PRACTICE PAPER

19

MATHEMATICS

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper* – 1.

Section–A

Choose the correct option:

1. **A is a scalar matrix, if**

   (*a*) $A = [a_{ij}]_n$ where $a_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ k & \text{if } i = j \end{cases}$ $k \in \mathbb{R}$.

   (*b*) $A = [a_{ij}]_n$, $a_{ij} = \begin{cases} k & \text{if } i \neq j \ \ k \in \mathbb{R} \\ 0 & \text{if } i = j \end{cases}$

   (*c*) $A = [a_{ij}]$, where $a_{ij} = k \ \forall \ i, j$

   (*d*) None of these

2. **Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, then**

   (*a*) $A^2 = A$ (*b*) $A^2 = 0$ (*c*) $A^2 = I$ (*d*) None of these

3. **If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, then $\lvert AB \rvert$ is**

   (*a*) 10 (*b*) –5 (*c*) 5 (*d*) 0

4. **If $y = e^{-x}$ then $\dfrac{d^2y}{dx^2}$ is equal to**

   (*a*) y (*b*) $-y$ (*c*) $\dfrac{1}{y}$ (*d*) $-\dfrac{1}{y}$

5. **The point on the curve $y = 12x - x^2$ where the slope of the tangent is zero will be**

   (*a*) (3, 9) (*b*) (2, 16) (*c*) (6, 36) (*d*) None of these

6. **The value of $\int \tan^2 x \, dx$ equals**

   (*a*) $\tan x + x + C$ (*b*) $\tan x - x + C$ (*c*) $\cot x + x + C$ (*d*) $-\tan x + x + C$

7. **The value of $\int \log x \, dx$ is**

   (*a*) $x \log x - x + C$ (*b*) $\log x - x + C$ (*c*) $x \log x + x + C$ (*d*) $\log x + C$

8. **The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, dx$ is**

   (*a*) $\dfrac{\pi}{2}$ (*b*) π (*c*) 0 (*d*) –1

9. **The value of $\int_{-1}^{1} \dfrac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$ is equal to**

(a) $\log 2$ (b) $2 \log 2$ (c) $\frac{1}{2}\log 2$ (d) $4 \log 2$

10. **Area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ is**

(a) 16π sq. units (b) 4π sq. units (c) 32π sq. units (d) 24π sq. units

11. **Solution of $x^2 + y^2 \frac{dy}{dx} = 4$ is**

(a) $x^2 + y^2 = 12x + C$ (b) $x^2 + y^2 = 3x + C$ (c) $x^3 + y^3 = 3x + C$ (d) $x^3 + y^3 = 12x + C$

12. **The solution of $xdx + ydy = x^2ydy - xy^2dx$ is**

(a) $x^2 - 1 = C(1 + y^2)$ (b) $x^2 + 1 = C(1 - y^2)$ (c) $x^3 - 1 = C(1 + y^3)$ (d) $x^3 + 1 = C(1 - y^3)$

13. **The point which does not lie in the half plane $2x + 3y - 12 \leq 0$ is**

(a) (1, 2) (b) (2, 1) (c) (2, 3) (d) (–3, 2)

14. **For the following distribution:**

X	– 4	– 3	– 2	– 1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

$E(X)$ is equal to

(a) – 2 (b) – 3 (c) – 2 (d) – 1.8

15. **For the binomial distribution $B(n, p)$, variance is**

(a) npq (b) $\sqrt{npq}$ (c) np (d) None of these

Section–B(B1)

16. **If $A = \{7, 8, 9\}$, then the relation $R = \{(8, 9)\}$ in A is**

(a) Symmetric only (b) Non-symmetric (c) Reflexive only (d) Equivalence

17. **Let f: (–1, 1) $\longrightarrow$ B, be a function defined by $f(x) = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$ then f is both one-one and onto when B is the interval**

(a) $\left[0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

18. **Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ then**

(a) $f(x)$ is one-one but not onto (b) $f(x)$ is neither one one nor onto

(c) $f(x)$ is many one and onto (d) $f(x)$ is one-one and onto

19. **If $a * b = a^2 + b^2$, then the value of $(4 * 5) * 3$ is**

(a) $(4^2 + 5^2) + 3^2$ (b) $(4 + 5)^2 + 3^2$ (c) $41^2 + 3^2$ (d) $(4 + 5 + 3)^2$

