Permutations & Combinations

Tip 1

- Permutations & Combinations, and Probability are key topics in CAT.
- You don't have to go too deep into these topics, but ensure that you learn the basics well.
- So look through this formula list a few times and understand the formulae.

Tip 2

- The best way to tackle this subject is by solving questions. The more questions you solve, the better you will get at this topic.

- Once you practice good number of sums, you will start to see that all of them are generally variations of the same few themes that are listed in the formula list.

- In this slide, we will look at the important formulae on P&C, and Probability.

Tip 3

- $N! = N(N-1)(N-2)(N-3)......1$
- $0! = 1! = 1$
- $^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$
- $^{n}P_{r} = \frac{n!}{(n-r)!}$

Tip 4

- **Arrangement** :

n items can be arranged in n! ways

- **Permutation** :

A way of selecting and arranging r objects out of a set of n objects, ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

- **Combination** :

A way of selecting r objects out of n (arrangement does not matter)

$${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

Tip 5

- Selecting r objects out of n is same as selecting (n-r) objects out of n, $^{n}C_{r} = {}^{n}C_{n-r}$

- Total selections that can be made from ‘n’ distinct items is given $\sum_{k=0}^{n} {}^{n}C_{k} = 2^{n}$

Tip 6

Partitioning :

- Number of ways to partition n identical things in r distinct slots is given by ${}^{n+r-1}C_{r-1}$
- Number of ways to partition n identical things in r distinct slots so that each slot gets at least 1 is given by ${}^{n-1}C_{r-1}$
- Number of ways to partition n distinct things in r distinct slots is given by r^n
- Number of ways to partition n distinct things in r distinct slots where arrangement matters = $\frac{(n+r-1)!}{(r-1)!}$

Tip 7

Arrangement with repetitions :

If x items out of n items are repeated, then the number of ways of arranging these n items is $\frac{n!}{x!}$ ways. If a items, b items and c items are repeated within n items, they can be arranged in $\frac{n!}{a!\,b!\,c!}$ ways.

Tip 8

Rank of a word :

- To get the rank of a word in the alphabetical list of all permutations of the word, start with alphabetically arranging the n letters. If there are x letters higher than the first letter of the word, then there are at least $x*(n-1)!$ Words above our word.

- After removing the first affixed letter from the set if there are y letters above the second letter then there are $y*(n-2)!$ words more before your word and so on. So rank of word = $x*(n-1)! + y*(n-2)! + .. +1$

Tip 9

Integral Solutions :

- Number of positive integral solutions to $x_1+x_2+x_3+.....+x_n= s$ where $s \geq 0$ is ${}^{s-1}C_{n-1}$

- Number of non-negative integral solutions to $x_1+x_2+x_3+.....+x_n = s$ where $s \geq 0$ is ${}^{n+s-1}C_{n-1}$

Tip 10

Circular arrangement :

Number of ways of arranging n items around a circle are 1 for n = 1,2 and (n-1)! for n≥3. If its a necklace or bracelet that can be flipped over, the possibilities are $\frac{(n-1)!}{2}$

Derangements :

If n distinct items are arranged, the number of ways they can be arranged so that they do not occupy their intended spot is

$$D = n!\left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots \frac{(-1)^n}{n!}\right)$$

Tip 11(i)

Bayes Theorem (Conditional Probability) for CAT:

Conditional probability is used in case of events which are not independent. In the discussion of probabilities all events can be classified into 2 categories: Dependent and Independent.

Independent events are those where the happening of one event does not affect the happening of the other. For example, if an unbiased coin is thrown 'n' times then the probability of head turning up in any of the attempts will be 1/2. It will not be dependent on the results of the previous outcomes.

Dependent events, on the other hand, are the events in which the outcome of the second event is dependent on the outcome of the first event.

For example, if you have to draw two cards from a deck one after the other, then the probability of second card being of a particular suit will depend on the which card was drawn in the first attempt.

Tip 11(ii)

Bayes Theorem (Conditional Probability) for CAT:

Let us first discuss the definition of conditional probability.
Let 'A' and 'B' be two events which are not independent then the probability of occurrence of B given that A has already occurred is given by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Here, $P(A \cap B)$ is nothing but the probability of occurrence of both A and B.
We often use Bayes theorem to solve problems on conditional probability.
Bayes theorem is defined as follows

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)}$$

Here, P(A|B) is the probability of occurrence of A given that B has already occurred.
P(A) is the probability of occurrence of A
P(B) is the probability of occurrence of B