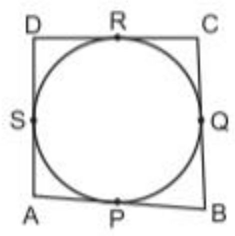
Circles

Case Study Based Questions

Case Study 1

In a park, four poles are standing at positions A, B, C and D around the fountain such that the cloth joining the poles AB, BC, CD and DA touches the fountain at P, Q, R and S respectively as shown in the figure.



Based on the above information, solve the following questions:

Q1. If O is the centre of the circular fountain, then <OSA =

a. 60°

b. 90°

c. 45°

d. All of these

Q2. Which of the following is correct?

a. AS = AP

b. BP = BQ

c. CQ = CR

d. None of these

Q3. If DR = 7 cm and AD = 11 cm, then AP =

a. 4 cm

b. 18 cm

c. 7 cm

d. 11 cm

Q4. If O is the centre of the fountain, with <QCR = 60°, then <QOR =

a. 60°

b. 120°

c. 90°

d. 30°

Q5. Which of the following is correct?

a. AB + BC = CD + DA

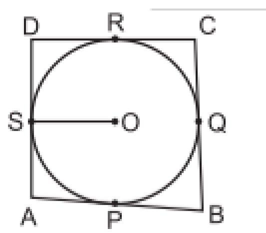
b. AB+ AD = BC + CD

c. AB + CD = AD + BC

d. All of these

Solutions

1. (b)



Here, OS the is radius of circle.
Since, radius at the point of contact is perpendicular to tangent.
So, <OSA = 90°
So, option (b) is correct.

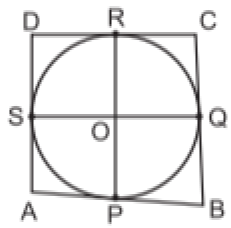
2. (d) Since, length of tangents drawn from an external point to a circle are equal.
.. ASAP, BP BQ,
CQ = CR and DR = DS ...(1)
So, option (d) is correct.

3. (a) AP AS AD-DS = AD-DR (using eq. (1))
= 11-7=4 cm
So, option (a) is correct.

4. (b) In quadrilateral OQCR, ZQCR = 60° (Given)
And <OQC = <ORC = 90°
(Since, radius at the point of contact is perpendicular to tangent.)
<QOR = 360°-90°-90°-60°

= 120°

So, option (b) is correct.



5. (c) From eq. (1), we have ASAP, DS = DR,

BQ=BP and CQCR

Adding all above equations, we get

AS+DS+BQ+CQ=AP+DR+BP+CR

= AD+BC AB + CD

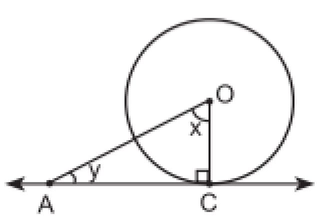
So, option (c) is correct.

Case Study 2

For class 10 students, a teacher planned a game for the revision of chapter circles with some questions written on the board, which are to be answered by the students. For each correct answer, a student will get a reward. Some of the questions are given below.

Based on the given information, solve the following questions:

Q1. In the given figure, x + y =



a. 60° b. 90° c. 120° d. 145°

Q2. If PA and PB are two tangents drawn to a circle with centre O from P such that PBA = 50°, then <OAB =

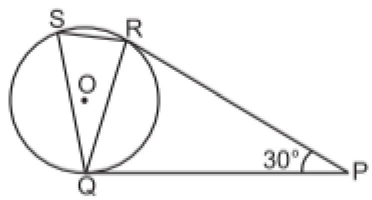
a. 50°

b. 25°

c. 40°

d. 130°

Q3. In the given figure, PQ and PR are two tangents to the circle, then <ROQ=



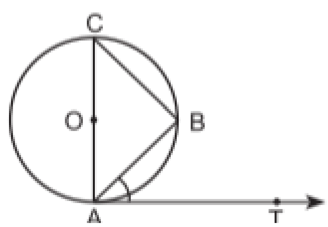
a. 30°

b. 60°

c. 105°

d. 150°

Q4. In the given figure, AB is a chord of the circle and AOC is its diameter such that ACB = 55°, then <BAT=



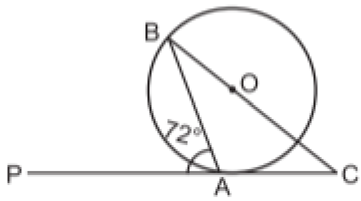
a. 35°

b. 55°

c. 125°

d. 110°

Q5. In the given figure, if PC is the tangent at A of the circle with <PAB 72° and <AOB = 132°, then <ABC =

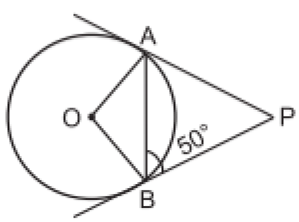


a. 18°
b. 30°
c. 60°
d. Can't be determined

Solutions

1. (b) In ΔOAC, <OCA = 90°
Since, radius at the point of contact is perpendicular to tangent.
.. <OAC + <AOC = 90° = x+y=90°
So, option (b) is correct.

2. (c)



Since, OB I PB (since, radius at the point of contact is perpendicular to tangent)
and <PBA 50° (Given)
<OBA=90° 50° = 40°
Also, OA = OB
.. <OAB <OBA = 40°
(radii of circle)
(angle opposite to equal sides are equal)
So, option (c) is correct.

3. (d) In quadrilateral OQPR,
<ROQ + <RPQ = 180°
(Angle between the two tangents drawn from an external point to a circle is

supplementary to the angle subtended by the line segment joining the point of contact at the centre)
..<ROQ = 180° -30° = 150°
So, option (d) is correct.