20. **Let $a*b$ denotes the bigger among a and b if $a.b = a*b + 3$, then 4.7 equals**

(a) 14 (b) 31 (c) 10 (d) 8

21. **The value of expression $\tan\left(\frac{\sin^{-1}x + \cos^{-1}x}{2}\right)$, where $x = \frac{\sqrt{3}}{2}$ is equal to**

(a) ∞ (b) 1 (c) – 1 (d) none of these

22. **If $\sin(\pi \cos x) = \cos(\pi \sin x)$ then x equals**

(a) $\frac{1}{2}\sin^{-1}\frac{3}{4}$ (b) $\frac{1}{2}\cos^{-1}\frac{3}{4}$

(c) $-\frac{1}{4}\sin^{-1}\frac{3}{4}$ (d) $-\frac{1}{2}\cos^{-1}\frac{3}{4}$

23. The domain of $\sin^{-1}[x]$ is given by

(a) $[-1, 1]$ (b) $[-1, 2)$ (c) $\{-1, 0, 1\}$ (d) None of these

24. $\cot^{-1}[(\cos\alpha)^{1/2}] + \tan^{-1}[(\cos\alpha)^{1/2}] = x$, then $\sin x$ equals

(a) 1 (b) $\cot^2(\alpha/2)$ (c) $\tan\alpha$ (d) $\cot(\alpha/2)$

25. If $A = \begin{bmatrix} 0 & \sin\theta \\ \cos\theta & 1 \end{bmatrix}$ and $A = A^T$ then θ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

26. If $A = \begin{bmatrix} 2 & 14 & 17 \\ 0 & \sin 2x & \cos 2x \\ 0 & \cos 2x & \sin 2x \end{bmatrix}$, then $|A|$ equals

(a) $\cos 2x$ (b) -2 (c) $-2\cos 4x$ (d) $\sin 4x$

27. The system of linear equations

$$x + 2y + z = 5$$
$$2x + 3y + z = 2$$
$$3x + 5y + 2z = 1$$

has

(a) a unique solution (b) no solution (c) exactly 3 solutions (d) infinite many solutions.

28. The function $f(x) = \begin{cases} \frac{\sin^3 x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is

(a) continuous but not derivable at $x = 0$
(b) neither continuous nor differentiable at $x = 0$
(c) continuous and differentiable at $x = 0$
(d) none of these

29. Let $f(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$ then $f(x)$ is

(a) differentiable at $x = 1$
(b) continuous $\forall\, x \in R$
(c) neither continuous nor differentiable at $x = 1$
(d) continuous but not differentiable at $x = 1$

30. Let $f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$ then $f(x)$ is

(a) differentiable at $x = 1$
(b) differentiable at $x = 2$
(c) differentiable at $x = 1$ and $x = 2$
(d) none of these

31. Let $f(x) = |\sin x|$ then $f(x)$ is

(a) continuous everywhere
(b) non-differentiable at odd and even multiple of p
(c) everywhere continuous but non-differentiable at $x = n\text{p}$, $n \in Z$
(d) all of these

32. The minimum value of $f(x) = 3\cos^2 x + 4\sin^2 x + \cos\frac{x}{2} + \sin\frac{x}{2}$ is

(a) 4 (b) $3 + \sqrt{2}$ (c) $4 + \sqrt{2}$ (d) none of these

33. $\int_{-1}^{1} [x]\, dx$, where [.] denotes the greatest integer function, is equal to

(a) 1 (b) -1 (c) 0 (d) none of these

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34. The value of $\int_0^{10\pi} |\sin x|\, dx$ is

(*a*) 15 (*b*) 20 (*c*) –10 (*d*) 7

35. $\int \tan^4 x\, dx = A \tan^3 x + B \tan x + f(x)$, then

(*a*) $A = 1/3, B = -1, f(x) = x + C$ (*b*) $A = 2/3, B = -1, f(x) = x + C$

(*c*) $A = 1/3, B = 1, f(x) = x + C$ (*d*) $A = 2/3, B = 1, f(x) = -x + C$

36. The area bounded by curves $y^2 = 16x$ and $x^2 = 16y$ is given by

(*a*) $\frac{64}{3}$ sq. units. (*b*) $\frac{256}{3}$ sq. units. (*c*) $\frac{16}{3}$ sq. units. (*d*) None of these

