CBSE Class 10th Mathematics
Basic Sample Paper - 02

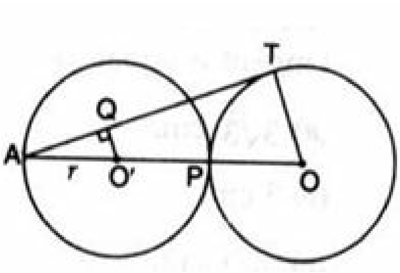
Maximum Marks:
Time Allowed: 3 hours

General Instructions:

a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

1. If $\sum f_i x_i = 1860$ and $\sum f_i = 30$, then the value of $\overline{x}$ is
   a. 26
   b. 63
   c. 64
   d. 62
2. Equal circles with centre O and O' touch each other at P. O and Q' touch each other at P. OO' is produced to meet circle (O', r) at A. AT is a tangent to the circle (O, r). O' Q is perpendicular to AT. Then the value of $\frac{AO'}{AO}$ is



a. $\frac{1}{2}$

b. $\frac{2}{3}$

c. $\frac{1}{4}$

d. $\frac{1}{3}$

3. The decimal expansion of $\frac{987}{10500}$ will terminate after:

   a. 2 decimal places

   b. 3 decimal places

   c. 1 decimal place

   d. None of these

4. A rational number can be expressed as a non-terminating repeating decimal if the denominator has the factors

   a. none of these

   b. other than 2 or 5 only

   c. 2 or 5 only

   d. 2 or 3 only

5. Let $\frac{p}{q}$ be a rational number. Then, the condition on q such that $\frac{p}{q}$ has a non-terminating but repeating decimal expansion is:

   a. $q = 2^m \times 5^n; m, n$ are whole numbers.

   b. $q \neq 2^m \times 3^n; m, n$ are whole numbers

c. $q = 2^m \times 3^n; m, n$ are whole numbers

d. $q \neq 2^m \times 5^n; m, n$ are whole numbers

6. If the polynomial $3x^3 - 4x^2 - 17x - k$ is exactly divisible by $x - 3$, then the value of 'k' is

 a. 6

 b. 5

 c. -5

 d. – 6

7. A quadratic polynomial with zeroes $\frac{1}{4}$ and – 1 is

 a. $4x^2 + 3x - 1$

 b. $4x^2 - 3x - 1$

 c. $4x^2 - 3x + 1$

 d. $4x^2 + 3x + 1$

8. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and these values are equally likely outcomes. The probability that it will point at a number greater than 5 is

 a. $\frac{1}{2}$

 b. $\frac{1}{4}$

 c. $\frac{1}{5}$

 d. $\frac{1}{3}$

9. If A and B are the points (– 6, 7) and (– 1, – 5) respectively, then the distance 2AB is equal to

 a. 20 units

b. 13 units

c. 15 units

d. 26 units

10. If the point R(x, y) divides the join of $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the given ratio $m_1 : m_2$, then the co-ordinates of the point R are

a. $\left(\frac{m_2 x_1 - m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 - m_1 y_2}{m_1 + m_2}\right)$

b. $\left(\frac{m_2 x_1 - m_1 x_2}{m_1 - m_2}, \frac{m_2 y_1 - m_1 y_2}{m_1 - m_2}\right)$

c. $\left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$

d. None of these

11. Fill in the blanks:

The perimeters of two similar $\triangle$ABC and $\triangle$PQR are respectively 18cm and 12cm. If PQ = 5cm, then AB is ________.

12. Fill in the blanks:

The coordinates of every point on the Y-axis are of the form ________.

OR

Fill in the blanks:

Three points are said to be collinear, if area of triangle formed by these points is ________.

13. Fill in the blanks:

If A and B are acute angles and sin A = cos B, then the value of (A + B) is ________.

14. Fill in the blanks:

The value of sinθ cosθ, for θ = 30^{0} is ________.

15. Fill in the blanks:

    The value of $3\sin 30^{o} - 4\sin^{3} 60^{o}$ is ________.

16. Form the top of a tower 50 m high the angles of depression of the top and bottom of a pole are observed to be 45^{o} and 60^{o} respectively. Find the height of the pole.

    OR

    Find the angle of elevation of the sun (sun's altitude) when the length of the shadow of a vertical pole is equal to its height.

17. Find k, if the given value of x is the k^{th} term of the given AP 25,50, 75,100,..., x = 1000.

18. The diameters of two circles are 38 cm and 18 cm. Find the diameter of the circle whose circumference is equal to sum of the circumferences of the two circles.

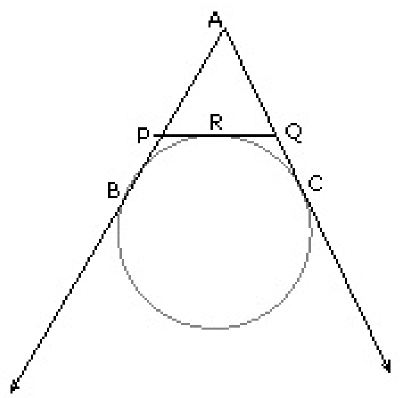
19. P and Q are the points on sides AB and AC, respectively of $\triangle ABC$, If AP = 3 cm , PB = 6 cm, AQ = 5 cm and QC = 10cm, Show that BC = 3PQ.

20. A bag contains 3 red and 5 black balls. A ball is drawn at random from the bag. What is the probability that the drawn ball is not red ?

Section B

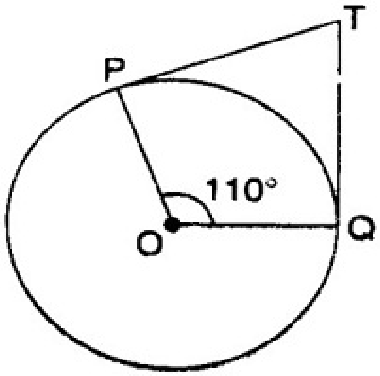
21. If α and β are the zeroes of the quadratic polynomial $f(s) = 3s^2 - 6s + 4$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

22. If AB, AC and PQ are tangents in the given figure and AB = 25 cm, Find the perimeter of $\triangle$ APQ.



OR

In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle$POQ = 110^{o}, then find $\angle$PTQ.



