Long Answer Type Questions

[4 marks]

Que 1. Using quadratic formula, solve the following equation for x:

$abx^2 + (b^2 - ac)\,x - bc = 0$

Sol. We have, $abx^2 + (b^2 - ac)\,x - bc = 0$

Here, A = ab, B = b^2 - ac, C = $-bc$

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4\,AC}}{2A}$$

$$\Rightarrow x = \frac{-\,(b^2 - ac) \pm \sqrt{(b^2 - ac)^2 - 4\,(ab)\,(-bc)}}{2ab}$$

$$\Rightarrow x = \frac{-\,(b^2 - ac) \pm \sqrt{(b^2 - ac)^2 - 4ab^2c)}}{2ab}$$

$$\Rightarrow x = \frac{-\,(b^2 - ac) \pm \sqrt{b^4 - 2ab^2c + a^2\,c^2 + 4ab^2c}}{2ab}$$

$$\Rightarrow x = \frac{-\,(b^2 - ac) \pm \sqrt{(b^2 - ac)^2}}{2ab} \qquad \Rightarrow x = \frac{-\,(b^2 - ac) \pm (b^2 + ac)}{2ab}$$

$$\Rightarrow x = \frac{-\,(b^2 - ac) + (b^2 + ac)}{2ab} \quad \text{or} \quad x = \frac{-\,(b^2 - ac) - (b^2 + ac)}{2ab}$$

$$\Rightarrow x = \frac{2ac}{2ab} \quad \text{or} \quad x = \frac{-\,2b^2}{2ab} \qquad \Rightarrow \; x = \frac{c}{b} \quad \text{or} \quad x = \frac{-\,c}{b}$$

Que 2. Find the value of p for which the quadratic equation
$(2p + 1)\,x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also find these roots.

Sol. Since the quadratic equation has equal roots, D = 0

$i.e.,$ $\quad b^2 - 4ac = 0$

In $(2p + 1)\,x^2 - (7p + 2)\,x + (7p - 3) = 0$

Here, a = (2p +1), b = -(7p + 2) , c = (7p - 3)

$$\therefore \quad (7p + 2)^2 - 4\,(2p + 1)\,(7p - 3) = 0$$

$$\Rightarrow \quad 49p^2 + 4 + 28p - (8p + 4)\,(7p - 3) = 0$$

$$\Rightarrow \quad 49p^2 + 4 + 28p - 56p^2 + 24p - 28p + 12 = 0$$

$$\Rightarrow \quad -7p^2 + 24p + 16 = 0 \qquad \Rightarrow \quad 7p^2 - 24p - 16 = 0$$

$$\Rightarrow \quad 7p^2 - 28p + 4p - 16 = 0 \qquad \Rightarrow \quad 7p\,(p - 4) + 4\,(p - 4) = 0$$

$$\Rightarrow \quad (7p + 4)\,(p - 4) = 0 \qquad \Rightarrow \quad p = -\frac{4}{7} \text{ or } p = 4$$

For $p = \frac{-4}{7}$

$\left(2 \times \frac{-4}{7} + 1\right) x^2 - \left(7 \times \frac{-4}{7} + 2\right) x + \left(7 \times \frac{-4}{7} - 3\right) = 0$

$\Rightarrow \quad \frac{-1}{7} x^2 + 2x - 7 = 0 \quad \Rightarrow \quad x^2 - 14x + 49 = 0$

$\Rightarrow \quad x^2 - 7x - 7x + 49 = 0 \quad \Rightarrow \quad x(x - 7) - 7(x - 7) = 0$

$\Rightarrow \quad (x - 7)^2 = 0 \quad \Rightarrow \quad x = 7, 7$

For p = 4,

$(2 \times 4 + 1) x^2 - (7 \times 4 + 2) x + (7 \times 4 - 3) = 0$

$\Rightarrow \quad 9x^2 - 30x + 25 = 0 \quad \Rightarrow \quad 9x^2 - 15x - 15x + 25 = 0$

$\Rightarrow \quad 3x(3x - 5) - 5(3x - 5) = 0 \quad \Rightarrow \quad (3x - 5)(3x - 5) = 0$

$\Rightarrow \quad x = \frac{5}{3}, \frac{5}{3}$

Que 3. Solve for x: $\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}; x \neq 5,7$

Sol. $\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3} \quad \Rightarrow \quad \frac{(x-4)(x-7)+(x-6)(x-5)}{(x-5)(x-7)} = \frac{10}{3}$

