Surface Areas & Volumes of Solids

Introduction to Volume

Observe the water tanks given below.

How would you decide which one is bigger?

We can do that by observing the space occupied by them or by comparing the maximum quantity of water they can hold.

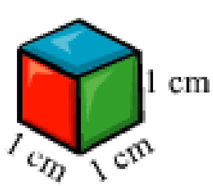
The space occupied by a solid shape is its volume, while the maximum quantity of liquid that it can hold shows its capacity.

We can also say that the capacity of a solid is equal to its volume.

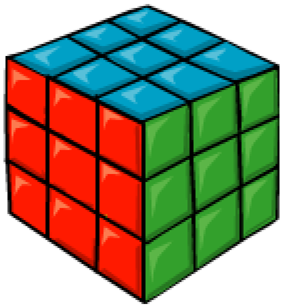
Two-dimensional shapes such as squares and rectangles do not have volume; only three-dimensional shapes have volumes.

Let us learn the concept by finding the volume of a cube.

When each side of a cube measures 1 cm, the space occupied by it i.e., its volume is said to be 1 cubic centimetre. This unit is written as c.c. or cm^3, which is the fundamental unit of volume. This cube is called the unit cube of side 1 cm.



Now, let us use some unit cubes to a make a bigger cube.



It can be observed that the above cube is made of 27 unit cubes of side 1 cm. Thus, its volume will be equal to the volume of 27 unit cubes.

Volume of bigger cube = 27 × Volume of unit cube = (27 × 1) cm^3 = 27 cm^3

Similarly, we can find the volumes of different cubes or cuboids by virtually breaking them into unit cubes.

Example 1:

By using the small cube in figure (a), find the volume of the solid in figure (b).

Figure (a)

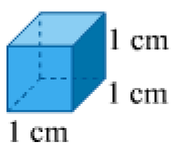
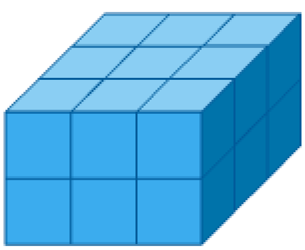


Figure (b)



Solution:

Volume of smaller cube in figure (a) = 1 cm^3

It can be observed that the solid in figure (b) consists of 18 cubes like that in figure (a).

∴ Volume of the solid = (18 × 1) cm^3 = 18 cm^3

Example 2:

By using the small cube in figure (a), find the volume of the solid in figure (b).

Figure (a)

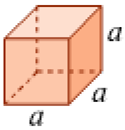
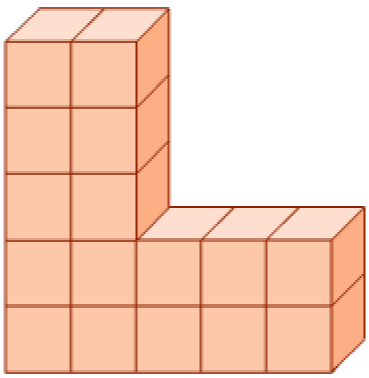


Figure (b)



Solution:

Volume of the smaller cube in figure (a) = a^3 cubic units

It can be observed that the solid in figure (b) consists of 16 cubes like that in figure (a).

∴ Volume of the solid = ($16 \times a^3$) cubic units = $16a^3$ cubic units

Volumes of a Cube and a Cuboid

Abhinav's mother gives him a container, asking him to go to the neighbouring milk booth and buy 2.5 L of milk. What does '2.5 L'represent? It represents the amount of milk that Abhinav needs to buy. In other words, it is the volume of milk that is to be bought.

After buying the milk, Abhinav notices that the container is full up to its brim. He says to himself, 'This container has no capacity to hold any more milk.' What does the word 'capacity' indicate? **The space occupied by a substance is called its volume.**

The capacity of a container is the volume of a substance that can fill the container completely. In this case, the volume and the capacity of the container are the same. The standard units which are used to measure the volume are **cm^3 (cubic centimetre)** and **m^3 (cubic metre).**

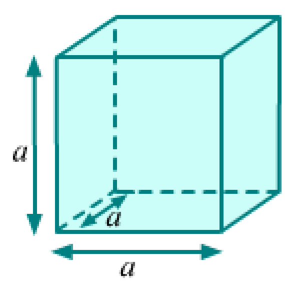
In this lesson, we will learn the formulae for the volumes or capacities of cubic and cuboidal objects. We will also solve examples using these formulae.

Did You Know?

A cube is one among the five platonic solids. This means that it is a regular and convex polyhedron with the same number of faces meeting at each vertex.

Formulae for the Volumes of a Cube and a Cuboid

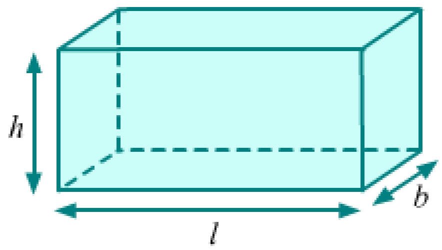
Consider a cube with an edge *a*.



