

Mathematics
Class XII
Sample Paper – 9 Solution

SECTION – A

1. a_{42} , means element at 3rd row and 2nd column

So,

$$a_{32} = 10$$

2. Differentiating w.r.t. x , we get,

$$\begin{aligned}\frac{d}{dx}(\sin(\cos x)) \\ &= \cos(\cos x) \frac{d}{dx}(\cos x) \\ &= -\sin x \cos(\cos x)\end{aligned}$$

3. DE:

$$x + \left(\frac{dy}{dx}\right) = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

squaring

$$x^2 + 2x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$x^2 + 2x\left(\frac{dy}{dx}\right) = 1$$

It is linear, since x is independent variable.

4. Let θ be the angles between, the given two lines

So, the angle between them given their direction cosines is given by

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

substituting we get

$$\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

OR

Let θ be the angles between, the given two lines

So, the angle between them given their direction cosines is given by

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

substituting we get

$$\theta = \cos^{-1} \left(\frac{10}{9\sqrt{22}} \right)$$

SECTION - B

5. $f: \mathbb{R}_+ \rightarrow [4, \infty)$ defined by $f(x) = x^2 + 4$

f is 1 - 1

Let $x_1, x_2 \in \mathbb{R}_+$ such that $f(x_1) = f(x_2)$

$$\Rightarrow x_1^2 + 4 = x_2^2 + 4$$

$$\Rightarrow x_1^2 = x_2^2$$

$$\Rightarrow x_1 = x_2 \quad \because x_1, x_2 \in \mathbb{R}_+$$

$\therefore f$ is 1 - 1

f is onto: Let $y \in [4, \infty)$

$$f(x) = y \Rightarrow x^2 + 4 = y$$

$$\Rightarrow x = \sqrt{y - 4}$$

Since $y \in [4, \infty) \Rightarrow x \in \mathbb{R}_+$

For $y \in [4, \infty)$ there is a $x \in \mathbb{R}_+$ such that $f(x) = y$.

So f is onto.

So, f is bijective function and hence f is invertible.

The inverse of f is defined by

$$f: [4, \infty) \rightarrow \mathbb{R}_+$$

$$f^{-1}(y) = \sqrt{y - 4}$$

6.

We have,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ and, } B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{diag}(3 \ 2 \ 1)$$

and,

$$3A + 4B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \text{diag}(11 \ 9 \ 2)$$

7. $I = \int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$

Let $e^x = t$ $e^x dx = dt$

Now integral I becomes,

$$I = \int \frac{dt}{\sqrt{5 - 4t - t^2}}$$

$$\Rightarrow I = \int \frac{dt}{\sqrt{5 + 4 - 4 - 4t - t^2}}$$

$$\Rightarrow I = \int \frac{dt}{\sqrt{9 - (4 + 4t + t^2)}}$$

$$\Rightarrow I = \int \frac{dt}{\sqrt{9 - (t + 2)^2}}$$

$$\Rightarrow I = \int \frac{dt}{\sqrt{3^2 - (t + 2)^2}}$$

$$\Rightarrow I = \sin^{-1} \frac{(t + 2)}{3} + C$$

$$\Rightarrow I = \sin^{-1} \frac{(e^x + 2)}{3} + C$$

8. $I = \int \frac{(x-4)e^x}{(x-2)^3} \cdot dx$

$$I = \int e^x \left(\frac{x-2}{(x-2)^3} - \frac{2}{(x-2)^3} \right) \cdot dx$$

$$I = \int e^x \left(\frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right) \cdot dx$$

Thus the given integral is of the form,

$$I = \int e^x [f(x) + f'(x)] dx \text{ where, } f(x) = \frac{1}{(x-2)^2}; f'(x) = \frac{-2}{(x-2)^3}$$

$$\begin{aligned} I &= \int \frac{e^x}{(x-2)^2} dx - \int \frac{2e^x}{(x-2)^3} dx \\ &= \int e^x \left[\frac{1}{(x-2)^2} + \frac{d}{dx} \left(\frac{1}{(x-2)^2} \right) \right] dx \end{aligned}$$

$$\text{So, } I = \frac{e^x}{(x-2)^2} + C$$

OR

$$\int \frac{x^2}{1+x^3} dx$$

$$\text{Let } 1+x^3 = t$$

$$\Rightarrow 0 + 3x^2 dx = dt$$

$$\Rightarrow x^2 dx = \frac{dt}{3}$$

$$\begin{aligned} \therefore \int \left(\frac{x^2}{1+x^3} \right) dx &= \int \frac{\frac{dt}{3}}{t} \\ &= \frac{1}{3} \int \frac{dt}{t} \\ &= \frac{1}{3} \log |t| + c \\ &= \frac{1}{3} \log |1+x^3| + c \end{aligned}$$

9. We have to differentiate it w.r.t. x two times

Differentiating

$$2y \frac{dy}{dx} = m(-2x)$$

$$y \frac{dy}{dx} = -mx \dots \dots \dots (1)$$

differentiating again

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = -m$$

from (1)

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = \frac{y}{x} \frac{dy}{dx}$$

which is the required differential equation

10.

$$\vec{r} \cdot 3\hat{i} + 4\hat{j} - 12\hat{k} + 13 = 0$$

$$\Rightarrow 3x + 4y - 12z + 13 = 0$$

Distance of point (1, 1, p) from the plane, is given by

$$\frac{|3 \times 1 + 4 \times 1 - 12 \times p + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}} = \frac{|20 - 12p|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$

