Waves

Waves - Transverse and Longitudinal

Waves

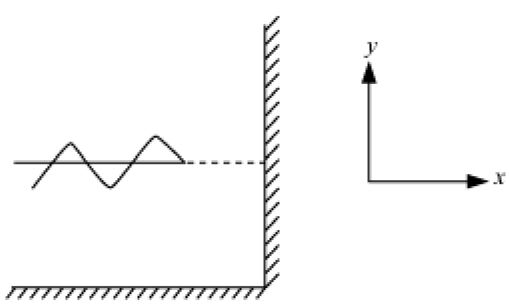
Continuous disturbance that transfers energy without any net displacement of the medium particles

Types of Wave

- **Mechanical wave**
- Requires a material medium for propagation
- Examples − sound waves, water waves, etc.
- **Electromagnetic wave**
- Does not require any material medium
- Examples − light wave, X-rays, etc.
- **Matter wave**
- Associated with electrons, protons, neutrons, atoms

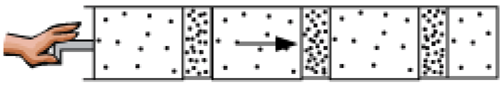
Types of Mechanical Wave

- **Transverse wave**
- Medium particles oscillate perpendicular to the direction of propagation of wave.



- Transverse waves are transmitted through solids and not through liquids and gases as the latter does not possess any internal transverse restoring force (shear strength).

- **Longitudinal waves**
- Medium particles oscillate along the direction of propagation of wave.



- Longitudinal waves can propagate through solids, liquids, and gases.

Displacement Relation in a Progressive Wave

- Representation of a sinusoidal wave travelling along the positive *x*-axis is

$y(x, t) = a \sin(kx - \omega t + \Phi)$

- The equation can also be represented as linear combination of sine and cosine function.

$y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$, where $a = \sqrt{A^2 + B^2}$ and $\Phi = \tan^{-1}\left(\frac{B}{A}\right)$

Where,

$y(x, t)$ → Displacement function of position x and time t

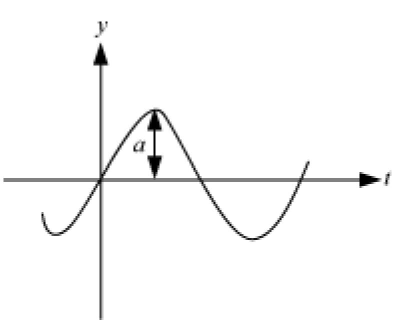
k → Propagation constant or angular wave number

ω → Angular frequency

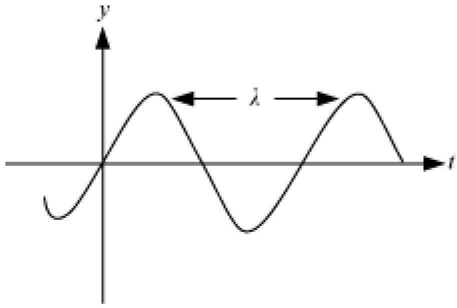
Φ → Initial phase angle

A, B, a → Amplitude

- Amplitude (a): Magnitude of the maximum displacement from the equilibrium position



- Wavelength (λ): Minimum distance between two consecutive crest or trough



- Angular wave number or propagation constant (k)

  Displacement at t = 0, taking Φ = 0

  $y(x, 0) = a \sin kx$

  Sine function repeats itself after every (2π).

  $$\therefore \sin kx = \sin(kx + 2n\pi) = \sin k\left(x + \frac{2n\pi}{k}\right)$$

  Therefore, the points at x and at $\left(x + \frac{2n\pi}{k}\right)$ are in the same phase.

  Therefore, for n = 1, the distance between the two consecutive points in phase is the minimum.

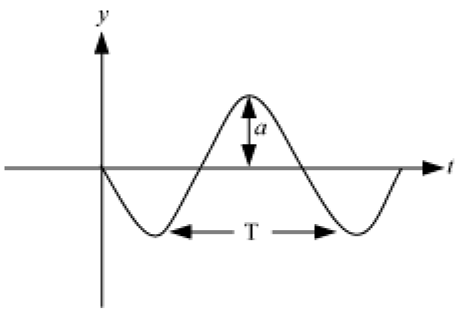
  $\therefore \lambda = \frac{2\pi}{k}$ or $k = \frac{2\pi}{\lambda}$ (Unit of k → rad m^{-1})

- Frequency (v) − Number of oscillations per second (Unit → hertz)

  $$v = \frac{\omega}{2\pi}$$

  ω is angular frequency; unit is rad/s.