The formula for the volume of this cube is given as follows:

Volume of the cube = a^3

Now, consider a cuboid with length l, breadth b and height h.



The formula for the volume of this cuboid is given as follows:

Volume of the cuboid = $l \times b \times h$

Concept Builder

The units of capacity and volume are interrelated as follows:

- 1 cm^3 = 1 mL
- 1000 cm^3 = 1 L
- 1 m^3 = 1 kL = 1000 L

Did You Know?

- A cube has the maximum volume among all cuboids with equal surface area.
- A cube has the minimum surface area among all cuboids with equal volume.

Solved Examples

Easy

Example 1:

Find the volumes of cubes of given sides.

(a) 2 cm (b) 5 m (c) 12 cm (d) 15 m

Solution:

(a)

Measure of side of cube = 2 cm

Volume of cube = $(\text{Side})^3 = 2^3 \text{ cm}^3 = 8 \text{ cm}^3$

(b)

Measure of side of cube = 5 m

Volume of cube = $(\text{Side})^3 = 5^3\ m^3 = 125\ m^3$

(c)

Measure of side of cube = 12 cm

Volume of cube = $(\text{Side})^3 = 12^3\ cm^3 = 1728\ cm^3$

(d)

Measure of side of cube = 15 m

Volume of cube = $(\text{Side})^3 = 15^3\ m^3 = 3375\ m^3$

Example 2:

Find the volumes of cuboids of given dimensions.

(a) length = 5 cm, breadth = 2 cm, height = 6 cm

(b) length = 15 cm, breadth = 10 cm, height = 30 cm

(c) length = 1 m, breadth = 0.5 m, height = 1.5 m

Solution:

(a)

We have

length = 5 cm, breadth = 2 cm, height = 6 cm

∴ Volume of cuboid = length × breadth × height

$= (5 \times 2 \times 6)\ cm^3$

$= 60\ cm^3$

(b)

We have

length = 15 cm, breadth = 10 cm, height = 30 cm

∴ Volume of cuboid = length × breadth × height

$= (15 \times 10 \times 30)\ cm^3$

$= 4500\ cm^3$

(c)

We have

length = 1 m, breadth = 0.5 m, height = 1.5 m

∴ Volume of cuboid = length × breadth × height

$= (1 \times 0.5 \times 1.5)\ m^3$

$= 0.75\ m^3$

Example 3:

If a cubical tank can contain 1331000 L of water, then find the edge of the tank.

Solution:

Capacity of the cubical tank = 1331000 L

$= 1331\ m^3$ (∵ 1000 L = $1\ m^3$)

Now, capacity of the tank = Volume of water that can be contained in the tank

We know that volume of water in the tank = $(\text{Edge})^3$

$\Rightarrow (\text{Edge})^3 = 1331\ m^3$

⇒ ∴ Edge = 11 m

Thus, the edge of the cubical tank is 11 m.

Example 4:

Find the height of the cuboid whose volume is 840 cm^3 and the area of whose base is 120 cm^2.

Solution:

Let the length, breadth and height of the cuboid be *l*, *b* and *h* respectively.

Area of the base of the cuboid = 120 cm^2

$\therefore l \times b = 120 \text{ cm}^2$

Volume of the cuboid = 840 cm^3

$\therefore l \times b \times h = 840 \text{ cm}^3$

$\Rightarrow 120 \text{ cm}^2 \times h = 840 \text{ cm}^3$ ($\because l \times b = 120 \text{ cm}^2$)

$$\Rightarrow h = \frac{840}{120} \text{ cm}$$
$$\Rightarrow \therefore h = 7 \text{ cm}$$

Thus, the height of the cuboid is 7 cm.

Example 5:

If the ratio of the edges of two cubes is 2 : 5, then find the ratio of their volumes.

Solution:

Let the edges of the cubes be $a = 2x$ and $b = 5x$.

Ratio of the volumes of the cubes $= \dfrac{\text{Volume of the first cube}}{\text{Volume of the second cube}}$

$$= \frac{a^3}{b^3}$$
$$= \frac{(2x)^3}{(5x)^3}$$
$$= \frac{8x^3}{125x^3}$$
$$= \frac{8}{125}$$

Thus, the volumes of the cubes are in the ratio 8 : 125.

Medium

Example 1:

A solid cube of edge 18 cm is cut into eight cubes of equal volume. Find the dimension of each new cube. Also find the ratio of the total surface area of the bigger cube to that of the new cubes formed.

Solution:

Let the edge of each new cube be *x*.

