# Derivative of Inverse Trigonometric Function

---

**Q.1.** Find dy/dx if $y = \tan^{-1} \{\sqrt{(1 + x^2)} - 1\}/x$ .

**Solution : 1**

We have $y = \tan^{-1} \{\sqrt{(1 + x^2)} - 1\}/x$ ,

Let $x = \tan\theta$ then, $\theta = \tan -1\ x$ ,

Therefore, $y = \tan^{-1} \{\sqrt{(1 + \tan^2\theta)} - 1\}/\tan\theta = \tan^{-1} [(\sec\theta - 1)/\tan\theta]$

$= \tan^{-1} [(1 - \cos\theta) / \sin\theta] = \tan -1 [\{2 \sin^2(\theta/2)\}/\{2 \sin(\theta/2) \cos(\theta/2)\}]$

$= \tan^{-1} [\tan(\theta/2)] = \theta/2 = 1/2 \tan -1\ x$ ,

Therefore, $dy/dx = 1/\{2(1 + x^2)\}$.

**Q.2.** If $y = \sin^{-1} /\sqrt{(1 - x^2)}$, prove that $(1 - x^2)\, dy/dx - xy = 1$.

**Solution : 2**

$y = \sin^{\mathbf{-1}} /\sqrt{(1 - x^{\mathbf{2}}}$

Differentiating both sides with respect to x we get,

$dy/dx = \sqrt{(1 - x^2)}\ d/dx(\sin^{-1}x) - \sin^{-1}x\ d/dx\{\sqrt{(1 - x^2)}\}/\{\sqrt{(1 - x^2)}\}^2$

$= \sqrt{(1 - x^2)} \times 1/\sqrt{(1 - x^2)} = \sin^{-1} x \times 1/[2\sqrt{(1 - x^2)}\ (-2x) / (1 - x^2)$

$(1 - x^2)\, dy/dx = 1 + (x \sin^{-1} x)/(\sqrt{(1 - x^2)}$

$(1 - x^2)\, dy/dx = 1 + xy$   $[y = \sin^{-1} x/\sqrt{(1 - x^2)}]$

$(1 - x^2)\, dy/dx - xy = 1$. **[Proved.]**

**Q.3.** If $y = \tan^{-1} [2x/(1 - x^2)]$ , prove that $dy/dx = 2/(1 + x^2)$.

**Solution : 3**

$y = \tan^{-1} [2x/(1 - x^2)]$ , put $x = \tan \theta$ , then $\theta = \tan^{-1} x$ .

Then , $y = \tan^{-1} [2 \tan \theta/(1 - \tan^2 \theta] = \tan^{-1} [\tan^2 \theta]$

$= 2 \theta = 2 \tan^{-1} x$

Therefore, $dy/dx = 2.[1/(1 + x^2)] = 2/(1 + x^2)$ . **[Proved.]**

**Q.4.** If $y = \sec^{-1} [(\sqrt{x} + 1)/\sqrt{x}^{-1})] + \sin^{-1} [(\sqrt{x}^{-1})/(\sqrt{x} +1)]$ , find dy/dx .

**Solution : 4**

$\sec^{-1} [(\sqrt{x} + 1)/(\sqrt{x}^{-1})] = \cos^{-1} [(\sqrt{x}^{-1})/(\sqrt{x} + 1)]$

Therefore , $y = \cos^{-1} [(\sqrt{x} - 1)/(\sqrt{x} + 1) + \sin^{-1} [(\sqrt{x} - 1)/(\sqrt{x} + 1)] = \pi/2$

Hence , $dy / dx = 0$ .

**Q.5.** If $y = \sin^{-1} \sqrt{(1 - x )} + \cos^{-1} (\sqrt{x})$ .

**Solution : 5**

Put $x = \cos^2 \theta$ , then

$y = \sin^{-1} \sqrt{( 1 - \cos^2\theta)} + \cos - 1 (\sqrt{\cos 2\theta})$

$= \sin^{-1} (\sin \theta) + \cos - 1 (\cos \theta)$

$= \theta + \theta = 2 \theta = 2 \cos - 1 (\sqrt{x})$ ,

Therefore , $dy/dx = 2 . (- 1)/\sqrt{\{1 - (\sqrt{x})\}^2} . d/dx (\sqrt{x})$

$= 2 . (- 1)/\sqrt{(1 - x )} . (- 1)/^2\sqrt{x}$

$= (- 1 )/\sqrt{\{x(1 - x )\}}$

$= (- 1 )/\sqrt{(x - x^2)}$ .