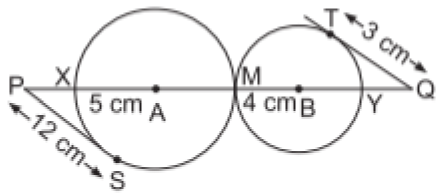
4. (b) Here, <ABC = 90° (angle in a semicircle)
Now, In ΔABC,
<BAC+ <ACB+ <ABC = 180°
(by angle sum property of triangle)
⇒ <BAC +55°+90° = 180°
⇒ <BAC=180°-145° = 35°
Also, <OAT 90° (radius at the point of contact is perpendicular to tangent)
⇒ <BAT+<OAB = 90°
⇒ <BAT = 90° -35° (: <CAB = <OAB)
= 55°
So, option (b) is correct.

5. (b) Here, <PAB = 72°
:- OAP = 90° (;- OA I AP)
<OAB + PAB = 90°
= <OAB = 90° - 72° = 18°
Also, <AOB=132° (given)
Now in ΔOAB,
<ABO+<BAO+<AOB = 180°
(by angle sum property of triangle)
<ABO = 180° - 132° -18° = 30°
:. <ABC + <ABO= 30°
So, option (b) is correct.

Case Study 3

In a math class-IX, the teacher draws two circles that touch each other externally at point

M with centres A and B and radii 5 cm and 4 cm respectively as shown in the figure.



Based on the above information, solve the following questions:

Q1. Find the value of PX.

Q2. Find the value of QY.

Q3. Show that PS^2 = PM-PX.

Or

Show that TQ^2 = YQ.MQ

Solutions

1. Here, AS = 5 cm and BT = 4 cm (*:* radii of circles)
Since, radius at point of contact is perpendicular to tangent.

$\therefore \quad AS \perp PS$

$\Rightarrow \quad \angle ASP = 90^\circ$

In right-angled ΔASP,

$PA^2 = PS^2 + AS^2$ (by Pythagoras theorem)

$\Rightarrow \quad PA^2 = (12)^2 + (5)^2$

$\Rightarrow \quad PA = \sqrt{144+25} = \sqrt{169} = 13\text{ cm}$

$\because \quad PX = PA - XA$

$\therefore \quad PX = 13 - 5 = 8\text{ cm}$ ($\because$ radius, XA = 5 cm)

2. $\because \quad BT \perp TQ$

$\Rightarrow \quad \angle BTQ = 90^\circ$

In right-angled ΔBTQ,

$BQ^2 = TQ^2 + BT^2$ (by Pythagoras theorem)

$\Rightarrow \quad BQ^2 = (3)^2 + (4)^2$

$\Rightarrow \quad BQ = \sqrt{9+16} = \sqrt{25} = 5\text{ cm}$

$\because \quad QY = BQ - BY$

$\therefore \quad QY = 5 - 4 = 1\text{ cm}$ ($\because$ radius, BY = 4 cm)

3. In right-angled ΔASP,

$PS^2=PA^2-A52$

$= PA^2-AM^2$ [:- AS = AM (radii)]

= (PA + AM) (PA - AM)

= (PA + AM) (PA - AX)

= PM-PX [:- AM = AX (radii)] **Hence proved.**

Or

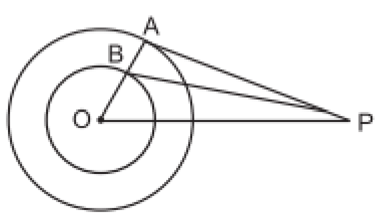
$TQ2^2=BQ^2-TB^2$
= (BQ - TB) (BQ + TB) (:- TB = MB (radii))
(BQ-MB) (BQ + MB)
(BQ-BY) MQ [:- MB = BY (radii)]
YQ.MQ (:- BQ+ MB = MQ, BQ - BY = YQ) **Hence proved.**

Case Study 4

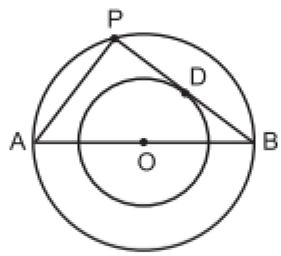
If a tangent is drawn to a circle from an external point, then the radius at the point of contact is perpendicular to the tangent.

Based on the above information, solve the following questions:

Q1. In the given figure, O is the centre of two concentric circles. From an external point P tangents PA and PB are drawn to these circles such that PA = 6 cm and PB = 8 cm. If OP = 10 cm, then find the value of AB.



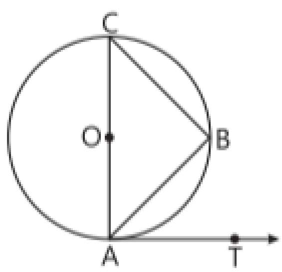
Q2. The diameter of two concentric circles are 10 cm and 6 cm. AB is a diameter of the bigger circle and BD is the tangent to the smaller circle touching it at D and intersecting the larger circle at P on producing. Find the length of BP.



Q3. Two concentric circles are such that the difference between their radii is 4 cm and the length of the chord of the larger circle which touches the smaller circle is 24 cm. Then find the radius of the smaller circle.

Or

If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in figure. Prove that <BAT = <ACB.



Solutions

1. Since, radius is perpendicular to the tangent.

.. OB I BP and OA I AP

Now in right-angled ΔOBP and ΔOAP,

Here, $OP^2 - PB^2 - 0B^2$ and $OP^2 - PA^2 = OA^2$

(by Pythagoras theorem)

$\therefore \quad OB = \sqrt{100 - 64} = \sqrt{36} = 6\text{ cm}$

($\because$ OP = 10 cm and PB = 8 cm)

and $OA = \sqrt{100 - 36} = \sqrt{64} = 8\text{ cm}$ ($\because$ PA = 6 cm)

$\therefore \quad AB = OA - OB = 8 - 6 = 2\text{ cm}$

2. Since, radius is perpendicular to the tangent

$\therefore$ OD $\perp$ BP

Given, AB = 10 cm

$\Rightarrow OB = 10/2 = 5$ cm and $OD = \frac{6}{2} = 3$ cm

Now in right-angled ΔODB,

$OB^2 = OD^2 + BD^2 \Rightarrow \quad BD = \sqrt{OB^2 - OD^2}$

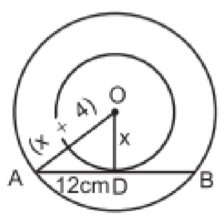
(by Pythagoras theorem)

$\Rightarrow BD = \sqrt{25-9} = \sqrt{16} = 4$ cm

Since, chord BP is bisected by radius OD.