23. Evaluate $\cos(40^\circ - \theta) - \sin(50^\circ + \theta) + \frac{\cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ}$.

OR

If secθ = x, write the value of tan θ.

24. The diameters of the front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that a rear wheel makes to cover the distance which the front wheel covers in 800 revolutions.

25. Harpreet tosses two different coins simultaneously. What is the probability that she gets:

   i. at least one head?
   ii. one head and one tail?

26. There are 30 cards of the same size in a bag in which the numbers 1 to 30 are written. One card is taken out from the bag at random. Find the probability that the number on the selected card is not divisible by 3.

Section C

27. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b.

28. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm. Draw its incircle and measure its radius. Write steps of construction also.

OR

Draw tangents from an external point P to a circle of radius 4 cm without using the centre.

29. A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to make the tank half-empty, if the tank is 3 m in diameter?

30. Prove the trigonometric identity:
$(\sec A + \tan A - 1)(\sec A - \tan A + 1) = 2\tan A$

OR

Prove that : $\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A}$ = cos A - sin A.

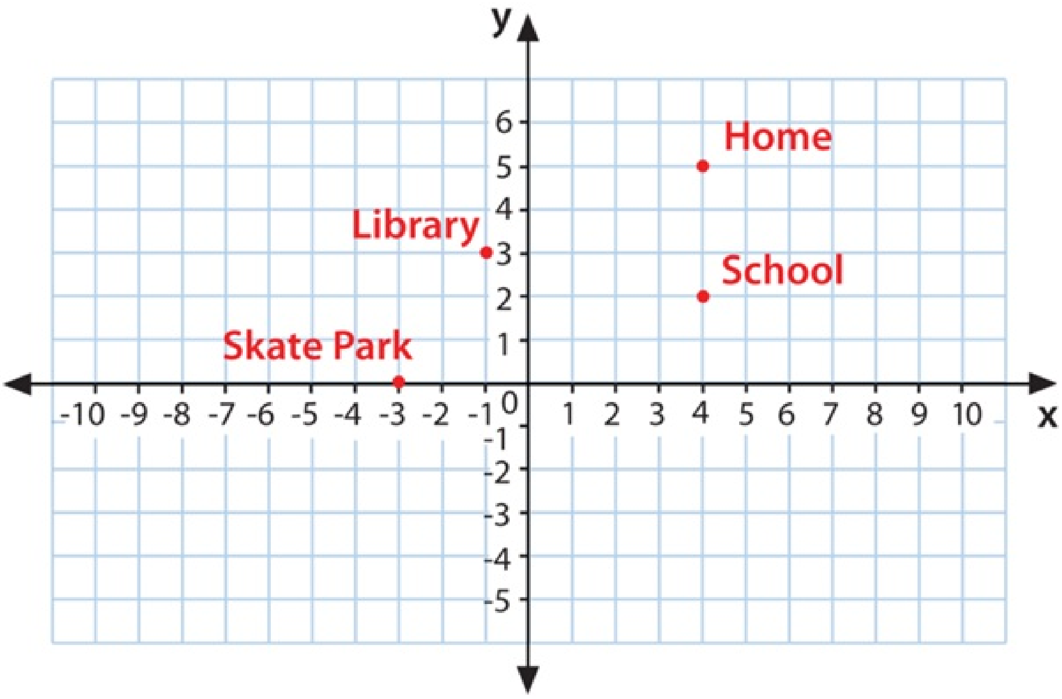
31. There are 156, 208 and 260 students in groups A, B and C respectively. Buses are to be hired to take them for a field trip. Find the minimum number of buses to be hired, if the same number of students should be accommodated in each bus.

OR

Prove that $6 + \sqrt{2}$ is irrational.

32. In two concentric circles, prove that a chord of larger circle which is tangent to smaller circle is bisected at the point of contact.

33. Two brothers Ramesh and Pulkit were at home and have to reach School. Ramesh went to Library first to return a book and then reaches School directly whereas Pulkit went to Skate Park first to meet his friend and then reaches School directly.



i. How far is School from their Home?

ii. What is the extra distance travelled by Ramesh in reaching his School?

iii. What is the extra distance travelled by Pulkit in reaching his School? (All distances are measured in metres as straight lines)

34. A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732]

Section D

35. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

36. A sequence is defined by $a_n = n^3 - 6n^2 + 11n - 6$. Show that the first three terms of the sequence are zero and all other terms are positive.

OR

Find the sum of all integers between 1 and 500 which are multiples of 2 as well as of 5.

37. Solve the following systems of linear equations graphically. Also find the coordinates

of the points where the lines meet axis of y.

2x - 5y + 4 = 0,

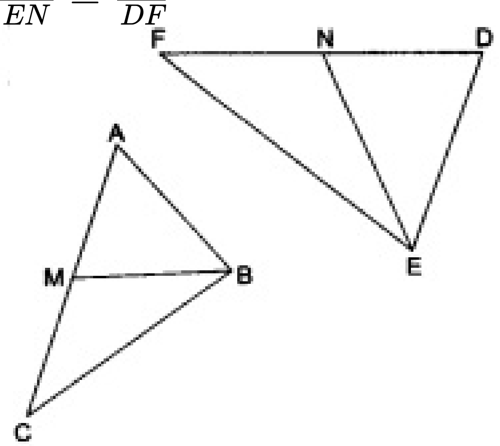
2x + y - 8 = 0

38. In ΔPQR, $QM \perp PR$ and PR^2 - PQ^2 = QR^2. Prove that QM^2 = PM $\times$ MR

OR

In the given figure, BM and EN are respectively the medians of $\triangle ABC$ and $\triangle DEF$. If $\triangle ABC \sim \triangle DEF$, prove that:

a. $\triangle AMB \sim \triangle DNE$
b. $\triangle CMB \sim \triangle FNE$
c. $\frac{BM}{EN} = \frac{AC}{DF}$



39. A cone of radius 10 cm is divided into two parts by a plane parallel to its base through the midpoint of its height. Compose the Volume of the two parts.

