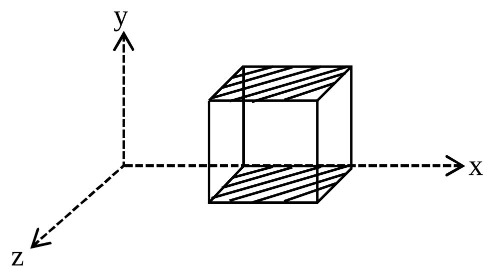
FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Wednesday 01st September, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is :



(1) 3.8×10^{-11} C (2) 8.3×10^{-11} C

(3) 3.8×10^{-12} C (4) 8.3×10^{-12} C

Official Ans. by NTA (2)

Sol. As electric field is in y-direction so electric flux is only due to top and bottom surface

Bottom surface y = 0

$\Rightarrow E = 0 \Rightarrow \phi = 0$

Top surface y = 0.5 m

$\Rightarrow E = 150\,(.5)^2 = \frac{150}{4}$

Now flux $\phi = EA = \frac{150}{4}(.5)^2 = \frac{150}{16}$

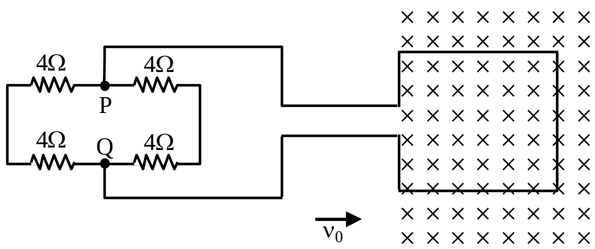
By Gauss's law $\phi = \frac{Q_{in}}{\epsilon_0}$

$\frac{150}{16} = \frac{Q_{in}}{\epsilon_0}$

$Q_{in} = \frac{150}{16} \times 8.85 \times 10^{-12} = 8.3 \times 10^{-11}$ C

Option (2)

2. A square loop of side 20 cm and resistance 1Ω is moved towards right with a constant speed v_0. The right arm of the loop is in a uniform magnetic field of 5T. The field is perpendicular to the plane of the loop and is going into it. The loop is connected to a network of resistors each of value 4Ω. What should be the value of v_0 so that a steady current of 2 mA flows in the loop ?

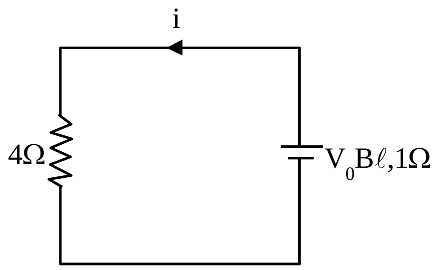


(1) 1 m/s (2) 1 cm/s

(3) 10^2 m/s (4) 10^{-2} cm/s

Official Ans. by NTA (2)

Sol. Equivalent circuit



$i = \frac{V_0B\ell}{4+1} \Rightarrow V_0 = \frac{5(2mA)}{5\times .2} = 10^{-2}$ m/s = 1 cm/s

Option (2)

3. The temperature of an ideal gas in 3-dimensions is 300 K. The corresponding de-Broglie wavelength of the electron approximately at 300 K, is :

[m_e = mass of electron = 9×10^{-31} kg

h = Planck constant = 6.6×10^{-34} Js

k_B = Boltzmann constant = 1.38×10^{-23} JK^{-1}]

(1) 6.26 nm (2) 8.46 nm

(3) 2.26 nm (4) 3.25 nm

Official Ans. by NTA (1)

Sol. De-Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

Where E is kinetic energy

$$E = \frac{3kT}{2} \text{ for gas}$$

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.6\times10^{-34}}{\sqrt{3\times9\times10^{-31}\times1.38\times10^{-23}\times300}}$$

$\lambda = 6.26 \times 10^{-9}$ m = 6.26 nm

Option (1)

4. A body of mass 'm' dropped from a height 'h' reaches the ground with a speed of $0.8\sqrt{gh}$. The value of workdone by the air-friction is :

(1) –0.68 mgh (2) mgh

(3) 1.64 mgh (4) 0.64 mgh

Official Ans. by NTA (1)

Sol. Work done = Change in kinetic energy

$$W_{mg} + W_{air\text{-}friction} = \frac{1}{2}m\left(.8\sqrt{gh}\right)^2 - \frac{1}{2}m(0)^2$$

$$W_{air-friction} = \frac{.64}{2}mgh - mgh = -0.68mgh$$

Option (1)

5. The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1, R_2 and H_1, H_2 respectively. Choose the correct option :

(1) $R_1 > R_2$ and $H_1 = H_2$ (2) $R_1 = R_2$ and $H_1 < H_2$

(3) $R_1 < R_2$ and $H_1 < H_2$ (4) $R_1 = R_2$ and $H_1 = H_2$

Official Ans. by NTA (2)

Sol. Range $R = \frac{u^2\sin 2\theta}{g}$ and same for θ and 90 – θ

So same for 42° and 48°

Maximum height $H = \frac{u^2\sin^2\theta}{2g}$

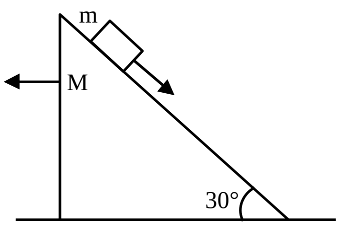
H is high for higher θ

So H for 48° is higher than H for 42°

Option (2)

6. A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is : Given m = 8 kg, M = 16 kg

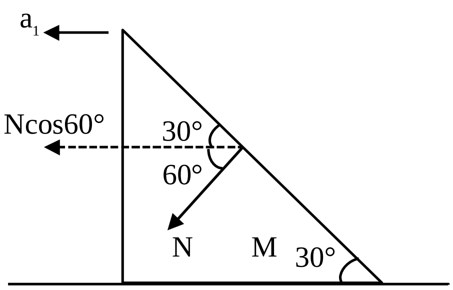
Assume all the surfaces shown in the figure to be frictionless.



(1) $\frac{4}{3}g$ (2) $\frac{6}{5}g$ (3) $\frac{3}{5}g$ (4) $\frac{2}{3}g$

Official Ans. by NTA (4)

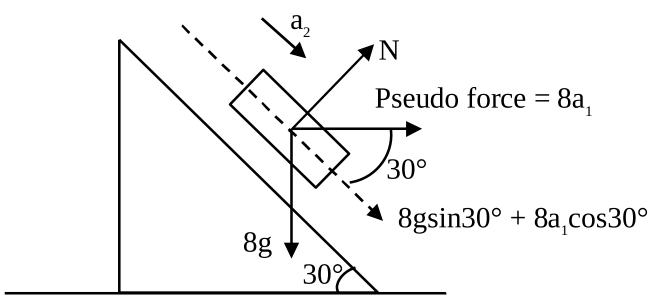
Sol. Let acceleration of wedge is a_1 and acceleration of block w.r.t. wedge is a_2



$$N\cos 60° = Ma_1 = 16a_1$$

$\Rightarrow N = 32a_1$

F.B.D. of block w.r.t wedge



$\perp$ to incline

$N = 8g\cos 30° - 8a_1\sin 30° \Rightarrow 32a_1 = 4\sqrt{3}g - 4a_1$

$$\Rightarrow a_1 = \frac{\sqrt{3}}{9}g$$

Along incline

$8g\sin 30° + 8a_1\cos 30° = ma_2 = 8a_2$

$$a_2 = g \times \frac{1}{2} + \frac{\sqrt{3}}{9} g . \frac{\sqrt{3}}{2} = \frac{2g}{3}$$

Option (4)

7. Due to cold weather a 1 m water pipe of cross–sectional area 1 cm^2 is filled with ice at –10°C. Resistive heating is used to melt the ice. Current of 0.5 A is passed through 4 kΩ resistance. Assuming that all the heat produced is used for melting, what is the minimum time required ?

(Given latent heat of fusion for water/ice $= 3.33 \times 10^5$ J kg^{-1}, specific heat of ice $= 2 \times 10^3$ J kg^{-1} and density of ice $= 10^3$ kg / m^3

(1) 0.353 s (2) 35.3 s

(3) 3.53 s (4) 70.6 s

Official Ans. by NTA (2)

Sol. mass of ice $m = \rho A\ell = 10^3 \times 10^{-4} \times 1 = 10^{-1}$ kg

Energy required to melt the ice

$Q = ms\Delta T + mL$

$= 10^{-1} (2 \times 10^3 \times 10 + 3.33 \times 10^5) = 3.53 \times 10^4$ J

$$Q = i^2RT \Rightarrow 3.53 \times 10^4 = \left(\frac{1}{2}\right)^2 \left(4 \times 10^3\right)(t)$$

Time = 35.3 sec

Option (2)

8. A student determined Young's Modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m/s^2, without any significant error, his observation are as following.

Physical Quantity	Least count of the Equipment used for measurement	Observed value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then the fractional error in the measurement of Y is :

(1) 0.0083 (2) 0.0155

(3) 0.155 (4) 0.083

Official Ans. by NTA (2)

Sol. $$y = \frac{MgL^3}{4bd^3\delta}$$

$$\frac{\Delta y}{y} = \frac{\Delta M}{M} + \frac{3\Delta L}{L} + \frac{\Delta b}{b} + \frac{3\Delta d}{d} + \frac{\Delta\delta}{\delta}$$

$$\frac{\Delta y}{y} = \frac{10^{-3}}{2} + \frac{3\times10^{-3}}{1} + \frac{10^{-2}}{4} + \frac{3\times10^{-2}}{4} + \frac{10^{-2}}{5}$$

$$= 10^{-3}\left[0.5 + 3 + 2.5 + 7.5 + 2\right] = 0.0155$$

Option (2)

9. Two resistors $R_1 = (4 \pm 0.8)\ \Omega$ and $R_2 = (4 \pm 0.4)\ \Omega$ are connected in parallel. The equivalent resistance of their parallel combination will be :

(1) (4 ± 0.4) Ω

(2) (2 ± 0.4) Ω

(3) (2 ± 0.3) Ω

(4) (4 ± 0.3) Ω

Official Ans. by NTA (3)

Sol. $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4} \Rightarrow R_{eq} = 2\Omega$$

Also $$\frac{\Delta R_{eq}}{R_{eq}^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\frac{\Delta R_{eq}}{4} = \frac{.8}{16} + \frac{.4}{16} = \frac{1.2}{16}$$

$\Delta R_{eq} = 0.3\Omega$

$R_{eq} = (2 \pm 0.3)\Omega$

Option (3)

10. The half life period of radioactive element x is same as the mean life time of another radioactive element y. Initially they have the same number of atoms. Then :

(1) x–will decay faster than y.

(2) y– will decay faster than x.

(3) x and y have same decay rate initially and later on different decay rate.

(4) x and y decay at the same rate always.

Official Ans. by NTA (2)

Sol. $(t_{1/2})_x = (\tau)_y$

$\Rightarrow \frac{\ell n2}{\lambda_x} = \frac{1}{\lambda_y} \Rightarrow \lambda_x = 0.693\ \lambda_y$

Also initially $N_x = N_y = N_0$

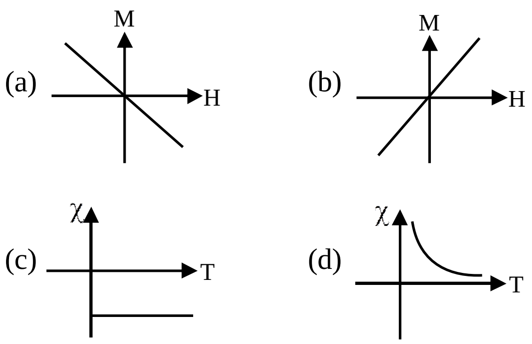
Activity $A = \lambda N$

As $\lambda_x < \lambda_y \Rightarrow A_x < A_y$

$\Rightarrow$ y will decay faster than x

Option (2)

11. Following plots show Magnetization (M) vs Magnetising field (H) and Magnetic susceptibility (χ) vs temperature (T) graph :



Which of the following combination will be represented by a diamagnetic material?