**Q.6.** If $y = \tan^{-1} [\{\sqrt{(1 + x)} - \sqrt{(1 - x)}\}/\{\sqrt{(1 + x)} + \sqrt{(1 - x)}\}]$ , find dy/dx .

**Solution : 6**

We have $y = \tan^{-1} [\{\sqrt{(1 + x)} - \sqrt{(1 - x)}\}/\{\sqrt{(1 + x)} + \sqrt{(1 - x)}\}]$ ,

Putting $x = \cos^2\theta$ , we get

$y = \tan^{-1} [\{\sqrt{(1 + \cos^2\theta)} - \sqrt{(1 - \cos^2\theta)}\}/ \{\sqrt{(1 + \cos^2\theta)} + \sqrt{(1 - \cos^2\theta)}\}]$

$= \tan^{-1} [\{\sqrt{(2\cos^2\theta)} - \sqrt{(2\sin^2\theta)}\}/\{\sqrt{(2\cos^2\theta)} + \sqrt{(2\sin^2\theta)}\}]$

$= \tan^{-1} [(\cos\theta - \sin\theta) / (\cos\theta + \sin\theta)]$

$= \tan^{-1} [(1 - \tan\theta)/(1 + \tan\theta)]$

$= \tan^{-1} [\tan(\pi/4 - \theta) ] = \pi/4 - \theta = \pi/4 - (1/2)\cos^{-1} x$,

Therefore, $dy/dx = - 1/2. (- 1)/\sqrt{(1 - x^2)} = 1/[2\sqrt{(1 - x^2)}]$ .

**Q.7.** If $y = \tan^{-1} [(\cos x + \sin x) / (\cos x - \sin x)]$ , find dy/dx.

**Solution : 7**

$y = \tan^{-1} [(\cos x + \sin x) / (\cos x - \sin x)]$

$= \tan^{-1} [(1 + \tan x) / (1 - \tan x)]$

$= \tan^{-1} [\tan (\pi/4 + x)] = \pi/4 + x$ ,

Therefore , $dy/dx = 0 + 1 = 1$ .

**Q.8.** Differentiate : $\tan^{-1} [\cos x/(1 + \sin x)]$ w. r. t. x.

**Solution : 8**

We have , $y = \tan^{-1} [\cos x/(1 + \sin x)]$

$= \tan^{-1} [\sin (\pi/2 - x)/\{1 + \cos (\pi/2 - x)\}]$

$= \tan^{-1} [\{2 \sin (\pi/4 - x/2) \cos (\pi/4 - x/2)\}/\{2 \cos^2 (\pi/4 - x/2\}]$

$= \tan^{-1} [\tan (\pi/4 - x/2)] = \pi/4 - x/2$ .

Therefore , $dy/dx = -1/2$.

**Q.9.** Differentiate : $\tan^{-1} [(1 - \cos x)/\sin x]$ w. r. t. x .

**Solution : 9**

We have , $y = \tan^{-1} [(1 - \cos x)/\sin x]$

$= \tan^{-1} [(2 \sin^2 x/2)/(2 \sin x/2 \cos x/2)]$

$= \tan^{-1} [\tan x/2] = x/2$ .

Therefore , $dy/dx = 1/2$ .

**Q.10.** Using a suitable substitution find the derivative of $\tan^{-1} [4\sqrt{x}/(1 - 4x)]$.

**Solution : 10**

Let $y = \tan^{-1} [4\sqrt{x}/(1 - 4x)]$ and let $2\sqrt{x} = \tan \theta$

Therefore, $y = \tan^{-1} [2\tan \theta/(1 - \tan^2\theta)]$

$= \tan^{-1} \tan 2\theta$

$= 2\theta$

$= 2 \tan^{-1} (2\sqrt{x})$

Then, dy/dx

$= \{2/(1 + 4x)\}(2/2\sqrt{x}) = 2/[\sqrt{x}(1 + 4x)]$.