Distance of point (-3, 0, 1) from the plane, is given by

$$\frac{|3 \times -3 + 4 \times 0 - 12 \times 1 + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}} = \frac{|-8|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$

The two distances are equal

$$\Rightarrow \frac{|20 - 12p|}{\sqrt{3^2 + 4^2 + (-12)^2}} = \frac{|-8|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$

$$\Rightarrow |20 - 12p| = |-8|$$

$$\Rightarrow 20 - 12p = \pm 8$$

$$\Rightarrow p = 1, \frac{7}{3}$$

OR

$$\begin{aligned}\text{L.H.S.} &= [\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] \\ &= (\vec{a} + \vec{b}) \cdot [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})] \\ &= (\vec{a} + \vec{b}) \cdot [\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{c} + \vec{c} \times \vec{a}] \\ &= (\vec{a} + \vec{b}) \cdot [\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a}] \\ &= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a}) + \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a}) \\ &= \vec{a} \cdot (\vec{b} \times \vec{c}) + 0 + 0 + 0 + 0 + \vec{b} \cdot (\vec{c} \times \vec{a}) \\ &= 2\vec{a} \cdot (\vec{b} \times \vec{c}) \\ &= 2[\vec{a} \ \vec{b} \ \vec{c}] = \text{R.H.S.}\end{aligned}$$

11. Let p be the probability that a student entering a university graduates,

So $p = 0.4$.

Since $p + q = 1$, thus, $q = 1 - 0.4 = 0.6$.

Let X denote the random variable representing the number of students who graduate out of the 3. Probability that r students graduate out of n entering the university is given by

$$\begin{aligned}P(X=r) &= {}^nC_r p^r q^{n-r} \\ &= {}^3C_r (0.4)^r (0.6)^{n-r} \quad \dots (1)\end{aligned}$$

(i)

Probability that none will graduate

$$= P(X=0)$$

$$= {}^3C_0 (0.4)^0 (0.6)^{3-0}$$

$$= (0.6)^3$$

$$= 0.216$$

$$\therefore \text{Probability that none will graduate} = 0.216$$

(ii)

Probability that one will graduate

$$= P(X = 1)$$

$$= {}^3C_1 (0.4)^1 (0.6)^{3-1}$$

$$= 3 \times (0.4) \times (0.36)$$

$$= 0.432$$

$\therefore$ Probability that only one will graduate = 0.432

12. Let E_1 , E_2 , and A be the events.

E_1 = white ball is transferred from 1st bag to 2nd bag.

E_2 = black ball is transferred from 1st bag to 2nd bag.

A = a black ball is drawn.

$$\text{So, } P(E_1) = \frac{5}{8}; \quad P(E_2) = \frac{3}{8};$$

Also, $P(A/E_1)$ = Probability of taking out black ball from bag 2 when white ball is already transferred from 1st bag to 2nd bag.

$$\Rightarrow P(A/E_1) = \frac{4}{8}$$

$P(A/E_2)$ = Probability of taking out black ball from bag 2 when black ball is transferred from 1st bag to 2nd bag.

$$\Rightarrow P(A/E_2) = \frac{5}{8}$$

By law of total probability

$$P(E_2 | A) = \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)}$$

$$= \frac{\frac{3}{8} \cdot \frac{5}{8}}{\frac{5}{8} \cdot \frac{4}{8} + \frac{3}{8} \cdot \frac{5}{8}} = \frac{15}{35} = \frac{3}{7}$$

OR

The events A, E₁, E₂, E₃, and E₄ are given by

A = event when doctor visits patients late

E₁ = doctor comes by train

E₂ = doctor comes by bus

E₃ = doctor comes by scooter

E₄ = doctor comes by other means of transport

$$\text{So, } P(E_1) = \frac{3}{10}, P(E_2) = \frac{1}{5}, P(E_3) = \frac{1}{10}, P(E_4) = \frac{2}{5}$$

$$P(A/E_1) = \text{Probability that the doctor arrives late, given that he is comes by train} = \frac{1}{4}$$

$$\text{Similarly } P(A/E_2) = \frac{1}{3}, P(A/E_3) = \frac{1}{12}, P(A/E_4) = 0$$

Required probability of the doctor arriving late by train by using Baye's theorem,

$$\begin{aligned} P(E_1/A) &= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + P(E_4)P(A/E_4)} \\ &= \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{3} + \frac{1}{10} \times \frac{1}{12} + \frac{2}{5} \times 0} \\ &= \frac{3}{40} \times \frac{120}{18} = \frac{1}{2} \end{aligned}$$

Hence the required probability is $\frac{1}{2}$.

SECTION - C

13.(i) $f: \mathbb{N} \rightarrow \mathbb{Z}$

$$f(x) = x$$

and $f: \mathbb{Z} \rightarrow \mathbb{Z}$ s.t.

$$g(x) = |x|$$

Clearly, g the absolute value function is not one-one.

$$\text{gof: } \mathbb{N} \rightarrow \mathbb{Z}$$

$$\text{gof}(x) = |x| \quad \forall x \in \mathbb{N}$$

$$g(f(x)) = g(x) = |x|$$

$$\text{Let } \text{gof}(x_1) = \text{gof}(x_2)$$

$$\Rightarrow g(f(x_1)) = g(f(x_2))$$

$$|x_1| = |x_2|$$

$$\Rightarrow x_1 = x_2$$

$$\text{since } x_1, x_2 \in \mathbb{N}$$

So gof is one-one.