OR

A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 24 cm with radii of its lower and upper ends as 7 cm and 14 cm, respectively. Find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 sq. cm.

40. The following distribution gives the distribution of lifetimes of washing machines of a certain company :

Life time	1000-	1200-	1400-	1600-	1800-	2000-	2200-

(in hours)	1200	1400	1600	1800	2000	2200	2400
Number of washing machines	15	60	68	86	75	61	45

Convert the above distribution into less than type' and draw its ogive.

Solution

Section A

1. (d) 62

   Explanation:

   $\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$ = $\frac{1860}{30}$ = 62

2. (d) $\frac{1}{3}$

   Explanation:

   $\frac{\text{AO'}}{\text{AO}} = \frac{r}{\text{AO'} + \text{O'P} + \text{OP}}$
   = $\frac{r}{r+r+r}$ (Radii of both circles are equal)
   $\Rightarrow \frac{\text{AO'}}{\text{AO}} = \frac{r}{3r}$ = $\frac{1}{3}$

3. (b) 3 decimal places

   Explanation:

   $\frac{987}{10500}$ = $\frac{47}{500}$ = $\frac{47}{2^2 \times 5^3}$ Here, in the denominator of the given fraction the highest power of prime factor 5 is 3, therefore, the decimal expansion of the rational number $\frac{47}{2^2 \times 5^3}$ will terminate after 3 decimal places.

4. (b) other than 2 or 5 only

   Explanation:

   A rational number can be expressed as a **non-terminating** repeating decimal if the denominator has the factors other than 2 or 5 only.

5. (d) $q \neq 2^m \times 5^n; m, n$ are whole numbers

   Explanation:

   $\frac{p}{q}$ has a non-terminating but repeating decimal expansion if $q \neq 2^m \times 5^n; m, n$ are whole numbers.

6. (d) – 6

   Explanation:

   If the polynomial $3x^3 - 4x^2 - 17x - k$ is exactly divisible by $x - 3$, then
   $p(3) = 0$ (By factor theorem)
   $\Rightarrow 3(3)^3 - 4(3)^2 - 17 \times 3 - k = 0$
   $\Rightarrow 81 - 36 - 51 - k = 0$
   $\Rightarrow -6 - k = 0$
   $\Rightarrow k = -6$

7. (a) $4x^2 + 3x - 1$

   Explanation:

   Explanation: x^2-$(\alpha + \beta)$x + $(\alpha\beta)$=0

   Here $\alpha + \beta = \frac{1}{4} + (-1) = \frac{1-4}{4} = \frac{-3}{4}$ And $\alpha\beta = \frac{1}{4} \times (-1) = \frac{-1}{4}$ = $\frac{c}{a}$

   x^2 -($\frac{-3}{4}$)x + ($\frac{-1}{4}$) = 0

   x^2 + $\frac{3}{4}$x - $\frac{1}{4}$ = 0

   $\frac{4x^2+3x-1}{4}$ = 0 (By L.C.M)

   $4x^2 + 3x - 1 = 0$

8. (a) $\frac{1}{2}$

   Explanation:

   Number of possible outcomes = {6, 7, 8, 9, 10} = 5
   Number of total outcomes = 10
   $\therefore$ Required Probability = $\frac{5}{10} = \frac{1}{2}$

9. (d) 26 units

   Explanation:

   2AB = $2\sqrt{(-1+6)^2 + (-5-7)^2}$

= $2\sqrt{25+144}$

= $2\sqrt{169}$

= 26 units

10. (c) $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

    Explanation:

    If the point R(x, y) divides the join of $P(x_1, y_1)$ and $Q(x_2, y_2)$
    internally in the given ratio $m_1 : m_2$,
    then the co – ordinates of the point R are $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$.

11. 7.5cm

12. (0, y)

    OR

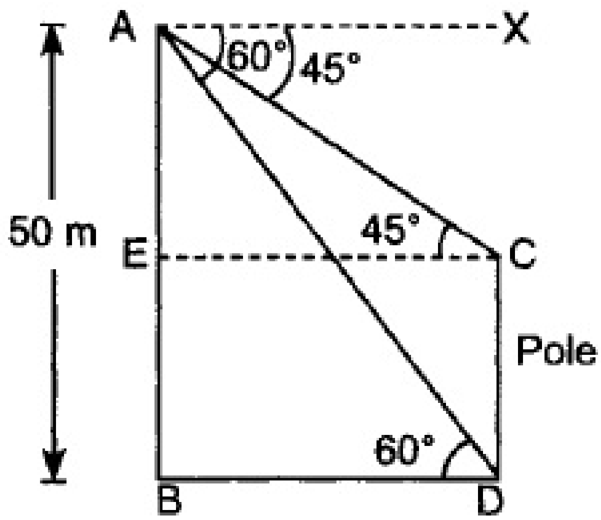
    zero

13. 90^{0}

14. $\frac{\sqrt{3}}{4}$

15. $\frac{3(1-\sqrt{3})}{2}$

16. In $\triangle$ABD, $\frac{BD}{AB} = \cot 60^{\circ}$



$\Rightarrow \frac{\mathrm{BD}}{50}=\frac{1}{\sqrt{3}} \Rightarrow \mathrm{BD}=\frac{50}{\sqrt{3}} \mathrm{~m}$

$\mathrm{BD}=\mathrm{EC} \Rightarrow \mathrm{EC}=\frac{50}{\sqrt{3}} \mathrm{~m}$

In $\triangle AEC, \frac{\mathrm{AE}}{\mathrm{EC}}=\tan 45^{\circ}$

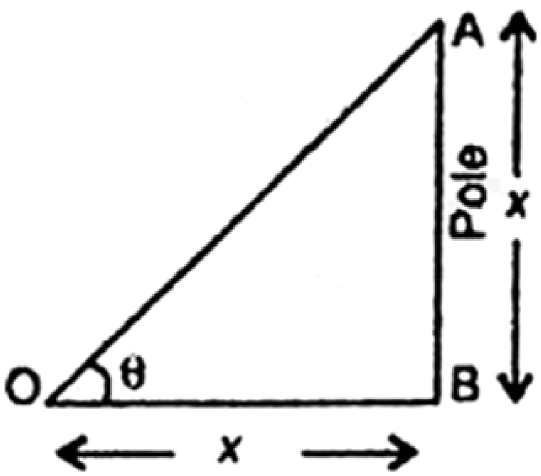
$\Rightarrow \quad \mathrm{AE}=\mathrm{EC} \Rightarrow \mathrm{AE}=\frac{50}{\sqrt{3}} \mathrm{~m}$