37. Which of the following differential equations has $y = C_1 e^x + C_2 e^{-x}$ as the general solution?

(*a*) $\frac{d^2y}{dx^2} + y = 0$ (*b*) $\frac{d^2y}{dx^2} - y = 0$

(*c*) $\frac{d^2y}{dx^2} + 1 = 0$ (*d*) $\frac{d^2y}{dx^2} - 1 = 0$

38. The order of differential equation $\left(\frac{d^2y}{dx^2}\right)^3 = \left(1 + \frac{dy}{dx}\right)^{1/2}$ is

(*a*) 2 (*b*) 3 (*c*) $\frac{1}{2}$ (*d*) 6

39. If points $A(60\hat{i} + 3\hat{j})$, $B(40\hat{i} - 8\hat{j})$ and $C(a\hat{i} - 52\hat{j})$ are collinear, then a is equal to

(*a*) 40 (*b*) 50 (*c*) –40 (*d*) none of these

40. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a} + \vec{b}$ is collinear with $\vec{c}$, $\vec{b} + \vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + \vec{b} + \vec{c} =$

(*a*) $\vec{c}$ (*b*) $\vec{b}$ (*c*) $\vec{a}$ (*d*) none of these

41. If $\vec{a}, \vec{b}$ are the vectors forming consecutive sides of a regular hexagon *ABCDEF*, then the vector representing side *CD* is

(*a*) $\vec{a} + \vec{b}$ (*b*) $-(\vec{a} + \vec{b})$ (*c*) $\vec{a} - \vec{b}$ (*d*) $\vec{b} - \vec{a}$

42. The vector $\cos\alpha \cos\beta\hat{i} + \cos\alpha \sin\beta\hat{j} + \sin\alpha\hat{k}$ is a

(*a*) Unit vector (*b*) Null vector (*c*) Constant vector (*d*) none of these

43. If the origin is shifted to (4, 5, – 3) without changing the direction of the axes then the new co-ordinates of the point (0, 8, 5) with respect to new frame is

(*a*) (– 4, 3, 8) (*b*) (4, – 3, – 8) (*c*) (4, 13, 2) (*d*) None of these

44. The direction ratios of a line segment joining the points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ are

(*a*) $\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}$ (*b*) $\frac{x - x_1}{x_2 - x_1}, \frac{y - y_1}{y_2 - y_1}, \frac{z - z_1}{z_2 - z_1}$

(*c*) $x_2 - x_1, y_2 - y_1, z_2 - z_1$ (*d*) None of these

45. The equation of the plane passing through the point (– 3, 1, – 2) which is the foot of the perpendicular drawn from the origin to the plane is

(*a*) $4x - 2y + 3z + 20 = 0$ (*b*) $3x - y + 2z + 14 = 0$

(*c*) $3x + 2y - 4z - 1 = 0$ (*d*) $2x - 3y + 4z + 17 = 0$

46. The angle between the line $\vec{r} = (2\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(3\hat{i} - 2\hat{j} + 5\hat{k}) = 4$ is

(*a*) $\sin^{-1}\left(\frac{11}{\sqrt{323}}\right)$ (*b*) $\sin^{-1}\left(\frac{22}{\sqrt{323}}\right)$

(*c*) $\sin^{-1}\left(\frac{22}{\sqrt{646}}\right)$ (*d*) None of these

47. A coin is tossed 10 times, probability that on the 10th throw to observe 5th head is

(*a*) ${}^9C_5 \times \frac{1}{2^5}$ (*b*) $\frac{{}^9C_4}{2^{10}}$ (*c*) $\frac{{}^9C_5}{2^9}$ (*d*) none of these

48. Let X denotes the number of time tail appear in n tosses of a fair coin.

If $P(X = 1)$, $P(X = 2)$ and $P(X = 3)$ are in AP, then the value of n is

(*a*) 9 (*b*) 2 (*c*) 7 (*d*) none of these

49. If A and B are events such that $P(A/B) = P(B/A)$, then

(*a*) $A \subseteq B$ but $A \neq B$ (*b*) $A = B$ (*c*) $A \cap B = \phi$ (*d*) $P(A) = P(B)$

50. If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of 3×3 matrix A then the sum of values of α for which $\det(A) + 1 = 0$ is

(*a*) 0 (*b*) -1 (*c*) 1 (*d*) 2