(ii) $f: \mathbb{N} \rightarrow \mathbb{N}$ such that

$$f(x) = x + 1 \text{ and}$$

$$g: \mathbb{N} \rightarrow \mathbb{N} \text{ such that } g(x) = \begin{cases} x-1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

Clearly f is not an onto function since set of natural numbers is infinite

$$\text{gof: } \mathbb{N} \rightarrow \mathbb{N} \text{ s.t.}$$

$$\text{gof}(x) = g(f(x)) = g(x+1) = x \quad \text{since } x \text{ is a natural number so } x+1 > 1$$

Now consider any natural number $x \in \mathbb{N}$, the codomain set of gof ,

Every natural number has a predecessor i.e every x in $\mathbb{N}$ can be mapped to $x-1$ in $\mathbb{N}$. So gof is onto.

OR

For one-one function

Let $x_1 = x_2 \in \mathbb{R}$ such that $f(x_1) = f(x_2)$

$$\begin{aligned}\frac{2x_1 - 1}{3} &= \frac{2x_2 - 1}{3} \\ \Rightarrow 2x_1 - 1 &= 2x_2 - 1 \\ \Rightarrow x_1 &= x_2\end{aligned}$$

So, f is a 1-1 function

For onto function

Let $y \in \mathbb{R}$ such that $f(x) = y$

$$\begin{aligned}\frac{2x - 1}{3} &= y \Rightarrow 2x - 1 = 3y \\ \Rightarrow x &= \frac{3y + 1}{2} \\ f\left(\frac{3y + 1}{2}\right) &= \frac{2\left(\frac{3y + 1}{2}\right) - 1}{3} \\ &= \frac{3y + 1 - 1}{3} = y\end{aligned}$$

Therefore, the function $f(x)$ is onto.

The function is bijective, therefore invertible.

For inverse function

Since, $f(x)$ is one-one and onto, therefore

$f^{-1}(x)$ exists.

Let $y = f(x)$

$$\begin{aligned}y &= \frac{2x - 1}{3} \\ \Rightarrow 3y &= 2x - 1 \\ \Rightarrow x &= \frac{3y + 1}{2} \\ \therefore f^{-1}(y) &= \frac{3y + 1}{2}\end{aligned}$$

$$\text{Hence, } f^{-1}(x) = \frac{3x + 1}{2}$$

14. Let $\sin^{-1}x = \theta$ or $\sin\theta = x$

So given equation becomes

$$\sin^{-1}(1-x) - 2\theta = \frac{\pi}{2}$$

$$\sin^{-1}(1-x) = \frac{\pi}{2} + 2\theta$$

$$1-x = \sin\left(\frac{\pi}{2} + 2\theta\right)$$

$$1-x = \cos 2\theta$$

$$1-x = 1 - 2\sin^2 \theta$$

$$\Rightarrow 1-x = 1 - 2x^2$$

$$\Rightarrow 2x^2 - x = 0$$

$$\Rightarrow x(2x-1) = 0$$

$$\Rightarrow x = 0, \quad x = \frac{1}{2}$$

If $x = 0$

$$\text{L.H.S of given equation} = \sin^{-1} 1 - 2\sin^{-1} 0 = \frac{\pi}{2} = \text{R.H.S.}$$

If $x = \frac{1}{2}$

$$\text{L.H.S. of given equation} = \sin^{-1} \frac{1}{2} - 2\sin^{-1} \frac{1}{2} = -\sin^{-1} \frac{1}{2} = -\frac{\pi}{6} \neq \text{R.H.S.}$$

$\therefore$ solution is $x = 0$

15.

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_1; R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz)(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} = 0$$

$$R_3 \rightarrow R_3 - R_2$$

$$\Rightarrow (1+xyz)(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x+y \\ 0 & 0 & z-y \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz)(x-y)(y-z)(z-x) = 0$$

$$\text{Given, } x, y, z \text{ are different } \Rightarrow 1+xyz=0 \Rightarrow xyz = -1$$

16.

$$y = x^{x^x}$$

then,

$$y = e^{\log x^{x^x}}$$

$$y = e^{x^x \log x}$$

differentiating w.r.t. x

$$\frac{dy}{dx} = e^{x^x \log x} \frac{d}{dx} (x^x \log x)$$

$$\frac{dy}{dx} = y \left(x^x \frac{d}{dx} \log x + \log x \frac{d}{dx} x^x \right)$$

$$\frac{dy}{dx} = y \left(x^x \frac{d}{dx} \log x + \log x \frac{d}{dx} e^{\log x^x} \right)$$

$$\frac{dy}{dx} = y \left(\frac{x^x}{x} + \log x \frac{d}{dx} e^{x \log x} \right)$$

$$\frac{dy}{dx} = y \left(\frac{x^x}{x} + \log x \frac{d}{dx} e^{x \log x} \right)$$

$$\frac{dy}{dx} = y \left(\frac{x^x}{x} + \log x \left(e^{x \log x} \frac{dy}{dx} x \log x \right) \right)$$

$$\frac{dy}{dx} = y \left(\frac{x^x}{x} + \log x \left(e^{x \log x} \left(x \times \frac{1}{x} + \log x \right) \right) \right)$$

$$\frac{dy}{dx} = y \left(\frac{x^x}{x} + \log x (x^x (1 + \log x)) \right)$$

$$\frac{dy}{dx} = x^{x^x} x^x \left(\frac{1}{x} + \log x (1 + \log x) \right)$$

OR

we have,

$$y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$$

$$y = e^{\tan x \cdot \log \sin x} + e^{\sec x \cdot \log \cos x}$$

differentiating

$$\frac{dy}{dx} = \frac{d}{dx}(e^{\tan x \cdot \log \sin x}) + \frac{d}{dx}(e^{\sec x \cdot \log \cos x})$$