.. BP = 2 BD = 2 x 4 = 8 cm.

3. Let x be the radius of smaller circle, then (x+4) be the radius of larger circle,



Since, radius is perpendicular to the tangent.

.. OD I AB

Now in right-angled ΔODA,

$OA^2 = OD^2$ + AD2 (by Pythagoras theorem)

$\Rightarrow (x+4)^2 = x^2 + 122$

>>8x+16=144

x= 16 cm

Or

Since, AC is a diameter, so the angle in a semi-circle will be 90°.

In ΔABC,

:- <ABC = 90°

<CAB+ <ABC + <ACB=180°

(sum of interior angles of a triangle)

= <CAB + <ACB=180°-90°-90° (1)

Since, the diameter of the circle is the perpendicular to the tangent.

i.e.. CA I AT

:- <CAT=90°

=> <CAB+ <BAT 90° ...(2)

From (1) and (2), we get

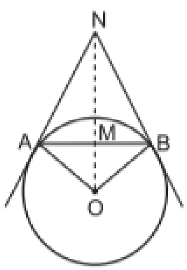
<CAB+<ACB= <CAB + <BAT

<ACB = <BAT Hence proved.

Case Study 5

Circles play an important part in our life. When a circular object is hung on the wall with a chord at nail N, the chords NA and NB work like tangents. Observe the figure, given that

<ANO = 30° and OA = 5 cm. [CBSE 2023]



Based on the above information, solve the following questions:

Q1. Find the distance AN.

Q2. Find the measure of <AOB.

Q3. Find the total length of chords NA, NB and the chord AB.

Or

Name the type of quadrilateral OANB. Justify your answer.

Solutions

1.

Here $OA \perp AN$

$\therefore \angle OAN = 90°$

Given, $\angle ANO = 30°$ and $OA = 5$ cm.

In right-angled ΔOAN,

$\tan \angle ANO = \frac{OA}{AN} \Rightarrow \tan 30° = \frac{5}{AN}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{AN}$

$\Rightarrow AN = 5\sqrt{3}$ cm

2. :- <ANO = <BNO = 30°

= <ANB=2x <ANO = 2 x 30° = 60°

:- OA I AN and OB I BN

:- <OAN = <OBN=90°

Now in quadrilateral OANB,

<AOB + <OAN + <OBN + <ANB = 360°

= <AOB+90°+90° +60° = 360°

:- <AOB = 360°-240° = 120°

3. In ΔAOB,

OA = OB (radii of circle)

= <OAB = <OBA 0 (Say)

:- <OAB + <OBA + <AOB = 180°

(by angle sum property]

= 0+0+120° 180° (:- ZAOB = 120°)

⇒20=60° = 0=30°

:- <OAB = <OBA = 30°

:- <OAN = <OAB + <BAN

:- 90° = 30°+<BAN

⇒ <BAN= 90° -30° = 60°

Similarly, <ABN = 60°

<ANB = <BAN = <ABN = 60°

:- ΔANB is an equilateral triangle.

:- Total length of chords = NA + NB+ AB

(:- AN = BN AB =5√3 cm)

=5√3+5√3+5√3

= 15√3 cm

From above parts,

<OAN = <OBN=90°

But <AOB = <ANB

Also, AN = BN = 5√3 cm

(the length of two tangents drawn from an external point of a circle are equal.]

and OA OB 5 cm (Radii)

In quadrilateral OANB,

longer diagonal ON bisect shorter diagonal AB perpendicularly.

(: the perpendicular from the centre of a circle to a chord bisect the chord)

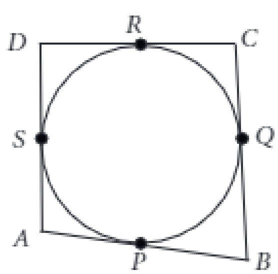
Hence, the special name of quadrilateral OANB is kite.

Solutions for Questions 6 to 15 are Given Below

Case Study 6

Park with Fountain

In a park, four poles are standing at positions A, B, C and D around the fountain such that the cloth joining the poles AB, BC, CD and DA touches the fountain at P, Q, R and S respectively as shown in the figure.



Based on the above information, answer the following questions.

(i) If O is the centre of the circular fountain, then $\angle OSA =$

(a) 60° (b) 90°
(c) 45° (d) None of these

(ii) Which of the following is correct?

(a) $AS = AP$ (b) $BP = BQ$
(c) $CQ = CR$ (d) All of these

(iii) If $DR = 7$ cm and $AD = 11$ cm, then $AP =$

(a) 4 cm (b) 18 cm
(c) 7 cm (d) 11 cm

(iv) If O is the centre of the fountain, with $\angle QCR = 60°$, then $\angle QOR =$

(a) 60° (b) 120°
(c) 90° (d) 30°

(v) Which of the following is correct?

(a) $AB + BC = CD + DA$ (b) $AB + AD = BC + CD$
(c) $AB + CD = AD + BC$ (d) All of these

Case Study 7

Smita always finds it confusing with the concepts of tangent and secant of a circle. But this time she has determined herself to get concepts easier. So, she started listing down the differences between tangent and secant of a circle along with their relation. Here, some points in question form are listed by Smita in her notes. Try answering them to clear your concepts also.

