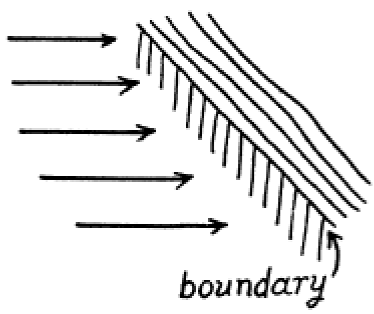
Polarization of Light (Part - 1)

Q.157. A plane monochromatic wave of natural light with intensity I_o falls normally on a screen composed of two touching Polaroid half-planes. The principal direction of one Polaroid is parallel, and of the other perpendicular, to the boundary between them. What kind of diffraction pattern is formed behind the screen? What is the intensity of light behind the screen at the points of the plane perpendicular to the screen and passing through the boundary between the Polaroids?

Ans. Natural light can be considered to be an incoherent mixture of two plane polarized light of intensity I_0/ 2 with mutually perpendicular planes of vibration. The screen cbnp sting of the two Polaroid half-planes acts as an opaque half-screen for one or the other of these light waves. The resulting diffraction pattern has the alterations in intensity (in the illuminated region) characteristic of a straight edge on both sides of the boundary. A t the boundary the intensity due to either component is



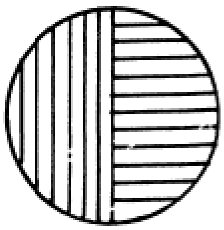
$$\frac{(I_0/\,2)}{4}$$

and the total intensity is $\frac{I_0}{4}$. (Recall that when light of intensity I_0 is incident on a straight edge, the illuminance in front of the edge is $I_0/4$) .

Q.158. A plane monochromatic wave of natural light with intensity Iofalls normally on an opaque screen with round hole corresponding to the first Fresnel zone for the observation point P. Find the intensity of light at the point P after the hole was covered with two identical Polaroids whose principal directions are mutually perpendicular and the boundary between them passes
(a) along the diameter of the hole;
(b) along the circumference of the circle limiting the first half of the Fresnel zone.

Ans. (a) Assume first that there is no Polaroid and the amplitude due to the entire hole which extends over the first Fresnel zone is A_1

Then, we know, as usual, $I_0 = \frac{A_1^2}{4}$,



When the Polaroid is introduced as shown above, each half transmits only the corresponding polarized light. If the full hole were covered by one Polaroid the amplitude transmitted w ill be $(A_1 / \sqrt{2})$.

Therefore the amplitude transmitted in the present case will be $\frac{A_1}{2\sqrt{2}}$ through either half.

Since these transmitted waves are polarized in mutually perpendicular planes, the total intensity will be

$$\left(\frac{A_1}{2\sqrt{2}}\right)^2 + \left(\frac{A_1}{2\sqrt{2}}\right)^2 = \frac{A_1^2}{4} = I_0 .$$

(b) We interpret the problem to mean that the two Polaroid pieces are separated along the circumference of the circle limiting the first half of the Fresnel zone. (This however is inconsistent with the polaroid's being identical in shape; however no other interpretation makes sense.) From (5 .103) and the previous problems we see that the amplitudes of the waves transmitted through the two parts is

$$\frac{A_1}{2\sqrt{2}}(1+i) \text{ and } \frac{A_1}{2\sqrt{2}}(1-i)$$

and the intensity is

$$\left|\frac{A_1^2}{2\sqrt{2}}(1+i)\right|^2 + \left|\frac{A_1}{2\sqrt{2}}(1-i)\right|^2$$

$$= \frac{A_1^2}{2} = 2I_0$$

Q.159. A beam of plane-polarized light falls on a polarizer which rotates about the axis of the ray with angular velocity ω = 21 rad/s. Find the energy of light passing through the polarizer per one revolution if the flux of energy of the, incident ray is equal to φ_0 = 4.0 mW.

Ans. When the polarizer rotates with angular velocity **ω** its instantaneous principal direction makes angle **ω**t from a reference direction which we choose to be along the direction of vibration of the plane polarized incident light the transmitted flux at this instant is

$$\Phi_0 \cos^2 \omega t$$

and the total energy passing through the polarizer per revolution is

$$\int_0^T \Phi_0 \cos^2 \omega t \, dt, \quad T = 2\pi/\omega$$

$$= \Phi_0 \frac{\pi}{\omega} = 0{\cdot}6 \text{ mJ}.$$

Q.160. A beam of natural light falls on a system of N = 6 Nicol prisms whose transmission planes are turned each through an angle φ = 30° with respect to that of the foregoing prism. What fraction of luminous flux passes through this system?

Ans. Let I_0 intensity of the incident beam.

Then the intensity of the beam transmitted through the first Nicol prism is

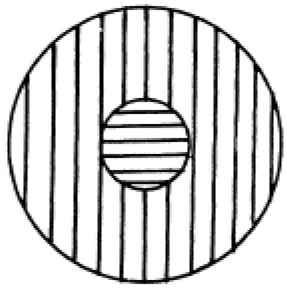
$$I_1 = \frac{1}{2} I_0$$

And through the 2nd prism is

$$I_2 = \left(\frac{1}{2} I_0\right) \cos^2 \varphi$$

Through the Nth prism it will be

$$I_N = I_{N-1} \cos^2 \varphi$$
$$= \frac{1}{2} I_0 \cos^{2(N-1)} \varphi$$



Hence fraction transmitted

$$= \frac{I_N}{I_0} = \eta = \frac{1}{2}\cos^{(2N-1)}\varphi = 0{\cdot}12 \quad \text{for} \quad N = 6\,.$$

and $\varphi = 30°$

Q.161. Natural light falls on a system of three identical in-line Polaroids, the principal direction of the middle Polaroid forming an angle φ = 60° with those of two other Polaroids. The maximum transmission coefficient of each Polaroid is equal to τ = 0.81 when plane-polarized light falls on them. How many times will the intensity of the light decrease after its passing through the system?

Ans. When natural light is incident on the first Polaroid, the fraction transmitted will be $\frac{1}{2}\tau$ (only the component polarized parallel to the principal direction of the Polaroid will go).

The emergent light will be plane polarized and on passing through the second Polaroid will be polarized in a different direction (corresponding to the principal direction of the 2nd polaroid) and the intensity will have decreased further by $\tau\cos^2\varphi$

In the third Polaroid the direction of polarization will again have to change by cp thus only a fraction $\tau\cos^2\varphi$ will go through.

Finally $I = I_0 \times \frac{1}{2}\tau^3\cos^4\varphi$

Thus the intensity will have decreased $\frac{I_o}{I} = \frac{2}{\tau^3\cos^4\varphi} = 60{\cdot}2$ times

for $\tau = 0{\cdot}81, \ \varphi = 60°$.

Q.162. The degree of polarization of partially polarized light is P = 0.25. Find the ratio of intensities of the polarized component of this light and the natural component.

Ans. Suppose the partially polarized light consists of natural light of intensity l_1 and plane polarized light of intensity I_2 with direction of vibration parallel to, say, x - axis. Then when a Polaroid is used to transmit it, the light transmitted will have a maximum intensity

$$\frac{1}{2}I_1 + I_2 ,$$

when the principal direction of the polaroid is parallel to x - axis, and will have a minimum

intensity $\frac{1}{2}I_1$ when the principal direction is $\perp^r$ to $x-$ axis.

Thus $$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{I_2}{I_1 + I_2}$$

so $$\frac{I_2}{I_1} = \frac{P}{1-P} = \frac{0{\cdot}25}{0{\cdot}75} = \frac{1}{3}.$$

Q.163. A Nicol prism is placed in the way of partially polarized beam of light. When the prism is turned from the position of maximum transmission through an angle ω = 60°, the intensity of transmitted light decreased by a factor of η = 3.0. Find the degree of polarization of incident light.

Ans. If, as above,
I_1 = intensity of natural component
I_2 = intensity of plane polarized component

Then $$I_{max} = \frac{1}{2}I_1 + I_2$$

And $$I = \frac{I_{max}}{\eta} = \frac{1}{2}I_1 + I_2 \cos^2 \varphi$$

so $$I_2 = I_{max}\left(1 - \frac{1}{\eta}\right) \operatorname{cosec}^2 \varphi$$

$$I_1 = 2I_{max}\left[1 - \left(1 - \frac{1}{\eta}\right)\operatorname{cosec}^2\varphi\right] = \frac{2I_{max}}{\sin^2\varphi}\left[\frac{1}{\eta} - \cos^2\varphi\right]$$

Then $$P = \frac{I_2}{I_1 + I_2} = \frac{1 - \frac{1}{\eta}}{2\left(\frac{1}{\eta} - \cos^2\varphi\right) + 1 - \frac{1}{\eta}} = \frac{\eta - 1}{1 - \eta\cos 2\varphi}$$

On putting $\eta = 3{\cdot}0,\ \varphi = 60°$

We get $$P = \frac{2}{1 + 3 \times \frac{1}{2}} = \frac{4}{5} = 0{\cdot}8$$

Q.164. Two identical imperfect polarizers are placed in the way of a natural beam of light. When the polarizers' planes are parallel, the system transmits η = 10.0 times more light than in the case of crossed planes. Find the degree of polarization of light produced
(a) by each polarizer separately;
(b) by the whole system when the planes of the polarizers are parallel.