Now $\mathrm{BE}=\mathrm{AB}-\mathrm{AE}=50-\frac{50}{\sqrt{3}}$

$=\frac{50(\sqrt{3}-1)}{\sqrt{3}} \mathrm{~m}$

$DC=BE$= $\frac{50(\sqrt{3}-1)}{\sqrt{3}} \mathrm{~m}$

OR



According to the question,

Let height of pole (AB) = x m

Then, length of shadow (OB) = x m

In ΔOAB

$\tan\theta = \frac{AB}{OB}$
$\Rightarrow \tan\theta = \frac{x}{x}$
$\Rightarrow \tan\theta = 1 = \tan 45^o$
$\Rightarrow \theta = 45^o$

17. $a = 25,\ d = 50 - 25 = 25,\ \ x = 1000$

A.T.Q. , $a_k = x$

$\Rightarrow$ a$+(k-1)d = 1000$

$\Rightarrow$ 25 + (k - 1)25 = 1000

$\Rightarrow$ (k - 1)25 = 975 $\Rightarrow$ k - 1 = $\frac{975}{25}$

$\Rightarrow$ k - 1 = 39 $\Rightarrow$ k = 40

18. D = 38 and R = 19

d = 18 and r = 9

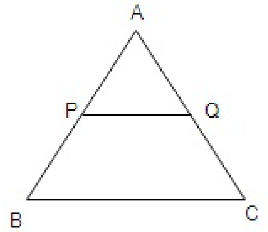
2πR + 2πr = 2πR'

2π(R + r) = 2πR'

R + r = R'

19 + 9 = R'

So, R' = 28.

19. 

$\frac{AP}{AB} = \frac{3}{9} = \frac{1}{3}$ ....(i)

$\frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$ ....(ii)

from eqn (i) and (2) we have

$\frac{AP}{AB} = \frac{AQ}{AC}$

Hence, PQ | | BC [Converse of basic proportionality theorem.]

So $\triangle ABC \sim \triangle APQ$

$\frac{AP}{AB} = \frac{AQ}{BC} = \frac{PQ}{BC}$

$\frac{3}{9} = \frac{5}{15} = \frac{PQ}{BC}$

$\frac{PQ}{BC} = \frac{1}{3}$

BC = 3 PQ

Hence proved.

20. Number of balls in the bag = 3 + 5 = 8

$probability \ = \ \frac{Number \ of \ \ favorable \ outcome}{Total \ number \ of \ outcome}$

P(that the drown ball is not red) = $\frac{5}{8}$

Section B

21. Since, α and β are the zeroes of the quadratic polynomial f(s) = $3s^2$ - 6s + 4

Compare f(s) with standard form f(x) = ax^2 + bx + c

a = 3, b = -6 and c=4

Sum of the zeroes = $\alpha + \beta = -\frac{b}{a} = \frac{6}{3}$

Product of the zeroes = $\alpha \times \beta = \frac{c}{a} = \frac{4}{3}$

Now,

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta.$

$= \frac{\alpha^2+\beta^2}{\alpha\beta} + 2\left[\frac{\alpha+\beta}{\alpha\beta}\right] + 3\alpha\beta$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} + 2\left[\frac{\alpha+\beta}{\alpha\beta}\right] + 3\alpha\beta$

By substituting the values of sum and product of the zeroes, we will get

$\frac{\alpha}{\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta = 8$

22. Perimeter of $\triangle$APQ = AP + AQ + PQ

= AP + AQ + PR + RQ

= AP + AQ + PB + CQ

= (AP + PB) + (AQ + QC)

= AB + AC

= 2AB = 2 $\times$ 25 = 50 cm

OR

$\angle$POQ = 110^o

$\angle$OPT = 90^o [Angle between tangent and radius through the point of contact]

$\angle$OQT = 90^o [Angle between tangent and radius through the point of contact]

In quadrilateral OPTQ,

$\angle$POQ + $\angle$OQT + $\angle$PTQ = 360^o

$\because$ The sum of all the angles of a quadrilateral is 360^o.

$\Rightarrow$ 110^o + 90^o + 90^o + $\angle$PTQ = 360^o

$\Rightarrow$ 290^o + $\angle$PTQ = 360^o

$\Rightarrow\angle$PTQ = 360^o - 290^o = 70^o

$\Rightarrow$ Hence, the $\angle$PTQ is 70^o.

23. We have,

$\cos(40^\circ - \theta) - \sin(50^\circ + \theta) + \frac{\cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ}$

$= \sin\{90^\circ - (40^\circ - \theta)\} - \sin(50^\circ + \theta) + \frac{\cos^2 40^\circ + \cos^2(90^\circ - 40^\circ)}{\sin^2 40^\circ + \sin^2(90^\circ - 40^\circ)}$

$[\because \cos\alpha = \sin(90^\circ - \alpha)]$

$= \sin(50^\circ + \theta) - \sin(50^\circ + \theta) + \frac{\cos^2 40^\circ + \sin^2 40^\circ}{\sin^2 40^\circ + \cos^2 40^\circ} = 0 + 1 = 1$

OR

According to the question, sec θ=x

We know that, tan θ= $\sqrt{(sec\theta)^2 - 1}$

Now, tan θ=$\sqrt{x^2 - 1}$.

