CBSE Class 12 - Mathematics
Sample Paper 14

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. The system of equations x + 2y = 11, -2 x – 4y = 22 has

   a. only one solution

   b. infinitely many solutions

   c. finitely many solutions

   d. no solution

2. If D = $\begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 3 & 4 & 5 \end{vmatrix}$, then $\begin{vmatrix} 1 & 6 & 3 \\ 4 & -6 & 0 \\ 3 & 12 & 5 \end{vmatrix}$ is equal to

a. 6D

b. 3D

c. 0

d. 2D

3. $\underset{x \to 0}{Lt} \quad \frac{x \cos x - \log(1+x)}{x^2}$ is equal to

a. None of these

b. 1

c. 0

d. $\frac{1}{2}$

4. A random variable is a real-valued function whose domain is the.

a. set of irrational numbers

b. set of integers

c. sample space of a random experiment

d. set of real numbers

5. Two coins are tossed once, where E : tail appears on one coin, F : one coin shows head. Find P(E/F)

a. 0.24

b. 0.33

c. 1

d. 0.23

6. Maximize Z = 3x + 4y, subject to the constraints: x + y ≤ 1, x ≥ 0, y ≥ 0.

a. 4

b. 5

c. 6

d. 3

7. The maximum value of sinx + cosx is

a. 2

b. 1

c. $\sqrt{2}$

d. $\frac{1}{\sqrt{2}}$

8. $\int_{0}^{2} \left[x^2\right] dx$ equals, where [.] denotes Greatest Integer Function

a. $5 - \sqrt{3} - \sqrt{2}$

b. 3

c. $\sqrt{5} - 4$

d. $\frac{8}{2}$

9. In the Cartesian form two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar if

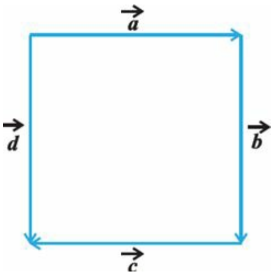
a. $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ -a_2 & b_2 & c_2 \end{vmatrix} = 0$

b. $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & -c_2 \end{vmatrix} = 0$

c. $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

d. $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & -b_2 & c_2 \end{vmatrix} = 0$

10. In the figure which are the Equal vectors?



a. vectors $\vec{b}$ *and* $\vec{c}$

b. vectors $\vec{a}$ *and* $\vec{d}$

c. vectors $\vec{b}$ *and* $\vec{d}$

d. vectors $\vec{c}$ *and* $\vec{d}$

11. Fill in the blanks:

A relation R from a set X to a set Y is defined as a ________ of the cartesian product X $\times$ Y.

12. Fill in the blanks:

Differential coefficient of sec ($\tan^{-1}$x) w.r.t. x is ________.

13. Fill in the blanks:

In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeros in one or more, then A^{-1} ________.

14. Fill in the blanks:

A straight line which is perpendicular to every line lying on a plane is called a ________ to the plane.

OR

Fill in the blanks:

The vector equation of the line through the points (3, 4, -7) and (1, -1, 6) is ________.

15. Fill in the blanks:

The vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{k}$ are the adhacent sides of a parallelogram. The acute angle between its diagonals is ________.

OR

Fill in the blanks:

The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ forming a right handed system is ________.

16. Let A be a square matrix of order 3 $\times$3. Write the value of $|2A|$, where $|A|$= 4.

17. Evaluate $\int \frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$

OR

Evaluate $\int \frac{dx}{\sin^2 x \cos^2 x}$.

18. Evaluate $\int_2^4 \frac{x}{x^2+1} dx.$

19. Find the area of the region enclosed by the lines y=x,x=e,the curve y= $\frac{1}{x}$ and the positive x-axis.

20. Find the differential equation of the family of lines through the origin.

Section B

21. Prove that $\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$

OR

Find gof and fog, if:

i. f(x) = |x| and g(x) = |5x - 2|
ii. f(x) = $8x^3$ and g(x) = $x^{\frac{1}{3}}$

22. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; xy = c^2$ to intersect orthogonally.

23. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} = -e^{x-y}$. Also, prove that $\frac{dy}{dx} = \frac{-e^{x-y}(1-e^y)}{1-e^x}$

24. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

OR

Find the direction cosines of the vector joining the points A(1,2, - 3) and B(-1, - 2,1) directed from A to B.

25. Find the equation of a plane which bisects perpendicularly the line joining the points $A\,(2, 3, 4)$ and $B\,(4, 5, 8)$ at right angles.

26. Suppose 10,000 tickets are sold in a lottery each for Re 1. For first position, there is a prize of Rs 3000. For second position, there is prize of Rs. 2000. For third position, there are three prizes each of Rs 500. If you buy one ticket, what is your expectation?

Section C

27. Show that the relation in the set R of real no. defined R = {(a, b) : a<b^2}, is neither reflexive nor symmetric nor transitive.