$\Rightarrow \frac{x^2-7x-4x+28+x^2-5x-6x+30}{x^2-7x-5x+35} = \frac{10}{3}$

$\Rightarrow \frac{2x^2-22x+58}{x^2-12x+35} = \frac{10}{3} \quad \Rightarrow \quad \frac{x^2-11x+29}{x^2-12x+35} = \frac{5}{3}$

$\Rightarrow 3x^2 - 33x + 87 = 5x^2 - 60x + 175 \quad \Rightarrow \quad 2x^2 - 27x + 88 = 0$

$\Rightarrow 2x^2 - 16x - 11x + 88 = 0 \quad \Rightarrow 2x(x - 8) - 11(x - 8) = 0$

$\Rightarrow (2x - 11)(x - 8) = 0 \quad \Rightarrow \quad 2x - 11 = 0$ or $x - 8 = 0$

$\Rightarrow x = \frac{11}{2}$ or $x = 8$

Que 4. The sum of the reciprocal of Rehman's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol. Let the present age of Rehman be x years.

So, 3 years ago, Rehman's age = $(x - 3)$ years

And 5 years from now, Rehman's age = $(x - 5)$ years

Now' according to question, we have

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \quad \Rightarrow \quad \frac{2x + 2}{(x - 3)(x + 5)} = \frac{1}{3}$

$\Rightarrow$ $6x + 6 = (x - 3)(x + 5)$ $\Rightarrow$ $6x + 6 = x^2 + 5x - 3x - 15$

$\Rightarrow$ $x^2 + 2x - 15 - 6x - 6 = 0$ $\Rightarrow$ $x^2 - 4x - 21 = 0$

$\Rightarrow$ $x^2 - 7x + 3x - 21 = 0$ $\Rightarrow$ $x(x - 7) + 3(x - 7) = 0$

$\Rightarrow$ $(x - 7)(x + 3) = 0$ $\Rightarrow$ $x = 7$ or $x = -3$

But $x \neq -3$ (age cannot be negative)
Therefore, present age of Rehman = 7 years.

Que 5. The different of two nature numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the number.

Sol. Let the Two nature numbers be x and y such that $x > y$.

According to the question
Difference of numbers, $x - y = 5$ $\Rightarrow$ $x = 5 + y$ … (i)
Difference of the reciprocals,

$$\frac{1}{y} - \frac{1}{x} = \frac{1}{10} \qquad \ldots (ii)$$

Putting the value of (i) in (ii)

$$\frac{1}{y} - \frac{1}{5+y} = \frac{1}{10} \quad \Rightarrow \quad \frac{5+y-y}{y(5+y)} = \frac{1}{10}$$

$\Rightarrow$ $50 = 5y + y^2$ $\Rightarrow$ $y^2 + 5y - 50 = 0$

$\Rightarrow$ $y^2 + 10y - 5y - 50 = 0$ $\Rightarrow$ $y(y + 10) - 5(y + 10) = 0$

$\Rightarrow$ $(y - 5)(y + 10) = 0$

$\therefore$ $y = 5$ or $y = -10$

$\therefore$ y is a nature number. $\therefore$ $y = 5$

Putting the value of y in (i), we have

$x = 5 + 5$ $\Rightarrow$ $x = 10$

The required numbers are 10 and 5.

Que 6. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Sol. Let the two consecutive odd numbers be x and $x + 2$.

$\Rightarrow$ $x^2 + (x + 2)^2 = 394$ $\Rightarrow$ $x^2 + x^2 + 4 + 4x = 394$

$\Rightarrow$ $2x^2 + 4x + 4 = 394$ $\Rightarrow$ $2x^2 + 4x - 390 = 0$

⇒ $x^2 + 2x - 195 = 0$ ⇒ $x^2 + 15x - 13x - 195 = 0$

⇒ $x(x + 15) - 13(x + 15) = 0$ ⇒ $(x - 13)(x + 15) = 0$

⇒ $x - 13 = 0$ or $x + 15 = 0$ ⇒ $x = 13$ or $x = -15$

Que 7. The sum of two number is 15 and the sum of their reciprocals is $\frac{3}{10}$. Find the numbers.

Sol. Let the numbers be x and $15 - x$.