24. Radius of the front wheel $= 40cm$

$= \frac{2}{5}m$

Circumference of the front wheel $= \left(2\pi \times \frac{2}{5}\right) m$

$= \frac{4\pi}{5}m$

Distance moved by it in 800 revolution

$= \left(\frac{4\pi}{5} \times 800\right) m$

$= (640\pi)m$

Circumference of rear wheel $= (2\pi 1)m = (2\pi)m$

Required number of revolutions $= \left(\frac{640\pi}{2\pi}\right)$

$= 320$

25. Number of possible outcomes are HH, TT, TH, HT

Number of possible outcomes = 4

i. P(atleast 2 head) = $\frac{3}{4}$
ii. P(one head and one tail) = $\frac{2}{4} = \frac{1}{2}$

26. The numbers which are multiple of 3 are divisible by 3 = 3, 6, 9 ,12 ,15 ,18 , 21, 24, 27,30

Total number of favourable outcomes = 30 -10 = 20

Total numbers = 30

$\therefore$ P(no. divisible by 3) = $\frac{20}{30} = \frac{2}{3}$

Section C

27. It is given that p(x) = $6x^4 + 8x^3 - 5x^2 + ax + b$ and g(x) = $2x^2 - 5$

$6x^4 + 8x^3 - 5x^2 + ax + b$ Divide by $2x^2 - 5$, we get

$$
\begin{array}{r}
3x^2 + 4x + 5 \\
2x^2 - 5 \overline{\smash{)}\, 6x^4 + 8x^3 - 5x^2 + ax + b} \\
\underline{\underset{-}{6x^4} \qquad \underset{+}{-}15x^2} \\
8x^3 + 10x^2 + ax + b \\
\underline{\underset{-}{8x^3} \qquad \underset{+}{-}20x} \\
10x^2 + 20x + ax + b \\
\underline{\underset{-}{10x^2} \qquad \underset{+}{-}25} \\
\underline{20x + ax + b + 25}
\end{array}
$$

$\because$ p(x) is exactly divisible by g(x).

Therefore, 20 x+ ax + b + 25 = 0

x (20 + a) + (b + 25) = 0x + 0

then, 20 + a = 0 and b + 25 = 0

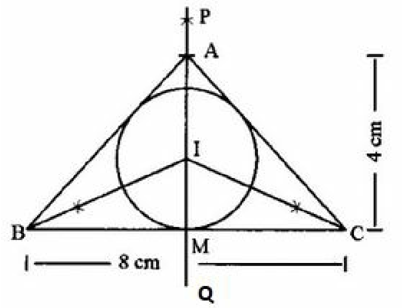
i.e., a = -20, b = -25

Hence the value of a and b are -20 and -25.

28. Steps of construction:

i. Draw BC = 8 cm.

ii. Draw PQ the perpendicular bisector of BC intersecting BC at M.
iii. Taking M as centre and MA as radius equal to the altitude of the triangle of 4 cm intersect PQ at A.
iv. Join A to B and C.
v. Draw bisectors of $\angle B$ and $\angle C$ intersecting AM at I.
vi. Taking I as centre and IM as radius draw an incircle which is the required circle.
vii. Measure radius IM which is equal to 1.7 cm approximately.



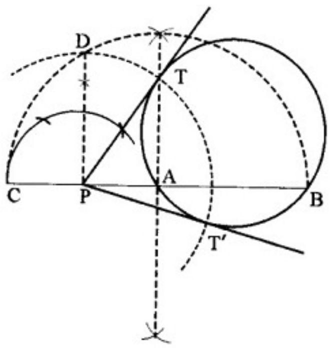
OR

Given: A circle of radius 4 cm.

Required: To draw two tangents from an external point P.

Steps of construction:

i. Draw a circle of radius 4 cm.
ii. Take a point P outside the circle and draw a secant PAB, intersecting the circle at A and B.
iii. Produce AP to C such that AP = CP.
iv. Draw a semi-circle with CB as diameter.
v. Draw,$PD \perp CB$ intersecting the semi-circle at D.
vi. With P as centre and PD as radius draw arcs to intersect the given circle at T and T'.
vii. Join PT and PT'. Then PT and PT' are the required tangents.



29. We have, Radius of hemispherical tank = $\frac{3}{2}m$

$\therefore$ Volume of the tank = $\frac{2}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\right)^3 \mathrm{m}^3 = \frac{99}{14}\mathrm{m}^3$

Volume of the water to be emptied =

$\frac{1}{2} \times \frac{99}{14}\mathrm{m}^3 = \frac{99}{28}\mathrm{m}^3 = \frac{99}{28} \times 1000 \text{ litres } = \frac{99000}{28}\text{litres}$

Since $\frac{25}{7}$ litres of water is emptied in one second. Therefore,

Total time taken to empty half the tank i.e $\frac{99000}{28}$ litres of water =

$= \frac{99000}{28} \div \frac{25}{7}\text{seconds}$

$= \frac{99000}{28} \times \frac{7}{25}\text{seconds}$

$= \frac{99000}{28} \times \frac{7}{25} \times \frac{1}{60}\text{minutes}$

= 16.5 minutes

30. We have, (secA + tanA - 1)(secA - tanA + 1)

$= \left(\sec A + \tan A - (\sec^2 A - \tan^2 A)\right)\left(\sec A - \tan A + (\sec^2 A - \tan^2 A)\right)$

$= (\sec A + \tan A - (\sec A + \tan A)(\sec A - \tan A))$

$(\sec A - \tan A + (\sec A + \tan A)(\sec A - \tan A))$

$= \{(\sec A + \tan A)(1 - (\sec A - \tan A))\}\{(\sec A - \tan A)(1 + (\sec A + \tan A)\}$

$= \{(\sec A + \tan A)(1 - \sec A + \tan A)\}\{(\sec A - \tan A)(1 + \sec A + \tan A)\}$

$= (\sec A + \tan A)(\sec A - \tan A)(1 - \sec A + \tan A)(1 + \sec A + \tan A)$

$= \left(\sec^2 A - \tan^2 A\right)(1 - \sec A + \tan A)(1 + \sec A + \tan A)$

$= 1\left(1 - \frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)\left(1 + \frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= 1\left(\frac{\cos A - 1 + \sin A}{\cos A}\right)\left(\frac{\cos A + 1 + \sin A}{\cos A}\right)$

$= \left(\frac{\cos A + \sin A - 1}{\cos A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$

$= \frac{(\cos A + \sin A)^2 - 1}{\cos^2 A}$

$= \frac{\cos^2 A + \sin^2 A + 2\cos A \sin A - 1}{\cos^2 A}$

$= \frac{1 + 2\cos A \sin A - 1}{\cos^2 A}\ \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \frac{2\cos A \sin A}{\cos A \cos A}$

$= 2\frac{\sin A}{\cos A} = 2\tan A = RHS.$

Hence proved.