28. Find all points of discontinuity of f where f is defined as follows,f(x) =
$$\begin{cases} |x| + 3, x \leq -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \geq 3 \end{cases}$$

OR

If f(x) = $\sqrt{x^2 + 1}$; g(x) = $\frac{x+1}{x^2+1}$ and h(x) = 2x - 3, then find f'[h'{g'(x)}].

29. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation: $xy = \log y + C : y' = \frac{y^2}{1-xy}(xy \neq 1)$

30. Evaluate $\int \frac{3x+1}{(x+1)^2(x+3)} dx.$

31. Three persons A, B and C apply for a job of Manager in a private company. Chances of their selection (A, B and C are in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

OR

Ten coins are tossed. What is the probability of getting at least 8 heads?

32. The objective of A diet problem is to ascertain the quantities of certain foods that should be eaten to meet certain nutritional requirement at minimum cost. The consideration is limited to milk, beef and eggs, and to vitamins A, B, C. The number of milligrams of each of these vitamins contained within A unit of each food is given below:

Vitamin	Litre of milk	Kg of beef	Dozen of eggs	Minimum daily requirements
A	1	1	10	1 mg
B	100	10	10	50 mg
C	10	100	10	10 mg
Cost	Rs.1.00	Rs.1.10	Rs.0.50	

What is the linear programming formulation for this problem?

Section D

33. Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

OR

Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 4 & 4 \\ 7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ find AB and use this result in solving the following system of equations.

$x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

34. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

35. Show that of all the rectangles with a given perimeter, the square has the largest area.

OR

A point on the hypotenuse of a right triangle is at distances a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$.

36. Find the shortest distance between the lines given by
$\overrightarrow{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and
$\overrightarrow{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

Solution

Section A

1. (d) no solution

   Explanation:

   For no solution , we have : $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$,for given system of equations we have : $\frac{1}{-2}=\frac{2}{-4}\neq\frac{11}{22}$.

2. (a) 6D

   Explanation:

   $\begin{vmatrix}1 & 2 & 3\\ 2 & -1 & 0\\ 3 & 4 & 5\end{vmatrix}$ = 1(-5) - 2(10) +3(11) = - 5 -20 +33 = 8

   $\begin{vmatrix}1 & 6 & 3\\ 4 & -6 & 0\\ 3 & 12 & 5\end{vmatrix}$ = 1(-30) - 6(20) + 3(66) = -30 -120 +198 = 48

   D = 8 $\Rightarrow$ 6D = 48

3. (d) $\frac{1}{2}$

   Explanation:

   $$\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}$$

   $=\lim_{x\to 0}\frac{\cos x-x\sin x-\frac{1}{1+x}}{2x}=\lim_{x\to 0}\frac{-sinx-(x\cos x+\sin x)+\frac{1}{(1+x)^2}}{2}=\frac{1}{2}$ (using L'Hospital Rule)

4. (c) sample space of a random experiment

   Explanation:

A random variable is a real valued function whose domain is the sample space of a random experiment .

5. (c) 1

   Explanation:

   S = {HH, HT, TH, TT}

   E = {HT , TH}

   F= {HT , TH}

   $\Rightarrow P(E)=\frac{2}{4}=\frac{1}{2}, P(F)=\frac{2}{4}=\frac{1}{2}, P(E\cap F)=\frac{1}{2}$

   $\Rightarrow P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{1/2}{1/2}=1$

6. (a) 4

   Explanation:

   Here , maximize , Z = 3x + 4y, subject to the constraints: x + y ≤ 1, x ≥ 0, y ≥ 0.

Corner points	Z = 3x +4 y
C(0, 0)	0
B (1,0)	3
D(0,1)	4

   Hence the maximum value is 4

7. (c) $\sqrt{2}$

   Explanation:

   Since, range of sine and cosine function is [-1,1]. But, sine is increasing function and cosine is decreasing function the highest that both together attain is 45^{0}

   $\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}\right)=\sqrt{2}$

8. (a) $5-\sqrt{3}-\sqrt{2}$

Explanation:

$$= \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2] dx + \int_{\sqrt{3}}^{2} [x^2] dx$$

$$= \int_0^1 0\ dx + \int_1^{\sqrt{2}} 1\ dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\ dx + \int_{\sqrt{3}}^{2} 3\ dx$$

$$\Rightarrow 0 + (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2} + 3(2 - \sqrt{3})$$

9. (c)

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Explanation:

In the Cartesian form two lines
$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$
and
$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$
are coplanar if
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

10. (c) vectors $\vec{b}\ and\ \vec{d}$

Explanation:

vectors $\vec{b}\ and\ \vec{d}$ are equal vectors because they have equal magnitude and same direction.