(1) (a), (c) (2) (a), (d)

(3) (b), (d) (4) (b), (c)

Official Ans. by NTA (1)

Sol. Conceptual question

Option (1)

12. A glass tumbler having inner depth of 17.5 cm is kept on a table. A student starts pouring water (μ = 4/3) into it while looking at the surface of water from the above. When he feels that the tumbler is half filled, he stops pouring water. Up to what height, the tumbler is actually filled ?

(1) 11.7 cm

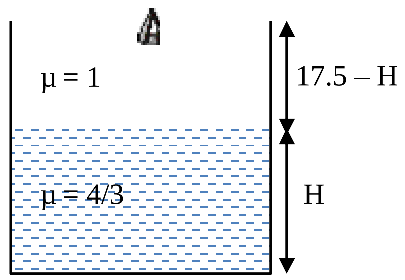
(2) 10 cm

(3) 7.5 cm

(4) 8.75 cm

Official Ans. by NTA (2)

Sol.



Height of water observed by observer

$= \frac{H}{\mu_w} = \frac{H}{(4/3)} = \frac{3H}{4}$

Height of air observed by observer = 17.5 – H

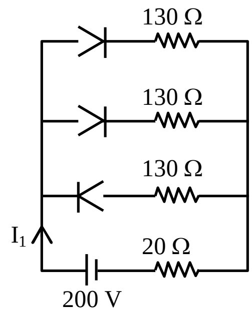
According to question, both height observed by observer is same.

$\frac{3H}{4} = 17.5 - H$

$\Rightarrow$ H = 10 cm

Option (2)

13. In the given figure, each diode has a forward bias resistance of 30Ω and infinite resistance in reverse bias. The current I_1 will be :

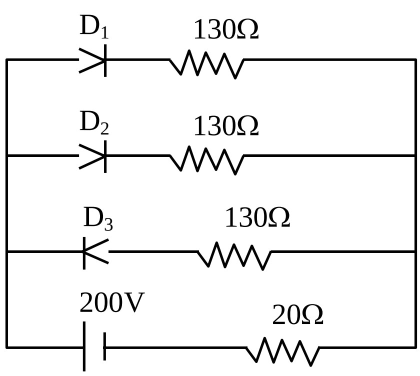


(1) 3.75 A (2) 2.35 A

(3) 2 A (4) 2.73 A

Official Ans. by NTA (3)

Sol.



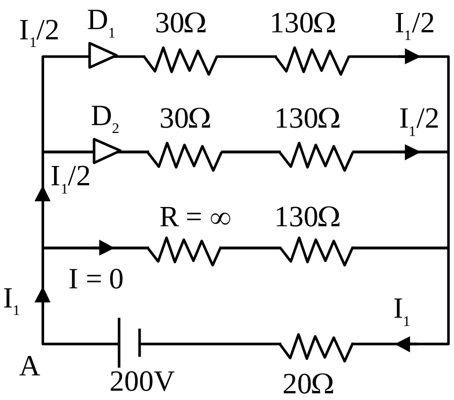
As per diagram,

Diode D_1 & D_2 are in forward bias i.e. R = 30Ω

whereas diode D_3 is in reverse bias i.e. R = infinite

$\Rightarrow$ Equivalent circuit will be

Applying KVL starting from point A



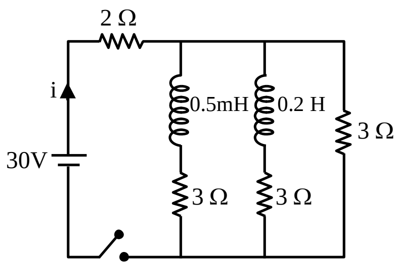
$$-\left(\frac{I_1}{2}\right)\times 30-\left(\frac{I_1}{2}\right)\times 130-I_1\times 20+200=0$$

$\Rightarrow -100\, I_1 + 200 = 0$

$I_1 = 2$

Option (3)

14. For the given circuit the current i through the battery when the key in closed and the steady state has been reached is__________.

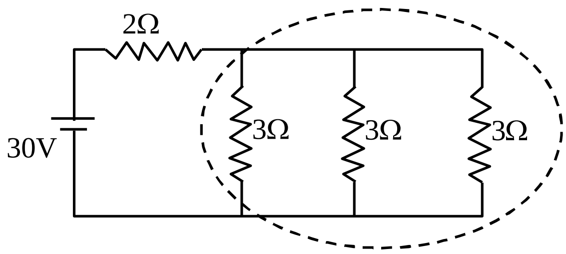


(1) 6 A (2) 25 A (3) 10 A (4) 0 A

Official Ans. by NTA (3)

Sol. In steady state, inductor behaves as a conducting wire.

So, equivalent circuit becomes

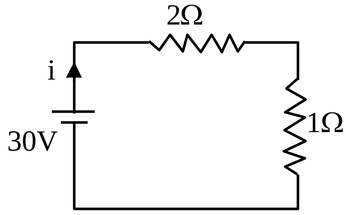


Parallel (R_{eq})

$$\frac{1}{R_{eq}}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

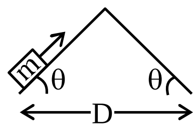
$\Rightarrow R_{eq} = 1\Omega$

$\Rightarrow$ Circuit becomes



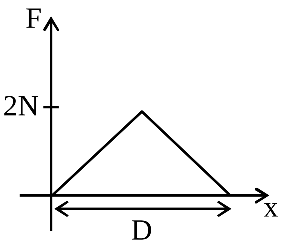
$$\Rightarrow\ i=\frac{30}{3}=10\text{A}$$

15. An object of mass 'm' is being moved with a constant velocity under the action of an applied force of 2N along a frictionless surface with following surface profile.

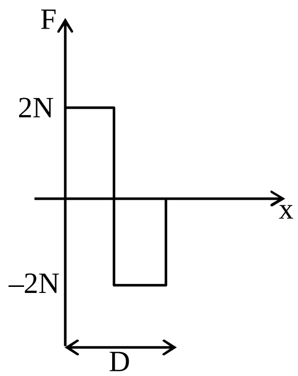


The correct applied force vs distance graph will be:

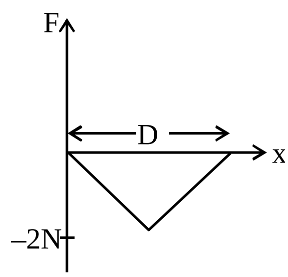
(1)

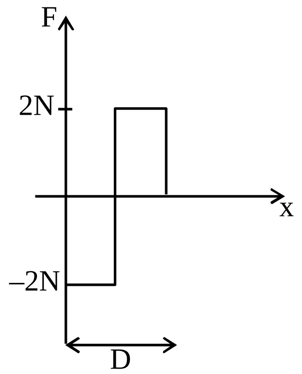


(2)



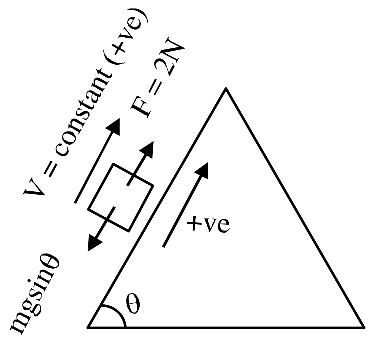
(3)



(4) 

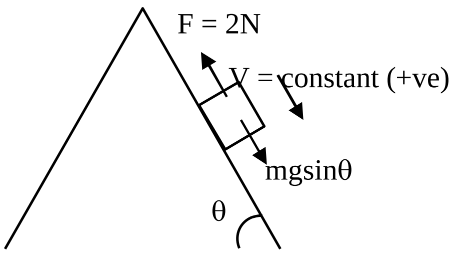
Official Ans. by NTA (2)

Sol. During upward motion



F = 2N = (+ve) constant

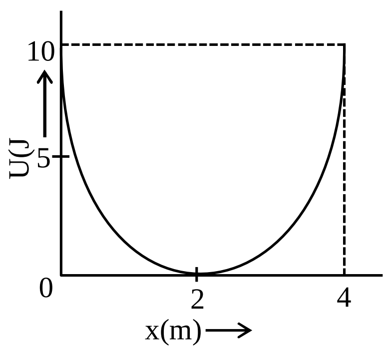
During downward motion



$\Rightarrow$ F = 2N = (–ve) constant

$\Rightarrow$ Best possible answer is option (2)

16. A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure. A simple pendulum of length 4 m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed?



(1) 10 m/s^2 (2) 5 m/s^2

(3) 4 m/s^2 (4) 9.8 m/s^2

Official Ans. by NTA (3)

Sol. From potential energy curve

$$U_{max} = \frac{1}{2}kA^2 \Rightarrow 10 = \frac{1}{2}k(2)^2$$

$$\Rightarrow k = 5$$

Now $T_{spring} = T_{pendulum}$

$$2\pi\sqrt{\frac{5}{5}} = 2\pi\sqrt{\frac{4}{g}}$$

$$\Rightarrow 1 = \sqrt{\frac{4}{g}} \Rightarrow g = 4 \text{ on planet}$$

Option (3)

17. A capacitor is connected to a 20 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 2 V in 1 μs. The capacitance of the capacitor is ....μF.

Given : $\ln\left(\frac{10}{9}\right) = 0.105$

(1) 9.52 (2) 0.95

(3) 0.105 (4) 1.85

Official Ans. by NTA (2)

Sol. $V = V_0\left(1-e^{-t/RC}\right)$

$$2 = 20\left(1-e^{-t/RC}\right)$$

$$\frac{1}{10} = 1 - e^{-t/RC}$$

$$e^{-t/RC} = \frac{9}{10}$$

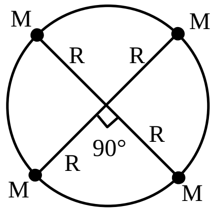
$$e^{t/RC} = \frac{10}{9}$$

$$\frac{t}{RC} = \ell n\left(\frac{10}{9}\right) \Rightarrow C = \frac{t}{R\ell n\left(\frac{10}{9}\right)}$$

$$C = \frac{10^{-6}}{10 \times .105} = .95\mu F$$

Option (2)

18. Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is :

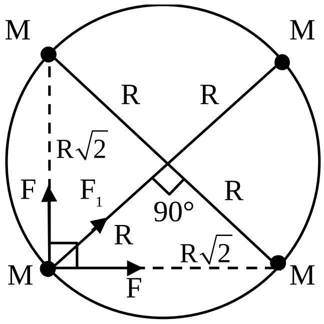


(1) $\frac{1}{2}\sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$ (2) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$

(3) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$ (4) $\sqrt{\frac{GM}{R}}$

Official Ans. by NTA (2)

Sol.



$$F_{net} = \frac{MV^2}{R}$$

$$\sqrt{2}F + F_1 = \frac{MV^2}{R}$$

$$\sqrt{2}\frac{GMM}{(\sqrt{2}R)^2} + \frac{GMM}{(2R)^2} = \frac{MV^2}{R}$$

$$\frac{GM}{R}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = V^2$$

$$\frac{GM}{R}\left(\frac{4+\sqrt{2}}{4\sqrt{2}}\right) = V^2$$

$$V = \sqrt{\frac{GM(4+\sqrt{2})}{R4\sqrt{2}}}$$

$$V = \frac{1}{2}\sqrt{\frac{GM(2\sqrt{2}+1)}{R}}$$

Option (2)

19. Electric field of plane electromagnetic wave propagating through a non–magnetic medium is given by $E = 20\cos(2 \times 10^{10}\, t - 200x)$ V/m. The dielectric constant of the medium is equal to : (Take $\mu_r = 1$)

(1) 9 (2) 2 (3) $\frac{1}{3}$ (4) 3

Official Ans. by NTA (1)

Sol. Speed of wave = $\frac{2\times10^{10}}{200} = 10^8\,m/s$

Refractive index = $\frac{3\times10^8}{10^8} = 3$

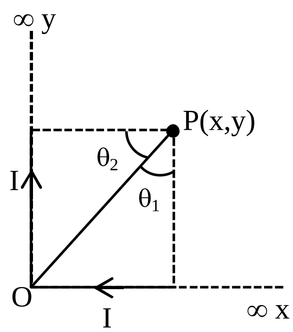
Now refractive index = $\sqrt{\varepsilon_r\mu_r}$

$3 = \sqrt{\varepsilon_r(1)}$

$\Rightarrow \varepsilon_r = 9$

Option (1)

20. There are two infinitely long straight current carrying conductors and they are held at right angles to each other so that their common ends meet at the origin as shown in the figure given below. The ratio of current in both conductor is 1 : 1. The magnetic field at point P is _____.