OR

LHS =

$\frac{\cos A}{1+\tan A}-\frac{\sin A}{1+\cot A}$

$=\frac{\cos A}{1+\frac{\sin A}{\cos A}}-\frac{\sin A}{1+\frac{\cos A}{\sin A}}$

$=\frac{\cos^2 A}{\cos A+\sin A}-\frac{\sin^2 A}{\sin A+\cos A}$

$=\frac{\cos^2 A-\sin^2 A}{(\sin A+\cos A)}$

$=\frac{(\cos A+\sin A)(\cos A-\sin A)}{\sin A+\cos A}\ [\because (a^2-b^2)=(a+b)(a-b)]$

= $cosA-sinA$

= RHS

Hence Proved.

31. Given numbers are 156, 208 and 260.

Here, 260 > 208> 156

Let us find the HCF of 260 and 208,

By using Euclid's division lemma for 260 and 208,

we get
260 = (208 × 1) + 52
Here , the remainder is 52, not zero.

On taking 208 as new dividend and 52 as new divisor and then apply Euclid's division lemma, we get
208 = (52 × 4) + 0
Here, the remainder is zero and the divisor is 52.

So, HCF of 208 and 260 is 52.

Now, 156 >52

Let us find the HCf of 52 and 156. By using Euclid's division lemma , we get
156 = (52 × 3) + 0
Here, the remiander is zero and the divisor is 52.

So, HCF of 52 amd 156 is 52.

Thus, HCf of 156, 208 and 260 is 52.

Hence, the minimum number of buses

$= \frac{156}{52} + \frac{208}{52} + \frac{260}{52} = \frac{156+208+260}{52} = \frac{624}{52} = 12$

The minimum number of buses is 12.

OR

We will prove $6 + \sqrt{2}$ irrational by contradiction.

Let us suppose that ($6 + \sqrt{2}$) is rational.

It means that we have co-prime integers *a* and *b* (*b* ≠ 0)

Such that

$6 + \sqrt{2} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$

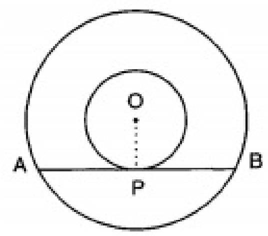
$\Rightarrow \sqrt{2} = \frac{a-6b}{b}$ .... (1)

a and b are integers.

It means L.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. ($6 + \sqrt{2}$) cannot be rational.

Hence, ($6 + \sqrt{2}$) is irrational.

32. 

In two concentric circles, We have to prove that a chord of larger circle which is tangent to smaller circle is bisected at the point of contact.

Let O be the common centre of two concentric circles, and let AB be a chord of the larger circle touching the smaller circle at P.

Join OP.

Since OP is the radius of the smaller circle and AB is a tangent to this circle at a point P.

$\therefore \quad OP \perp AB$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle, bisects the chord. So,

$OP \perp AB$

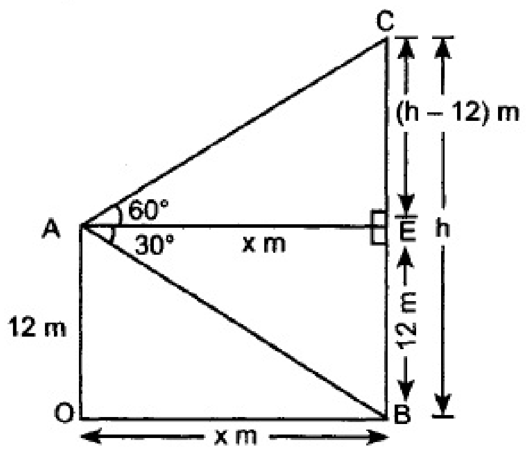
$\Rightarrow$ AP = BP

Hence, AB is bisected at P.

33. Let Home represented by point H(4, 5), Library by point L(-1, 3), Skate Park by point P(3, 0) and School by S(4, 2).

    i. Distance between Home and School, HS = $\sqrt{(4-4)^2+(2-5)^2} = 3\ metres$
    ii. Now, HL = $\sqrt{(-1-4)^2+(3-5)^2} = \sqrt{25+4} = \sqrt{29}$
    LS = $\sqrt{[4-(-1)]^2+(2-3)^2} = \sqrt{25+1} = \sqrt{26}$
    Thus, HL + LS = $\sqrt{29}+\sqrt{26} = 10.48\ metres$
    So, extra distance covered by Ramesh is = HL + LS - HS = 10.48 - 3 = 7.48 metres
    iii. Now, HP = $\sqrt{(3-4)^2+(0-5)^2} = \sqrt{1+25} = \sqrt{26}$
    PS = $\sqrt{[4-3]^2+(2-0)^2} = \sqrt{1+4} = \sqrt{5}$
    Thus, HP + PS = $\sqrt{26}+\sqrt{5} = 7.33\ metres$
    So, extra distance covered by Ramesh is = HP + PS - HS = 7.33 - 3 = 4.33 metres

34. A is the position of the man, OA = 12 m, BC is cliff.



BC = h m and CE = (h - 12)m

Let $\mathrm{AE} = \mathrm{OB} = x$ m

In right angled triangle AEB,

$\frac{\mathrm{AE}}{\mathrm{BE}} = \cot 30^\circ \Rightarrow \mathrm{AE} = 12 \times \sqrt{3}$

$= 12 \times 1.732\mathrm{m} = 20.78\mathrm{m}$

$\therefore$ Distance of ship from cliff = 20.78 m.