11. subset

12. $\frac{x}{\sqrt{1+x^2}}$

13. does not exist

14. normal

OR

$(x-3)\hat{i}+(y-4)\hat{j}+(z+7)\hat{k}$ = $\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$

15. $\frac{\pi}{4}$

OR

$\hat{k}$

16. We are given a square matrix A of order 3 $\times$3. We have to find the value of $|2A|$, where $|A|$= 4.

We know that, for a square matrix A of order n,
$|\,kA|=k^n.\,|A|$
Here, $|2A|=2^3|A|$ [$\because$ order of A is 3×3]
$=2^3\times4=8\times4=32$ [Put |A| = 4]

17. $I=\int\frac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}$

Put tan $\sqrt{x}=t$

$\sec^2\sqrt{x}\,.\,\frac{1}{2\sqrt{x}}dx=dt$

$\frac{\sec^2\sqrt{x}}{\sqrt{x}}dx=2dt$

$I=2\int t^4dt=2\frac{t^5}{5}+c$

$=\frac{2}{5}\tan^5\sqrt{x}+c$

OR

Let $I=\int\frac{dx}{\sin^2x\cos^2x}$
$=\int\frac{(\sin^2x+\cos^2x)}{\sin^2x\cdot\cos^2x}dx\ \left[\because\sin^2\theta+\cos^2\theta=1\right]$
$=\int\left(\sec^2x+\ cosec^2x\right)dx$
$=\int\sec^2xdx+\int\csc^2xdx$

= tan x - cot x + C

18. Let $I = \int_2^4 \frac{x}{x^2+1} dx$

Put x^2 + 1 = t $\Rightarrow 2xdx = dt \Rightarrow xdx = \frac{dt}{2}$

Lower limit when x = 2, then t = 2^2 + 1 = 5

Upper limit when x = 4, then t = 4^2 + 1 = 17.

$\therefore \quad I = \int_5^{17} \frac{dt}{2t} = \frac{1}{2} \int_5^{17} \frac{1}{t} dt = \frac{1}{2} [\log |t|]_5^{17}$

$= \frac{1}{2} [\log 17 - \log 5] = \frac{1}{2} \log\left(\frac{17}{5}\right) \left[\because \log m - \log n = \log\left(\frac{m}{n}\right)\right]$

19. The point of intersection of y = x and y = 1/x is (1,1) .Also the meeting point of x = e and y = 1/x is (e, 1/e). The limits on the x axis for the required shaded region extends from 1 to e split as follows.

0-1 below the line y = x and 1-e below the curve y = 1/x. Hence,the required area =

$\int_0^1 xdx + \int_1^e \frac{1}{x}\ dx$

$= \frac{1}{2} + 1$

$= \frac{3}{2}$ square units.

20. Let y = mx be the family of lines through origin. Therefore, $\frac{dy}{dx} = m$.

Eliminating m, we get $y = \frac{dy}{dx} . x$ or $x\frac{dy}{dx} - y = 0$

Section B

21. L.H.S $= \tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2}$

$= \tan^{1} \left[\frac{x + \frac{2x}{1-x^2}}{1 - x\left(\frac{2x}{1-x^2}\right)} \right]$

$= \tan^{-1} \left[\frac{\frac{x - x^3 + 2x}{1-x^2}}{\frac{(1-x^2) - 2x^2}{1-x^2}} \right]$

$= \tan^{-1} \left[\frac{3x - x^3}{1 - 3x^2} \right]$

L.H.S = R.H.S

OR

To find: gof and fog

1. f(x) = |x| and g(x) = |5x - 2|
   gof = g[f(x)] = g[|x|] = |5|x|-2| and fog=f(g(x))=f(|5x-2|) =||5x-2|| = |5x-2|
2. f(x) = $8x^3$ and g(x) = $x^{\frac{1}{3}}$
   gof = g[f(x)] = g[$8x^3$] $\left(8x^3\right)^{\frac{1}{3}}$ = 2x
   and fog = f[g(x)] $= f\left[\left(x^{\frac{1}{3}}\right)\right] = 8\left(x^{\frac{1}{3}}\right)^3$ = 8x

22. Let the curves intersect at (x_1, y_1). Therefore,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{b^2x}{a^2y}$$

$\Rightarrow$ slope of tangent at the point of intersection is $m_1 = \frac{b^2x_1}{a^2y_1}$

Again $xy = c^2 \Rightarrow x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x} \Rightarrow m_2 = \frac{-y_1}{x_1}$.

For orthoganality, $m_1 \times m_2 = -1$.

$$\Rightarrow \frac{b^2x_1}{a^2y_1} \times \frac{-y_1}{x_1} = -1$$

$$\Rightarrow \frac{b^2}{a^2} = 1 \; or \; a^2 - b^2 = 0$$

23. Given that $e^x + e^y = e^{x+y}$

Differentiating both sides w.r.t. x, we have

$$e^x + e^y\frac{dy}{dx} = e^{x+y}\left(1 + \frac{dy}{dx}\right)$$

or $\left(e^y - e^{x+y}\right)\frac{dy}{dx} = e^{x+y} - e^x$,

Which implies that $\frac{dy}{dx} = \frac{e^{x+y}-e^x}{e^y-e^{x+y}} = \frac{e^x.e^y-e^x}{e^y.e^x-e^y} = \frac{-e^x(e^x-1)}{e^y(e^x-1)} = -e^{x-y}$

$$\frac{dy}{dx} = \frac{e^{x+y}-e^x}{e^y-e^{x+y}} = \frac{e^x.e^y-e^x}{e^y-e^x.e^y} = \frac{e^x(e^y-1)}{e^y(1-e^x)} = \frac{-e^{x-y}(1-e^y)}{1-e^x}$$

24. Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$

Projection of vector $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}$

$= \frac{(1)(7)+(3)(-1)+7(8)}{\sqrt{(7)^2+(-1)^2+(8)^2}}$

$= \frac{7-3+56}{\sqrt{49+61+64}} = \frac{60}{\sqrt{114}}$

OR

Given: Points A(1, 2, -3) and B(-1, - 2,1)

$\therefore$ Position vector of point $A = \overrightarrow{OA} = \hat{i} + 2\hat{j} - 3\hat{k}$

And Position vector of point $B = \overrightarrow{OB} = -\hat{i} - 2\hat{j} + \hat{k}$

$\therefore$ Vector $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\hat{i} - 2\hat{j} + \hat{k} - \hat{i} - 2\hat{j} + 3\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$

Now $\left|\overrightarrow{AB}\right| = \sqrt{(-2)^2 + (-4)^2 + (4)^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$

$\therefore$ A unit vector along $\overrightarrow{AB} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{-2\hat{i}-4\hat{j}+4\hat{k}}{6}$

$= \frac{-2}{6}\hat{i} - \frac{4}{6}\hat{j} + \frac{4}{6}\hat{k} = \frac{-1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

Therefore, the direction cosines of vector $\overrightarrow{AB} = \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

25. Since, the plane is bisecting perpendicularly the line joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ at right angles.

So, mid-point of AB is $\left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2}\right)$ i.e., (3, 4, 6).

Also, $\vec{N} = (4-2)\hat{i} + (5-3)\hat{j} + (8-4)\hat{k} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

So, the required equation of the plane is $(\vec{r} - \vec{a}).\vec{N} = 0$

$\Rightarrow \left[(x-3)\hat{i} + (y-4)\hat{j} + (z-6)\hat{k}\right].\left(2\hat{i} + 2\hat{j} + 4\hat{k}\right) = 0$

$\left[\because \vec{a} = 3\hat{i} + 4\hat{j} + 6\hat{k}\right]$

$\Rightarrow 2x - 6 + 2y - 8 + 4z - 24 = 0$

$\Rightarrow 2x + 2y + 4z = 38$

$\therefore x + y + 2z = 19$

26. Let x is the random variable for the prize.

X	0	500	2000	3000
P(X)	$\frac{9995}{10000}$	$\frac{3}{10000}$	$\frac{1}{10000}$	$\frac{1}{10000}$

Since, $E(X) = \sum XP(X)$
$\therefore E(X) = 0 \times \frac{9995}{10000} + \frac{1500}{10000} + \frac{2000}{10000} + \frac{3000}{10000}$
$= \frac{1500+2000+3000}{10000}$
$= \frac{6500}{10000} = \frac{13}{20} = Rs\ 0.65$

Section C

27.
1. (a, a) $\notin$ R as a<a is false,therefore R is not reflexive.
2. $a < b^2\ and\ b < a^2$ Which is false,therefore R is not symmetric.
3. $a < b^2, b < c^2, then\ a < c^2$ Which is false. Therefore R is not transitive.

   $\therefore$ f is neither reflexive nor symmetric nor transitive.

28. Given function is

f(x) = $\begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$ = $\begin{cases} -x + 3, x \leq -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \geq 3 \end{cases}$

First, we verify continuity at x = - 3 and then at x = 3

Continuity at x = -3

LHL = $\lim\limits_{x \to (-3)^-} f(x) = \lim\limits_{x \to (-3)^-} (-x + 3)$

$\Rightarrow \quad \text{LHL} = \lim\limits_{h \to 0} [-(-3 - h) + 3]$

$= \lim\limits_{h \to 0} (3 + h + 3)$

= 3 + 3 = 6

and RHL = $\lim_{x \to (-3)^+} f(x) = \lim_{x \to (-3)^+} (-2x)$

$\Rightarrow \quad RHL = \lim_{h \to 0} [-2(-3+h)]$

$= \lim_{h \to 0} (6 - 2h)$

$\Rightarrow$ RHL = 6

Also, f(-3) = value of f(x) at x = - 3

=-(-3) + 3

= 3 + 3 = 6

$\because$ LHL = RHL = f(-3)

$\therefore$f(x) is continuous at x = - 3 So, x = - 3 is the point of continuity.

