1. ખાલી જગ્યાઓ પૂરો : $\int \frac{\sin x}{3 + 4\cos^2 x}\, dx = ...+ C$

➥ $I = \int \frac{\sin x}{3 + 4\cos^2 x}\, dx$

હવે $\cos x = t$ આદેશ લેતાં,

$\therefore -\sin x\, dx = dt$

$\therefore I = -\int \frac{dt}{3 + 4t^2}$

$= -\left\{ \frac{1}{\sqrt{3(4)}} \tan^{-1}\left(\sqrt{\frac{4}{3}} \cdot t \right) \right\} + C$

$I = \frac{-1}{2\sqrt{3}} \tan^{-1}\left(\frac{2\cos x}{\sqrt{3}} \right) + C$

2. ખાલી જગ્યાઓ પૂરો : જો $\int_0^a \frac{1}{1 + 4x^2}\, dx = \frac{\pi}{8}$ હોય તો $a = ...$

➥ $I = \int_0^a \frac{1}{1 + 4x^2}\, dx = \frac{\pi}{8}$

$\therefore \int_0^a \frac{1}{4\left(\frac{1}{4} + x^2\right)}\, dx = \frac{\pi}{8}$

$\therefore \frac{1}{4} \int_0^a \frac{1}{x^2 + \left(\frac{1}{2}\right)^2}\, dx = \frac{\pi}{8}$

$\therefore \frac{1}{4} \left\{ \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{x}{\frac{1}{2}} \right) \right\}_0^a = \frac{\pi}{8}$

$\therefore \frac{2}{4} \left[\tan^{-1}(2x) \right]_0^a = \frac{\pi}{8}$

$\therefore \tan^{-1}(2a) - 0 = \frac{\pi}{4}$

$\therefore 2a = \tan\frac{\pi}{4}$

$\therefore 2a = 1$

$\therefore a = \frac{1}{2}$

અન્ય રીત :

$$\int_0^a \frac{1}{1+4x^2}\,dx = \frac{\pi}{8}$$

$$\therefore \frac{1}{(1)(4)} \tan^{-1}\left(\sqrt{\frac{4}{1}} \cdot x\right)_0^a = \frac{\pi}{8}$$

$$\left(\because \int \frac{1}{a+bx^2}\,dx = \frac{1}{\sqrt{ab}} \tan^{-1}\left(\sqrt{\frac{b}{a}}\; x\right) \text{ છે.}\right)$$

$$\therefore \frac{1}{2}\left(\tan^{-1}(2x)\right)_0^a = \frac{\pi}{8}$$

$$\therefore \tan^{-1}(2a) - \tan^{-1}(0) = \frac{\pi}{4}$$

➥ $$I = \int_0^a \frac{1}{1+4x^2}\,dx = \frac{\pi}{8}$$

$$\therefore \int_0^a \frac{1}{4\left(\frac{1}{4}+x^2\right)}\,dx = \frac{\pi}{8}$$

$$\therefore \frac{1}{4}\int_0^a \frac{1}{x^2+\left(\frac{1}{2}\right)^2}\,dx = \frac{\pi}{8}$$

$$\therefore \frac{1}{4}\left\{\frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{x}{\frac{1}{2}}\right)\right\}_0^a = \frac{\pi}{8}$$

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$$\therefore \tan^{-1}(2a) - \tan^{-1}(0) = \frac{\pi}{4}$$

3. ખાલી જગ્યાઓ પૂરો : $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x \, dx = \text{.........}$

➥ $I = \int_{-\pi}^{\pi} \sin^3 x \cos^2 x \, dx$

અહીં $f(x) = \sin^3 x \cos^2 x$

$\therefore f(-x) = \sin^3(-x) \cos^2(-x)$

$= -\sin^3 x \cos^2 x$

$= -f(x)$

$\therefore f(x)$ અયુગ્મ વિધેય છે.

$\therefore \int_{-\pi}^{\pi} f(x) \, dx = 0$