(i) A line that intersects a circle exactly at two points is called

(a) Secant (b) Tangent (c) Chord (d) Both (a) and (b)

(ii) Number of tangents that can be drawn on a circle is

(a) 1 (b) 0 (c) 2 (d) Infinite

(iii) Number of tangents that can be drawn to a circle from a point not on it, is

(a) 1 (b) 2 (c) 0 (d) Infinite

(iv) Number of secants that can be drawn to a circle from a point on it is

(a) infinite (b) 1 (c) 2 (d) 0

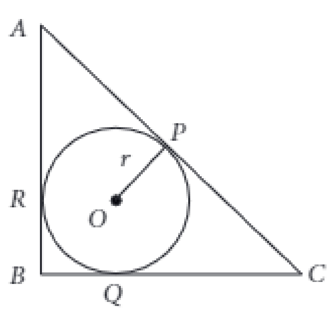
(v) A line that touches a circle at only one point is called

(a) Secant (b) Chord (c) Tangent (d) Diameter

Case Study 8

Triangular Backyard

A backyard is in the shape of a triangle with right angle at B, $AB = 6$ m and $BC = 8$ m. A pit was dig inside it such that it touches the walls AC, BC and AB at P, Q and R respectively such that $AP = x$ m.



Based on the above information, answer the following questions.

(i) The value of AR =

(a) $2x$ m (b) $x/2$ m (c) x m (d) $3x$ m

(ii) The value of BQ =

(a) $2x$ m (b) $(6 - x)$ m (c) $(2 - x)$ cm (d) $4x$ m

(iii) The value of CQ =

(a) $(4 + x)$ m (b) $(10 - x)$ m (c) $(2 + x)$ m (d) both (b) and (c)

(iv) Which of the following is correct?

(a) Quadrilateral $AROP$ is a square. (b) Quadrilateral $BROQ$ is a square.

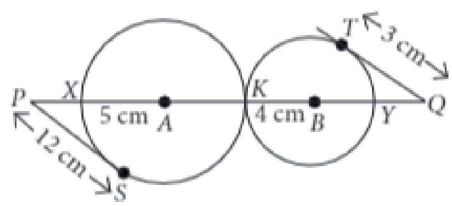
(c) Quadrilateral $CQOP$ is a square. (d) None of these

(v) Radius of the pit is

(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

Case Study 9

In a maths class, the teacher draws two circles that touch each other externally at point K with centres A and B and radii 5 cm and 4 cm respectively as shown in the figure.



Based on the above information, answer the following questions.

(i) The value of PA =

(a) 12 cm (b) 5 cm (c) 13 cm (d) Can't be determined

(ii) The value of BQ =

(a) 4 cm (b) 5cm (c) 6 cm (d) None of these

(iii) The value of PK =

(a) 13 cm (b) 15 cm (c) 16 cm (d) 18 cm

(iv) The value of QY =

(a) 2 cm (b) 5 cm (c) 1 cm (d) 3 cm

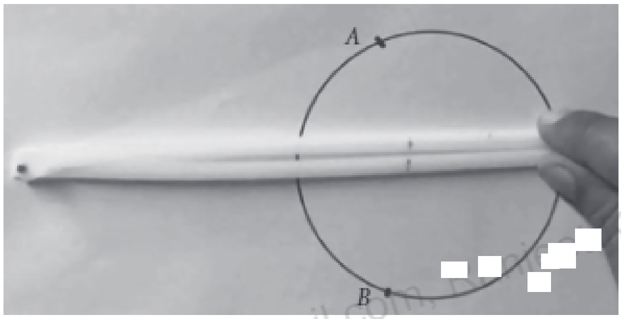
(v) Which of the following is true?

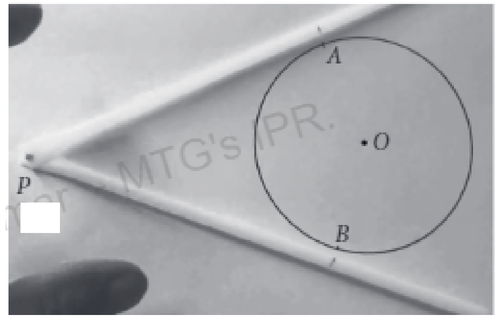
(a) $PS^2 = PA \cdot PK$ (b) $TQ^2 = QB \cdot QK$ (c) $PS^2 = PX \cdot PK$ (d) $TQ^2 = QA \cdot QB$

Case Study 10

Activity using Straws and Nail

Prem did an activity on tangents drawn to a circle from an external point using 2 straws and a nail for maths project as shown in figure.





Based on the above information, answer the following questions.

(i) Number of tangents that can be drawn to a circle from an external point is

(a) 1 (b) 2
(c) infinite (d) any number depending on radius of circle

(ii) On the basis of which of the following congruency criterion, $\Delta OAP \cong \Delta OBP$?

(a) ASA (b) SAS (c) RHS (d) SSS

(iii) If $\angle AOB = 150°$, then $\angle APB =$

(a) 75° (b) 30° (c) 60° (d) 100°

(iv) If $\angle APB = 40°$, then $\angle BAO =$

(a) 40° (b) 30° (c) 50° (d) 20°

(v) If $\angle ABO = 45°$, then which of the following is correct option?

(a) $AP \perp BP$ (b) $PAOB$ is square (c) $\angle AOB = 90°$ (d) All of these

Case Study 11

Online Test

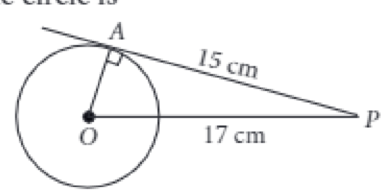
In an online test, Ishita comes across the statement - If a tangent is drawn to a circle from an external point, then the square of length of tangent drawn is equal to difference of squares of distance of the tangent from the centre of circle and radius of the circle.

