{-1}\frac{3}{5} = y \ldots.(2)$

Then, $\sin y = \frac{3}{5}$; $\cos y = \sqrt{1 - y^2}$

$$\Rightarrow \cos y = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \cos y = \sqrt{\frac{16}{25}}$$

$$\Rightarrow \cos y = \frac{4}{5}$$

$$\therefore \tan y = \frac{\sin y}{\cos y}$$

$$\Rightarrow \tan y = \frac{\frac{3}{5}}{\frac{4}{5}}$$

$$\Rightarrow \tan y = \frac{3}{4}$$

$$\Rightarrow y = \tan^{-1}\left(\frac{3}{4}\right) \ldots.(3)$$

From equations (2) and (3), we have,

$$\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

Now consider $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)$:

From equations (1) and (3), we have,

$$\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{8}{15}\right)+\tan^{-1}\left(\frac{3}{4}\right)$$

$$=\tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}\right)\quad\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan^{-1}\left(\frac{32+45}{60-24}\right)$$

$$\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{77}{36}\right).....(4)$$

Now, we have:

Let $\tan^{-1}\left(\frac{77}{36}\right)=z.$

Then $\tan z=\frac{77}{36}$

$$\Rightarrow \sec z=\sqrt{1+\left(\frac{77}{36}\right)^2}\quad\left[\because \sec\theta=\sqrt{1+\tan^2\theta}\right]$$

$$\Rightarrow \sec z=\sqrt{\frac{1296+5929}{1296}}$$

$$\Rightarrow \sec z=\sqrt{\frac{7225}{1296}}$$

$$\Rightarrow \sec z=\frac{85}{36}$$

We know that $\cos z=\frac{1}{\sec z}$

Thus, $\sec z=\frac{85}{36},\cos z=\frac{36}{85}$

$$\Rightarrow z=\cos^{-1}\left(\frac{36}{85}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{77}{36}\right)=\cos^{-1}\left(\frac{36}{85}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)\quad[\because \text{from equation (4)}]$$

Hence proved.

17. The equation of the given curve is $y = x^3 - 11x + 5$.

The equation of the tangent to the given curve is given as $y = x - 11$ (which is of the form $y = mx + c$).

∴Slope of the tangent = 1

Now, the slope of the tangent to the given curve at the point (x, y) is given by,

$$\frac{dy}{dx} = 3x^2 - 11$$

Then, we have:

$3x^2 - 11 = 1$

$\Rightarrow 3x^2 = 12$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

When $x = 2$, $y = (2)^3 - 11(2) + 5 = 8 - 22 + 5 = -9$.

When $x = -2$, $y = (-2)^3 - 11(-2) + 5 = -8 + 22 + 5 = 19$.

Hence, the required points are (2, −9) and (−2, 19).

OR

Consider $y = \sqrt{x}$, Let $x = 49$ and $\Delta x = 0.5$.

Then,

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$
$$= \sqrt{49.5} - \sqrt{49}$$
$$= \sqrt{49.5} - 7$$
$$\Rightarrow \sqrt{49.5} = 7 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x$$
$$= \frac{1}{2\sqrt{x}}(0.5) \quad \left[\because y = \sqrt{x}\right]$$
$$= \frac{1}{2\sqrt{49}}(0.5)$$
$$= \frac{1}{14}(0.5)$$
$$= 0.035$$

Hence the approximate value of $\sqrt{49.5}$ is 7 + 0.035 = 7.035

18. It is known that, $\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$

$$\therefore \int \sin x \sin 2x \sin 3x\, dx = \int \left[\sin x \times \frac{1}{2}\left(\cos(2x-3x) - \cos(2x+3x)\right)\right]$$

$$= \frac{1}{2}\int \left(\sin x \cos(-x) - \sin x \cos 5x\right) dx$$

$$= \frac{1}{2}\int \left(\sin x \cos x - \sin x \cos 5x\right) dx$$

$$= \frac{1}{2}\int \frac{\sin 2x}{2} dx - \frac{1}{2}\int \sin x \cos 5x$$

$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int \frac{1}{2}\left\{\sin(x+5x) + \sin(x-5x)\right\} dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\int \left(\sin 6x + \sin(-4x)\right) dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{6} + \frac{\cos 4x}{4}\right] + C$$

$$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$$

$$= \frac{-6\cos 2x}{48} - \frac{1}{8}\left[\frac{-2\cos 6x + 3\cos 4x}{6}\right] + C$$

$$= \frac{1}{48}\left[2\cos 6x - 3\cos 4x - 6\cos 2x\right] + C$$

OR

Let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$

$$2 = A(1+x^2) + (Bx+C)(1-x)$$

$$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$$

Equating the coefficient of x^2, x, and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these equations, we obtain