According to the question, we have:

Volumes of 8 cubes each of edge *x* = Volume of cube of edge 18 cm

$$\Rightarrow 8 \times x^3 = (18\text{ cm})^3$$
$$\Rightarrow x^3 = \frac{18\text{ cm} \times 18\text{ cm} \times 18\text{ cm}}{8} = 729\text{ cm}^3$$
$$\Rightarrow x^3 = (9\text{ cm})^3$$
$$\Rightarrow \therefore x = 9\text{ cm}$$

Thus, the edge of each new cube is 9 cm.

Total surface area (S_1) of the bigger cube = 6 × $(18\text{ cm})^2$

Total surface area of 8 cubes (S_2) each of edge 9 cm = 8 × [6 × $(9\text{ cm})^2$]

$$\therefore \frac{S_1}{S_2} = \frac{6 \times 18^2}{8 \times 6 \times 9^2} = \frac{1}{2}$$

Hence, the required ratio is 1 : 2.

Example 2:

A hostel having strength of 300 students requires on an average 36000 L of water per day. It has a tank measuring 10 m × 8 m × 9 m. For how many days will the water in the tank filled to capacity last?

Solution:

Let the cuboidal tank have length *l*, breadth *b* and height *h*.

It is given that l = 10 m, b = 8 m and h = 9 m.

Capacity of the tank = $l \times b \times h$ = 10 m × 8 m × 9 m = 720 m^3

∴ Amount of water in the tank filled to capacity = 720 m^3 = 720000 L (∵ 1000 L = 1 m^3)

Amount of water used by 300 students in 1 day = 36000 L

Number of days for which the water in the full tank will last $= \frac{\text{Amount of water in the full tank}}{\text{Amount of water used in a day}}$

$$= \frac{720000}{36000}$$
$$= 20$$

Thus, the water in the tank filled to capacity will last for 20 days.

Example 3:

The dimensions of a wall in a godown are 25 m × 0.3 m × 10 m. How many bricks of dimensions 25 cm × 10 cm × 5 cm were used to construct the wall?

Solution:

Length (L) of the wall = 25 m = (25 × 100) cm = 2500 cm

Breadth (B) of the wall = 0.3 m = (0.3 × 100) cm = 30 cm

Height (H) of the wall = 10 m = (10 × 100) cm = 1000 cm

∴ Volume of the wall = $L \times B \times H$ = (2500 × 30 × 1000) cm^3

Length (l) of one brick = 25 cm

Breadth (b) of one brick = 10 cm

Height (h) of one brick = 5 cm

∴ Volume of one brick = $l \times b \times h$ = (25 × 10 × 5) cm^3

Number of bricks used to construct the wall $= \frac{\text{Volume of the wall}}{\text{Volume of one brick}}$

$$= \frac{2500 \times 30 \times 1000}{25 \times 10 \times 5}$$
$$= 60000$$

Thus, 60000 bricks of dimensions 25 cm × 10 cm × 5 cm were used to construct the wall.

Example 4:

A storeroom is in the form of a cuboid with dimensions 90 m × 150 m × 120 m. How many cubical boxes of edge 60 dm can be stored in the room?

Solution:

Length (l) of the storeroom = 90 m

Breadth (b) of the storeroom = 150 m

Height (h) of the storeroom = 120 m

∴ Volume of the storeroom = $l \times b \times h$ = (90 × 150 × 120) m^3

Edge (a) of one cubical box $= 60 \text{ dm} = \left(\frac{60}{10}\right) \text{ m} = 6 \text{ m}$

∴ Volume of one box = $a^3 = (6)^3 \text{ m}^3$

Number of boxes that can be stored in the room $= \frac{\text{Volume of the storeroom}}{\text{Volume of one box}}$

$$= \frac{90 \times 150 \times 120}{6 \times 6 \times 6}$$
$$= 7500$$

Thus, 7500 cubical boxes of edge 60 dm can be stored in the room.

Hard

Example 1:

A man-made canal is 5 m deep and 60 m wide. The water in the canal flows at the rate of 3 km/h. The canal empties its water into a reservoir. How much water will fall into the reservoir in 10 minutes?

Solution:

Depth (h) of the canal = 5 m

Width (b) of the canal = 60 m

Length (l) of the canal is the rate of water flowing per hour = 3 km = 3000 m

Amount of water flowing per hour = $l \times b \times h$ = (3000 × 60 × 5) m^3 = 900000 m^3 = 900000 kL ($\because$ 1 m^3 = 1 kL)

$\therefore$ Amount of water flowing in 60 min = 900000 kL

$\Rightarrow$ Amount of water flowing in 1 minute $= \left(\frac{900000}{60}\right) \text{kL}$

$\Rightarrow$ Amount of water flowing in 10 minutes $= \left(\frac{900000}{60} \times 10\right) \text{kL}$

= 150000 kL

Thus, 150000 kL of water will fall into the reservoir in 10 minutes.