According to given condition,

$$\frac{1}{x} + \frac{1}{15 - x} = \frac{3}{10} \Rightarrow \frac{15 - x + x}{x(15 - x)} = \frac{3}{10}$$

⇒ $150 = 3x(15 - x)$ ⇒ $50 = 15x - x^2$

⇒ $x^2 - 15x + 50 = 0$ ⇒ $x^2 - 5x - 10x + 50 = 0$

⇒ $x(x - 5) - 10(x - 5) = 0$ ⇒ $(x - 5)(x - 10) = 0$

⇒ $x = 5$ or 10.

When $x = 5$, then $15 - x = 15 - 5 = 10$

When $x = 10$, then $15 - x = 15 - 10 = 5$

Hence, the two numbers are 5 and 10.

Que 8. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects.

Sol. Let Shefali's marks in Mathematics be x.

Therefore, Shefali's marks in English is $(30 - x)$.

Now, according to question,

$(x + 2)(30 - x - 3) = 210$ ⇒ $(x + 2)(27 - x) = 210$

⇒ $27x - x^2 + 54 - 2x = 210$ ⇒ $25x - x^2 + 54 - 210 = 0$

⇒ $25x - x^2 - 156 = 0$ ⇒ $-(x^2 - 25x + 156) = 0$

⇒ $x^2 - 25x + 156 = 0$ ⇒ $x^2 - 13x - 12x + 156 = 0$

⇒ $x(x - 13) - 12(x - 13) = 0$ ⇒ $(x - 13)(x - 12) = 0$

Either $x - 13 = 0$ or $x - 12 = 0$

∴ $x = 13$ or $x = 12$

Therefore, Shefali's marks in Mathematics = 13

Marks in English = 30 - 13 = 17

or Shefali's marks in Mathematics = 12, marks in English = 30 – 12 = 18.

Que 9. A train travels 360 km at a uniform speed. If the speed has been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol. Let the uniform speed of the train be x km/h.

Then, time taken to cover 360 km = $\frac{360}{x}$ h

Now, new increased speed = (x + 5) km/h

So, time taken to cover 360 km = $\frac{360}{x+5}$ h

According to question, $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow$ $360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$ $\Rightarrow$ $\frac{360\,(x+5-x)}{x\,(x+5)} = 1$

$\Rightarrow$ $\frac{360 \times 5}{x\,(x+5)} = 1$ $\Rightarrow$ $1800 = x^2 + 5x$

$\therefore$ $x^2 + 5x - 1800 = 0$ $\Rightarrow$ $x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow$ $x\,(x + 45) - 40\,(x + 45) = 0$ $\Rightarrow$ $(x + 45)\,(x - 40) = 0$

Either $x + 45 = 0$ or $x - 40 = 0$

$\therefore$ $x = -45$ or $x = 40$

But x cannot be negative, so $x \neq -45$

Therefore, $x = 40$

Hence, the uniform speed of train is 40 km/h.

Que 10. The sum of the areas of two squares is 468 m2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Sol. Let x be the length of the side of First Square and y be the length of side of the second square.

Then, $x^2 + y^2 = 468$ … (i)

Let x be the length of the side of the bigger square.

$4x - 4y = 24$

$\Rightarrow$ $x - y = 6$ or $x = y + 6$ … (ii)

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y + 6)^2 + y^2 = 468$

$\Rightarrow$ $y^2 + 12y + 36 + y^2 = 468$ or $2y^2 + 12y - 432 = 0$

$\Rightarrow$ $y^2 + 6y - 216 = 0$ $\Rightarrow$ $y^2 + 18y - 12y - 216 = 0$

$\Rightarrow$ $y(y + 18) - 12(y + 18) = 0$ $\Rightarrow$ $(y + 18)(y - 12) = 0$

Either $y + 18 = 0$ or $y - 12 = 0$

$\Rightarrow$ $y = -18$ or $y = 12$

But, sides cannot be negative, so y = 12

Therefore, $x = 12 + 6 = 18$

Hence, sides of two squares are 18 m and 12 m.

Que 11. Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two-fifth of Varun's age. Find their present ages.

Sol. Seven years ago, let Swati's age be x years. Then, seven years ago Varun's age was $5x^2$ years.