Ans. Let us represent the natural light as a sum of two mutually perpendicular components, both with intensity I_0. Suppose that each polarizer transmits a fraction α_1 of the light with oscillation plane parallel to the principal direction of the polarizer and a fraction α_2 with oscillation plane perpendicular to the principal direction of the polarizer. Then the intensity of light transmitted through the two polarizers is equal to

$$I_\parallel = \alpha_1^2 I_0 + \alpha_2^2 I_0$$

when their principal direction are parallel and

$$I_\perp = \alpha_1\alpha_2 I_0 + \alpha_2\alpha_1 I_0 = 2\alpha_1\alpha_2 I_0$$

when they are crossed. But

$$\frac{I_\perp}{I_\parallel} = \frac{2\alpha_1\alpha_2}{\alpha_1^2 + \alpha_2^2} = \frac{1}{\eta}$$

So $$\frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} = \sqrt{\frac{\eta - 1}{\eta + 1}}$$

(a) Now the degree of polarization produced by either polarizer when used singly is

$$P_0 = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2}$$

(assuming, of course, $\alpha_1 > \alpha_2$)

$$P_0 = \sqrt{\frac{\eta - 1}{\eta + 1}} = \sqrt{\frac{9}{11}} = 0{\cdot}905$$

(b) When both polarizer are used with their principal directions parallel, the transmitted light, when analyzed, has

maximum intensity, $I_{max} = \alpha_1^2 I_0$ and minimum intensity, $I_{min} = \alpha_2^2 I_0$

so $$P = \frac{\alpha_1^2 - \alpha_2^2}{\alpha_1^2 + \alpha_2^2} = \frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} \cdot \frac{(\alpha_1 + \alpha_2)^2}{\alpha_1^2 + \alpha_2^2}$$

maximum intensity, $I_{max} = \alpha_1^2 I_0$ and minimum intensity, $I_{min} = \alpha_2^2 I_0$

$$P = \frac{\alpha_1^2 - \alpha_2^2}{\alpha_1^2 + \alpha_2^2} = \frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} \cdot \frac{(\alpha_1 + \alpha_2)^2}{\alpha_1^2 + \alpha_2^2}$$

$$= \sqrt{\frac{\eta - 1}{\eta + 1}} \cdot \left(1 + \frac{2\alpha_1 \alpha_2}{\alpha_1^2 + \alpha_2^2}\right)$$

$$= \sqrt{\frac{\eta - 1}{\eta + 1}} \left(1 + \frac{1}{\eta}\right) = \frac{\sqrt{\eta^2 - 1}}{\eta} = \sqrt{1 - \frac{1}{\eta^2}} = 0.995.$$

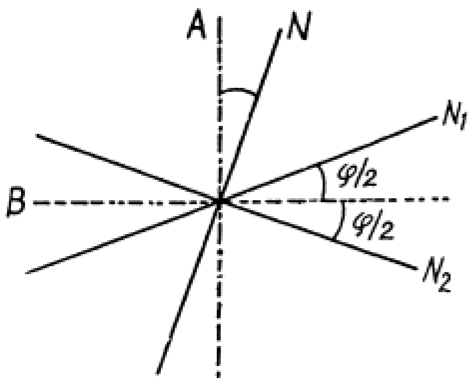
Q.165. Two parallel plane-polarized beams of light of equal intensity whose oscillation planes N_1 and N_2 form a small angle ω between them (Fig. 5.30) fall on a Nicol prism. To equalize the intensities of the beams emerging behind the prism, its principal direction N must be aligned along the bisecting line A or B. Find the value of the angle φ at which the rotation of the Nicol prism through a small angle $\delta\varphi \ll \varphi$ from the position A results in the fractional change of intensities of the beams AIII by the value $\eta = 100$ times exceeding that resulting due to rotation through the same angle from the position B.

Ans. If the principal direction N o f the Nicol is along A or B , the intensity of light transmitted is the same whether the light incident is one with oscillation plane N_1 or one with N_2. If N makes an angle $\delta\varphi$ with A as shown then the fractional difference in intensity transmitted (when the light incident is N_1 or N_2) is

$$\left(\frac{\Delta I}{I}\right)_A = \frac{\cos^2\left(90° - \frac{\varphi}{2} - \delta\varphi\right) - \cos^2\left(90° + \frac{\varphi}{2} - \delta\varphi\right)}{\cos^2\left(90° - \frac{\varphi}{2}\right)}$$

$$= \frac{\sin^2\left(\frac{\varphi}{2} + \delta\varphi\right) - \sin^2\left(\frac{\varphi}{2} - \delta\varphi\right)}{\sin^2\frac{\varphi}{2}}$$

$$= \frac{2\sin\frac{\varphi}{2}\cdot 2\cos\frac{\varphi}{2}\,\delta\varphi}{\sin^2\frac{\varphi}{2}} = 4\cot\frac{\varphi}{2}\,\delta\varphi$$



If N makes an angle $\delta\varphi$ ($<< \varphi$) with B then

$$\left(\frac{\Delta I}{I}\right)_B = \frac{\cos^2(\varphi/2 - \delta\varphi) - \cos^2(\varphi/2 + \delta\varphi)}{\cos^2\varphi/2}$$

$$= \frac{2\cos\frac{\varphi}{2}\cdot 2\sin\varphi/2\,\delta\varphi}{\cos^2\varphi/2} = 4\tan\varphi/2\,\delta\varphi$$

Thus $\eta = \left(\frac{\Delta I}{I}\right)_A \Big/ \left(\frac{\Delta I}{I}\right)_B = \cot^2\varphi/2$

or $\varphi = 2\tan^{-1}\frac{1}{\sqrt{\eta}}$

This gives $\varphi = 11{\cdot}4°$ for $\eta = 100$.

Q.166. Resorting to the Fresnel equations, demonstrate that light reflected from the surface of dielectric will be totally polarized if the angle of incidence θ_1, satisfies the condition $\tan\theta_1 = n$, where n is the refractive index of the dielectric. What is in this case the angle between the reflected and refracted rays?

$\delta\varphi \ll \varphi$

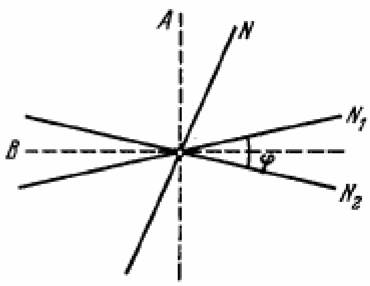


Fig. 5.30.

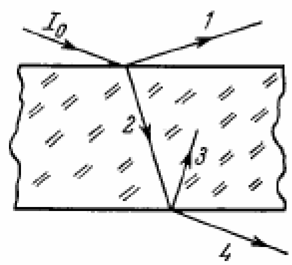


Fig. 5.31.

Ans. Fresnel equations read

$$I'_\perp = I_\perp \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \text{ and } I'_{||} = I_{||} \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)}$$

At the boundary between vacuum and a dielectric $\theta_1 \neq \theta_2$ since by Snell's law

$$\sin\theta_1 = n \sin\theta_2$$

Thus $I'_\perp / I_\perp$ cannot be zero. However, if $\theta_1 + \theta_2 = 90°$, $I'_{||} = 0$ and the reflected light is polarized in this case. The condition for this is

$$\sin\theta_1 = n\sin\theta_2\,, \quad = n\sin(90° - \theta_1)$$

or $\tan\theta_1 = n$ θ_1 is called Brewsta's angle.

The angle between reflected light and refracted light is 90° in this case.

Q.167. Natural light falls at the Brewster angle on the surface of glass. Using the Fresnel equations, find (a) the reflection coefficient; (b) the degree of polarization of refracted light.

Ans. (a) From Fresnel's equations

$$\left.\begin{aligned} I'_{\perp} &= I_{\perp}\frac{\sin^2(\theta_1-\theta_2)}{\sin^2(\theta_1+\theta_2)} \\ I'_{||} &= 0 \end{aligned}\right\} \text{at Brewste's angle}$$

$$I'_{\perp} = I_{\perp}\sin^2(\theta_1-\theta_2)$$

$$= \frac{1}{2}I(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)^2$$

Now

$$\tan\theta_1 = n,\ \sin\theta_1 = \frac{n}{\sqrt{n^2+1}}$$

$$\cos\theta_1 = \frac{1}{\sqrt{n^2+1}},\ \sin\theta_2 = \cos\theta_1$$

$$\cos\theta_2 = \sin\theta_1$$

$$I'_{\perp} = \frac{1}{2}I\left(\frac{n^2-1}{n^2+1}\right)^2$$

Thus reflection coefficient

$$= \rho = \frac{I'_{\perp}}{I}$$

$$= \frac{1}{2}\left(\frac{n^2-1}{n^2+1}\right)^2 = 0{\cdot}074$$

On putting n = 1.5

(b) For the refracted light

$$I''_{\perp} = I_{\perp} - I'_{\perp} = \frac{1}{2}I\left\{1-\left(\frac{n^2-1}{n^2+1}\right)^2\right\}$$

$$= \frac{1}{2}I\frac{4n^2}{(n^2+1)^2}$$

$$I'_{||} = \frac{1}{2}I$$

At the Brewster’s angle.

Thus the degree of polarization of the refracted light is

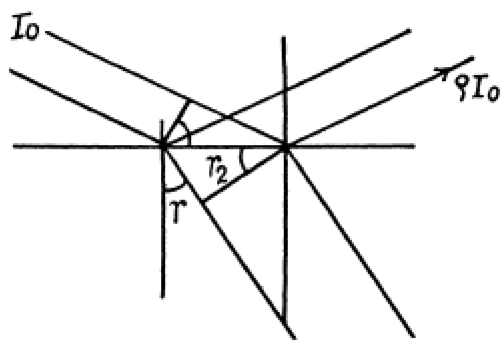
$$P = \frac{I''_{||} - I''_{\perp}}{I''_{||} + I''_{\perp}} = \frac{(n^2+1)^2 - 4n^2}{(n^2+1)^2 + 4n^2}$$

$$= \frac{(n^2-1)^2}{2(n^2+1)^2 - (n^2-1)^2} = \frac{\rho}{1-\rho}$$

On putting p = 0.074 we get P = 0.080.