Example 2:

The external length, breadth and height of a closed rectangular wooden box are 9 cm, 5 cm and 3 cm respectively. The thickness of the wood used is 0.25 cm. The box weighs 7.5 kg when empty and 50 kg when it is filled with sand. Find the weights of one cubic cm of wood and one cubic cm of sand.

Solution:

External length (L) of the wooden box = 9 cm

External breadth (B) of the wooden box = 5 cm

External height (H) of the wooden box = 3 cm

$\therefore$ External volume of the wooden box = $L \times B \times H$ = (9 × 5 × 3) cm^3 = 135 cm^3

Thickness of the wood used = 0.25 cm

Internal length (l) of the wooden box = 9 cm − (0.25 cm + 0.25 cm) = 8.5 cm

Internal breadth (b) of the wooden box = 5 cm − (0.25 cm + 0.25 cm) = 4.5 cm

Internal height (h) of the wooden box = 3 cm − (0.25 cm + 0.25 cm) = 2.5 cm

∴ Internal volume of the wooden box = $l \times b \times h$ = (8.5 × 4.5 × 2.5) cm^3 = 95.625 cm^3

Now, volume of the wood = External volume of the box − Internal volume of the box

= (135 − 95.625) cm^3

= 39.375 cm^3

Weight of the empty box = 7.5 kg

⇒ Weight 39.375 cm^3 of wood = 7.5 kg

∴ Weight of 1 cm^3 of wood $= \left(\frac{7.5}{39.375}\right)$ kg = 0.19 kg

Now, volume of sand = Internal volume of the box = 95.625 cm^3

Weight of sand = Weight of the box filled with sand − Weight of the empty box

= (50 − 7.5) Kg

= 42.5 Kg

⇒ Weight of 95.625 cm^3 of sand = 42.5 kg

∴ Weight of 1 cm^3 of sand $= \left(\frac{42.5}{95.625}\right)$ kg = 0.44 kg

Surface Areas of a Cube and a Cuboid

We give gifts to our friends and relatives at one time or another. We usually wrap our gifts in nice and colourful wrapping papers. Look, for example, at the nicely wrapped and tied gift shown below.

Clearly, the gift is packed in box that is cubical or shaped like a **cube**. Suppose you have a gift packed in a similar box. How would you determine the amount of wrapping paper needed to wrap the gift?

You could do so by making an estimate of the surface area of the box. In this case, the total area of all the faces of the box will tell us the area of the wrapping paper needed to cover the box.

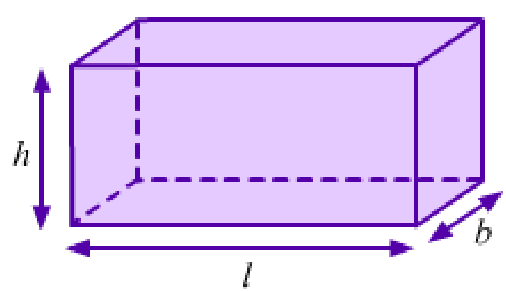
Knowledge of surface areas of the different solid figures proves useful in many real-life situations where we have to deal with them. In this lesson, we will learn the formulae for the surface areas of a cube and a **cuboid**. We will also solve some examples using these formulae.

Did You Know?

- The word 'cuboid' is made up of 'cube' and '-oid' (which means 'similar to'). So, a cuboid indicates something that is similar to a cube.
- A cuboid is also called a 'rectangular prism' or a 'rectangular parallelepiped'.

Formulae for the Surface Area of a Cuboid

Consider a cuboid of length *l*, breadth *b* and height *h*.



The formulae for the surface area of this cuboid are given as follows:

Lateral surface area of the cuboid = 2*h* (*l* + *b*)

Total surface area of the cuboid = 2 (*lb* + *bh* + *hl*)

Here, lateral surface area refers to the area of the solid excluding the areas of its top and bottom surfaces, i.e., the areas of only its four standing faces are included. Total surface area refers to the sum of the areas of all the faces.

Did You Know?

Two mathematicians named Henri Lebesgue and Hermann Minkowski sought the definition of surface area at around the twentieth century.

Know Your Scientist

Henri Lebesgue (1875−1941) was a French mathematician who is famous for his theory of integration. His contribution is one of the major achievements of modern analysis which greatly expands the scope of Fourier analysis. He also made important contributions to topology, the potential theory, the *Dirichlet* problem, the calculus of variations, the set theory, the theory of surface area and the dimension theory.

Hermann Minkowski (1864−1909) was a Polish mathematician who developed the geometry of numbers and made important contributions to the number theory, mathematical physics and the theory of relativity. His idea of combining time with the three dimensions of space, laid the mathematical foundations for Albert Einstein's theory of relativity.

