MIND MAP : LEARNING MADE SIMPLE CHAPTER - 3

Trigonometric Functions

Important Trigonometric Ratios values

- $\sin 15^\circ$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$ or $\cos\frac{5\pi}{12}$
- $\cos 15^\circ$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$ or $\sin\frac{5\pi}{12}$
- $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ$
- $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$
- $\sin\frac{\pi}{10}$ or $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$
- $\cos\frac{\pi}{5}$ or $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

Factorisation of the Sum or Difference of sine and cosine with two variables

- $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

Transformation of Product into Sum or Difference of Sines or Cosines

- $2\sin A \sin B = \sin(A+B) + \sin(A-B)$
- $2\cos A \sin B = \sin(A+B) - \sin(A-B)$
- $2\cos A \cos B = \cos(A+B) + \cos(A-B)$
- $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

Trigonometrical Function sum and Difference of Two Angles

- $\sin(A\pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A\pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A\pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\cot(A\pm B) = \frac{\cot A \cot B \mp 1}{\cot A \pm \cot B}$
- $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$
- $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B)\cos(A-B)$

Standard General Solutions of Trigonometrical ratios

General solution: The solution consisting of all possible solutions of a trigonometric equation is called its General Solution.

- $\sin\theta = 0 \Leftrightarrow \theta = n\pi$, $\cos\theta = 0 \Leftrightarrow \theta = (2n+\frac{\pi}{2})$
- $\tan\theta = 0 \Leftrightarrow \theta = n\pi$, $\sin\theta = \sin\alpha \Leftrightarrow \theta = n\pi + (-1)^n\alpha$, where $\alpha \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
- $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi \pm \alpha$, where $\alpha \in [0,\pi]$
- $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha$, where $\alpha \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
- $\sin^2\theta = \sin^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$, $\cos^2\theta = \cos^2\alpha \Leftrightarrow n\pi \pm \alpha$
- $\tan^2\theta = \tan^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$, $\sin\theta = 1 \Leftrightarrow \theta = (4n+1)\frac{\pi}{2}$
- $\cos\theta = 1 \Leftrightarrow \theta = 2n\pi$, $\cos = -1 \Leftrightarrow \theta = (2n+1)\pi$
- $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi + \alpha$, $\eta \in z$

Degree & Radian Measures

If in a circle of radius r, an arc of length l subtends an angle of θ radians , then $l = r\theta$.

- Radian Measure $= \frac{\pi}{180} \times$ Degree Measure
- Degree Measure $= \frac{180}{\pi} \times$ Radian Measure

Trigonometric Identities

- $\sin^2\theta + \cos^2\theta = 1$, $-1 \le \sin\theta \le 1$, $-1 \le \cos\theta \le 1$ $\forall\theta \in R$
- $\sec^2\theta - \tan^2\theta = 1$, $|\sec\theta| \ge 1$, $\forall\theta \in R$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$, $|\operatorname{cosec}\theta| \ge 1$, $\theta \in R$

Trigonometrical Ratio of allied Angles

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$
- $\cot(-\theta) = -\cot\theta$
- $\sec(-\theta) = \sec\theta$
- $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

- $\cos(\pi \mp \theta) = -\cos\theta$
- $\sin(\pi \mp \theta) = \pm\sin\theta$
- $\tan(\pi \mp \theta) = \mp\tan\theta$
- $\cot(\pi \mp \theta) = \mp\cot\theta$
- $\sec(\pi \mp \theta) = -\sec\theta$
- $\operatorname{cosec}(\pi \mp \theta) = \pm\operatorname{cosec}\theta$

- $\sin\left(\frac{3\pi}{2} \mp \theta\right) = -\cos\theta$
- $\cos\left(\frac{3\pi}{2} \mp \theta\right) = \mp\sin\theta$
- $\tan\left(\frac{3\pi}{2} \mp \theta\right) = \pm\cot\theta$
- $\cot\left(\frac{3\pi}{2} \mp \theta\right) = \pm\tan\theta$
- $\sec\left(\frac{3\pi}{2} \mp \theta\right) = \mp\operatorname{cosec}\theta$
- $\operatorname{cosec}\left(\frac{3\pi}{2} \mp \theta\right) = \sec\theta$

- $\sin\left(\frac{\pi}{2} \mp \theta\right) = \cos\theta$
- $\cos\left(\frac{\pi}{2} \mp \theta\right) = \pm\sin\theta$
- $\tan\left(\frac{\pi}{2} \mp \theta\right) = \pm\cot\theta$
- $\cot\left(\frac{\pi}{2} \mp \theta\right) = \pm\tan\theta$
- $\sec\left(\frac{\pi}{2} \mp \theta\right) = \pm\operatorname{cosec}\theta$
- $\operatorname{cosec}\left(\frac{\pi}{2} \mp \theta\right) = \sec\theta$

- $\sin(2\pi \mp \theta) = \mp\sin\theta$
- $\cos(2\pi \mp \theta) = \cos\theta$
- $\tan(2\pi \mp \theta) = \mp\tan\theta$
- $\cot(2\pi \mp \theta) = \mp\cot\theta$
- $\sec(2\pi \mp \theta) = \sec\theta$
- $\operatorname{cosec}(2\pi \mp \theta) = \mp\operatorname{cosec}\theta$

Multiple and Half Angles

- $\sin 2A = 2\sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
  $= 2\cos^2 A - 1$
  $= 1 - 2\sin^2 A$
- $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
- $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$
- $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$
- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$
- $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
- $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$
- $\tan\theta = 2\tan\frac{\theta}{2} / \left(1-\tan^2\frac{\theta}{2}\right)$

Trigonometrical Equations

The equation involving trigonometric functions of unknown angles are known as Trigonometric equations.
e.g $\cos\theta = 0$, $\cos^2\theta - 4\cos\theta = 1$
A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. $\sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \ldots$