(1) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}+(x+y)\right]$

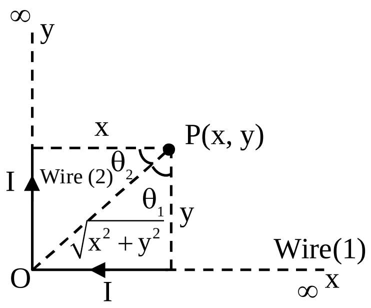
(2) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(3) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(4) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}+(x+y)\right]$

Official Ans. by NTA (1)

Sol.



$$B_{\text{due to wire (1)}} = \frac{\mu_o I}{4\pi y}\left[\sin 90 + \sin\theta_1\right]$$

$$= \frac{\mu_0}{4\pi}\frac{I}{y}\left(1 + \frac{x}{\sqrt{x^2+y^2}}\right) \quad ......(1)$$

$$B_{\text{due to wire (2)}} = \frac{\mu_0}{4\pi}\frac{I}{x}\left(\sin 90° + \sin\theta_2\right)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{x}\left(1 + \frac{y}{\sqrt{x^2+y^2}}\right) \quad .....(2)$$

Total magnetic field

$B = B_1 + B_2$

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{1}{y} + \frac{x}{y\sqrt{x^2+y^2}} + \frac{1}{x} + \frac{y}{x\sqrt{x^2+y^2}}\right]$$

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{x+y}{xy} + \frac{x^2+y^2}{xy\sqrt{x^2+y^2}}\right]$$

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{x+y}{xy} + \frac{\sqrt{x^2+y^2}}{xy}\right]$$

$$B = \frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2} + (x+y)\right]$$

Option (1)

SECTION-B

1. The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0 °C without changing the pressure of the gas. The molecules in the gas rotate but do not oscillate. If the ratio of change in internal energy of the gas to the amount of workdone by the gas is $\frac{x}{10}$. Then the value of x (round off to the nearest integer) is _______.

(Given R = 8.31 J mol^{-1} K^{-1})

Official Ans. by NTA (25)

Sol. Pressure is not changing ⇒ isobaric process

$$\Rightarrow \Delta U = nC_v\Delta T = \frac{5nR\Delta T}{2}$$

and W = nRΔT

$$\frac{\Delta U}{W} = \frac{5}{2} = \frac{x}{10} \Rightarrow x = 25.00$$

2. The width of one of the two slits in a Young's double slit experiment is three times the other slit. If the amplitude of the light coming from a slit is proportional to the slit-width, the ratio of minimum to maximum intensity in the interference pattern is x : 4 where x is _______.

Official Ans. by NTA (1)

Sol. Given amplitude ∝ slit width

Also intensity ∝ $(\text{Amplitude})^2$ ∝ $(\text{Slit width})^2$

$$\frac{I_1}{I_2} = \left(\frac{3}{1}\right)^2 = 9 \Rightarrow I_1 = 9I_2$$

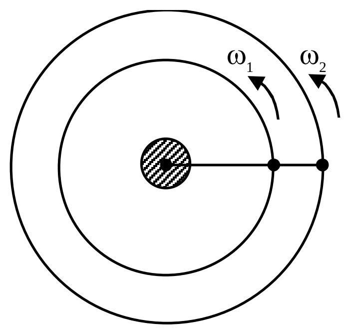
$$\frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1}-\sqrt{I_2}}{\sqrt{I_1}+\sqrt{I_2}}\right)^2 = \left(\frac{3-1}{3+1}\right)^2 = \frac{1}{4} = \frac{x}{4}$$

⇒ x = 1.00

3. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period of revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is 2 × 10^3 km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x}$ rad h^{-1} where x is ....

Official Ans. by NTA (3)

Sol.



T_1 = 1 hour

⇒ ω_1 = 2π rad/hour

T_2 = 8 hours

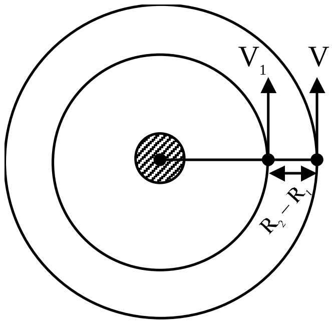
$\Rightarrow \omega_2 = \frac{\pi}{4}$ rad / hour

$R_1 = 2 \times 10^3$ km

As $T^2 \propto R^3$

$$\Rightarrow \left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2$$

$$\Rightarrow \frac{R_2}{R_1} = \left(\frac{8}{1}\right)^{2/3} = 4 \Rightarrow R_2 = 8 \times 10^3 \text{ km}$$



$$V_1 = \omega_1 R_1 = 4\pi \times 10^3 \text{ km / h}$$

$$V_2 = \omega_2 R_2 = 2\pi \times 10^3 \text{ km / h}$$

$$\text{Relative } \omega = \frac{V_1 - V_2}{R_2 - R_1} = \frac{2\pi \times 10^3}{6 \times 10^3}$$

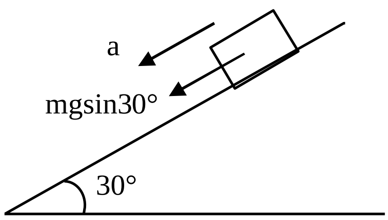
$= \frac{\pi}{3}$ rad / hour

x = 3

4. When a body slides down from rest along a smooth inclined plane making an angle of 30° with the horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time αT, where α is a constant greater than 1. The co-efficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2 - 1}{\alpha^2}\right)$ where x = ...

Official Ans. by NTA (3)

Sol.

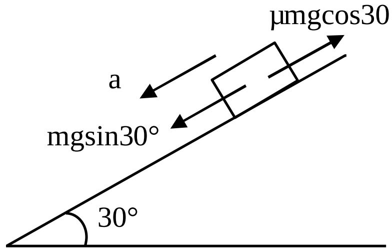


On smooth incline

a = g sin30°

by $S = ut + \frac{1}{2}at^2$

$$S = \frac{1}{2}\frac{g}{2}T^2 = \frac{g}{4}T^2 \text{ ....(i)}$$



On rough incline

a = g sin30° – μg cos 30°

by $S = ut + \frac{1}{2}at^2$

$$S = \frac{1}{4}g\left(1 - \sqrt{3}\mu\right)(\alpha T)^2 \text{ ...(ii)}$$

By (i) and (ii)

$$\frac{1}{4}gT^2 = \frac{1}{4}g\left(1 - \sqrt{3}\mu\right)\alpha^2 T^2$$

$$\Rightarrow 1 - \sqrt{3}g = \frac{1}{\alpha^2} \Rightarrow g = \left(\frac{\alpha^2 - 1}{\alpha^2}\right) \cdot \frac{1}{\sqrt{3}}$$

$\Rightarrow$ x = 3.00

5. The average translational kinetic energy of N_2 gas molecules at ....°C becomes equal to the K.E. of an electron accelerated from rest through a potential difference of 0.1 volt.

(Given $k_B = 1.38 \times 10^{-23}$ J/K)

(Fill the nearest integer).

Official Ans. by NTA (500)

Sol. Given

Translation K.E. of N_2 = K.E. of electron

$$\frac{3}{2}kT = eV$$

$$\frac{3}{2} \times 1.38 \times 10^{-23} T = 1.6 \times 10^{-19} \times 0.1$$

$$\Rightarrow T = 773k$$

T = 773 – 273 = 500°C

6. A uniform heating wire of resistance 36 Ω is connected across a potential difference of 240 V. The wire is then cut into half and potential difference of 240 V is applied across each half separately. The ratio of power dissipation in first case to the total power dissipation in the second case would be 1 : x, where x is...........

Official Ans. by NTA (4)

Sol. First case $P_1 = \frac{V^2}{R} = \frac{(240)^2}{36}$

Second case Resistance of each half = 18 Ω

$$P_2 = \frac{(240)^2}{18} + \frac{(240)^2}{18} = \frac{(240)^2}{9}$$

$$\frac{P_1}{P_2} = \frac{1}{4}$$

x = 4.00

7. A steel rod with y = 2.0 × 10^{11} Nm^{-2} and α = 10^{-5} $°C^{-1}$ of length 4 m and area of cross-section 10 cm^2 is heated from 0° C to 400°C without being allowed to extend. The tension produced in the rod is x × 10^5 N where the value of x is ....

Official Ans. by NTA (8)

Sol. Thermal force F = Ay∝ΔT

$F = (10 \times 10^{-4})(2 \times 10^{11})(10^{-5})(400)$

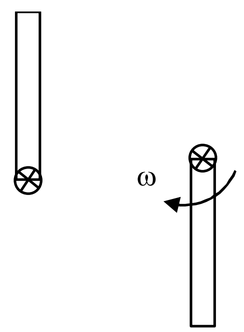
$F = 8 \times 10^5$ N

$\Rightarrow x = 8$

8. A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity, Ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is ....ms^{-1}. (Take g = 10 ms^{-2})

Official Ans. by NTA (6)

Sol.



by energy conservation $mg\ell = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{m\ell^2\omega^2}{3}$

$$\Rightarrow \omega = \sqrt{\frac{6g}{\ell}}$$

Speed $v = \omega r = \omega\ell = \sqrt{6g\ell}$

$v = \sqrt{6 \times 10 \times .6} = 6m/s$

9. A carrier wave with amplitude of 250 V is amplitude modulated by a sinusoidal base band signal of amplitude 150 V. The ratio of minimum amplitude to maximum amplitude for the amplitude modulated wave is 50 : x, then value of x is ....

Official Ans. by NTA (200)

Sol. $A_{max} = A_C + A_m = 250 + 150 = 400$

$A_{min} = A_C - A_m = 250 - 150 = 100$

$$\frac{A_{min}}{A_{max}} = \frac{100}{400} = \frac{1}{4} = \frac{50}{200}$$

x = 200

10. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40,000 kg is moving with a speed of 72 kmh^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is .... × 10^5 N/m.