$$\frac{dy}{dx} = e^{\tan x \cdot \log \sin x} \frac{d}{dx}(\tan x \cdot \log \sin x) + e^{\sec x \cdot \log \cos x} \frac{d}{dx}(\sec x \cdot \log \cos x)$$

$$\begin{aligned} \frac{dy}{dx} = (\sin x)^{\tan x} & \left(\tan x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} \tan x \right) + \\ & (\cos x)^{\sec x} \frac{d}{dx} \left(\sec x \frac{d}{dx} \log \cos x + \log \cos x \frac{d}{dx} \sec x \right) \end{aligned}$$

$$\frac{dy}{dx} = (\sin x)^{\tan x} (\sec^2 x \cdot \log \sin x + 1) + (\cos x)^{\sec x} (\sec x \cdot \tan x \cdot \log \cos x - \sec x \cdot \tan x)$$

17. Let

$$y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$$

$$y = e^{\cot x \cdot \log x} + \frac{2x^2 - 3}{x^2 + x + 2}$$

differentiate

$$\frac{dy}{dx} = e^{\cot x \cdot \log x} \times \frac{d}{dx}(\cot x \cdot \log x) + \frac{d}{dx} \left(\frac{2x^2 - 3}{x^2 + x + 2} \right)$$

$$\frac{dy}{dx} = x^{\cot x} \times \left(\cot x \times \frac{1}{x} + \log x \times -\operatorname{cosec}^2 x \right) + \frac{(x^2 + x + 2)4x - (2x^2 - 3)(2x + 1)}{(x^2 + x + 2)^2}$$

$$\frac{dy}{dx} = x^{\cot x} \times \left(\cot x \times \frac{1}{x} + \log x \times -\operatorname{cosec}^2 x \right) + \frac{2x^2 + 14x + 3}{(x^2 + x + 2)^2}$$

18.

$$y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta, 0 \leq \theta \leq \pi,$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(2 + \cos \theta)(4 \cos \theta) + 4 \sin \theta(\sin \theta)}{(2 + \cos \theta)^2} - 1 \\ &= \frac{8 \cos \theta + 4(\cos^2 \theta + \sin^2 \theta) - 4 - \cos^2 \theta - 4 \cos \theta}{(2 + \cos \theta)^2} \\ &= \frac{4 \cos \theta - \cos^2 \theta}{(2 + \cos \theta)^2} \\ &= \frac{\cos \theta(4 - \cos \theta)}{(2 + \cos \theta)^2} \end{aligned}$$

Since, $(2 + \cos \theta)^2 > 0$ for all θ

$4 - \cos \theta > 0$ for all θ as $|\cos \theta| \leq 1$

$$\therefore \frac{dy}{d\theta} > 0 \quad \text{if} \quad \cos \theta > 0$$

$$\therefore \text{If } \theta \in \left[0, \frac{\pi}{2}\right] \quad \frac{dy}{d\theta} > 0$$

$$\therefore y \text{ is increasing in } \left[0, \frac{\pi}{2}\right]$$

19.

$$\begin{aligned} & \int \frac{x^2}{x^4 + x^2 - 2} dx \\ &= \int \frac{x^2}{x^2 - 1} \cdot \frac{1}{x^2 + 2} dx \\ &= \int \frac{x^2}{x - 1} \cdot \frac{1}{x + 1} \cdot \frac{1}{x^2 + 2} dx \end{aligned}$$

Using partial fraction,

$$\begin{aligned} \frac{x^2}{(x-1)(x+1)(x^2+2)} &= \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{Cx+D}{(x^2+2)} \\ \frac{x^2}{(x-1)(x+1)(x^2+2)} &= \frac{A(x+1)(x^2+2) + B(x-1)(x^2+2) + Cx+D}{(x-1)(x+1)(x^2+2)} \end{aligned}$$

Equating the coefficients from both the numerators we get,

$$A + B + C = 0 \dots\dots(1)$$

$$A - B + D = 1 \dots\dots(2)$$

$$2A + 2B - C = 0 \dots\dots(3)$$

$$2A - 2B - D = 0 \dots\dots(4)$$

Solving the above equations we get,

$$A = \frac{1}{6}, B = -\frac{1}{6}, C = 0, D = \frac{2}{3}$$

Our Integral becomes,

$$\begin{aligned} \int \frac{x^2}{(x-1)(x+1)(x^2+2)} dx &= \int \frac{1}{6(x-1)} - \frac{1}{6(x+1)} + \frac{2}{3(x^2+2)} dx \\ &= \frac{1}{6} \log(x-1) - \frac{1}{6} \log(x+1) + \frac{2}{3} \times \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) + C \\ &= \frac{1}{6} \left[\log(x-1) - \log(x+1) + 2\sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) \right] + C \end{aligned}$$