- Period



$T \rightarrow$ represents the time period

Unit $\rightarrow$ s (second)

Speed of a Transverse Wave on a Stretched String

- **Speed of a Travelling Wave**

Suppose a transverse wave is travelling on a string in the positive direction of *x*-axis. Assume that during a small time interval Δt, the wave moves by a distance Δx.

Wave speed, $v = \frac{\Delta x}{\Delta t} = \frac{\text{Distance}}{\text{Time interval}}$

From the equation of displacement of wave motion, we have

$y(x, t) = a \sin(kx - \omega t)$

Consider fixed phase point on the wave

$kx - \omega t = \text{Constant}$ …. (*i*)

Differentiating both sides of the equation (*i*) w.r.t. time,

$$\frac{d}{dt}(kx - \omega t) = 0$$

$$or \quad k\frac{dx}{dt} - \omega = 0$$

$$\Rightarrow \omega = k\frac{dx}{dt} \; or \; \frac{\omega}{k} = \frac{dx}{dt} = v$$

$$\therefore v = \frac{\omega}{k}$$

Putting $\omega = \frac{2\pi}{T}$ and $k = \frac{2\pi}{\lambda}$, we get,

$$v = \frac{\lambda}{T}$$

- **Speed of a Transverse Wave: Derivation Using Dimensional Analysis**

  Speed (*v*) of a transverse wave on a string depends upon:

- Restoring force of medium: It is provided by the tension (*T*) of the string. It is directly related to speed.
- Inertial property of medium: It is the linear mass density (μ) of the string. It is inversely related to speed.

$$\nu \propto \mu^a T^b$$

Where, a and b are dimensions of μ and T.

$$\nu = k\mu^a T^b \qquad .....(i)$$

Where, *k* is the dimensionless constant of proportionality

$$\mu = \left[ML^{-1}\right]$$

$$T = \left[MLT^{-2}\right]$$

$$v = \left[LT^{-1}\right]$$

Putting in equation (i),

$$[M^0 L^1 T^{-1}] = [M^1 L^{-1} T^0]^a \, [M^1 L^1 T^{-2}]^b = M^{a+b} L^{-a+b} T^{-2b}$$

On applying the principle of homogeneity of dimensions, we get

a + b = 0

−a + b =1

Hence, $\text{a} = -\frac{1}{2} \text{and b} = \frac{1}{2}$

Putting these values in equation (*i*), we get

$$v = k\mu^{-\frac{1}{2}}T^{\frac{1}{2}} = k\sqrt{\frac{T}{\mu}}$$

Power Transmitted by a String Wave,

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

Speed of a Longitudinal Wave- Sound Wave

Speed (v) is determined by

- restoring force of medium, i.e. bulk modulus (B) of the medium; it is directly related to speed
- inertial property of medium, i.e. mass density (ρ); it is inversely related to speed
- Derivation using dimensional analysis:

$B = [ML^{-1} T^{-2}]$

$\rho = [ML^{-3}]$

$v = [LT^{-1}]$

$\therefore \frac{B}{\rho} = \frac{[ML^{-1}T^{-2}]}{[ML^{-3}]}$

$\frac{B}{\rho} = [L^2T^{-2}]$

$\because v = [LT^{-1}]$

$\therefore v = C\sqrt{\frac{B}{\rho}}$

Here, C is a constant.

Taking C = 1,

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

- Speed of longitudinal waves in a solid bar:

$$v = \sqrt{\frac{Y}{\rho}}$$

Here, Y is the Young's modulus of the bar.

- Speed of sound in gas:

For ideal gas,

$PV = NK_BT$

Here,

$P \rightarrow$ Pressure

$V \rightarrow$ Volume

$N \rightarrow$ Number of molecules

$K_B \rightarrow$ Boltzmann constant

$T \rightarrow$ Temperature

An isothermal change gives $V\Delta P + P\Delta V = 0$

$$-\frac{\Delta P}{\Delta V/V} = P$$

$\therefore$ Bulk modulus, $B = P$

$\because\ v = \sqrt{\frac{B}{\rho}}$

$\therefore\ v = \sqrt{\frac{P}{\rho}}$

This equation is also called Newton’s formula.