Official Ans. by NTA (16)

Sol. Work = ΔK.E.

$$W_{friction} + W_{spring} = 0 - \frac{1}{2}mv^2$$

$$-\frac{90}{100}\left(\frac{1}{2}mv^2\right) + W_{Spring} = -\frac{1}{2}mv^2$$

$$W_{spring} = -\frac{10}{100} \times \frac{1}{2}mv^2$$

$$-\frac{1}{2}kx^2 = -\frac{1}{20}mv^2$$

$$\Rightarrow k = \frac{40000 \times (20)^2}{10 \times (1)^2} = 16 \times 10^5$$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Wednesday 01st September, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Water sample is called cleanest on the basis of which one of the BOD values given below

(1) 11 ppm (2) 15 ppm

(3) 3 ppm (4) 21 ppm

Official Ans. by NTA (3)

Sol. Clean water could have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

2. Calamine and Malachite, respectively, are the ores of :

(1) Nickel and Aluminium

(2) Zinc and Copper

(3) Copper and Iron

(4) Aluminium and Zinc

Official Ans. by NTA (2)

Sol. Calamine $\Rightarrow ZnCO_3$

Malachite $\Rightarrow Cu(OH)_2 \cdot CuCO_3$

3. Experimentally reducing a functional group **cannot** be done by which one of the following reagents ?

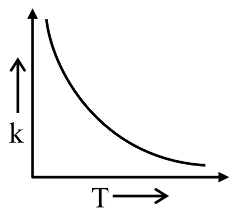
(1) Pt-C/H_2 (2) Na/H_2

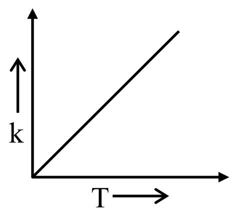
(3) Pd-C/H_2 (4) Zn/H_2O

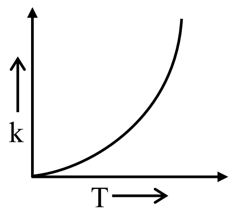
Official Ans. by NTA (2)

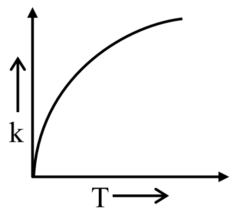
Sol. Solution NaH_2 is not reducing agent

4. Which one of the following given graphs represents the variation of rate constant (k) with temperature (T) for an endothermic reaction ?

(1) 

(2) 

(3) 

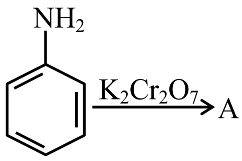
(4) 

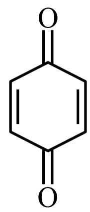
Official Ans. by NTA (3)

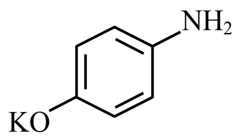
Sol. By observation we get this plot during measurable temperatures

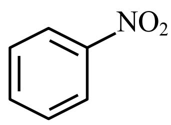
Ans. 3rd Option.

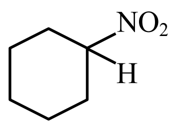
5. Identify A in the following reaction.



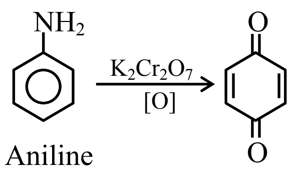
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. 

Aniline [A]

6. In the following sequence of reactions a compound **A**, (molecular formula $C_6H_{12}O_2$) with a straight chain structure gives a C_4 carboxylic acid. **A** is :

$$\mathbf{A} \xrightarrow[H_3O^+]{LiAlH_4} \mathbf{B} \xrightarrow{\text{Oxidation}} C_4 - \text{carboxylic acid}$$

(1) $CH_3 - CH_2 - COO - CH_2 - CH_2 - CH_3$

(2) $CH_3{-}CH_2{-}CH(OH){-}CH_2{-}O{-}CH{=}CH_2$

(3) $CH_3 - CH_2 - CH_2 - COO - CH_2 - CH_3$

(4) $CH_3 - CH_2 - CH_2 - O - CH = CH - CH_2 - OH$

Official Ans. by NTA (3)

$$CH_3-CH_2-CH_2-\underset{\underset{O}{\|}}{C}-O-CH_2-CH_3 \quad (A)\ [C_6H_{12}O_2]$$

$$\downarrow \text{(1) } LiAlH_4 \text{ (2) } H_3O^+$$

$$CH_3-CH_2-CH_2-CH_2-OH + CH_3-CH_2-$$

(B)

$$\downarrow [O]$$

$$CH_3-CH_2-CH_2-\underset{\underset{O}{\|}}{C}-OH \quad [C_4 \text{ carboxylic acid}]$$

7. Match **List – I** with **List - II**.

List -I (Colloid Preparation Method)		List -II (Chemical Reaction)	
(a)	Hydrolysis	(i)	$2AuCl_3 + 3HCHO + 3H_2O \rightarrow 2Au(sol) + 3HCOOH + 6HC1$
(b)	Reduction	(ii)	$As_2O_3 + 3H_2S \rightarrow As_2S_3(sol) + 3H_2O$
(c)	Oxidation	(iii)	$SO_2 + 2H_2S \rightarrow 3S(sol) + 2H_2O$
(d)	Double Decomposition	(iv)	$FeCl_3 + 3H_2O \rightarrow Fe(OH)_3(sol) + 3HCl$

Choose the most appropriate answer from the options given below.

(1) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(2) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii)

(3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Official Ans. by NTA (2)

Sol. According to type of reactions for preparation, colloids have been classified

8. The Crystal Field Stabilization Energy (CFSE) and magnetic moment (spin-only) of an octahedral aqua complex of a metal ion (M^{z+}) are $-0.8\ \Delta_0$ and 3.87 BM, respectively. Identify (M^{Z+}) :

(1) V^{3+} (2) Cr^{3+}

(3) Mn^{4+} (4) Co^{2+}

Official Ans. by NTA (4)

Sol. $V^{3+} \Rightarrow$ [ ][ ] e_g $= 2 \times 0.4\ \Delta_0$

[1][1][ ] t_{2g} $= -0.8\ \Delta_0$

$= 2$ unpaired e^-

$\mu = 2.89$ Bm

$Co^{2+} \Rightarrow$ [1][1] e_g $[2 \times 0.6\ \Delta_0 - 5 \times 0.4\ \Delta_0]$

$= -0.8\ \Delta_0$

[⥮][⥮][1] t_{2g} 3 unpaired $e^- \Rightarrow \mu = 3.87$ BM

hence d^7 configuration is of Co^{2+} Ans.

9. Monomer units of Dacron polymer are :

(1) ethylene glycol and phthalic acid

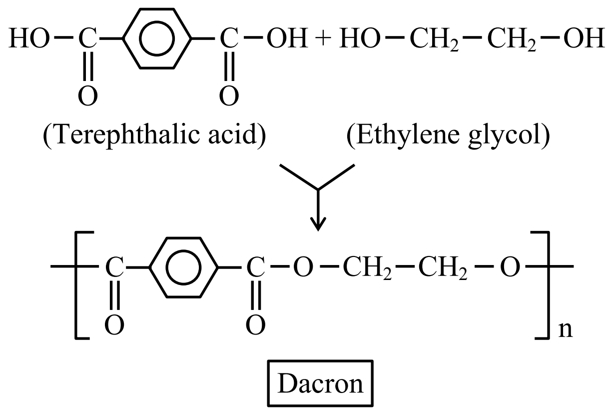
(2) ethylene glycol and terephthalic acid

(3) glycerol and terephthalic acid

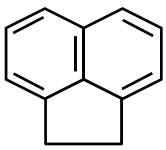
(4) glycerol and phthalic acid

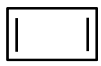
Official Ans. by NTA (2)

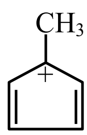
Sol.

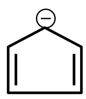


10. Which one of the following compounds is aromatic in nature ?

(1) 

(2) 

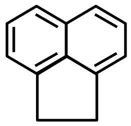
(3) 

(4) 

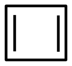
Official Ans. by NTA (4)

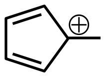
Allen Ans. (1,4)

Sol. (1) (Acenaphthene)

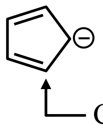

$\rightarrow$ 10 πe^- in cyclic conjugation

$\rightarrow$ $\boxed{\text{Aromatic}}$

(2)  $\rightarrow$ 4πe^- in ring conjugation $\Rightarrow$ Anti Aromatic

(3)  $\Rightarrow$ 4πe^- in ring conjugation $\Rightarrow$ Antiaromatic

(4)

 $\Rightarrow$ 6πe^- in ring conjugation $\Rightarrow$ $\boxed{\text{Aromatic}}$

Cyclopentadienyl anion

11. In the given chemical reaction, colors of the Fe^{2+} and Fe^{3+} ions, are respectively :

$5Fe^{2+} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$

(1) Yellow, Orange (2) Yellow, Green

(3) Green, Orange (4) Green, Yellow

Official Ans. by NTA (4)

Sol. Colour of Fe^{2+} is observed green and Fe^{3+} is yellow

12. The stereoisomers that are formed by electrophilic addition of bromine to trans-but-2-ene is/are :

(1) 2 enantiomers and 2 mesomers

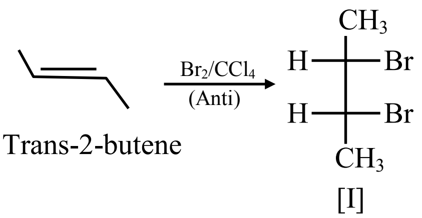
(2) 2 identical mesomers

(3) 2 enantiomers

(4) 1 racemic and 2 enantiomers

Official Ans. by NTA (2)

Sol.



meso product

13. Hydrogen peroxide reacts with iodine in basic medium to give :

(1) IO_4^- (2) IO^- (3) I^- (4) IO_3^-

Official Ans. by NTA (3)

Sol. $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$

14. In the following sequence of reactions,

$C_3H_6 \xrightarrow{H^+/H_2O} \textbf{A} \xrightarrow[\text{dil KOH}]{\text{KIO}} \textbf{B} + \textbf{C}$

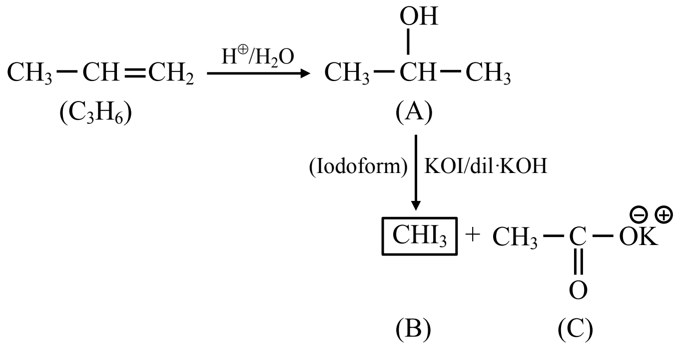
The compounds **B** and **C** respectively are :

(1) CI_3COOK, HCOOH (2) CI_3COOK, CH_3I

(3) CH_3I, HCOOK (4) CHI_3, CH_3COOK

Official Ans. by NTA (4)

Sol.



15. Given below are **two** statements :

Statement I : The nucleophilic addition of sodium hydrogen sulphite to an aldehyde or a ketone involves proton transfer to form a stable ion.

Statement II : The nucleophilic addition of hydrogen cyanide to an aldehyde or a ketone yields amine as final product.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true.

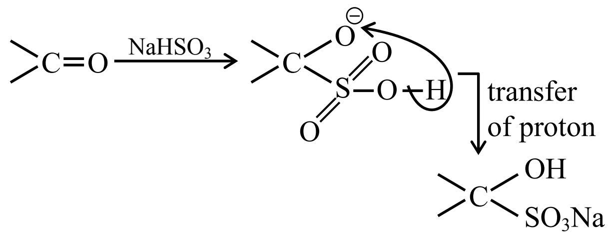
(2) **Statement I** is true but **Statement II** is false.

(3) **Statement I** is false but **Statement II** is true.

(4) Both **Statement I** and **Statement II** are false.