In right angled triangle AEC,

$\frac{CE}{AE} = \tan 60^{\circ} \Rightarrow \frac{h-12}{12\sqrt{3}} = \sqrt{3}$

$h - 12 = 36 \Rightarrow h = 48\text{m}$

Section D

35. Let the larger number be x,

Then, (smaller number)^2 = 8(larger number)

= 8x $\Rightarrow$ smaller number $= \sqrt{8x}$

According to the question, $x^2 - 8x = 180$

$\Rightarrow x^2 - 8x - 180 = 0$

Using the quadratic formula, $= \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

we get $= \frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-180)}}{2(1)}$

$= \frac{8\pm\sqrt{64+720}}{2} = \frac{8\pm\sqrt{784}}{2} = \frac{8\pm28}{2}$

$= \frac{8+28}{2}, \frac{8-28}{2} \Rightarrow x = 18, -10$

x = -10 is inadmissible

Then smaller number $= \sqrt{8(-10)} = \sqrt{-80}$

which does not exist.

$\therefore x = 18 \therefore \sqrt{8x} = \sqrt{8 \times 18} = \sqrt{144} = \pm 12$

Hence, the two numbers are 18, 12 or 18, -12.

36. We have, $a_n = n^3 - 6n^2 + 11n - 6$

Putting, $n = 1, 2, 3$

$a_1 = 1^3 - 6(1^2) + 11(1) - 6 = 1 - 6 + 11 - 6 = 12 - 12 = 0$

$a_2 = 2^3 - 6(2^2) + 11(2) - 6 = 8 - 24 + 22 - 6 = 30 - 30 = 0$

$a_3 = 3^3 - 6(3^2) + 11(3) - 6 = 27 - 54 + 33 - 6 = 60 - 60 = 0$

Thus, we have

$a_1 = a_2 = a_3$

We observe that $a_n = n^3 - 6n^2 + 11n - 6$ is a cubic polynomial in n and it vanishes for n = 1, 2, and 3. Therefore, by factor theorem (n -1), (n - 2) and (n- 3) are factors of a_n.

Thus, we have

a_n = (n- 1)(n - 2)(n - 3)

In this expression, if we substitute any value of n which is greater than 3, then each factor on the RHS is positive. Therefore, $a_n > 0$ for all n > 3.

Hence, first three terms of the sequence are zero and all other terms are positive.

OR

Integers between 1 and 500 which are multiples of 2 as well as 5 are
10, 20, 30,..........., 490
This forms an A.P. with a = 10, d = 10, and l = 490
Let the number of these terms be n.
Then,
$a_n = 490$
$\Rightarrow$a + (n - 1)d = 490
$\Rightarrow$10 + (n - 1)(10) = 490
$\Rightarrow$(n - 1)(10) = 480
$\Rightarrow$n - 1 = 48
$\Rightarrow$ n = 49
$\Rightarrow S_{49} = \frac{49}{2}[2 \times 10 + 48 \times 10]$
$= \frac{49}{2} \times [20 + 480]$
$= \frac{49}{2} \times 500$
= 12250

37. On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and the y-axis respectively.

We have, 2x - 5y + 4= 0
2x - 5y + 4 = 0 $\Rightarrow$ y = $\frac{2x+4}{5}$ . ...(i)
We have the following table for 2x - 5y + 4 = 0

x	-2	3	8
y	0	2	4

Now, plot the points $A(-2,0), B(3,2)\ and\ C(8,4)$ on the graph paper.
Join AB and BC to get the graph line AC.
Thus, the line AC is the graph of the equation 2x - 5y + 4 = 0

Now, $2x+y-8=0$

$2x+y-8=0$

$\Rightarrow y=8-2x$........(ii)

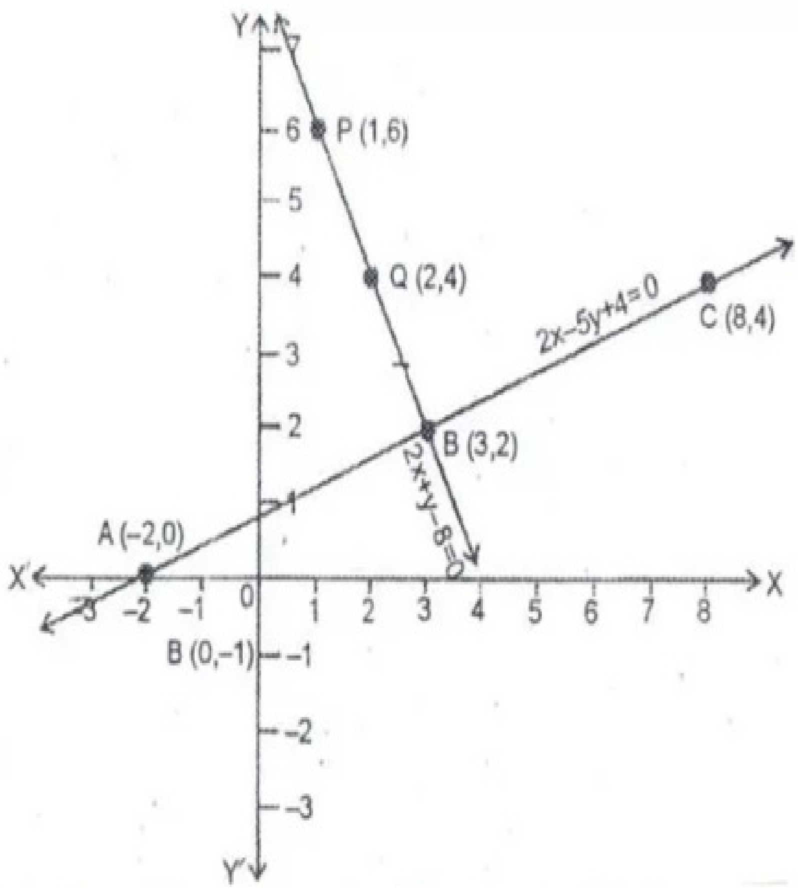
We have the following table for $2x+y-8=0$

x	1	3	2
y	6	2	4

Now, plot the points $P(1,6)\ and\ Q(2,4)$ on the same graph paper.

The point $B(3,2)$ has already been plotted.

Join PQ and QB to obtain the line PB.