Q.168. A plane beam of natural light with intensity I_0 falls on the surface of water at the Brewster angle. A fraction p = 0.039 of luminous flux is reflected. Find the intensity of the refracted beam.

Ans. he energy transmitted is, by conservation of energy, the difference between incident energy and the reflected energy. However the intensity is affected by the change of the cross section of the beam by refraction. Let A_i, A_r, A_t be the cross sections of the incident, reflected and transmitted beams. Then



$$A_i = A_r$$

$$A_t = A_i \frac{\cos r}{\cos i}$$

But at Brewster's angle $r = 90 - i$

so $\quad A_t = A_i \tan i = n A_i$

Thus $\quad I_t = \frac{(1-\rho) I_0}{n}$

Q.169. A beam of plane-polarized light falls on the surface of water at the Brewster angle. The polarization plane of the electric vector of the electromagnetic wave makes an angle λ = 45° with the incidence plane. Find the reflection coefficient.

Ans. The amplitude of the incident component whose oscillation vector is perpendicular to the plane of incidence is

$$A_\perp = A_0 \sin\varphi$$

and similarly

$$A_{||} = A_0 \cos\varphi$$

Then

$$I'_\perp = I_0 \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \sin^2\varphi$$

$$= I_0 \left[\frac{\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2}{\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2}\right]^2 \sin^2\varphi$$

$$= I_0 \left[\frac{n^2 - 1}{n^2 + 1}\right]^2 \sin^2\varphi$$

Hence

$$\rho = \frac{I'_\perp}{I_0} = \left[\frac{n^2 - 1}{n^2 + 1}\right]^2 \sin^2\varphi$$

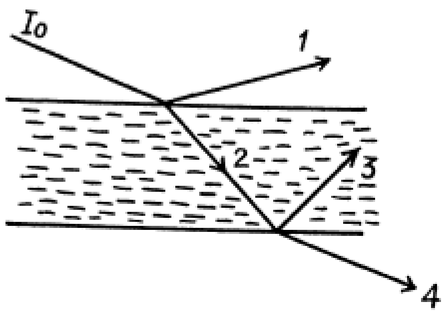
Putting n = 1.33 for water we get p = 0.0386

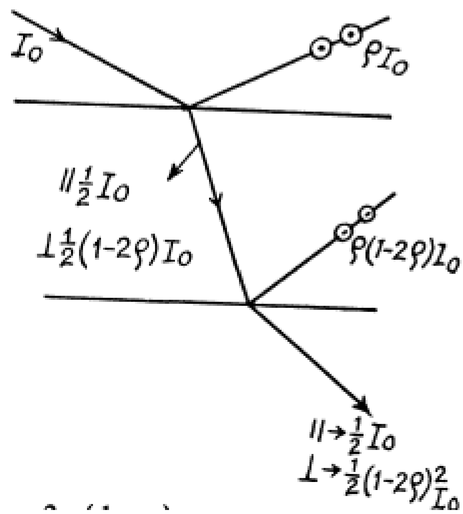
Q.170. A narrow beam of natural light falls on the surface of a thick transparent plane-parallel plate at the Brewster angle. As a result, a fraction p = 0.080 of luminous flux is reflected from its top surface. Find the degree of polarization of beams 1-4 (Fig. 5.31)

Ans. Since natural light is incident at the Brewster's angle, the reflected light 1 is completely polarized and $P_1 = 1$.
Similarly the ray 2 is incident on glass air surface at

Brewster's angle $\left(\tan^{-1}\frac{1}{n}\right)$ sc 3 is also completely polarized. Thus $P_3 = 1$
Now as in 5.167 (b)

$$P_2 = \frac{\rho}{1-\rho} = 0{\cdot}087 \quad \text{if} \quad \rho = 0{\cdot}080$$





Finally as shown in the figure

$$P_4 = \frac{\frac{1}{2} - \frac{1}{2}(1-2\rho)^2}{\frac{1}{2} + \frac{1}{2}(1-2\rho)^2} = \frac{2\rho(1-\rho)}{1-2\rho(1-\rho)} = 0{\cdot}173$$

Q.171. A narrow beam of light of intensity I_0 falls on a plane parallel glass plate (Fig. 5.31) at the Brewster angle. Using the Fresnel equations, find:
(a) the intensity of the transmitted beam 1:4 if the oscillation plane of the incident plane-polarized light is perpendicular to the incidence plane;
(b) the degree of polarization of the transmitted light if the light falling on the plate is natural.

Ans. (a) In this case from Fresnel's equations

$$I'_\perp = I_\perp \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)}$$

we get $$I_1 = \left(\frac{n^2-1}{n^2+1}\right)^2 I_0 = \rho I_0 \text{ say}$$

then $$I_2 = (1-\rho) I_0, \; I_3 = \rho(1-\rho) I_0$$

(ρ is invariant under the substitution $n \rightarrow \frac{1}{n}$)

Finally $$I_4 = (1-\rho)^2 I_0 = \frac{16 n^4}{(n^2+1)^4} I_0 = 0{\cdot}726\, I_0 .$$

(b) Suppose p' = coefficient of reflection for the component of light whose electric vector oscillates at right angles to the incidence plane.

From Fresnel's equations $\rho' = \left(\frac{n^2-1}{n^2+1}\right)^2$

Then in the transmitted beam we have a partially polarized beam which is a

superposition of two $(|| \,\&\, \perp)$ components with intensities

$$\frac{1}{2}I_0 \,\&\, \frac{1}{2}I_0(1-\rho')^2$$

Thus $$P = \frac{1-(1-\rho')^2}{1+(1-\rho')^2} = \frac{(n^2+1)^4-16n^4}{(n^2+1)^4+16n^4} = \frac{1-0{\cdot}726}{1+0{\cdot}726} \approx 0{\cdot}158$$

Q.172. A narrow beam of natural light falls on a set of N thick plane-parallel glass plates at the Brewster angle. Find: (a) the degree P of polarization of the transmitted beam; (b) what P is equal to when N = 1, 2, 5, and 10.

Ans. (a) When natural light is incident on a glass plate at Brewster's angle, the transmitted light has

$$I_{||}'' = I_0/2 \text{ and } I_{\perp}'' = \frac{16n^4}{(n^2+1)^4}I_0/2 = \alpha^4 I_0/2$$

where I_0 is the incident intensity (see 5.171 a)

After passing through the 2nd plate we find

$$I_{||}'''' = \frac{1}{2}I_0 \quad \text{and} \quad I_{|}'''' = (\alpha^4)^2\frac{1}{2}I_0$$

Thus After N Plates

$$I_{||}^{trans} = \frac{1}{2}I_0$$

$$I_{\perp}^{trans} = \alpha^{4N}\,\frac{1}{2}I_0$$

Hence $$P = \frac{1-\alpha^{4N}}{1+\alpha^{4N}} \text{ where } \alpha = \frac{2n}{1+n^2}$$

(b) $\alpha^4 = 0{\cdot}726$ for $n = \frac{3}{2}$.

Thus

$P(N = 1) = 0{\cdot}158$, $P(N = 2) = 0{\cdot}310$
$P(N = 5) = 0{\cdot}663$, $P(N = 10) = 0{\cdot}922$.

Q.173. Using the Fresnel equations, find:
(a) the reflection coefficient of natural light falling normally on the surface of glass;
(b) the relative loss of luminous flux due to reflections of a paraxial ray of natural light passing through an aligned optical system comprising five glass lenses (secondary reflections of light are to be neglected).

Ans.

(a) We decompose the natural light into two components with intensity $I_{||} = \frac{1}{2} I_0 = I_{\perp}$ where $||$ has its electric vector oscillating parallel to the plane of incidence and $\perp$ has the same $\perp^r$ to it.

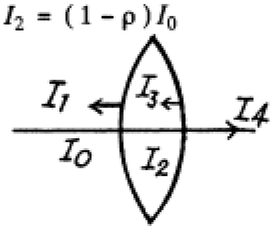
By Fresnel's equations for normal incidence

$$\frac{I'_{\perp}}{I_{\perp}} = \lim_{\theta_1 \to 0} \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} = \lim_{\theta_1 \to 0} \left(\frac{\theta_1 - \theta_2}{\theta_1 + \theta_2}\right)^2 = \left(\frac{n-1}{n+1}\right)^2 = \rho$$

Similarity $\quad \dfrac{I'_{||}}{I_{||}} = \rho = \left(\dfrac{n-1}{n+1}\right)^2$

Thus $\quad \dfrac{I'}{I} = \rho = \left(\dfrac{0{\cdot}5}{2{\cdot}5}\right)^2 = \dfrac{1}{25} = 0{\cdot}04$

(b) The reflected light at the first surface has the intensity
$I_1 = \rho I_0$
Then the transmitted light has the intensity
$I_2 = (1 - \rho) I_0$



At the second surface where light emerges from glass into air, the reflection coefficient is again p because

ρ is invariant under the substitution $n \to \frac{1}{n}$.

Thus $\quad I_3 = \rho(1 - \rho) I_0$ and $I_4 = (1 - \rho)^2 I_0$.

For N lenses the loss in luminous flux is then

$$\frac{\Delta \Phi}{\Phi} = 1 - (1 - \rho)^{2N} = 0.335 \text{ for } N = 5$$

Polarization of Light (Part - 2)

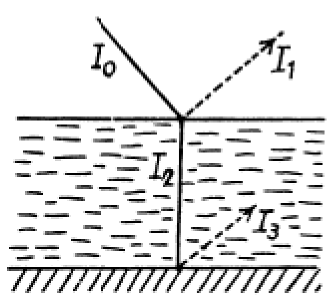
Q.174. A light wave falls normally on the surface of glass coated with a layer of transparent substance. Neglecting secondary reflections, demonstrate that the amplitudes of light waves reflected from the two surfaces of such a laver will be equal under the condi- tion $n' = \sqrt{n}$, where n' and n are the refractive indices of the layer and the glass respectively.