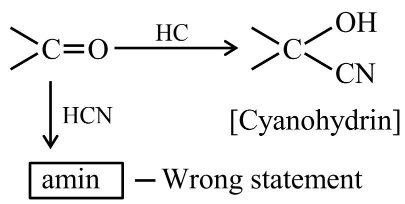
Official Ans. by NTA (2)

Sol. **Statement I :** Correct



(White crystalline soluble ppt)

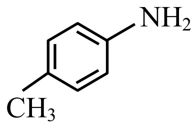
Statement II :

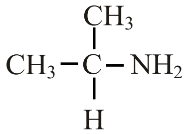
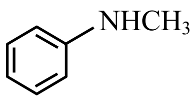


— Wrong statement

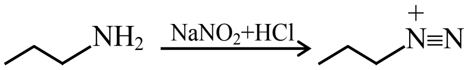
(Amine not formed)

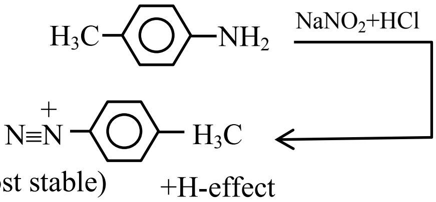
16. Which one of the following gives the most stable Diazonium salt ?

(1) $CH_3–CH_2–CH_2–NH_2$ (2) 

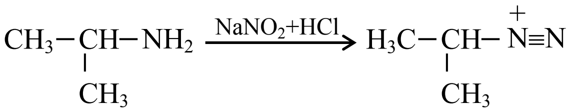
(3)  (4) 

Official Ans. by NTA (2)

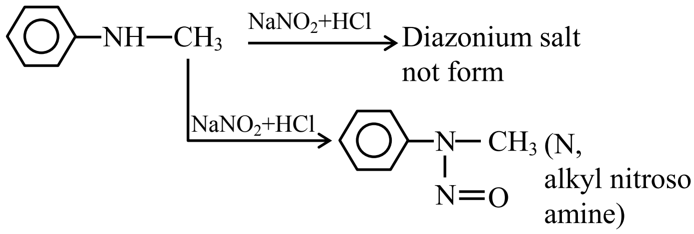
Sol. (1) 

(2) 

(3)



(4)



17. The potassium ferrocyanide solution gives a Prussian blue colour, when added to :

(1) $CoCl_3$ (2) $FeCl_2$

(3) $CoCl_2$ (4) $FeCl_3$

Official Ans. by NTA (4)

Sol. $FeCl_3 + K_4[Fe(CN)_6] \rightarrow Fe_4[Fe(CN)_6]_3$

Prussian blue

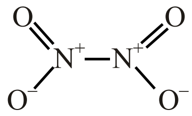
18. The oxide **without** nitrogen-nitrogen bond is :

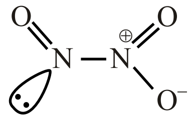
(1) N_2O (2) N_2O_4

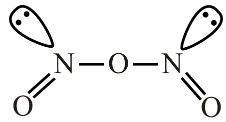
(3) N_2O_3 (4) N_2O_5

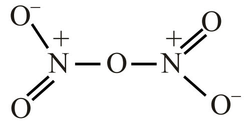
Official Ans. by NTA (4)

Sol. (1) $N\equiv N^+—O^-$

(2) 

(3)  unsymmetrical

 symmetrical

(4) 

19. Number of paramagnetic oxides among the following given oxides is ___________.

Li_2O, CaO, Na_2O_2, KO_2, MgO and K_2O

(1) 1 (2) 2

(3) 3 (4) 0

Official Ans. by NTA (1)

Sol. $Li_2O \Rightarrow 2Li^+ \quad O^{2-}$ $\qquad MgO \Rightarrow Mg^{2+} \quad O^{2-}$

$CaO \Rightarrow Ca^{2+} \quad O^{2-}$ $\qquad K_2O \Rightarrow 2K^+ \quad O^{2-}$

$Na_2O_2 \Rightarrow 2Na^+ \quad O_2^{2-}$

$KO_2 \Rightarrow K^+ \quad O_2^-$

$O_2^- \Rightarrow$ Complete octet, diamagnetic

$O^{2-} \Rightarrow \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2px}^2\ \pi_{2py}^2 \simeq \pi_{2pz}^2\ \pi_{2py}^{*2} \simeq \pi_{2pz}^{*2}$ (dia)

$O_2^- \Rightarrow \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2px}^2\ \pi_{2py}^2 \simeq \pi_{2pz}^2\ \pi_{2py}^{*2} \simeq \pi_{2pz}^{*1}$ (para)

20. Identify the element for which electronic configuration in +3 oxidation state is $[Ar]3d^5$:

(1) Ru (2) Mn

(3) Co (4) Fe

Official Ans. by NTA (4)

Sol. Fe^{3+} $[Ar]\ 3d^5$

SECTION-B

1. An empty LPG cylinder weighs 14.8 kg. When full, it weighs 29.0 kg and shows a pressure of 3.47 atm. In the course of use at ambient temperature, the mass of the cylinder is reduced to 23.0 kg. The final pressure inside of the cylinder is ______atm. (Nearest integer)

(Assume LPG of be an ideal gas)

Official Ans. by NTA (2)

Sol. Initial mass of gas = 29 – 14.8 = 14.2 Kg
mass of gas used = 29 – 23 = 6 Kg
gas left = 14.2 – 6 = 8.2 Kg

(1) $3.47 \times V = \left(\frac{14.2 \times 10^3}{M}\right) \times R \times T$

(2) $p \times V = \left(\frac{8.2 \times 10^3}{M}\right) \times R \times T$

Divide :

$\frac{(1)}{(2)} \Rightarrow \frac{3.47}{P} = \frac{14.2}{8.2}$

P = 2.003

2. The molar solubility of $Zn(OH)_2$ in 0.1 M NaOH solution is $x \times 10^{-18}$ M. The value of x is _____(Nearest integer)

(Given : The solubility product of $Zn(OH)_2$ is 2×10^{-20})

Official Ans. by NTA (2)

Sol. $Zn(OH)_2\,(s) \rightleftharpoons Zn^{+2}\,(aq) + 2OH^-\,(aq)$

$\qquad\qquad S \qquad (0.1 + 2s) \simeq 0.1$

$K_{sp} = S(0.1)^2$

$2 \times 10^{-20} = s \times 10^{-2} \Rightarrow s = 2 \times 10^{-18}$

$= x \times 10^{-18}$

x = 2

3. For the reaction $2NO_2(g) \rightleftharpoons N_2O_4(g)$, when $\Delta S = -176.0\ JK^{-1}$ and $\Delta H = -57.8\ kJ\ mol^{-1}$, the magnitude of ΔG at 298 K for the reaction is_____ $kJ\ mol^{-1}$. (Nearest integer)

Official Ans. by NTA (5)

Sol. $\Delta G = \Delta H - T\Delta S$

$\Delta G = 57.8 - \frac{298(-176)}{1000}$

$\Delta G = -5.352$ kJ/mole

|Nearest integer value| = 5

4. The sum of oxidation states of two silver ions in $[Ag(NH_3)_2]\,[Ag(CN)_2]$ complex is _______.

Official Ans. by NTA (2)

Sol. $[Ag(NH_3)_2]^+\,[Ag(CN)_2]^-$

+1 +1

5. The number of atoms in 8 g of sodium is $x \times 10^{23}$. The value of x is________.(Nearest integer)

[Given : $N_A = 6.02 \times 10^{23}\ mol^{-1}$

Atomic mass of Na = 23.0 u]

Official Ans. by NTA (2)

Sol. No. of atoms $= \frac{8}{23} \times 6.02 \times 10^{23} = 2.09 \times 10^{23}$

$\simeq 2 \times 10^{23}$

$= x \times 10^{23}$

x = 2

6. If 80 g of copper sulphate $CuSO_4{\cdot}5H_2O$ is dissolved in deionised water to make 5 L of solution. The concentration of the copper sulphate solution is $x \times 10^{-3}\ mol\ L^{-1}$. The value of x is _________.

[Atomic masses Cu : 63.54 u, S : 32 u, O : 16 u, H : 1 u]

Official Ans. by NTA (64)

Sol. Moles of $CuSO_4{\cdot}5H_2O = \frac{80}{249.54}$

Molarity $= \frac{\frac{80}{249.54}}{5} = 64.117 \times 10^{-3}$

Nearest integer, x = 64

7. A 50 watt bulb emits monochromatic red light of wavelength of 795 nm. The number of photons emitted per second by the bulb is $x \times 10^{20}$. The value of x is __________.

[Given : $h = 6.63 \times 10^{-34}$ Js and $c = 3.0 \times 10^8\ ms^{-1}$]

Official Ans. by NTA (2)

Sol. Total energy per sec. = 50 J

$$50 = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^{8}}{795 \times 10^{-9}}$$

$n = 1998.49 \times 10^{17}$ [n = no. of photons per second]

$= 1.998 \times 10^{20}$

$\simeq 2 \times 10^{20}$

$= x \times 10^{20}$

$x = 2$

8. The spin-only magnetic moment value of B_2^+ species is ________$\times 10^{-2}$ BM. (Nearest integer)

[Given : $\sqrt{3} = 1.73$]

Official Ans. by NTA (173)

Sol. $B_2^+ \Rightarrow \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2py}^1 \simeq \pi_{2pz}^0$

$\Rightarrow 9e^-$

$\mu = \sqrt{1(1+2)} = \sqrt{3}$ BM

$= 1.73$ BM

$= 1.73 \times 10^{-2}$ BM

9. If the conductivity of mercury at 0°C is 1.07×10^6 S m^{-1} and the resistance of a cell containing mercury is 0.243 Ω, then the cell constant of the cell is $x \times 10^4$ m^{-1}. The value of x is _______.(Nearest integer)

Official Ans. by NTA (26)

Sol. $k = 1.07 \times 10^6$ Sm^{-1}, $R = 0.243\ \Omega$

$$G = \frac{1}{R} = \frac{1}{0.243}\Omega^{-1}$$

$k = G \times G^*$

$$G^* = \frac{k}{G} = \frac{1.07 \times 10^6}{\frac{1}{0.243}} \simeq 26 \times 10^4\ \text{m}^{-1}$$

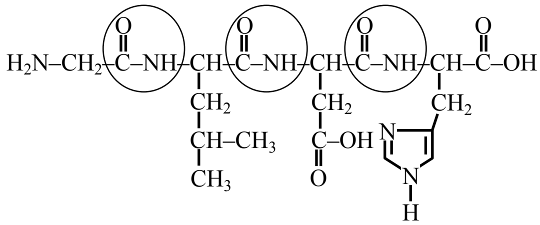
10. A peptide synthesized by the reactions of one molecule each of Glycine, Leucine, Aspartic acid and Histidine will have ________ peptide linkages.