Thus, the line PB is the graph of the equation $2x+y-8=0$

Two graphs lines intersect at the point $C(3,2)$

$\therefore x=3, y=2$ is the solution of the given system of equations.

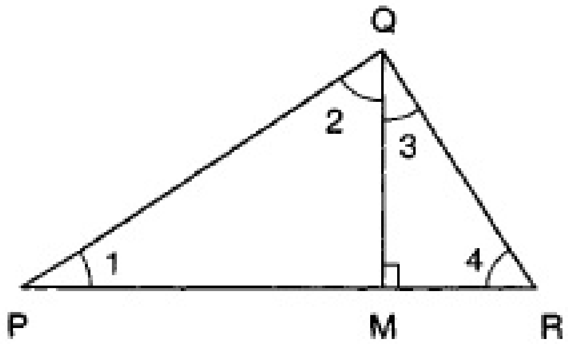
38. In ΔPQR,

$PR^2 - PQ^2 = QR^2$

$\therefore$ PR^2 = PQ^2 + QR^2

$\Rightarrow$ ΔPQR is a right triangle right-angled at Q.

$\Rightarrow$ $\angle$2 + $\angle$3 = 90°



Also, $\angle$1 + $\angle$2 = 90° [$\because$ $\angle$PMQ = 90°]

$\therefore$ $\angle$1 = $\angle$3

Similarly, we have

$\angle$2 = $\angle$4

Thus, in Δ's PMQ and QMR, we have,

$\angle$1 = $\angle$3 and $\angle$2 = $\angle$4

So, by AA-criterion for similarity, we obtain,

ΔPMQ ~ ΔQMR

$\Rightarrow \quad \frac{PM}{QM} = \frac{MQ}{MR}$

$\Rightarrow \quad QM^2 = PM \times MR$

OR

$\triangle ABC \sim \triangle DEF$ (given)

$\therefore \quad \angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$ ...(i)

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ ...(ii)

Since BM and EN are medians, we have

CA = 2AM = 2CM

and FD = 2DN = 2FN.

$\therefore$ from (ii), we have

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2AM}{2DN} = \frac{2CM}{2FN}$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{AM}{DN} = \frac{CM}{FN}$ ...(iii)

a. In $\triangle AMB$ and $\triangle DNE$, we have

$\angle BAM = \angle EDN$ [$\because \angle A = \angle D$ from (i)]

and $\frac{AB}{DE} = \frac{AM}{DN}$ [from (iii)].

$\therefore \quad \triangle AMB \sim \triangle DNE$ [by SAS-similarity]

b. In $\triangle CMB$ and $\triangle FNE$, we have

$\angle BCM = \angle EFN$ [$\because \angle C = \angle F$ from (i)]

and $\frac{BC}{EF} = \frac{CM}{FN}$ [from (iii)]

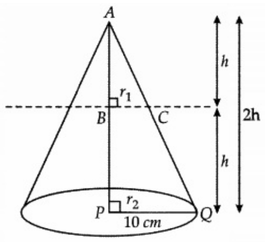
$\therefore \quad \triangle CMB \sim \triangle FNE$ [by SAS-similarity].

c. As proved above, $\triangle AMB \sim \triangle DNE$ and so

$\frac{AB}{DE} = \frac{BM}{EN}$ ...(iv)

From (ii) and (iv), we get

$\frac{BM}{EN} = \frac{AC}{FD}$.

39. 

According to the question,A cone of radius 10 cm is divided into two parts by a plane parallel to its base through the midpoint of its height.

$\triangle ABC \sim \triangle APQ$

$\Rightarrow \quad \frac{h}{2h} = \frac{r_1}{10}$

$\Rightarrow \quad r_1 = 5\text{cm}$

Volume of smaller cone $= \frac{1}{3}\pi(5)^2 \times h$

Volume of frustum $= \frac{1}{2}\pi \times h\left(5^2 + 10 + 5 \times 10\right)$

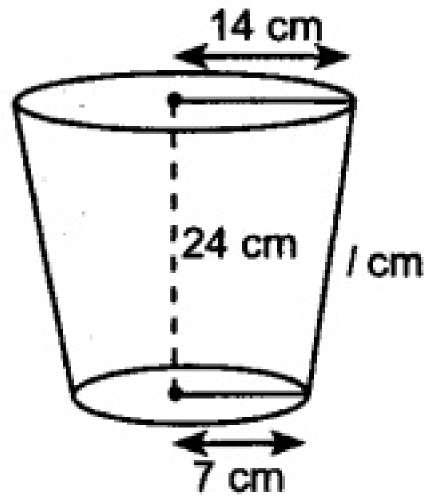
$= \frac{1}{3}\pi \times h \times 175$

Required ratio $= \frac{\frac{1}{3}\times\pi\times25\times h}{\frac{1}{3}\times\pi\times h\times175} = \frac{1}{7}$

OR

Given,

r_2 = 7 cm, r_1 = 14 cm, h = 24 cm



Slant height(l) = $\sqrt{(24)^2 + (14-7)^2}$ cm

$= \sqrt{576 + 49}$ cm

$= \sqrt{625}$ cm

= 25 cm

Area of metal sheet used

$= \pi l\,(r_1 + r_2) + \pi r_2^2$

$= \left[\frac{22}{7} \times 25(14+7) + \frac{22}{7} \times (7)^2\right]$ cm^2

$= \left[\frac{22}{7} \times 25 \times 21 + \frac{22}{7} \times 7 \times 7\right]$ cm^2

= (1650 + 154) cm^2 = 1804 cm^2

Cost of Metal sheet = Rs: 8 per 100 sq. cm.

Cost of metal sheet for = 1804 $\times \frac{8}{100}$

= Rs 144.32

40.

Life time	c.f.
Less than 1200	15
Less than 1400	15 + 60 = 75
Less than 1600	75 + 68 = 143
Less than 1800	143 + 86 = 229
Less than 2000	229 + 75 = 304
Less than 2200	304 + 61 = 365

Less than 2400	365 + 45 = 410

Units: x-axis 1 cm = 1200

y-axis 1 cm = 50