$A = 1$, $B = 1$, and $C = 1$

$$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

$$\Rightarrow \int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$=-\int\frac{1}{x-1}dx+\frac{1}{2}\int\frac{2x}{1+x^2}dx+\int\frac{1}{1+x^2}dx$$

$$=-\log|x-1|+\frac{1}{2}\log\left|1+x^2\right|+\tan^{-1}x+C$$

19.

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^3 & b^3 & c^3\end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$, we have:

$$\Delta=\begin{vmatrix}1-1 & 1-1 & 1\\ a-c & b-c & c\\ a^3-c^3 & b^3-c^3 & c^3\end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & 1\\ a-c & b-c & c\\ a-c\;\; a^2+ac+c^2 & b-c\;\; b^2+bc+c^2 & c^3\end{vmatrix}$$

$$=\; c-a\;\; b-c\begin{vmatrix}0 & 0 & 1\\ -1 & 1 & c\\ -\;\; a^2+ac+c^2 & b^2+bc+c^2 & c^3\end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$, we have:

$$\Delta=\; c-a\;\; b-c\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ b^2-a^2\;\; +\;\; bc-ac & b^2+bc+c^2 & c^3\end{vmatrix}$$

$$=\; b-c\;\; c-a\;\; a-b\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ -\;\; a+b+c & b^2+bc+c^2 & c^3\end{vmatrix}$$

$$=\; a-b\;\; b-c\;\; c-a\;\; a+b+c\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ -1 & b^2+bc+c^2 & c^3\end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = a-b \quad b-c \quad c-a \quad a+b+c \quad -1 \begin{vmatrix} 0 & 1 \\ 1 & c \end{vmatrix}$$

$$= a-b \quad b-c \quad c-a \quad a+b+c$$

Hence proved.

20. It is given that, y = 3cos(log x) + 4sin(log x)
Then,

$$\frac{dy}{dx} = 3\times\frac{d}{dx}\left[\cos \ \log x \ \right]+4\times\frac{d}{dx}\left[\sin \ \log x \ \right]$$

$$=3\times\left[-\sin \ \log x \ \times\frac{d}{dx} \ \log x \ \right]+4\times\left[\cos \ \log x \ \times\frac{d}{dx} \ \log x \ \right]$$

$$=\frac{-3\sin \ \log x}{x}+\frac{4\cos \ \log x}{x}=\frac{4\cos \ \log x \ -3\sin \ \log x}{x}$$

$$\frac{d^2y}{dx^2}=$$

$$=\frac{d}{dx}\left(\frac{4\cos(\log x)-3\sin(\log x)}{x}\right)$$

$$=\frac{x\{4\cos(\log x)-3\sin(\log x)\}'-\{4\cos(\log x)-3\sin(\log x)\}(x)'}{x^2}$$

$$=\frac{x[-4\sin(\log x)\times(\log x)'-3\cos(\log x)\times(\log x)']-4\cos(\log x)+3\sin(\log x)}{x^2}$$

$$=\frac{-4\sin(\log x)-3\cos(\log x)-4\cos(\log x)+3\sin(\log x)}{x^2}$$

$$=\frac{-\sin(\log x)-7\cos(\log x)}{x^2}$$

$$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y$$

$$=x^2\left(\frac{-\sin(\log x)-7\cos(\log x)}{x^2}\right)+x\left(\frac{4\cos(\log x)-3\sin(\log x)}{x}\right)+3\cos(\log x)+4\sin(\log x)$$

$$=-\sin(\log x)-7\cos(\log x)+4\cos(\log x)-3\sin(\log x)+3\cos(\log x)+4\sin(\log x)$$

$$=0$$

Hence proved.