Official Ans. by NTA (3)

Sol. $\boxed{\text{Glycine — leucine — Aspartic acid — Histidine}}$

peptide link   peptide link   peptide link

Total (3) peptide linkages are present



3 peptide linkage

Ans. (3)

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Wednesday 01st September, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f : \mathbf{R} \to \mathbf{R}$ be a continuous function. Then $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\frac{\pi}{4}\int_{2}^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}}$ is equal to :

(1) $f(2)$ (2) $2f(2)$

(3) $2f(\sqrt{2})$ (4) $4f(2)$

Official Ans. by NTA (2)

Sol. $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\frac{\pi}{4}\int_{2}^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}}$

$\lim\limits_{x\to\frac{\pi}{4}} \frac{\pi}{4}.\dfrac{\left[f\left(\sec^2 x\right).2\sec x.\sec x\tan x\right]}{2x}$

$\lim\limits_{x\to\frac{\pi}{4}} \frac{\pi}{4} f\left(\sec^2 x\right).\sec^3 x.\dfrac{\sin x}{x}$

$\frac{\pi}{4} f(2).\left(\sqrt{2}\right)^3.\frac{1}{\sqrt{2}} \times \frac{4}{\pi}$

$\Rightarrow 2f(2)$

2. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ is equal to :

(The inverse trigonometric functions take the principal values)

(1) $3\pi - 11$ (2) $4\pi - 9$

(3) $4\pi - 11$ (4) $3\pi + 1$

Official Ans. by NTA (3)

Sol. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$

$\Rightarrow (2\pi - 5) + (6 - 2\pi) - (12 - 4\pi)$

$\Rightarrow 4\pi - 11.$

3. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in \mathbf{R}$ for which the system is inconsistent and S_2 be the set of all $a \in \mathbf{R}$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

(1) $n(S_1) = 2, n(S_2) = 2$ (2) $n(S_1) = 1, n(S_2) = 0$

(3) $n(S_1) = 2, n(S_2) = 0$ (4) $n(S_1) = 0, n(S_2) = 2$

Official Ans. by NTA (3)

Sol. $\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$

$= -1(a^2 + 10) - 1(-3a - 10) + 2(-6 + 2a)$

$= -a^2 - 10 + 3a + 10 - 12 + 4a$

$\Delta = -a^2 + 7a - 12$

$\Delta = -[a^2 - 7a + 12]$

$\Delta = -[(a - 3)(a - 4)]$

$\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$

$= 0 - 1(-a - 35) + 2(-2 + 7a)$

$\Rightarrow a + 35 - 4 + 14a$

$15a + 31$

Now $\Delta_1 = 15a + 31$

For inconsistent $\Delta = 0 \therefore a = 3, a = 4$

and for a = 3 and 4 $\Delta_1 \neq 0$

$n(S_1) = 2$

For infinite solution : $\Delta = 0$

and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

Not possible

$\therefore n(S_2) = 0$

4. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P?

(1) $\left(3, 1, -\frac{1}{2}\right)$ (2) $\left(-2, 0, -\frac{1}{2}\right)$

(3) (0, 2, –4) (4) (4, 0, – 2)

Official Ans. by NTA (2)

Sol. $P_1 : x - 2y - 2z + 1 = 0$

$P_2 : 2x - 3y - 6z + 1 = 0$

$$\left|\frac{x-2y-2z+1}{\sqrt{1+4+4}}\right| = \left|\frac{2x-3y-6z+1}{\sqrt{2^2+3^2+6^2}}\right|$$

$$\frac{x-2y-2z+1}{3} = \pm\frac{2x-3y-6z+1}{7}$$

Since $a_1a_2 + b_1b_2 + c_1c_2 = 20 > 0$

$\therefore$ Negative sign will give

acute bisector

$7x - 14y - 14z + 7 = -[6x - 9y - 18z + 3]$

$\Rightarrow 13x - 23y - 32z + 10 = 0$

$\left(-2, 0, -\frac{1}{2}\right)$ satisfy it $\therefore$ Ans (2)

5. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

(1) $\sim (q \rightarrow p)$ (2) $\sim p \rightarrow \sim q$

(3) $\sim (p \rightarrow \sim q)$ (4) $\sim (p \rightarrow q)$

Official Ans. by NTA (4)

Sol.

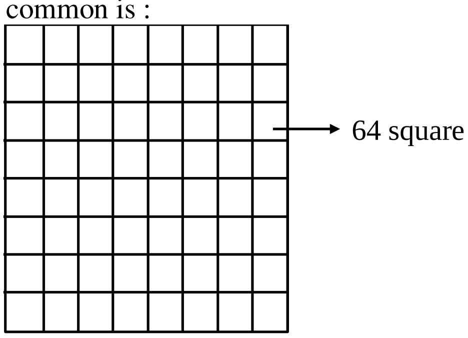
p	q	$\sim p$	$\sim q$	$p - q$	$\sim(p \rightarrow q)$	$q \rightarrow p$	$\sim(q \rightarrow p)$
T	T	F	F	T	F	T	F
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	F	T	F

$p \wedge \sim q$	$\sim p \rightarrow \sim q$	$p \rightarrow \sim q$	$\sim(p \rightarrow \sim q)$
F	T	F	T
T	T	T	F
F	F	T	F
F	T	T	F

$p \wedge \sim q \equiv \sim (p \rightarrow q)$

Option (4)

6. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is :



(1) $\frac{2}{7}$ (2) $\frac{1}{18}$

(3) $\frac{1}{7}$ (4) $\frac{1}{9}$

Official Ans. by NTA (2)

Sol. Total ways of choosing square $= {}^{64}C_2$

$$= \frac{64 \times 63}{2 \times 1} = 32 \times 63$$

ways of choosing two squares having common side $= 2\,(7 \times 8) = 112$

Required probability $= \frac{112}{32 \times 63} = \frac{16}{32 \times 9} = \frac{1}{18}$.

Ans. (2)

7. If $y = y\,(x)$ is the solution curve of the differential equation $x^2dy + \left(y - \frac{1}{x}\right)dx = 0$; $x > 0$ and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (2) $3 + \frac{1}{\sqrt{e}}$

(3) $3 + e$ (4) $3 - e$

Official Ans. by NTA (4)

Sol. $x^2dy + \left(y - \frac{1}{x}\right)dx = 0 \ : x > 0, y(1) = 1$

$$x^2dy + \frac{(xy-1)}{x}dx = 0$$

$$x^2dy = \frac{(xy-1)}{x}dx$$

$$\frac{dy}{dx} = \frac{1-xy}{x^3}$$

$$\frac{dy}{dx} = \frac{1}{x^3} - \frac{y}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2}.y = \frac{1}{x^3}$$

If $e^{\int \frac{1}{x^2}dx} = e^{-\frac{1}{x}}$

$$ye^{-\frac{1}{x}} = \int \frac{1}{x^3}.e^{-\frac{1}{x}}$$

$$ye^{-\frac{1}{x}} = e^{-x}\left(1 + \frac{1}{x}\right) + C$$

$$1.e^{-1} = e^{-1}(2) + C$$

$$C = -e^{-1} = -\frac{1}{e}$$

$$ye^{-\frac{1}{x}} = e^{-\frac{1}{x}}\left(1 + \frac{1}{x}\right) - \frac{1}{e}$$

$$y\left(\frac{1}{2}\right) = 3 - \frac{1}{e} \times e^2$$

$$y\left(\frac{1}{2}\right) = 3 - e$$

8. If n is the number of solutions of the equation

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0,\pi]$$

and S is the sum of all these solutions, then the ordered pair (n, S) is :

(1) $(3, 13\pi/9)$ (2) $(2, 2\pi/3)$

(3) $(2, 8\pi/9)$ (4) $(3, 5\pi/3)$

Official Ans. by NTA (1)

Sol. $$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1$$

$$2\cos x\left(4\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)-1\right)=1$$

$$2\cos x\left(4\left(\frac{1}{2}-\sin^2 x\right)-1\right)=1$$

$$2\cos x\left(2-4\sin^2 x-1\right)=1$$

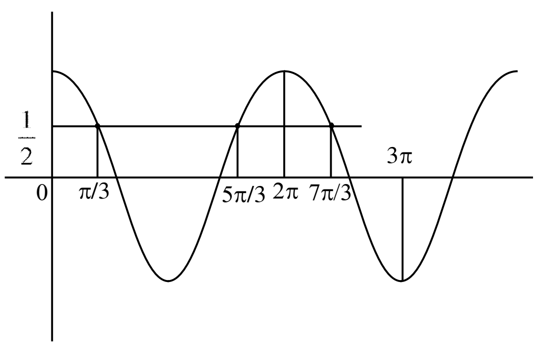
$$2\cos x\left(1-4\sin^2 x\right)=1$$

$$2\cos x\left(4\cos^2 x-3\right)=1$$

$$4\cos^3 x-3\cos x=\frac{1}{2}$$

$$\cos 3x=\frac{1}{2}$$

$x \in [0, \pi] \therefore 3x \in [0, 3\pi]$



9. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements.

(S1) there exists $x_1, x_2 \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) there exists $x_3, x_4 \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$.

Then

(1) both (S1) and (S2) are true

(2) (S1) is false and (S2) is true

(3) both (S1) and (S2) are false

(4) (S1) is true and (S2) is false

Official Ans. by NTA (1)

Sol. $f(x) = x^3 - 6x^2 + ax + b$

$f(2) = 8 - 24 + 2a + b = 0$

$2a + b = 16$ ...(1)

$f(4) = 64 - 96 + 4a + b = 0$

$4a + b = 32$ ...(2)

Solving (1) and (2)

$a = 8, b = 0$

$$\boxed{f(x) = x^3 - 6x^2 + 8x}$$

$f(x) = x^3 - 6x^2 + 8x$

$f'(x) = 3x^2 - 12x + 8$

$f''(x) = 6x - 12$

$\Rightarrow f'(x)$ is $\uparrow$ for $x > 2$, and $f'(x)$ is $\downarrow$ for $x < 2$

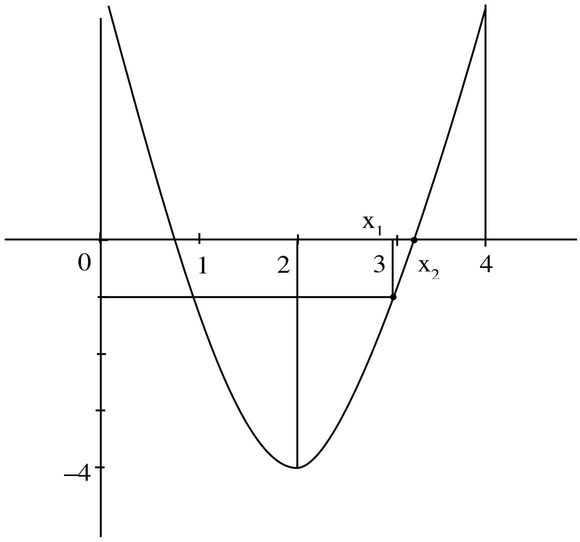
$f'(2) = 12 - 24 + 8 = -4$

$f'(4) = 48 - 48 + 8 = 8$

$f'(x) = 3x^2 - 12x + 8$

vertex $(2, -4)$

$f'(2) = -4, f'(4) = 8, f'(3) = 27 - 36 + 8$



$f'(x_1) = -1$, then $x_1 = 3$

$f'(x_2) = 0$

Again $f'(x) < 0$ for $x \in (2, x_4)$

$f'(x) > 0$ for $x \in (x_4, 4)$

$x_4 \in (3, 4)$

$f(x) = x^3 - 6x^2 + 8x$

$f(3) = 27 - 54 + 24 = -3$

$f(4) = 64 - 96 + 32 = 0$

For $x_4(3, 4)$

$f(x_4) < -3\sqrt{3}$

and $f'(x_3) > -4$

$2f'(x_3) > -8$

So, $2f'(x_3) = \sqrt{3}\ f(x_4)$

Correct Ans. (1)

10. Let $J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m - 1} dx, \quad \forall\ n > m$ and $n, m \in N$.

Consider a matrix $A = [a_{ij}]_{3\times 3}$ where

$a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$. Then $|adjA^{-1}|$ is :

(1) $(15)^2 \times 2^{42}$ (2) $(15)^2 \times 2^{34}$

(3) $(105)^2 \times 2^{38}$ (4) $(105)^2 \times 2^{36}$

Official Ans. by NTA (3)