- Laplace correction:

  It was introduced because it was found that sound waves travel under adiabatic conditions, not under isothermal conditions.

- Reason for this consideration:

  Sound waves travel in air rapidly.

  Gas is a bad conductor of heat.

  For adiabatic process,

  PV^{γ}= constant

  Here, γ is the ratio of specific heat.

  $\therefore \Delta(PV^{\gamma}) = 0$

  $P\gamma V^{\gamma-1}\Delta V + V^{\gamma}\Delta P = 0$

  $$-\frac{\Delta P}{\Delta V/V} = \gamma P$$

  $\therefore \gamma P = B$ (Bulk modulus)

  $$\because v = \sqrt{\frac{B}{\rho}}, \boxed{\therefore v = \sqrt{\frac{\gamma P}{\rho}}}$$

  Factors affecting velocity of sound:

  (i) Effect of density:
  Velocity of sound,

  $$v = \sqrt{\frac{\gamma P}{\rho}}$$

  Hence, velocity of sound is inversely proportional to the square root of density of gas.

  (ii) Effect of pressure:

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\because \rho = \frac{M}{V}$$

$$\therefore v = \sqrt{\frac{\gamma PV}{M}}$$

Boyle's Law is obeyed when temperature *T* is constant, i.e. *PV* = constant.

As *M* and *γ* are also constant,
v = constant

Thus, velocity of sound is independent of pressure, provided the temperature is constant.

(iii) Effect of temperature:

Velocity of sound,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

According to the standard gas equation,

$$P = \frac{RT}{V}$$

Putting the value of *P*, we get:

$$v = \sqrt{\frac{\gamma RT}{\rho \times V}}$$

$$\Rightarrow v = \sqrt{\frac{\gamma RT}{M}} \ (\because \rho V = M)$$

Clearly, $v \propto \sqrt{T}$

So, velocity of sound in a gas is directly proportional to the square root of absolute temperature.

(iv) Effect of humidity:

If ρ_m represents the density of moist air, velocity of sound is given by:

$$v_m = \sqrt{\frac{\gamma P}{\rho_m}} \quad \ldots\ldots(1)$$

If ρ_d represents the density of dry air, velocity of sound is given by:

$$v_d = \sqrt{\frac{\gamma P}{\rho_d}} \quad \ldots\ldots(2)$$

Dividing equation (1) by (2) we get:

$$\frac{v_m}{v_d} = \sqrt{\frac{\rho_d}{\rho_m}}$$

Since, water vapour reduces the density of air,

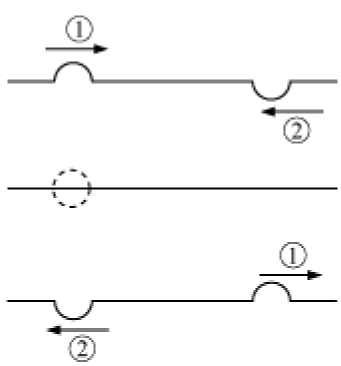
$$\rho_m < \rho_d$$

So, $v_m > v_d$

Hence, velocity of sound in moist air is greater than velocity of sound in dry air.

The Principle of Superposition of Waves

- A wave preserves its identity while travelling in space.
- Two pulses crossing each other combine to produce a resultant pulse.
- If y_1, y_2, y_3,… are the displacements of waves, then the resultant displacement (y) is $y = y_1 + y_2 + y_3 + \ldots$



Consider two waves differing by some initial phase.

$y_1(x, t) = a \sin(kx - \omega t)$ (1)

$y_2(x, t) = a \sin(kx - \omega t + \Phi)$ (2)

Net displacement, $y(x, t) = a \sin(kx - \omega t) + a \sin(kx - \omega t + \Phi)$

$$\therefore y(x,t) = a\left[2\sin\left[\frac{(kx-\omega t)+(kx-\omega t+\phi)}{2}\right]\cos\frac{\phi}{2}\right]$$

$$= 2a\cos\frac{\phi}{2}\sin\left[kx-\omega t+\frac{\phi}{2}\right]$$

From phase, $\Phi = 0$.