Help Ishita, in answering the following questions based on the above statement.

(i) If AB is a tangent to a circle with centre O at B such that $AB = 10$ cm and $OB = 5$ cm, then $OA =$

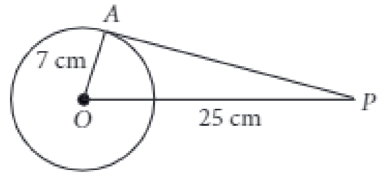
(a) $3\sqrt{5}$ cm (b) $5\sqrt{5}$ cm (c) $4\sqrt{5}$ cm (d) $6\sqrt{5}$ cm

(ii) In the adjoining figure, radius of the circle is



(a) 8 cm (b) 7 cm (c) 9 cm (d) 10 cm

(iii) In the adjoining figure, length of tangent AP is

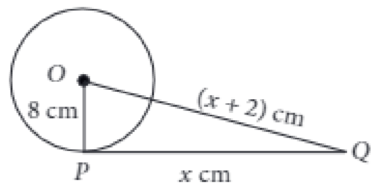


(a) 12 cm (b) 24 cm (c) 30 cm (d) None of these

(iv) PT is a tangent to a circle with centre O and diameter = 40 cm. If PT = 21 cm, then OP =

(a) 33 cm (b) 29 cm (c) 37 cm (d) None of these

(v) In the adjoining figure, the length of the tangent is



(a) 15 cm (b) 9 cm (c) 8 cm (d) 10 cm

Case Study 12

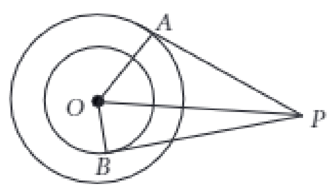
Theorem on Circles

If a tangent is drawn to a circle from an external point, then the radius at the point of contact is perpendicular to the tangent. Answer the following questions using the above condition.

(i) Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

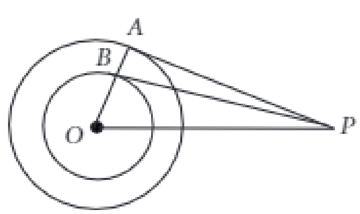
(a) 8 cm (b) 4 cm (c) 10 cm (d) 6 cm

(ii) In the given figure, O is the centre of two concentric circles of radii 5 cm and 3 cm. From an external point P tangents PA and PB are drawn to these circles. If PA = 12 cm, then PB =



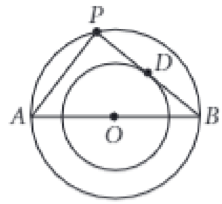
(a) $2\sqrt{10}$ cm (b) $2\sqrt{5}$ cm (c) $4\sqrt{10}$ cm (d) $4\sqrt{5}$ cm

(iii) In the given figure, O is the centre of two concentric circles. From an external point P tangents PA and PB are drawn to these circles such that PA = 6 cm and PB = 8 cm. If OP = 10 cm, then AB =



(a) 1 cm (b) 2 cm (c) 4 cm (d) Can't be determined

(iv) The diameter of two concentric circles are 10 cm and 6 cm. AB is a diameter of the bigger circle and BD is the tangent to the smaller circle touching it at D and intersecting the larger circle at P on producing. Find the length of BP.



(a) 4 cm (b) 16 cm (c) 10 cm (d) 8 cm

(v) Two concentric circles are such that the difference between their radii is 4 cm and the length of the chord of the larger circle which touches the smaller circle is 24 cm. Then the radius of the smaller circle is

(a) 16 cm (b) 20 cm (c) 18 cm (d) None of these

Case Study 13

Assessment Test on Circles

Following are questions of section-A in assessment test on circle that Eswar attend last month in school. He scored 5 out of 5 in this section. Answer the questions and check your score if 1 mark is allotted to each question.

(i) If two tangents *AB* and *CD* drawn to a circle with centre *O* at *P* and *Q* respectively, are parallel to each other, then which of the following is correct?

(a) $\angle POQ = 180°$ (b) *PQ* is a diameter

(c) $\angle APQ = \angle PQD = 90°$ (d) All of these

(ii) If *l* is a tangent to the circle with centre *O* and line *m* is passing through *O* intersects the tangent *l* at point of contact, then

(a) $l \parallel m$ (b) $l \perp m$

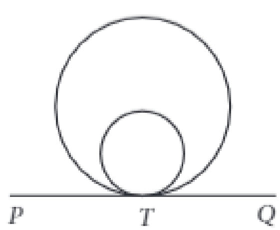
(c) line *l* and line *m* intersects and makes an angle of 60°

(d) can't be determined

(iii) Number of tangents that can be drawn to a circle from a point inside it, is

(a) 1 (b) 2 (c) infinite (d) 0

(iv) Which of the following is true?



(a) PQ is a tangent to both the circles
(b) Two circles are concentric
(c) PQ is a tangent to bigger circle only
(d) PQ is a tangent to smaller circle only

(v) A parallelogram circumscribing a circle is called a
(a) rhombus
(b) rectangle
(c) square
(d) none of these

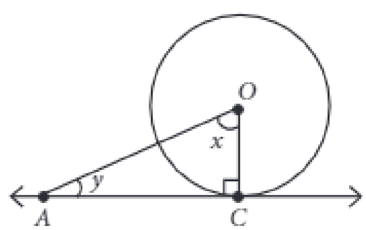
Case Study 14

Teaching through Play-Way Method

For class 10 students, a teacher planned a game for the revision of chapter circles with some questions written on the board, which are to be answered by the students. For each correct answer, a student will get a reward. Some of the questions are given below.

Answer these questions to check your knowledge.