Sol. $\begin{array}{ccc} \surd & \surd & \surd \end{array}$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$J_{6+i,3} - J_{i+3,3}$; $i \le j$

$$\Rightarrow \int_0^{\frac{1}{2}} \frac{x^{6+i}}{x^3-1} - \int_0^{\frac{1}{2}} \frac{x^{i+3}}{x^3-1}$$

$$\Rightarrow \int_0^{1/2} \frac{x^{i+3}\left(x^3-1\right)}{x^3-1}$$

$$\Rightarrow \frac{x^{3+i+1}}{3+i+1} = \left(\frac{x^{4+i}}{4+i}\right)_0^{1/2}$$

$$a_{ij} = j_{6+i,3} - j_{i+3,3} = \frac{\left(\frac{1}{2}\right)^{4+i}}{4+i}$$

$$a_{11} = \frac{\left(\frac{1}{2}\right)^5}{5} = \frac{1}{5.2^5}$$

$$a_{12} = \frac{1}{5.2^5}$$

$$a_{13} = \frac{1}{5.2^5}$$

$$a_{22} = \frac{1}{6.2^6}$$

$$a_{23} = \frac{1}{6.2^6}$$

$$a_{33} = \frac{1}{7.2^7}$$

$$A = \begin{bmatrix} \frac{1}{5.2^5} & \frac{1}{5.2^5} & \frac{1}{5.2^5} \\ 0 & \frac{1}{6.2^6} & \frac{1}{6.2^6} \\ 0 & 0 & \frac{1}{7.2^7} \end{bmatrix}$$

$$|A| = \frac{1}{5.2^5}\left[\frac{1}{6.2^6} \times \frac{1}{7.2^7}\right]$$

$$|A| = \frac{1}{210.2^{18}}$$

$$\left|adjA^{-1}\right| = \left|A^{-1}\right|^{n-1} = \left|A^{-1}\right|^2 = \frac{1}{\left(|A|\right)^2}$$

$\Rightarrow (210.2^{18})^2$

$(105)^2 \times 2^{38}$

11. The area, enclosed by the curves y = sin x + cos x and $y = |\cos x - \sin x|$ and the lines $x = 0,\ x = \frac{\pi}{2}$, is :

(1) $2\sqrt{2}\left(\sqrt{2}-1\right)$ (2) $2\left(\sqrt{2}+1\right)$

(3) $4\left(\sqrt{2}-1\right)$ (4) $2\sqrt{2}\left(\sqrt{2}+1\right)$

Official Ans. by NTA (1)

Sol. $A = \int_0^{\pi/2}\left((\sin x + \cos x) - |\cos x - \sin x|\right)dx$

$$A = \int_0^{\pi/2}\left((\sin x + \cos x) - (\cos x - \sin x)\right)dx$$

$$+\int_{\pi/4}^{\pi/2}\left((\sin x + \cos x) - (\sin x - \cos x)\right)dx$$

$$A = 2\int_0^{\pi/2}\sin x dx + 2\int_{\pi/4}^{\pi/2}\cos x dx$$

$$A = -2\left(\frac{1}{\sqrt{2}}-1\right) + 2\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$A = 4 - 2\sqrt{2} = 2\sqrt{2}\,(\sqrt{2} - 1)$$

Option (1)

12. The distance of line 3y – 2z – 1 = 0 = 3x – z + 4 from the point (2, – 1, 6) is :

(1) $\sqrt{26}$ (2) $2\sqrt{5}$

(3) $2\sqrt{6}$ (4) $4\sqrt{2}$

Official Ans. by NTA (3)

Sol. $3y - 2z - 1 = 0 = 3x - z + 4$

$3y - 2z - 1 = 0$ D.R′s $\Rightarrow (0, 3, -2)$

$3x - z + 4 = 0$ D.R′s $\Rightarrow (3, -1, 0)$

Let DR′s of given line are a, b, c

Now 3b – 2c = 0 & 3a – c = 0

$\therefore$ 6a = 3b = 2c

a : b : c = 3 : 6 : 9

Any pt on line

3K – 1, 6K + 1, 9K + 1

Now 3(3K – 1) + 6(6K + 1)1 + 9(9K + 1) =0

$\Rightarrow K = \frac{1}{3}$

Point on line $\Rightarrow$ (0, 3, 4)

Given point (2, –1, 6)

$\Rightarrow$ Distance = $\sqrt{4+16+4} = 2\sqrt{6}$

Option (3)

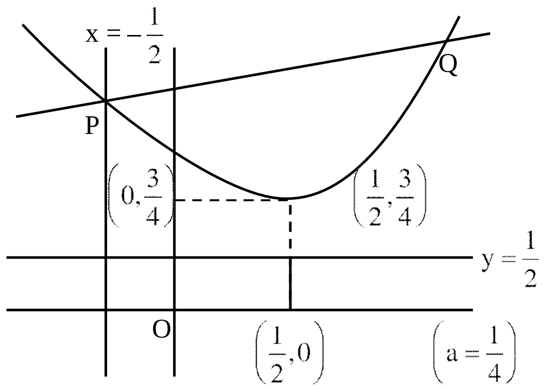
13. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to :

(1) $\frac{75}{8}$ (2) $\frac{125}{16}$

(3) $\frac{25}{2}$ (4) $\frac{15}{2}$

Official Ans. by NTA (2)

Sol.



$$\left(y - \frac{3}{4}\right) = \left(x - \frac{1}{2}\right)^2 \ \ldots (1)$$

For $x = -\frac{1}{2}$

$$y - \frac{3}{4} = 1 \Rightarrow y = \frac{7}{4} \Rightarrow P\left(-\frac{1}{2}, \frac{7}{4}\right)$$

Now $y' = 2\left(x - \frac{1}{2}\right)$ At $x = -\frac{1}{2}$

$\Rightarrow m_T = -2,\ m_N = \frac{1}{2}$

Equation of Normal is

$$y-\frac{7}{4}=\frac{1}{2}\left(x+\frac{1}{2}\right)$$

$$y = \frac{x}{2}+2$$

Now put y in equation (1)

$$\frac{x}{2}+2-\frac{3}{4}=\left(x-\frac{1}{2}\right)^2$$

$$\Rightarrow x = 2\ \&\ -\frac{1}{2}$$

$\Rightarrow Q(2, 3)$

Now $(PQ)^2 = \frac{125}{16}$

Option (2)

14. The numbers of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is :

(1) 6 (2) 2

(3) 4 (4) 8

Official Ans. by NTA (1)

Sol. Consider the equation $x^2 + ax + b = 0$

If has two roots (not necessarily real α & β)

Either $\alpha = \beta$ or $\alpha \neq \beta$

Case (1) If $\alpha = \beta$, then it is repeated root. Given that $\alpha^2 - 2$ is also a root

So, $\alpha = \alpha^2 - 2 \Rightarrow (\alpha + 1)(\alpha - 2) = 0$

$\Rightarrow \alpha = -1$ or $\alpha = 2$

When $\alpha = -1$ then (a, b) = (2, 1)

$\alpha = 2$ then (a, b) = (–4, 4)

Case (2) If $\alpha \neq \beta$ Then

(I) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$

Here $(\alpha, \beta) = (2, -1)$ or $(-1, 2)$

Hence $(a, b) = (-(\alpha + \beta), \alpha\beta)$

$= (-1, -2)$

(II) $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$

Then $\alpha - \beta = \beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \alpha)$

Since $\alpha \neq \beta$ we get $\alpha + \beta = \beta^2 + \alpha^2 - 4$

$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta - 4$

Thus $-1 = 1 - 2\alpha\beta - 4$ which implies

$\alpha\beta = -1$ Therefore $(a, b) = (-(\alpha + \beta), \alpha\beta)$

$= (1, -1)$

(III) $\alpha = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$

$\Rightarrow \alpha = -\beta$

Thus $\alpha = 2, \beta = -2$

$\alpha = -1, \beta = 1$

Therefore (a, b) = (0, –4) & (0, –1)

(IV) $\beta = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$ is same as (III)

Therefore we get 6 pairs of (a, b)

Which are (2, 1), (–4, 4), (–1, –2), (1, –1) (0, –4)

Option (1)

15. Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \ldots + (n-1) \cdot 1,\ n \geq 4$.

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!}-\frac{1}{(n-2)!}\right)$ is equal to :

(1) $\frac{e-1}{3}$ (2) $\frac{e-2}{6}$

(3) $\frac{e}{3}$ (4) $\frac{e}{6}$

Official Ans. by NTA (1)

Sol. Let $T_r = r(n-r)$

$T_r = nr - r^2$

$$\Rightarrow S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n}\left(nr - r^2\right)$$

$$S_n = \frac{n.(n)(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{n(n-1)(n+1)}{6}$$

Now $\displaystyle\sum_{r=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$

$$= \sum_{r=4}^{\infty}\left(2.\frac{n(n-1)(n+1)}{6.n(n-1)(n-2)!} - \frac{1}{(n-2)!}\right)$$

$$= \sum_{r=4}^{\infty}\left(\frac{1}{3}\left(\frac{n-2+3}{(n-2)!}\right) - \frac{1}{(n-2)!}\right)$$

$$= \sum_{r=4}^{\infty}\frac{1}{3}\cdot\frac{1}{(n-3)!} = \frac{1}{3}(e-1)$$

Option (1)

16. Let P_1, P_2, ..., P_{15} be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is :

(1) 12 (2) 419 (3) 443 (4) 455

Official Ans. by NTA (3)

Sol. Total Number of Triangles = ${}^{15}C_3$

$i + j + k = 15$ (Given)

5 Cases			4 Cases			3 Cases			1 Cases		
i	j	k	i	j	k	i	j	k	i	j	k
1	2	12	2	3	10	3	4	8	4	5	6
1	3	11	2	4	9	3	5	7			
1	4	10	2	5	8						
1	5	9	2	6	7						
1	6	8									

Number of Possible triangles using the vertices P_i, P_j, P_k such that $i + j + k \neq 15$ is equal to ${}^{15}C_3 - 12 = 443$

Option (3)

17. The range of the function,

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$$

is :

(1) $\left(0, \sqrt{5}\right)$ (2) $[-2, 2]$

(3) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$ (4) $[0, 2]$

Official Ans. by NTA (4)

Sol. $f(x) = \log_{\sqrt{5}}$

$$\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$$

$$f(x) = \log_{\sqrt{5}}\left[3 + 2\cos\left(\frac{\pi}{4}\right)\cos(x) - 2\sin\left(\frac{3\pi}{4}\right)\sin(x)\right]$$

$$f(x) = \log_{\sqrt{5}}\left[3 + \sqrt{2}(\cos x - \sin x)\right]$$

Since $-\sqrt{2} \leq \cos x - \sin x \leq \sqrt{2}$

$$\Rightarrow \log_{\sqrt{5}}\left[3 + \sqrt{2}(-\sqrt{2}) \leq f(x) \leq \log_{\sqrt{5}}\left[3 + \sqrt{2}(\sqrt{2})\right]\right]$$

$$\Rightarrow \log_{\sqrt{5}}(1) \leq f(x) \leq \log_{\sqrt{5}}(5)$$

So Range of f(x) is [0, 2]

Option (4)

18. Let $a_1, a_2, \ldots\ldots, a_{21}$ be an AP such that $\displaystyle\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}} = \frac{4}{9}$.