21. We know that, equation of a line passing through (x_1, y_1, z_1) with direction ratios a, b, c is given by $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

So, the required equation of a line passing through $(-1,3,-2)$ is:

$$\frac{x+1}{a} = \frac{y-3}{b} = \frac{z+2}{c} \text{------}(1)$$

Given that line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is perpendicular to line (1), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$a(1) + b(2) + c(3) = 0$$

$$a + 2b + 3c = 0 \qquad \text{---}(2)$$

And line $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$ is perpendicular to line (1), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$a(-3) + b(2) + c(5) = 0$$

$$-3a + 2b + 5c = 0 \qquad \text{---}(3)$$

Solving equation (2) and (3) by cross multiplication,

$$\frac{a}{(2)(5) - (2)(3)} = \frac{b}{(-3)(3) - (1)(5)} = \frac{c}{(1)(2) - (-3)(2)}$$

$$\Rightarrow \frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6}$$

$$\Rightarrow \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{-7} = \frac{c}{4} = \lambda \text{ (say)}$$

$$\Rightarrow a = 2\lambda,\ b = -7\lambda,\ c = 4\lambda$$

Putting the value of a, b, and c in (1) gives

$$\frac{x+1}{2\lambda} = \frac{y-3}{-7\lambda} = \frac{z+2}{4\lambda}$$

$$\Rightarrow \frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

22. $(x+1)\frac{dy}{dx}=2e^{-y}-1$

$\Rightarrow \frac{dy}{2e^{-y}-1}=\frac{dx}{x+1}$

$\Rightarrow \frac{e^{y}}{2-e^{y}}=\frac{dx}{x+1}$

Integrating both sides, we get:

$\int \frac{e^{y}dy}{2-e^{y}}=\log|x+1|+\log C \qquad ...(1)$

Let $2-e^{y}=t$.

$\therefore \frac{d}{dy}\left(2-e^{y}\right)=\frac{dt}{dy}$

$\Rightarrow -e^{y}=\frac{dt}{dy}$

$\Rightarrow e^{y}dy=-dt$

Substituting this value in equation (1), we get:

$\int \frac{-dt}{t}=\log|x+1|+\log C$

$\Rightarrow -\log|t|=\log|C(x+1)|$

$\Rightarrow -\log\left|2-e^{y}\right|=\log|C(x+1)|$

$\Rightarrow \frac{1}{2-e^{y}}=C(x+1)$

$\Rightarrow 2-e^{y}=\frac{1}{C(x+1)} \qquad ...(2)$

Now, at x=0 and y=0, equation (2) becomes:

$\Rightarrow 2-1=\frac{1}{C}$

$\Rightarrow C=1$

Substituting C = 1 in equation (2), we get:

$2-e^{y}=\frac{1}{x+1}$

$\Rightarrow e^{y}=2-\frac{1}{x+1}$

$\Rightarrow e^{y}=\frac{2x+2-1}{x+1}$

$\Rightarrow e^{y}=\frac{2x+1}{x+1}$

$$\Rightarrow y=\log\left|\frac{2x+1}{x+1}\right|,(x\neq -1)$$

This is the required particular solution of the given differential equation.

SECTION – C

23. The given system of equation can be written in the form of AX = B, where

$$A=\begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}, X=\begin{bmatrix}X\\ Y\\ Z\end{bmatrix} \text{ and } B=\begin{bmatrix}7\\ -5\\ 12\end{bmatrix}.$$

Now,

$$|A|=1(12-5)+1(9+10)+2(-3-8)=7+19-22=4\neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11}=7$, $A_{12}=-19$, $A_{13}=-11$

$A_{21}=1$, $A_{22}=-1$, $A_{23}=-1$

$A_{31}=-3$, $A_{32}=11$, $A_{33}=7$

$$\therefore A^{-1}=\frac{1}{|A|}(\text{adj}A)=\frac{1}{4}\begin{bmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{bmatrix}$$

OR

Consider the given matrix.

$$\text{Let } A=\begin{bmatrix}-1 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$$