$y(x, t) = 2a \sin(kx - \omega t)$ (3)

$\Phi = \pi$

$y(x, t) = 0$ (4)

Maximum amplitude = $2a$

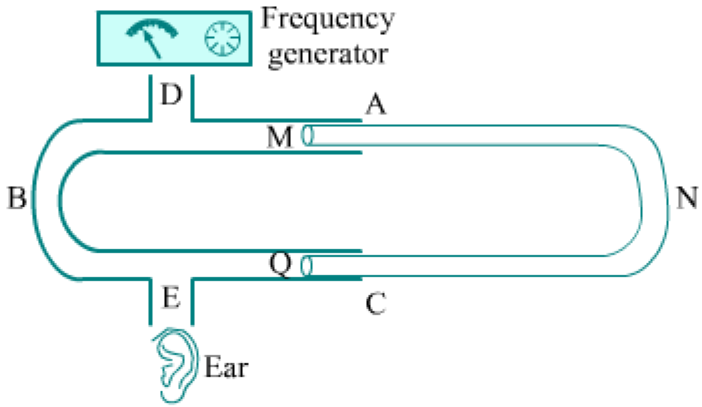
Equation (3) represents constructive interference.

Equation (4) represents destructive interference.

Change of Phase

- A wave, whether transverse or longitudinal, while travelling in a certain medium undergoes a change in phase when it is incident on the boundary of another medium.
- A wave travelling in a rarer medium suffers a change in phase by ππ radians when it is incident on the boundary of a denser medium.
- In the above mentioned case, a compression of a longitudinal wave is reflected back as a compression and it travelled back. Similarly, a rarefaction is reflected back as a rarefaction.
- In case of a transverse wave, a crest is reflected back as a trough and vice versa.
- For a wave travelling in a denser medium like water, there is practically no resistance when it is incident on the boundary of a rarer medium like air.
- For a transverse wave, a crest is reflected as a crest and a trough is reflected as a trough when the wave is travelling from high density medium to low density medium.
- In case of a longitudinal wave, a compression is reflected as a rarefaction and a rarefaction is reflected as a compression at rearer boundary of another medium.

Quincke's Tube Experiment



- In this experiment, a sound wave is divided into two components having the same frequency and zero phase difference and the interference is observed. If the path of one of the component waves is altered, the interference pattern is disturbed.

- The apparatus consists of two U tubes—ABC and MNQ—whose limbs can be inserted into each other and can be pulled or pushed out.

- Tube ABC is provided with two side tubes: D and E. A sound source is placed near to the side tube D. The ear of the listener is held near E.

- Initially, path lengths ABC and MNQ are made equal. The waves arriving at E along the two paths are in phase. Hence, a loud sound is heard.

- Now, tube MNQ is gradually pulled out. When the path difference between the waves becomes $\lambda/2$, the waves reaching E via paths ABC and MNQ becomes 180°, out of phase and cancel each other.

- The compression due to one wave overlaps the rarefaction due to the other wave.

- On increasing the path difference, a loud sound is heard again when the path difference between the travelling waves becomes λ.

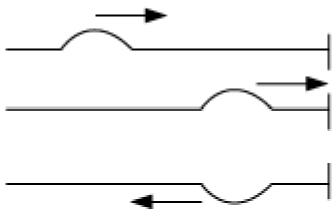
- Whenever the path difference becomes $n\lambda$, where $n = 0, 1, 2, 3, \ldots$, the intensity of sound becomes maximum.

- Whenever the path difference becomes $(n + 1/2)$, where $n = 0, 1, 2, 3, \ldots$, the intensity of sound becomes minimum.
- This experiment, besides illustrating interference, provides a good laboratory method of measuring the velocity of sound in air.

Standing Wave and Normal Modes in a Stretched String

Reflection of Waves

- At a rigid boundary, a pulse gets reflected.
- At the interface of elastic media, a part of the pulse is reflected and the rest is transmitted.



- For a rigid boundary, the reflected pulse differs by a phase of π.
- For a non-rigid boundary, the reflected pulse has the same phase.
- When a longitudinal wave is incident on a rigid wall, the compression of the wave is reflected back by the wall as a compression in the opposite direction.
- If a longitudinal wave is incident on a rarer medium, the compression reaching the boundary at the other end will be reflected as rarefaction.