Ans. Suppose the incident light can be decomposed into waves with intensity $I_{||}$ & $I_{\perp}$ with oscillations of the electric vectors parallel and perpendicular to the plane of incidence.
For normal incidence we have from Fresnel equations

$$I_{\perp}' = I_{\perp}\left(\frac{\theta_1 - \theta_2}{\theta_1 + \theta_2}\right)^2 \longrightarrow I_{\perp}\left(\frac{n-1}{n+1}\right)^2$$

where we have used $\sin\theta \approx \theta$ for small θ.

Similarly $I'_{||} = I_{||}\left(\frac{n'-1}{n'+1}\right)^2$



Then the refracted wave will be

$$I''_{||} = I_{||}\frac{4n'}{(n'+1)^2} \quad \text{and} \quad I_{\perp}'' = I_{\perp}\frac{4n'}{(n'+1)^2}$$

At the interface with glass

$$I_{\perp}''' = I_{\perp}''\left(\frac{n'-n}{n'+n}\right)^2, \text{ similarly for } I_{||}'''$$

we see that

$$\frac{I_\perp'}{I_\perp} = \frac{I_\perp'''}{I_\perp''} \quad \text{if} \quad n' = \sqrt{n}, \text{ similarly for } || \text{ component.}$$

This shows that the light reflected as a fraction of the incident light is the same on the two surfaces if $n' = \sqrt{n}$.

Note:- The statement of the problem given in the book is incorrect. Actual amplitudes are not equal; only the reflectance is equal.

Q.175. A beam of natural light falls on the surface of glass at an angle of 45°. Using the Fresnel equations, find the degree of polarization of
(a) reflected light;
(b) refracted light.

Ans. Here

$\theta_1 = 45°$

$$\sin\theta_2 = \frac{1}{\sqrt{2}} \times \frac{1}{n} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3} = 0{\cdot}4714$$

$$\theta_2 = \sin^{-1} 0{\cdot}4714 = 28{\cdot}1°$$

Hence

$$I'_\perp = I_\perp \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)}$$

$$= \frac{1}{2} I_0 \left(\frac{\sin 16{\cdot}9°}{\sin 73{\cdot}1°}\right)^2 = \frac{1}{2} I_0 \times 0{\cdot}0923$$

Thus $$I'_{||} = \frac{1}{2} I_0 \left(\frac{\tan 16{\cdot}9}{\tan 73{\cdot}1}\right)^2 = \frac{1}{2} I_0 \times 0{\cdot}0085$$

(1) Degree of polarization P of the reflected light $= \frac{0{\cdot}0838}{0{\cdot}1008} = 0{\cdot}831$

(b) By conservation of energy

$$I_{\perp}'' = \frac{1}{2} I_0 \times 0{\cdot}9077$$

$$I_{||}'' = \frac{1}{2} I_0 \times 0{\cdot}9915$$

Thus $$P = \frac{0{\cdot}0838}{1{\cdot}8982} = 0{\cdot}044$$

Q.176. Using Huygens's principle, construct the wave fronts and the propagation directions of the ordinary and extraordinary rays in a positive uniaxial crystal whose optical axis
(a) is perpendicular to the incidence plane and parallel to the surface of the crystal;
(b) lies in the incidence plane and is parallel to the surface of the crystal;

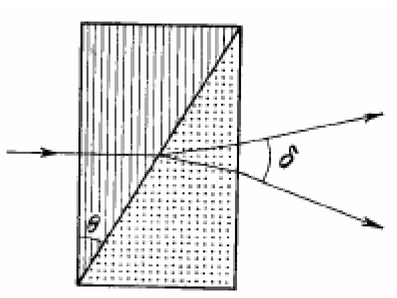
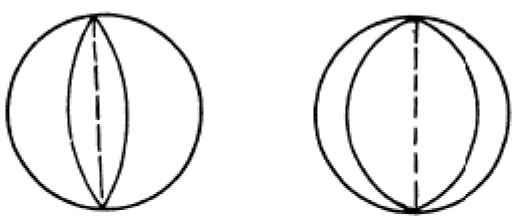


Fig. 5.32.

(c) lies in the incidence plane at an angle of 45° to the surface of the crystal, and light falls at right angles to the optical axis.

Ans. The wave surface of a uniaxial crystal consists of two sheets of which one is a sphere while the other is an ellipsoid of revolution.

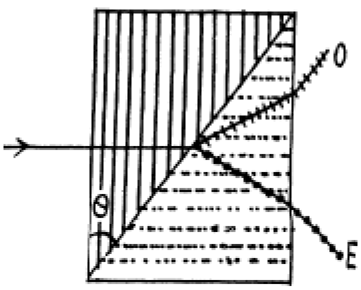


The optic axis is the line joining the points of contact.
To makes the appropriate Huygens's construction we must draw the relevant section of the wave surface inside the crystal and determine the directions of the ordinary and extraordinary rays. The result is as shown in Fig. 42 (a, b & c) of the answers

Q.177. A narrow beam of natural light with wavelength λ = 589 nm falls normally on the surface of a Wollaston polarizing prism made of Iceland spar as shown in Fig. 5.32. The optical axes of the two parts of the prism are mutually

perpendicular. Find the angle 8 between the directions of the beams behind the prism if the angle 0 is equal to 30°.

Ans. In a uniaxial crystal, an unpolarized beam of light (or even a polarized one) splits up into O (for ordinary) and E (for extraordinary) light waves. The direction of vibration in the O and E waves are most easily specified in terms of the O and E principal planes. The principal plane of the ordinary wave is defined as the plane containing the O ray and the optic axis. Similarly the principal plane of the E wave is the plane containing the E ray and the optic axis.
In terms of these planes the following is true : The O vibrations are perpendicular to the principal plane of the O ray while the E vibrations are in the principal plane of the E ray.



When we apply this definition to the Wollaston prism we find the following :
When unpolarized light enters from the left the O and E waves travel in the same direction but with different speeds. The O ray on the left has its vibrations normal to the plane of the paper and it becomes E ray on crossing the diagonal boundary of the two prism similarly the E ray on the left becomes O ray on the right In this case Snell’s law is applicable only approximately. The two rays are incident on the boundary at an angle θ and in the right prism the ray which we have called O ray on the right emerges at

$$\sin^{-1}\frac{n_e}{n_0}\sin\theta = \sin^{-1}\frac{1{\cdot}658}{1{\cdot}486}\times\frac{1}{2} = 33{\cdot}91°$$

where we have used

$$n_e = 1{\cdot}1658\,,\ n_0 = 1{\cdot}486 \text{ and } \theta = 30°.$$

Similarly the E ray on the right emerges within the prism at

$$\sin^{-1}\frac{n_0}{n_e}\sin\theta = 26{\cdot}62°$$

This means that the O ray is incident at the boundary between the prism and air at

$$33{\cdot}91 - 30° = 3{\cdot}91°$$

and will emerge into air with a deviation of

$$\sin^{-1} n_0 \sin 3{\cdot}91°$$

$$= \sin^{-1}(1{\cdot}658 \sin 3{\cdot}91°) = 6{\cdot}49°$$

The E ray will emerge with an opposite deviation of

$$\sin^{-1}(n_e \sin(30° - 26{\cdot}62°))$$

$$= \sin^{-1}(1{\cdot}486 \sin 3{\cdot}38°) = 5{\cdot}03°$$

Hence $\delta = 6{\cdot}49° + 5{\cdot}03° = 11{\cdot}52°$

This result is accurate to first order in $(n_e - n_0)$ because Snell's law holds when $n_e = n_0$.

Q.178. What kind of polarization has a plane electromagnetic wave if the projections of the vector E on the x and y axes are perpendicular to the propagation direction and are defined by the following equations:

(a) $E_x = E\cos(\omega t - kz)$, $E_y = E\sin(\omega t - kz)$;
(b) $E_x = E\cos(\omega t - kz)$, $E_y = E\cos(\omega t - kz + \pi/4)$;
(c) $E_x = E\cos(\omega t - kz)$, $E_y = E\cos(\omega t - kz + \pi)$?

Ans. The wave is moving in the direction of z – axis

(a) Here $E_x = E\cos(\omega t - kz)$, $E_y = E\sin(\omega t - kz)$

$$\frac{E_x^2}{E^2} + \frac{E_y^2}{E^2} = 1$$

so the of the electric vector moves along a circle. For the right handed coordinate system this represents circular anticlockwise polarization when observed towards the incoming wave

(b) $E_x = E\cos(\omega t - kz)$, $E_y = E\cos\left(\omega t - kz + \frac{\pi}{4}\right)$

so $$\frac{E_y}{E} = \frac{1}{\sqrt{2}}\cos(\omega t - kz) - \frac{1}{\sqrt{2}}\sin(\omega t - kz)$$

or $$\left(\frac{E_y}{E} - \frac{1}{\sqrt{2}}\frac{E_x}{E}\right)^2 = \frac{1}{2}\left(1 - \frac{E_x^2}{E^2}\right)$$

or $$\frac{E_y^2}{E^2} + \frac{E_x^2}{E^2} - \sqrt{2}\,\frac{E_y E_x}{E^2} = \frac{1}{2}$$

This is clearly an ellipse. By comparing with the previous case (compare the phase of E_y in the two cases) we see this represents elliptical clockwise polarization when

viewed towards the incoming wave.
We write the equations as

$$E_x + E_y = 2E\cos\left(\omega t - kz + \frac{\pi}{8}\right)\cos\frac{\pi}{8}$$

$$E_x - E_y = +2E\sin\left(\omega t - kz + \frac{\pi}{8}\right)\sin\frac{\pi}{8}$$

Thus $$\left(\frac{E_x + E_y}{2E\cos\frac{\pi}{8}}\right)^2 + \left(\frac{E_x - E_y}{2E\sin\frac{\pi}{8}}\right)^2 = 1$$

Since $\cos\frac{\pi}{8} > \sin\frac{\pi}{8}$, the major axis is in the direction of the straight line y = x.