We know that, $A=I_nA$

Perform sequence of elementary row operations on A on the left hand side and the term I_n on the right hand side till we obtain the result,

$I_n=BA$

Thus, $B=A^{-1}$

$$\text{Here, } I_3=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Thus, we have,

$$\begin{bmatrix}-1 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \rightarrow R_2 + R_1$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_1 \rightarrow R_1 + R_2$

$$\begin{bmatrix} 1 & 5 & 8 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_1 \rightarrow R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_2 \rightarrow \frac{R_2}{3}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{5}{3} \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_3 \rightarrow R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{5}{3} & \frac{-4}{3} & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -4 & 3 \end{bmatrix} A$$

$$R_2 \to R_2 - \frac{5}{3}R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Thus the inverse of the matrix A is given by

$$\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$

24. Let the manufacturer produce x packages of nuts and y packages of bolts.
Therefore, $x \geq 0$ and $y \geq 0$.
The given information can be complied in a table as follows.

	Nuts	Bolts	Availability
Machine A (h)	1	3	12
Machine B (h)	3	1	12

The profit on a package of nuts is Rs. 17.50 and on a package of bolts is Rs. 7.
Therefore, the constraints are

$x + 3y \leq 12$

$3x + y \leq 12$

Total profit, $Z = 17.5x + 7y$

The mathematical formulation of the given problem is

Maximise $Z = 17.5x + 7y$ ...(1)

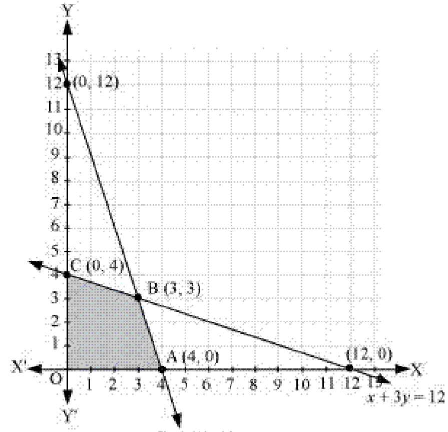
Subject to the constraints,

$x + 3y \leq 12$ ... (2)

$3x + y \leq 12$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows:



The corner points are A(4, 0), B(3, 3), and C(0, 4).
The values of Z at these corner points are as follows:

Corner point	Z = 17.5x + 7y	
O(0, 0)	0	
A(4, 0)	70	
B(3, 3)	73.5	⇒Maximum
C(0, 4)	28	

The maximum value of Z is Rs. 73.50 at (3, 3).
Thus, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of Rs. 73.50.

25. We know that, equation of a plane passing through 3 points,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$

$$\Rightarrow x-3 \quad 12-0 \; - \; y+1 \quad 8+8 \; + \; z-2 \quad 0+12 \; = 0$$

$$\Rightarrow 12x-36-16y-16+12z-24=0$$

$$\Rightarrow 12x-16y+12z-76=0$$

$$\Rightarrow 3x-4y+3z-19=0$$

Also ,perpendicular distance of $P(6, 5, 9)$ to the plane $3x-4y+3z-19=0$

$$= \frac{|3\times 6-4\times 5+3\times 9-19|}{\sqrt{9+16+9}}$$

$$= \frac{6}{\sqrt{34}} \text{ units}$$

26.

$$\int_0^{\pi/4} \ \sqrt{\tan x}+\sqrt{\cot x} \ \ dx$$

$$=\int_0^{\pi/4}\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$$

$$=\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{\sqrt{\sin x\cos x}}\right)dx$$

$$=\sqrt{2}\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{\sqrt{2\sin x\cos x}}\right)dx$$

$$=\sqrt{2}\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{\sqrt{1- \ \sin x-\cos x \ ^2}}\right)dx$$