At a rigid boundary

Incident travelling wave,

$$y_i(x,t) = a\sin(kx - \omega t)$$

Reflected wave is,

$$y_r(x,t) = a\sin(kx - \omega t + \pi)$$
$$= -a\sin(kx - \omega t)$$

At the boundary:

$$y = y_i(x,t) + y_r(x,t) = 0$$

Standing Wave and Normal Modes in a Stretched String

- Steady wave pattern in a string is called stationary wave.
- Incident and reflected waves are represented as

$y = a \sin(\omega t - kx)$ ...(1)

$y' = -a \sin(\omega t + kx)$ ...(2)

- Applying the principle of superposition,

  $y = y_1 + y_2$

$$
\begin{aligned}
&= a[\sin(\omega t - kx) - \sin(\omega t + kx)] \\
&\therefore y = +2a\cos\omega t \sin(-kx) \\
&\quad = -2a\sin kx \cos t\omega t \qquad (3)
\end{aligned}
$$

- Amplitude = 2*a* sin *kx* (varies from point to point)
- String vibrates in phase with different amplitudes at different points. This give rise to standing waves.
- **Nodes −** points of zero amplitude
- **Anti-nodes −** points of largest amplitude
- A system has a fixed set of natural frequencies called normal modes of oscillation.
- Positions of nodes,

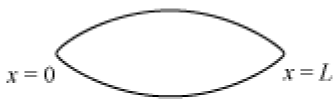
$$
\begin{aligned}
&\sin kx = 0 \\
&\Rightarrow kx = n\pi \qquad (n = 0, 1, 2, 3, \ldots) \\
&\because k = \frac{2\pi}{\lambda} \\
&\therefore x = \frac{n\lambda}{2}
\end{aligned}
$$

- Distance between two successive nodes = $\frac{\lambda}{2} \qquad (n = 1)$.
- Position of anti-nodes,

$$
\begin{aligned}
&|\sin kx = 1| \\
&\Rightarrow kx = \left(n + \frac{1}{2}\right)\pi \qquad (n = 0, 1, 2, 3, \ldots) \\
&x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}
\end{aligned}
$$

- Distance between two successive anti-nodes $= \frac{\lambda}{2}$.

- Boundary condition of a string:



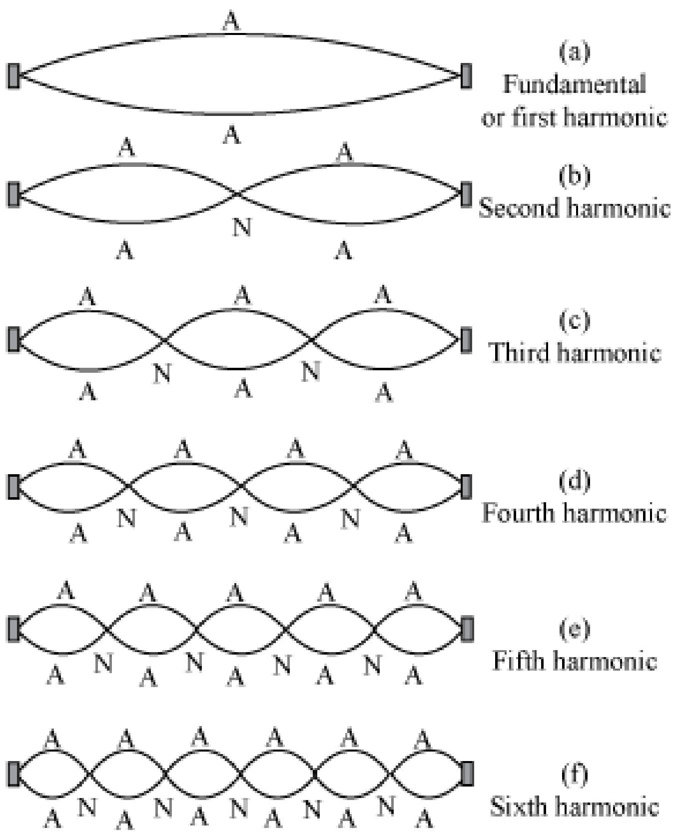
L = Length of the string

∴ From equation (3),

$$\sin kL = 0$$
$$\Rightarrow kL = n\pi$$
$$k = \frac{n\pi}{L} \qquad (n = 1, 2, 3, \ldots)$$
$$\because k = \frac{2\pi}{\lambda}$$
$$\lambda = \frac{2L}{n}$$