20. To evaluate: $\int_0^2 (x^2 + e^x) dx$

Here $f(x) = x^2 + e^x$, $a = 0$, $b = 2$

So, $nh = b - a = 2$

Now $f(0) = 0 + e^0 = 1$

$f(0 + h) = f(h) = h^2 + e^h$

$f(0 + 2h) = f(2h) = 2^2 h^2 + e^{2h}$

$f(0 + (n - 1)h) = f((n - 1)h) = (n - 1)^2 h^2 + e^{(n-1)h}$

Now $\int_0^2 f(x) dx = \lim_{h \rightarrow 0} \left[f(0) + f(0 + h) + f(0 + 2h) + \dots + f(0 + (n - 1)h) \right]$

$$= \lim_{h \rightarrow 0} h \left[\left(h^2 + 2^2 h^2 + \dots + (n - 1)^2 h^2 \right) + \left(1 + e^h + e^{2h} + \dots + e^{(n-1)h} \right) \right]$$

$$= \lim_{h \rightarrow 0} h \left[h^2 \left(1^2 + 2^2 + \dots + (n - 1)^2 \right) + 1 \cdot \left(\frac{(e^h)^n - 1}{e^h - 1} \right) \right]$$

$$= \lim_{h \rightarrow 0} \left[h^3 \frac{n(n - 1)(2n - 1)}{6} + \frac{h(e^{nh} - 1)}{e^h - 1} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{nh(nh - h)(2nh - h)}{6} \right] + \lim_{h \rightarrow 0} (e^{nh} - 1) \times \lim_{h \rightarrow 0} \frac{h}{e^h - 1}$$

$$= \frac{2(2 - 0)(4 - 0)}{6} + (e^2 - 1) \times 1$$

$$= \frac{8}{3} + e^2 - 1 = \frac{5}{3} + e^2$$

21. The given differential equation is

$$(x+y+1)^2 \frac{dy}{dx} = 1$$

$$\text{sub } x+y+1 = v$$

$$\frac{dy}{dx} + 1 = \frac{dv}{dx}$$

so,

$$\Rightarrow v^2 \left(\frac{dv}{dx} - 1 \right) = 1$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{v^2} + 1$$

$$\Rightarrow \frac{dv}{dx} = \frac{1+v^2}{v^2}$$

$$\Rightarrow \int \frac{v^2}{1+v^2} dv = \int dx$$

$$\Rightarrow \int \left(1 - \frac{1}{1+v^2} \right) dv = x + c$$

$$\Rightarrow v - \tan^{-1} v = x + c$$

$$\Rightarrow x+y+1 - \tan^{-1}(x+y+1) = x + c$$

given that $x = -1$, then $y = 0$

we get

$$c = 1$$

so,

$$y = \tan^{-1}(x+y+1)$$

$$\tan y = x+y+1$$

OR

The given differential equation is

$$(x - y)(dx + dy) = dx - dy$$

$$\Rightarrow (x - y - 1)dx = -(x - y + 1)dy$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x - y + 1}{x - y - 1}$$

$$\text{let } x - y = v$$

$$\frac{dy}{dx} = 1 - \frac{dv}{dx}$$

so,

$$1 - \frac{dv}{dx} = -\frac{v - 1}{v + 1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{2v}{v + 1}$$

$$\Rightarrow \frac{v + 1}{v} dv = 2 dx$$

$$\Rightarrow \int \left(1 + \frac{1}{v}\right) dv = \int dx$$

$$\Rightarrow v + \log|v| = 2x + c$$

$$\Rightarrow x - y + \log|x - y| = 2x + c$$

$$\Rightarrow \log|x - y| = x + y + c$$

given that

$$x = 0, \text{ then } y = -1$$

so, substituting we get

$$c = 1$$

$$\log|x - y| = x + y + 1$$

$$x - y = \pm e^{x+y+1}$$

22. $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

The vector which is perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ is in the direction of

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix} = 32\hat{i} - \hat{j} - 14\hat{k}$$

Since $\vec{d}$ is perpendicular to $\vec{a}$ and $\vec{b}$

$\vec{a} \times \vec{b}$ is also perpendicular to both $\vec{a}$ and $\vec{b}$

$\Rightarrow \vec{d}$ is parallel to $\vec{a} \times \vec{b}$.

$$\Rightarrow \vec{d} = \lambda(\vec{a} \times \vec{b})$$

$$\Rightarrow \vec{d} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}$$

$$\text{Also } \vec{c} \cdot \vec{d} = 15 \Rightarrow 2 \cdot 32\lambda + (-1)(-\lambda) + 4(-14\lambda) = 15$$

$$\Rightarrow \lambda = \frac{5}{3}$$

$$\Rightarrow \vec{d} = \frac{5}{3}(32\hat{i} - \hat{j} - 14\hat{k})$$

23.

$$\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$$

and

$$\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$$

Let us rewrite the equations of the given lines as follows:

$$\frac{-(x-1)}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$$

and

$$\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{-(z-6)}{7}$$

That is we have,

$$\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$$

and

$$\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-7}$$

The lines are perpendicular so angle between them is 90°

So, $\cos\theta = 0$

Here $(a_1, b_1, c_1) = (-3, 2\lambda, 2)$ and $(a_2, b_2, c_2) = (3\lambda, 1, -7)$

For perpendicular lines

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow -9\lambda + 2\lambda - 14 = 0$$

$$\Rightarrow -7\lambda - 14 = 0$$

$$\Rightarrow -7\lambda = 14$$

$$\Rightarrow \lambda = \frac{14}{-7}$$

$$\Rightarrow \lambda = -2$$

SECTION - D

24.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(A+B)^T =$$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 1+1 & 1 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

OR

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}$$

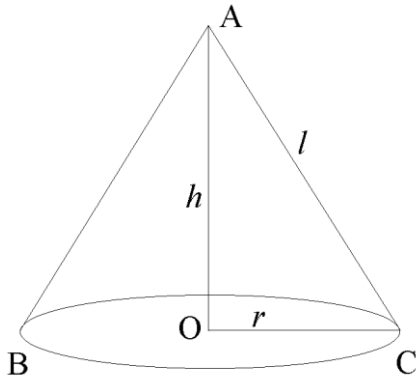
$$AB = \begin{bmatrix} 4-1-2 & -2+1+1 & -1+1 \\ 8-2-6 & -4+2+3 & -2+2 \\ -4+4 & 2-2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & -2 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

25.