(c) $E_x = E\cos(\omega t - kz)$

$E_y = E\cos(\omega t - kz + \pi) = -E\cos(\omega t - kz)$

Thus the top of the electric vector traces the curve

$$E_y = -E_x$$

which is a straight line ($y = -x$). It corresponds to plane polarization.

Q.179. One has to manufacture a quartz plate cut parallel to its optical axis and not exceeding 0.50 mm in thickness. Find the maximum thickness of the plate allowing plane-polarized light with wavelength λ = 589 nm
(a) to experience only rotation of polarization plane;
(b) to acquire circular polarization after passing through that plate.

Ans.

For quartz

$$\left.\begin{aligned} n_e &= 1{\cdot}553 \\ n_0 &= 1{\cdot}544 \end{aligned}\right\} \text{ for } \lambda = 589\text{ nm}.$$

In a quartz plate cut parallel to its optic axis, plane polarized light incident normally from the left divides itself into O and E waves which move in the same direction with different speeds and as a result acquire a phase difference. This phase difference is

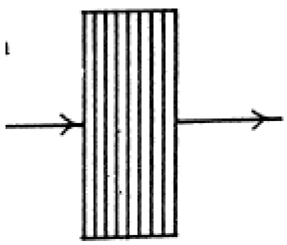
$$\delta = \frac{2\pi}{\lambda}(n_e - n_0)\,d$$

where d = thickness of the plate. In general this makes the emergent light elliptically polarized.
(a) For emergent light to experience only rotation of polarization plane

$\delta = (2k+1)\pi, \ k = 0, 1, 2, 3 \ldots$

For this $d = (2k+1)\dfrac{\lambda}{2(n_e - n_0)}$

$= (2k+1)\dfrac{\cdot 589}{2 \times \cdot 009}\mu\text{m} \quad = (2k+1)\dfrac{\cdot 589}{18} m\text{m}$



The maximum value of (2k+1) for which this is

less than 0·50 is obtained from

$$\frac{0\cdot 50 \times 18}{0\cdot 589} = 15\cdot 28$$

Then we must take $k = 7$ and $d = 15 \times \dfrac{\cdot 589}{18} = 0\cdot 4908$ mm

(b) For circular polarization $\delta = \dfrac{\pi}{2}$

Modulo 2π i.e. $\delta = (4k+1)\dfrac{\pi}{2}$

So $d = (4k+1)\dfrac{\lambda}{4(n_e - n_0)} = (4k+1)\dfrac{0\cdot 589}{36}$

Now $\dfrac{0\cdot 50 \times 36}{0\cdot 589} = 30\cdot 56$

The nearest integer less than this which is of the form $4k+1$ is 29 for $k = 7$. For this d = 0.4749 mm

Q.180. A quartz plate cut parallel to the optical axis is placed between two crossed Nicol prisms. The angle between the principal directions of the Nicol prisms and the plate is equal to 45°. The thickness of the plate is d = 0.50 mm. At what wavelengths in the interval from 0.50 to 0.60 µm is the intensity of light which passed through that system independent of rotation of the rear prism? The difference of refractive indices for ordinary and extraordinary rays in that wavelength interval is assumed to be Δn = 0.0090.

Ans. As in the previous problem the quartz plate introduces a phase difference δ between the O & E components. When $\delta = \pi/2 \text{ (modulo } \pi \text{)}$ the resultant wave is circularly polarized.

In this case intensity is independent of the rotation of the rear prism. Now

$$\begin{aligned}\delta &= \frac{2\pi}{\lambda}(n_e - n_0)\,d \\ &= \frac{2\pi}{\lambda} 0{\cdot}009 \times 0{\cdot}5 \times 10^{-3}\,\text{m} \\ &= \frac{9\pi}{\lambda},\ \lambda \text{ in } \mu\text{m}\end{aligned}$$

For $\lambda = 0{\cdot}50\,\mu m$. $\delta = 18\pi$. The relevant values of δ have to be chosen in the form

$\left(k + \frac{1}{2}\right)\pi$. For $k = 17, 16, 15$ we get

$\lambda = 0{\cdot}5143\,\mu m,\ 0{\cdot}5435\,\mu\text{m}$ and $0{\cdot}5806\,\mu\text{m}$

These are the values of δ which lie between 0.50µm and 0.60µm .

Q.181. White natural light falls on a system of two crossed Nicol prisms having between them a quartz plate 1.50 mm thick, cut parallel to the optical axis. The axis of the plate forms an angle of 45° with the principal directions of the Nicol prisms. The light transmitted through that system was split into the spectrum. How many dark fringes will be observed in the wavelength interval from 0.55 to 0.66 µm? The difference of refractive indices for ordinary and extraordinary rays in that wavelength interval is assumed to be equal to 0.0090.

Ans. As in the previous two problems the quartz plate will introduce a phase difference

δ. The light on passing through the plate will remain plane polarized only for the latter case the plane of polarization of the light incident on the plate will be rotated by 90° by it so light passing through the analyzer (which was originally crossed) will be a maximum. Thus dark bands will be observed only for those λ for which

$$\delta = 2k\pi$$

Now $\delta = \frac{2\pi}{\lambda}(n_e - n_0)\,d \doteq \frac{2\pi}{\lambda} \times {\cdot}009 \times 1{\cdot}5 \times 10^{-3}\,\text{m}$

$$= \frac{27\pi}{\lambda}\ (\lambda \text{ in } \mu\text{m})$$

For $\lambda = 0{\cdot}55$ we get $\delta = 49{\cdot}09\,\pi$

Choosing $\delta = 48\pi,\ 46\pi,\ 44\pi,\ 42\pi$ we get $\lambda = 0{\cdot}5625\ \mu m$, $\lambda = 0{\cdot}5870\ \mu m$, $\lambda = 0{\cdot}6136\ \mu m$ and $\lambda = 0{\cdot}6429\ \mu m$. These are the only values between $0{\cdot}55\ \mu m$ and $0{\cdot}66\ \mu m$. Thus there are four bands.

Q.182. A crystalline plate cut parallel to its optical axis is 0.25 mm thick and serves as a quarter-wave plate for a wavelength λ = 530 nm. At what other wavelengths of visible spectrum will it also serve as a quarter-wave plate? The difference of refractive indices for extraordinary and ordinary rays is assumed to be constant and equal to $n_e - n_o = 0.0090$ at all wavelengths of the visible spectrum.

Ans. Here

$$\delta = \frac{2\pi}{\lambda} \times 0{\cdot}009 \times 0{\cdot}25\ \text{m}$$

$$= \frac{4{\cdot}5\,\pi}{\lambda},\ \lambda \text{ in } \mu\text{m}.$$

We check that for

$$\lambda = 428{\cdot}6\ \text{nm} \quad \delta = 10{\cdot}5\,\pi$$
$$\lambda = 529{\cdot}4\ \text{nm} \quad \delta = 8{\cdot}5\,\pi$$
$$\lambda = 692{\cdot}3\ \text{nm} \quad \delta = 6{\cdot}5\,\pi$$

These are the only values of λ, for which the plate acts as a quarter wave plate.

Q.183. A quartz plate cut parallel to its optical axis is placed between two crossed Nicol prisms so that their principle directions form an angle of 45° with the optical axis of the plate. What is the minimum thickness of that plate transmitting light of wavelength λ_1 = 643 nm with maximum intensity while greatly reducing the intensity of transmitting light of wavelength λ_2 = 564 nm? The difference of refractive indices for extraordinary and ordinary rays is assumed to be equal to $n_e - n_o = 0.0090$ for both wavelengths.

Ans. Between crossed Nicols, a quartz plate, whose optic axis makes 45° with the principal directions of the Nicols, must introduce a phase difference of (2k + 1) ji so as to transmit the incident light (of that wavelength) with maximum intensity. For in this case the plane of polarization of the light emerging from the polarizer will be rotated by 90° and will go through the analyzer undiminished. Thus we write for light of wavelengths 643 nm

$$\delta = \frac{2\pi \times 0{\cdot}009}{0{\cdot}643 \times 10^{-6}} \times d\,(\text{mm}) \times 10^{-3}$$

$$= \frac{18\,\pi\, d}{0{\cdot}643} = (2k+1)\,\pi \qquad (1)$$

To nearly block light of wavelength 564 nm we require

$$\frac{18\pi d}{0\cdot564} = (2k')\pi \quad (2)$$

We must have $2k' > 2k+1$. For the smallest value of d we take $2k' = 2k+2$.