Put $\sin x-\cos x=t \Rightarrow (\cos x+\sin x)dx=dt$

If $x=0, t=0-1=-1$

and if $x=\frac{\pi}{4}, t=\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=0$

$$\therefore \int_0^{\pi/4} \ \sqrt{\tan x}+\sqrt{\cot x} \ \ dx=\sqrt{2}\int_{-1}^{0}\frac{dt}{\sqrt{1-t^2}}$$

$$=\sqrt{2}\left[\sin^{-1} t\right]_{-1}^{0}$$

$$=\sqrt{2}\left[\sin^{-1} 0-\sin^{-1}(-1)\right]$$

$$=\sqrt{2}\left[0+\frac{\pi}{2}\right]$$

$$=\sqrt{2}\times\frac{\pi}{2}$$

OR

$\int_1^3 (2x^2 + 5x)\, dx$

Here, $a = 1, b = 3, f(x) = 2x^2 + 5x$

$\therefore nh = b - a = 3 - 1 = 2$

Now $\int_a^b f(x)dx = \lim\limits_{h \to 0} h\, [f(a) + f(a+h) + f(a+2h) + ... + f(a+(n-1)h)]$

$\therefore \int_1^3 (2x^2 + 5x)\, dx$

$= \lim\limits_{h \to 0} h\, [f(1) + f(1+h) + f(1+2h) + ... + f(1+(n-1)h)]$

$$= \lim_{h \to 0} h \left[\begin{aligned} &(2(1)^2 + 5(1)) + (2(1+h)^2 + 5(1+h)) + (2(1+2h)^2 + 5(1+2h)) + ... \\ &+ (2(1+(n-1)h)^2 + 5(1+(n-1)h)) \end{aligned} \right]$$

$$= \lim_{h \to 0} h \left[\begin{aligned} &7 + (2h^2 + 9h + 7) + (8h^2 + 18h + 7) + ... \\ &+ (2(n-1)^2 h^2 + 9(n-1)h + 7) \end{aligned} \right]$$

$$= \lim_{h \to 0} h \left[7n + 2h^2 (1^2 + 2^2 + ... + (n-1)^2) + 9h (1 + 2 + ... + (n-1))\right]$$

$$= \lim_{h \to 0} h \left[7n + 2h^2 \frac{n(n-1)(2n-1)}{6} + 9h \frac{n(n-1)}{2}\right]$$

$$= \lim_{h \to 0} \left[7nh + 2 \frac{nh(nh-h)(2nh-h)}{6} + 9 \frac{nh(nh-h)}{2}\right]$$

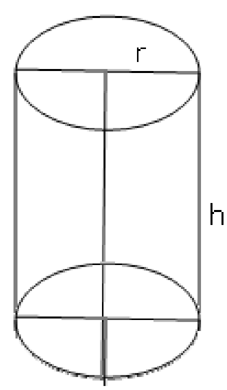
$$= \lim_{h \to 0} \left[14 + 2 \frac{2(2-h)(4-h)}{6} + 9 \frac{2(2-h)}{2}\right]$$

$$= 14 + \frac{16}{3} + 18 = \frac{112}{3}$$

27. Let r and h be the radius and height of the cylinder. Then,

$A = 2\pi rh + 2\pi r^2$ (Given)

$$\Rightarrow h = \frac{A - 2\pi r^2}{2\pi r}$$



Now, $\text{Volume}(V) = \pi r^2 h$

$$\Rightarrow V = \pi r^2 \left(\frac{A - 2\pi r^2}{2\pi r} \right) = \frac{1}{2} (Ar - 2\pi r^3)$$

$$\Rightarrow \frac{dV}{dr} = \frac{1}{2} (A - 6\pi r^2) \qquad ...(1)$$

$$\Rightarrow \frac{d^2V}{dr^2} = \frac{1}{2} (-12\pi r) \qquad ...(2)$$

$$\text{Now, } \frac{dV}{dr} = 0 \Rightarrow \frac{1}{2}\left(A - 6\pi r^2\right) = 0$$

$$\Rightarrow r^2 = \frac{A}{6\pi} \Rightarrow r = \sqrt{\frac{A}{6\pi}}$$

$$\text{Now, } \left|\frac{d^2V}{dr^2}\right|_{r=\sqrt{\frac{A}{6\pi}}} = \frac{1}{2}\left(-12\pi\sqrt{\frac{A}{6\pi}}\right) < 0$$

Therefore, Volume is maximum at $r = \sqrt{\frac{A}{6\pi}}$

$$\Rightarrow r^2 = \frac{A}{6\pi} \Rightarrow 6\pi r^2 = A$$

$$\Rightarrow 6\pi r^2 = 2\pi rh + 2\pi r^2$$

$$\Rightarrow 4\pi r^2 = 2\pi rh \Rightarrow 2r = h$$

Hence, the volume is maximum if its height is equal to its diameter.