Here, Volume 'V' of the cone is $V = \frac{1}{3} \pi r^2 h \Rightarrow r^2 = \frac{3V}{\pi h}$... (1)

Surface area $S = \pi r l = \pi r \sqrt{h^2 + r^2}$... (2)

Where h = height of the cone

r = radius of the cone

l = Slant height of the cone

$S^2 = \pi^2 r^2 (h^2 + r^2)$ from equation (2)

Let, $S_1 = S^2$

Substituting the value of r^2 from equation (1), we have,

$$S_1 = \frac{3\pi V}{h} \left(h^2 + \frac{3V}{\pi h} \right) = 3\pi V h + \frac{9V^2}{h^2}$$

Differentiating S_1 with respect to h, we get

$$\frac{dS_1}{dh} = 3\pi V + 9V^2 \left(\frac{-2}{h^3} \right)$$

$$\frac{dS_1}{dh} = 0 \text{ for maxima/minima}$$

$$3\pi V + 9V^2 \left(\frac{-2}{h^3} \right) = 0$$

$$\Rightarrow 3\pi V = 9V^2 \left(\frac{2}{h^3} \right)$$

$$\Rightarrow h^3 = \frac{6V}{\pi}$$

$$\frac{d^2S_1}{dh^2} = \frac{54V^2}{h^4}$$

$$\frac{d^2S_1}{dh^2} > 0 \text{ at } h^3 = \frac{6V}{\pi^2}$$

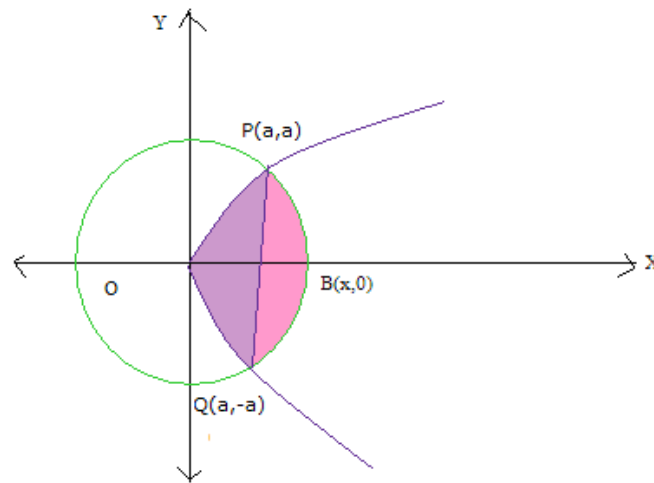
Therefore curved surface area is minimum at $\frac{\pi h^3}{6} = V$

$$\text{Thus, } \frac{\pi h^3}{6} = \frac{1}{3}\pi r^2 h \Rightarrow h^2 = 2r^2$$

$$\Rightarrow h = \sqrt{2}r$$

Hence for least curved surface the altitude is $\sqrt{2}$ times radius.

26.



The circle is $x^2 + y^2 = 2a^2 \Rightarrow C(0, \sqrt{2}a)$

The parabola is $y^2 = ax, a > 0 \Rightarrow y^2 = 4 \frac{1}{4} ax, a > 0$

Their point of intersection is given by : $x^2 + ax = 2a^2$

$$\Rightarrow x^2 + ax - 2a^2 = 0$$

$$\Rightarrow (x + 2a)(x - a) = 0$$

$$\Rightarrow x = a, -2a$$

$$\Rightarrow x = a$$

$$\Rightarrow y^2 = a^2 \Rightarrow y = \pm a$$

$\Rightarrow$ shade region is the smaller of the two areas

in which the circle is divided by the parabola

$$\begin{aligned} A &= 2 \left[\int_0^a \sqrt{ax} dx + \int_a^{\sqrt{2}a} (2a^2 - x^2)^{1/2} dx \right] \\ &= 2\sqrt{a} \left[\frac{x^{3/2}}{\frac{3}{2}} \right]_0^a + 2 \left[\frac{x}{2} (2a^2 - x^2)^{1/2} + \frac{2a^2}{2} \sin^{-1} \frac{x}{\sqrt{2}a} \right]_{a}^{\sqrt{2}a} \\ &= \frac{4}{3} \sqrt{a} \left[a^{3/2} \right] + \left[x (2a^2 - x^2)^{1/2} + 2a^2 \sin^{-1} \frac{x}{\sqrt{2}a} \right]_{a}^{\sqrt{2}a} \end{aligned}$$

$$= \frac{4}{3} \sqrt{a} \left[a^{3/2} \right] + \left[2a^2 \sin^{-1} \frac{\sqrt{2}a}{\sqrt{2}a} - a \left(2a^2 - a^2 \right)^{1/2} - 2a^2 \sin^{-1} \frac{a}{\sqrt{2}a} \right]$$