Continuity at x = 3

LHL = $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} [-(2x)]$

$\Rightarrow \quad LHL = \lim_{h \to 0} [-2(3-h)]$

$= \lim_{h \to 0} (-6 + 2h)$

$\Rightarrow$ LHL = -6

and RHL = $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x + 2)$

$\Rightarrow \quad \text{RHL} = \lim_{h \to 0} [6(3+h) + 2]$

$\Rightarrow$ RHL = 20

$\because \quad \text{LHL} \neq \text{RHL}$

$\therefore$ f is discontinuous at x = 3 Now, as f (x) is a polynomial function for x < -3 , -3< x < 3 and x > 3 so it is continuous in these intervals.

Hence, only x = 3 is the point of discontinuity of f(x).

OR

Given, f(x) = $\sqrt{x^2+1}$; g(x) = $\frac{x+1}{x^2+1}$ and h(x) = 2x - 3

Therefore,on differentiating above functions w.r.t x, we get

f'(x) = $\frac{1}{2\sqrt{1+x^2}} \times 2x = \frac{x}{\sqrt{1+x^2}}$ [By using chain rule of derivative]

g'(x) = $\frac{(x^2+1) \cdot 1 - (x+1)2x}{(x^2+1)^2}$ [By using quotient rule of derivative]

$= \frac{x^2+1-2x^2-2x}{(x^2+1)^2} = \frac{-x^2-2x+1}{(x^2+1)^2}$

and h'(x) = 2

Therefore, f'[h'{g'(x)}].= $f'\left[h'\left(\frac{-x^2-2x+1}{(x^2+1)^2}\right)\right]$
= f'(2) $[\because h'(x)=2]$
$=\frac{2}{\sqrt{1+4}}=\frac{2}{\sqrt{5}}\left[\because f'(x)=\frac{x}{\sqrt{1+x^2}}\right]$

29. Given: xy = log y + C ...(i)

To prove:y given by eq. (i) is a solution of differential equation $y'=\frac{y^2}{1-xy}$ ....(ii)

Proof: Differentiating both sides of eq. (i) w.r.t x, we have

$xy'+y(1)=\frac{1}{y}y'+0$

$\Rightarrow xy'-\frac{y'}{y}=-y$

$\Rightarrow y'\left(x-\frac{1}{y}\right)=-y$

$\Rightarrow y'\left(\frac{xy-1}{y}\right)=-y$

$\Rightarrow y'(xy-1)=-y^2$

$\Rightarrow y'=\frac{-y^2}{xy-1}$

$\Rightarrow y'=\frac{-y^2}{-(1-xy)}=\frac{y^2}{1-xy}$

Hence, function (implicit) given by eq. (i) is a solution of $y'=\frac{y^2}{1-xy}$.

30. Given, $I=\int\frac{3x+1}{(x+1)^2(x+3)}dx$

By using partial fractions ,
let $\frac{3x+1}{(x+1)^2(x+3)}=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x+3)}$ .....(i)
$\Rightarrow 3x+1=A(x+1)(x+3)+B(x+3)+C(x+1)^2$
$\Rightarrow 3x+1=A(x^2+4x+3)+B(x+3)+C(x^2+1+2x)$
$\Rightarrow 3x+1=(A+C)x^2+(4A+B+2C)x+3A+3B+C$
Comparing coefficient of x^2, x and constant terms from both sides we get ,
$\Rightarrow A+C=0$ ...(ii)

$\Rightarrow 4A + B + 2C = 3$ ...(iii)

$\Rightarrow 3A + 3B + C = 1$ ...(iv)

On solving equations (ii) , (iii) and (iv), we get A = 2, B = -1 and C = -2

$\therefore$ Equation (i) becomes

$$\frac{3x+1}{(x+1)^2(x+3)} = \frac{2}{(x+1)} - \frac{1}{(x+1)^2} - \frac{2}{(x+3)}$$

Integrating both sides w.r.t. x we get ,

$$\Rightarrow \int \frac{3x+1}{(x+1)^2(x+3)} dx = \int \frac{2}{(x+1)} dx - \int \frac{1}{(x+1)^2} dx - \int \frac{2}{(x+3)} dx$$

$$\Rightarrow \quad I = 2\log|x+1| - \frac{(x+1)^{-2+1}}{(-2+1)} - 2\log|x+3| + C$$

$$\Rightarrow \quad I = 2\log|x+1| - 2\log|x+3| + (x+1)^{-1} + C$$

$$= 2\log\left|\frac{x+1}{x+3}\right| + \frac{1}{(x+1)} + C\left[\because \log m - \log n = \log\frac{m}{n}\right]$$

31. Let us define the following events

A = selecting person A

B = selecting person B

C = selecting person C

$P(A) = \frac{1}{1+2+4}, P(B) = \frac{2}{1+2+4}$

and $P(C) = \frac{4}{1+2+4}$

$P(A) = \frac{1}{7}, P(B) = \frac{2}{7}$

and $P(C) = \frac{4}{7}$

Let E = Event to introduce the changes in their profit.