Thus $\quad 0\cdot643\,(2k+1) = 0\cdot564 \times (2k+2)$

so $\quad 0\cdot079 \times 2k = 0\cdot564 \times 2 - 0\cdot643$

or $\quad 2k = 6\cdot139$

This is not quite an integer but is close to one. This means that if we take 2 k = 6 equations (1) can be satisfied exactly while equation (2) will hold approximately. Thus

$$d = \frac{7 \times 0\cdot643}{18} = 0\cdot250 \text{ mm}$$

Q.184. A quartz wedge with refracting angle $\Theta = 3.5°$ is inserted between two crossed Polaroids. The optical axis of the wedge is parallel to its edge and forms an angle of 45° with the principal directions of the Polaroids. On transmission of light with wavelength λ = 550 nm through this system, an interference fringe pattern is formed. The width of each fringe is Δx = 1.0 mm. Find the

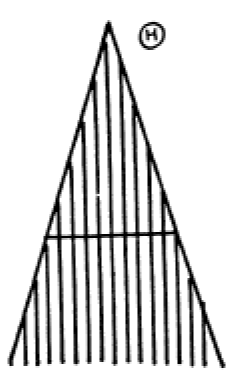
$$I = \frac{1}{2} I_0 [\cos^2(\varphi - \varphi') - \sin 2\varphi \cdot \sin 2\varphi' \ \sin^2(\delta/2)],$$

Difference of refractive indices of quartz for ordinary and extraordinary rays at the wavelength indicated above.

Ans. If a ray traverses the wedge at a distance x below the joint, then the distance that the ray moves in the wedge is

$2x \tan\frac{\Theta}{2}$ and this cause a phase difference

$$\delta = \frac{2\pi}{\lambda}(n_e - n_0)\, 2x \tan\frac{\Theta}{2}$$



between the E and O wave components of the ray. For a general x the resulting light is elliptically polarized and is not completely quenched by the analyzer polaroid. The condition for complete quenching is

$$\delta = 2k\pi \text{— dark fringe}$$

That for maximum brightness is

$$\delta = (2k+1)\pi - \text{bright fringe.}$$

The fringe width is given by

$$\Delta x = \frac{\lambda}{2(n_e - n_0)\tan\frac{\Theta}{2}}$$

Hence $$(n_e - n_0) = \frac{\lambda}{2\,\Delta x \tan\Theta/2}$$

using $$\tan(\Theta/2) = \tan 175° = 0{\cdot}03055\,,$$

$$\lambda = 0{\cdot}55\,\mu m \quad \text{and} \quad \Delta x = 1\,mm\text{, we get}$$

$$n_e - n_0 = 9{\cdot}001 \times 10^{-3}$$

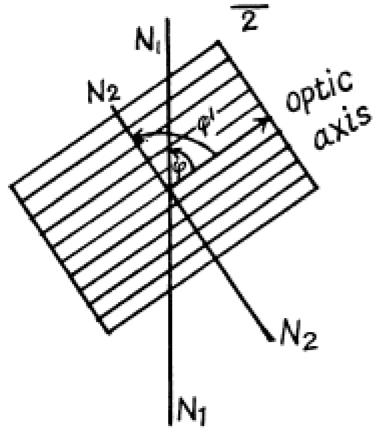
Q.185. Natural monochromatic light of intensity I_0 falls on a system of two Polaroids between which a crystalline plate is inserted, cut parallel to its optical axis. The plate introduces a phase difference δ between the ordinary and extraordinary rays. Demonstrate that the intensity of light transmitted through that system is equal to

$$I = I_0\,(1 + \sin 2\varphi \cdot \sin \delta),$$

Where φ and φ' are the angles between the optical axis of the crystal and the principal directions of the Polaroids. In particular, consider the cases of crossed and parallel Polaroids.

Ans. Light emerging from the first polaroid is plane polarized with amplitude A where N_1 is the principal direction of the polaroid and a vibration of amplitude can be resolved into two vibration: E wave with vibration along the optic axis of amplitude A cosφ and the O wave with vibration perpendicular to the optic axis and having an amplitude A sinφ. These acquire a phase difference δ on passing through the plate. The second Polaroid transmits the components

~~we get~~ $A^2 = \frac{I_0}{2}$



$$A \cos\varphi \cos\varphi'$$

and $$A \sin\varphi \sin\varphi'$$

What emerges from the second Polaroid is a set of two plane polarized waves in the same direction and same plane of polarization but phase difference δ. They interfere and produce a wave of amplitude squared

$$R^2 = A^2\left[\cos^2\varphi\cos^2\varphi' + \sin^2\varphi\sin^2\varphi' + 2\cos\varphi\cos\varphi'\sin\varphi\sin\varphi'\cos\delta\right],$$

using $$\cos^2(\varphi - \varphi') = (\cos\varphi\cos\varphi' + \sin\varphi\sin\varphi')^2$$
$$= \cos^2\varphi\cos^2\varphi' + \sin^2\varphi\sin^2\varphi' + 2\cos\varphi\cos\varphi'\sin\varphi\sin\varphi'$$

we easily find

$$R^2 = A^2\left[\cos^2(\varphi - \varphi') - \sin 2\varphi \sin 2\varphi' \sin^2\frac{\delta}{2}\right]$$

Now $A^2 = I_0/2$ and $R^2 = I$ so the result is

$$I = \frac{1}{2}I_0\left[\cos^2(\varphi - \varphi') - \sin 2\varphi\sin 2\varphi'\sin^2\frac{\delta}{2}\right]$$

Special cases :- Crossed polaroids : Here $\varphi - \varphi' = 90°$ or $\varphi' = \varphi - 90°$ and $2\varphi' = 2\varphi - 180°$

Thus in this case

$$I = I_{\perp} = \frac{1}{2}I_0 \sin^2 2\varphi \sin^2\frac{\delta}{2}$$

Parallel Polaroid’s:

Here $\varphi = \varphi'$ and

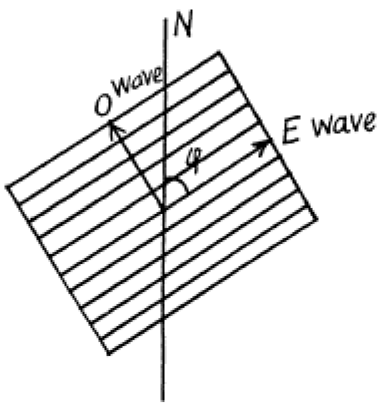
$$I = I_{||} = \frac{1}{2} I_0 \left(1 - \sin^2 2\varphi \sin^2 \frac{\delta}{2} \right)$$

With $\delta = \frac{2\pi}{\lambda} \Delta$, the conditions for the maximum and minimum are easily found to be that shown in the answer.

Polarization of Light (Part - 3)

Q.186. Monochromatic light with circular polarization falls normally on a crystalline plate cut parallel to the optical axis. Behind the plate there is a Nicol prism whose principal direction forms an angle φ with the optical axis of the plate. Demonstrate that the intensity of light transmitted through that system is equal to Where δ is the phase difference between the ordinary and extraordinary rays which is introduced by the plate.

Ans. Let the circularly polarized light be resolved into plane polarized components of amplitude A_0 with a phase



Difference $\frac{\pi}{2}$ between then

On passing through the crystal the phase difference becomes $\delta + \frac{\pi}{2}$ and the components

of the E and O wave in the direction N are respectively $A_0 \cos \varphi$ and $A_0 \sin \varphi$

They interfere to produce the amplitude squared

$$R^2 = A_0^2 \cos^2 \varphi + A_0^2 \sin^2 \varphi + 2 A_0^2 \cos \varphi \sin \varphi \cos \left(\delta + \frac{\pi}{2}\right)$$

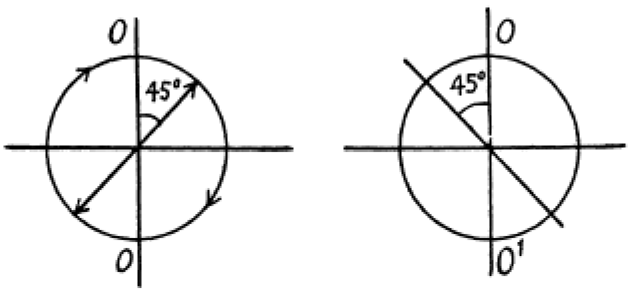
$$= A_0^2 (1 + \sin 2\varphi \sin \delta)$$

Hence $$I = I_0 (1 + \sin 2\varphi \sin \delta)$$

Here I_0 is the intensity of die light transmitted by the Polaroid when there is no crystal plate

Q.187. Explain how, using a Polaroid and a quarter-wave plate made of positive uniaxial crystal $(n_e > n_o)$, o distinguish
(a) light with left-hand circular polarization from that with right-hand polarization;
(b) natural light from light with circular polarization and from the composition of natural light and that with circular polarization.

Ans. (a) The light with right circular polarization (viewed against the oncoming light, this means that the light vector is moving clock wise.) becomes plane polarized on passing through a quarter-wave plate. In this case the direction of oscillations of the electric vector of the electromagnetic wave forms an angle of + 45° with the axis of the crystal OO' (see Fig (a) below). In the case of left hand circular polarizations, this angle will be - 45° (Fig (b).



(b) If for any position of the plate the rotation of the Polaroid (located behind the plate) does not bring about any variation in the intensity of the transmitted light, the incident legit is unpolarized (i.e. natural). If the intensity of the transmitted light can drop to zero on rotating the analyzer Polaroid for some position of the quarter wave plate, the incident light is circularly polarized. If it varies but does not drop to zero, it must be a mixture of natural and circularly polarized light.