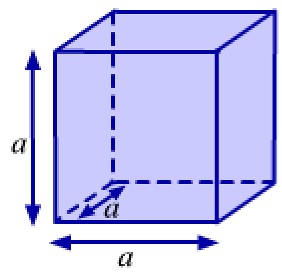
Example Based on the Surface Area of a Cuboid

Did You Know?

The concept of surface area is widely used in chemical kinetics, regulation of digestion, regulation of body temperature, etc.

Formulae for the Surface Area of a Cube

Consider a cube with edge *a*.



The formulae for the surface area of this cube are given as follows:

Lateral surface area of the cube = $4a^2$

Total surface area of the cube = $6a^2$

Here, lateral surface area refers to the area of the solid excluding the areas of its top and bottom surfaces, i.e., the areas of only its four standing faces are included. Total surface area refers to the sum of the areas of all the faces.

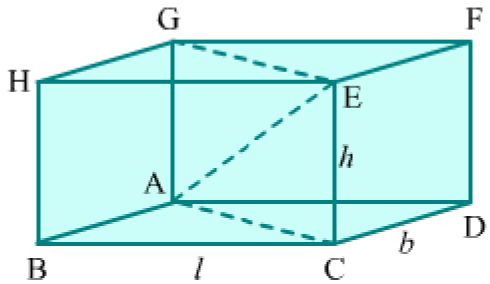
Did You Know?

- A cube can have 11 different nets.
- Cubes and cuboids are **convex polygons** that satisfy **Euler's formula**, i.e., **F + V − E = 2**.

Know More

Length of the diagonal in a cube and in a cuboid

A cuboid has four diagonals (say AE, BF, CG and DH). The four diagonals are equal in length.



Let us consider the diagonal AE.

In rectangle ABCD, length of diagonal AC = $\sqrt{l^2 + b^2}$

Now, ACEG is a rectangle with length AC and breadth CE or *h*.

So, length of diagonal AE = $\sqrt{AC^2 + CE^2}$

$$= \sqrt{\left(\sqrt{l^2 + b^2}\right)^2 + h^2}$$
$$= \sqrt{l^2 + b^2 + h^2}$$

∴ **Length of the diagonal of a cuboid =** $\boldsymbol{\sqrt{l^2 + b^2 + h^2}}$

A cube is a particular case of cuboid in which the length, breadth and height are equal to *a*.

∴ **Length of the diagonal of a cube** = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \boldsymbol{\sqrt{3}a}$

Solved Examples

Easy

Example 1:

There are twenty-five cuboid-shaped pillars in a building, each of dimensions 1 m ×1 m × 10 m. Find the cost of plastering the surface of all the pillars at the rate of Rs 16 per m^2.

Solution:

Length (l) of one pillar = 1 m

Breadth (b) of one pillar = 1 m

Height (h) of one pillar = 10 m

∴ Lateral surface area of one pillar= $2h(l + b)$

$= 2 \times 10 \times (1 + 1)$ m^2

= 40 m^2

⇒ Lateral surface area of twenty-five pillars = (25×40) m^2 = 1000 m^2

Cost of plastering 1 m^2 of surface = Rs 16

⇒ Cost of plastering 1000 m^2 of surface = Rs (16×1000) = Rs 16000

Thus, the cost of plastering the twenty-five pillars of the building is Rs 16000.

Example 2:

Find the length of the diagonal of a cube whose surface area is 294 m^2.

Solution:

Let the edge of the given cube be a.

∴ Surface area of the cube = $6a^2$

It is given that the surface area of the cube is 294 m^2.

So, $6a^2 = 294$

$\Rightarrow a^2 = 49 \text{ m}^2$

$\Rightarrow \therefore a = \sqrt{49} \text{ m} = 7 \text{ m}$

Now, length of the diagonal of the cube = $\sqrt{3}a = 7\sqrt{3}$ m

Medium

Example 1:

A metallic container (open at the top) is a cuboid of dimensions 7 cm × 5 cm × 8 cm. What amount of metal sheet went into making the container? Also, find the cost required for painting the outside of the container, excluding the base, at the rate of Rs 17 per 3 cm^2.

Solution:

Length (l) of the container = 7 cm

Breadth (b) of the container = 5 cm

Height (h) of the container = 8 cm

The container is open at the top. Therefore, while calculating the amount of metal sheet used, we will exclude the top part.

∴ Amount of metal sheet used = Total surface area − Area of the top part

$= 2\,(lb + bh + lh) - lb$

$= [2 \times (7 \times 5 + 5 \times 8 + 7 \times 8) - 7 \times 5]\ \text{cm}^2$

$= [2 \times (35 + 40 + 56) - 35]\ \text{cm}^2$

$= (2 \times 131 - 35)\ \text{cm}^2$

$= 227\ \text{cm}^2$

Thus, 227 cm^2 of metal went into making the given container.