28. Consider the following events:

E_1= Getting 5 OR 6 in a single throw of the die

E_2= Getting 1, 2, 3 OR 4 in a single throw of the die

A = Getting exactly 2 heads

We have to find, $P(E_2/A)$.

$$\text{Since } P(E_2/A) = \frac{P(A/E_2)P(E_2)}{P(A/E_1)P(E_1) + P(A/E_2)P(E_2)}$$

$$\text{Now, } P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = \frac{4}{6} = \frac{2}{3}$$

Also,

$$P(A/E_1) = \text{Probability of getting exactly 2 heads when a coin is tossed 3 times} = \frac{3}{8}$$

$$\text{And, } P(A/E_2) = \text{Probability of getting 2 heads when a coin is tossed 2 times} = \frac{1}{2}$$

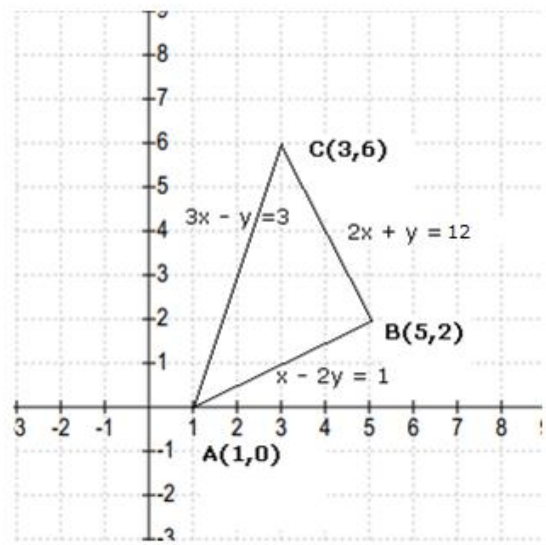
$$\therefore P(E_2/A) = \frac{\frac{1}{2}\times\frac{2}{3}}{\frac{3}{8}\times\frac{1}{3} + \frac{1}{2}\times\frac{2}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}\left(\frac{3}{8}+1\right)} = \frac{\frac{1}{3}}{\frac{1}{3}\left(\frac{8+3}{8}\right)} = \frac{8}{11}$$

29. Given equations are:

$3x - y = 3$ ... (1)

$2x + y = 12$ ... (2)

$x - 2y = 1$ ... (3)



To Solve (1) and (2),

(1) + (2) $\Rightarrow 5x = 15 \Rightarrow x = 3$

(2) $\Rightarrow y = 12 - 6 = 6$

Thus (1) and (2) intersect at C(3, 6).

To solve (2) and (3),

(2) – 2(3) $\Rightarrow 5y = 10 \Rightarrow y=2$

(2) $\Rightarrow 2x = 12 - 2 = 10 \Rightarrow x = 5$

Thus (2) and (3) intersect at B(5, 2).

To solve (3) and (1),

2(1) – (3) $\Rightarrow 5x = 5 \Rightarrow x = 1$

(3) $\Rightarrow 1 - 2y = 1 \Rightarrow y = 0$

Thus (3) and (1) intersect at A(1, 0).

$$\text{Area} = \int_1^3 (3x-3)dx + \int_3^5 (12-2x)dx - \int_1^5 \frac{1}{2}(x-1)dx$$

$$= 3\left[\frac{x^2}{2} - x\right]_1^3 + \left[12x - x^2\right]_3^5 - \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^5$$

$$= 3\left[\left(\frac{9}{2} - 3\right) - \left(\frac{1}{2} - 1\right)\right] + \left[(60-25) - (36-9)\right] - \frac{1}{2}\left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= 3\left[\frac{3}{2} + \frac{1}{2}\right] + [35 - 27] - \frac{1}{2}\left[\frac{15}{2} + \frac{1}{2}\right]$$

$$= 6 + 8 - 4 = 10 \text{ sq. units}$$