$\therefore$ Swat's present age = $(x + 7)$ years

Varun's present age = $(5x^2 + 7)$ years

Three years hence,

Swati's age = $(x + 7 + 3)$ years = $(x + 10)$ years

Varun's age = $(5x^2 + 7 + 3)$ years = $(5x^2 + 10)$ years

According to the questions,

$$x + 10 = \frac{2}{5}(5x^2 + 10) \quad \Rightarrow x + 10 = \frac{2}{5} \times 5\,(x^2 + 2)$$

$$\Rightarrow x + 10 = 2x^2 + 4 \quad \Rightarrow \quad 2x^2 - x - 6 = 0$$

$$\Rightarrow 2x^2 - 4x + 3x - 6 = 0 \quad \Rightarrow \quad 2x\,(x - 2) + 3\,(x - 2) = 0$$

$$\Rightarrow (2x + 3)\,(x - 2) = 0$$

$$\Rightarrow x - 2 = 0 \text{ or } 2x + 3 = 0$$

$$\Rightarrow x = 2 \qquad [\because 2x + 3 \neq 0 \text{ as } x > 0]$$

Hence, Swati's present age = (2+7) years = 9 years

and Varun's present age = $(5 \times 2^2 + 7)$ years = 27 years

Que 12. A train takes 2 hours less for a journey of 300 km, if its speed is increased by 5 km/h from its usual speed. Find the usual speed of the train.

Sol. Let the usual speed of the train = x km/h.

Therefore, time taken to cover 300 km = $\frac{300}{x}$ hours … (i)

When its speed is increased by 5 km/h, then time taken by the train to cover the distance of 300 km = $\frac{300}{x+5}$ hour … (ii)

According to the question, $\left(\frac{300}{x} - \frac{300}{x+5}\right)$ hours = 2 hours

$$\Rightarrow \frac{300\,(x+5) - 300\,x}{x\,(x+5)} = 2 \quad \Rightarrow \quad \frac{300\,\{(x+5) - x\}}{x\,(x+5)} = 2$$

$$\Rightarrow 2x\,(x + 5) = 300 \times 5 \quad \Rightarrow \quad 2x^2 + 10x - 1500 = 0$$

$$\Rightarrow x^2 + 5x - 750 = 0 \quad \Rightarrow \quad x^2 + 30x - 25x - 750 = 0$$

$$\Rightarrow x\,(x + 30) - 25\,(x + 30) = 0 \quad \Rightarrow \quad (x - 25)\,(x + 30) = 0$$

$\Rightarrow$ $x = 25$ or $x = -30$

$\Rightarrow$ $x = 25$ ($\because$ speed cannot be negative)

Therefore, the usual speed of the train 25 km/h.

Que 13. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Sol. Let the digit at tens place be x.

Then, digit at unit place = $\frac{18}{x}$

$\therefore$ Number = $10x + \frac{18}{x}$

And number obtained by interchanging the digits = $10 \times \frac{18}{x} + x$

According to the question,

$\left(10x + \frac{18}{x}\right) - 63 = 10 \times \frac{18}{x} + x$ $\Rightarrow$ $\left(10x + \frac{18}{x}\right) - \left(10 \times \frac{18}{x} + x\right) = 63$

$\Rightarrow$ $10x + \frac{18}{x} - \frac{180}{x} - x = 63$ $\Rightarrow$ $9x - \frac{162}{x} - 63 = 0$

$\Rightarrow$ $9x^2 - 63x - 162 = 0$ $\Rightarrow$ $x^2 - 7x - 18 = 0$

$\Rightarrow$ $x^2 - 9x + 2x - 18 = 0$ $\Rightarrow$ $x(x - 9) + 2(x - 9) = 0$

$\Rightarrow$ $(x - 9)(x + 2) = 0$ $\Rightarrow$ $x = 9$ or $x = -2$

$\Rightarrow$ $x = 9$ [$\because$ a digit can never be negative]

Hence, the required number = $10 \times 9 + \frac{18}{9} = 92$.

Que 14. If twice the area of a smaller square is subtracted from the area of a larger square; the result is 14 cm². However, if twice the area of the larger square is added to three times the area of the smaller square, the result is 203 cm². Determine the sides of the two squares.

Sol. Let the sides of the larger and smaller squares be x and y respectively. Then

$x^2 - 2y^2 = 14$ … (i)

and $2x^2 + 3y^2 = 203$ … (ii)

Operating $(ii) - 2 \times (i)$, we get

$2x^2 + 3y^2 - (2x^2 - 4y^2) = 203 - 2 \times 14$

$\Rightarrow$ $2x^2 + 3y^2 - 2x^2 - 4y^2 = 203 - 28$

$\Rightarrow$ $7y^2 = 175$ $\Rightarrow$ $y^2 = 25$ $\Rightarrow$ $y = \pm 5$

$\Rightarrow$ $y = 5$ [$\because$ side cannot be negative]

By putting the value of y in equation (i), we get

$x^2 - 2 \times 5^2 = 14$ ⇒ $x^2 - 50 = 14$ or $x^2 = 64$

∴ $x = \pm 8$ or $x = 8$

∴ x side of the two squares are 8 cm and 5 cm.