Also given $P\left(\frac{E}{A}\right) = 0.8, P\left(\frac{E}{B}\right) = 0.5$ and $P\left(\frac{E}{C}\right) = 0.3$

$$\Rightarrow \quad P\left(\frac{\overline{E}}{A}\right) = 1 - 0.8 = 0.2, P\left(\frac{\overline{E}}{B}\right) = 1 - 0.5 = 0.5$$

and $P\left(\frac{\overline{E}}{C}\right) = 1 - 0.3 = 0.7$

The probability that change does not take place by the appointment of C,

$$P\left(\frac{C}{E}\right) = \frac{P(C)\cdot P\left(\frac{\overline{E}}{C}\right)}{P(A)\times P\left(\frac{\overline{E}}{A}\right) + P(B)\times P\left(\frac{\overline{E}}{B}\right) + P(C)\times P\left(\frac{\overline{E}}{C}\right)}$$

$$= \frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2 + \frac{2}{7}\times 0.5 + \frac{4}{7}\times 0.7}$$

$$= \frac{2.8}{0.2+1.0+2.8} = \frac{2.8}{4} = 0.7$$

OR

In this case, we have to find out the probability of getting at least 8 heads. Let X is the random variable for getting a head.

Here, $n=10, r \geqslant 8,$

i.e., r = 8, 9, 10, $p=\frac{1}{2}, q=\frac{1}{2}$

we know that, P(X = r) $={}^{n}C_{r} p^{r} q^{n-r}$

$\therefore$ P(X = r) = P(r = 8) + P(r = 9) + P(r = 10)

$={}^{10}C_{8}\left(\frac{1}{2}\right)^{8}\left(\frac{1}{2}\right)^{10-8}+{}^{10}C_{9}\left(\frac{1}{2}\right)^{9}\left(\frac{1}{2}\right)^{10-9}+{}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10-10}$

$=\frac{10!}{8!2!}\left(\frac{1}{2}\right)^{10}+\frac{10!}{9!1!}\left(\frac{1}{2}\right)^{10}+\frac{10!}{0!10!}\left(\frac{1}{2}\right)^{10}$

$=\left(\frac{1}{2}\right)^{10}\left[\frac{10\times 9}{2}+10+1\right]$

$=\left(\frac{1}{2}\right)^{10}\cdot 56=\frac{1}{2^{7}\cdot 2^{3}}\cdot 56=\frac{7}{128}$

32. Let the daily diet consists of x litres of milk, y kgs of beaf and z dozens of eggs. Then,

Total cost per day = Rs.(x + 1.10y + 0.50z)

Let Z denote the total cost in Rs. Then, Z = x + 1.10y + 0.50z

Total amount of vitamin A in the daily diet is (x + y + 10z) mg

But, the minimum requirement is 1 mg of vitamin A.

$\therefore$ x + y + 10z $\geq$ 1

Similarly, total amounts of vitamins B and C in the daily diet are (100x + 10y + 10z) mg and (10x + 100y + 10z) mg respectively and their minimum requirements are of 50 mg and 10 mg

respectively.

$\therefore$ 100x + 10y + 10z $\geq$ 50 and, 10x + 100y + 10z $\geq$ 10

Finally, the quantity of milk, kgs of beaf and dozens of eggs cannot assume negative values.

$\therefore$ x $\geq$ 0, y $\geq$ 0, z $\geq$ 0

Hence, the mathematical formulation of the given LPP is

Minimize Z = x + 1.10y + 0.50z subject to

x + y + 10z $\geq$ 1

100x + 10y + 10z $\geq$ 50

10x + 100y + 10z $\geq$ 10

and, x $\geq$ 0, y $\geq$ 0, z $\geq$ 0

Section D

33. We have, $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}_{2\times2} A\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}_{2\times2}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}_{2\times2}$

Let $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$

$\therefore \begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2a+c & 2b+d\\3a+2c & 3b+2d\end{bmatrix}\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}-6a-3c+10b+5d & 4a+2c-6b-3d\\-9a+6c+15b+10d & 6a+4c-9b-6d\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\Rightarrow$ -6a - 3c + 10b + 5d = 1 ....(i)

$\Rightarrow$ 4a + 2c - 6b - 3d = 0 ....(ii)

$\Rightarrow$ -9a - 6c + 15b + 10d = 0 ....(iii)

$\Rightarrow$ 6a + 4c - 9b - 6d = 1 ....(iv)

On adding Eqs. (i) and (iv), we get

c + b - d = 2 $\Rightarrow$ d = c + b - 2 ....(v)

On adding Eqs. (ii) and (iii), we get

-5a - 4c + 9b + 7d = 0 ....(vi)

On adding Eqs. (vi) and (iv), we get

a + 0 + 0 + d = 1 $\Rightarrow$ d = 1 - a ....(vii)

From Eqs. (v) and (vii)

$\Rightarrow$ c + b - 2 = 1 - a $\Rightarrow$ a + b + c = 3 ....(viii)