Now, area to be painted = Lateral surface area of the cuboid

$= 2h\,(l + b)$

$= [2 \times 8 \times (7 + 5)]\ \text{cm}^2$

$= (16 \times 12)\ \text{cm}^2$

$= 192\ \text{cm}^2$

Cost of painting 3 cm^2 of surface = Rs 17

⇒ Cost of painting 1 cm^2 of surface = Rs 17/3

⇒ Cost of painting 192 cm^2 of surface = $\text{Rs}\left(192 \times \frac{17}{3}\right)$ = Rs 1088

Therefore, the cost of painting the outside of the container is Rs 1088.

Example 2:

If the total surface area of a cube is $24x^2$, then find the surface area of the cuboid formed by joining

i)two such cubes.

ii)three such cubes.

Solution:

Total surface area of cube = $6a^2$

It is given that the total surface area of the cube is $24x^2$.

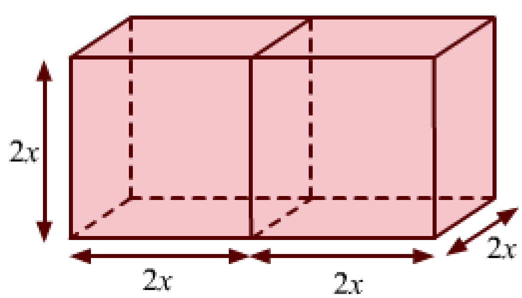
So, $6a^2 = 24x^2$

$\Rightarrow a^2 = 4x^2$

$\Rightarrow \therefore a = 2x$

So, the edge of the cube is $2x$.

i) When two cubes with edge $2x$ are joined, we obtain the following cuboid.



Length (l) of the cuboid = $2x + 2x = 4x$

Breadth (b) of the cuboid = $2x$

Height (h) of the cuboid = $2x$

$\therefore$ Surface area of the cuboid = $2\ (lb + bh + lh)$

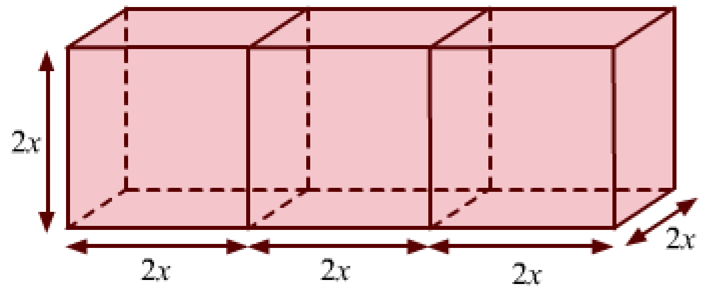
$= 2 \times (4x \times 2x + 2x \times 2x + 4x \times 2x)$

$= 2 \times (8x^2 + 4x^2 + 8x^2)$

$= 40x^2$

Thus, the surface area of the cuboid formed according to the given specifications is $40x^2$.

ii)When three cubes with edge $2x$ are joined, we obtain the following cuboid.



Length (l) of the cuboid = $2x + 2x + 2x = 6x$

Breadth (b) of the cuboid = $2x$

Height (h) of the cuboid = $2x$

∴ Surface area of the cuboid = $2(lb + bh + lh)$

$= 2 \times (6x \times 2x + 2x \times 2x + 6x \times 2x)$

$= 2 \times (12x^2 + 4x^2 + 12x^2)$

$= 56x^2$

Thus, the surface area of the cuboid formed according to the given specifications is $56x^2$.

Hard

Example 1:

The cost of flooring a twenty-metre-long room at Rs 5 per square metre is Rs 1000. If the cost of painting the four walls of the room at Rs 15 per square metre is Rs 1800, then find the height of the room.

Solution:

The length (l) of the room is given as 20 m. Let b and h be its breadth and height respectively.

Area of the floor = $l \times b$

Cost of flooring at Rs 5 per m^2 = Rs 1000

So, $5 \times l \times b = 1000$

$$\Rightarrow 5 \times 20 \times b = 1000$$
$$\Rightarrow \therefore b = \frac{1000}{100} = 10$$

Area of the four walls = $2(bh + lh)$

Cost of painting the four walls at Rs 15 per m^2 = Rs 1800

So, $15 \times [2(bh + lh)] = 1800$

$\Rightarrow 15 \times [2 \times (10 \times h + 20 \times h)] = 1800$

$$\Rightarrow 30h = \frac{1800}{15 \times 2}$$
$$\Rightarrow \therefore h = \frac{1800}{15 \times 2 \times 30} = 2$$

Thus, the height of the room is 2 m.

Example 2:

The internal measures of a cuboidal room are 20 m × 15 m × 12 m. Dinesh wants to paint the four walls of the room with orange colour and the roof of the room with white colour. 100 m^2 of surface can be painted using each can of orange paint and 125 m^2 of surface can be painted using each can of white paint. How many cans of each colour will be required? If the orange and white paints are available at Rs 250 per can and Rs 300 per can respectively, then how much money will be spent by Dinesh to paint the room?