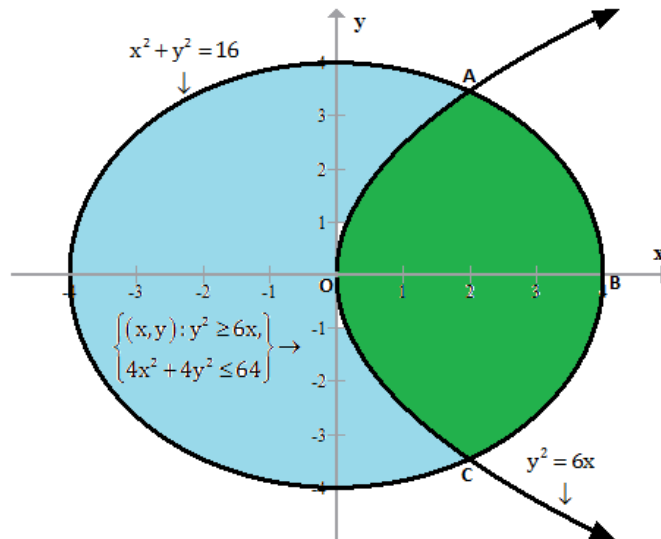
$$= \frac{4}{3} \sqrt{a} \left[a^{3/2} \right] + \left[2a^2 \sin^{-1} 1 - a \left(a^2 \right)^{1/2} - 2a^2 \sin^{-1} \frac{1}{\sqrt{2}} \right]$$

$$= \frac{4}{3} \sqrt{a} \left[a^{3/2} \right] + \left[2a^2 \frac{\pi}{2} - a^2 - 2a^2 \frac{\pi}{4} \right]$$

$$= \frac{4}{3} \sqrt{a} \left[a^{3/2} \right] + 2a^2 \frac{\pi}{4} - a^2$$

$$= \frac{4}{3} \sqrt{a} \left[a^{3/2} \right] + a^2 \frac{\pi}{2} - a^2 \text{sq. units}$$

OR



$$4x^2 + 4y^2 = 64$$

$$\Rightarrow x^2 + y^2 = 16$$

The points of intersection of the two curves $x^2 + y^2 = 16$ and $y^2 = 6x$

$$x^2 + 6x = 16 \Rightarrow x^2 + 6x - 16 = 0 \Rightarrow (x+8)(x-2) = 0 \Rightarrow x = -8, 2$$

But x is non negative so $x = 2$

Required area (Blue shaded portion)

= Ar (Circle) - Ar (Green Shaded portion)

$$\Rightarrow \text{Required area} = \pi(4)^2 - 2 \left[\int_0^2 \sqrt{6x} dx + \int_2^4 \sqrt{16-x^2} dx \right]$$

$$= 16\pi - 2\sqrt{6} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^2 - 2 \left[\frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_2^4$$

$$= 16\pi - \frac{4\sqrt{6}}{3} \left(\frac{3}{2} - 0 \right) - 2 \left[8\sin^{-1}(1) - \sqrt{16-4} - 8\sin^{-1}\left(\frac{1}{2}\right) \right]$$

$$= 16\pi - \frac{4\sqrt{6} \times 2\sqrt{2}}{3} - 2 \left[8 \times \frac{\pi}{2} - \sqrt{12} - 8 \times \frac{\pi}{6} \right]$$

$$= 16\pi - \frac{16\sqrt{3}}{3} - \frac{16\pi}{3} + 4\sqrt{3} = \frac{32\pi}{3} - \frac{4\sqrt{3}}{3} = \frac{4}{3}(8\pi - \sqrt{3}) \text{ sq. units}$$

27. Let $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$

$$x = -2 + 3\lambda, y = -1 + 2\lambda, z = 3 + 2\lambda$$

Therefore, a point on this line is: $\{(-2+3\lambda), (-1+2\lambda), (3+2\lambda)\}$

The distance of the point $\{(-2+3\lambda), (-1+2\lambda), (3+2\lambda)\}$ from point $(1, 2, 3) = 3\sqrt{2}$

$$\therefore \sqrt{(-2+3\lambda-1)^2 + (-1+2\lambda-2)^2 + (3+2\lambda-3)^2} = 3\sqrt{2}$$

$$\Rightarrow -3+3\lambda^2 + -3+2\lambda^2 + 2\lambda^2 = 18$$

$$\Rightarrow 9+9\lambda^2 - 18\lambda + 9+4\lambda^2 - 12\lambda + 4\lambda^2 = 18$$

$$17\lambda^2 - 30\lambda = 0$$

$$\lambda = 0, \lambda = \frac{30}{17}$$

$$\text{When } \lambda = \frac{30}{17},$$

$$x = -2 + 3\lambda = -2 + 3\left(\frac{30}{17}\right) = -2 + \frac{90}{17} = \frac{56}{17}$$

$$y = -1 + 2\lambda = -1 + 2\left(\frac{30}{17}\right) = -1 + \frac{60}{17} = \frac{43}{17}$$

$$z = 3 + 2\lambda = 3 + 2\left(\frac{30}{17}\right) = \frac{51+60}{17} = \frac{111}{17}$$

Thus, when $\lambda = \frac{30}{17}$, the point is $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$

and when $\lambda = 0$, the point is $(-2, -1, 3)$.