$\Rightarrow$ a = 3 - b - c

Now, using the values of a and d in Eq. (iii) , we get

-9(3 - b - c) - 6c + 15b + 10(-2 + b + c) = 0

$\Rightarrow$ -27 + 9b + 9c - 6c + 15b - 20 + 10b + 10c = 0

$\Rightarrow$ 34b + 13c = 47

Now, using the values of a and d in Eq. (ii), we get

4(3 - b - c) + 2c - 6b - 3(b + c - 2) = 0

$\Rightarrow$ 12 - 4b - 4c + 2c - 6b - 3b - 3c + 6 = 0

$\Rightarrow$ -13b + 5c = 18 ....(x)

On multiplying Eq. (ix) by 5 and Eq. (x) by 13, then adding, we get

$$\begin{array}{r} -169b-65c=-234 \\ 170b+65c=235 \\ \hline b=1 \end{array}$$

$\Rightarrow$ -13 $\times$ 1 - 5c = -18 [from Eq. (x)]

$\Rightarrow$ -5c = -18 + 13 = -5 $\Rightarrow$ c = 1

a = 3 - 1 - 1 = 1 and d = 1 - 1 = 0

$$A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$

OR

$$x-y+z=4$$

$$x-2y-2z=9$$

$$2x+y+3z=1$$

Let $A=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix} X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} C=\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$

Then, given system of equations can be rewritten as,

AX = C

Now, $AB=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$

$=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}$

AB = 8I

$$A^{-1}=\frac{1}{8}B\begin{bmatrix}\because A^{-1}AB=8A^{-1}I\\ B=8A^{-1}\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{8}\begin{bmatrix}\frac{-1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{-7}{8} & \frac{1}{8} & \frac{3}{8}\\ \frac{5}{8} & \frac{-3}{8} & \frac{-1}{8}\end{bmatrix}$$

Now, AX = C,

$\Rightarrow$ X = $A^{-1}C$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{-7}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{5}{8} & \frac{-3}{8} & \frac{-1}{8} \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-4}{2} + \frac{9}{2} + \frac{1}{2} \\ \frac{-28}{8} + \frac{9}{8} + \frac{3}{8} \\ \frac{20}{8} + \frac{-27}{8} + \frac{-1}{8} \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

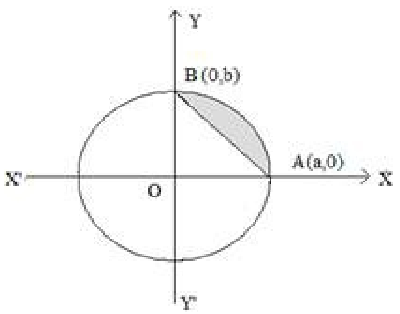
$$\Rightarrow x = 3, y = -2, z = -1$$

34. The given curves are;

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Straight Line: $\frac{x}{a} + \frac{y}{b} = 1$

The required area bounded by given curves is shown in fig. below by shaded portion;



Now area of bounded region is given as;

$A = \int y dx$

Here A = (Area of ellipse in Ist quadrant) - (Area of triangle OAB)

From given ellipse we have;

$y = \frac{b}{a}\sqrt{a^2 - x^2}$

And from given line we have;

$y = \frac{b}{a}(a - x)$

Therefore the required area may be calculated as;

Area $= \int_0^a \frac{b}{a}\sqrt{(a^2 - x^2)}dx - \int_0^a \frac{b}{a}(a - x)\, dx$

$= \frac{b}{a}\left[\frac{x}{a}\sqrt{(a^2 - x^2)} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a - \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$

$= \frac{b}{a}\left[\frac{a^2}{2}\frac{\pi}{2} - 0\right] - \frac{b}{a}\left[\frac{a^2}{2}\right]$
$= \frac{ab}{4}[\pi - 2]$
Which is the required area.

35. Let x and y be the lengths of two sides of a rectangle. Again, let P denotes its perimeter and 'A' be the area of rectangle

Then, P = 2 (x + y) [∵ perimeter of rectangle = 2 (l + b)]
$\Rightarrow \quad P = 2x + 2y$
$\Rightarrow \quad y = \frac{P-2x}{2}$ ...(i)
We know that, area of rectangle is given by

A = xy
$\Rightarrow \quad A = x\left(\frac{P-2x}{2}\right)$ [by using Eq. (i)]
On differentiating both sides w.r.t. x, we get
$\frac{dA}{dx} = \frac{P-4x}{2}$
For maxima or minima put $\frac{dA}{dx} = 0$
$\Rightarrow \quad \frac{P-4x}{2} = 0 \Rightarrow P = 4x$
$\Rightarrow \quad 2x + 2y = 4x \quad [\because P = 2x + 2y]$
$\Rightarrow \quad x = y$
So, the rectangle is a square.
Also, $\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{P-4x}{2}\right)$
$= -\frac{4}{2} = -2 < 0$
$\Rightarrow$ A is maximum.

Hence, area is maximum, when rectangle is a square.