- Corresponding frequencies,

$$\nu = \frac{nv}{2L} \qquad (n = 1, 2, 3, \ldots)$$



Vibrating String					
Mode of vibration	Harmonic	Tone	Nodes	Anti-nodes	Frequency
First or fundamental	First	Fundamental	2	1	$\frac{v}{2L}$
Second	Second	First	3	2	$\frac{2v}{2L}$

n^{th}	n^{th}	$(n - 1)$ tones	$n + 1$	n	$\frac{nv}{2L}$

- Fundamental mode − Lowest possible natural frequency of a system

Laws of Vibrating Strings

The fundamental frequency of the vibrations in a stretched string,

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Here, L is the length of the vibrating string
T is the tension in the vibrating string
μ is the linear mass density of the vibrating string

1. **Law of length:** The fundamental frequency of transverse vibration of a stretched string is inversely proportional to the length of the vibrating string. This is subject to the condition that the tension (T) in the string and the linear density (μ) of the string remain constant.

$$f \propto \frac{1}{L}$$
$$f_1 L_1 = f_2 L_2 = \text{constant}$$

2. **Law of tension:** The fundamental frequency of transverse vibration is directly proportional to the square root of the tension (T) in the string if the length of the vibrating string (L) and linear density (μ) are kept constant.

$$f \propto \sqrt{T}$$
$$\frac{f_1}{\sqrt{T_1}} = \frac{f_2}{\sqrt{T_2}} = \text{constant}$$

3. **Law of linear density:** The fundamental frequency of transverse vibration of a stretched string is inversely proportional to the square root of the linear density of the string if the length of the vibrating string (L) and tension (T) are constant.

$$f \propto \frac{1}{\sqrt{\mu}}$$

$$f\sqrt{\mu} = \text{constant}$$

$$f_1\sqrt{\mu_1} = f_2\sqrt{\mu_2} = \text{constant} \quad \ldots . (1)$$

But $\mu = \frac{M}{l} = \frac{\rho V}{l} = \frac{\rho(\pi r^2 l)}{l} = \rho\pi r^2$.

From equation (1),

$$f_1\sqrt{\rho_1 \pi {r_1}^2} = f_2\sqrt{\rho_2 \pi {r_2}^2}$$

If the strings of same material are used $(\rho_1 = \rho_2)$, then

$$f_1 r_1 = f_2 r_2$$

$$f \propto \frac{1}{r}$$

If the strings of different materials are used $(\rho_1 \neq \rho_2)$, then

$$f \propto \frac{1}{\sqrt{\rho}} \quad (r_1 = r_2)$$

Standing Wave and Normal Modes in an Air Column and Beat

Normal Modes of Oscillation of an Air Column (One end closed and other end open)

- Closed end → Pressure is highest → Displacement of air particles is minimum (zero) → Node is formed ($x = 0$)
- Open end → Pressure is least → Displacement is maximum → Anti-node is formed ($x = L$)
- $$L = \left(n + \frac{1}{2}\right)\frac{\lambda}{2} \qquad (n = 0, 1, 2, \ldots)$$

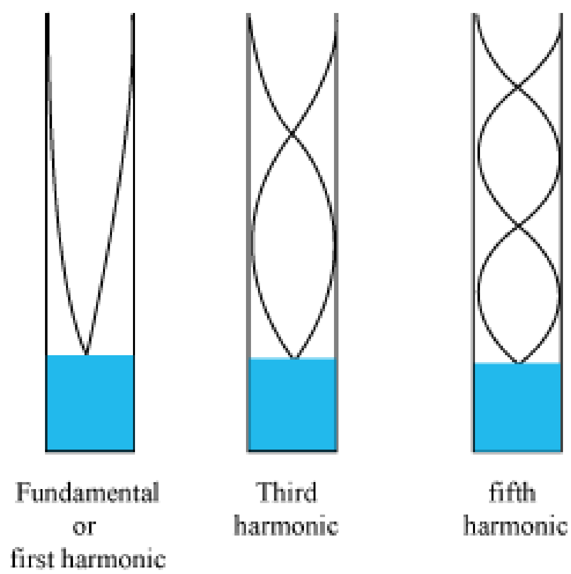
  $$\therefore \lambda = \frac{2L}{(n + 1/2)}$$
- Natural frequencies − normal modes