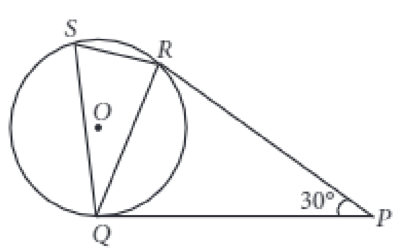
(i) In the given figure, $x + y =$



(a) 60°
(b) 90°
(c) 120°
(d) 145°

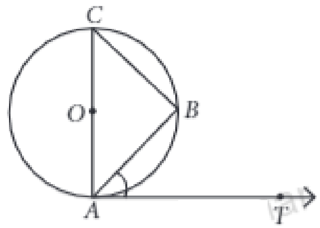
(ii) If PA and PB are two tangents drawn to a circle with centre O from P such that $\angle PBA = 50°$, then $\angle OAB =$
(a) 50°
(b) 25°
(c) 40°
(d) 130°

(iii) In the given figure, PQ and PR are two tangents to the circle, then $\angle ROQ =$



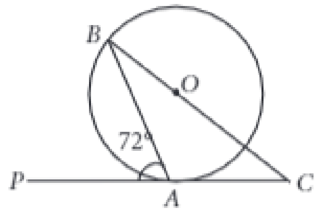
(a) 30°
(b) 60°
(c) 105°
(d) 150°

(iv) In the adjoining figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 55°$, then $\angle BAT =$



(a) 35° (b) 55° (c) 125° (d) 110°

(v) In the adjoining figure, if PC is the tangent at A of the circle with $\angle PAB = 72°$ and $\angle AOB = 132°$, then $\angle ABC =$

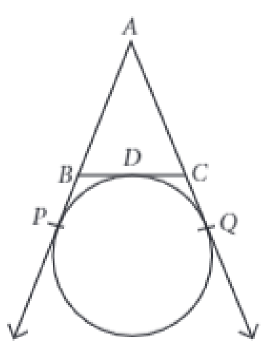


(a) 18° (b) 30° (c) 60° (d) can't be determined

Case Study 15

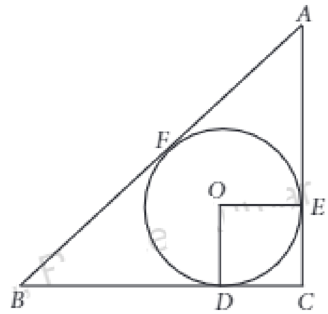
Raghav loves geometry. So he was curious to know more about the concepts of circle. His father is a mathematician. So, he reached to his father to learn something interesting about tangents and circles. His father gave him knowledge on circles and tangents and ask him to solve the following questions.

(i) In the given figure, AP, AQ and BC are tangents to the circle such that $AB = 7$ cm, $BC = 5$ cm and $AC = 8$ cm, then AP is equal to



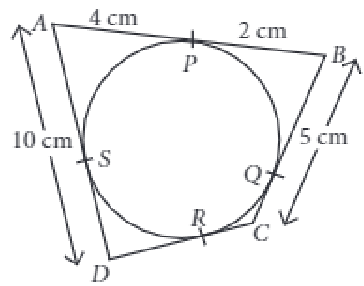
(a) 12 cm (b) 15 cm (c) 13 cm (d) 10 cm

(ii) A circle of radius 3 cm is inscribed in a right angled triangle BAC such that $BD = 9$ cm and $DC = 3$ cm. Find the length of AB.



(a) 6 cm (b) 12 cm (c) 15 cm (d) 10 cm

(iii) In the given figure, the length of *CD* is

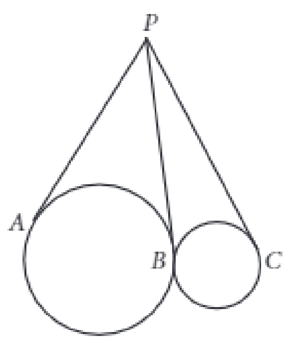


(a) 11 cm (b) 9 cm (c) 7 cm (d) 13 cm

(iv) If *PA* and *PB* are two tangents to a circle with centre *O* from an external point *P* such that $\angle OPB = 40°$, then $\angle BPA =$

(a) 60° (b) 50° (c) 120° (d) 80°

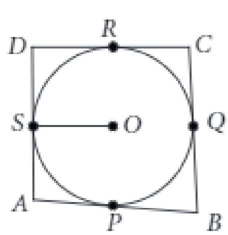
(v) In the given figure, *P* is an external point from which tangents are drawn to two externally touching circles. If $PA = 7$ cm, then $PC =$



(a) 3.5 cm (b) 4 cm (c) 7 cm (d) Can't be determined

HINTS & EXPLANATIONS

6. **(i) (b):**



Here, *OS* the is radius of circle.

Since radius at the point of contact is perpendicular to tangent.

So, $\angle OSA = 90°$

(ii) (d): Since, length of tangents drawn from an external point to a circle are equal.

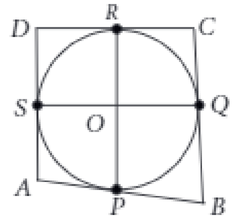
$\therefore \quad AS = AP, BP = BQ,$

$CQ = CR$ and $DR = DS$ ...(1)

(iii) (a): $AP = AS = AD - DS = AD - DR$ (Using (1)

$= 11 - 7 = 4$ cm

(iv) (b): In quadrilateral $OQCR$, $\angle QCR = 60°$ (Given)
And $\angle OQC = \angle ORC = 90°$
[Since, radius at the point of contact is perpendicular to tangent.]