OR

Let P be a point on the hypotenuse AC of right-angled ΔABC, Such that $PL \perp AB$ and PL=a and $PM \perp BC$ and PM=b.
Let $\angle APL = \angle ACB = \theta$[say]
Then, $AP = a\sec\theta, PC = b\ cosec\theta$
Let l be the length of the hypotenuse, then
l = AP+ PC
$\Rightarrow l = a\sec\theta + b\ cosec\theta, 0 < \theta < \frac{\pi}{2}$

On differentiating both sides w.r.t. θ, we get,

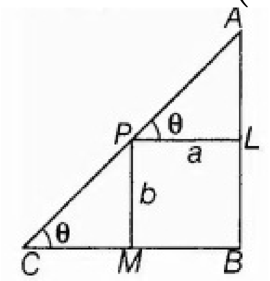
$\frac{dl}{d\theta} = a\sec\theta\tan\theta - b\ cosec\theta\cot\theta$.........(i)

For maxima or minima, put $\frac{dl}{d\theta} = 0$

$\Rightarrow a\sec\theta\tan\theta = b\ cosec\theta\cot\theta$

$\Rightarrow \quad \frac{a\sin\theta}{\cos^2\theta} = \frac{b\cos\theta}{\sin^2\theta}$

$\Rightarrow \quad \tan\theta = \left(\frac{b}{a}\right)^{1/3}$



Again, on differentiating both sides of Eq.(i) w.r.t. θ we get

$\frac{d^2l}{d\theta^2} = a\left(\sec\theta\times\sec^2\theta + \tan\theta\times\sec\theta\tan\theta\right) - b\left[\ cosec\theta\left(-\ cosec^2\theta\right)\right.$

$\left. + \cot\theta(-\ cosec\theta\cot\theta)\right]$

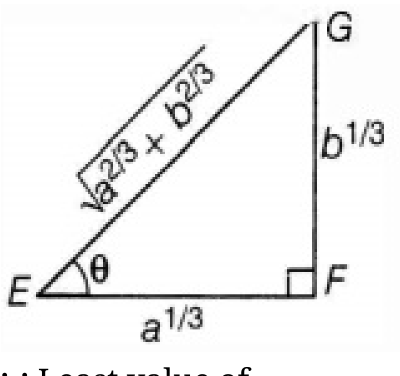
$= a\sec\theta\left(\sec^2\theta + \tan^2\theta\right) + b\ cosec\theta\left(\ cosec^2\theta + \cot^2\theta\right)$

For $0 < \theta < \frac{\pi}{2}$, all trigonometric ratios are positive

Also, a > 0 and b > 0

$\because \frac{d^2l}{d\theta^2}$ is positive.

Thus, l is least when $\tan\theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$



$\because$ Least value of,

$l = a\sec\theta + b\ cosec\theta$

$= a\frac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} + b\frac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}$

$= \sqrt{a^{2/3}+b^{2/3}}\left(a^{2/3}+b^{2/3}\right) = \left(a^{2/3}+b^{2/3}\right)^{3/2}$

$$\left[\therefore \text{ in } \Delta EFG, \tan\theta = \frac{b^{1/3}}{a^{1/3}}, \sec\theta = \frac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} \text{ and } \cos ec\theta = \frac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}\right]$$

36. We have, $\overrightarrow{r} = (8+3\lambda)\hat{i} + (-9-16\lambda)\hat{j} + (10+7\lambda)\hat{k}$

$= 8\hat{i} - 9\hat{j} + 10\hat{k} + 3\lambda\hat{i} - 16\lambda\hat{j} + 7\lambda\hat{k}$

$= 8\hat{i} - 9\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

$\Rightarrow \overrightarrow{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k}$ and $\overrightarrow{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$ .... (i)

Also, $\overrightarrow{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

$\Rightarrow \overrightarrow{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k}$ and $\overrightarrow{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$ .... (ii)

Now, shortest distance between two lines is given by $\left|\frac{(\overrightarrow{b}_1 \times \overrightarrow{b}_2)\cdot(\overrightarrow{a}_1 - \overrightarrow{a}_2)}{\left|\overrightarrow{b}_1 \times \overrightarrow{b}_2\right|}\right|$

$\therefore \overrightarrow{b}_1 \times \overrightarrow{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$

$= \hat{i}(80-56) - \hat{j}(-15-21) + \hat{k}(24+48)$

$= 24\hat{i} + 36\hat{j} + 72\hat{k}$

Now, $\left|\overrightarrow{b}_1 \times \overrightarrow{b}_2\right| = \sqrt{(24)^2 + (36)^2 + (72)^2}$

$= 12\sqrt{2^2 + 3^2 + 6^2} = 84$

And $\left(\overrightarrow{a}_2 - \overrightarrow{a}_1\right) = (15-8)\hat{i} + (29+9)\hat{j} + (5-10)\hat{k}$

$= 7\hat{i} + 38\hat{j} - 5k$

$\therefore$ Shortest distance $= \left|\frac{(24i+36\hat{j}+72\hat{k})\cdot(7\hat{i}+38\hat{j}-5\hat{k})}{84}\right|$

$= \left|\frac{168+1368-360}{84}\right| = \left|\frac{1176}{84}\right| = 14$ units