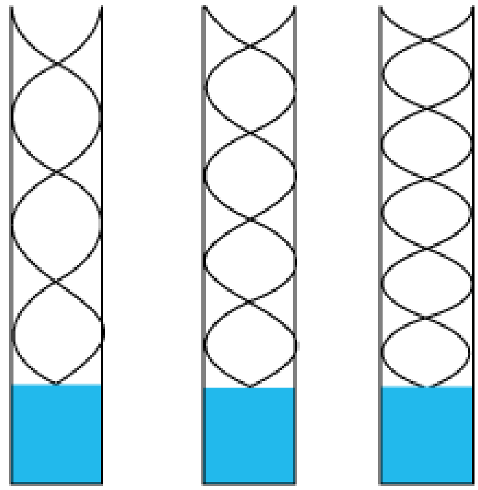
$$\nu = \left(n + \frac{1}{2}\right)\frac{v}{2L}\,(n = 0, 1, 2, \ldots)$$

- Fundamental frequency → for *n* = 0 → $\frac{v}{4L}$
- Odd harmonics → Higher frequencies → Odd multiples of fundamental frequencies

$$\left(\frac{3v}{4L}, \frac{5v}{4L}, \text{etc}\right)$$

- When external frequency is close to any of the natural frequencies, system is in resonance.





Normal modes of an air column − (Only one end open)

Beats

- Two sound waves of nearly the same frequency and amplitude produce beats.
- The resultant sound has alternate maxima and minima.

Analytical derivation

Consider two harmonic sound waves:

$S_1 = a \cos \omega_1 t$ (1)

$S_2 = a \cos \omega_2 t$ (2)

- Amplitude (a) is equal.
- Angular frequencies: $\omega_1 > \omega_2$)
- Displacement is longitudinal.

Applying the principle of superposition,

$S = S_1 + S_2$

$= a (\cos \omega_1 t + \cos \omega_2 t)$

$$= 2a\cos\left(\frac{(\omega_1 - \omega_2)t}{2}\right)\cos\left(\frac{(\omega_1 + \omega_2)t}{2}\right)$$

Let

$$\frac{(\omega_1 + \omega_2)}{2} = \omega_a$$
$$\frac{(\omega_1 - \omega_2)t}{2} = \omega_b$$

$$\therefore S = (2a\cos\omega_a t)\cos\omega_a t \qquad (3)$$
$$\left|\omega_1 - \omega_2\right| << \omega_1$$
$$\therefore \omega_a >> \omega_b$$

From equation (3),

- Angular frequency of resultant wave is ω_a.
- Amplitude is ($2a$ cos $\omega_b t$). Therefore, it is not constant. It varies from +1 to −1.

$$\omega_b = \omega_1 - \omega_2 \left(\omega = 2\pi\nu\right)$$

v → Frequency

Beat frequency $\nu_{\text{beat}} = \nu_1 - \nu_2$

End Correction

For the vibration of an air column in a pipe that is open at both the ends or closed at one end, the nodes should be formed at the closed end and the anti-nodes should be formed at the open end.

But practically, the anti-nodes are not formed at the open end but at a little distance beyond it. The distance between the open end and the closed end is known as end correction.

- The end correction is numerically expressed as e =0.3 d. Here, d is the inner diameter of the tube.
- The corrected length of the air column is the sum of the length of the air column in the pipe and the end correction

$L = l + 0.3d$

- For fundamental mode of vibration of an air column in a pipe closed at one end, $v = n\lambda = 4nL$ or

$v = 4n(l + 0.3d)$.

Cause of End Correction

- The cause of end correction is that the air particles in the plane of the open end of the tube are not free to move in all directions. Therefore, reflection takes place at the plane at a small distance outside the tube.

Calculation of End Correction

(i) Pipe closed at one end:

(i) Pipe closed at one end:

$$v = 4n_1L_1 = 4n_2L_2$$
$$n_1L_1 = n_2L_2$$
$$n_1\left(l_1 + e\right) = n_2\left(l_2 + e\right)$$
$$\Rightarrow e = \frac{(n_1l_1 - n_2l_2)}{(n_1 - n_2)}$$

(ii) Pipe open at both ends:

$$v = 2n_1L_1 = 2n_2L_2$$
$$n_1L_1 = n_2L_2$$
$$n_1\left(l_1 + e\right) = n_2\left(l_2 + e\right)$$
$$e = \frac{(n_1l_1 - n_2l_2)}{2(n_1 - n_2)}$$

Limitations of End Correction:

The various limitations of end correction are:

- Inner diameter of the tube must be constant throughout the length of the tube.
- Effects of air outside and that of the temperature of the air outside are to be neglected.
- The tuning fork must be held in a way such that the tip of its prong must be horizontal, at the centre and at a small distance above the open end of the tube.