Q.188. Light with wavelength λ falls on a system of crossed polarizer P and analyzer A between which a Babinet compensator C

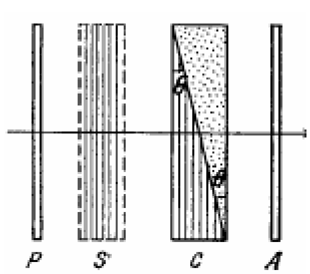
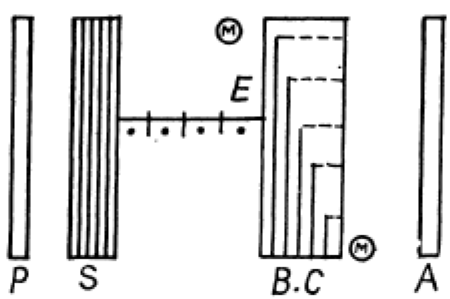


Fig. 5.33.

is inserted (Fig. 5.33). The compensa- tor consists of two quartz wedges with the optical axis of one of them being parallel to the edge, and of the other,

perpendicular to it. The principal direc- tions of the polarizer and the analyzer form an angle of 45° with the optical axes of the compensator. The refracting angle of the wedges is equal to Θ $(\Theta \ll 1)$
the difference of refractive indices of quartz is $n_e - n_o$. The insertion of investigated birefringent sample S, with the optical axis oriented as shown in the figure, results in displacement of the fringes upward by δx mm. Find:
(a) the width of the fringe Δx;
(b) the magnitude and the sign of the optical path difference of ordinary and extraordinary rays, which appears due to the sample S.

Ans. The light from P is plane polarized with its electric vector vibrating at 45° with the plane of the paper. At first the sample S is absent Light from P can be resolved into components vibrating in and perpendicular to the plane of the paper. The former is the E ray in the left half of the Babinet compensator and the latter is the O ray. In the right half the nomenclature is the opposite. In the compensator the two components acquire a pahse difference which depends on the relative position of the ray. If the ray is incident at a distance x above the central line through the compensator then the E ray acquires a phase



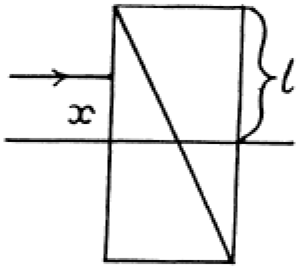
$$\frac{2\pi}{\lambda}\left(n_E(l-x)+n_0(l+x)\right)\tan\Theta$$

while the O ray acquires

$$\frac{2\pi}{\lambda}\left(n_o(l-x)+n_E(l+x)\right)\tan\Theta$$

so the phase difference between the two reays is

$$\frac{2\pi}{\lambda}(n_o-n_E)\,2x\tan\Theta = \delta$$



we get dark fringes when ever $\delta = 2k\pi$
because then the emergent light is the same as that coming from the polarizer and is quenched by the analyser. {If $\delta = (2k+1)\pi$, we get bright fringes because in this case, the plane of polarizaton of the emergent hight has rotated by 90° and is therefore fully transmitted by the analyser.} If follows that the fringe width

Δx is given by

$$\Delta x = \frac{\lambda}{2\,|n_0 - n_E|\tan\Theta}$$

(b) If the fringes are displaced upwards by δx, then the path difference introduced by the sample between the O and the E rays must be such as to be exactly cancelled by the compensator. Thus

$$\frac{2\pi}{\lambda}\left[d(n'_O - n'_E) + (n_E - n_O)\,2\,\delta x \tan\Theta\right] = 0$$

or

$$d(n'_O - n'_E) = -2(n_E - n_O)\,\delta x\,\Theta$$

using $\tan\Theta \approx \Theta$.

Q.189. Using the tables of the Appendix, calculate the difference of refractive indices of quartz for light of wavelength λ = 589.5 nm with right-hand and left-hand circular polarizations.

Ans. Light polarized along the x-direction (i.e. one whose electric vector has only an x component) and propagating along the z-direction can be decomposed into left and right circularly polarized light in accordance with the formula

$$E_x = \frac{1}{2}(E_x + iE_y) + \frac{1}{2}(E_x - iE_y)$$

On passing through a distance l of an active medium these acquire the phases $\delta_R = \frac{2\pi}{\lambda} n_R\, l$

and $\delta_l = \frac{2\pi}{\lambda} n_l l$ so we get for the complex amplitude

$$E' = \frac{1}{2}(E_x + iE_y)e^{i\delta_R} + \frac{1}{2}(E_x - iE_y)e^{i\delta_L}$$
$$= e^{i\frac{\delta_R+\delta_l}{2}}\left[\frac{1}{2}(E_x + iE_y)e^{i\delta/2} + \frac{1}{2}(E_x - iE_y)e^{-i\delta/2}\right]$$

$$= e^{i\frac{\delta_R+\delta_l}{2}}\left[E_x \cos\frac{\delta}{2} - E_y \sin\frac{\delta}{2}\right], \ \delta = \delta_R - \delta_L.$$

Apart from an over all phase $(\delta_R + \delta_l)/2$ (which is irrelevant) this represents a wave whose plane of polarization has rotated by

$$\frac{\delta}{2} = \frac{\pi}{\lambda}(\Delta n)l, \quad \Delta n = |n_R - n_l|$$

By definition this equals αl so

$$\Delta n = \frac{\alpha\lambda}{\pi}.$$
$$= \frac{589{\cdot}5 \times 10^{-6}\ \text{m m} \times 21{\cdot}72\ \text{deg/m m}}{\pi} \times \frac{\pi}{180}(\text{rad})$$
$$= \frac{{\cdot}5895 \times 21{\cdot}72}{180} \times 10^{-3}$$
$$= 0{\cdot}71 \times 10^{-4}$$

Q.190. Plane-polarized light of wavelength 0.59 ton falls on a trihedral quartz prism P (Fig. 5.34) with refracting angle $\Theta = 30^\circ$. . Inside the prism light propagates along the optical axis whose direction is shown by hatching.

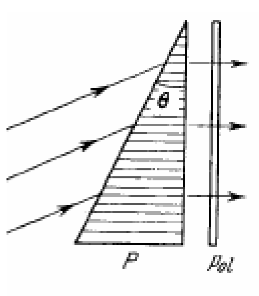
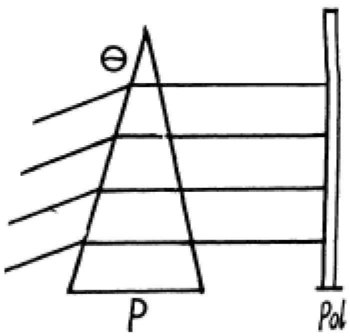


Fig. 5.34.

Behind the Polaroid Pol an interference pattern of bright and dark fringes of width Δx = 15.0 mm is observed. Find the specific rotation constant of quartz and the distribution of intensity of light behind the Polaroid.

Ans. Plane polarized light on entering the wedge decomposes into right and left circularly polarized light which travel with different speeds in P and the emergent light gets its plane of polarization rotated by an angle which depends on the distance travelled.

Given that Δ x = fringe width



$\Delta x \tan\theta =$ difference in the path length traversed by two rays which form successive bright or dark fringes.

Thus $$\frac{2\pi}{\lambda}|n_R - n_l|\Delta x \tan\theta = 2\pi$$

Thus $$\alpha = \frac{\pi \Delta n}{\lambda} = \pi/\Delta x \tan\theta$$

Let x = distance on the polaroid Pol as measured from a maximum. Then a ray that falls at this distance traverses an extra distance equal to

$$\pm x \tan\theta$$

and hence a rotation of $$\pm \alpha x \tan\theta = \pm \frac{\pi x}{\Delta x}$$

By Malus' law the intensity at this point will be $\sim \cos^2\left(\frac{\pi x}{\Delta x}\right)$.

Q.191. Natural monochromatic light falls on a system of two crossed Nicol prisms between which a quartz plate cut at right angles to its optical axis is inserted. Find the minimum thickness of the plate at which this system will transmit η = 0.30 of luminous flux if the specific rotation constant of quartz is equal to α= 17 ang.deg/mm.

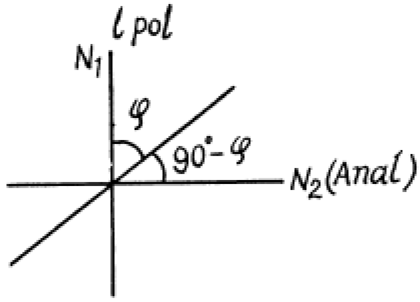
Ans. If I_0 intensity of natural light then

$\frac{1}{2}I_0 =$ intensity of light emerging from the polarizer nicol.

Suppose the quartz plate rotates this light by φ, then the analyser will transmit

$$\frac{1}{2} I_0 \cos^2 (90 - \varphi)$$

$$= \frac{1}{2} I_0 \sin^2 \varphi$$



of this intensity. Hence $\quad \eta I_0 = \frac{1}{2} I_0 \sin^2 \varphi$

or $\quad \varphi = \sin^{-1} \sqrt{2\eta}$

But $\quad \varphi = \alpha d$ so

$$d_{min} = \frac{1}{\alpha} \sin^{-1} \sqrt{2\eta}$$

For minimum d we must take the principal value of inverse sine. Thus using $\alpha = 17$ ang deg/m m.

$$d_{min} = 2 \cdot 99 \text{ m m}.$$

Q.192. Light passes through a system of two crossed Nicol prisms between which a quartz plate cut at right angles to its optical axis is placed. Determine the minimum thickness of the plate which allows light of wavelength 436 nm to be completely cut off by the system and transmits half the light of wavelength 497 nm. The specific rotation constant of quartz for these wavelengths is equal to 41.5 and 31.1 angular degrees per mm respectively.

Ans. For light of wavelength 436 nm

$$41 \cdot 5° \times d = k \times 180° = 2k \times 90°$$

(Light will be completely cut off when the quartz plate rotates the plane of polarization by a multiple of 180°.) Here d = thickness of quartz plate in mm.