$\therefore\ \angle QOR = 360° - 90° - 90° - 60° = 120°$

(v) (c): From (1), we have $AS = AP$, $DS = DR$, $BQ = BP$ and $CQ = CR$
Adding all above equations, we get
$AS + DS + BQ + CQ = AP + DR + BP + CR$
$\Rightarrow\ AD + BC = AB + CD$

7. (i) (a) (ii) (d) (iii) (b)
(iv) (a) (v) (c)

8. Here in right angled triangle ABC, $AB = 6$ m and $BC = 8$ cm.

$\therefore$ By Pythagoras theorem, $AC = \sqrt{(AB)^2 + (BC)^2}$
$= \sqrt{(8)^2 + (6)^2} = \sqrt{100} = 10$ m

Also, $AP = x$ m.

(i) (c): $AR = AP = x$ m ...(1)
[Since, length of tangents drawn from an external point are equal]

(ii) (b): $BQ = BR = AB - AR = (6 - x)$ m (Using (1))

(iii) (d): $CQ = CP = AC - AP = (10 - x)$ m
Also, $CQ = BC - BQ = BC - BR = 8 - (6 - x) = 2 + x$

(iv) (b): Since, $CQ = 10 - x = 2 + x$
$\Rightarrow\ 8 = 2x \ \Rightarrow\ x = 4$
$\therefore\ AR = AP = 4$ m, $BR = BQ = 2$ m
and $CP = CQ = 6$ m
Also, $OQ \perp BQ$ and $OR \perp BR$
$\therefore\ BROQ$ is a square.

(v) (a): Radius of the circle, $OR = BR = 2$ cm

9. Here, $AS = 5$ cm, $BT = 4$ cm [$\because$ Radii of circles]

(i) (c): Since, radius at point of contact is perpendicular to tangent.

$\therefore$ By Pythagoras theorem, we have
$PA = \sqrt{PS^2 + AS^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$ cm

(ii) (b): Again by Pythagoras theorem, we have
$BQ = \sqrt{TQ^2 + BT^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ cm

(iii) (d): $PK = PA + AK = 13 + 5 = 18$ cm

(iv) (c): $QY = BQ - BY = 5 - 4 = 1$ cm

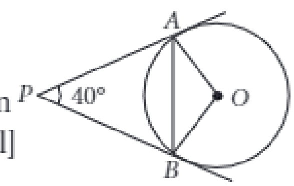
(v) (c): $PS^2 = PA^2 - AS^2 = PA^2 - AK^2$
$= (PA + AK)(PA - AK) = PK \cdot PX$ [$\because AK = AX$]

10. (i) (b)

(ii) (c): In ΔOAP and ΔOBP,
$\angle OAP = \angle OBP = 90°$
[Since, radius at the point of contact is perpendicular to tangent]
$OP = OP$ (Common)
$OA = OB$ (Radii of circle)
So, $\Delta OAP \cong \Delta OBP$ (By RHS congruency criterion)

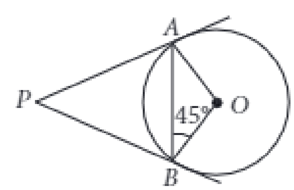
(iii) (b): In quadrilateral $OAPB$, $\angle AOB = 150°$ [Given]
$\angle OAP = \angle OBP = 90°$
$\therefore\ \angle APB = 360° - 90° - 90° - 150° = 30°$

(iv) (d): We have, $\angle APB = 40°$
Now, $PA = PB$
[Since, length of tangents drawn from an external point are equal]



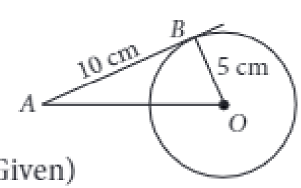
In ΔPAB, $\angle PAB = \angle PBA = 70°$
[Angles opposite to equal sides are equal]
Also, $\angle PAB + \angle BAO = 90°$
[Since, radius at the point of contact is perpendicular to tangent]
$\Rightarrow\ \angle BAO = 90° - 70° = 20°$

(v) (d): We have, $\angle ABO = 45°$
$\because\ AO = OB$ (Radii of circle)
$\therefore\ \angle BAO = \angle ABO = 45°$
[Angles opposite to equal sides are equal]



Now, in ΔOAB,
$\angle AOB = 180° - 45° - 45° = 90°$
Since, $\angle APB = 360° - 90° - 90° - 90° = 90°$ *i.e.*, $AP \perp BP$
So, $OAPB$ is a square.

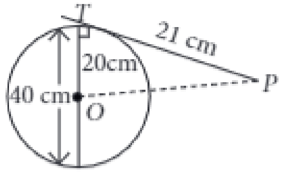
11. (i) (b): $OA^2 = AB^2 + OB^2$
$\Rightarrow\ OA = \sqrt{10^2 + 5^2} = 5\sqrt{5}$ cm



(ii) (a): $OA = \sqrt{OP^2 - AP^2}$ (Given)
$= \sqrt{17^2 - 15^2} = \sqrt{64} = 8$ cm

(iii) (b): Length of tangent, $AP = \sqrt{OP^2 - OA^2}$ (Given)
$= \sqrt{25^2 - 7^2} = \sqrt{576} = 24$ cm

(iv) (b):



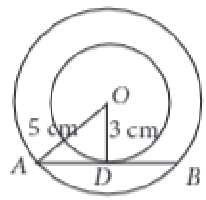
Since, $OP = \sqrt{(PT)^2 + (OT)^2} = \sqrt{21^2 + 20^2} = 29$ cm

(v) (a): Since, $OP^2 + PQ^2 = OQ^2$

$\Rightarrow 8^2 + x^2 = (x+2)^2 \Rightarrow 64 = 4x + 4 \Rightarrow x = 15$ cm

So, length of tangent, $PQ = 15$ cm

12. (i) (a): Here, $OA^2 = OD^2 + AD^2$



$\Rightarrow AD = \sqrt{25-9} = 4$ cm

As OD bisects AB, then

$AB = 2AD = 2 \times 4 = 8$ cm

(ii) (c): Here, $PB^2 + OB^2 = OP^2 = PA^2 + OA^2$

Then $PB^2 + 9 = 144 + 25 \Rightarrow PB^2 = 160$

$\Rightarrow PB = 4\sqrt{10}$ cm

(iii) (b): Here, $OP^2 - PB^2 = OB^2$ and $OP^2 - PA^2 = OA^2$

$\therefore OB = \sqrt{100-64} = \sqrt{36} = 6$ cm

and $OA = \sqrt{100-36} = \sqrt{64} = 8$ cm

$\therefore AB = OA - OB = 8 - 6 = 2$ cm

(iv) (d): Here, in right angled ΔOBD, $OB = 5$ cm and $OD = 3$ cm.