Solution:

Length (l) of the room = 20 m

Breadth (b) of the room = 15 m

Height (h) of the room = 12 m

Area of the room to be painted using orange colour = Area of the four walls of the room

= Lateral surface area of the room

$= 2h\,(l + b)$

$= [2 \times 12\,(20 + 15)]\ \text{m}^2$

$= (24 \times 35)\ \text{m}^2$

$= 840\ \text{m}^2$

It is given that 100 m^2 of surface can be painted using each can of orange paint.

∴ Number of cans of orange paint required $= \dfrac{\text{Area of the room painted using orange colour}}{\text{Area that can be painted using each can}}$

$= \dfrac{840}{100}$

$= 8.4$

= 9 (∵ 8 cans will be insufficient for the job)

Thus, 9 cans of orange paint will be required for painting the four walls of the room.

Area of the room to be painted using white colour = Area of the roof

$= l \times b$

$= (20 \times 15)\ \text{m}^2$

$= 300\ \text{m}^2$

It is given that 125 m^2 of surface can be painted using each can of white paint.

∴ Number of cans of white paint required $= \dfrac{\text{Area of the room painted using white colour}}{\text{Area that can be painted using each can}}$

$= \dfrac{300}{125}$

$= 2.4$

= 3 (∵ 2 cans will be insufficient for the job)

Thus, 3 cans of white paint will be required for painting the roof of the room.

Cost of each can of orange paint = Rs 250

⇒ Cost of 9 cans of orange paint = 9 × Rs 250 = Rs 2250

Cost of each can of white paint = Rs 300

⇒ Cost of 3 cans of white paint = 3 × Rs 300 = Rs 900

Thus, total money that will be spent in painting the room = Rs 2250 + Rs 900 = Rs 3150

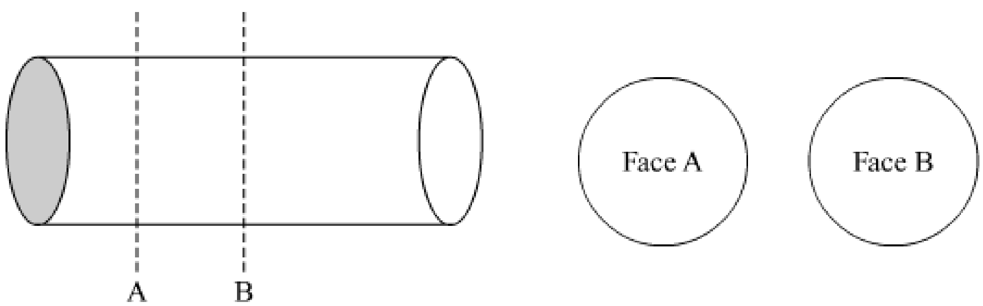
Cross-section of Solids

Cross Section

It is a cut which is made through a solid perpendicular to its length.
Cross Section is of two types:

(i) Uniform Cross Section:

A solid is said to have Uniform cross section, if the perpendicular cut is of the same shape and size at each point of its length.
Example:
When a cylinder is cut through the points A and B perpendicular to its length, the faces obtained as the cross section at both of the points are of same shape and size.

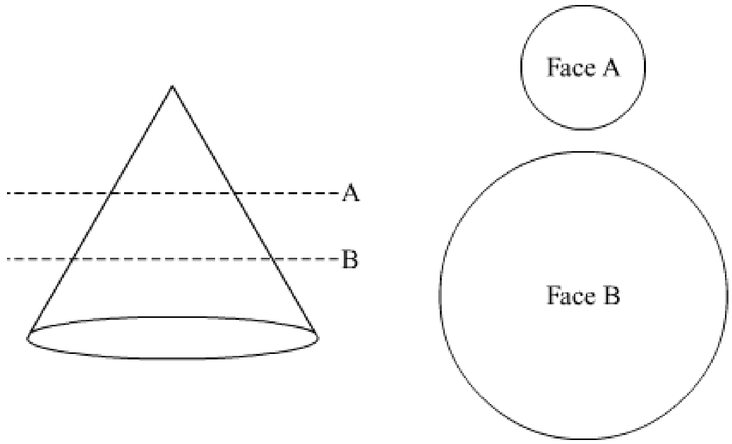


So, cylinder has a uniform cross section.

(ii) Non- uniform Cross Section:

A solid is said to have Non-Uniform cross section, if the perpendicular cut is not of the same shape and size at each point of its length.

Example:
When a cone is cut through the points A and B perpendicular to its length, the faces obtained as the cross section at both of the points are not of same shape and size.