For natural incident light, half the light will be transmitted when the quartz rotates light by an odd multiple of 90°. Thus

$$31 \cdot 1° \times d = (2k' + 1) \times 90°$$

Now $\quad \frac{41 \cdot 5}{31 \cdot 1} = 1 \cdot 3344 \approx \frac{4}{3}$

Thus $\quad k = 2$ and $k' = 1$ and

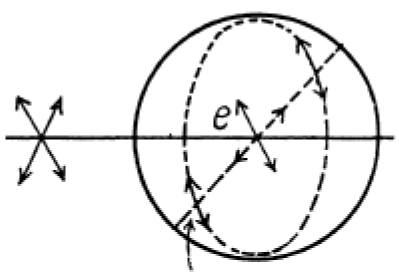
$$d = \frac{4 \times 90}{41 \cdot 5} = 8 \cdot 67 \text{ m m}.$$

Q.193. Plane-polarized light of wavelength 589 nm propagates along the axis of a cylindrical glass vessel filled with slightly turbid sugar solution of concentration

500 g/l. viewing from the side, one can see a system of helical fringes, with 50 cm between neighbouring dark fringes along the axis. Explain the emergence of the fringes and determine the specific rotation constant of the solution.

Ans. Two effects are involved here : rotation of plane of polarization by sugar solution and the effect of that rotation on the scattering of light in the transverse direction. The latter is shown in the figure given below. It is easy to see trom the figure that there will be no scattering of light in this transverse direction if the incident light has its electric vector parallel to the line of sight. In such a situation, we expect fringes to occur in the given experiment.

From the given data we see that in a distance of 50 cm, the rotation of plane of polarization must be 180°. Thus the specific rotation constant of sugar



$$= \frac{\text{rotation constant}}{\text{concentration}}$$

$$= \frac{180/50}{500\,\frac{g}{l}} \text{ ang/deg/cm} = \frac{180}{5{\cdot}0 \text{ d m} \times (\cdot 500 \text{ gm/cc})}$$

$$= 72^\circ \text{ ang deg/(dm} \cdot \text{gm/cc)} \quad (1 \text{ d m} = 10 \text{ cm})$$

Q.194. A Kerr cell is positioned between two crossed Nicol prisms so that the direction of electric field E in the capacitor forms an angle of 45° with the principal directions of the prisms. The capacitor has the length l = 10 cm and is filled up with nitrobenzene. Light of wavelength λ = 0.50 [tm passes through the system. Taking into account that in this case the Kerr constant is equal to B = $2.2.10^{-10}$ cm/V^2, find:

(a) the minimum strength of electric field E in the capacitor at which the intensity of light that passes through this system is independent of rotation of the rear prism;

(b) how many times per second light will be interrupted when a sinusoidal voltage of frequency v = 10 MHz and strength amplitude Em = 5 0 kV/cm is applied to the capacitor.

Note. The Kerr constant is the coefficient B in the equation $n_e - n_o = B\lambda E^2$.

Ans. (a) in passing through the Kerr cell the two perpendicular components of the electric field will acquire a phase difference. When this phase difference equals 90° the emergent light will be circularly polarized because the two perpendicular components O & E have the same magnitude since it is given that the direction of electric field E in the capacitor forms an angle of 45° with the principal directions of then icols. In this case the intensity of light that emerges from this system will be independent of the rotation of the analyser prism.

Now the phase difference introduced is given by

$$\delta = \frac{2\pi}{\lambda}(n_e - n_0)\, l$$

In the present case $\delta = \frac{\pi}{2}$ (for minimum electric field)

$$n_e - n_0 = \frac{\lambda}{4l}$$

Now $$n_e - n_0 = B\lambda E^2$$

so $$E_{min} = \sqrt{\frac{1}{4Bl}}\,. = 10^5/\sqrt{88} = 10{\cdot}66 \text{ kV/cm}.$$

(b) If the applied electric field is

$E = E_m \sin \omega t,\ \omega = 2\pi\nu$

than the Kerr cell introduces a time varying phase difference

$$\delta = 2\pi B \mid E_m^2 \sin^2 \omega t$$
$$= 2\pi \times 2{\cdot}2 \times 10^{-10} \times 10 \times (50 \times 10^3)^2 \sin^2 \omega t$$
$$= 11\pi \sin^2 \omega t$$

In one half-cycle $\left(\text{i.e. in time } \frac{\pi}{\omega} = T/2 = \frac{1}{2\nu}\right)$.

this reaches the value 2kπ when

$$\sin^2 \omega t = 0, \frac{2}{11}, \frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \frac{10}{11}$$
$$\frac{2}{11}, \frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \frac{10}{11}$$

i.e. 11 times. On each of these occasions light will be interrupted. Thus light will be interrupted

$2\nu \times 11 = 2{\cdot}2 \times 10^8$ times per second

(Light will be interrupted when the Kerr cell (placed between crossed Nicols) introduces a phase difference oil kit and in no other case.)

Q.195. Monochromatic plane-polarized light with angular frequency ω passes through a certain substance along a uniform magnetic field H. Find the difference of refractive indices for right-hand and left-hand components of light beam with circular polarization if the Verdet constant is equal to V.

Ans. From problem 189, we know that

$$\Delta n = \frac{\alpha \lambda}{\pi}$$

where α is the rotation constant Thus

$$\Delta n = \frac{2\alpha}{2\pi/\lambda} = \frac{2\alpha c}{\omega}$$

On the other hand $\alpha_{mag} = VH$

Thus for the magnetic rotations $\Delta n = \frac{2cVH}{\omega}$.

Q.196. A certain substance is placed in a longitudinal magnetic field of a solenoid located between two Polaroids. The length of the tube with substance is equal to l = 30 cm. Find the Verdet constant if at a field strength H = 56.5 kA/m the angle of rotation of polarization plane is equal to $\varphi_1 = +5°10'$ or one direction of the field and to $\varphi_2 = -3°20'$ the opposite direction.

Ans. Part of the rotations is due to Faraday effect and part of it is ordinary optical rotation. The latter does not change sign when magnetic field is reversed. Thus

$$\varphi_1 = \alpha l + VlH$$

$$\varphi_2 = \alpha l - VlH$$

Hence $2VlH = (\varphi_1 - \varphi_2)$

Or $V = \left(\frac{\varphi_1 - \varphi_2}{2}\right) / lH$

Putting the values $V = \frac{510 \text{ ang min}}{2 \times \cdot 3 \times 56{\cdot}5} \times 10^{-3} \text{ per } A = 0{\cdot}015 \text{ ang min/A}$

Q.197. A narrow beam of plane-polarized light passes through dextrorotatory positive compound placed into a longitudinal magnetic field as shown in Fig. 5.35. Find the angle through which the

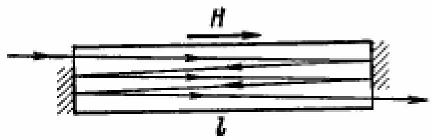


Fig. 5.35.

polarization plane of the transmitted beam will turn if the length of the tube with the compound is equal to 1, the specific rotation constant of the compound is equal to α, the Verdet constant is V, and the magnetic field strength is H.

Ans. We write

$$\varphi = \varphi_{\text{chemical}} + \varphi_{\text{magnetic}}$$

We look against the transmitted beam and count the positive direction clockwise. The chemical part of the rotation is annulled by reversal of wave vector up on reflection . Thus

$$\varphi_{\text{chemical}} = \alpha l$$

Since in effect there is a single transmission.
On the other hand

$$\varphi_{\text{mag}} = -NHVl$$

To get the signs right recall that dextro rotatory compounds rotate the plane of vibration in a clockwise direction on looking against the oncoming beam. The sense of rotation of light vibration in Faraday effect is defined in terms of the direction of the field, positive rotation being that of a right handed screw advancing in the direction of the field. This is the opposite of the definition of $\varphi_{\text{chemical}}$ for the present case. Finally

$$\varphi = (\alpha - VNH)l$$

(Note : If plane polarized light is reflected back & forth through the same active medium in a magnetic Held, the Faraday rotation increases with each traveresal.)

Q.198. A tube of length l = 26 cm is filled with benzene and placed in a longitudinal magnetic field of a solenoid positioned between two Polaroids. The angle between the principle directions of the Polaroids is equal to 45°. Find the minimum strength of the magnetic field at which light of the wavelength 589 nm propagates through that system only in one direction (optical valve). What happens if the direction of the given magnetic field is changed to the opposite one?

Ans. There must be a Faraday rotation by 45° in through the second polaroid. Thus

$$VlH_{\text{min}} = \pi/4$$

or

$$H_{min} = \frac{\pi/4}{Vl} = \frac{45 \times 60}{2.59 \times 0.26} \frac{A}{m}$$

$$= 4.01 \frac{kA}{m}$$

If the direction of magnetic field is changed then the sense of rotation will also change. Light will be completely quenched in the above case.

Q.199. Experience shows that a body irradiated with light with circular polarization acquires a torque. This happens because such a light possesses an angular momentum whose flow density in vacuum is equal to M = l/ω, where I is the intensity of light, ω is the angular oscillation frequency. Suppose light with circular polarization and wavelength λ = 0.70 lam falls normally on a uniform black disc of mass m = 10 mg which can freely rotate about its axis. How soon will its angular velocity become equal to ωo = 1.0 rad/s provided I = 10 W/cm2?

Ans. Let r - radius of the disc

then its moment of inertia about its axis $= \frac{1}{2} m r^2$

In time t the disc will acquire an angular momentum $t \cdot \pi r^2 \cdot \frac{I}{\omega}$

When circularly polarized light of intensity l falls on it. By conservation of angular momentum this must equal

$$\frac{1}{2} m r^2 \cdot \omega_0$$

where ω_0 = final angular velocity.

Equating $$t = \frac{m \omega \omega_0}{2 \pi I}.$$

But $$\frac{\omega}{2\pi} = \nu = \frac{c}{\lambda} \quad \text{so } t = \frac{m c \omega_0}{I \lambda}.$$

Substituting the values of the various quantities we get
t = 11.9 hours