OR

The equation of line L_1 :

$$\frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$$
$$\Rightarrow \frac{x-1}{-3} = \frac{y-2}{\frac{p}{7}} = \frac{z-3}{2} \dots (1)$$

The equation of line L_2 :

$$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$
$$\Rightarrow \frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} \dots (2)$$

Since line L_1 and L_2 are perpendicular to each other, we have

$$-3 \times \left(\frac{-3p}{7} \right) + \frac{p}{7} \times 1 + 2 \times (-5) = 0$$
$$\Rightarrow \frac{9p}{7} + \frac{p}{7} = 10$$
$$\Rightarrow 10p = 70$$
$$\Rightarrow p = 7$$

Thus equations of lines L_1 and L_2 are:

$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-3}{2}$$
$$\frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$$

Thus the equation of the line passing through the point $(3, 2, -4)$

and parallel to the line L_1 is:

$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$$

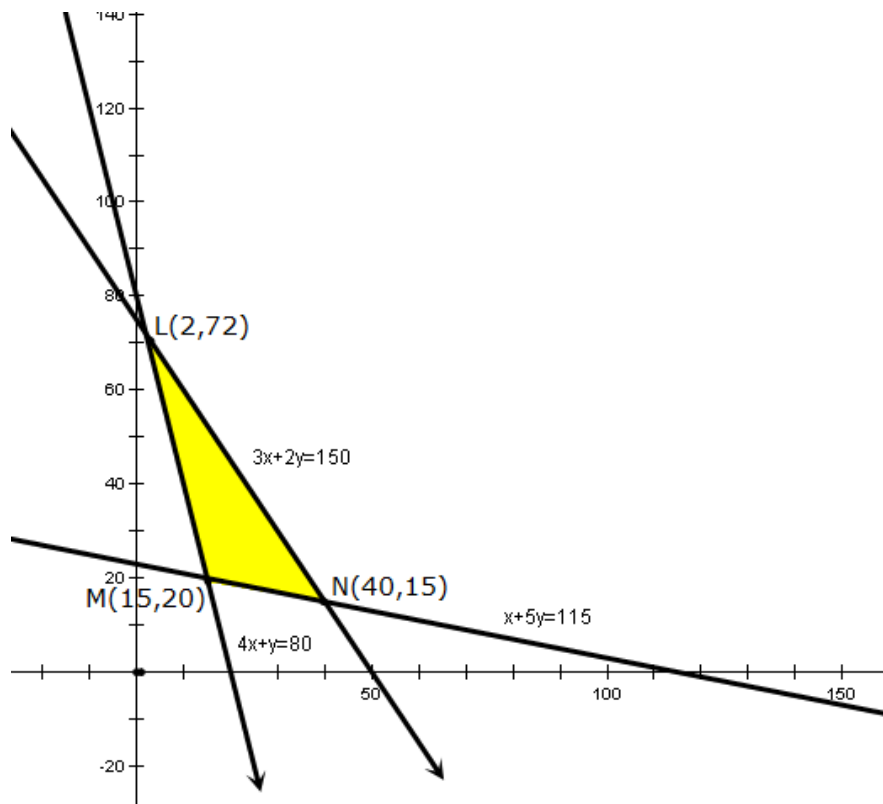
28. Let x and y be number of packets of food P and Q respectively.

Linear programming problem is

Minimize $Z = 6x + 3y$ (Vitamins A)

$$\begin{aligned} \text{s.t.} \quad & 12x + 3y \geq 240 & \text{or} & \quad 4x + y \geq 80 \\ & 4x + 20y \geq 460 & \text{or} & \quad x + 5y \geq 115 \\ & 6x + 4y \leq 300 & \text{or} & \quad 3x + 2y \leq 150 \\ & x \geq 0, y \geq 0, \end{aligned}$$

Graphically the problem can be represented as



Co-ordinates of corner points L, M, N are $(2, 72)$, $(15, 20)$ and $(40, 15)$, we have

Corner points	$z = 6x + 3y$
L $(2, 72)$	228
M $(15, 20)$	150 $\rightarrow$ Minimum
N $(40, 15)$	285

Hence, minimum vitamin A is used at point $(15, 20)$ i.e. when 15 packets of food P and 20 packets of food Q are used.

29. This is a case of Bernoulli's trials.

Let Success: Getting a purple ball on a draw

Failure: Getting a pink ball on a draw

$$p = P(\text{success}) = \frac{10}{25} = \frac{2}{5} \Rightarrow q = \frac{3}{5}$$

$$(i) P(6\text{success}) = {}^6C_6 p^6 q^0 = 1 \times \left(\frac{2}{5}\right)^6 \times 1 = \left(\frac{2}{5}\right)^6$$

$$(ii) P(\text{ not more than 2 failures }) = P(\text{ not less than 4 success}) = P(4) + P(5) + P(6)$$

$$= {}^6C_4 p^4 q^2 + {}^6C_5 p^5 q^1 + {}^6C_6 p^6 q^0 = 15 \times \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2 + 6 \times \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right) + 1 \times \left(\frac{2}{5}\right)^6$$

$$= \left(\frac{2}{5}\right)^4 \left[15 \times \left(\frac{3}{5}\right)^2 + 6 \times \left(\frac{2}{5}\right) \left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)^2 \right]$$

$$= \left(\frac{2}{5}\right)^4 \left[\frac{135}{25} + \frac{36}{25} + \frac{4}{25} \right]$$

$$= \left(\frac{2}{5}\right)^4 \left[\frac{175}{25} \right] = 7 \times \left(\frac{2}{5}\right)^4$$

$$(iii) P(3\text{success } 3\text{failures}) = P(3) = {}^6C_3 p^3 q^3 = 15 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 = \frac{648}{3125}$$

$$(iv) P(\text{atleast 1 failure}) = P(\text{ at most 5 success})$$

$$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = 1 - P(6) = 1 - {}^6C_6 p^6 q^0$$

$$= 1 - 1 \times \left(\frac{2}{5}\right)^6 \times 1 = 1 - \left(\frac{2}{5}\right)^6 = 1 - \frac{64}{15625} = \frac{15561}{15625}$$