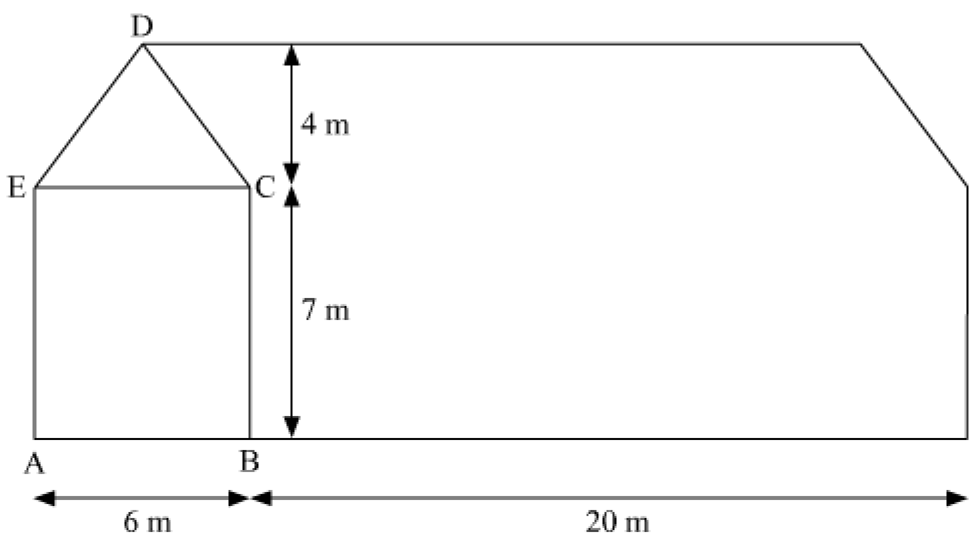
Let us find out volume and surface area of **uniform** cross section body.

1) Volume = Area of cross section × length

2) Surface area (excluding cross-section) = Perimeter of cross section × length

Example: 1

Consider the figure:

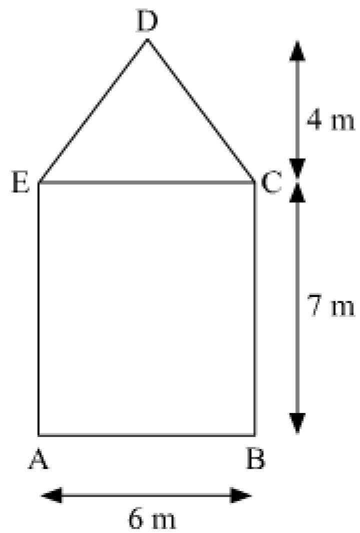


i) Find the volume of the hut.

ii) Find the Total surface area of the hut.

Solution: 1

i) First find out Area of cross section i.e area of ABCDE.

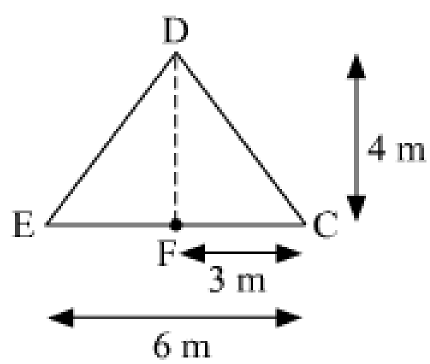


area of ABCDE = Area of Triangle DEC + Area of the rectangle ABCE

$$= \left[\frac{1}{2} \times 9 \times 4\right] + [9 \times 7]$$
$$= 18 + 63$$
$$= 81\ m^2$$

Now,
Volume = Area of cross section × length
= 81 × 20 = 1620 m^3

ii)
First find out the perimeter of cross section i.e length of ABCDE.



In right angled triangle EFC,

By pythagoras theorem,

$DC^2 = DF^2 + FC^2$
$\Rightarrow DC^2 = 4^2 + 3^2$
$\Rightarrow DC^2 = 16 + 9$
$\Rightarrow DC^2 = 25$
$\Rightarrow DC = 5\ m$

Since, Surface area (excluding cross-section) = Perimeter of cross section × length

Therefore, Surface area (excluding cross-section) is 33 × 20 = 660 m^2

Total surface area = Surface area (excluding cross-section) + 2 (area of cross-section)
= 660 + 2 (81)
= 660 + 162
= 822 m^2

Flow of Water (or any other liquid)

If water is flowing through a pipe of uniform cross section, then the volume of the water flowing out in unit time is Area of the cross section multiplied by speed of the flow.

i.e volume of the water flowing out = Area of the cross section × speed of the flow

Example: 2

How many litres of water flows out of the pipe of a cross section area 9 cm^2 in 1 minute, if the speed of the water in the pipe is 45 cm/s?

Solution: 2

Volume of water flowing in 1 sec. = 45 × 9 = 405 cm^3

Therefore,

Volume of water flowing in 1 minute = 405 × 60 cm^3 = 24300 cm^3

Since, 1 litre = 1000 cm^3

So, Volume of water flowing in 1 minute is 24.3 litres.