Que 15. If Zeba was younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

Sol. Let the present age of Zeba be x years

Age before 5 years = (x - 5) years

According to given condition,

⇒ $(x - 5)^2 = 5x + 11$ ⇒ $x^2 + 25 - 10x = 5x + 11$

⇒ $x^2 - 10x - 5x + 25 - 11 = 0$ ⇒ $x^2 - 15x + 14 = 0$

⇒ $x^2 - 14x - x + 14 = 0$ ⇒ $x(x - 14) - 1(x - 14) = 0$

⇒ $(x - 1)(x - 14) = 0$ ⇒ $x - 1 = 0$ or $x - 14 = 0$

$x = 1$ or $x = 14$

But present age cannot be 1 year.

∴ Present age of Zeba is 14 years.

Que 16. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Let the speed of the stream be x km/h.

Speed of motor boat in still water = 18 km/h

Speed of motor boat in upstream = (18 - x) km/h

Speed of motor boat in downstream = (18 + x) km/h

Distance travelled = 24 km.

Time taken by motor boat to travel upstream = $\frac{24}{18 - x}$

Time taken by motor boat to travel downstream = $\frac{24}{18 + x}$

$\frac{24}{18 - x} = \frac{24}{18 + x} + 1$ ⇒ $24(18 + x) = (18 - x)(24 + 18 + x)$

⇒ $432 + 24x = (18 - x)(42 + x)$ ⇒ $432 + 24x = 756 + 18x - 42x - x^2$

⇒ $x^2 + 48x - 324 = 0$ ⇒ $x^2 + 54x - 6x - 324 = 0$

⇒ $x(x + 54) - 6(x + 54) = 0$ ⇒ $(x - 6)(x + 54) = 0$

$x = 6$ or $x = -54$

speed of motorboat = 6 km/h.

Que 17. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

Sol. Let the natural number be x

According to the question,

$$x + 12 = \frac{160}{x}$$

$\Rightarrow$ $x^2 + 12x - 160 = 0$ $\Rightarrow$ $x^2 + 20x - 8x – 160 = 0$

$\Rightarrow$ $x(x + 20) – 8(x + 20) = 0$ $\Rightarrow$ $(x + 20)(x – 8) = 0$

$\Rightarrow$ $x = -20$ (Not possible) or $x = 8$

Hence, the required natural number is 8.

Que 18. The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples.

Sol. Let the two consecutive multiples of 7 be x and $x + 7$.

$$x^2 + (x + 7)^2 = 637$$

$\Rightarrow$ $x^2 + x^2 + 49 + 14x = 637$ $\Rightarrow$ $2x^2 + 14x – 588 = 0$

$\Rightarrow$ $x^2 + 7x – 294 = 0$ $\Rightarrow$ $x^2 + 21x - 14x – 294 = 0$

$\Rightarrow$ $x(x + 21) – 14(x + 21) = 0$ $\Rightarrow$ $(x – 14)(x + 21) = 0$

$x = 14$ or $x = - 21$

The multiples are 14 and 21.

Que 19. Solve for x: $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$, $x \neq - 1, - 2, - 4$

Sol. $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$\Rightarrow$ $\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$ $\Rightarrow$ $(x + 4)(x + 2 + 2x + 2) = 4(x + 1)(x + 2)$

$\Rightarrow$ $(x + 4)(3x + 4) = 4(x^2 + 3x + 2)$ $\Rightarrow$ $x^2 - 4x – 8 = 0$

$\Rightarrow$ $x = \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

Que 20. Find the positive value(s) of k for which both quadratic equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots.

Sol. (i) For $x^2 + kx + 64 = 0$ to have real roots

$K^2 – 4(l)(64) \geq 0$ $i.e.$, $k^2 – 256 \geq 0$ $\Rightarrow$ $k \geq \pm 16$

(ii) For $x^2 - 8x + k = 0$ to have real roots

$(-8)^2 – 4(k) \geq 0$ $i.e.$, $64 – 4k \geq 0$ $\Rightarrow$ $k \leq \pm 16$

For (i) and (ii) to hold simultaneously k = 16