Doppler Effect

Relative motion between a source of sound and a listener causes the apparent frequency of the sound heard by the listener to be different from the frequency of the sound emitted by the source.

Consider,

S → Source of sound

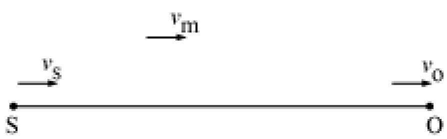
O → Observer

v → Frequency of sound emitted by the source

λ → Wavelength of the emitted sound

Suppose, the source and observer are at rest.

Velocity of sound in air, $v = v\lambda$.



Here,

v_s → Velocity of the source

v_m → Velocity of the medium

v_0 → Velocity of the observer

Let the distance between the source and observer = v (That is, v waves reach the observer from the source in 1 s).

∴ Frequency of the sound heard by the listener = v .

When the medium is in motion, the distance travelled by the sound wave = $v + v_m$.

Distance moved by source in one second, $SS' = v_S$.

Distance covered by the sound in 1 s = (v + v_m) − v_s (relative to the source).

Frequency (v) remains the same.

∴ Apparent wavelength, $\lambda' = \frac{[v + v_m - v_s]}{v} \qquad (1)$

Distance travelled by the observer $OO' = v_0$ (in 1 s).

Distance available in 1 s to $\lambda' = (v + v_m) - v_0$.

Apparent frequency of sound heard, $v' = \frac{(v + v_m) - v_0}{\lambda'}$.

From equation (1),

$$\boxed{v' = \frac{[(v + v_m) - v_0]}{v + v_m - v_s}}$$

Sign Conversion

Source → Listener (+ ve velocities)

Listener → Source (− ve velocity)

Medium at rest, v_m = 0.

$$\boxed{\therefore v' = \frac{v - v_0}{v - v_s} \times v}$$

Special Cases

- Source moving towards observer − Observer stationary:

  ∴v_s = + v_e and v_0 = 0

$$\therefore \nu' = \frac{v}{v - v_s} \times \nu$$

- Source moving away from observer − Observer stationary:

  $v_s = -v_e$ and $v_0 = 0$

$$\therefore \nu' = \frac{v}{v + v_s} \times \nu$$

- Source stationary − Observer moving away from source:

  $v_s = 0$ and $v_0 = +v_e$

$$\therefore \nu' = \frac{v - v_0}{v} \times \nu$$

- Source stationary − Observer moving towards source:

  $v_s = 0$ and $v_0 = -v_e$

$$\therefore \nu' = \frac{v - (-v_0)}{v} \times \nu = \frac{v + v_0}{v} \times \nu$$

- Source and observer approaching each other:

  $v_s = +v_e$ and $v_0 = -v_e$

$$\therefore \nu' = \frac{v + v_0}{v - v_s} \times \nu$$

- Source and observer moving away from each other:

  $v_s = -v_e$ and $v_0 = +v_e$

$$\therefore \nu' = \frac{v - v_o}{v + v_s} \times \nu$$

Applications of Doppler's effect:

- Doppler's effect is used to measure the velocities of moving objects in diverse areas such as military, medical science, astrophysics, etc.

- It is also used by police to check over-speeding of vehicles by using speed guns that produce an alarm when the speed exceeds the limit.

- It is used at airports to guide aircraft and in the military to detect enemy aircraft.

- Astrophysicists use it to measure the velocities of stars and also in determining the speed of gases in sun

- Doctors use it to study heart beat and blood flow in different parts of the body.
- When ultrasonic waves enter the body of a person, some of them are reflected back and give information about the motion of blood and pulsation of heart valves, as well as pulsation of the heart of a foetus in the womb of a pregnant woman.

Limitations of Doppler's effect:

- It is applicable when the velocities of the sources of sound and observer are much lower than the velocity of sound.

- The source and the observer must move in the same direction.

- The medium must be in rest; otherwise the formulae have to be modified.