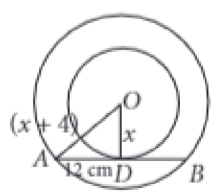
$\therefore BD = \sqrt{25-9} = \sqrt{16} = 4$ cm

Since, chord BP is bisected by radius OD.

$\therefore BP = 2BD = 8$ cm

(v) (a): Let x be the radii of smaller circle.

Now, $OA^2 = OD^2 + AD^2$

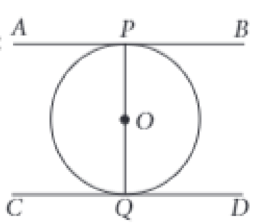


$\Rightarrow (x+4)^2 = x^2 + 12^2$

$\Rightarrow 8x + 16 = 144$

$\Rightarrow x = 16$ cm

13. (i) (d):



Two tangents of a circle are parallel only when they are drawn at ends of a diameter.

So, PQ is the diameter of the circle.

(ii) (b) **(iii) (d)**

(iv) (a): Here, the two circles have a common point of contact T and PQ is the tangent at T. So, PQ is the tangent to both the circles.

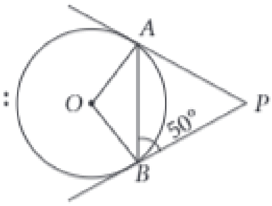
(v) (a)

14. (i) (b): In ΔOAC, $\angle OCA = 90°$

[Since, radius at the point of contact is perpendicular to tangent]

$\therefore \angle OAC + \angle AOC = 90° \Rightarrow x + y = 90°$

(ii) (c):



Since, $OB \perp PB$ [Since, radius at the point of contact is perpendicular to tangent]

and $\angle PBA = 50°$ (Given)

$\therefore \angle OBA = 90° - 50° = 40°$

Also, $OA = OB$ [Radii of circle]

$\therefore \angle OAB = \angle OBA = 40°$

[Angle opposite to equal sides are equal]

(iii) (d): In quadrilateral $OQPR$,

$\angle ROQ + \angle RPQ = 180°$

[$\because$ Angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the point of contact at the centre]

$\Rightarrow \angle ROQ = 180° - 30° = 150°$

(iv) (b): Here, $\angle ABC = 90°$ (Angle in a semicircle)

So, in ΔABC, $\angle BAC = 180° - 90° - 55° = 35°$

Also, $\angle OAT = 90°$

$\Rightarrow \angle BAT + \angle OAB = 90° \Rightarrow \angle BAT = 90° - 35° = 55°$

(v) (b): Here, $\angle PAB = 72°$

$\therefore \angle OAB = 90° - 72° = 18°$

Also, $\angle AOB = 132°$ [Given]

Now, in ΔOAB, $\angle ABC = 180° - 132° - 18° = 30°$

15. (i) (d): We have, $AP = AQ$, $BP = BD$, $CQ = CD$ ...(i)

[$\because$ Tangents drawn from an external points are equal in length]

Now, $AB + BC + AC = 7 + 5 + 8 = 20$ cm

$\Rightarrow AB + BD + CD + AC = 20$ cm

$\Rightarrow AP + AQ = 20$ cm $\Rightarrow 2AP = 20$ cm $\Rightarrow AP = 10$ cm

(ii) (c): Let $AF = AE = x$ cm

[$\because$ Tangents drawn from an external point to a circle are equal in length]

Given, $BD = FB = 9$ cm, $CD = CE = 3$ cm

In ΔABC, $AB^2 = AC^2 + BC^2$

$\Rightarrow \quad (AF + FB)^2 = (AE + EC)^2 + (BD + CD)^2$

$\Rightarrow \quad (x + 9)^2 = (x + 3)^2 + 12^2$

$\Rightarrow \quad 18x + 81 = 6x + 9 + 144$

$\Rightarrow \quad 12x = 72 \quad \Rightarrow \quad x = 6$ cm

$\therefore \quad AB = 6 + 9 = 15$ cm

(iii) (b): Here, $AP = AS = 4$ cm

$\therefore \quad DS = DR = 10 - 4 = 6$ cm

And $BP = BQ = 2$ cm. So, $CR = CQ = 5 - 2 = 3$ cm

So, $CD = DR + CR = 6 + 3 = 9$ cm

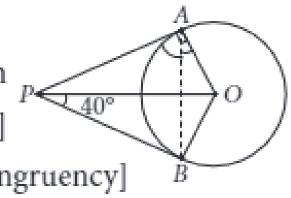
(iv) (d): Here $\angle OAP = 90°$

In ΔAOP and ΔBOP

$\angle OAP = \angle OBP$ [90° each]

$OA = OB$ [Radii of circle]

$PA = PB$ [Tangents drawn from an external point are equal]



$\therefore \quad \Delta AOP \cong \Delta BOP$ [By SAS congruency]

$\therefore \quad \angle APO = \angle OPB$ [C.P.C.T]

$= 40°$

$\therefore \quad \angle BPA = 40° + 40° = 80°$

(v) (c): For bigger circle, $PA = PB$ …(i)

[$\because$ Tangents drawn from an external point are equal in length]

Similarly, for smaller circle, $PB = PC$ ...(ii)

From (i) and (ii), we get

$PA = PB = PC = 7$ cm