If the sum of this AP is 189, then $a_6 a_{16}$ is equal to :

(1) 57 (2) 72

(3) 48 (4) 36

Official Ans. by NTA (2)

Sol. $\displaystyle\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}} = \sum_{n=1}^{20}\frac{1}{a_n(a_n + d)}$

$$= \frac{1}{d}\sum_{n=1}^{20}\left(\frac{1}{a_n} - \frac{1}{a_n + d}\right)$$

$$\Rightarrow \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{21}}\right) = \frac{4}{9} \text{ (Given)}$$

$$\Rightarrow \frac{1}{d}\left(\frac{a_{21} - a_1}{a_1 a_{21}}\right) = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left(\frac{a_1 + 20d - a_1}{a_1 a_2}\right) = \frac{4}{9} \Rightarrow a_1 a_2 = 45 \text{ .... (1)}$$

Now sum of first 21 terms = $\frac{21}{2}$ $(2a_1 + 20d) = 189$

$\Rightarrow a_1 + 10d = 9$ ... (2)

For equation (1) & (2) we get

$a_1 = 3$ & $d = \frac{3}{5}$

OR

$a_1 = 15$ & $d = -\frac{3}{5}$

So, $a_6.a_{16} = (a_1 + 5d)(a_1 + 15d)$

$\Rightarrow a_6 a_{16} = 72$

Option (2)

19. The function f(x), that satisfies the condition $f(x) = x + \int_0^{\pi/2} \sin x \cdot \cos y f(y) dy$, is :

(1) $x + \frac{2}{3}(\pi - 2)\sin x$ (2) $x + (\pi + 2) \sin x$

(3) $x + \frac{\pi}{2}\sin x$ (4) $x + (\pi - 2) \sin x$

Official Ans. by NTA (4)

Sol. $f(x) = x + \int_0^{\pi/2} \sin x \cos y f(y) dy$

$$f(x) = x + \sin x \underbrace{\int_0^{\pi/2} \cos y f(y) dy}_{K}$$

$\Rightarrow f(x) = x + K \sin x$

$\Rightarrow f(y) = y + K \sin y$

Now $K = \int_0^{\pi/2} \cos y (y + K \sin y) dy$

$$K = \underbrace{\int_0^{\pi/2} y \cos dy}_{\text{Apply IBP}} + \underbrace{\int_0^{\pi/2} \cos y \sin y dy}_{\text{Put } \sin y = t}$$

$$K = (y \sin y)_0^{\pi/2} - \int_0^{\pi/2} \sin dy + K\int_0^1 t\,dt$$

$\Rightarrow K = \frac{\pi}{2} - 1 + K\left(\frac{1}{2}\right)$

$\Rightarrow K = \pi - 2$

So $f(x) = x + (\pi - 2) \sin x$

Option (4)

20. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to :

(1) $\frac{5}{2\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{4}{\sqrt{3}}$ (4) 2

Official Ans. by NTA (2)

Sol. The point of intersection of the curves $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and $x^2 + y^2 = 3$ in the first quadrant is $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

Now slope of tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ at $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ is

$$m_1 = -\frac{1}{3\sqrt{3}}$$

And slope of tangent to the circle at $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ is m_2

$= -\sqrt{3}$

So, if angle between both curves is θ then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\frac{1}{3\sqrt{3}} + \sqrt{3}}{1 + \left(-\frac{1}{3\sqrt{3}}(-\sqrt{3})\right)}\right|$$

$$= \frac{2}{\sqrt{3}}$$

Option (2)

SECTION-B

1. Let X be a random variable with distribution.

x	–2	–1	3	4	6
P(X = x)	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\sigma^2$ is equal to :

Official Ans. by NTA (781)

Sol.

x	–2	–1	3	4	6
P(X = x)	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

$\overline{X} = 2.3$

$-a + 6b = \frac{9}{10}$ .... (1)

$\sum P_i = \frac{1}{5} + a + \frac{1}{3} + \frac{1}{5} + b = 1$

$a + b = \frac{4}{15}$ .... (2)

From equation (1) and (2)

$a = \frac{1}{10}, \quad b = \frac{1}{6}$

$\sigma^2 = \Sigma p_i x_i^2 - (\overline{X})^2$

$\frac{1}{5}(4) + a(1) + \frac{1}{3}(9) + \frac{1}{5}(16) + b(36) - (2.3)^2$

$= \frac{4}{5} + a + 3 + \frac{16}{5} + 36b - (2.3)^2$

$= 4 + a + 3 + 36b - (2.3)^2$

$= 7 + a + 36b - (2.3)^2$

$= 7 + \frac{1}{10} + 6 - (2.3)^2$

$= 13 + \frac{1}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{131}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{1310 - (23)^2}{100}$

$= \frac{1310 - 529}{100}$

$\sigma^2 = \frac{781}{100}$

$100\sigma^2 = 781$

2. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3$, $x \in \mathbf{R}$. Then the natural number n for which $\lim_{x\to 1} \frac{x^n f(1) - f(x)}{x - 1} = 44$ is________.

Official Ans. by NTA (7)

Sol. $f(n) = x^6 + 2x^4 + x^3 + 2x + 3$

$\lim_{x\to 1} \frac{x^n f(1) - f(x)}{x - 1} = 44$

$\lim_{x\to 1} \frac{9x^n - (x^6 + 2x^4 + x^3 + 2x + 3)}{x - 1} = 44$

$\lim_{x\to 1} \frac{9nx^{n-1} - (6x^5 + 8x^3 + 3x^2 + 2)}{1} = 44$

$\Rightarrow 9n - (19) = 44$

$\Rightarrow 9n = 63$

$\Rightarrow n = 7$

3. If for the complex numbers z satisfying $|z - 2 - 2i| \le 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to ________.

Official Ans. by NTA (5)

Sol. $|z - 2 - 2i| \le 1$

$|x + iy - 2 - 2i| \le 1$

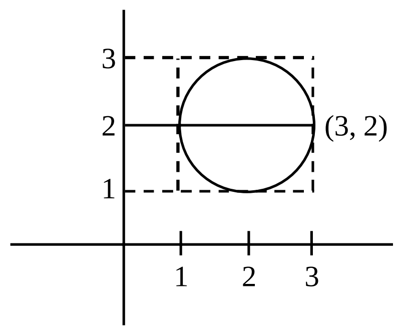
$|(x - 2) + i(y - 2)| \le 1$

$(x - 2)^2 + (y - 2)^2 \le 1$

$|3iz + 6|_{max}$ at $a + ib$

$|3i| \left|z + \frac{6}{3i}\right|$

$3|z - 2i|_{max}$



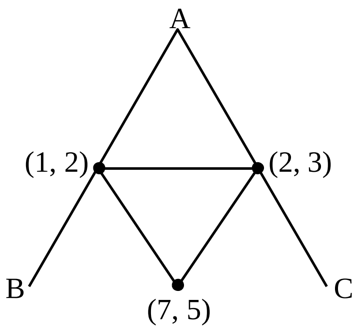
From Figure maximum distance at 3 + 2i

$a + ib = 3 + 2i = a + b = 3 + 2 = 5$ **Ans.**

4. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is ________.

Official Ans. by NTA (6)

Sol. intersection point of give lines are (1, 2), (7, 5), (2,3)



$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & 5 & 1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(5-3) - 2(7-2) + 1(21-10)]$$

$$= \frac{1}{2}[2 - 10 + 11]$$

$$\Delta DEF = \frac{1}{2}(3) = \frac{3}{2}$$

$$\Delta ABC = 4\ \Delta DEF = 4\left(\frac{3}{2}\right) = 6$$

5. Let f(x) be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for k = 2, 3, 4, 5. Then the value of $52 - 10\ f(10)$ is equal to :

Official Ans. by NTA (26)

Sol. $k\ f(k) + 2 = \lambda\ (x-2)(x-3)(x-4)(x-5)$ …(1)

put x = 0

we get $\lambda = \frac{1}{60}$

Now put λ in equation (1)

$\Rightarrow kf(k) + 2 = \frac{1}{60}(x-2)(x-3)(x-4)(x-5)$

Put x = 10

$\Rightarrow 10f(10) + 2 = \frac{1}{60}(8)(7)(6)(5)$

$\Rightarrow 52 - 10f(10) = 52 - 26 = 26$

6. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is ______.

Official Ans. by NTA (77)

Sol. FARMER (6)

A, E, F, M, R, R

A					
E					
F	A	E			
F	A	M			
F	A	R	E		
F	A	R	M	E	R

$\frac{\lfloor 5}{\lfloor 2} - \lfloor 4 = 60 - 24 = 36$

$\frac{\lfloor 3}{\lfloor 2} - \lfloor 2 = 3 - 2 = 1$

= 1

= 2

= 1

77

7. If the sum of the coefficients in the expansion of $(x + y)^n$ is 4096, then the greatest coefficient in the expansion is _______.

Official Ans. by NTA (924)

Sol. $(x + y)^n \Rightarrow 2^n = 4096$ $\quad 2^{10} = 1024 \times 2$

$\Rightarrow 2^n = 2^{12}$ $\quad 2^{11} = 2048$

$n = 12$ $\quad 2^{12} = \underline{4096}$

$${}^{12}C_6 = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$= 11 \times 3 \times 4 \times 7$

$= 924$

8. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to _____.

Official Ans. by NTA (1494)

Sol. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{c} = 3\hat{i} + 2\hat{j} - \hat{k}$

$\vec{v} = x\vec{a} + y\vec{b}$ $\quad\quad$ $\vec{v}\left(3\hat{i} + 2\hat{j} - k\right) = 0$

$\vec{v}\cdot\hat{a} = 19$

$\vec{v} = \lambda\vec{c} \times \left(\vec{a} \times \vec{b}\right)$

$\vec{v} = \lambda\left[\left(\vec{c}.\vec{b}\right)\vec{a} - \left(\vec{c}\cdot\vec{a}\right)\vec{b}\right]$

$= \lambda[(3+4+1)\left(2\hat{i} - \hat{j} + 2\hat{k}\right) - \left(\frac{6-2-2}{2}\right)\left(\hat{i} + 2\hat{j} + \hat{k}\right)$

$= \lambda[16\hat{i} - 8\hat{j} + 16\hat{k} - 2\hat{i} - 4\hat{j} + 2\hat{k}]$

$\vec{v} = \lambda\left[14\hat{i} - 12\hat{j} + 18\hat{k}\right]$

$\lambda[14\hat{i} - 12\hat{j} + 18\hat{k}] \cdot \frac{\left(2\hat{i} - \hat{j} + 2\hat{k}\right)}{\sqrt{4+1+4}} = 19$

$\lambda\frac{[28+12+36]}{3} = 19$

$\lambda\left(\frac{76}{3}\right) = 19$

$4\lambda = 3 \Rightarrow \lambda = \frac{3}{4}$

$|2v^2| = \left|2 \times \frac{3}{4}\left(14\hat{i} - 12\hat{j} + 18\hat{k}\right)\right|^2$

$\frac{9}{4} \times 4\left(7\hat{i} - 6\hat{j} + 9\hat{k}\right)^2$

$= 9\ (49 + 36 + 81)$

$= 9\ (166)$

$= 1494$

9. Let [t] denote the greatest integer $\leq$ t. The number of points where the function

$$f(x) = [x]\left|x^2 - 1\right| + \sin\left(\frac{\pi}{[x]+3}\right) - [x+1], x \in (-2, 2)$$

is not continuous is ______.

Official Ans. by NTA (2)

Sol. $f(x) = [x]\ |x^2 - 1| + \sin\frac{\pi}{[x+3]} - [x+1]$

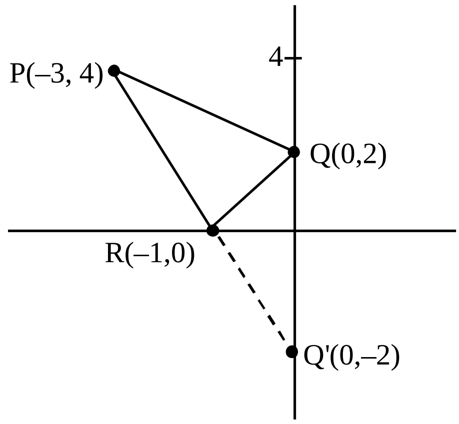
$$f(x) = \begin{cases} 3 - 2x^2, & -2 < x < -1 \\ x^2, & -1 \leq x < 0 \\ \frac{\sqrt{3}}{2} + 1 & 0 \leq x < 1 \\ x^2 + 1 + \frac{1}{\sqrt{2}}, & 1 \leq x < 2 \end{cases}$$

discontinuous at x=0 , 1

10. A man starts walking from the point P(–3,4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50\left((PR)^2 + (RQ)^2\right)$ is equal to _____.

Official Ans. by NTA (1250)

Sol.



$50(PR^2 + RQ^2)$

$50\ (20 + 5)$

$50(25)$

$= 1250$